Exercise 1 - Introduction

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DHBW
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1 Linear Algebra Refresher

a) Matrix Operations (1 point)

A fellow student suggests that matrix addition and multiplication are very similar to scalar addition and multiplication, i. e. commutative, associative and distributive. Is this a correct statement? Prove it mathematically or disprove it by providing at least one counter example per property (commutativity, associativity, distributivity).

Solution:

b) Matrix Inverse (1 point)

What is a matrix inverse? How can you build the inverse of a non-square matrix? You would like to invert a matrix $M \in \mathbb{R}^{2\times 3}$ - write down the equation for computing it and specify the dimensionality of the matrices after each single operation (e.g. multiplication, inverse).

Solution:

c) Eigenvectors and Eigenvalues (1 point)

Explain what eigenvectors and eigenvalues of a matrix M are. Why are they relevant in machine learning?

Solution:

2 Statistics Refresher

a) Terminology (1 point)

What is a random variable? What is a probability density function (PDF)? What is a probability mass function (PMF)? What do a PDF and a PMF tell us about a random variable?

Solution:

b) Expectation and Variance (1 point)

State the general definition of expectation and variance for the probability density $f : \Omega \to \mathbb{R}$ of a continuous random variable. What do expectation and variance express?

Solution:

3 Optimization

a) Numerical Optimization - Gradient Descent (5 points)

Implement a simple gradient descent algorithm for finding a minimum of the Rosenbrock function with $n = 2$ using Python and NumPy:

$$f(x) = \sum_{i=0}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right]$$

Submit your code and a plot of the learning curve for the best run of your gradient descent implementation. Which learning rate worked best? (Hint: You need to find the first derivative(s) of $f(x)$ for $n = 2$ and iteratively evaluate them during gradient descent. Automatic differentiation tools are not allowed for this exercise.)

Solution: