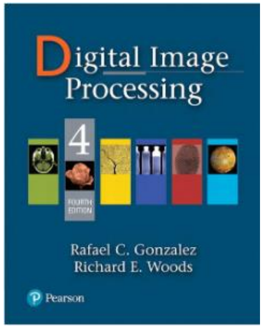


Digital Image Processing

Image Enhancement (I)

Dr. Tun-Wen Pai

1) Concept of image enhancement
2) Intensity Transformation
3) Histogram Transformation
4) Histogram Equalization
5) Histogram Matching (Specification)

Image Enhancement

- To process an image so that output is "visually better" than the input, for a specific application.
- Enhancement is therefore, very much dependent on the particular problem/image at hand.
- Enhancement can be done in either:
  - Spatial domain: operate on the original image

    $$g(x,y) = T[f(x,y)]$$

  - Frequency domain: operate on the DFT of the original image

    $$G(u, v) = T[F(u, v)], \text{ where}$$

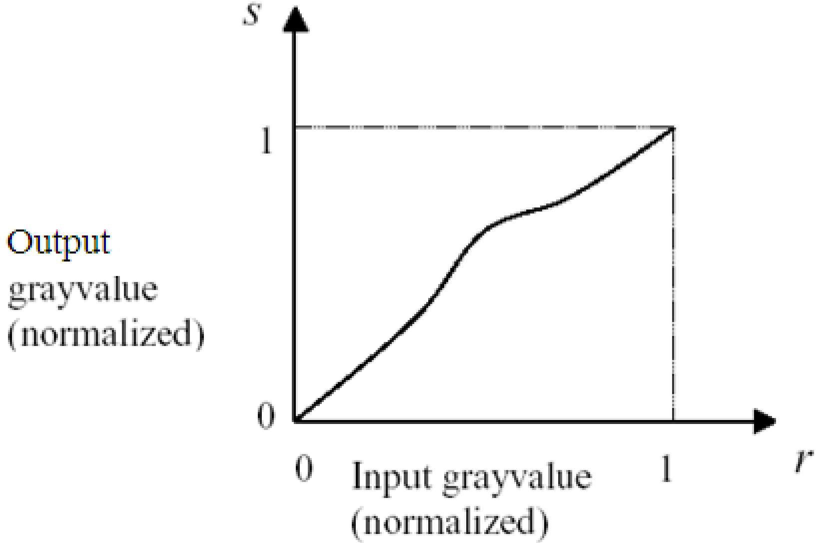
    $$F(u, v) = \mathcal{F}[f(x,y)], \text{ and } G(u, v) = \mathcal{F}[g(x,y)]$$

Image Enhancement Techniques

Point Operations	Mask Operations	Transform Operations	Coloring Operations
•Image Negative •Contrast Stretching •Compression of dynamic range •Graylevel slicing •Image Subtraction •Image Averaging •Histogram operations	•Smoothing operations •Median Filtering •Sharpening operations •Derivative operations •Histogram operations	•Low pass filtering •High pass filtering •Band pass filtering •Homomorphic filtering •Histogram operations	•False coloring •Full color processing

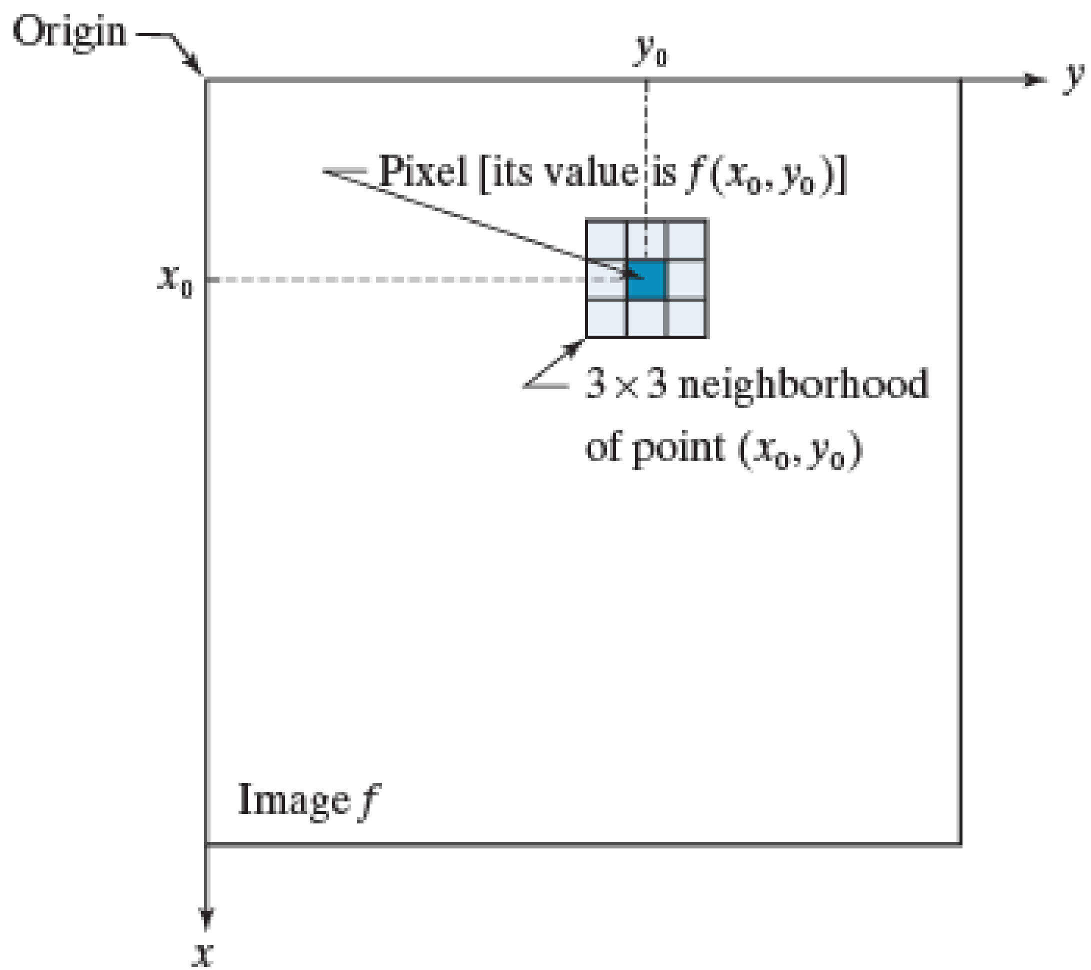
Point Operation

- Output pixel value g(x,y) at pixel (x,y) depends only on the input pixel value at f(x,y) at (x,y) (and not on the neighboring pixel values).
- We normally write s = T(r), where s is the output pixel value and r is the input pixel value.



- T is any increasing function that maps [0, 1] into [0, 1].

FIGURE 3.1 A 3×3 neighborhood about a point (x_0, y_0) in an image. The neighborhood is moved from pixel to pixel in the image to generate an output image. Recall from Chapter 2 that the value of a pixel at location (x_0, y_0) is $f(x_0, y_0)$, the value of the image at that location.



a b

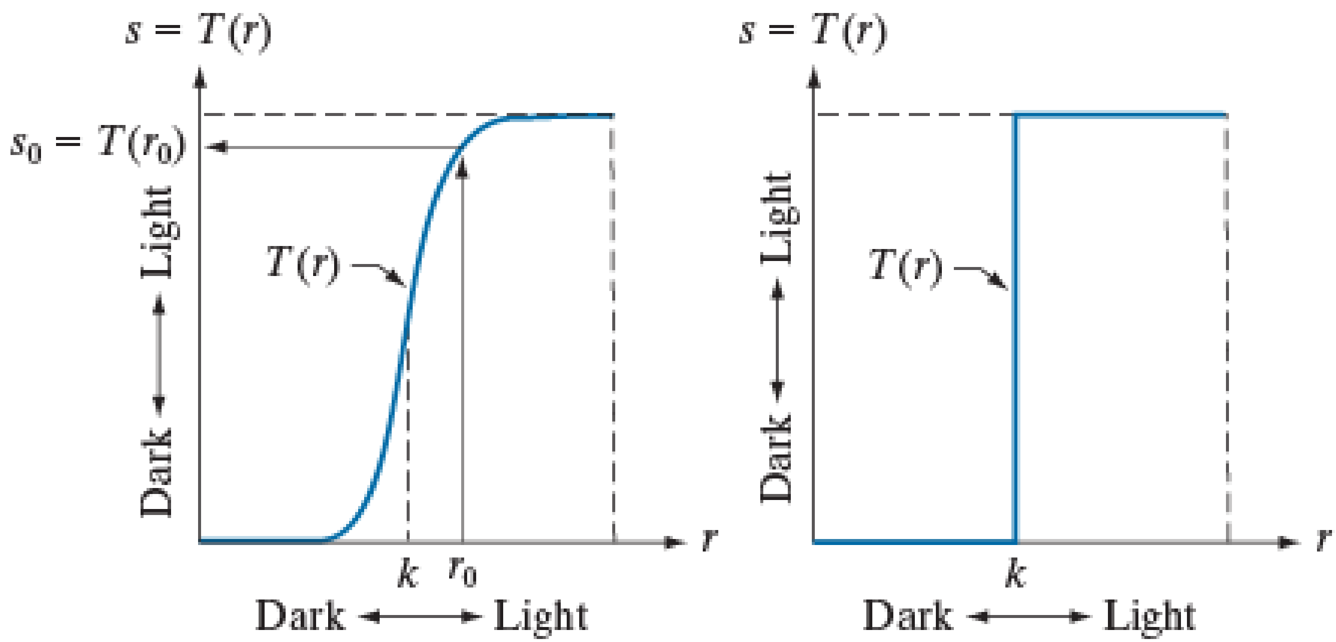


FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

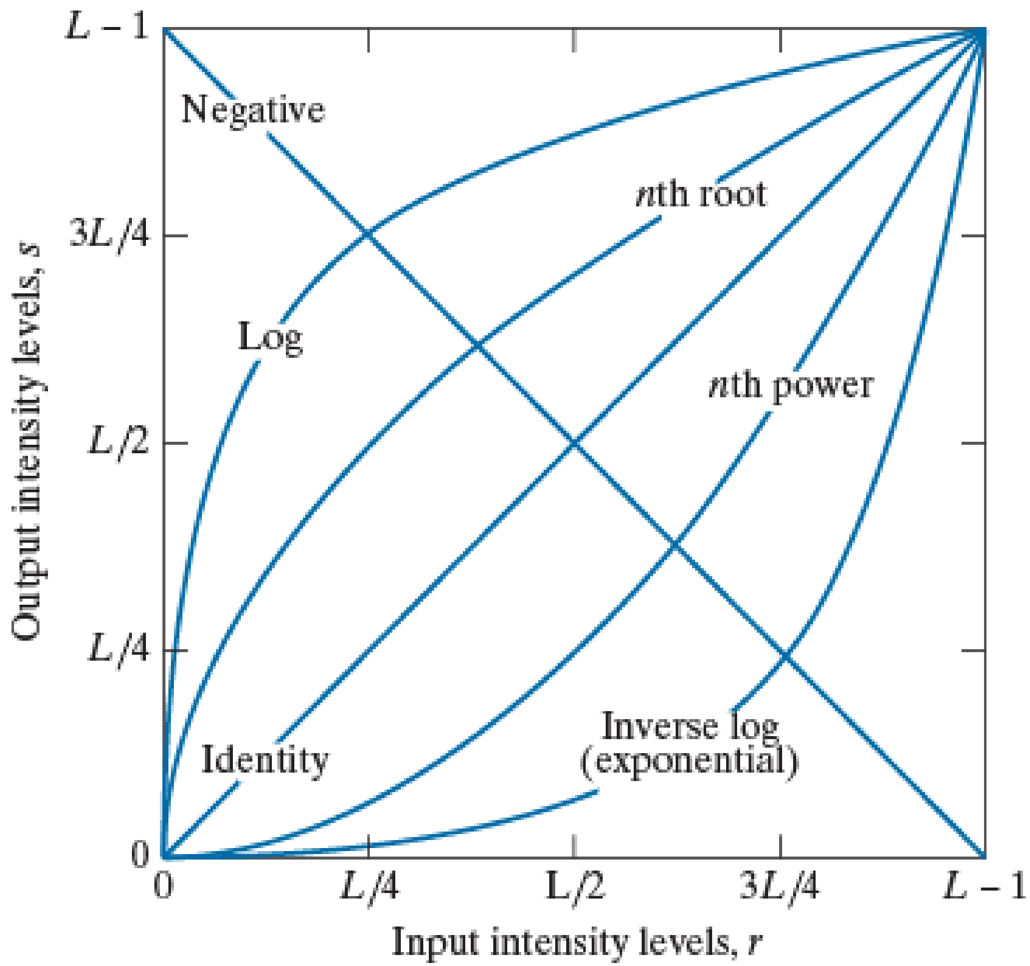
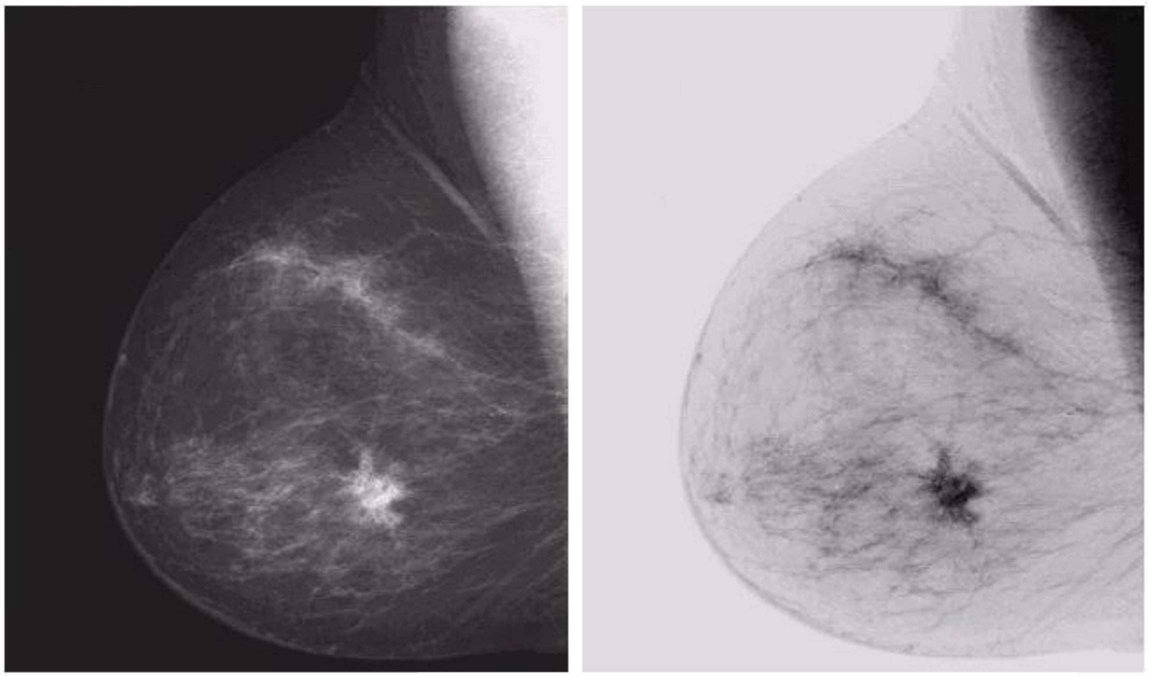


