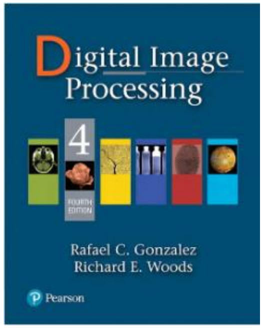


Digital Image Processing

Image Restoration and Reconstruction (I)

Dr. Tun-Wen Pai

1) **Concept of image restoration**
2) **Linear degradation model**
3) **Noise model /periodic Interference**
4) **Restoration in the presence of only noise**
5) **Bandreject/bandpass/notch filters**

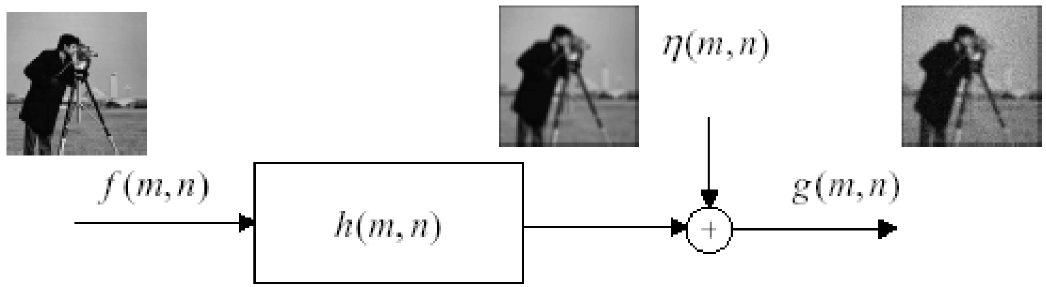
Image restoration

- Most images obtained by optical, electronic, or electro-optic means is likely to be degraded.
- The degradation can be due to camera misfocus, relative motion between camera and object, noise in electronic sensors, atmospheric turbulence, etc.
- The goal of image restoration is to obtain a relatively “clean” image from the degraded observation.
- It involves techniques like filtering, noise reduction etc.

- **Restoration:**
  - A process that attempts to reconstruct or recover an image that has been degraded by using some prior knowledge of the degradation phenomenon.
  - Involves modeling the degradation process and applying the inverse process to recover the original image.
  - A criterion for “goodness” is required that will recover the image in an optimal fashion with respect to that criterion.
  - Ex. Removal of blur by applying a deblurring function.

- **Enhancement:**
  - Manipulating an image in order to take advantage of the psychophysics of the human visual system.
  - Techniques are usually “heuristic”.
  - Ex. Contrast stretching, histogram equalization.

(Linear) Degradation Model



$$g(m,n) = (f(m,n) * h(m,n)) + \eta(m,n)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

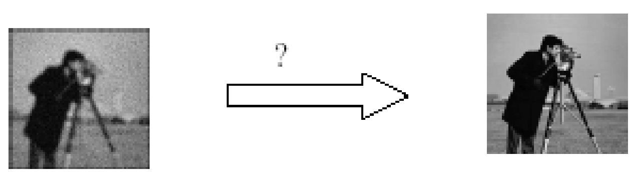
$f(m,n)$: Degradation free image

$g(m,n)$: Observed image

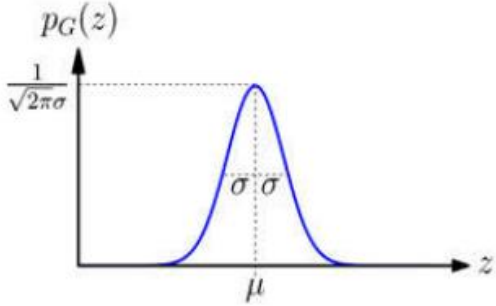
$h(m,n)$: PSF of blur degradation

$\eta(m,n)$: Additive Noise

Problem: Given an observed image g(m, n), to recover the origianl image f(m, n), using knowledge about the blur function h(m, n) and the characteristics of the noise η(m, n).



- We need to find an image $\hat{f}(m,n)$, such that the error $f(m,n) - \hat{f}(m,n)$ is “small”



Noise Models

- With the exception of periodic interference, we will assume that noise values are uncorrelated from pixel to pixel and with the (uncorrupted) image pixel values.
- These assumptions are usually met in practice and simplify the analysis.
- With these assumptions in hand, we need to only describe the statistical properties of noise; i.e., its probability density function (PDF).

Gaussian Noise

- Mathematically speaking, it is the most tractable noise model. Therefore, it is often used in practice, even in situations where they are not well justified from physical principles.
- The pdf of a Gaussian random variable z is given by:

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2}$$

where z represents (noise) gray value, u is the mean, and σ is its standard deviation. The squared standard deviation σ^2 is usually referred to as variance.

- For a Gaussian pdf, approximately 68.3% of the values are within one standard deviation of the mean and 95% of the values are within two standard deviations of the mean.

Rayleigh noise

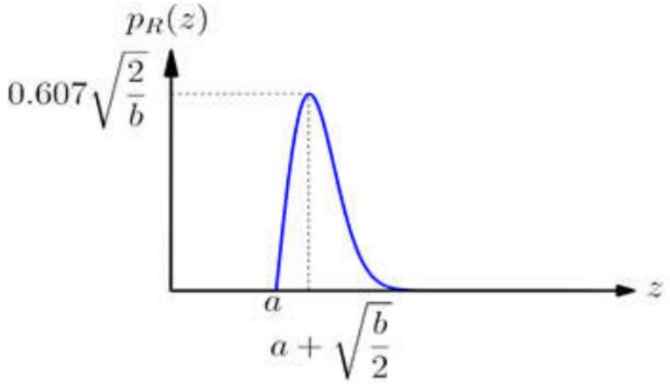
- The pdf of a Rayleigh noise is given by:

$$p(z) = \begin{cases} \dfrac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

- The mean and variance are given by:

$$\mu = a + \sqrt{\pi b/4}$$

$$\sigma = \frac{b(4-\pi)}{4}$$



- This noise is “one-sided” and the density function is skewed.

Erlang(Gamma) noise

- The pdf of Erlang noise is given by:

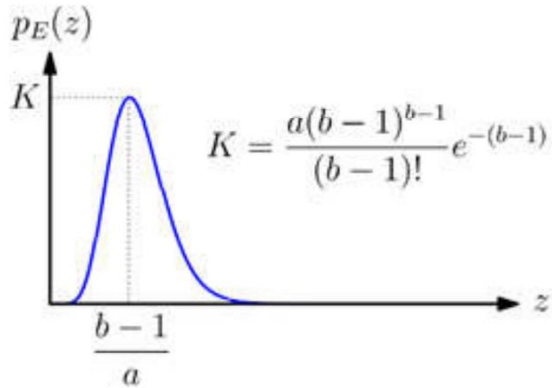
$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

where, a > 0, b is an integer and “!” represents factorial.

- The mean and variance are given by:

$$\mu = \frac{b}{a}$$

$$\sigma^2 = \frac{b}{a^2}$$



- This noise is “one-sided” and the density function is skewed.

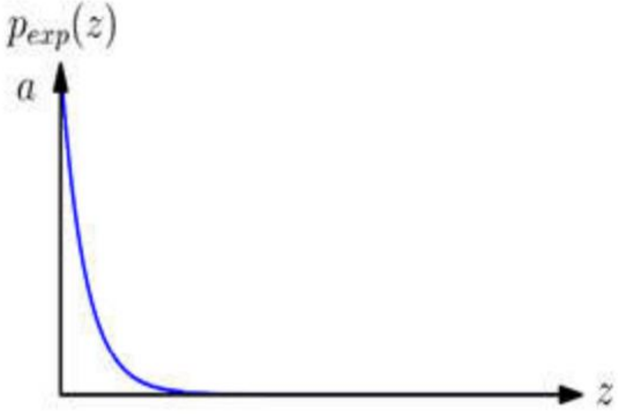
Exponential noise

- The pdf of exponential noise is given by:

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$
$$where, a > 0$$

The mean and variance are given by:

$$\mu = \frac{1}{a}$$
$$\sigma^2 = \frac{1}{a^2}$$



- This is a special case of Erlang density with b=1.

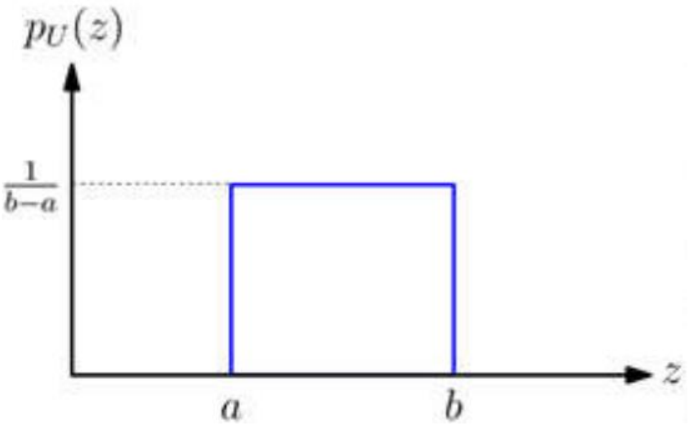
Uniform noise

- The pdf of uniform noise is given by:

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

- The mean and variance are given by:

$$\mu = \frac{a+b}{2}$$
$$\sigma^2 = \frac{(b-a)^2}{12}$$



Impulse (salt-and-pepper) noise

- The pdf of uniform noise is given by:

$$p(z)=\begin{cases} P_a & \text{for } z=a \\ P_b=1-P_a & \text{for } z=b \\ 0 & \text{otherwise} \end{cases}$$

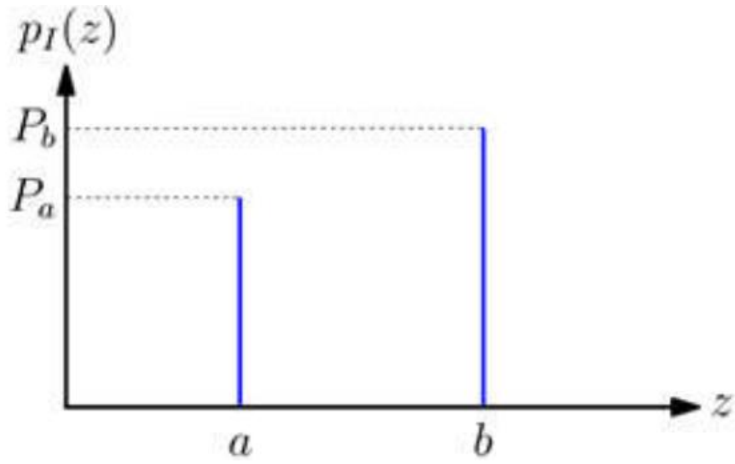


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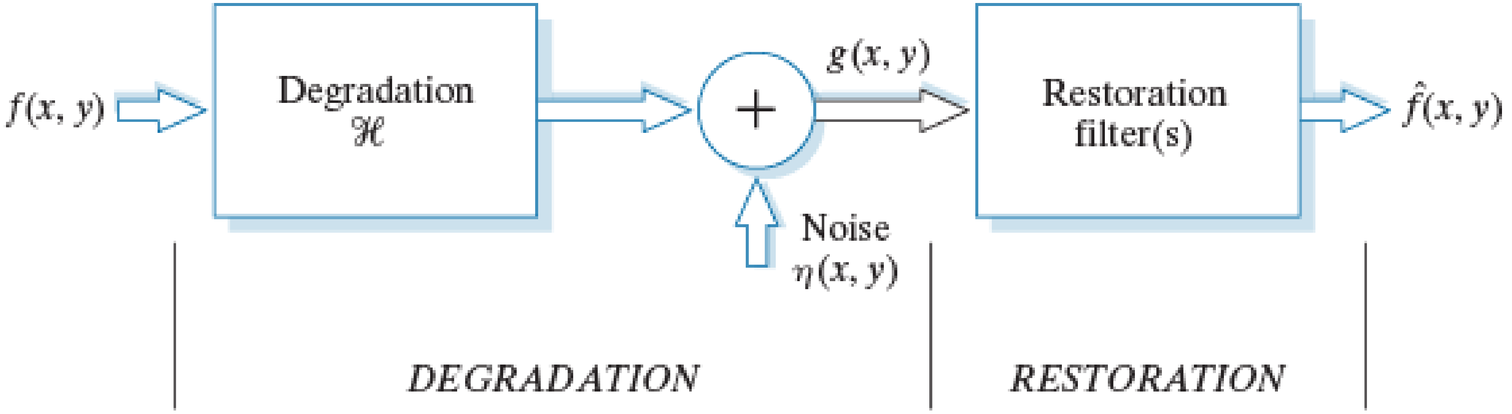
Image restoration

FIGURE 5.1
A model of the image degradation/restoration process.



