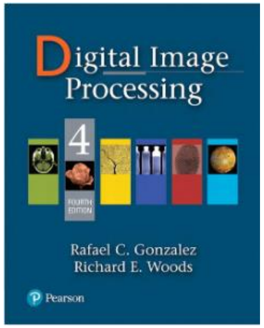


Fall, 2022

Digital Image Processing

Image Restoration and Reconstruction (II)

Dr. Tun-Wen Pai

1) **Linear, position-invariant degradation**
2) **Inverse Filters**
3) **Matrix Formulation of Image Restoration Problem**
4) **Constrained least squares filtering**
5) **Pseudo-inverse filtering**
6) **Minimum Mean Square Error (Wiener) Filter**

Dec.5, 2022

Linear, position-invariant degradation

- We will now consider the general degradation equation

$$g(m,n) = h(m,n) * f(m,n) + \eta(m,n)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

- This consists of a "blurring" function h(m, n), in addition to the random noise component η(m, n). (assume η(m, n)= 0)
- The blurring function h(m, n) is usually referred to as a point-spread function (PSF) and represents the observed image corresponding to imaging an impulse or point source of light.

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)\,d\alpha\,d\beta$$

Assuming again that $\eta(x,y) = 0$

$$g(x,y) = \mathcal{H}[f(x,y)] = \mathcal{H}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)\,d\alpha\,d\beta\right]$$

If $\mathcal{H}$ is a linear operator and we extend the additivity property to integrals, then

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{H}[f(\alpha,\beta)\delta(x-\alpha, y-\beta)]\,d\alpha\,d\beta$$

Because $f(\alpha,\beta)$ is independent of x and y, and using the homogeneity property, it follows that

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\mathcal{H}[\delta(x-\alpha, y-\beta)]\,d\alpha\,d\beta$$

The term

impulse response of $\mathcal{H}$ $\quad h(x,\alpha,y,\beta) = \mathcal{H}[\delta(x-\alpha, y-\beta)]$

Using Image observation

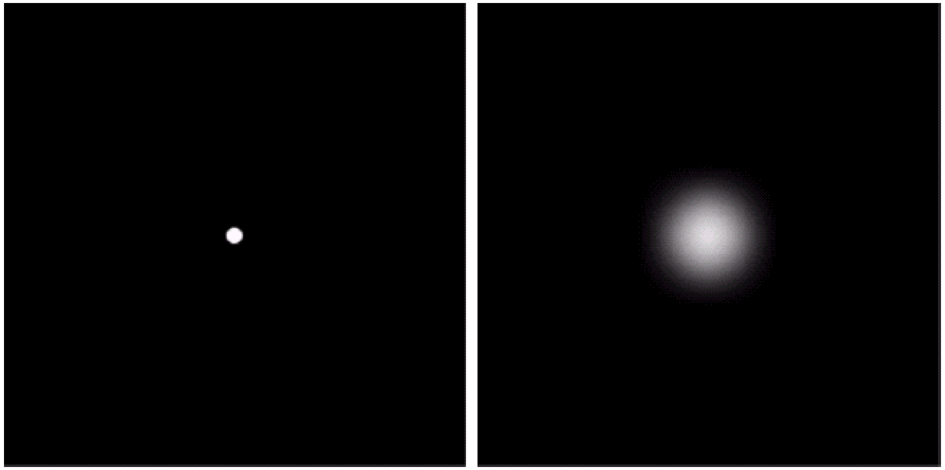
- Identify portions of the observed image (sub-image) that are relatively noise-free and which corresponds to some simple structures.

- We can then obtain $H_s(u,v) = \dfrac{G_s(u,v)}{\hat{F}_s(u,v)}$, where $G_s(u, v)$ is the spectrum of the observed subimage, $\hat{F}_s(u,v)$ is our estimate of the spectrum of the original image (based on the simple structure that the subimage represents).
- Based on the characteristic of the function $H_s(u, v)$, once can rescale to obtain the overall PSF H(u, v).

Experimentation

- If feasible, image a known object, usually a point source of light, using the given imaging equipment and setup.
- If A is the intensity of light source and G(u, v) is the observed spectrum, we have

$$H(u,v) = \frac{G(u,v)}{A}$$



a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Modeling

- A physical model is often used to obtain the PSF.
- Blurring due to atmospheric turbulence can be modeled by the transfer function:

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

  where k is a constant that depends on the nature of the turbulence.
- Note that this is similar to a Gaussian low-pass filter.
- Gaussian low-pass filter is also often used to model mild uniform blurring.