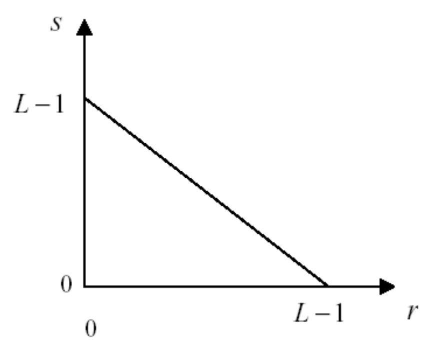
FIGURE 3.3 Some basic intensity transformation functions. Each curve was scaled *independently* so that all curves would fit in the same graph. Our interest here is on the *shapes* of the curves, not on their relative values.

Chapter 3
Image Enhancement in the Spatial Domain



a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)



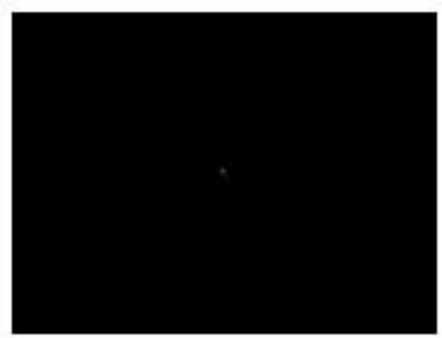
$$T(r) = s = L - 1 - r, \quad L : \text{max. grayvalue}$$

Comprssion of Dynamic Range

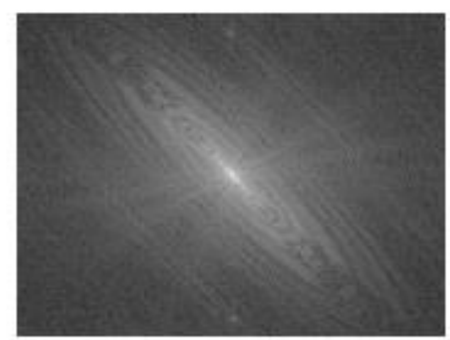
- When the dynamic range of the input grayvlauels is large compared to that of the display, we need to “compress” the grayvalue range --- example: Fourier transform magnitude.
- Typically we use a log scale.

$$s = T(r) = c\log(1 + |r|)$$

Saturn Image



Mag. Spectrum



Mag. Spectrum in log scale

a b

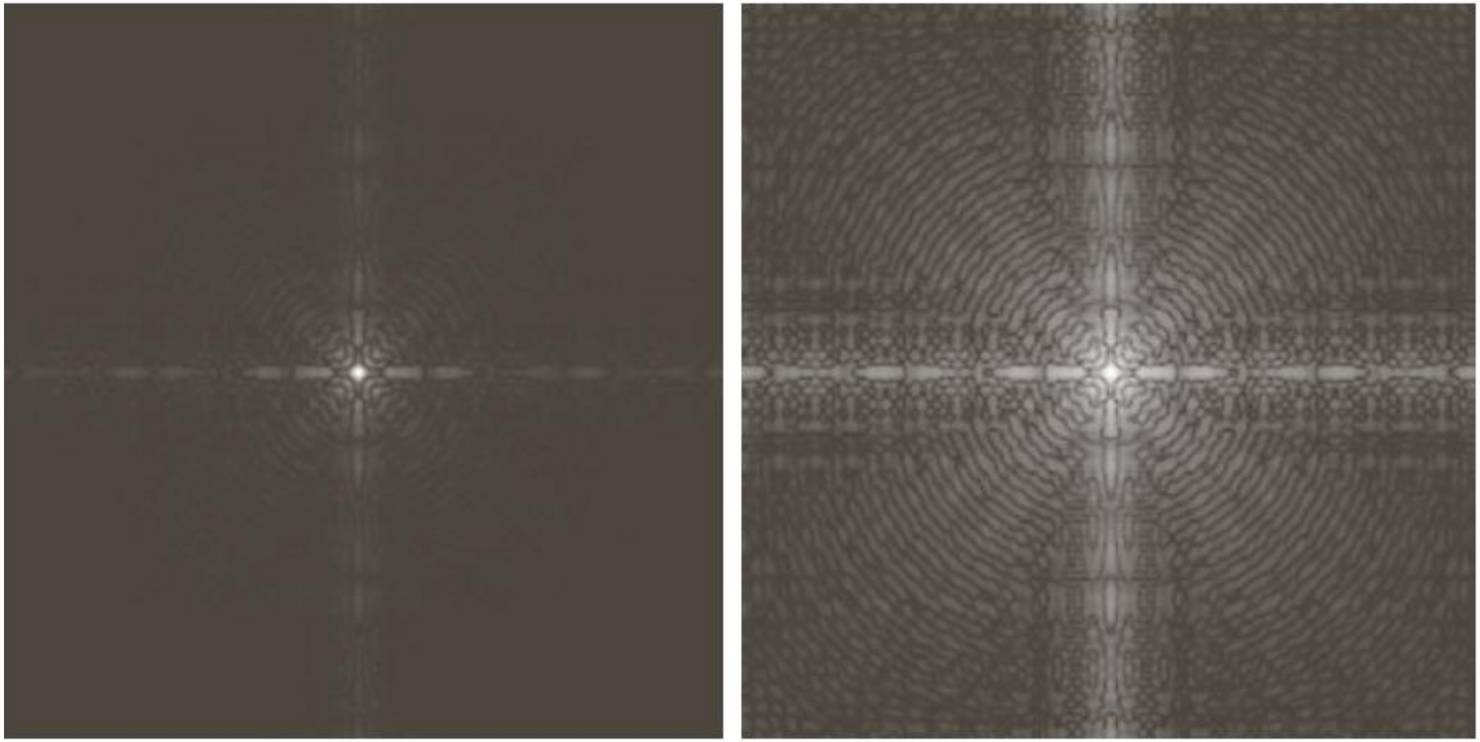


FIGURE 3.5 (a) Fourier spectrum displayed as a grayscale image. (b) Result of applying the log transformation in Eq. (3-4) with $c = 1$. Both images are scaled to the range [0, 255].

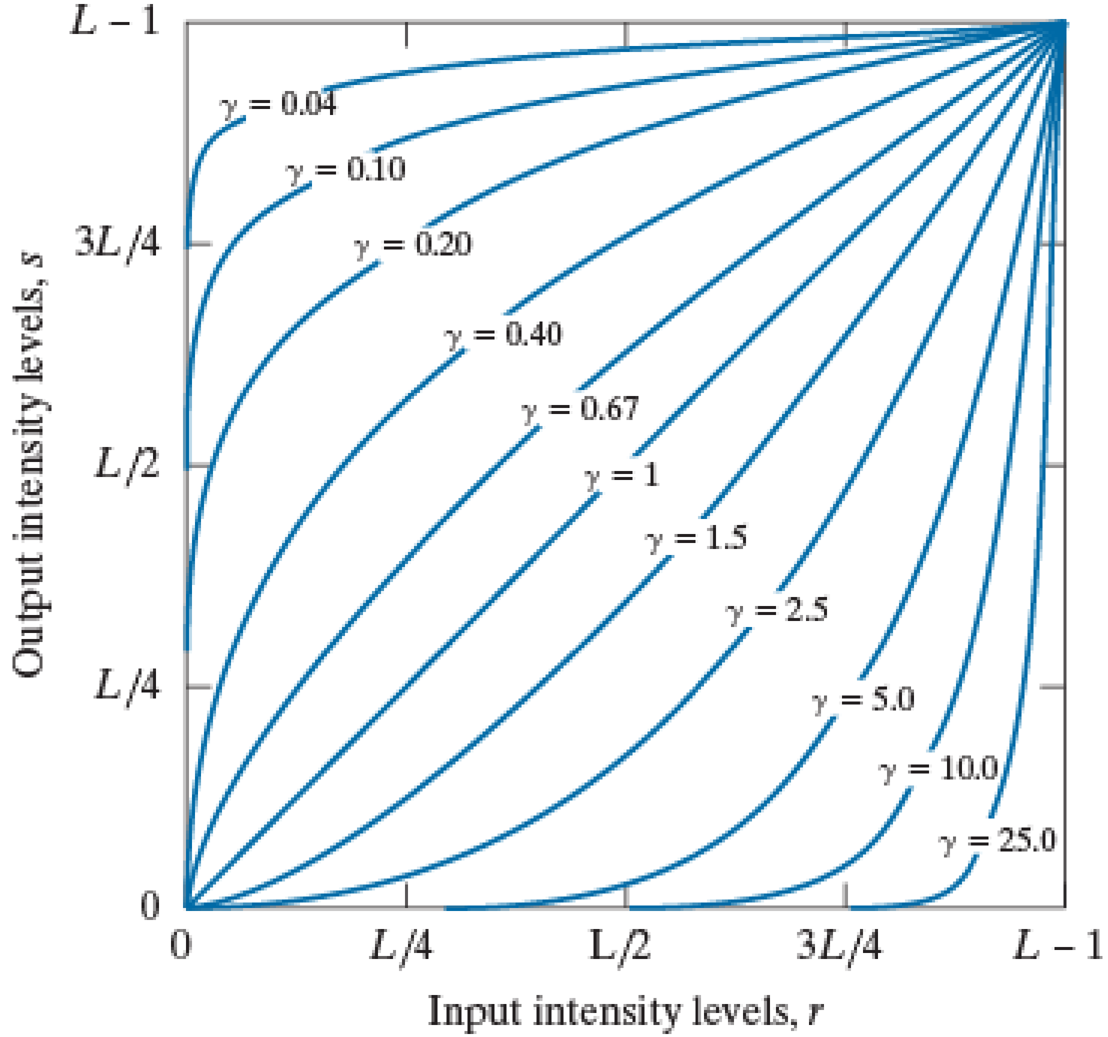
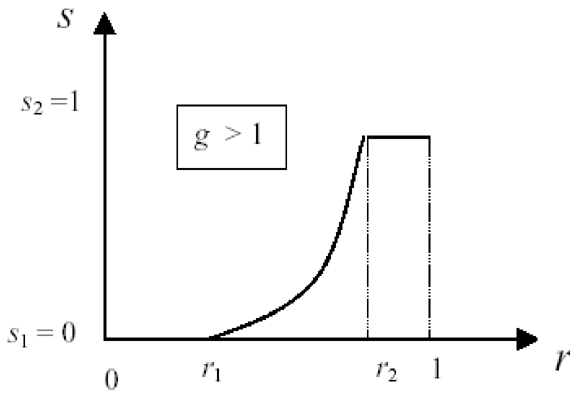


FIGURE 3.6 Plots of the gamma equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). Each curve was scaled *independently* so that all curves would fit in the same graph. Our interest here is on the *shapes* of the curves, not on their relative values.

