*** Applied Machine Learning Fundamentals *** Regression

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SAPSE / DHBW Mannheim

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Lecture Overview

Unit I Machine Learning Introduction

Unit II Mathematical Foundations

Unit III Bayesian Decision Theory

Unit IV Probability Density Estimation

Unit V Regression

Unit VI Classification I

Unit VII Evaluation

Unit VIII Classification II

Unit IX Clustering

Unit X Dimensionality Reduction



Introduction Solutions to Regression Probabilistic Regression Basis Function Regression Wrap-Up

Agenda for this Unit

Introduction

What is Regression? Least Squares Error Function

Solutions to Regression

Closed-Form Solutions and Normal Equation Gradient Descent

3 Probabilistic Regression

Underlying Assumptions
Maximum Likelihood Solution

A Basis Function Regression

General Idea Polynomial Basis Functions Radial Basis Functions Regularization Techniques

6 Wrap-Up

Summary
Self-Test Questions
Lecture Outlook
Recommended Literature and further Reading
Meme of the Day



Section: Introduction



Regression

Type of target variable

Continuous

Type of training information

Supervised

Example Availability

Batch learning

Algorithm sketch: Given the training data \mathcal{D} , the algorithm derives a function of the type

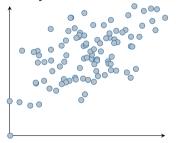
$$h_{\theta}(\mathbf{x}) = \theta_0 + \theta_1 x_1 + \dots + \theta_m x_m \qquad \mathbf{x} \in \mathbb{R}^m, \theta \in \mathbb{R}^{m+1}$$
 (1)

from the data. θ is the parameter vector containing the coefficients to be estimated by the regression algorithm. Once θ is learned, it can be used for prediction.

Wrap-Up

Example Data Set: Revenues

Revenue y



Marketing Expenses x_1

Find a linear function:

$$h_{\theta}(\mathbf{x}) = \theta_0 + \theta_1 x_1 + \dots + \theta_m x_m$$

• Usually: $x_0 = 1$:

$$\widehat{\mathbf{x}} \in \mathbb{R}^{m+1} = [1 \ \mathbf{x}]^{\mathsf{T}}$$

$$h_{\boldsymbol{\theta}}(\widehat{\mathbf{x}}) = \sum_{j=0}^{m} \theta_{j} x_{j} = \boldsymbol{\theta}^{\mathsf{T}} \widehat{\mathbf{x}}$$



Error Function for Regression

• We need an error function $\mathcal{J}(\boldsymbol{\theta})$ in order to know how good the function fits:

$$\mathcal{J}(\theta) = \frac{1}{2n} \sum_{i=1}^{n} (h_{\theta}(\widehat{\mathbf{x}}^{(i)}) - y^{(i)})^{2}$$
 (2)

• We want to minimize $\mathcal{J}(\boldsymbol{\theta})$:

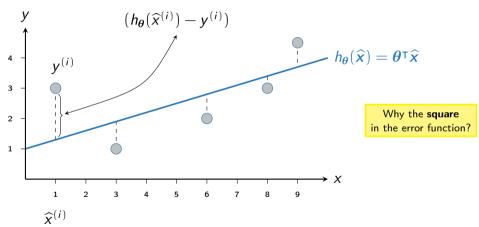
$$\min_{\theta} \frac{1}{2n} \sum_{i=1}^{n} (h_{\theta}(\widehat{\mathbf{x}}^{(i)}) - y^{(i)})^{2}$$

This is ordinary least squares (OLS)



Wrap-Up

Error Function Intuition



Section: Solutions to Regression



Closed-Form Solutions

• Usual approach (for two unknowns): Calculate θ_0 and θ_1 according to

sample mean \overline{x}

$$\theta_0 = \overline{y} - \theta_1 \overline{x} \qquad \qquad \theta_1 = \frac{\sum_{i=1}^n (x^{(i)} - \overline{x}) \cdot (y^{(i)} - \overline{y})}{\sum_{i=1}^n (x^{(i)} - \overline{x})^2}$$
(3)

'Normal equation' (scales to arbitrary dimensions):

$$\theta = (\widehat{X}^{\mathsf{T}}\widehat{X})^{-1}\widehat{X}^{\mathsf{T}}y$$
Moore-Penrose
pseudo-inverse
(4)

