Exercise 5 - Backpropagation and Unsupervised Learning

January 31, 2020

1 Backpropagation

a) Backpropagation by Hand (5 points)

You are given the following neural network. Compute one forward pass and backward pass based on the labeled training example $(\mathbf{x} = (\mathbf{0.05}, \mathbf{0.10}), \mathbf{y} = (\mathbf{0.01}, \mathbf{0.99}))$. The weight matrices and bias values are initialized as follows (where for instance the weight W_{12} connects the input x_1 with hidden layer neuron 2 and V_{21} connects hidden layer neuron 1 with output layer neuron 2):

$$W = \begin{pmatrix} 0.15 & 0.20 \\ 0.25 & 0.30 \end{pmatrix}$$
$$b_1 = 0.35$$
$$V = \begin{pmatrix} 0.40 & 0.45 \\ 0.50 & 0.55 \end{pmatrix}$$
$$b_2 = 0.60$$

Use the activation functions mentioned above the hidden layer (ReLU) and output layer (Sigmoid). Show your calculations, annotate the visual illustration of the network with the gradients at the single neurons (derivative of the activations) and state the gradients for the weights w_{12}, v_{21}.

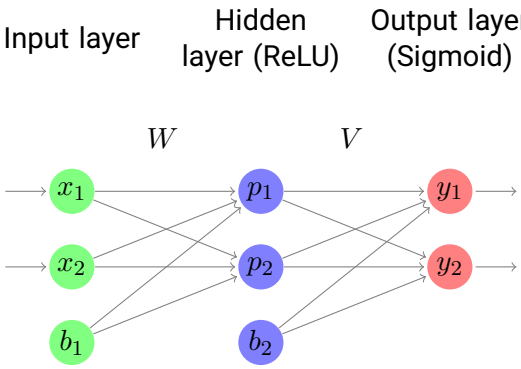

Figure 1: Multi-layer perceptron to be used for one forward and backward pass

2 Unsupervised Learning

a) k-Means clustering (4 points)

Implement k-Means clustering for image compression. Compress the example image file *data/d-hbw.jpg*. You find an explanation of how image compression with k-Means works under the following link[1].

b) PCA (1 point)

Explain how to choose the number of eigenvectors to use for principal component analysis. Why does this work?

c) Bonus Question: Spectral clustering (1 point)

How can you automatically choose the number of clusters for spectral clustering?

[1] https://medium.com/@agarwalvibhor84/image-compression-using-k-means-clustering-8c0ec055103f