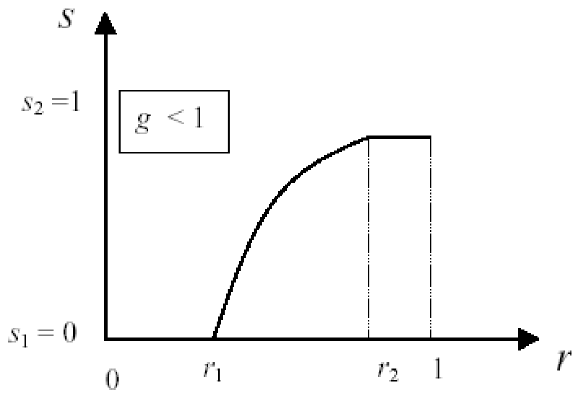
Gamma correction:

$$s_1 = 0, \quad s_2 = 1, \text{ and}$$

$$T(r) = \begin{Bmatrix} 0, \quad r < r_1 \\ \left(\dfrac{r - r_1}{r_2 - r_1}\right)^g, r_1 \le r \le r_2 \\ 1, \quad r > r_2 \end{Bmatrix}$$



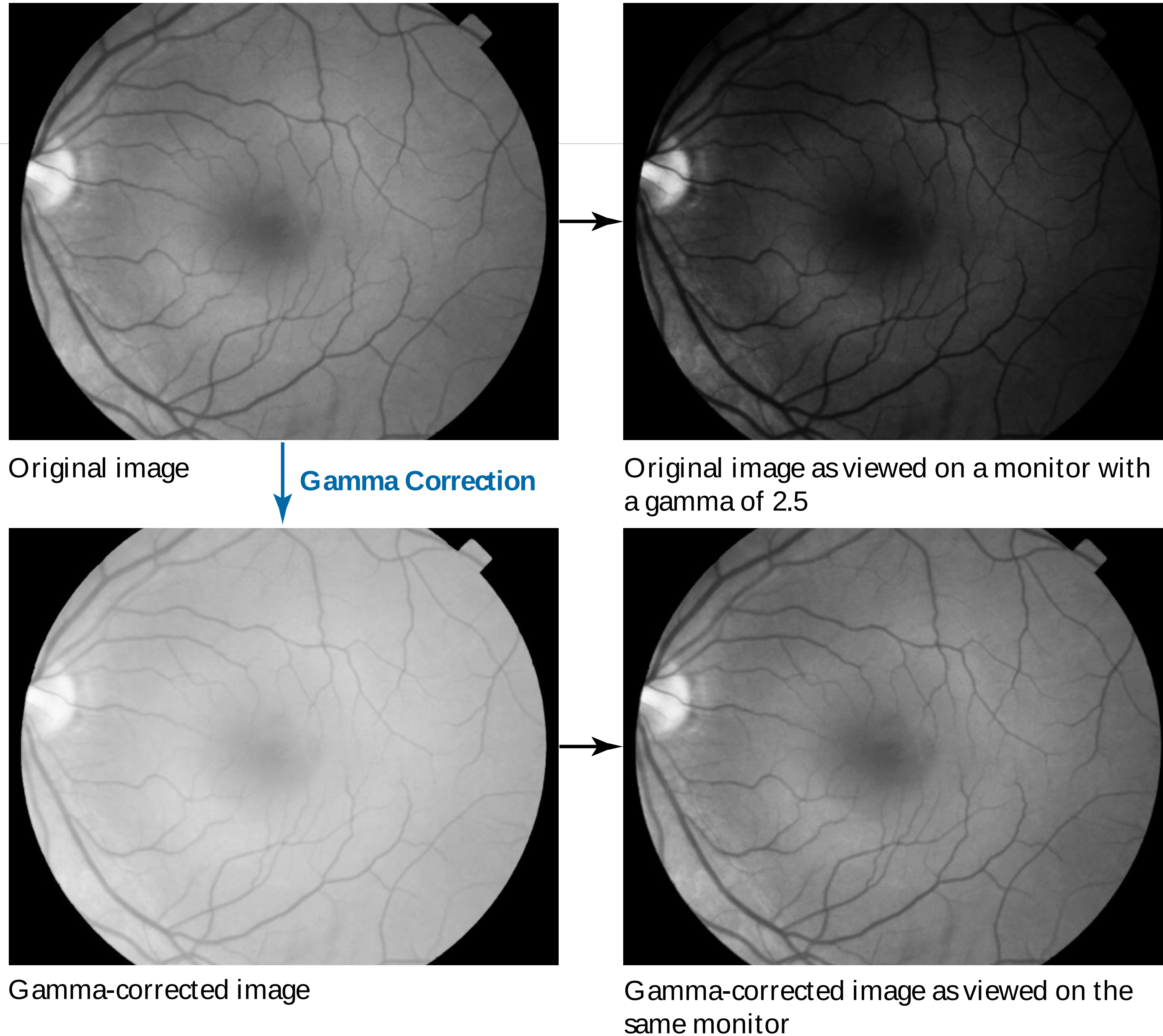
Output Image is “darker”

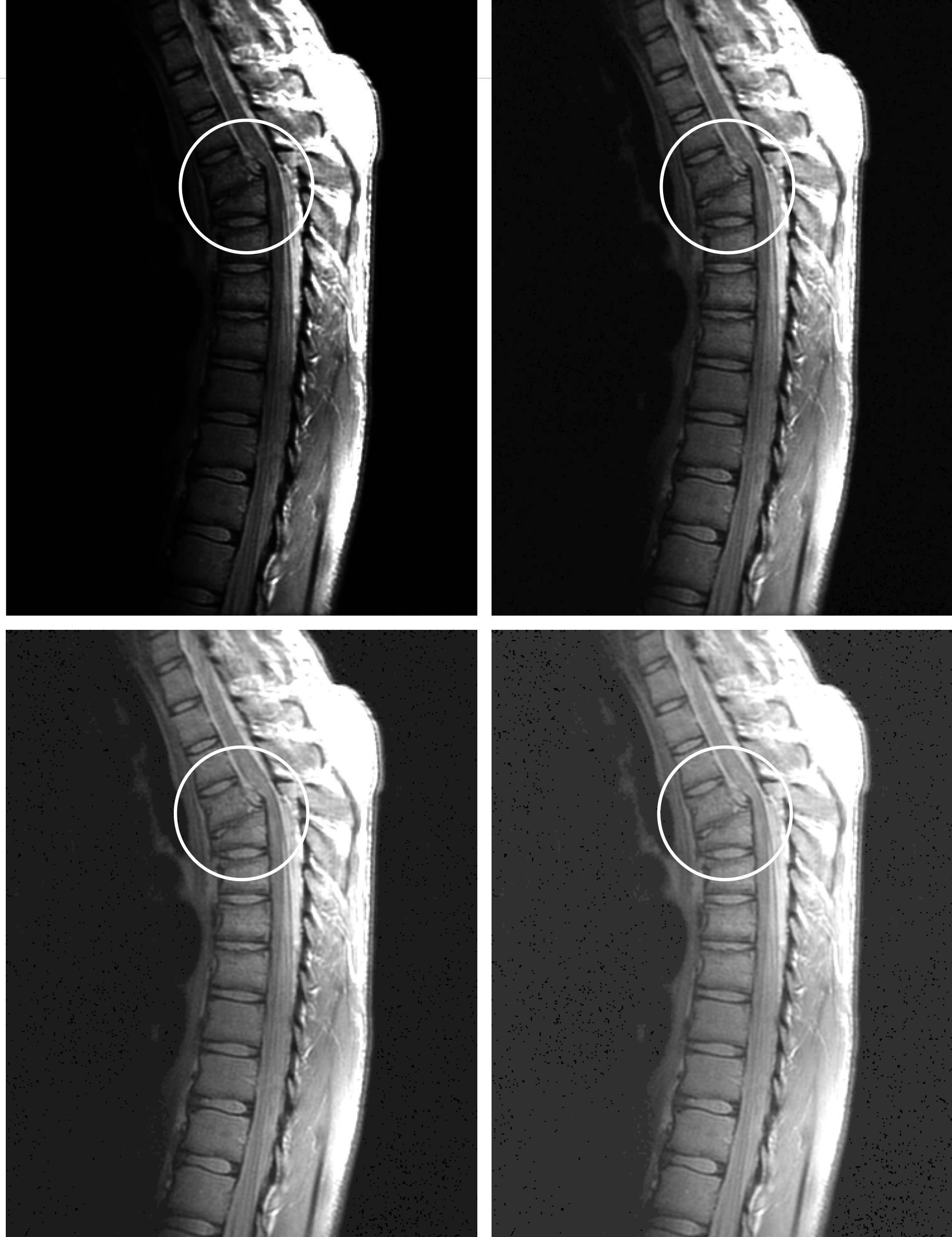


Output Image is “brighter”

a b
c d

FIGURE 3.7
(a) Image of a human retina.
(b) Image as as it appears on a monitor with a gamma setting of 2.5 (note the darkness).
(c) Gamma-corrected image.
(d) Corrected image, as it appears on the same monitor (compare with the original image).
(Image (a) courtesy of the National Eye Institute, NIH)





a b
c d

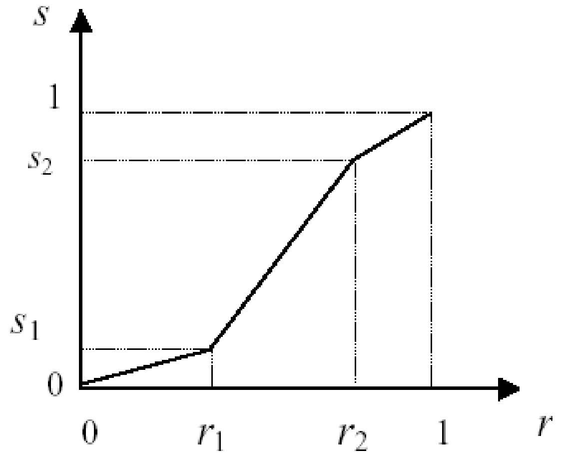
FIGURE 3.8
(a) Magnetic resonance image (MRI) of a fractured human spine (the region of the fracture is enclosed by the circle). (b)–(d) Results of applying the transformation in Eq. (3-5) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3-5) with $\gamma = 3.0, 4.0$, and 5.0, respectively. ($c = 1$ in all cases.) (Original image courtesy of NASA.)