 $\widehat{\boldsymbol{X}}$ is called 'design matrix' or 'regressor matrix'



Design Matrix / Regressor Matrix

• The design matrix $\widehat{\mathbf{X}} \in \mathbb{R}^{n \times (m+1)}$ looks as follows:

$$\widehat{\mathbf{X}} = \begin{pmatrix} 1 & x_1^{(1)} & x_2^{(1)} & \cdots & x_m^{(1)} \\ 1 & x_1^{(2)} & x_2^{(2)} & \cdots & x_m^{(2)} \\ 1 & x_1^{(3)} & x_2^{(3)} & \cdots & x_m^{(3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1^{(n)} & x_2^{(n)} & \cdots & x_m^{(n)} \end{pmatrix}$$

In the following $\widehat{\mathbf{X}} \equiv \mathbf{X}$

(5)

• And the $n \times 1$ label vector:

$$\mathbf{y} = (y^{(1)}, y^{(2)}, y^{(3)}, \dots, y^{(n)})^{\mathsf{T}}$$





Derivation of the Normal Equation

- The derivation involves a bit of linear algebra
- Step **①**: Rewrite $\mathcal{J}(\boldsymbol{\theta})$ in matrix-vector notation:

$$\mathcal{J}(\boldsymbol{\theta}) = \frac{1}{2} (\boldsymbol{X}\boldsymbol{\theta} - \boldsymbol{y})^{\mathsf{T}} (\boldsymbol{X}\boldsymbol{\theta} - \boldsymbol{y})
= \frac{1}{2} ((\boldsymbol{X}\boldsymbol{\theta})^{\mathsf{T}} - \boldsymbol{y}^{\mathsf{T}}) (\boldsymbol{X}\boldsymbol{\theta} - \boldsymbol{y})
= \frac{1}{2} ((\boldsymbol{X}\boldsymbol{\theta})^{\mathsf{T}} \boldsymbol{X}\boldsymbol{\theta} - (\boldsymbol{X}\boldsymbol{\theta})^{\mathsf{T}} \boldsymbol{y} - \boldsymbol{y}^{\mathsf{T}} (\boldsymbol{X}\boldsymbol{\theta}) + \boldsymbol{y}^{\mathsf{T}} \boldsymbol{y})
= \frac{1}{2} (\boldsymbol{\theta}^{\mathsf{T}} \boldsymbol{X}^{\mathsf{T}} \boldsymbol{X}\boldsymbol{\theta} - 2(\boldsymbol{X}\boldsymbol{\theta})^{\mathsf{T}} \boldsymbol{y} + \boldsymbol{y}^{\mathsf{T}} \boldsymbol{y})$$

To be continued...





Derivation of the Normal Equation (Ctd.)

• Step **2**: Calculate the derivative of $\mathcal{J}(\boldsymbol{\theta})$ and set it to zero:

$$\nabla_{\boldsymbol{\theta}} \mathcal{J}(\boldsymbol{\theta}) = \frac{1}{2} (2\boldsymbol{X}^{\mathsf{T}} \boldsymbol{X} \boldsymbol{\theta} - 2\boldsymbol{X}^{\mathsf{T}} \boldsymbol{y}) \stackrel{!}{=} 0$$
$$\Leftrightarrow \boldsymbol{X}^{\mathsf{T}} \boldsymbol{X} \boldsymbol{\theta} = \boldsymbol{X}^{\mathsf{T}} \boldsymbol{y}$$

• If $X^{T}X$ is invertible, we can multiply both sides by $(X^{T}X)^{-1}$:

Normal equation:

$$\boldsymbol{\theta} = (\boldsymbol{X}^{\intercal} \boldsymbol{X})^{-1} \boldsymbol{X}^{\intercal} \boldsymbol{y}$$



Problems with Matrix Inversion?

