

# Exercise 1 - Math Refresher

Winter term 2019/2020

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## Important

Please solve the assignments in groups of 3 to 4 students. The solutions are going to be presented and discussed after the submission deadline. Sample solutions will not be uploaded. However, you are free to share correct solutions with your colleagues **after they have been graded**. Please submit your solutions via Moodle **and** in printed form. Only one member of the group has to submit the solutions. Therefore, make sure to specify the names of all group members. Please do not submit hand-written solutions, rather use proper type-setting software like L<sup>A</sup>T<sub>E</sub>X or other comparable programs.

Your homework will be corrected and given back to you. Correct solutions are rewarded with a bonus for the exam (max. 10 percent, if all solutions submitted are correct). **Please note:** You have to pass the exam **without the bonus points!** (*i.e. it is not possible to turn 5.0 into 4.0*) The solutions have to be your own work. If you plagiarize, you will lose all bonus points!

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## Further remarks:

- Code assignments have to be done in Python
- The following packages are allowed: `numpy`, `pandas`  
(please ask, if you want to use a specific package not mentioned here)
- **Do not use already implemented models** (e.g. from `scikit-learn`)

# 1 Linear Algebra Refresher

## a) Matrix Operations (1 point)

A fellow student suggests that matrix addition and multiplication are very similar to scalar addition and multiplication, i. e. commutative, associative and distributive. Is this a correct statement? Prove it mathematically or disprove it by providing at least one counter example per property (commutativity, associativity, distributivity).

**Solution:**

## b) Matrix Inverse (1 point)

What is a matrix inverse? How can you build the inverse of a non-square matrix? You would like to invert a matrix  $M \in \mathbb{R}^{2 \times 3}$ , write down the equation for computing it and specify the dimensionality of the matrices after each single operation (e. g. multiplication, inverse).

**Solution:**

c) Eigenvectors and Eigenvalues (1 point)

Explain what eigenvectors and eigenvalues of a matrix  $A$  are. Why are they relevant in machine learning?

**Solution:**

## 2 Statistics Refresher

a) Terminology (1 point)

What is a random variable? What is a probability density function (PDF)? What is a probability mass function (PMF)? What do a PDF and a PMF tell us about a random variable?

**Solution:**

b) Expectation and Variance (1 point)

State the general definition of expectation and variance for the probability density  $f : \Omega \rightarrow \mathbb{R}$  of a continuous random variable. What do expectation and variance express?

**Solution:**

### 3 Optimization

a) Numerical Optimization - Gradient Descent (5 points)

Implement a simple gradient descent algorithm for finding a minimum of the Rosenbrock function with  $n = 2$  using Python and NumPy:

$$f(\mathbf{x}) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$$

Submit your code and a plot of the learning curve for the best run of your gradient descent implementation. Which learning rate worked best? (Hint: You need to find the first derivative(s) of  $f(\mathbf{x})$  for  $n = 2$  and iteratively evaluate them during gradient descent. Automatic differentiation tools are not allowed for this exercise. Choose a random starting point for the parameters, for example  $\mathbf{x} \in [-2, 2]$ .)

**Solution:**