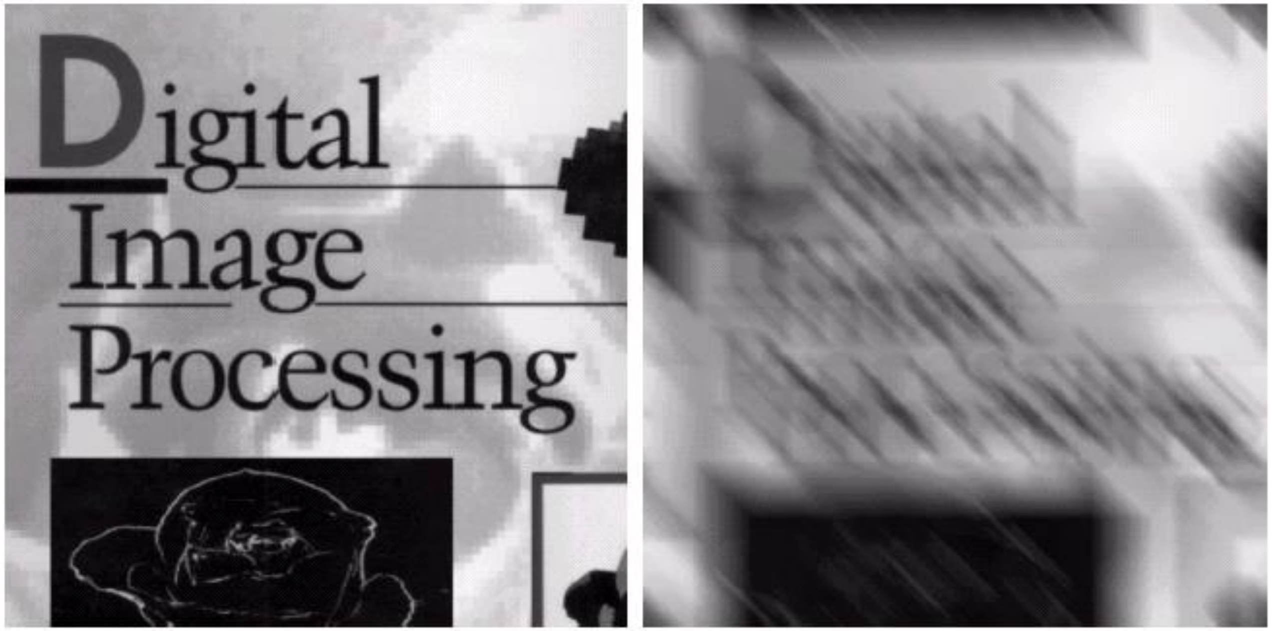
a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, k = 0.0025. (c) Mild turbulence, k = 0.001. (d) Low turbulence, k = 0.00025. (Original image courtesy of NASA.)

- Precise mathematical modeling of the blurring process is sometime used. For example, blurring due to uniform motion is modeled as:

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin\left[\pi(ua+vb)\right] e^{-j\pi(ua+vb)}$$

Where T is the duration of exposure and a and b are the displacements in the x- and y-directions, respectively, during this time T.



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq.(5-77) with a = b = 0.1 and T = 1.

An image has been blurred by **uniform linear motion** between the image and the sensor during image acquisition.

$$g(x,y) = \int_0^T f[x - x_0(t), y - y_0(t)]dt$$

T is the duration of the exposure.

where $g(x,y)$ is the blurred image.

The continuous Fourier transform of this expression is

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}dx\,dy$$

$$\Rightarrow \quad G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_0^T f[x - x_0(t), y - y_0(t)]dt\right]e^{-j2\pi(ux+vy)}dx\,dy$$

$$\Rightarrow \quad G(u,v) = \int_0^T\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x - x_0(t), y - y_0(t)]e^{-j2\pi(ux+vy)}dx\,dy\right]dt$$

$$\Rightarrow \quad G(u,v) = \int_0^T F(u,v)\mathrm{e}^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$= F(u,v)\int_0^T \mathrm{e}^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$H(u,v) = \int_0^T \mathrm{e}^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$\Rightarrow \quad G(u,v) = H(u,v)F(u,v)$$

uniform linear motion in the x-direction only (i.e., $y_0(t) = 0$) at a rate $x_0(t) = at/T$

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t) + vy_0(t)]}\,dt$$

$$G(u,v) = H(u,v)F(u,v)$$

$$\Rightarrow \quad H(u,v) = \int_0^T e^{-j2\pi ux_0(t)}\,dt = \int_0^T e^{-j2\pi uat/T}\,dt$$

$$= \frac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}$$

If $x_0(t) = at/T$, $y_0(t) = bt/T$

$$\Rightarrow \quad H(u,v) = \frac{T}{\pi(ua+vb)}\sin\left[\pi(ua+vb)\right]e^{-j\pi(ua+vb)}$$

Inverse Filter

- The simplest approach to restore an image is to directly apply inverse filtering. This is obtained as follows:

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$\Rightarrow \hat{F}(u,v) = R(u,v)G(u,v), \;\; u,v = 0,1,\ldots,N-1$$

where

$$R(u,v) = \frac{1}{H(u,v)}$$

- We can rewrite this in the spatial domain as follows:

$$\hat{f}(m,n) = g(m,n) * r(m,n) = IDFT\left\{\frac{G(u,v)}{H(u,v)}\right\}$$

- In practice, we actually use a slightly modified filter:

$$R(u,v) = \begin{cases} \dfrac{1}{H(u,v)}, & |H(u,v)| > \varepsilon \\ 0, & \text{otherwise} \end{cases}$$

where ε is a small value. This avoids numerical problems when |H(u, v)| is small.