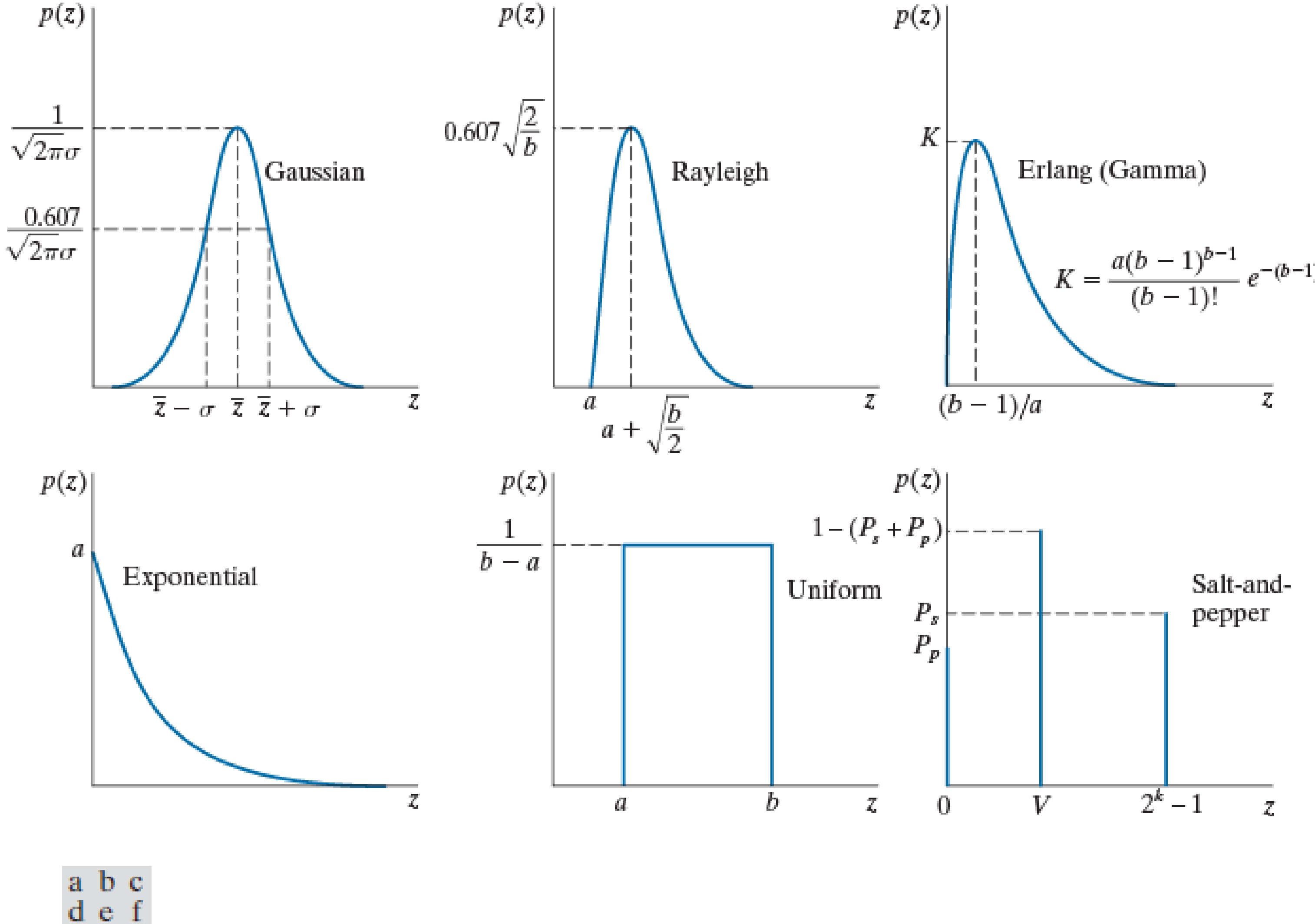


FIGURE 5.2 Some important probability density functions.

(new)Chapter 5
Image Restoration

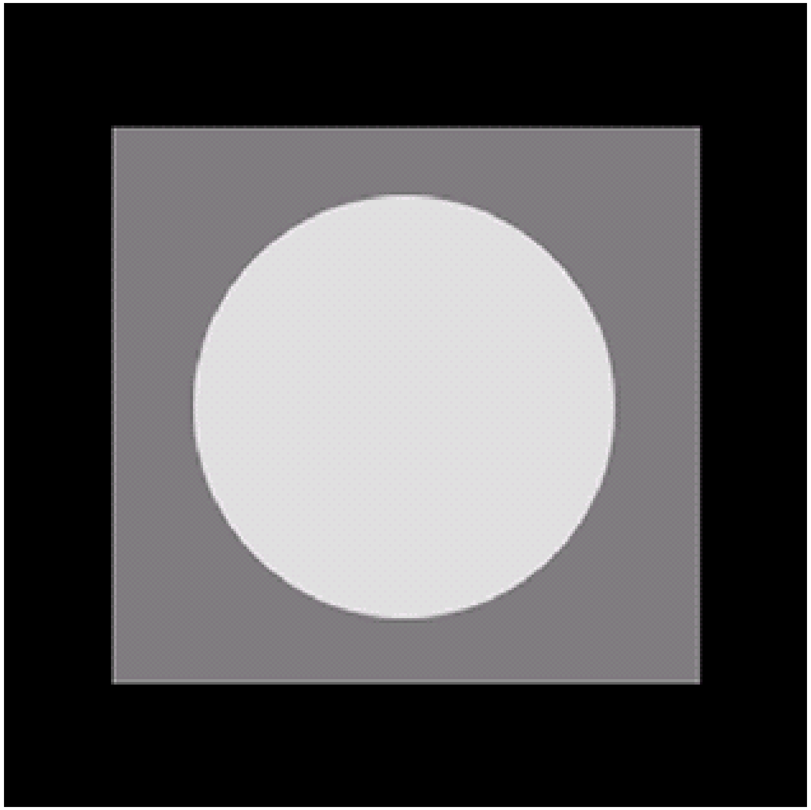
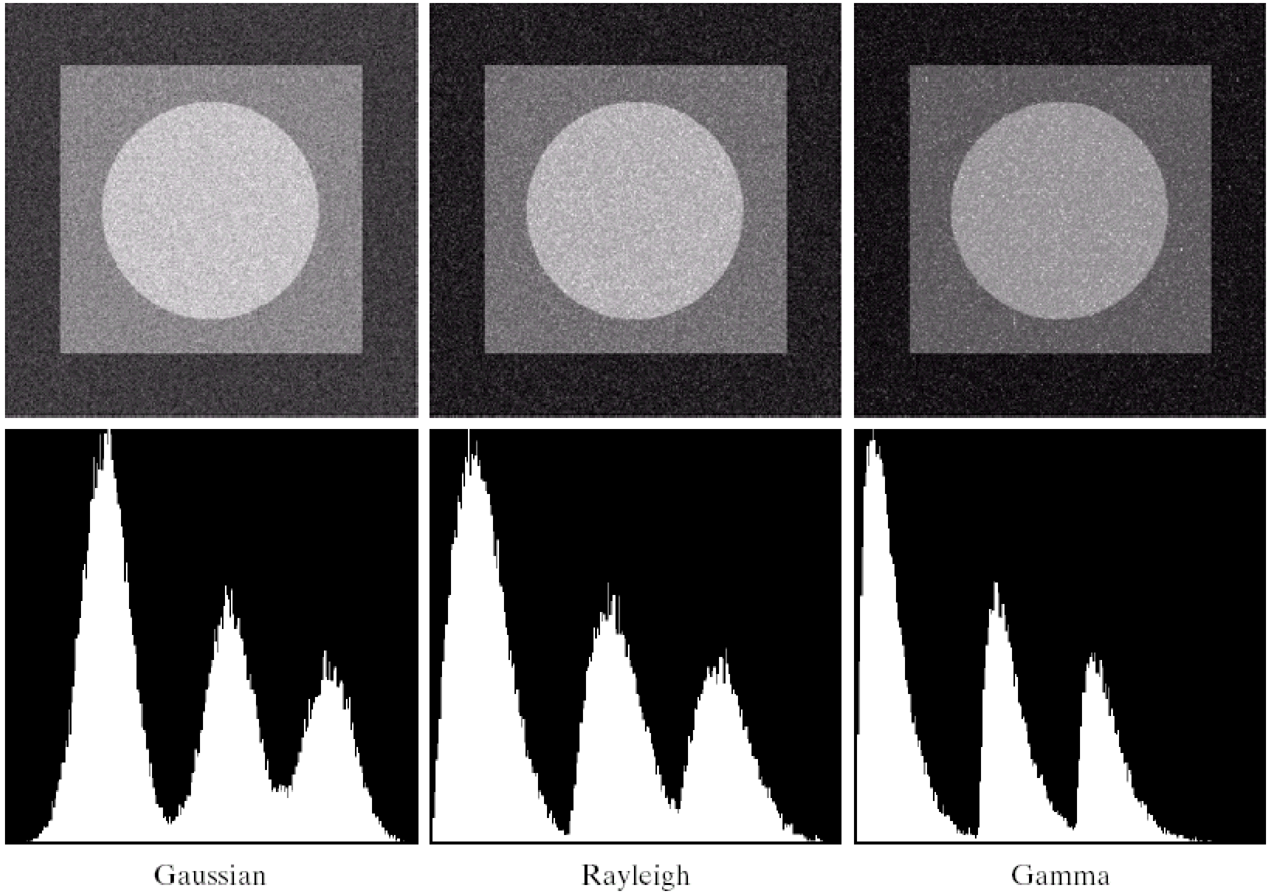


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.



a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and Erlanga noise to the image in Fig. 5.3.

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3

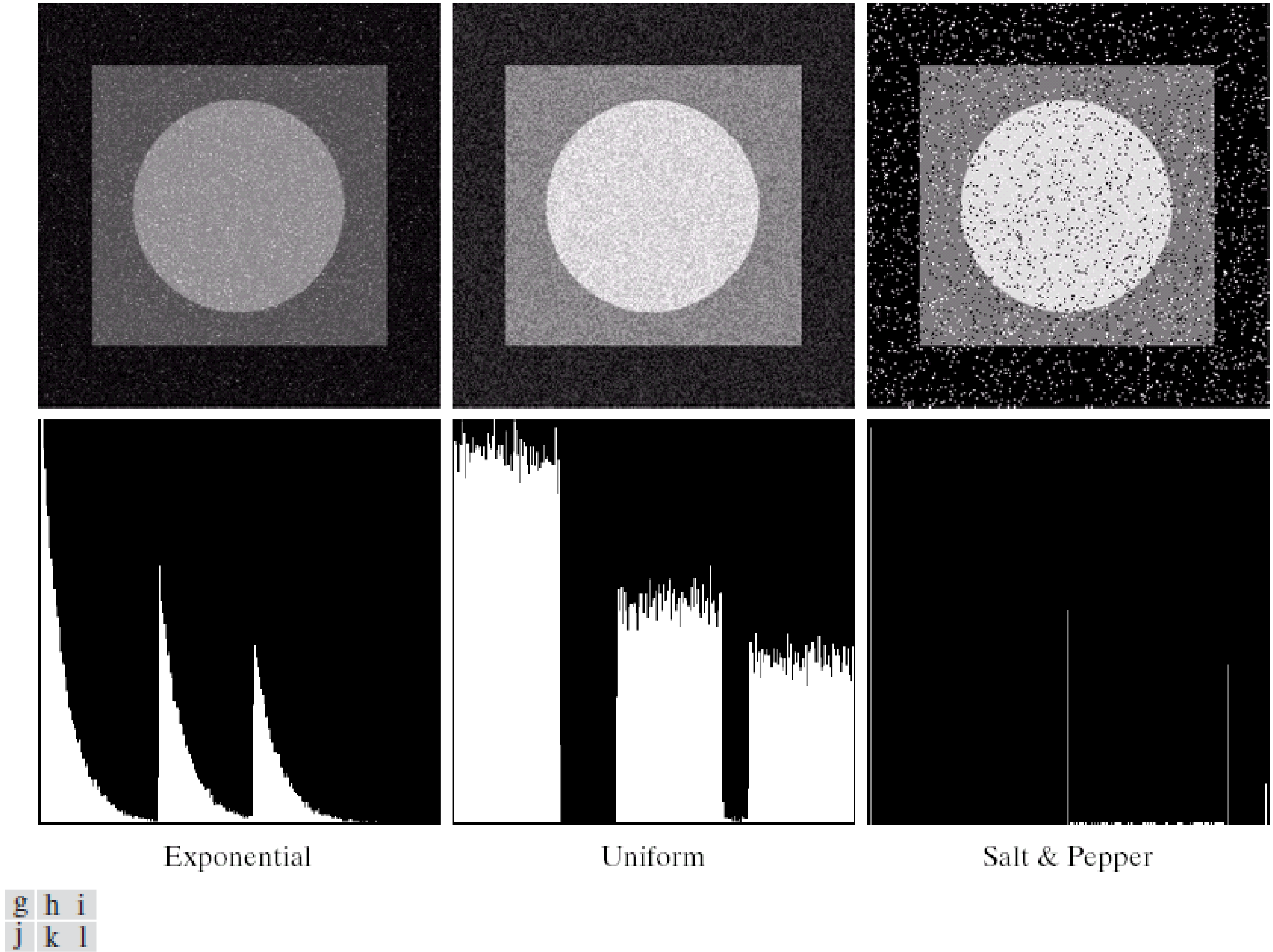
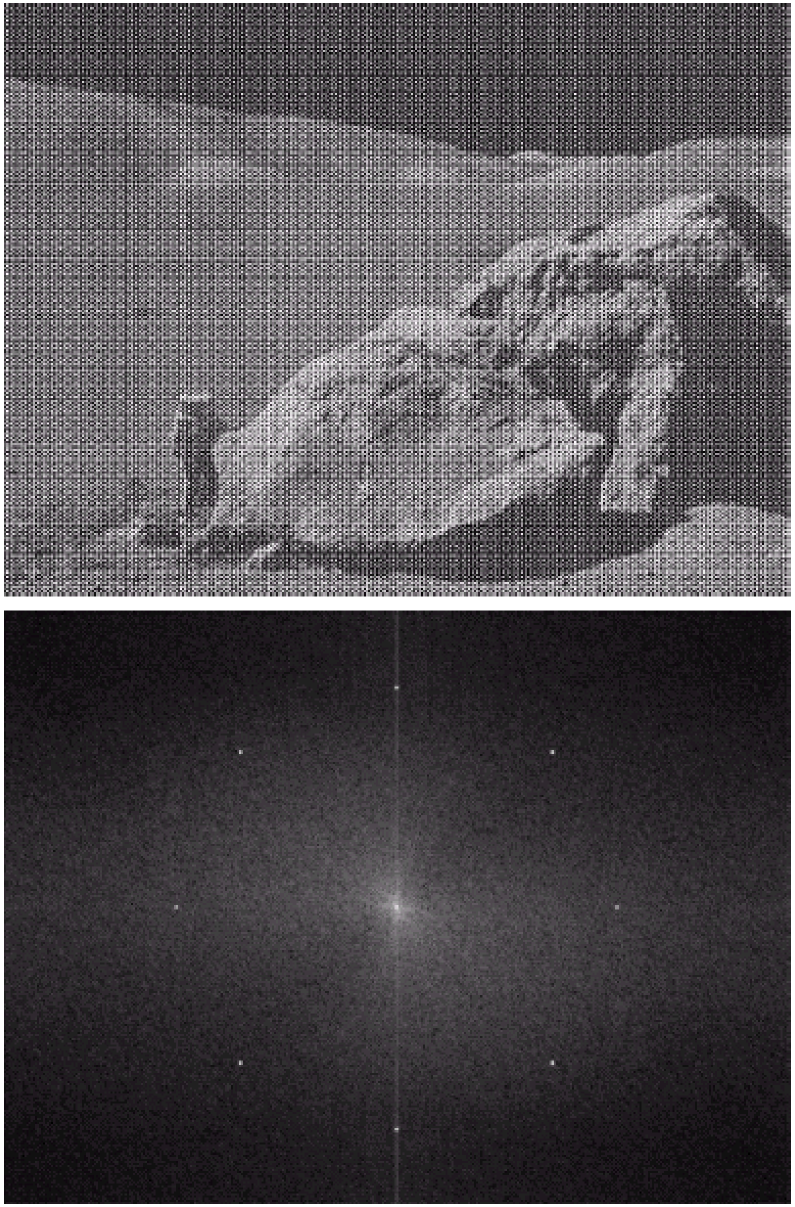


FIGURE 5.4 (*continued*) Images and histograms resulting from adding exponential, uniform, and salt-and-pepper noise to the image in Fig. 5.3. In the salt-and-pepper histogram, the peaks in the origin (zero intensity) and at the far end of the scale are shown displaced slightly so that they do not blend with the page background.

Periodic Interference/Noise

- Periodic noise or interference occurs in images due to electrical or electromechanical interference during image acquisition.
- It is an example of spatially dependent noise.
- This type of noise can be very effectively removed using frequency domain filtering. Recall that the spectrum of a pure sinusoid would be a simple impulse at the appropriate frequency location.

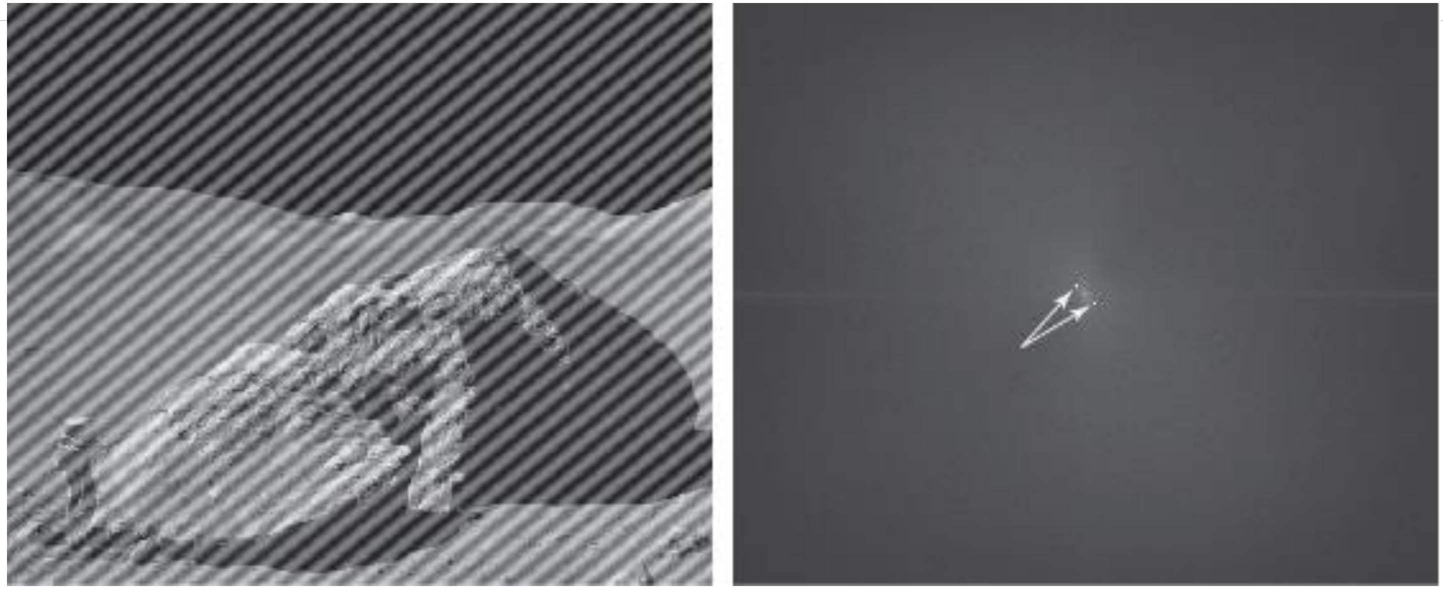


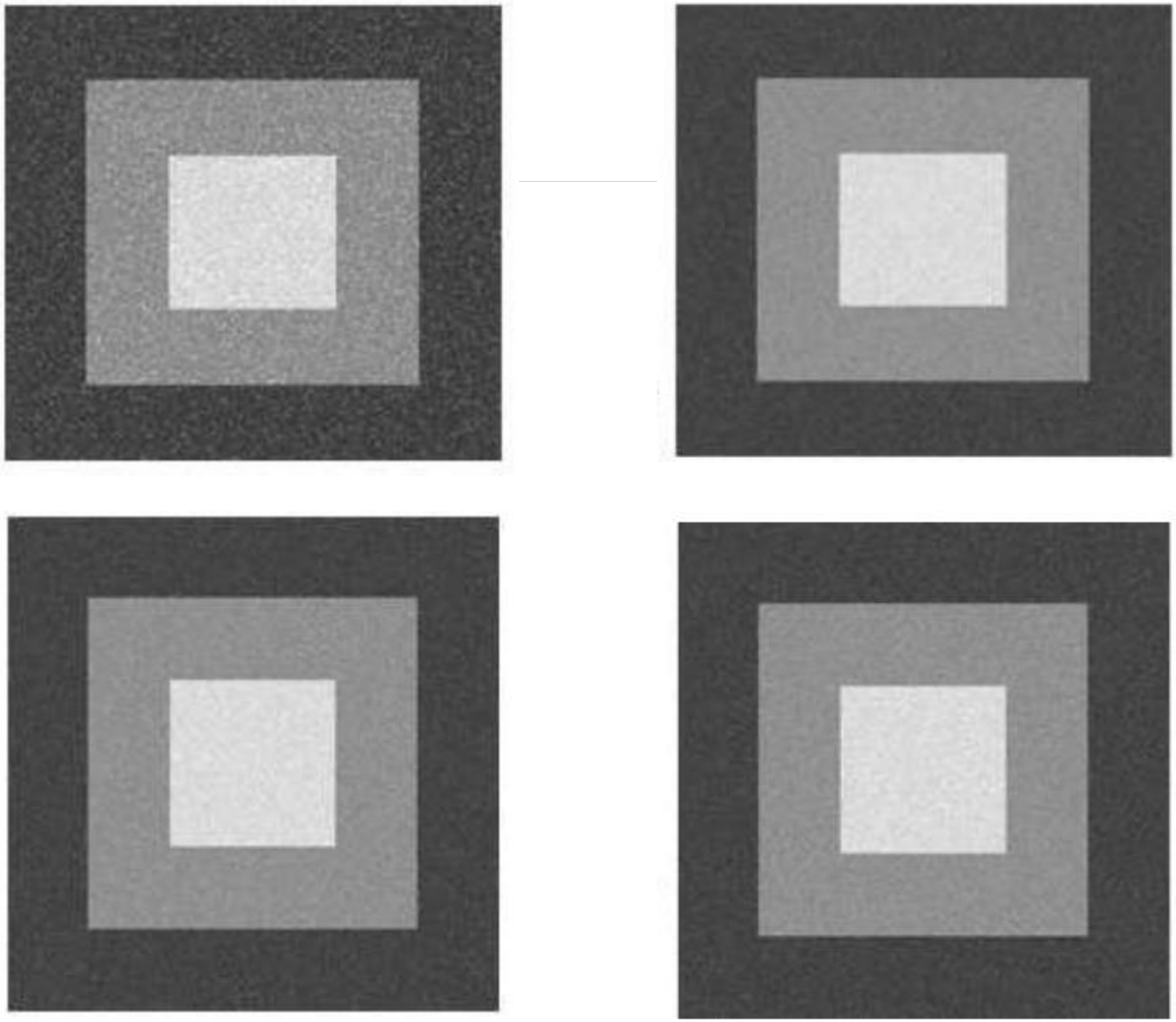
a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

a b

FIGURE 5.5 (a) Image corrupted by additive sinusoidal noise. (b) Spectrum showing two conjugate impulses caused by the sine wave. (Original image courtesy of NASA.)