- What if $(X^{T}X)^{-1}$ does not exist?
- Problems and solutions:
 - 1 Linearly dependent (redundant) features or design matrix does not have full rank? (E.g. size in m² and size in feet²)
 - ⇒ Delete correlated features
 - 2 Too many features (m > n)?
 - ⇒ Delete features (e.g. using PCA) / add training examples
 - 3 Other numerical instabilities?
 - ⇒ Add a regularization term (later)
 - 4 Computationally too expensive?
 - ⇒ Use gradient descent



Gradient Descent

• We want to minimize a smooth function $\mathcal{J}: \mathbb{R}^{m+1} \to \mathbb{R}$:

$$\min_{oldsymbol{ heta} \in \mathbb{R}^{m+1}} \mathcal{J}(oldsymbol{ heta})$$

Update the parameters iteratively:

$$\boldsymbol{\theta}^{(t+1)} \longleftarrow \boldsymbol{\theta}^{(t)} - \alpha \nabla_{\boldsymbol{\theta}} \mathcal{J}(\boldsymbol{\theta}^{(t)}) \tag{6}$$

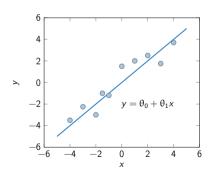
• where $\alpha > 0$ (learning rate) and $\nabla_{\theta} \mathcal{J}(\theta)$ is the gradient of $\mathcal{J}(\theta)$ w.r.t. θ :

$$abla_{m{ heta}}\mathcal{J}(m{ heta}) = \left(rac{\partial \mathcal{J}(m{ heta})}{\partial m{ heta}_0}, rac{\partial \mathcal{J}(m{ heta})}{\partial m{ heta}_1}, \ldots, rac{\partial \mathcal{J}(m{ heta})}{\partial m{ heta}_m}
ight)^{\mathsf{T}}$$

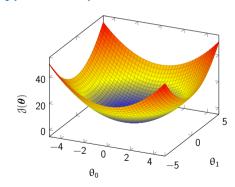


Data Input Space vs. Hypothesis Space

Data input space



Hypothesis space \mathcal{H}



Data Input Space vs. Hypothesis Space (Ctd.)

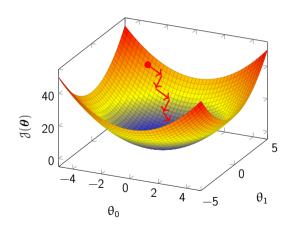
- Data input space
 - Determined by the m+1 attributes of the data set $x_0, x_1, x_2, \ldots, x_m$
 - Often high-dimensional
- Hypothesis space ${\mathcal H}$
 - Determined by the number of parameters of the model
 - Each point in the hypothesis space corresponds to a specific assignment of model parameters
 - The error function gives information about how good this assignment is
 - Gradient descent is applied in the hypothesis space ${\mathcal H}$





Data Input Space vs. Hypothesis Space (Ctd.)

Visualization of Gradient Descent in 3 Dimensions



Versions of Gradient Descent

- Assume some training data \mathcal{D} : $\{x^{(i)}, y^{(i)}\}_{i=1}^n$
- Squared error for a **single** example: $\ell(y_{pred}, y_{true}) = (y_{pred} y_{true})^2$
- Our objective is to minimize the **total** error:

$$\min_{\boldsymbol{\theta} \in \mathbb{R}^{m+1}} \mathcal{J}(\boldsymbol{\theta}) = \min_{\boldsymbol{\theta} \in \mathbb{R}^{m+1}} \sum_{i=1}^{n} \ell(h_{\boldsymbol{\theta}}(\boldsymbol{x}^{(i)}), \boldsymbol{y}^{(i)})$$

- Three versions of gradient descent:
 - Batch gradient descent
 - 2 Stochastic gradient descent
 - 3 Mini-batch gradient descent

Versions of Gradient Descent (Ctd.)