- The inverse filter works fine provided there is **no noise**. This is illustrated in the following example.
- Let us now analyze the performance of the inverse filter in the presence of noise. Indeed, in this case:

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

which gives

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

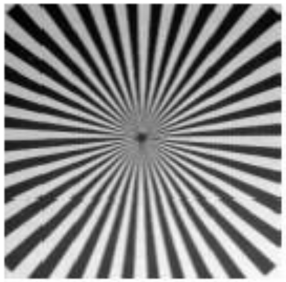
- Hence noise actually gets **amplified** at frequencies where |H(u, v)| is zero or very small. In fact, the contribution from the noise term dominates at there frequencies.

- As illustrated by an example, the inverse filter fails miserably in the presence of noise. It is therefore, seldom used in practice, in the presence of noise.

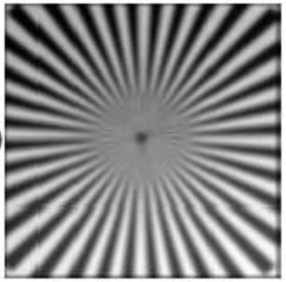
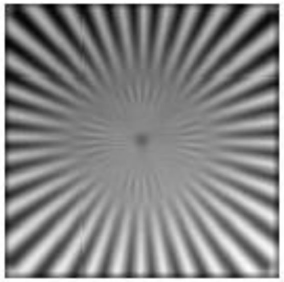
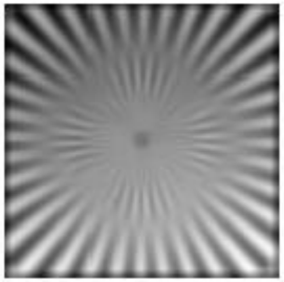
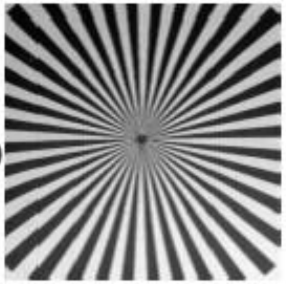
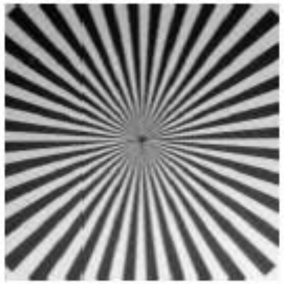
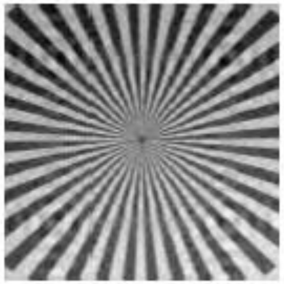
Inverse Filtering example (no noise)

$$h = \frac{1}{N^2}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}_{N\times N}$$

$\varepsilon = 0.001$



$f(m,n)$

	$N=7$	$N=11$	$N=15$
$g(m,n)$			
	MSE $=0.014$	MSE $=0.03$	MSE $=0.05$
$\hat{f}(m,n)$			
	MSE $=3.6\times10^{-5}$	MSE $=2.3\times10^{-4}$	MSE $=0.0029$

Inverse Filtering example (no noise)

$$H(u,v)=\frac{1}{1+\left[\sqrt{\frac{u^2+v^2}{r_o}}\right]^2}$$

$f(m,n)$

$g(m,n)$ $\hat{f}(m,n)$

$r_0=11$

MSE $=0.02$

MSE $=0.008$

$r_0=15$

MSE $=0.017$

MSE $=0.005$

$r_0=23$

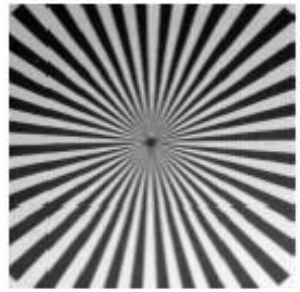
MSE $=0.013$

MSE $=0.0016$

Inverse Filtering example (additive noise)

$$h=\frac{1}{25}\begin{bmatrix}1 & 1 & \cdots & 1\\ 1 & 1 & \cdots & 1\\ \vdots & \vdots & \ddots & \vdots\\ 1 & 1 & \cdots & 1\end{bmatrix}_{5\times5}$$

