W3WI DS304.1 Applied Machine Learning Fundamentals

Lagrange Optimization (Example)

Optimize the following function subject to the given constraint:

$$\max_{x_1,x_2} \mathcal{J}(x_1, x_2) = 1 - x_1^2 - x_2^2$$

subject to:

$$f(x_1, x_2) = x_1 + x_2 - 1 = 0$$

Step 1) Compute the Lagrange function $\mathcal{L}$. It is given by:

$$\mathcal{L}(x_1, x_2, \lambda) := 1 - x_1^2 - x_2^2 + \lambda(x_1 + x_2 - 1)$$

Step 2) Compute the partial derivatives and set them to zero:

Equation I

$$\frac{\partial}{\partial x_1}\mathcal{L}(x_1, x_2, \lambda) = -2x_1 + \lambda \quad \overset{!}{=} 0$$

Equation II

$$\frac{\partial}{\partial x_2}\mathcal{L}(x_1, x_2, \lambda) = -2x_2 + \lambda \quad \overset{!}{=} 0$$

Equation III

$$\frac{\partial}{\partial \lambda}\mathcal{L}(x_1, x_2, \lambda) = x_1 + x_2 - 1 \overset{!}{=} 0$$

Step 3) Solve:

From I:

$$x_1 = \frac{1}{2}\lambda$$

From II:

$$x_2 = \frac{1}{2}\lambda$$

Now substitute into equation III:

$$\frac{1}{2}\lambda + \frac{1}{2}\lambda = 1 \Longleftrightarrow \lambda = 1$$

Substituting $\lambda = 1$ into equations I and II yields the solution: $x_1^\star = x_2^\star = \frac{1}{2}$