• Batch gradient descent: Compute gradient based on ALL data points

$$\boldsymbol{\theta}^{(t+1)} \longleftarrow \boldsymbol{\theta}^{(t)} - \alpha \sum_{i=1}^{n} \nabla \ell(h_{\boldsymbol{\theta}^{(t)}}(\boldsymbol{x}^{(i)}), \boldsymbol{y}^{(i)})$$
 (7)

- Stochastic gradient descent: Compute gradient based on a <u>SINGLE</u> data point (pick training example randomly and not sequentially!)
- For $i \in \{1, ..., n\}$ do:

$$\boldsymbol{\theta}^{(t+1)} \longleftarrow \boldsymbol{\theta}^{(t)} - \alpha \nabla \ell(h_{\boldsymbol{\theta}^{(t)}}(\boldsymbol{x}^{(i)}), \boldsymbol{y}^{(i)}) \tag{8}$$



Solving linear Regression using Gradient Descent

- ullet Randomly initialize $oldsymbol{ heta}$
- ullet To minimize the error, keep changing $oldsymbol{ heta}$ according to:

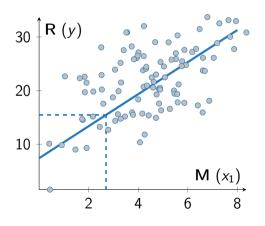
$$\boldsymbol{\theta}^{(t+1)} \longleftarrow \boldsymbol{\theta}^{(t)} - \alpha \nabla_{\boldsymbol{\theta}} \mathcal{J}(\boldsymbol{\theta}^{(t)}) \tag{9}$$

• We need to calculate $\nabla_{\theta_i} \mathcal{J}(\boldsymbol{\theta}^{(t)})$: (based on a single example)

$$\frac{\partial}{\partial \theta_j} \mathcal{J}(\boldsymbol{\theta}) = \frac{\partial}{\partial \theta_j} \frac{1}{2} (h_{\boldsymbol{\theta}}(\boldsymbol{x}) - y)^2 = 2 \cdot \frac{1}{2} (h_{\boldsymbol{\theta}}(\boldsymbol{x}) - y) \cdot \frac{\partial}{\partial \theta_j} (h_{\boldsymbol{\theta}}(\boldsymbol{x}) - y)$$
(10)

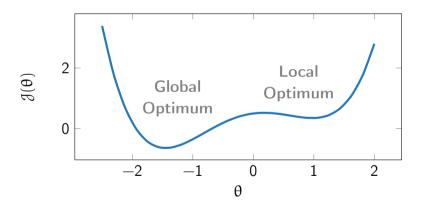
$$= (h_{\theta}(\mathbf{x}) - y) \cdot \frac{\partial}{\partial \theta_i} (\theta_0 x_0 + \dots + \theta_m x_m - y) = \left[(h_{\theta}(\mathbf{x}) - y) x_j \right]$$
(11)

Solving the introductory Example



- $\theta_0 \approx 7.4218$
- $\theta_1 \approx 2.9827$
- $\mathcal{J}(\boldsymbol{\theta}) \approx 446.9584$
- $h_{\theta}(\mathbf{x}) = 7.4218 + 2.9827 \cdot x_1$
- $R = h_{\theta}(2.7) = 15.4750$

Disadvantage of Gradient Descent



Section: Probabilistic Regression



Probabilistic Regression

 Assumption 1: The target function values are generated by adding noise to the function estimate:

$$y = h_{\theta}(\mathbf{x}) + \varepsilon \tag{12}$$

Assumption 2: The noise is a Gaussian random variable:

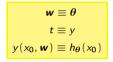
$$\beta \equiv \text{precision} \qquad \qquad \epsilon \sim \mathcal{N}(0, \beta^{-1}) \tag{13}$$

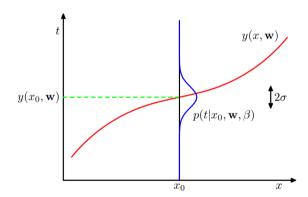
$$\beta = \frac{1}{\sigma^2} \qquad \qquad p(y|x, \theta, \beta) = \mathcal{N}(y|h_{\theta}(x), \beta^{-1}) \tag{14}$$

• y is now a random variable!



Probabilistic Regression (Ctd.)





cf. [1], p. 29; probabilistic regression

Maximum Likelihood Regression

- **Given:** A labeled set of training data points $\{(x^{(i)}, y^{(i)})\}_{i=1}^n$
- Conditional likelihood (assuming the data is i. i. d.):

$$p(\mathbf{y}|\mathbf{X},\boldsymbol{\theta},\boldsymbol{\beta}) = \prod_{i=1}^{n} \mathcal{N}(\mathbf{y}^{(i)}|h_{\boldsymbol{\theta}}(\mathbf{x}^{(i)}),\boldsymbol{\beta}^{-1})$$
 (15)

$$= \prod_{i=1}^{n} \mathcal{N}(y^{(i)} | \boldsymbol{\theta}^{\mathsf{T}} \boldsymbol{x}^{(i)}, \beta^{-1})$$
 (16)

• Maximize the likelihood w.r.t. θ and β



Maximum Likelihood Regression (Ctd.)