$\varepsilon = 0.01$

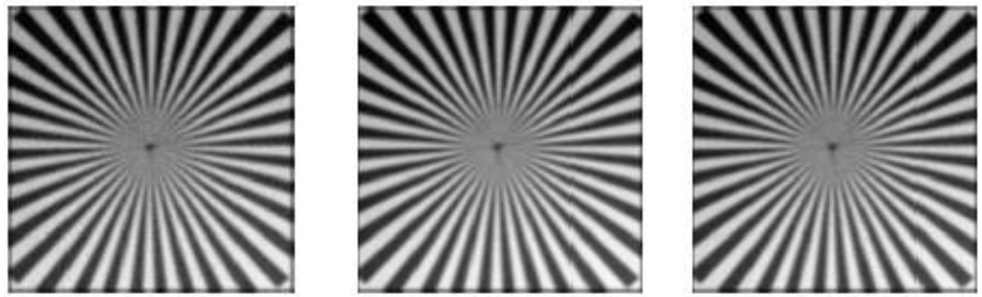


$f(m,n)$

Zero-mean Gaussian noise with variance σ^2

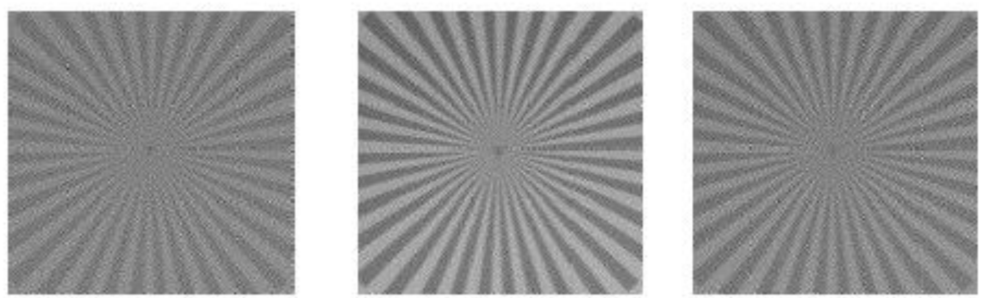
$\sigma = 0.03$ | $\sigma = 0.01$ | $\sigma = 0.02$

$g(m,n)$



MSE $= 0.008$ | MSE $= 0.007$ | MSE $= 0.0075$

$\hat{f}(m,n)$



MSE $= 0.9$ | MSE $= 0.09$ | MSE $= 0.47$

$$H(u,v) = e^{-k\left[(u + M/2)^2 + (v - N/2)^2\right]^{5/6}} \quad \text{with } k = 0.0025$$

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) using Eq. (5-78). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40. (c) Result with H cut off outside a radius of 70. (d) Result with H cut off outside a radius of 85.

MINIMUM MEAN SQUARE ERROR (WIENER) FILTERING

This error measure is defined as

$$e^2 = E\left\{(f - \hat{f})^2\right\}$$

where $E\{\bullet\}$ is the expected value of the argument.

$$\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v) \\
&= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v) \\
&= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)
\end{aligned}$$

minimum mean square error filter or the *least square error filter*

$$
\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v) S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2 + S_\eta(u,v)} \right] G(u,v) \\
&= \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v) \\
&= \left[\frac{1}{H(u,v)} \frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)
\end{aligned}
$$

1. $\hat{F}(u,v)$ = Fourier transform of the estimate of the undegraded image.
2. $G(u,v)$ = Fourier transform of the degraded image.
3. $H(u,v)$ = degradation transfer function (Fourier transform of the spatial degradation).
4. $H^*(u,v)$ = complex conjugate of $H(u,v)$.
5. $\left|H(u,v)\right|^2 = H^*(u,v)H(u,v)$.
6. $S_\eta(u,v) = \left|N(u,v)\right|^2$ = power spectrum of the noise
7. $S_f(u,v) = \left|F(u,v)\right|^2$ = power spectrum of the undegraded image.

frequency domain

$$\mathrm{SNR} = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} |F(u,v)|^2 \Big/ \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} |N(u,v)|^2$$

$$\mathrm{MSE} = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f(x,y) - \hat{f}(x,y)\right]^2$$

spatial domain

$$\mathrm{SNR} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \hat{f}(x,y)^2 \Big/ \sum_{x=0}^{M-1}\sum_{y=0}^{M-1}\left[f(x,y) - \hat{f}(x,y)\right]^2$$

When dealing with white noise, the spectrum is a constant, which simplifies things considerably.

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right]G(u,v)$$

where K is a specified constant that is added to all terms of $|H(u,v)|^2$.

