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# Artificial Intelligence and Machine Learning

## Exercises – Support Vector Machines

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### Question 1 (Computing a support vector machine by hand)

The aim of this question is to compute a hard-margin support vector machine (SVM) by hand. For this, let a dataset consisting of the two training examples

$$\left(x^1 := \begin{pmatrix} -2 & -1 \end{pmatrix}^\top, y_1 := +1\right) \quad \text{and} \quad \left(x^2 := \begin{pmatrix} 1 & 1 \end{pmatrix}^\top, y_2 := -1\right)$$

be given. *Admittedly, this dataset is not very useful in practical applications, but using such a small dataset makes it feasible to work through the computations by hand. This enhances your understanding of support vector machines.*

Please work through the following tasks to train the SVM:

1. Write down the hard-margin SVM dual optimization problem for the dataset above.
2. State the *Karush-Kuhn-Tucker (KKT)* conditions for this optimization problem. Are the KKT conditions sufficient for a solution in this case? Is the solution unique?
3. Compute the optimal Lagrange multipliers  $\alpha_1$  and  $\alpha_2$  by solving the KKT system which you have specified in task 2.
4. Compute the optimal model parameters  $w$  and  $b$ .
5. Let the test example  $x' := \begin{pmatrix} -1/2 & 1 \end{pmatrix}^\top$  be given. Use the model parameters you have computed in task 4 to classify this new example.