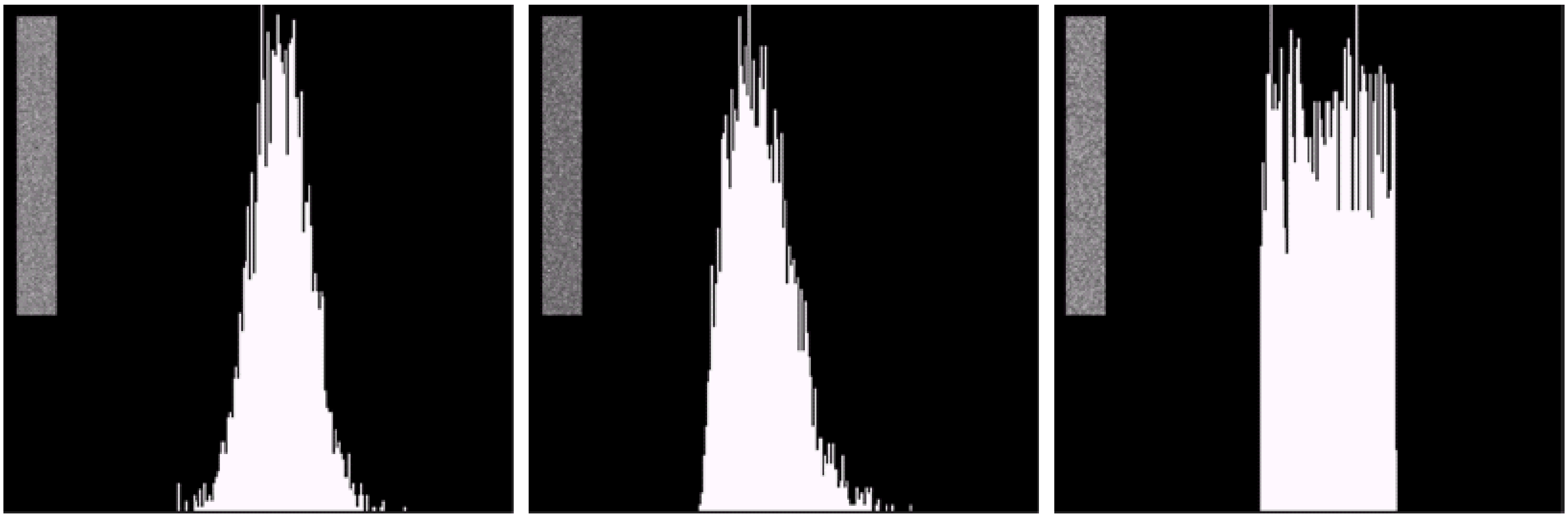
Estimation of noise parameters

- The noise pdf is usually available from sensor specifications.
  Sometimes, the form of the pdf is known from physical modeling.
- The pdf (or parameters of the pdf) are also often estimated from the image.
- Typically, if feasible, **a flat uniformly illuminated surface** is imaged using the imaging system. The histogram of the resulting image is usually a good indicator of the noise pdf.
- If that is not possible, we can usually choose **a small patch** of an image that is relatively uniform and compute a histogram of the image over that region.
- Using the histogram, we can estimate the noise mean and variance as follows:

$$\mu = \sum_{t \in S} z_t p(z_t)$$

$$\sigma^2 = \sum_{t \in S} (z_t - \mu)^2 p(z_t)$$

  where z_t is the gray-value of pixel I in S, and $p(z_t)$ is the histogram value.
- The shape of the histogram identifies the closet pdf match.
- The mean and variance are used to solve for the parameters a and b in the density function.



a b c

FIGURE 5.6 (a) Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Restoration in the presence of only noise

- In this case, the degradation equation becomes:

$$g(m,n) = f(m,n) + \eta(m,n)$$
$$G(u,v) = F(u,v) + N(u,v)$$

- Spatial filtering is usually the best method to restore images corrupted purely by additive noise. The process is similar to that of image enhancement.

Mean filter

Arithmetic mean

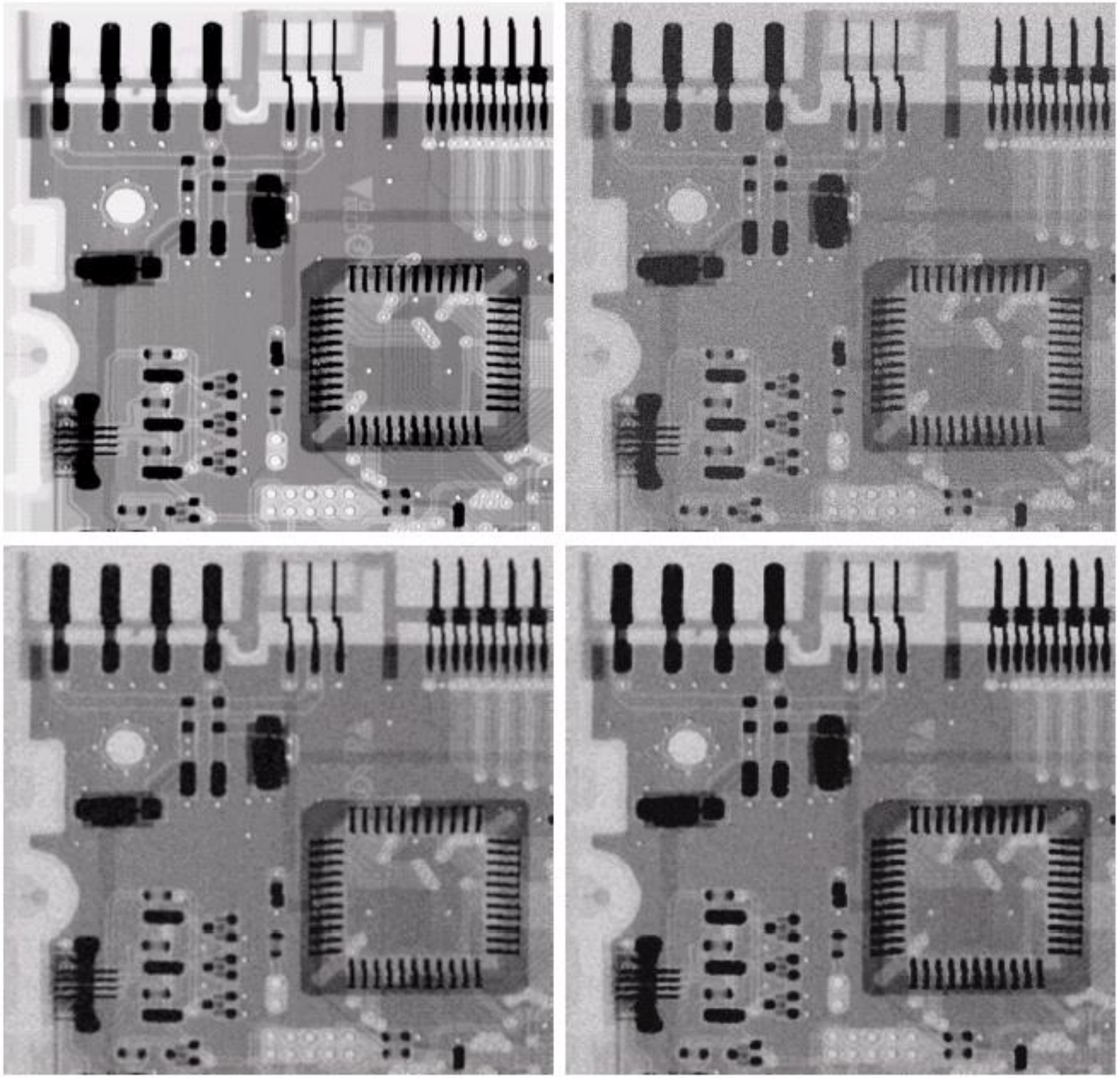
- Let S_{ab} be a rectangular window of size *a* x *b*. The arithmetic mean filter computes the average value of the pixels in $g(m,n)$ over the window S_{ab}

$$\hat{f}(m,n) = \frac{1}{ab} \sum_{(s,t)\in S_{ab}} g(s,t)$$

- This operation can be thought of as a convolution with a uniform rectangular mask of size a x b, each of whose values is 1/ab.
- This smoothes out variations and noise is reduced.

Geometric mean

- The geometric mean filter computes the geometric mean of the pixels in g(*m*, *n*) over the window S_{ab}
- $$\hat{f}(m,n) = \left[\prod_{(s,t)\in S_{ab}} g(s,t) \right]^{1/ab}$$
- This usually results in similar results as the arithmetic mean filter, with possibly less loss of image detail.



a	b
c	d

FIGURE 5.7 (a) X-ray image, (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 x 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Harmonic mean

- The harmonic mean filter computes the harmonic mean of the pixels in g(m, n) over the window S_{ab}.

$$\hat{f}(m,n) = \frac{ab}{\sum_{S_{ab}} \frac{1}{g(s,t)}}$$

- This works well for salt noise, but fails for pepper noise. It also works well with Gaussian noise.

Contraharmonic mean

- The contraharmonic mean filter is given by the expression:

$$\hat{f}(m,n) = \frac{\sum_{S_{ab}} g(s,t)^{Q+1}}{\sum_{S_{ab}} g(s,t)^{Q}}$$

where Q is called order of the filter.

- This yields the arithmetic mean filter for Q=0 and the harmonic mean filter for Q=-1.
- For positive values of Q, it reduces pepper noise and for negative values of Q, it reduces salt noise. It cannot do both simultaneously.

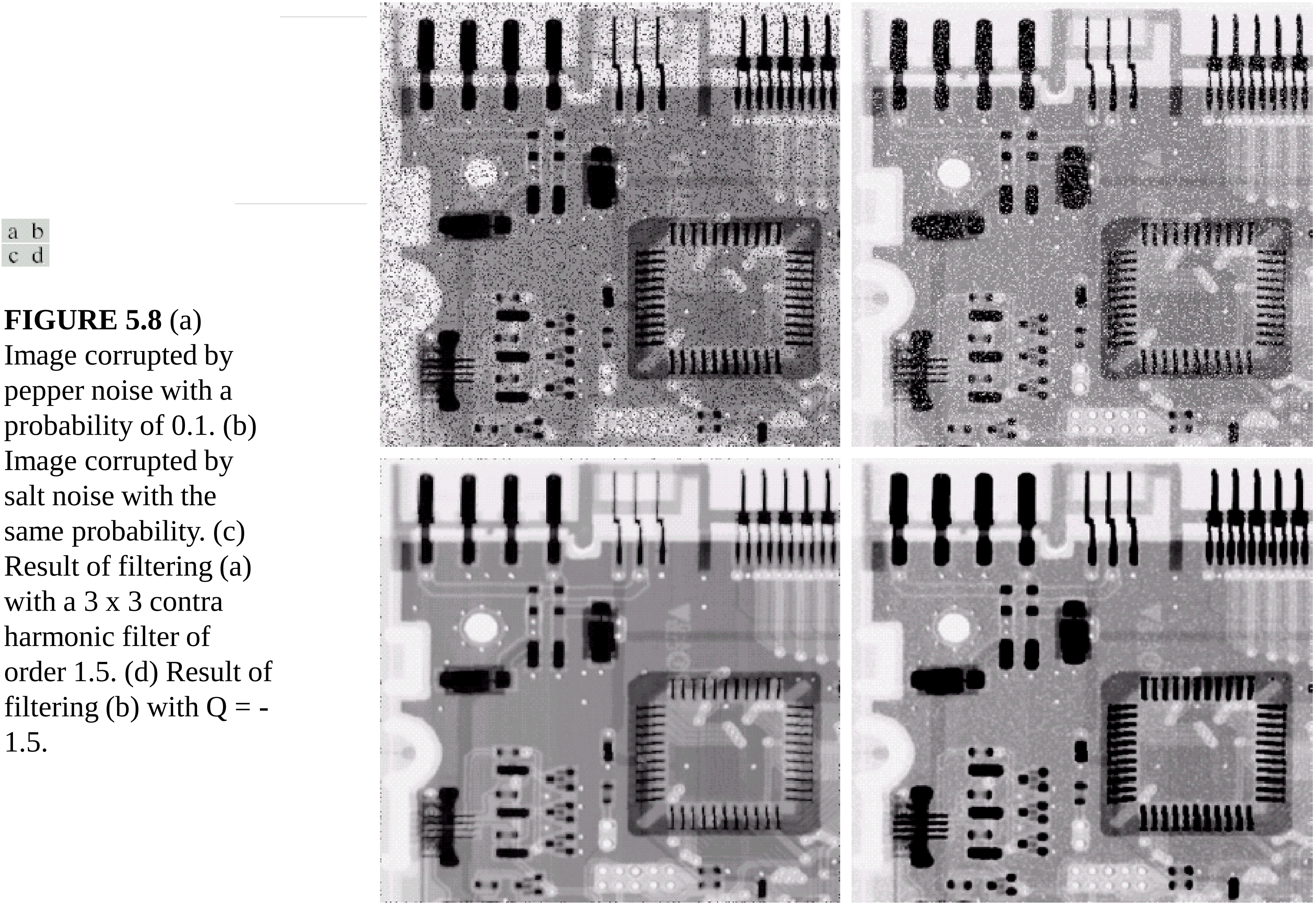
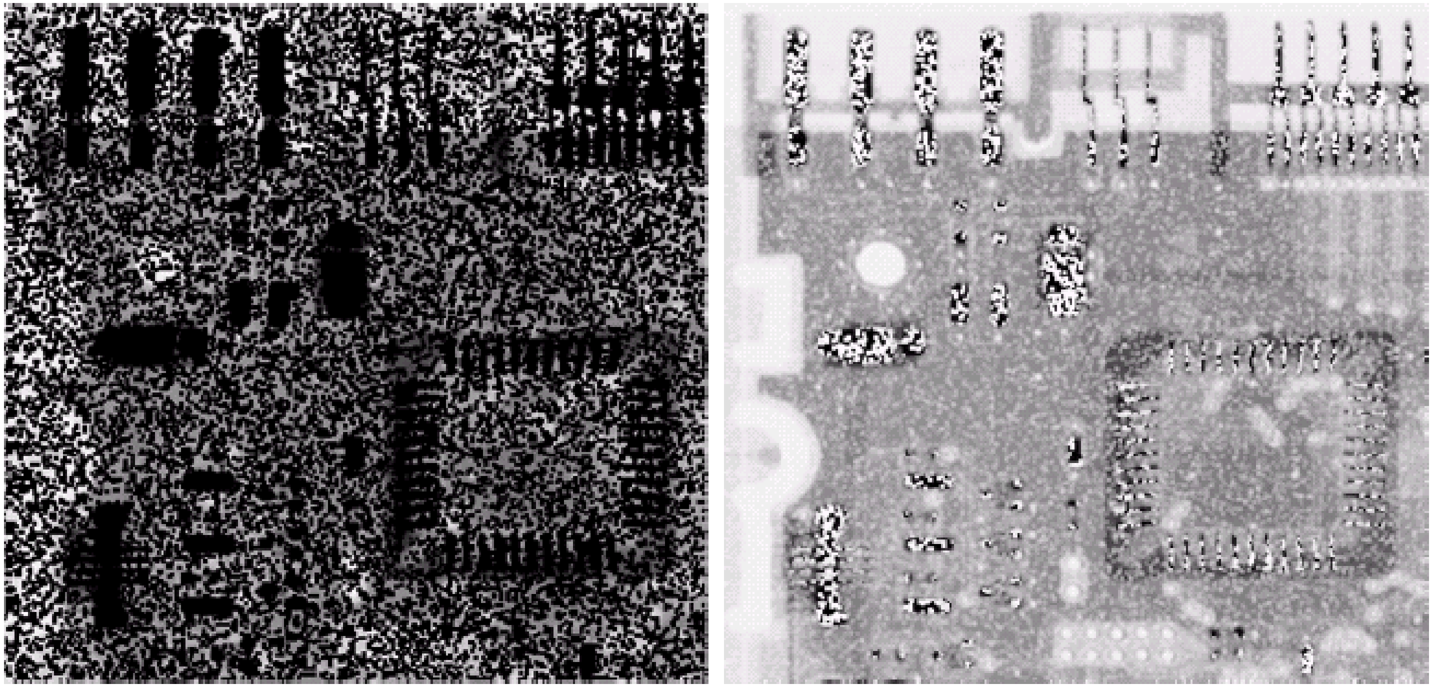


FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 x 3 contra harmonic filter of order 1.5. (d) Result of filtering (b) with Q = -1.5.



a b

FIGURE 5.9
Results of selecting the wrong sign in contraharmonic filtering. (a) result of filtering Fig.5.8 (a) with a contraharmonic filter of size 3 x 3 and Q = -1.5. (b) Result of filtering 5.8 (b) with Q = 1.5.

Order Statistic filters

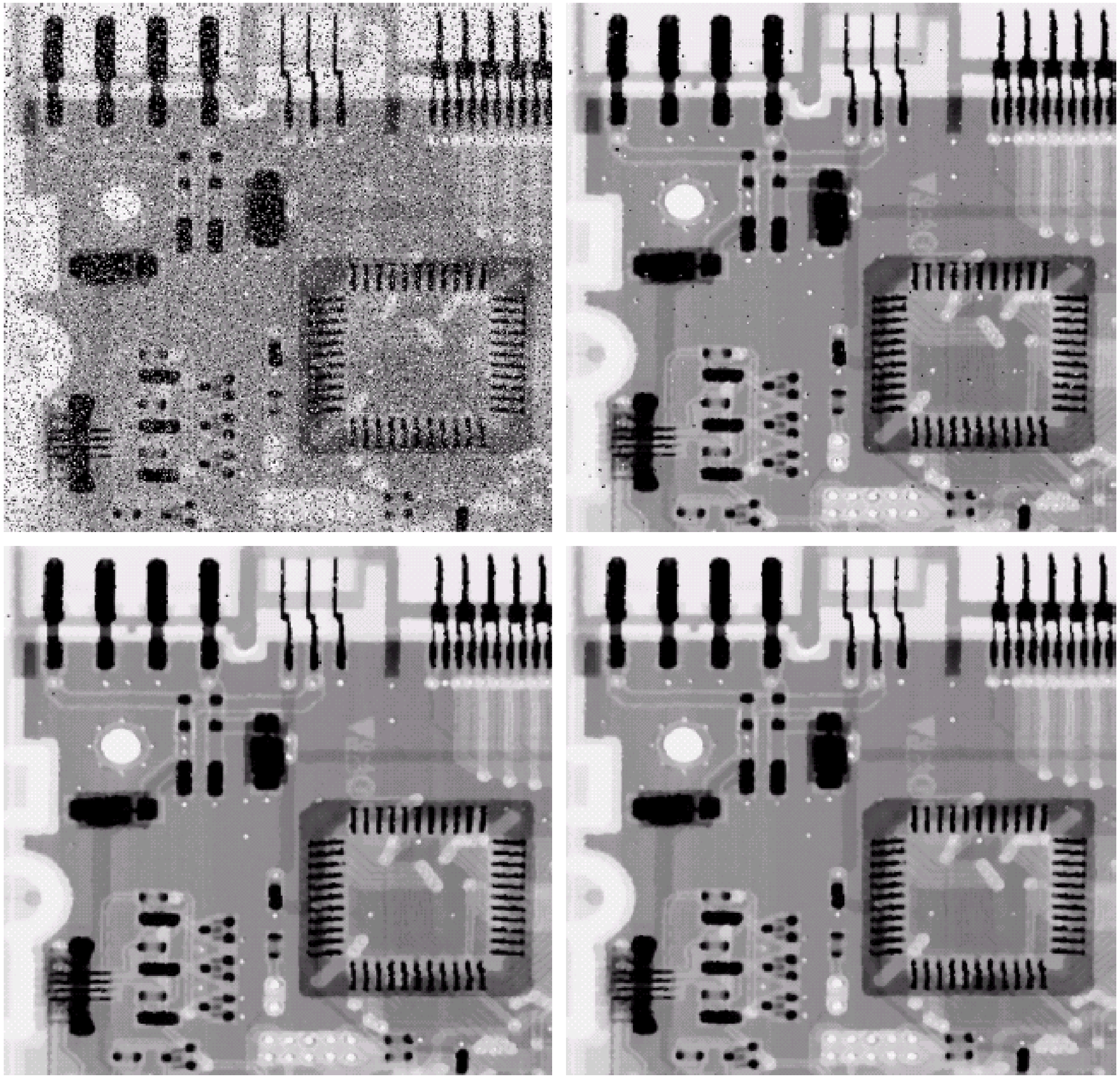
- Order statistic filters are obtained by first ordering (or ranking) the pixel values in a window S_{ab} around a given pixel.

Median Filter

- It replaces the values of a pixel by the median of the grayvalues in a neighborhood S_{ab} of the pixel.

$$\hat{f}(m,n) = \underset{\{s,t\}\in S_{ab}}{median}\{g(s,t)\}$$

- Median filters are particularly suited for impulsive noise. They often result in much less loss of sharp edges in the original image.



a b
c d

FIGURE 5.10
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3 x 3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

Max and Min Filter

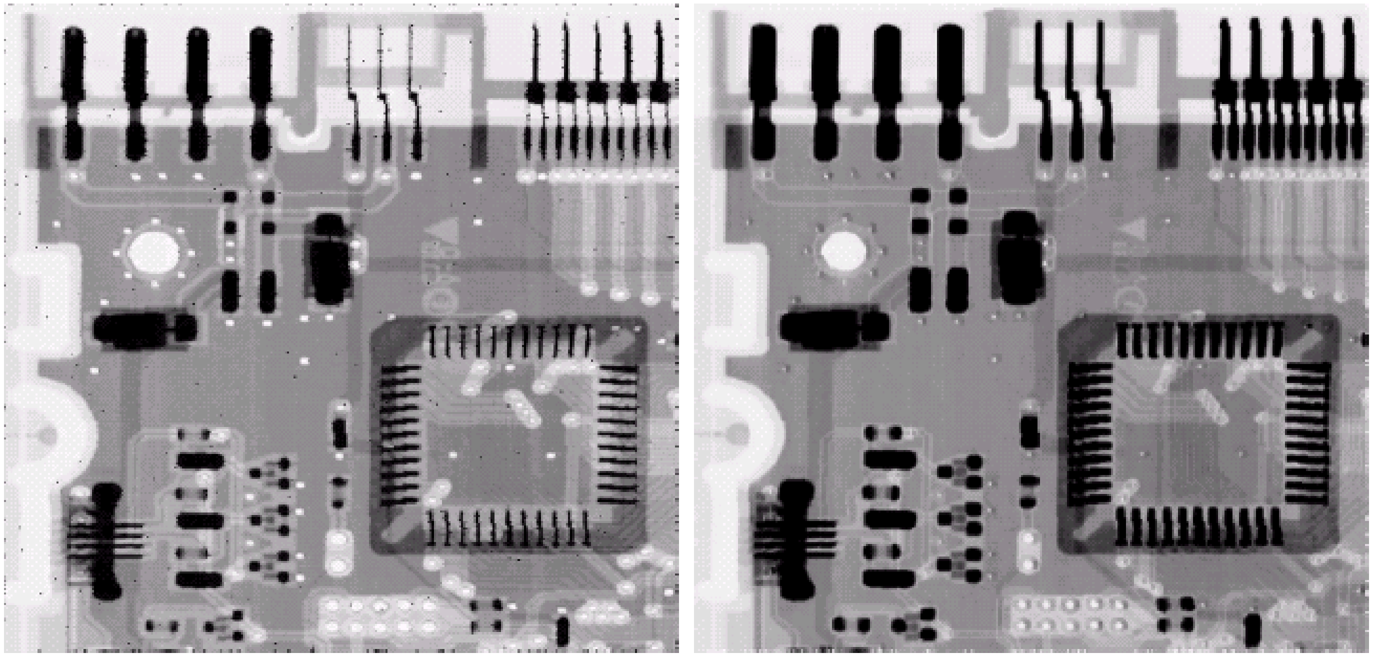
- The Max filter replaces the values of a pixel by the maximum of the grayvalues in a neighborhood S_{ab} of the pixel.

$$\hat{f}(m,n) = \max_{\{s,t\} \in S_{ab}} \{g(s,t)\}$$

- It is used to reduce pepper noise and to find the bright spots in an image.
- The Min filter replaces the values of a pixel by the minimum of the grayvalues in a neighborhood S_{ab} of the pixel.

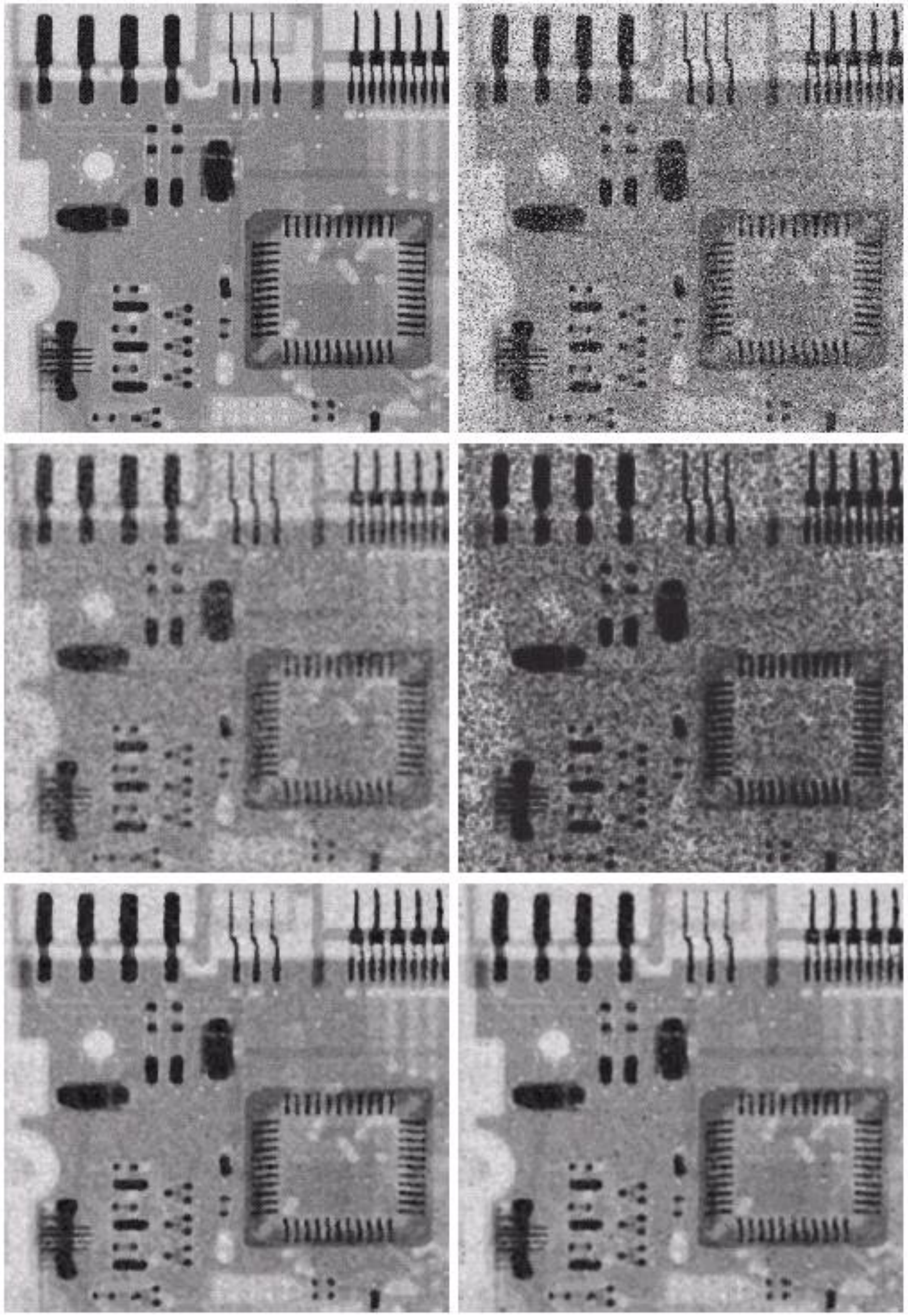
$$\hat{f}(m,n) = \min_{\{s,t\} \in S_{ab}} \{g(s,t)\}$$

- It is used to reduce salt noise and to find the dark spots in an image.
- Usually, the max and min filters are used in conjunction.



a b

FIGURE 5.11
(a) Result of filtering Fig.5.8 (a) with a max filter of size 3 x 3. (b) Result of filtering 5.8 (b) with a min filter of the same size.



a	b
c	d
e	f

FIGURE 5.12
(a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5 x 5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with d = 5.