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Constrained least squares filtering (restoration)

- Recall that the knowledge of blur function h(m, n) is essential to obtain a meaningful solution to the restoration problem.
- Often knowledge of h(m, n) is not perfect and subject to errors.
- One way to alleviate sensitivity of the result to errors in h(m, n) is to base optimality of restoration on a measure of smoothness, such as the second derivative of the image.
- We will approximate the second derivative (Laplacian) by a matrix Q. Indeed, we will first formulate the constrained restoration problem and obtain its solution in terms of a general matrix Q.
- Later different choices of matrix Q will be considered, each giving rise to a different restoration filter.
- Suppose Q is any matrix (of appropriate dimension). In constrained image restoration, we choose $\hat{f}$ to minimize $\left\|Q\hat{f}\right\|^2$, subject to the constraint,

$$\left\|g - H\hat{f}\right\|^2 = \left\|n\right\|^2 \text{ (Recall the degradation euqation } g = H\hat{f} + n \Rightarrow g - H\hat{f} = n)$$

- Introduction of matrix Q allows considerable flexibility in the design of appropriate restoration filters (we will discuss specific choices of Q later). So our problem is formulated as follows:

$$\min \left\|Q\hat{f}\right\|^2$$

$$\text{subject to } \left\|g - H\hat{f}\right\|^2 = \left\|n\right\|^2 \; or \left\|g - H\hat{f}\right\|^2 - \left\|n\right\|^2 = 0$$

A brief review of matrix differentiation

- Suppose

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and $f(x_1, x_2)$ is a function of two variables. Then

$$\frac{\partial f(x_1, x_2)}{\partial x} = \begin{bmatrix} \dfrac{\partial f(x_1, x_2)}{\partial x_1} \\ \dfrac{\partial f(x_1, x_2)}{\partial x_2} \end{bmatrix}$$

- If

$$f(x_1, x_2) = \|Ax - b\|^2 = (Ax - b)^T (Ax - b)$$

for some matrix A and some vector b, then

$$\frac{\partial f(x_1, x_2)}{\partial x} = 2A^T (Ax - b)$$

where superscript T denotes matrix transpose.

- Recall from calculus that such a constrained minimization problem can be solved by means of Lagrange multipliers. We need to minimize the augmented objective function $J(\hat{f})$:

$$J(\hat{f}) = \left\|Q\hat{f}\right\|^2 + \alpha\left(\left\|g - H\hat{f}\right\|^2 - \|n\|^2\right)$$

where α is a Lagrange multiplier.

- We set the derivative of $J(\hat{f})$ with respect to $\hat{f}$ to zero.

$$\nabla J(\hat{f}) = 2Q^T Q\hat{f} - 2\alpha \mathrm{H}^{\mathrm{T}}(g - H\hat{f}) = 0$$

$$\Rightarrow (Q^T Q + \alpha H^T H)\hat{f} = \alpha \mathrm{H}^{\mathrm{T}} g$$

Therefore,

$$\hat{f} = (Q^T Q + \alpha H^T H)^{-1} \alpha \mathrm{H}^{\mathrm{T}} g$$

$$= (\frac{1}{\alpha} Q^T Q + H^T H)^{-1} \mathrm{H}^{\mathrm{T}} g$$

$$= (\gamma Q^T Q + H^T H)^{-1} \mathrm{H}^{\mathrm{T}} g$$

where $\gamma = \frac{1}{\alpha}$ is chosen to satisfy the constraint $\left\|g - \mathrm{H}\hat{f}\right\|^2 = \|n\|^2$

- We will now use the above formulation to derive a number of restoration filters.

Pseudo-inverse Filtering

- The pseudo-inverse filter tries to avoid the pitfalls of applying an inverse filter in the presence of noise.
- Consider the constrained restoration solution,

$$\hat{\mathrm{f}} = (\gamma \mathrm{Q}^{\mathrm{T}}\mathrm{Q} + \mathrm{H}^{\mathrm{T}}\mathrm{H})^{-1} H^{T} g$$

with $Q = I$, This gives,

$$\hat{\mathrm{f}} = (\gamma I + \mathrm{H}^{\mathrm{T}}\mathrm{H})^{-1} H^{T}$$

- It can be implemented in the Fourier domain by the following equation:

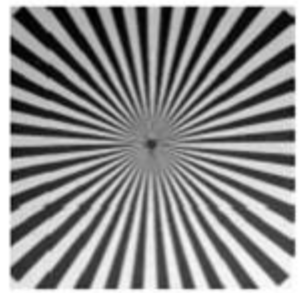
$$\hat{\mathrm{F}}(\mathrm{u, v}) = R(u,v)G(u,v), where$$

$$R(u,v) = \frac{H^{*}(u,v)}{|H(u,v)|^{2} + \gamma} = \frac{1}{H(u,v)}\left[\frac{|H(u,v)|^{2}}{|H(u,v)|^{2} + \gamma}\right]$$

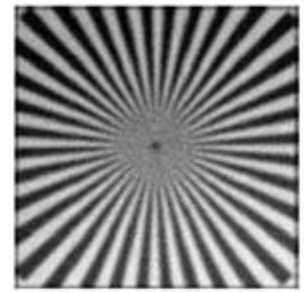
- The parameter γ is a constant to be chosen.
- Note that $\gamma = 0$ gives us back the inverse filter. For $\gamma > 0$, the denominator of R(u, v) is strictly positive and the pseudo-inverse filter is well defined.