Contrast Stretching

- Increase the dynamic range of grayvalues in the input image.
- Suppose you are interested in stretching the input intensity values in the interval $[r_1, r_2]$:



- Note that $(r_2 - r_1) < (s_2 - s_1)$. The grayvalues in the range $[r_1, r_2]$ is stretched into the range $[s_1, s_2]$
- **Special cases:**
  - **Thresholding or binarization**

$$r_1 = r_2,\ s_1 = 0 \text{ and } s_2 = 1$$

a b
c d

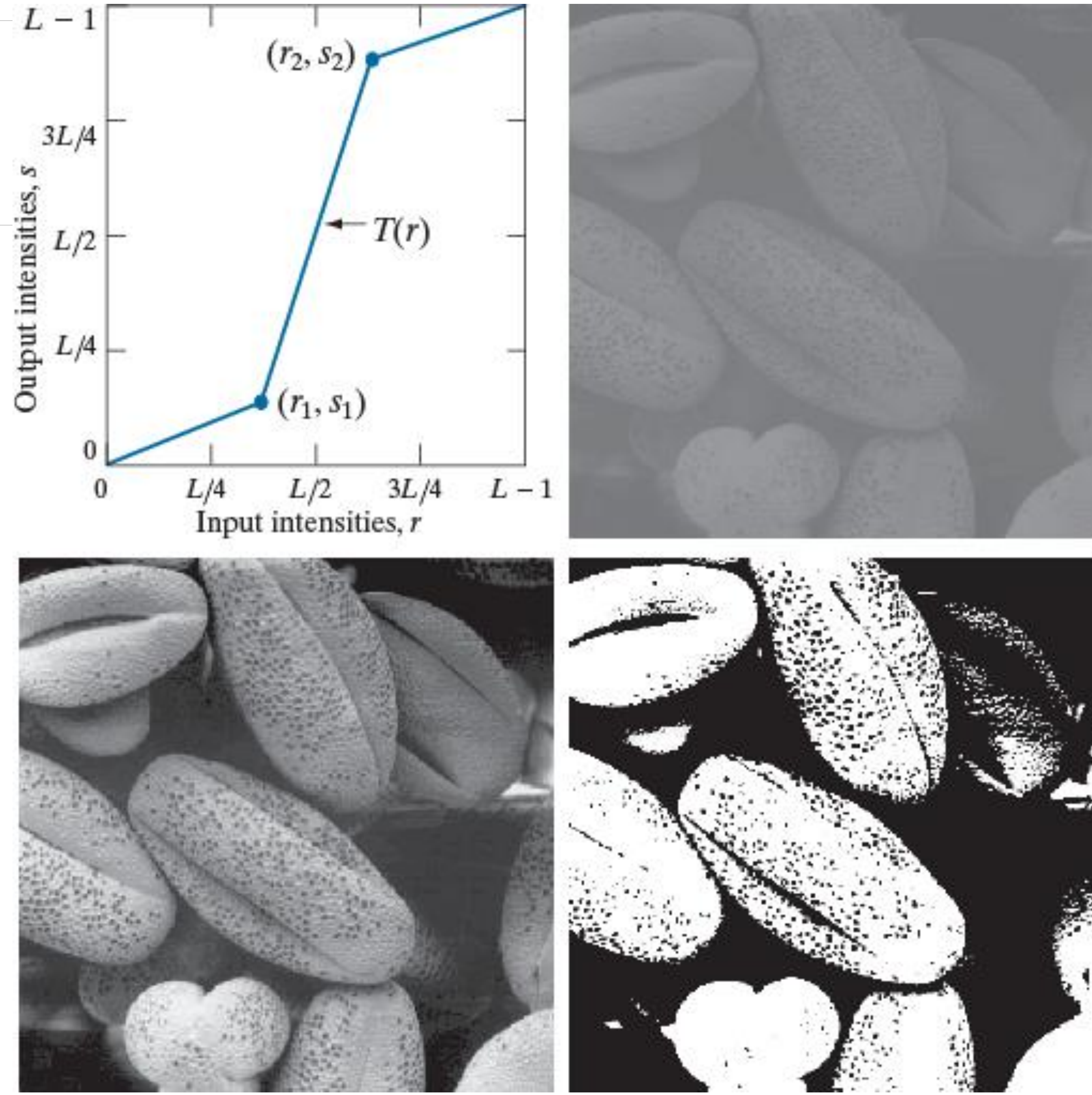
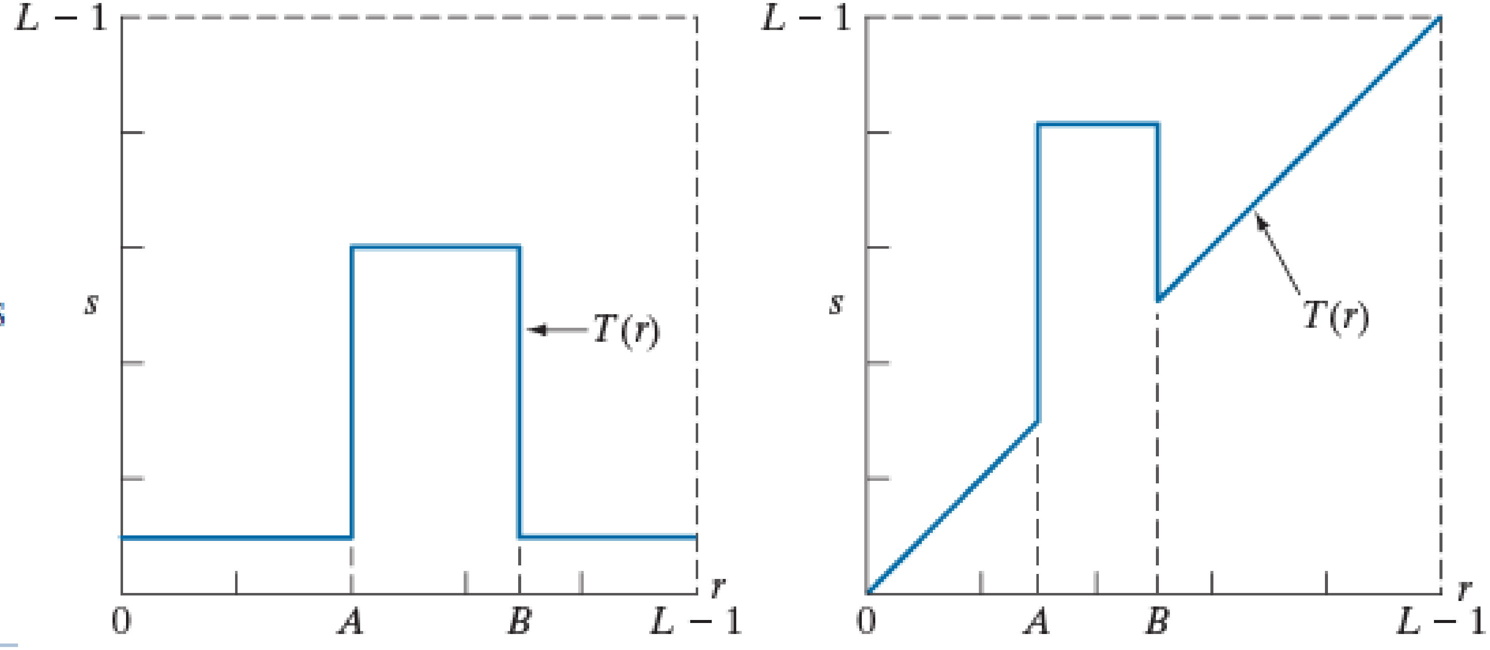
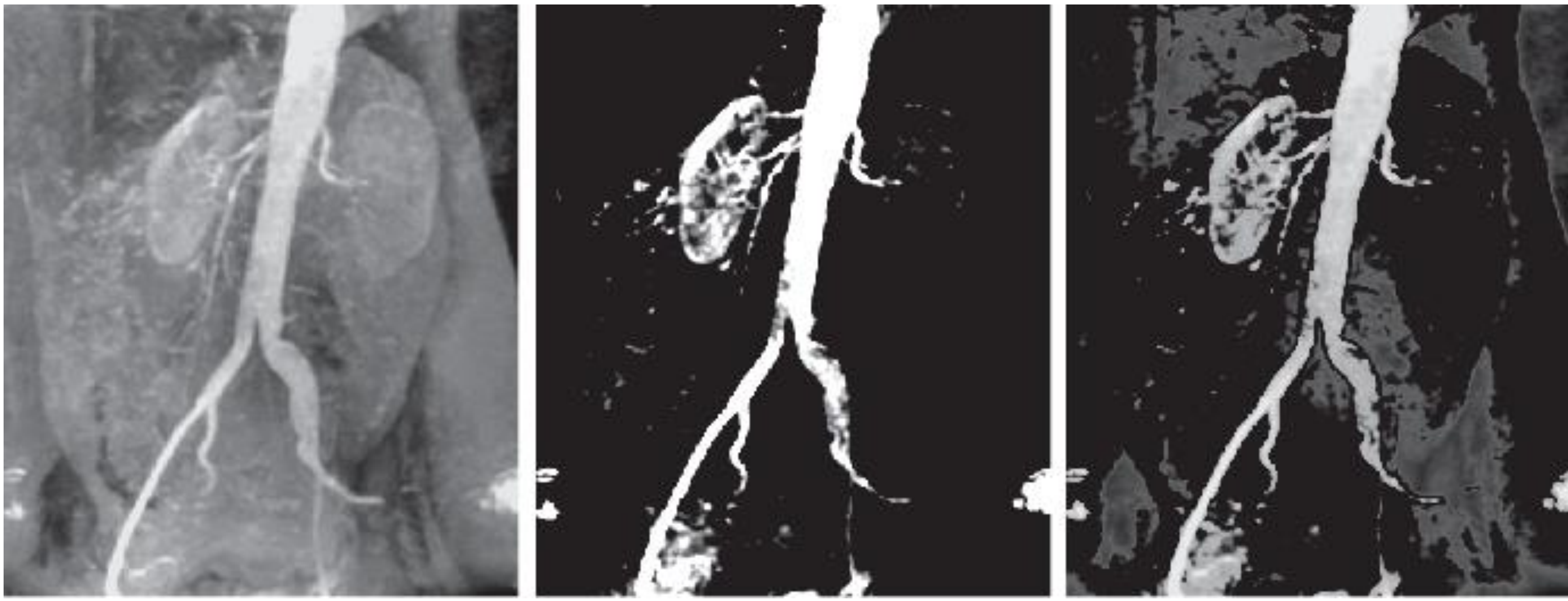


FIGURE 3.10 Contrast stretching. (a) Piecewise linear transformation function. (b) A low-contrast electron microscope image of pollen, magnified 700 times. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

a b

FIGURE 3.11
(a) This transformation function highlights range $[A, B]$ and reduces all other intensities to a lower level. (b) This function highlights range $[A, B]$ and leaves other intensities unchanged.





a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected range set near black, so that the grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