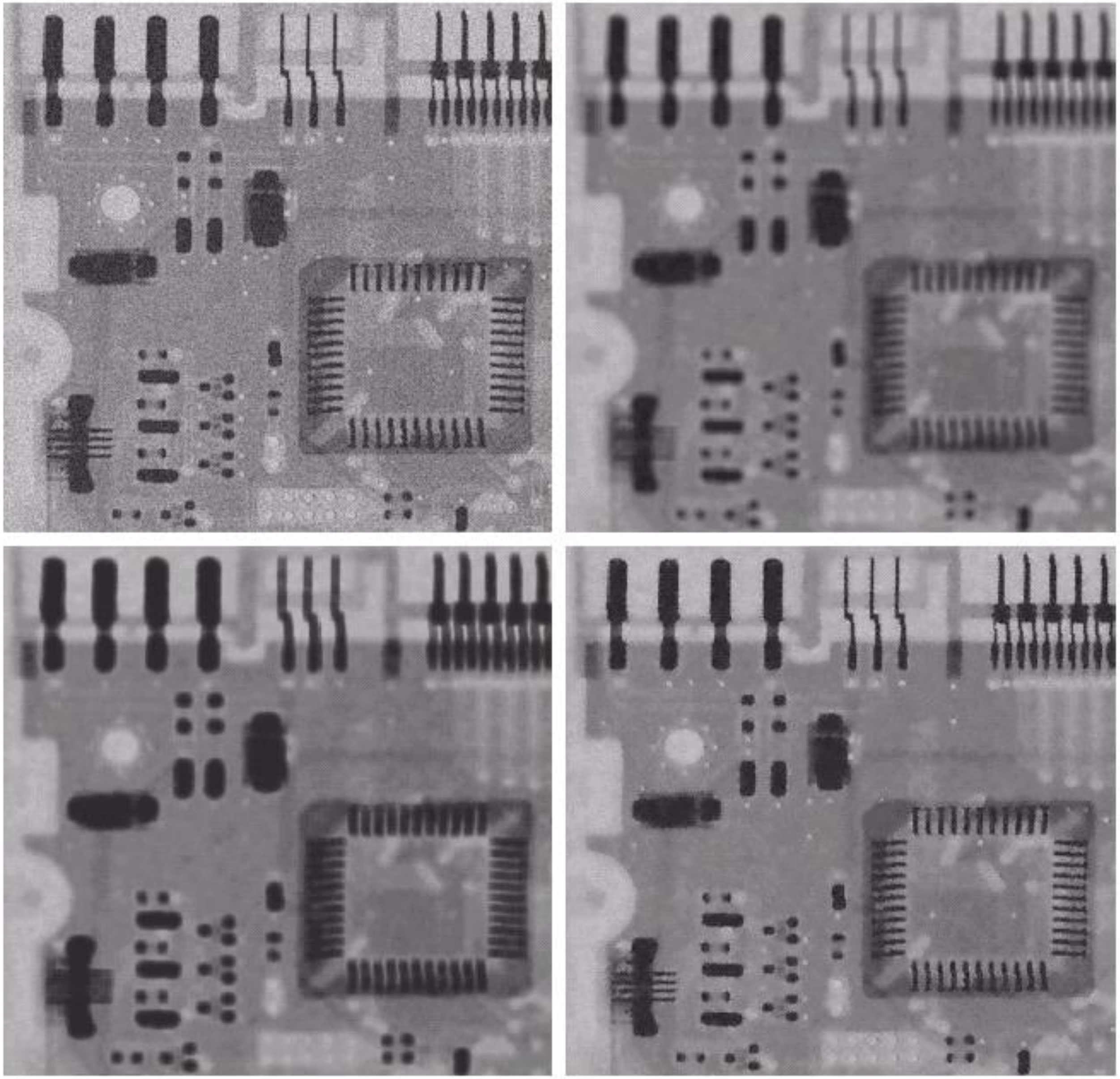
Image restoration
Adaptive local noise reduction filter

- Filter operation is not uniform at all pixel locations but depends on the local characteristics (local mean, local variance) of the observed image.
- Consider an observed image g(m, n) and an a x b window S_{ab}. Let σ_η^2 be the noise variance and m_L(m, n), $\sigma_L^2(m,n)$ be the local mean and variance of g(m, n) over an a x b window around (m, n).
- The adaptive filter is given by:

$$\hat{f}(m,n) = g(m,n) - \frac{\sigma_\eta^2}{\sigma_L^2(m,n)}(g(m,n) - m_L(m,n))$$

- Usually, we need to be careful about the possibility of $\sigma_L^2(m,n) < \sigma_\eta^2$ in which case , we could potentially get a negative output gray value.
- This filter does the following:
  - If $\sigma_\eta^2 = 0$ (or is small), the filter simply returns the value of g(m, n).
  - If the local variance $\sigma_L^2(m,n)$ is high relative to the noise variance σ_η^2 , the filter returns a value close to g(m, n). This usually corresponds to a location associated with edges in the image.
  - If the two variances are roughly equal , the filter does a simple averaging over window S_{ab}.

1. If σ_η^2 is zero, the filter should return simply the value of g at (x, y). This is the trivial, zero-noise case in which g is equal to f at (x, y).
2. If the local variance $\sigma_{S_{xy}}^2$ is high relative to σ_η^2, the filter should return a value close to g at (x, y). A high local variance typically is associated with edges, and these should be preserved.
3. If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in S_{xy}. This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced by averaging.



a b
c d

FIGURE 5.13
(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 x 7.

Image restoration
Adaptive Median filter

Adaptive median filtering can handle noise with probabilities P_s and P_p larger than 0.2.

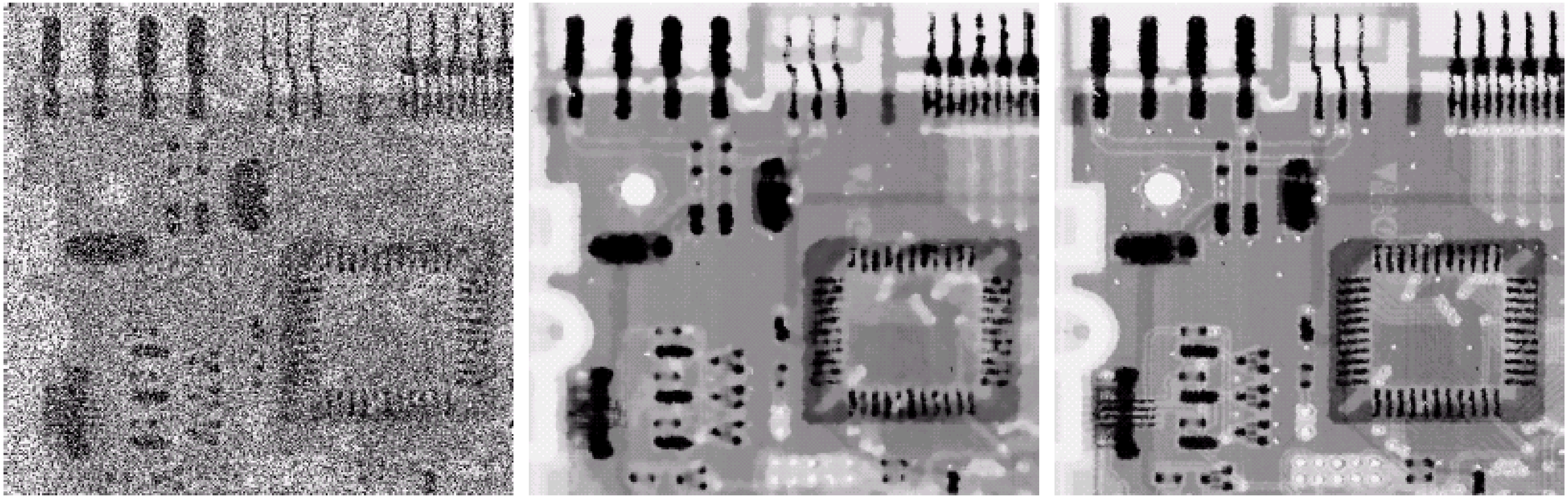
$$z_{\min} = \text{minimum intensity value in } S_{xy}$$
$$z_{\max} = \text{maximum intensity value in } S_{xy}$$
$$z_{\text{med}} = \text{median of intensity values in } S_{xy}$$
$$z_{xy} = \text{intensity at coordinates } (x, y)$$
$$S_{\max} = \text{maximum allowed size of } S_{xy}$$

The adaptive median-filtering algorithm uses two processing levels, denoted level A and level B, at each point (x, y):

Level A: If $z_{\min} < z_{\text{med}} < z_{\max}$, go to Level B
Else, increase the size of S_{xy}
If $S_{xy} \leq S_{\max}$, repeat level A
Else, output z_{med}.

Level B: If $z_{\min} < z_{xy} < z_{\max}$, output z_{xy}
Else output z_{med}.

where S_{xy} and $S_{\max}$ are odd, positive integers greater than 1. Another option in the last step of level A is to output z_{xy} instead of z_{med}. This produces a slightly less blurred result, but can fail to detect salt (pepper) noise embedded in a constant background having the same value as pepper (salt) noise.



a b c

FIGURE 5.14

(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7 x 7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Notch filters

- It is a kind of bandreject/bandpass filter that rejects/passes a very narrow set of frequencies, around a center frequency.
- Due to symmetry considerations, the notches must occur in symmetric pairs about the origin of the frequency plane.
- The transfer function of an ideal notch-reject filter of radius D_0 with center frequency (u_0, v_0) is given by

$$H_{\mathrm{NR}}(u,v) = \prod_{k=1}^{Q} H_k(u,v) H_{-k}(u,v)$$

where $H_k(u,v)$ and $H_{-k}(u,v)$ are highpass filter transfer functions whose centers are at (u_k, v_k) and $(-u_k, -v_k)$, respectively.

$$D_k(u,v) = \left[(u - M/2 - u_k)^2 + (v - N/2 - v_k)^2\right]^{1/2}$$

$$D_{-k}(u,v) = \left[(u - M/2 + u_k)^2 + (v - N/2 + v_k)^2\right]^{1/2}$$

- The transfer function of a Butterworth notch reject filter of order n is given by

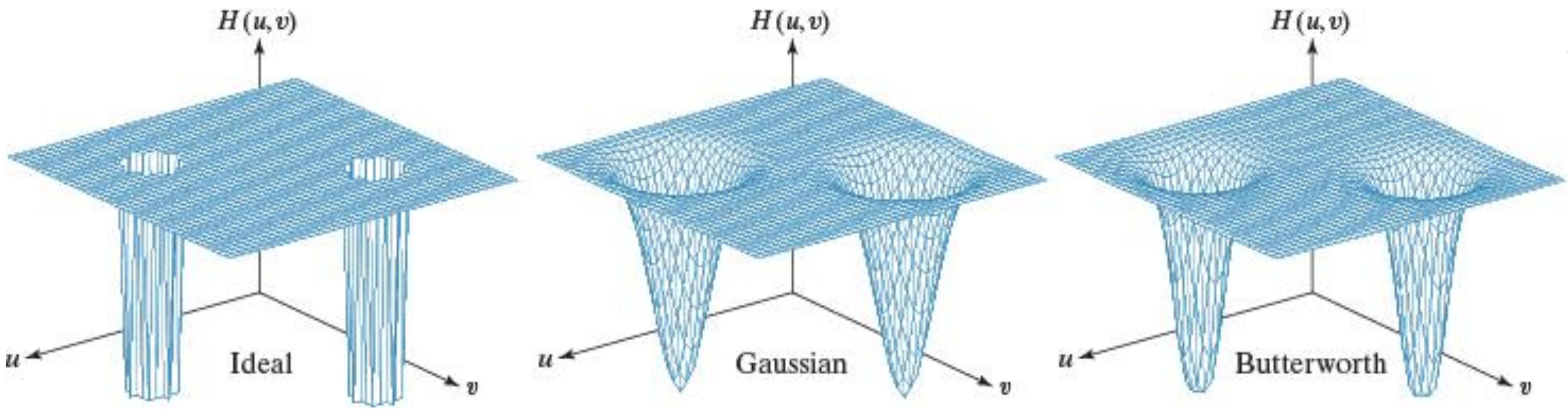
$$H_{\mathrm{NR}}(u,v) = \prod_{k=1}^{3}\left[\frac{1}{1 + [D_{0k}/D_k(u,v)]^n}\right]\left[\frac{1}{1 + [D_{0k}/D_{-k}(u,v)]^n}\right]$$

- A Gaussian notch reject filter is given by

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v) D_2(u,v)}{D_0^2}\right]^2}$$

- A notch pass filter can be obtained from a notch reject filter using:

$$H_{np}(u,v) = 1 - H_{nr}(u,v)$$



a b c

FIGURE 5.15 Perspective plots of (a) ideal, (b) Gaussian, and (c) Butterworth notch reject filter transfer functions.