Pseudo-inverse Filtering example

$$h = \frac{1}{25}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}_{5\times 5}$$



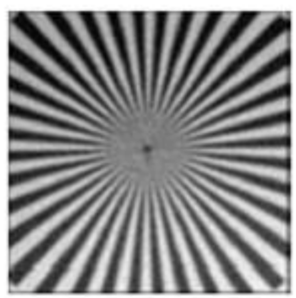
$f(m,n)$



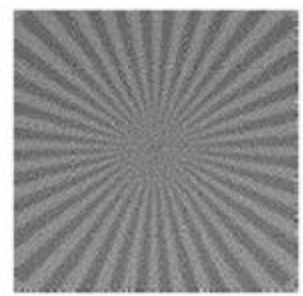
$g(m,n)$ MSE $= 0.01$

Zero-mean Gaussian noise with variance $\sigma^2 = 0.003$

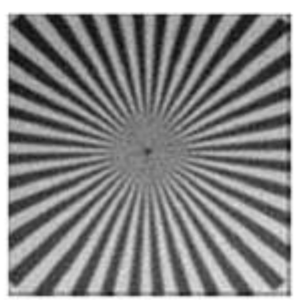
$\hat{f}(m,n)$



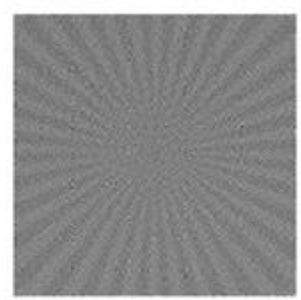
$\gamma = 0.075$ MSE $= 0.0076$



$\gamma = 0.001$ MSE $= 0.3308$



$\gamma = 0.5$ MSE $= 0.0447$



$\gamma = 0$ (Inv. Filter) MSE $= 2.7$

Minimum Mean Square Error (Wiener) Filter

- This is a restoration technique based on the statistics (mean and correlation) of the image and noise.
- We consider each element of f and n as random variables. Define the correlation matrices

$$R_f = E\{ff^T\} = \begin{bmatrix} E(f_0 f_0) & E(f_0 f_1) & \cdots & E(f_0 f_{MN-1}) \\ E(f_1 f_0) & E(f_1 f_1) & \cdots & E(f_1 f_{MN-1}) \\ \vdots & \vdots & \ddots & \vdots \\ E(f_{MN-1} f_0) & E(f_{MN-1} f_1) & \cdots & E(f_{MN-1} f_{MN-1}) \end{bmatrix} \quad and$$

$$R_n = E\{nn^T\} = \begin{bmatrix} E(n_0 n_0) & E(n_0 n_1) & \cdots & E(n_0 n_{MN-1}) \\ E(n_1 n_0) & E(n_1 n_1) & \cdots & E(n_1 n_{MN-1}) \\ \vdots & \vdots & \ddots & \vdots \\ E(n_{MN-1} n_0) & E(n_{MN-1} n_1) & \cdots & E(n_{MN-1} n_{MN-1}) \end{bmatrix}$$

- The matrices $\mathbf{R_f}$ and $\mathbf{R_n}$ are real and symmetric, with all eigenvalues being non-negative.
- The 2D-DFT of the correlations $\mathbf{R_f}$ and $\mathbf{R_n}$ are called the power spectra and are denoted by $S_f(u, v)$ and $S_n(u, v)$, respectively.
- Recall the constrained restoration solution given by

$$\hat{f} = (H^T H + \gamma Q^T Q)^{-1} H^T g$$

- Choose matrix Q such that

$$Q^T Q = R_f^{-1} R_n$$

- In a sense, we are trying to minimize the noise-to-signal ratio.
- The constrained restoration is then given by

$$\hat{f} = (H^T H + \gamma R_f^{-1} R_n)^{-1} H^T g$$

- This can be implemented using DFT as

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |S_\eta(u,v)/S_f(u,v)|} \right] G(u,v) = R(u,v)G(u,v)$$

$$where \;\; R(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |S_\eta(u,v)/S_f(u,v)|} \right]$$

$$= \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \gamma |S_\eta(u,v)/S_f(u,v)|}$$

- Here $S_f(u,v) = E\left\lfloor |F(u,v)^2| \right\rfloor$ is the power spectral density of the image f(m, n) and

  $S_\eta(u,v) = E\left\lfloor |N(u,v)^2| \right\rfloor$ is the power spectral density of the noise $\eta(m,n)$
- The restoration filter R(u, v) is called the parametric Wiener filter, with parameter γ

- Special cases:
  - γ= 1: Wiener Filter
  - γ= 0: Inverse Filter
  - γ≠ 0,1: Parametric Wiener Filter

- According to the constrained restoration filter derived earlier, parameter γ should be chosen to satisfy $\left\| g - H\hat{f} \right\| = \|n\|$

- However, choice of γ= 1 yields an optimal filter in the sense of minimizing the error function $e^2 = E\left\{\left[f(m,n) - \hat{f}(m,n)\right]^2\right\}$. In other words, setting γ= 1 yields a statistically optimal restoration.
- Implementation of the parametric Wiener filter requires knowledge of the image and noise power spectra $S_f(u, v)$ and $S_n(u, v)$. In particular, we need the so called signal-to-noise ratio (SNR) $\rho(u, v) = S_f(u, v)/ S_n(u, v)$.