Simplify using the log-likelihood:

$$\log p(\mathbf{y}|\mathbf{X}, \boldsymbol{\theta}, \boldsymbol{\beta}) = \sum_{i=1}^{n} \log \mathcal{N}(\mathbf{y}^{(i)}|\boldsymbol{\theta}^{\mathsf{T}}\mathbf{x}^{(i)}, \boldsymbol{\beta}^{-1})$$
(17)

$$= \sum_{i=1}^{n} \left[\log \left(\frac{\sqrt{\beta}}{\sqrt{2\pi}} \right) - \frac{\beta}{2} (y^{(i)} - \boldsymbol{\theta}^{\mathsf{T}} \boldsymbol{x}^{(i)})^{2} \right]$$
 (18)

Remember log-rules?

$$= \frac{n}{2} \log \beta - \frac{n}{2} \log(2\pi) - \frac{\beta}{2} \sum_{i=1}^{n} (y^{(i)} - \theta^{\mathsf{T}} \mathbf{x}^{(i)})^{2}$$
 (19)

Maximum Likelihood Regression (Ctd.)

• Compute the gradient w.r.t. θ :

$$\nabla_{\boldsymbol{\theta}} \log p(\mathbf{y}|\mathbf{X}, \boldsymbol{\theta}, \boldsymbol{\beta}) = 0$$
$$-\beta \sum_{i=1}^{n} (\mathbf{y}^{(i)} - \boldsymbol{\theta}^{\mathsf{T}} \mathbf{x}^{(i)}) \mathbf{x}^{(i)} = 0$$

$$oldsymbol{ heta}_{ml} = (oldsymbol{X}^\intercal oldsymbol{X})^{-1} oldsymbol{X}^\intercal oldsymbol{y}$$

• Same result as in least squares regression



. . .



We have derived the squared Error!

Minimizing the squared error gives the maximum likelihood solution for the parameters θ assuming Gaussian noise.

- The maximum likelihood approach gives rise to the squared error
- But it is much more powerful than regular least squares ⇒ We can estimate the uncertainty β

$$\beta_{ml} = \left(\frac{1}{n} \sum_{i=1}^{n} (y^{(i)} - \theta_{ml}^{\mathsf{T}} \mathbf{x}^{(i)})^{2}\right)^{-1}$$
 (20)

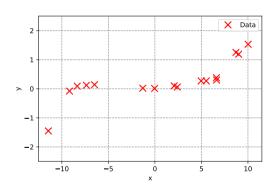
Section: Basis Function Regression



What if the Data is non-linear?

- So far we have fitted straight lines
- What if the data is not linear...?

The best-fitting function is obviously **not a straight line!**What would you do?



Basis Functions

- Remember: 'When stuck switch to a different perspective'
- We can add **higher-order** features using basis functions φ :

We assume 1-D data
$$h_{\boldsymbol{\theta}}(x) = \sum_{j=0}^{p} \theta_{j} \varphi_{j}(x) \tag{21}$$

- There exist several types of basis functions:
 - linear: $\varphi_0(x) = 1$ and $\varphi_1(x) = x$
 - polynomial ⇒ see below
 - radial basis functions (RBFs) ⇒ see below
 - Fourier basis



New Design Matrix

Applying the basis functions to X we get the new design matrix Φ :

$$\boldsymbol{\Phi} = \begin{pmatrix} \varphi_0(x^{(1)}) & \varphi_1(x^{(1)}) & \varphi_2(x^{(1)}) & \dots & \varphi_p(x^{(1)}) \\ \varphi_0(x^{(2)}) & \varphi_1(x^{(2)}) & \varphi_2(x^{(2)}) & \dots & \varphi_p(x^{(2)}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \varphi_0(x^{(n)}) & \varphi_1(x^{(n)}) & \varphi_2(x^{(n)}) & \dots & \varphi_p(x^{(n)}) \end{pmatrix}$$
(22)

The model is still linear in the parameters, so we can still use the same algorithm as before. This is still linear regression (!!!)