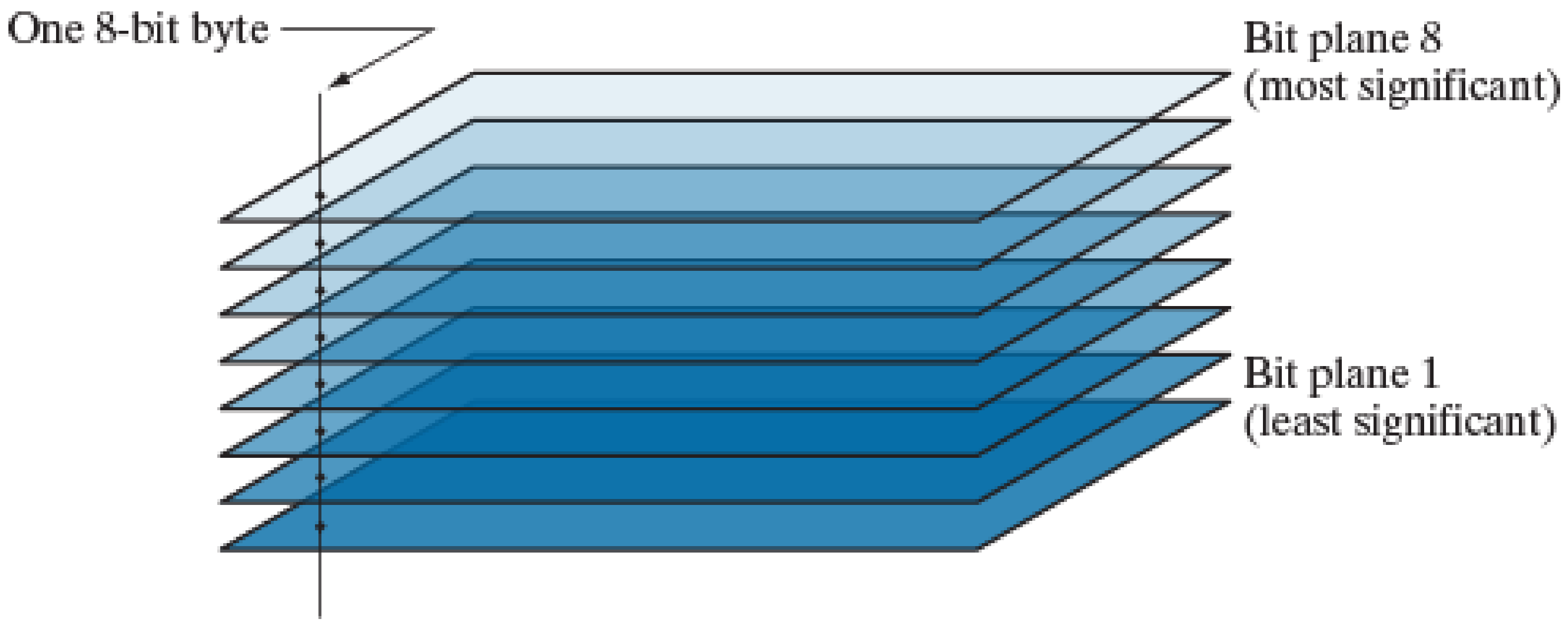
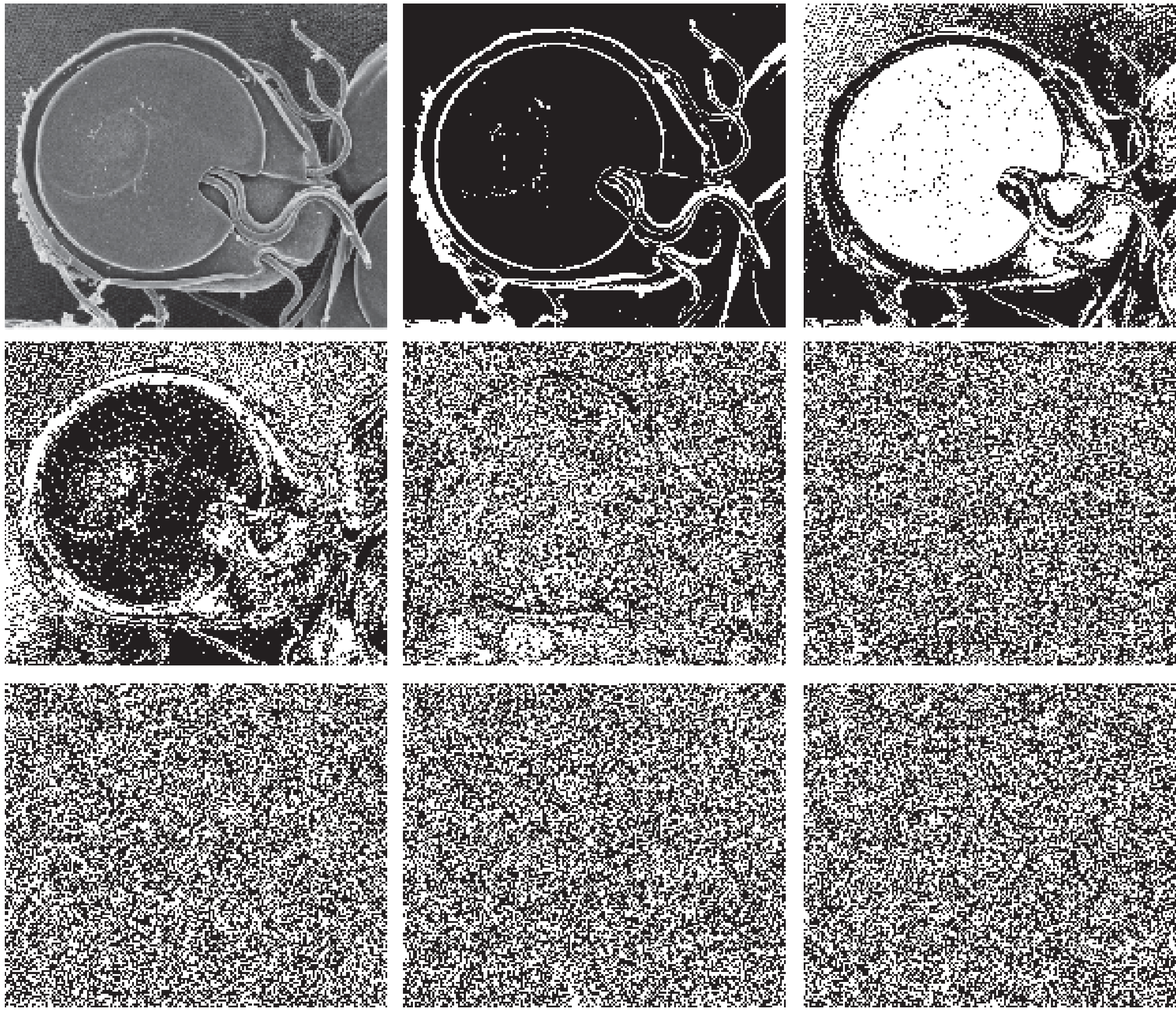


FIGURE 3.13
Bit-planes of an 8-bit image.

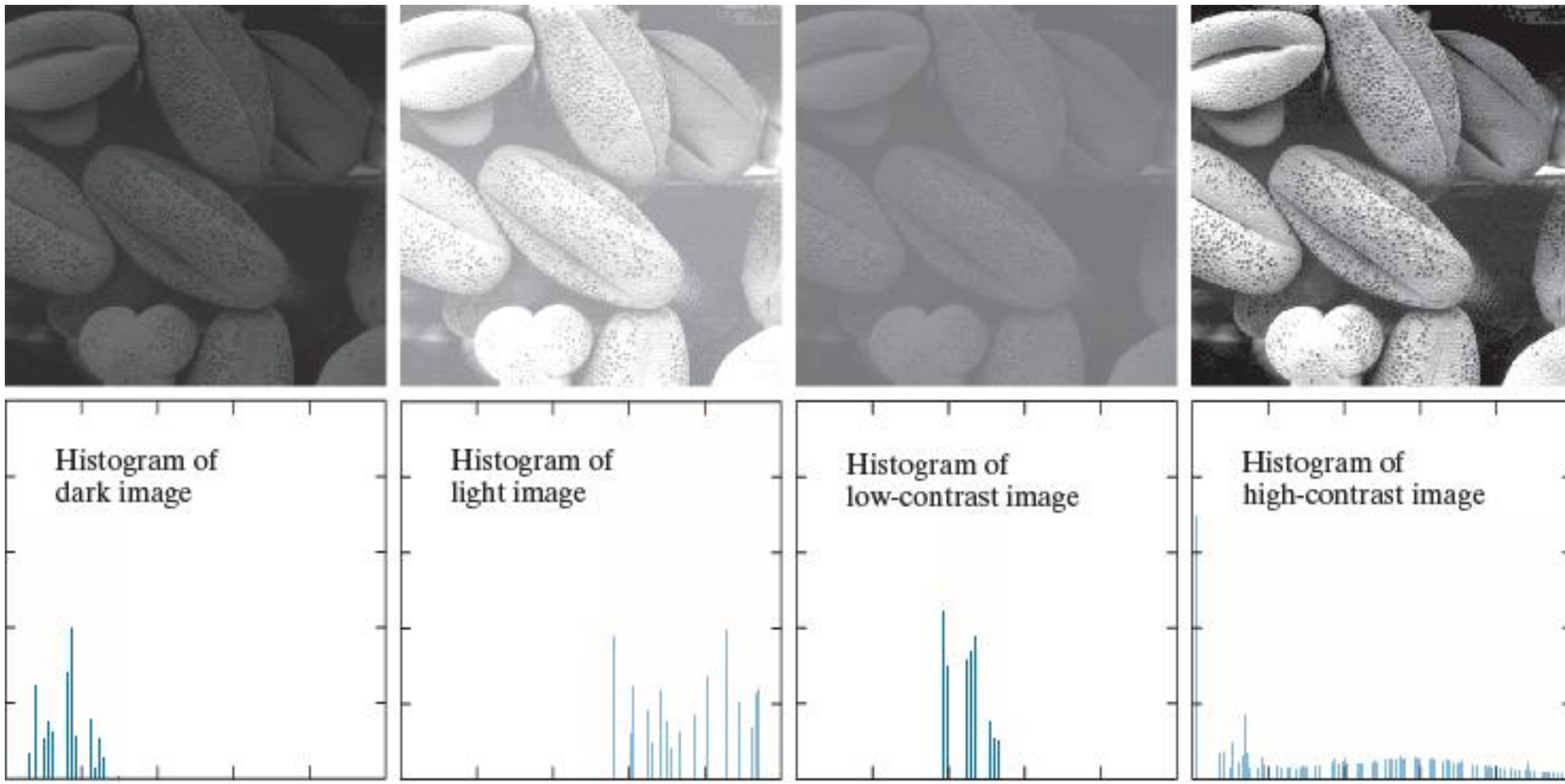
a b c
d e f
g h i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 837×988 pixels. (b) through (i) Bit planes 8 through 1, respectively, where plane 1 contains the least significant bit. Each bit plane is a binary image. Figure (a) is an SEM image of a trophozoite that causes a disease called *giardiasis*. (Courtesy of Dr. Stan Erlandsen, U.S. Center for Disease Control and Prevention.)



a b c

FIGURE 3.15 Image reconstructed from bit planes: (a) 8 and 7; (b) 8, 7, and 6; (c) 8, 7, 6, and 5.



a b c d

FIGURE 3.16 Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of r_k and the vertical axis are values of $p(r_k)$.

a b

FIGURE 3.17
(a) Monotonic increasing function, showing how multiple values can map to a single value. (b) Strictly monotonic increasing function. This is a one-to-one mapping, both ways.

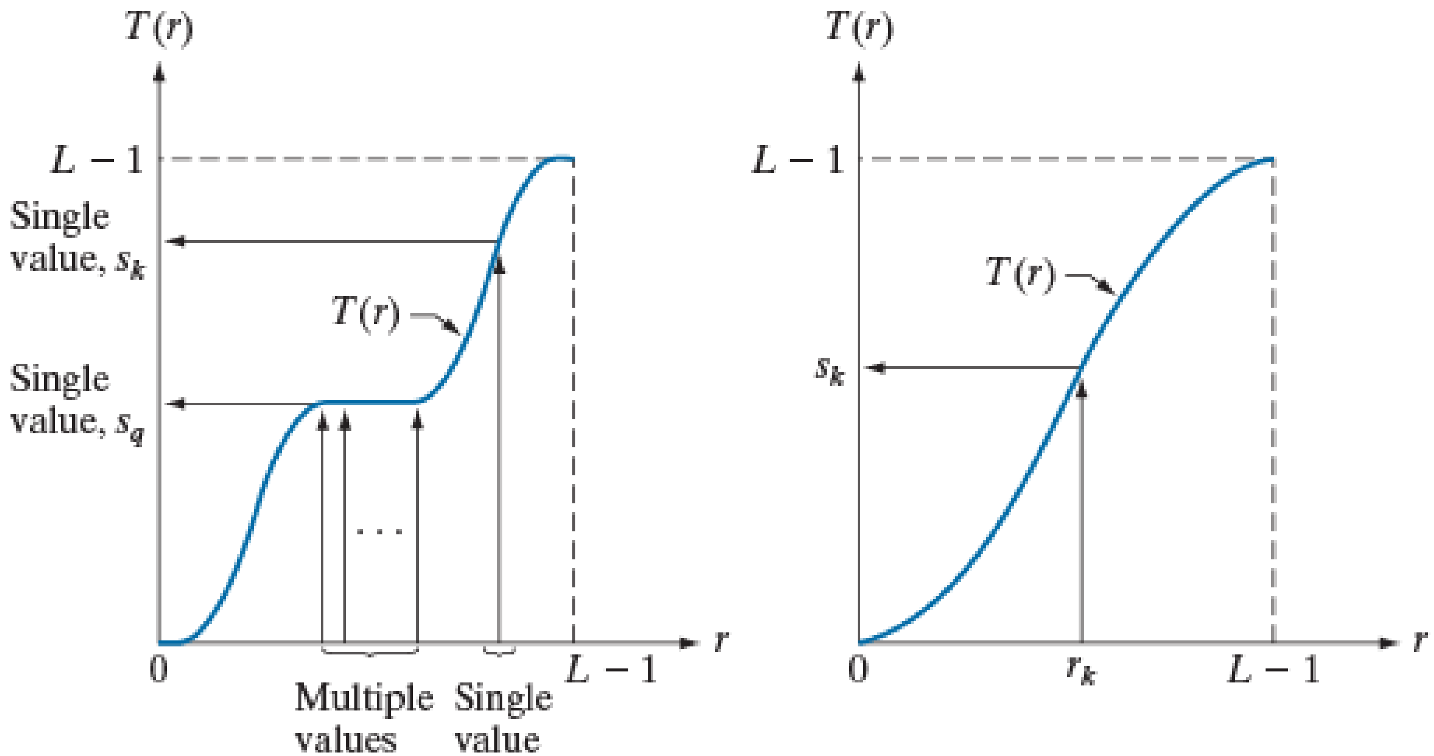


Image Enhancement: Histogram-based methods

- The histogram of a digital image with grayvalues $r_0, r_1, \ldots, r_{L-1}$ is the discrete funciotn

pixels with value r_k

$$p(r_k) = \frac{n_k}{n}$$

Total # pixels in image

- The function $p(r_k)$ represents the fraction of the total number of pixels with grayvalue r_k.