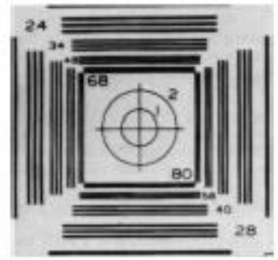
- This is not always available and a simple approximation is to replace ρ(u, v) by a constant ρ. In this case, the Wiener filter is given by

$$R(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{\gamma}{\rho}} \right]$$

- Note that as ρ→∞ (no noise), the Wiener filter tends to the inverse filter.

Wiener filter example

$$H(u,v)=\frac{1}{1+\left[\frac{\sqrt{u^2+v^2}}{r_0}\right]^2}$$



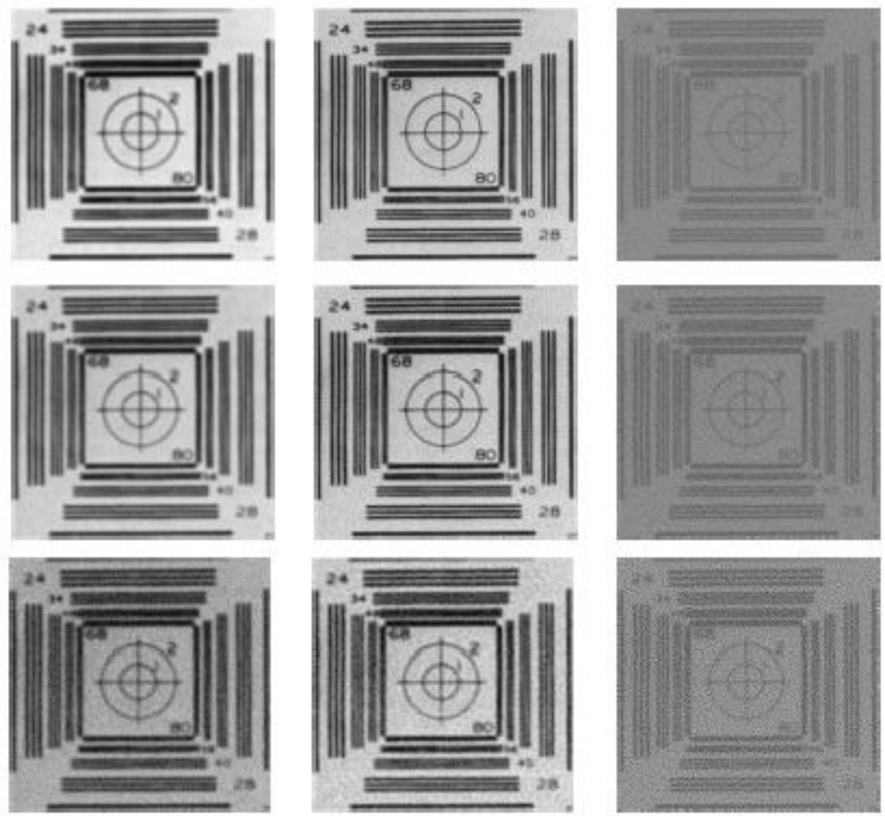
$$\rho=\frac{\sigma_f^2}{\sigma_n^2} \quad \text{or} \quad 10\log\left(\frac{\sigma_f^2}{\sigma_n^2}\right)dB$$

$f(m,n)$

$\rho=25.9dB$

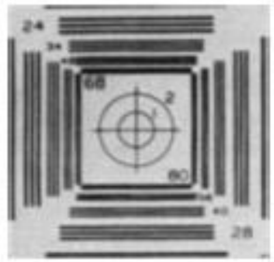
$\rho=15.9dB$

$\rho=5.9dB$



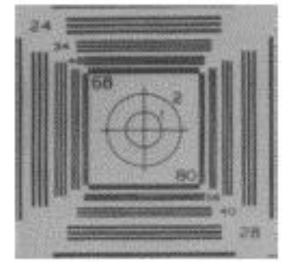
$g(m,n)$ $\quad$ $\hat{f}(m,n),\ \gamma=1$ $\quad$ $\hat{f}(m,n),\ \gamma=0.01$

Parametric Wiener Filter example (effect of parameter γ)