Polynomial Basis Functions

A quite frequently used basis function: The polynomial basis

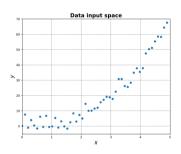
$$\varphi_0(x) = 1$$
$$\varphi_i(x) = x^j$$

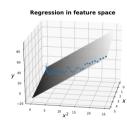
For *N*-D data we would also include cross-terms!

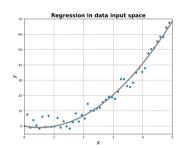
$$h_{\theta}(x) = \sum_{j=0}^{\rho} \theta_j \varphi_j(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \dots + \theta_{\rho} x^{\rho}$$

- Here, p is the degree of the polynomial
- Here: $\varphi(x) = [1, x, x^2, x^3, \dots, x^p]$

It is still linear!

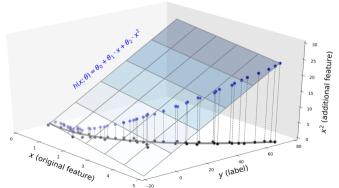






It is still linear! (Ctd.)

Basis function regression



Basis Functions: Radial Basis Functions

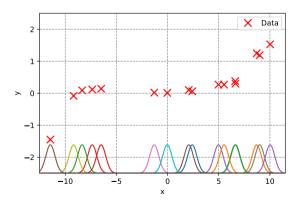
• Yet another possible choice of basis function: Radial basis functions

$$\varphi_0(x) = 1 \tag{23}$$

$$\varphi_j(x) = \exp\left\{-\frac{1}{2}\|x - z_j\|^2 / 2\sigma^2\right\}$$
 (24)

- $\{z_j\}$ are the centers of the radial basis functions
- p denotes the number of centers / number of radial basis functions
- Often we take each data point as a center, so p = n

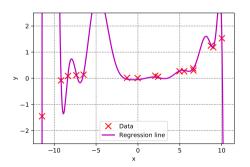
Radial Basis Functions (Ctd.)



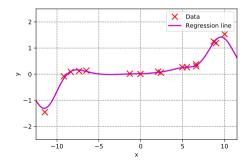


The Danger of too expressive Models...

Polynomial of degree p = 16 (g severe overfitting g)



RBF with $\sigma = 1.00$, p = n (About right)



Overfitting vs. Underfitting

Underfitting

- The model is not complex enough to fit the data well ⇒ High bias
- Make the model more complex; adding new examples does not help

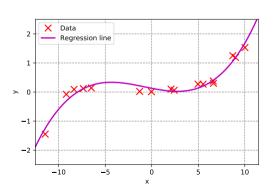
Overfitting

- The model predicts the training data perfectly
- But it fails to generalize to unseen instances ⇒ High variance
- Decrease the degree of freedom or add more training examples
- Also: Try regularization
- Bias-Variance trade-off



First Solution: Smaller Degree

One solution: Use a smaller degree (here: p = 3)



Much better :)

Second Solution: Regularization

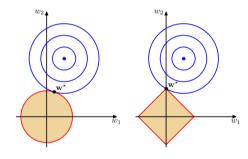
- Enrich $\mathcal{J}(\boldsymbol{\theta})$ with a regularization term
- This can **prevent overfitting** and results in a smoother function (large values for θ_i are prevented)
- Two forms of regularization, L1 and L2:

$$\begin{split} \min_{\boldsymbol{\theta}} \mathcal{J}(\boldsymbol{\theta}) + \lambda |\boldsymbol{\theta}| & \rightarrow (\mathbf{L1}) & \min_{\boldsymbol{\theta}} \mathcal{J}(\boldsymbol{\theta}) + \lambda ||\boldsymbol{\theta}||^2 & \rightarrow (\mathbf{L2}) \\ |\boldsymbol{\theta}| &= \sum_{j=1}^m |\theta_j| & ||\boldsymbol{\theta}||^2 &= \sum_{j=1}^m \theta_j^2 \end{split}$$