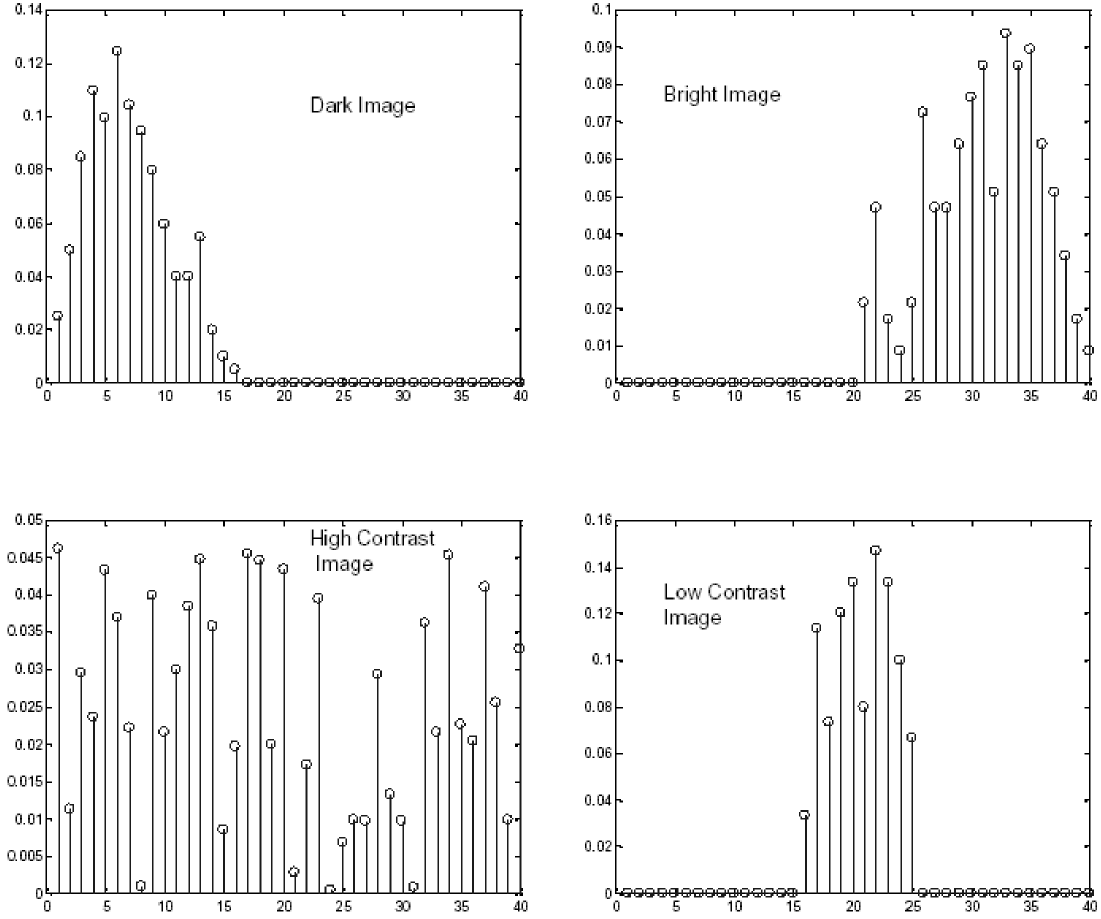
- Histogram provides a global description of the appearance of the image.

- If we consider the grayvalues in the image as realizations of a random variable R, with some probability density, histogram provides an approximation to this probability density. In other words,

$$\mathrm{p_r}[R = r_k] \approx p(r_k)$$

Some typical Histograms

- The shape of a histogram provides useful information for contrast enhancement.



Histogram Equalization

- Let us assume for the moment that the input image to be enhanced has continuous grayvalues, with r = 0 representing black and r = 1 representing white.
- We need to design a grayvalue transformation s = T(r), based on the histogram of the input image, which will enhance the image.
- As before, we assume that:
  - T(r) is a monotonically increasing function for $0 \leq r \leq 1$ (preserves order from black to white)
  - T(r) maps [0, 1] into [0, 1] (preserves the range of allowed grayvalues).

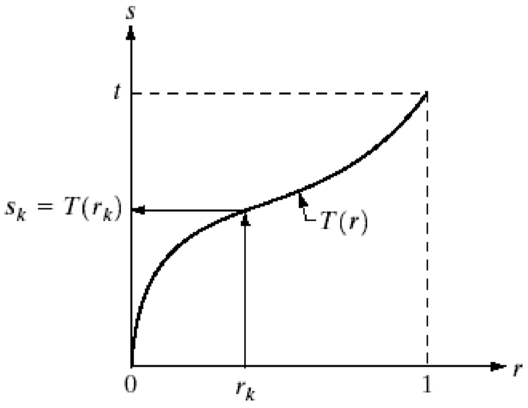


FIGURE 3.16 A gray-level transformation function that is both single valued and monotonically increasing.

Histogram Equalization

- Let us denote the inverse transformation by $r = T^{-1}(s)$. We assume that the inverse transformation also satisfies the above two conditions.
- We consider the grayvalues in the input image and output image as random variables in the interval [0, 1].
- Let $p_{in}(r)$ and $p_{out}(s)$ denote the probability density of the grayvalues in the input and output images.
- If $p_{in}(r)$ and $T(r)$ are known, and $T^{-1}(s)$ satisfies condition 1, we write (result from probability theory):

$$p_{out}(s) = [p_{in}(r)\frac{dr}{ds}]_{r=T^{-1}(s)}$$

- One way to enhance the image is to design a transformation T(.) such that the grayvalues in the output is uniformly distributed in [0, 1], i.e.

$$p_{out}(s) = 1,\ 0 \le s \le 1$$

- In terms of histograms, the ouput image will have al grayvalues in “equal proportion.”

Histogram Equalization

- Consider the transformation

$$s = T(r) = \int_0^r p_{in}(w)dw, \;\; 0 \le r \le 1$$

- Note that this is the cumulative distribution function (CDF) if $p_{in}(r)$ and satisfies the previous two conditions.

- From the previous equation and using the fundamental theorem of calculus,

$$\frac{ds}{dr} = p_{in}(r)$$

- Therefore, the output histogram is given by

$$p_{out}(s) = [p_{in}(r)\frac{1}{p_{in}(r)}]_{r=T^{-1}(s)}$$
$$= [1]_{r=T^{-1}(s)} = 1, \text{ for } 0 \le \text{s} \le 1$$

- The output probability density function is uniform, regardless of the input.

An example of histogram equalization in a continuous pdf.

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2} & \text{for } 0 \le r \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw = \frac{2}{L-1}\int_0^r w\,dw = \frac{r^2}{L-1}$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = \frac{2r}{(L-1)^2}\left|\left[\frac{ds}{dr}\right]^{-1}\right|$$

$$= \frac{2r}{(L-1)^2}\left|\left[\frac{d}{dr}\frac{r^2}{L-1}\right]^{-1}\right| = \frac{2r}{(L-1)^2}\left|\frac{(L-1)}{2r}\right| = \frac{1}{L-1}$$

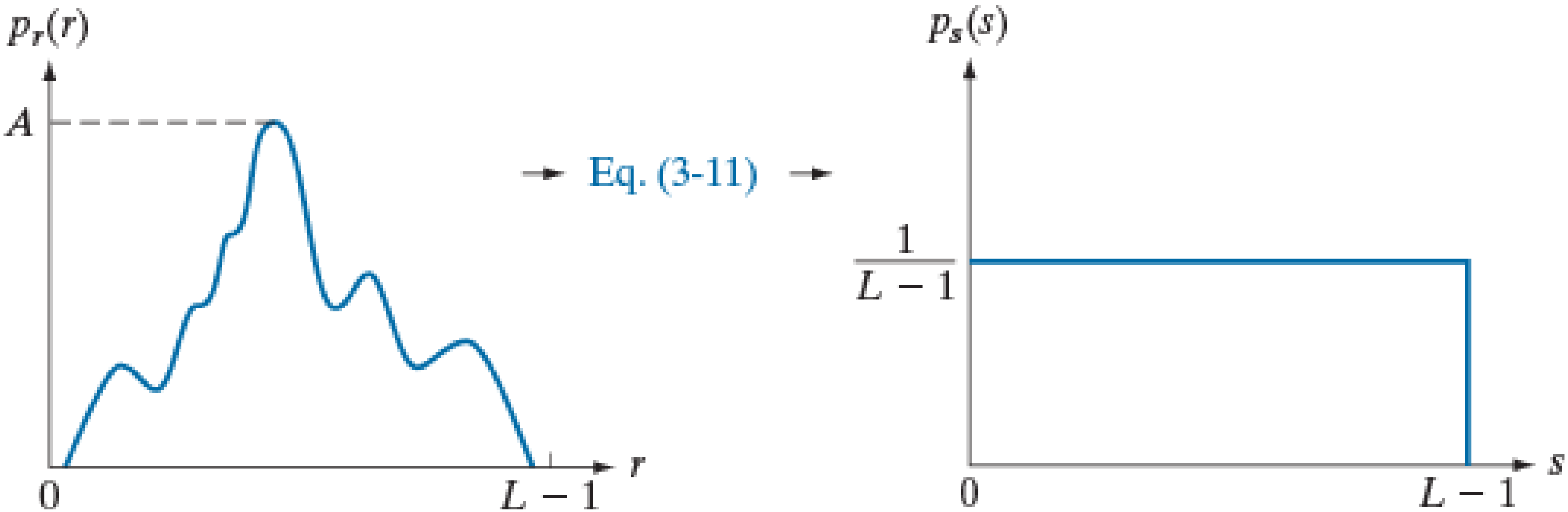


FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying Eq. (3-11) to the input PDF. The resulting PDF is always uniform, independently of the shape of the input.

Histogram Equalization

- Thus, using a transformation function equal to the CDF of input grayvalues r, we can obtain an image with uniform grayvalues.

- This usually results in an enhanced image, with an increase in the dynamic range of pixel values.

- For images with discrete grayvalues, we have

$$p_{in}(r_k) = \frac{n_k}{n}, \text{ for } 0 \le r_k \le 1, \text{ and } 0 \le \mathrm{k} \le \mathrm{L} - 1$$

  - L: Total number of graylevels
  - n_k:Number of pixels with grayvalue r_k
  - n: Total number of pixels in the image

- The discrete version of the previous transformation based on CDF is given by:

$$s_k = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n} = \sum_{j=0}^{k} p_{in}(r_j), \text{ for } 0 \le \mathrm{k} \le \mathrm{L}-1$$

An example of histogram equalization in a discrete pdf.

TABLE 3.1
Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

r_k	n_k	$p_r(r_k) = n_k/MN$	
$r_0 = 0$	790	0.19	1
$r_1 = 1$	1023	0.25	3
$r_2 = 2$	850	0.21	5
$r_3 = 3$	656	0.16	6
$r_4 = 4$	329	0.08	6
$r_5 = 5$	245	0.06	7
$r_6 = 6$	122	0.03	7
$r_7 = 7$	81	0.02	7

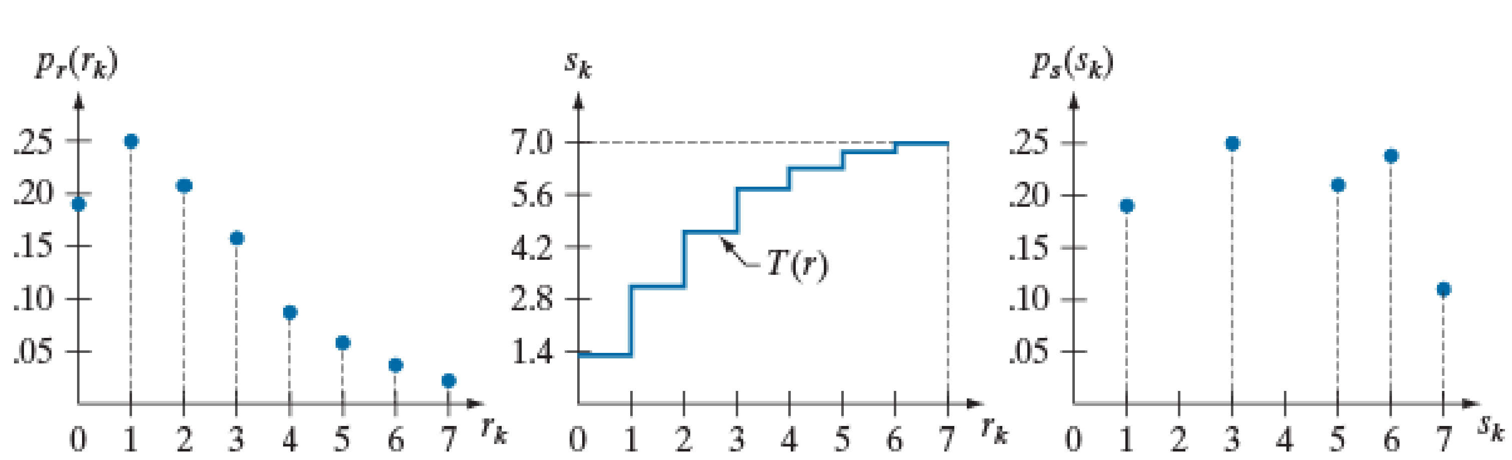


FIGURE 3.19 Histogram equalization. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