Example

- Band reject filters are ideally suited for filtering out periodic interference.
- Recall that the Fourier transform of a pure sine or cosine function is just a pair of impulses.
- Therefore the interference is “localized” in the spectral domain and one can easily identify this region and filter it out.

a b
c d

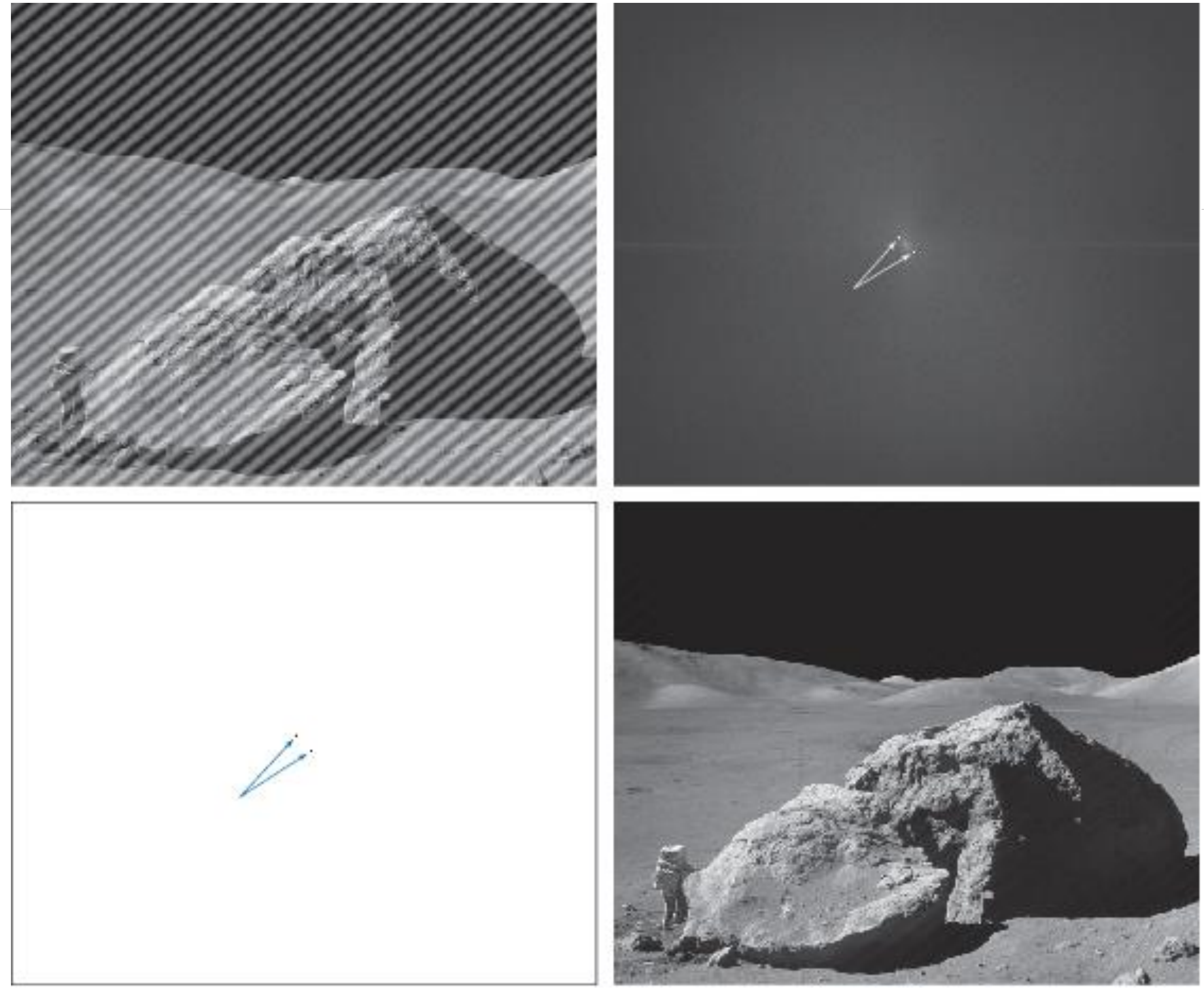


FIGURE 5.16 (a) Image corrupted by sinusoidal interference. (b) Spectrum showing the bursts of energy caused by the interference. (The bursts were enlarged for display purposes.) (c) Notch filter (the radius of the circles is 2 pixels) used to eliminate the energy bursts. (The thin borders are not part of the data.) (d) Result of notch reject filtering. (Original image courtesy of NASA.)

Bandpass filters

- Bandpass filters are the exact opposite of bandreject filters. They pass a band of frequencies, around some frequency, say D_0 (rejecting the rest).

- One can write:

$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

- Bandp ass filter is usually used to isolate components of an image that correspond to a band of frequencies.

- It can also be used to isolate noise interference, so that more detailed analysis of the interference can be performed, independent of the image.

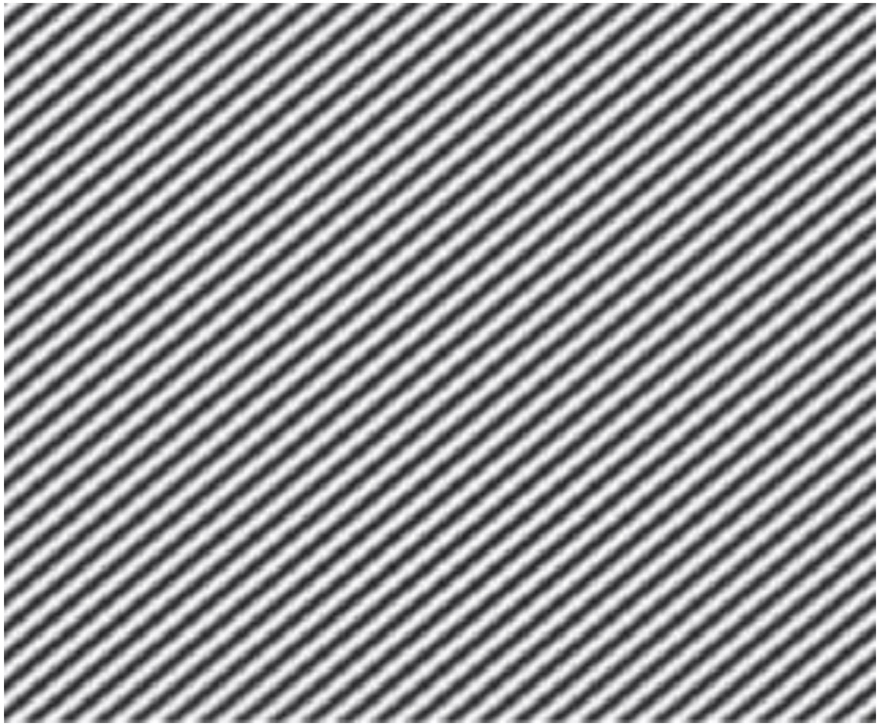
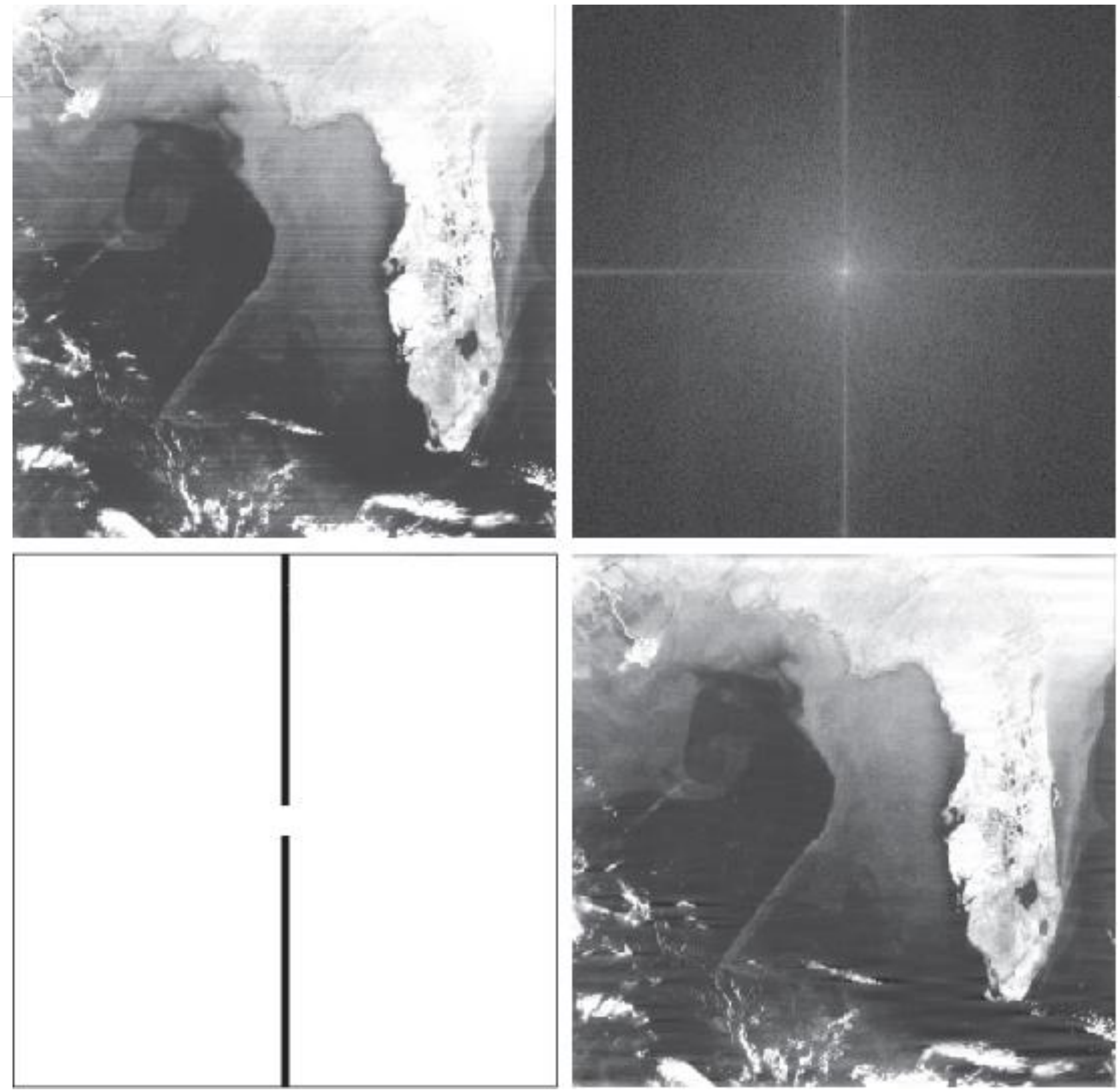


FIGURE 5.17 Sinusoidal pattern extracted from the DFT of Fig. 5.16(a) using a notch pass filter.



a b
c d

FIGURE 5.18
(a) Satellite image of Florida and the Gulf of Mexico. (Note horizontal sensor scan lines.) (b) Spectrum of (a). (c) Notch reject filter transfer function. (The thin black border is not part of the data.) (d) Filtered image. (Original image courtesy of NOAA.)