• $\lambda \ge 0$ controls the degree of regularization

Regularization visualized

- Here: $\mathbf{w} \equiv \mathbf{\theta}$
- L1-Regularization
 - ⇒ Lasso regression
 (least abs. shrinkage and select. operator)
- L2-Regularization
 - ⇒ Ridge regression (Tikhonov regularization)
- The combination of both is called elastic net



cf. [1], p. 146; left: L2, right: L1

Incorporating Regularization

 Normal equation with regularization: (ridge regression) The regularization also helps to overcome numerical issues!

$$\boldsymbol{\theta} = (\boldsymbol{X}^{\mathsf{T}}\boldsymbol{X} + \lambda \boldsymbol{I})^{-1}\boldsymbol{X}^{\mathsf{T}}\boldsymbol{y} \tag{25}$$

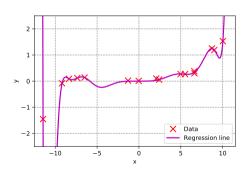
Regularized gradient descent update rule:

$$\boldsymbol{\theta}^{(t+1)} \longleftarrow \boldsymbol{\theta}^{(t)} - \alpha \nabla_{\boldsymbol{\theta}} \mathcal{J}(\boldsymbol{\theta}^{(t)})$$

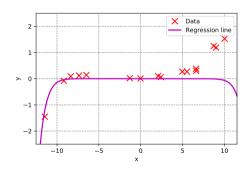
$$\frac{\partial}{\partial \theta_i} \mathcal{J}(\boldsymbol{\theta}) = (h_{\boldsymbol{\theta}}(\boldsymbol{x}) - \boldsymbol{y}) x_j + \lambda \theta_j$$

Polynomial Regression with Regularization

At least better



Way too much regularization



Section: Wrap-Up



Summary

- Regression predicts continuous target variables
- The algorithm minimizes the (mean) squared error
- Minimizing the squared error gives the maximum likelihood solution
- Two approaches:
 - Normal equation
 - (Batch / stochastic / mini-batch) gradient descent
- Probabilistic regression allows to quantify the uncertainty of the model
- Use basis functions to fit non-linear regression lines
- Regularization is important



Introduction
Solutions to Regression
Probabilistic Regression
Basis Function Regression
Wrap-Up

Summary
Self-Test Questions
Lecture Outlook
Recommended Literature and further Reading
Meme of the Day

Self-Test Questions

- What is the goal of regression?
- 2 What can you do if matrix inversion fails for the normal equation?
- 3 What is a suitable cost function for regression? Where does it come from?
- 4 Does gradient descent give the exact solution?
- **5** What is the advantage of probabilistic regression?
- 6 What are basis functions? Why use them? State some examples.
- What is overfitting / underfitting?
- 8 What is regularization? Why should you apply it?



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What's next...?

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Unit III Bayesian Decision Theory

Unit IV Probability Density Estimation

Unit V Regression

Unit VI Classification I

Unit VII Evaluation

Unit VIII Classification II

Unit IX Clustering

Unit X Dimensionality Reduction



Recommended Literature and further Reading I



Christopher Bishop. Springer. 2006.

- ightarrow Link, cf. chapter 3.1
- [2] Machine Learning: A Probabilistic Perspective Kevin Murphy. MIT Press. 2012.
 - ightarrow Link, cf. chapters 1.4.5 and 1.4.7
- [3] Stanford CS229 course notes
 Andrew Ng. Stanford University. 2019.
 - $\to \underline{\mathtt{Link}}$



Summary Self-Test Questions Lecture Outlook Recommended Literature and further Reading Meme of the Day

Recommended Literature and further Reading II

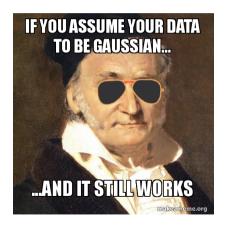


[4] Stanford CS229 course recording

Andrew Ng. Stanford University. 2008.

 $ightarrow \underline{ t Link}$

Meme of the Day



Thank you very much for the attention!

Topic: *** Applied Machine Learning Fundamentals *** Regression

Term: Winter term 2020/2021

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Do you have any questions?