$s_0 = 1.33 \rightarrow 1$ $\quad s_2 = 4.55 \rightarrow 5$ $\quad s_4 = 6.23 \rightarrow 6$ $\quad s_6 = 6.86 \rightarrow 7$

$s_1 = 3.08 \rightarrow 3$ $\quad s_3 = 5.67 \rightarrow 6$ $\quad s_5 = 6.65 \rightarrow 7$ $\quad s_7 = 7.00 \rightarrow 7$

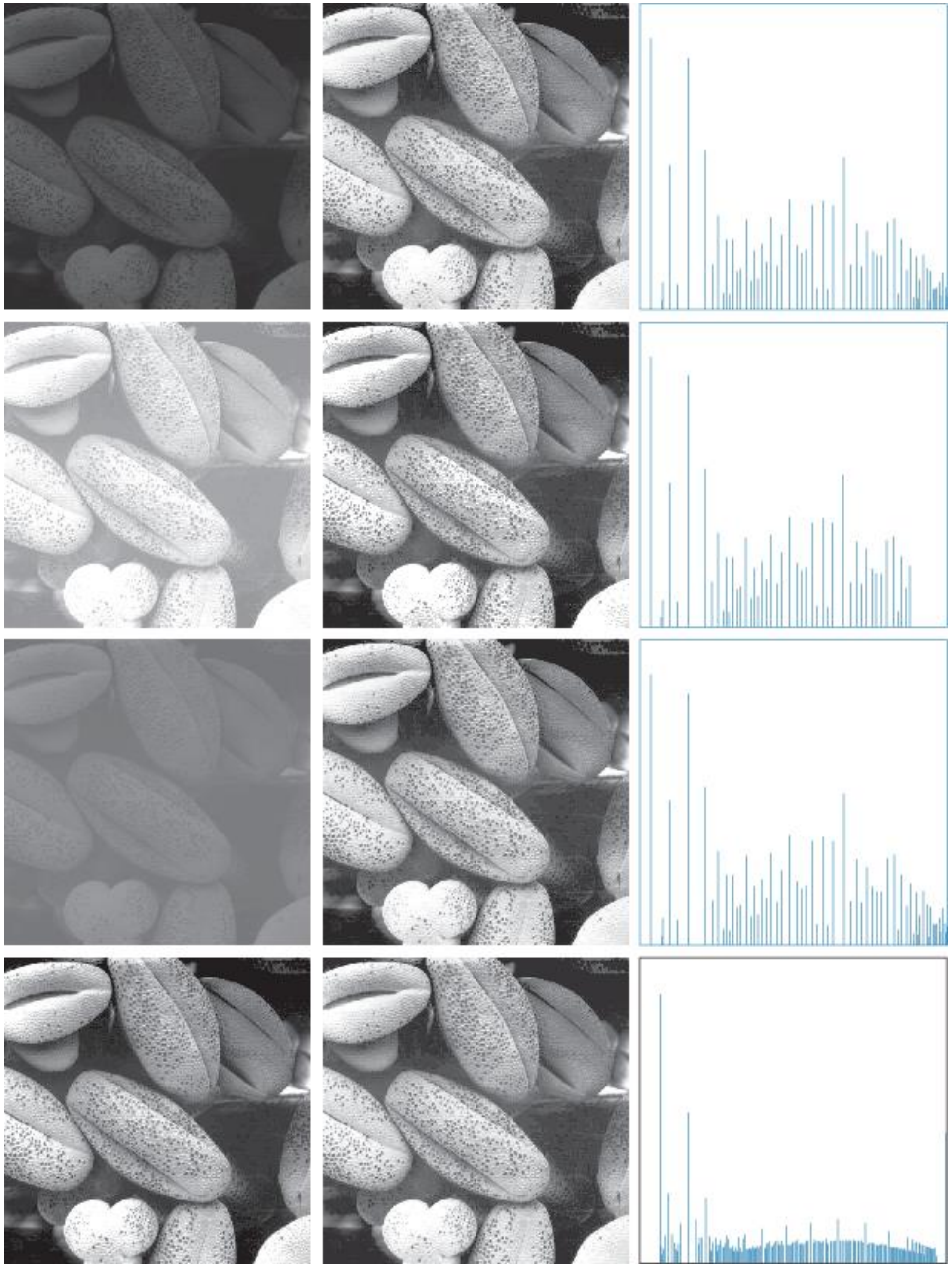
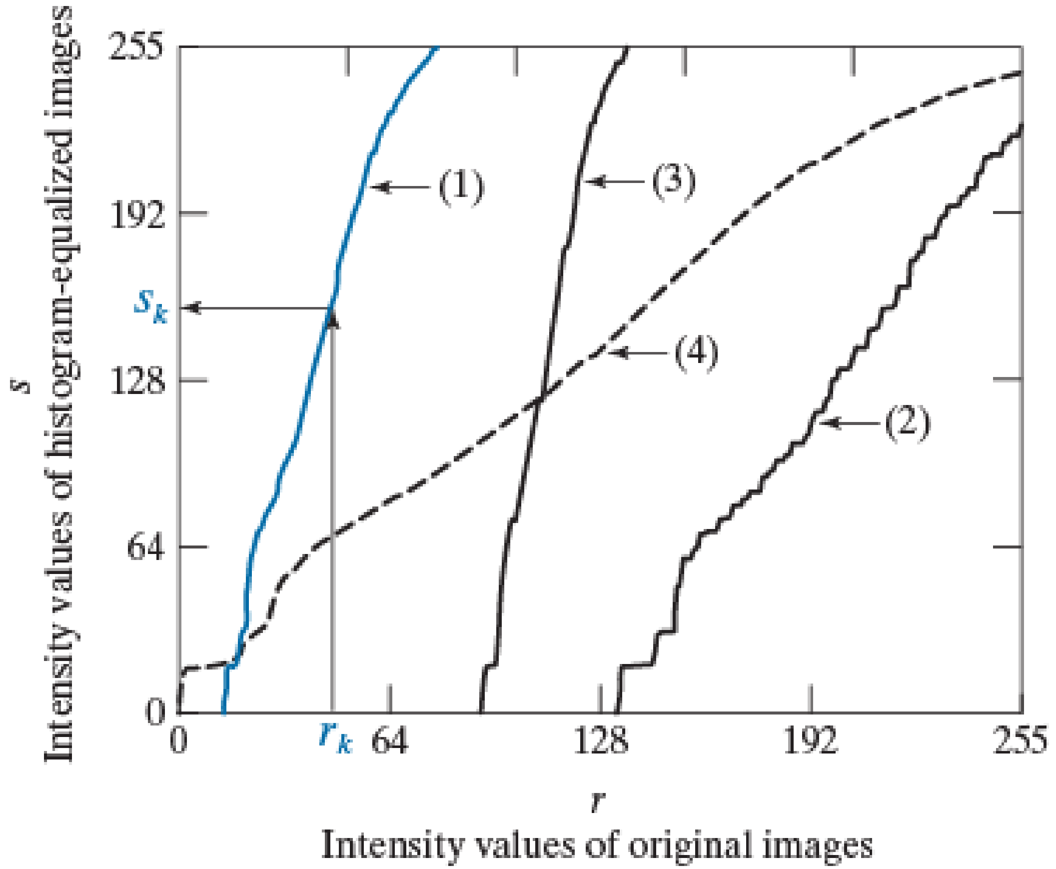
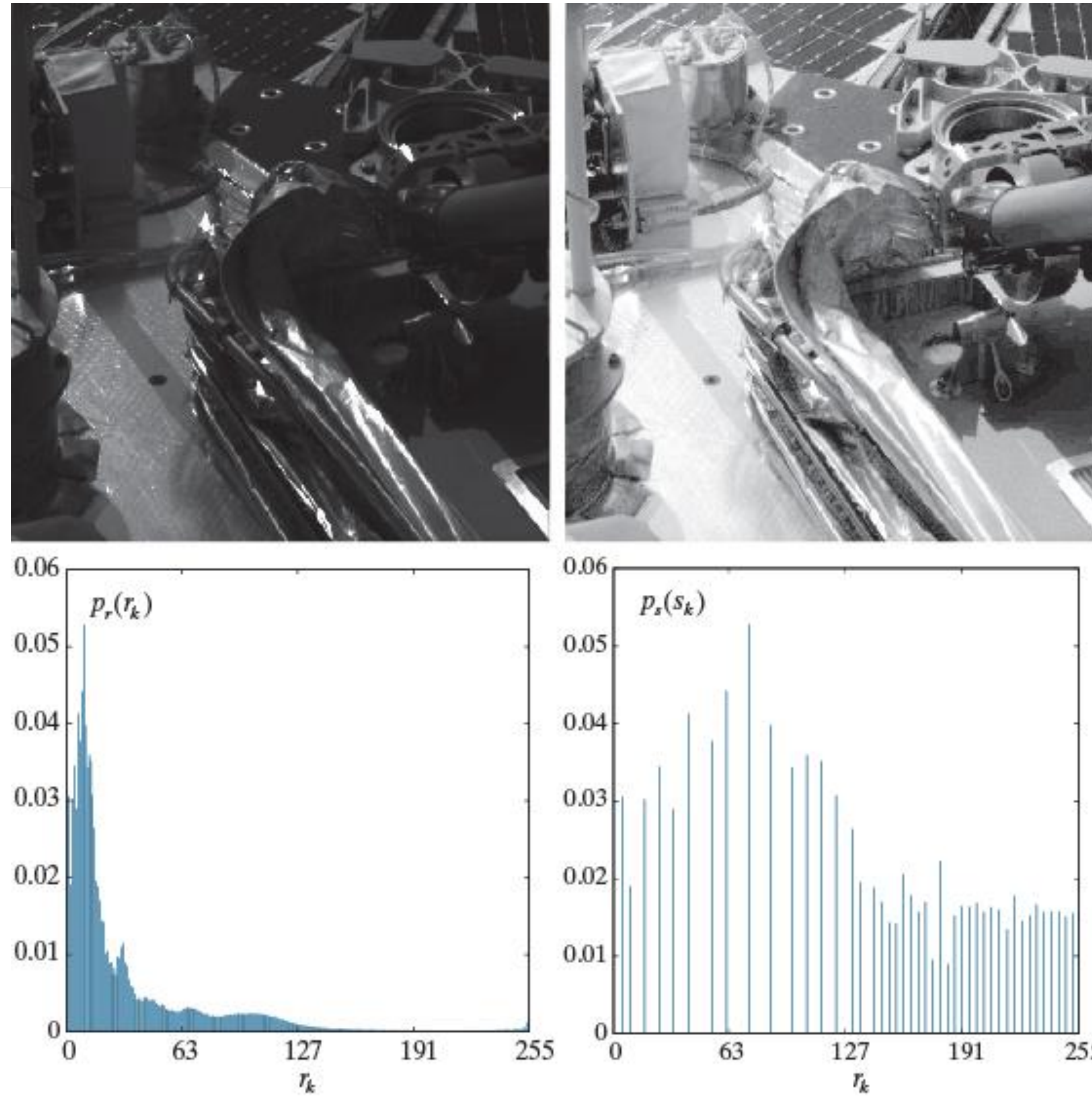


FIGURE 3.20 Left column: Images from Fig. 3.16. Center column: Corresponding histogram-equalized images. Right column: histograms of the images in the center column (compare with the histograms in Fig. 3.16).

FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained using Eq. (3-15) and the histograms of the images on the left column of Fig. 3.20. Mapping of one intensity value r_k in image 1 to its corresponding value s_k is shown.





a b
c d

FIGURE 3.22
(a) Image from Phoenix Lander. (b) Result of histogram equalization. (c) Histogram of image (a). (d) Histogram of image (b). (Original image courtesy of NASA.)

Histogram Specification

- Histogram equalization yields an image whose pixels are (in theory) uniformly distributed among all graylevels.

- Sometimes, this may not be desirable. Instead, we may want a transformation that yields an output image with a pre-specified histogram. This technique is called **histogram specification**.

- Again, we will assume, for the moment, continuous-grayvalues.

- Suppose, the input image has probability density $p_{in}(r)$. We want to find a transformation z = H(r), such that the probability density of the new image obtained by this transformation is $p_{out}(z)$, which is not necessarily uniform.