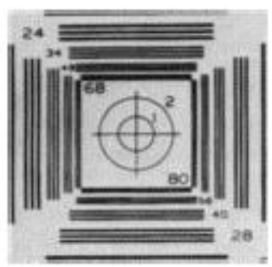


$g(m,n)$

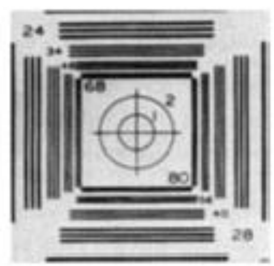
$\hat{f}(m,n),\ \gamma = 0.01$



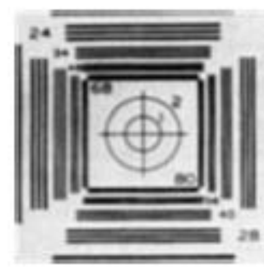
$\hat{f}(m,n),\ \gamma = 0.1$



$\hat{f}(m,n),\ \gamma = 1$



$\hat{f}(m,n),\ \gamma = 5$



$\hat{f}(m,n),\ \gamma = 50$

- Small values of γ result in better “blur removal” and poor noise filtering.
- Large values of γ result in poor blur removal“ and better noise filtering.

Chapter 5
Image Restoration

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

a	b	c
d	e	f
g	h	i

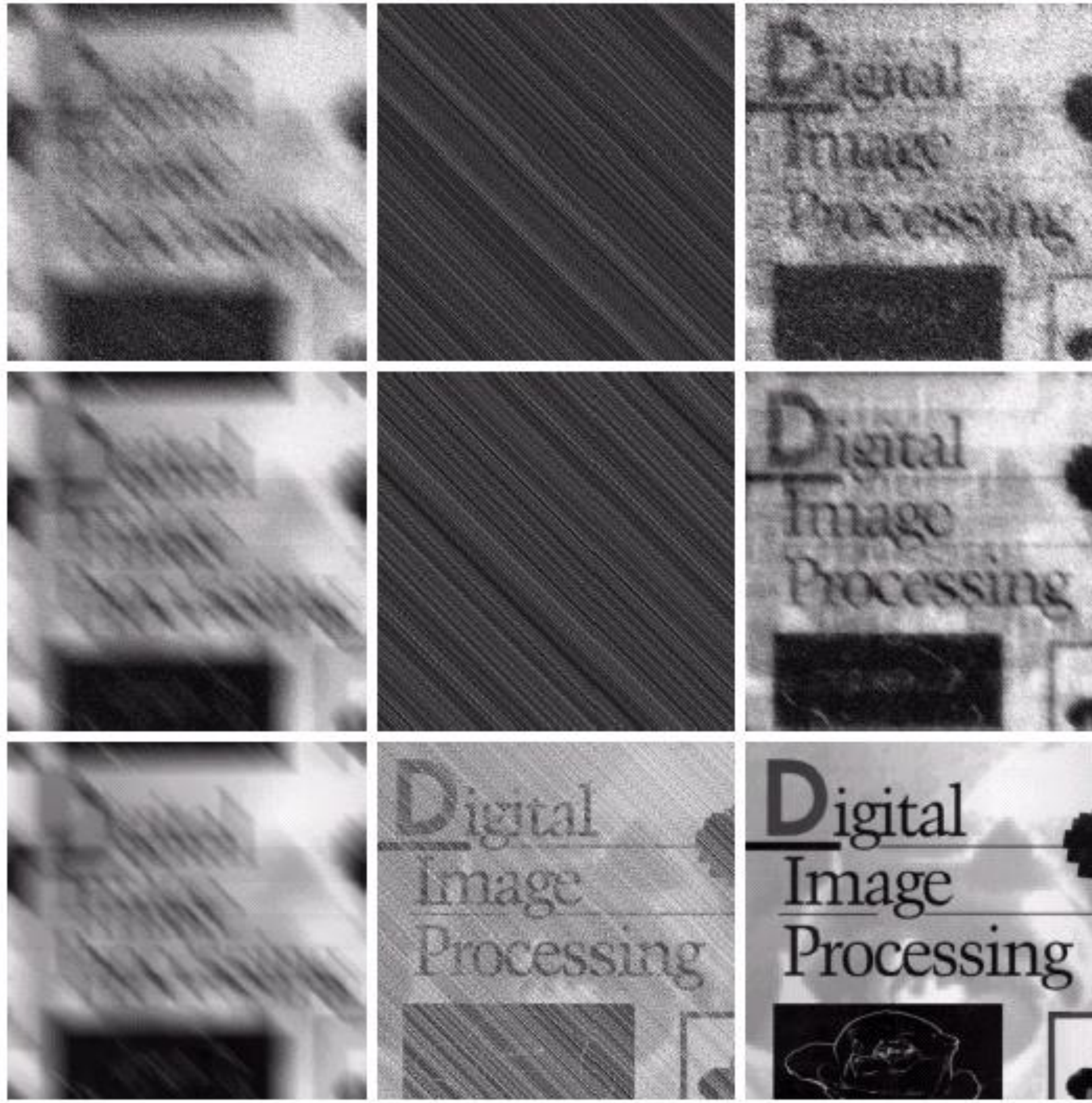


FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)-(f). Same sequence, but with noise variance one order of magnitude less. (g)-(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.25(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.