Optimum Notch Filtering

- When interference patterns are more complicated, the preceding filters tend to reject more image information in an attempt to filter out the noise.
- In this case, we first filter out the noise interference using a notch pass filter:

$$N(u,v) = H(u,v)G(u,v)$$

$$\eta(m,n) = \mathcal{F}^{-1}\{N(u,v)\}$$

- The image $\eta(m, n)$ yields a rough estimate of the interference pattern.
- We can then subtract off a weighted portion of $\eta(m, n)$ from the image $g(m, n)$ to obtain our restored image:

$$\hat{f}(m,n) = g(m,n) - w(m,n)\eta(m,n)$$

- It is possible to design the **weighting function or modulation function w(m,n)** in an optimal fashion. See section 5.4.4 (page 251, 252) of text for details.

One approach is to select $w(x, y)$ so that the variance of $\hat{f}(x, y)$ is minimized over a specified neighborhood of every point (x, y).

Consider a neighborhood S_{xy} of (odd) size $m \times n$, centered on (x, y). The "local" variance of $\hat{f}(x, y)$ at point (x, y) can be estimated using the samples in S_{xy}, as follows:

$$\sigma^2(x, y) = \frac{1}{mn} \sum_{(r,c) \in S_{xy}} [\hat{f}(r, c) - \bar{\hat{f}}]^2$$

where $\bar{\hat{f}}$ is the average value of $\hat{f}$ in neighborhood S_{xy}; that is,

$$\bar{\hat{f}} = \frac{1}{mn} \sum_{(r,c) \in S_{xy}} \hat{f}(r, c)$$

$$\sigma^2(x, y) = \frac{1}{mn} \sum_{(r,c) \in S_{xy}} \left\{ [g(r, c) - w(r, c)\eta(r, c)] - [\bar{g} - \overline{w\eta}] \right\}^2$$

where $\bar{g}$ and $\overline{w\eta}$ denote the average values of g and of the product $w\eta$ in neighborhood S_{xy}, respectively.

To minimize $\sigma^2(x, y)$ with respect to $w(x, y)$ we solve $\dfrac{\partial \sigma^2(x, y)}{\partial w(x, y)} = 0 \Rightarrow w(x, y) = \dfrac{\overline{g\eta} - \bar{g}\,\bar{\eta}}{\overline{\eta^2} - \bar{\eta}^2}$

(new)Chapter 5
Image Restoration

a b

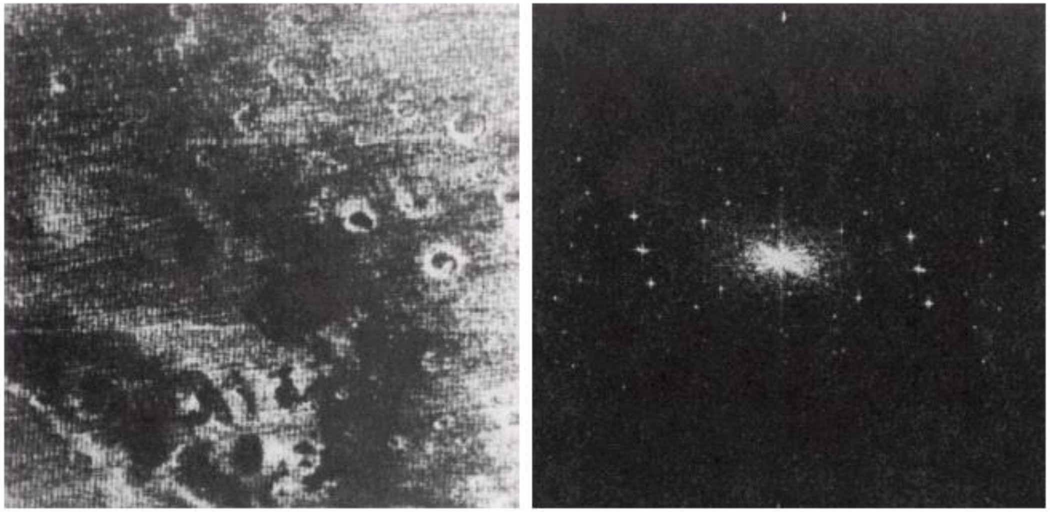


FIGURE 5.20
(a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

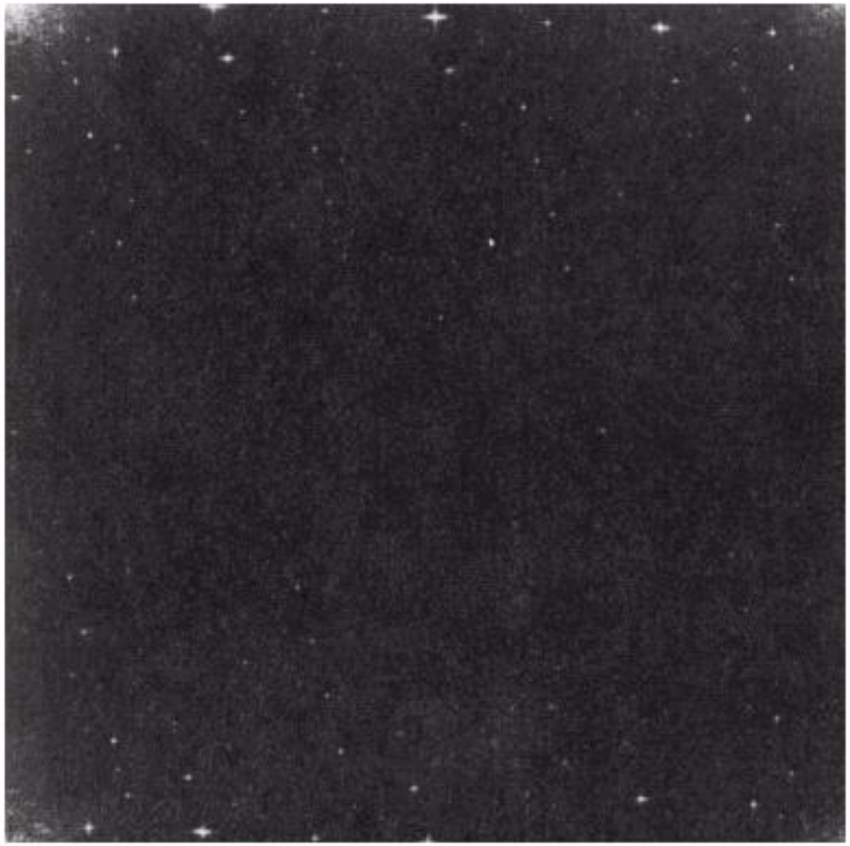
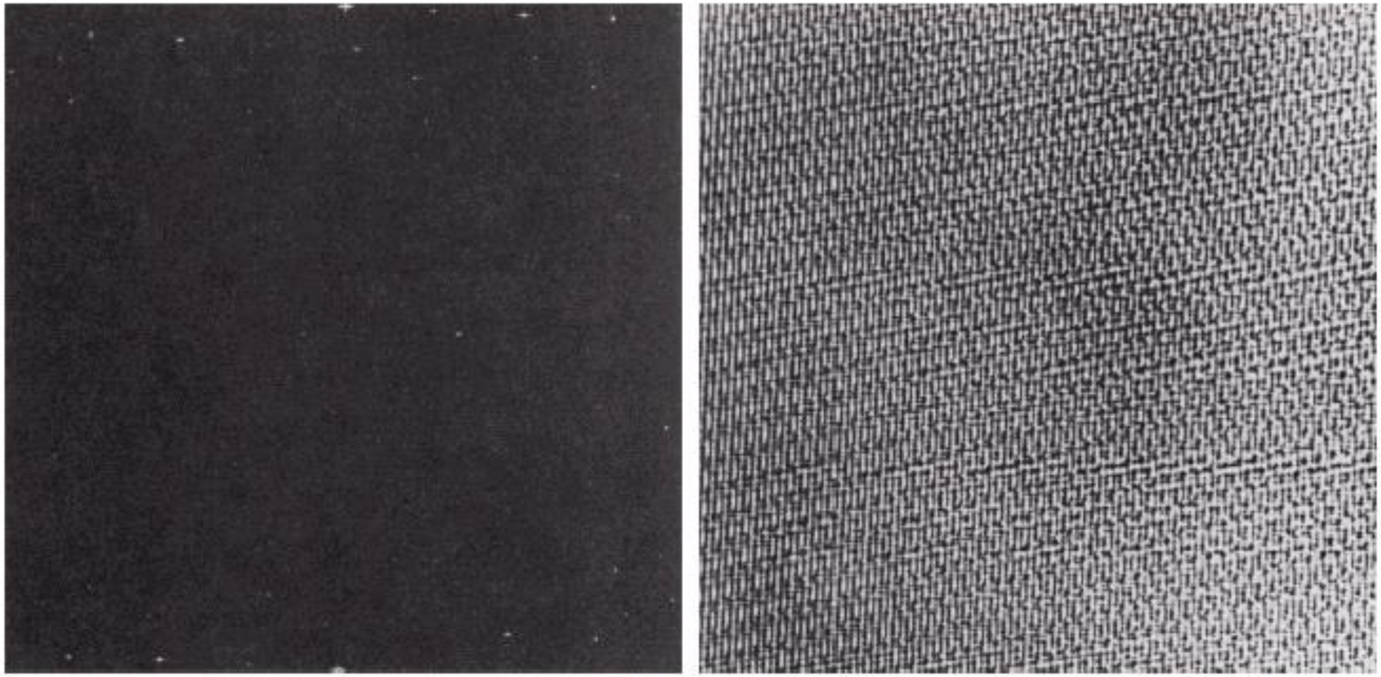


FIGURE 5.21
Fourier spectrum (without shifting) of the image shown in Fig.5.20(a). (Courtesy of NASA.)



a b

FIGURE 5.22 (a) Fourier spectrum of N(u, v), and (b) corresponding noise interference pattern η(x, y). (Courtesy of NASA.)

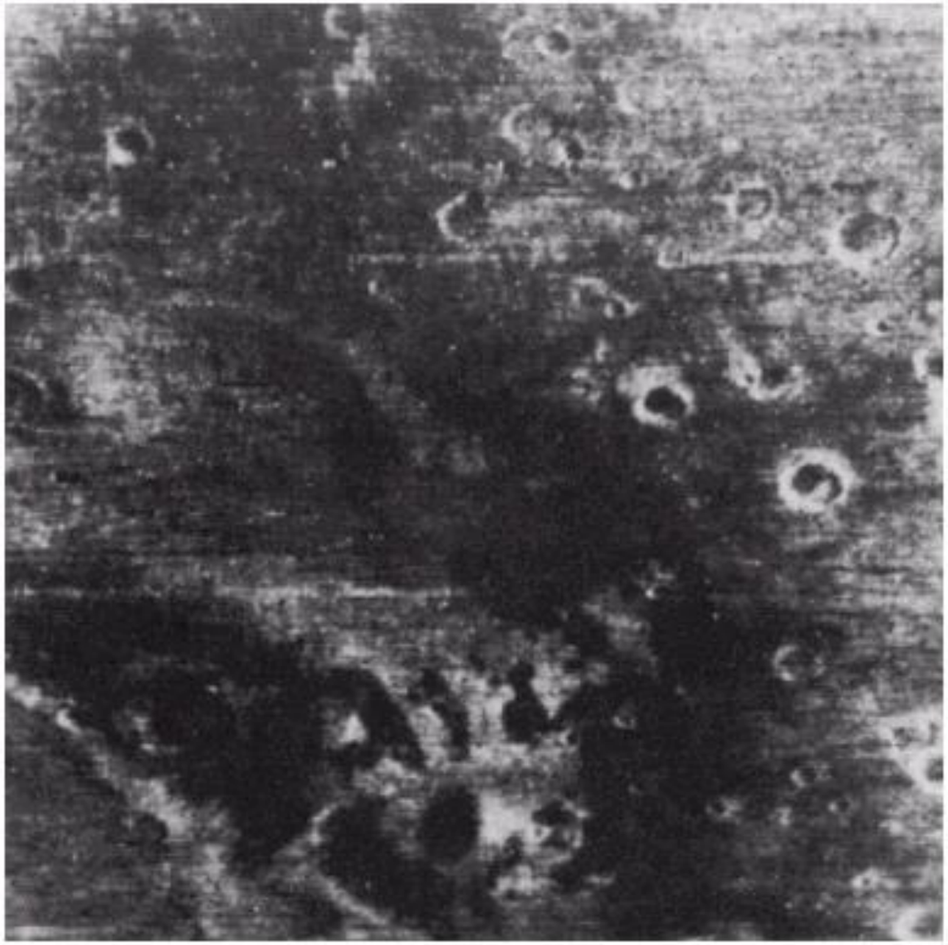


FIGURE 5.23 Processed image. (Courtesy of NASA.)

$$N(u,v) = H(u,v)G(u,v)$$
$$\eta(m,n) = \mathcal{F}^{-1}\{N(u,v)\}$$

$$\Leftrightarrow \quad w(x,y) = \frac{\overline{g\eta} - \bar{g}\,\bar{\eta}}{\overline{\eta^2} - \bar{\eta}^2}$$

$$\Rightarrow \quad \hat{f}(m,n) = g(m,n) - w(m,n)\eta(m,n)$$