- First apply the transformation

- $$s = T(r) = \int_0^r p_{in}(w)dw, \;\; 0 \le r \le 1 \quad (*)$$

  This gives an image with a uniform probability density.

- If the desired output image were available, then the following transformation would generate an image with uniform density

$$v = G(z) = \int_0^z p_{out}(w)dw, \quad 0 \le z \le 1 \quad (**)$$

- From the grayvalues *v* we can obtain the grayvalues z by using the inverse tranformation , $z = G^{-1}(v)$.

- If instead of using the grayvalues *v* obtained from (**), we use the grayvalues *s* obtained from (*) above (both are uniformly distributed!), then the point transformation will generate an image with the specified density $p_{out}(z)$, from an input image with density $p_{in}(r)$!

- For discrete graylevels, we have

$$s_k = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n}, \text{ for } 0 \le \text{k} \le \text{L -1 and}$$

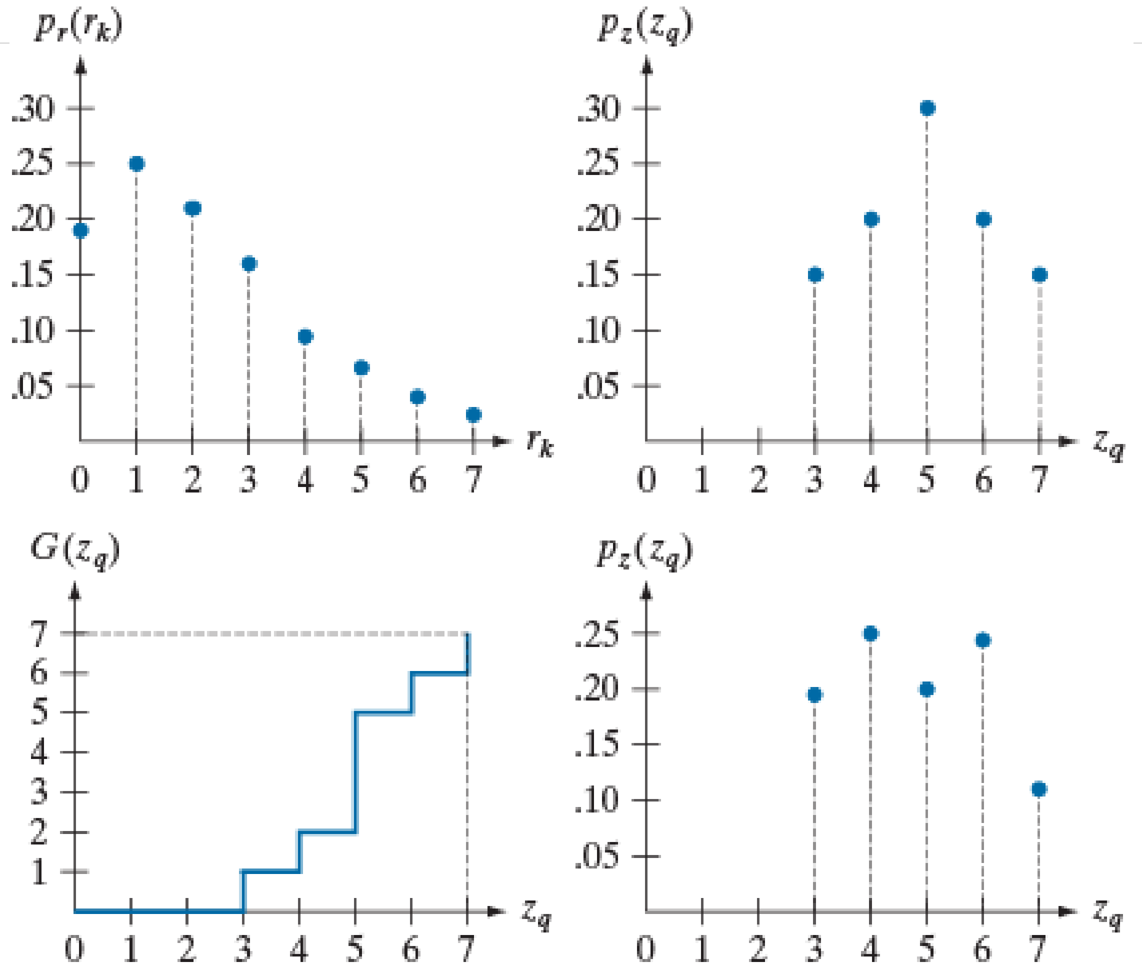
$$v_k = G(z_k) = \sum_{j=0}^{k} p_{out}(z_j), \text{ for } 0 \le \text{k} \le \text{L -1}$$

- $z_k = G^{-1}(v_k) = G^{-1}(s_k) = G^{-1}(\,T(r_k)\,)$

- Dealing with a small set of discrete grayvalues.

a b
c d

FIGURE 3.23
(a) Histogram of a 3-bit image.
(b) Specified histogram.
(c) Transformation function obtained from the specified histogram.
(d) Result of histogram specification. Compare the histograms in (b) and (d).



$s_0 = 1.33 \rightarrow 1 \quad s_2 = 4.55 \rightarrow 5 \quad s_4 = 6.23 \rightarrow 6 \quad s_6 = 6.86 \rightarrow 7$

$s_1 = 3.08 \rightarrow 3 \quad s_3 = 5.67 \rightarrow 6 \quad s_5 = 6.65 \rightarrow 7 \quad s_7 = 7.00 \rightarrow 7$

TABLE 3.2
Specified and actual histograms (the values in the third column are computed in Example 3.7).

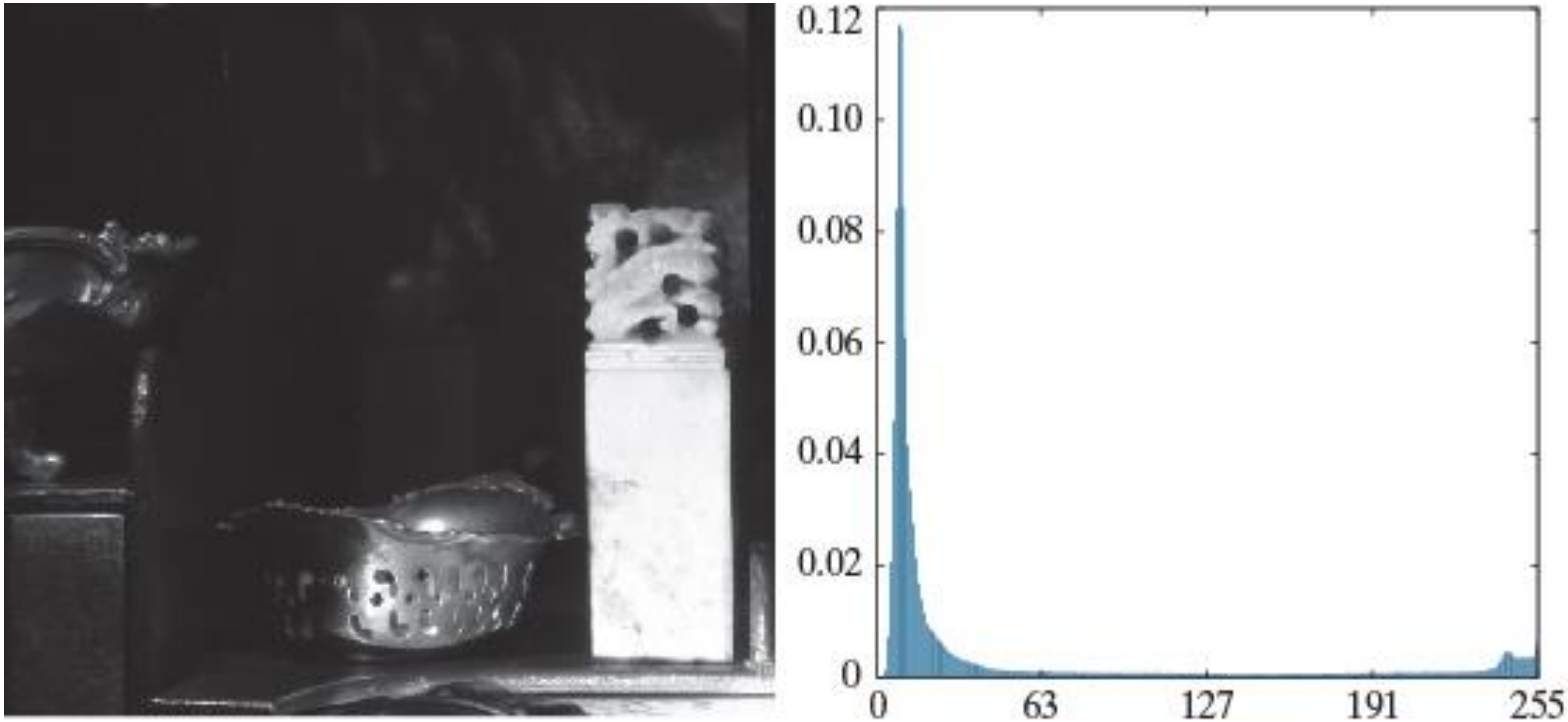
z_q	Specified $p_z(z_q)$		Actual $p_z(z_q)$	
$z_0 = 0$	0.00		0.00	
$z_1 = 1$	0.00		0.00	
$z_2 = 2$	0.00		0.00	
$z_3 = 3$	0.15	1	0.19	1
$z_4 = 4$	0.20	2	0.25	3
$z_5 = 5$	0.30	5	0.21	5
$z_6 = 6$	0.20	6	0.24	6
$z_7 = 7$	0.15	7	0.11	7

TABLE 3.3
Rounded values of the transformation function $G(z_q)$.

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

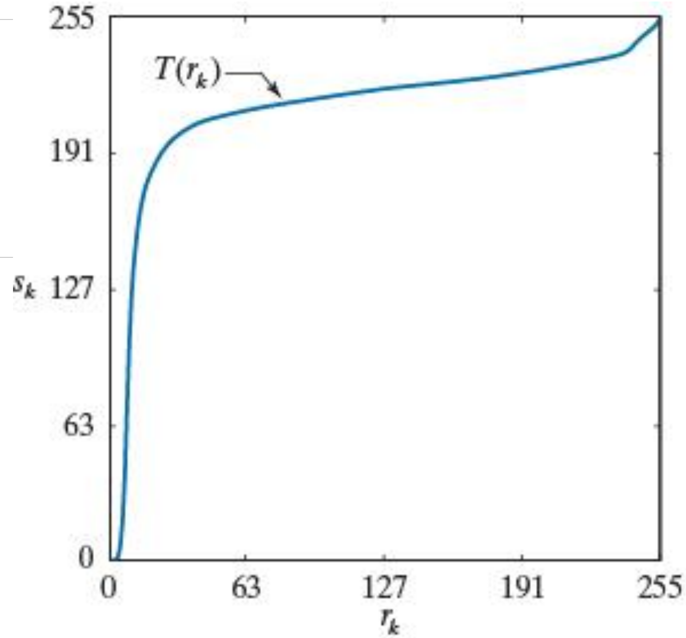
TABLE 3.4
Mapping of values s_k into corresponding values z_q.

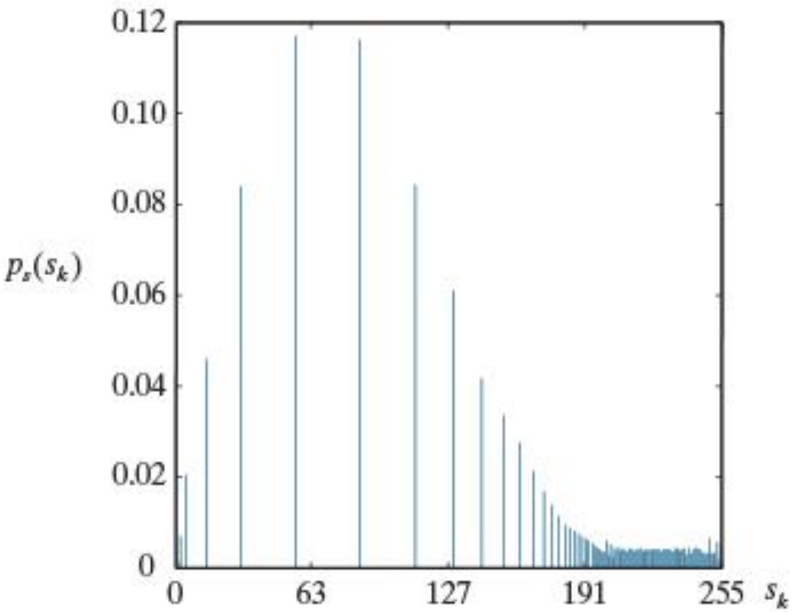
s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7



a b

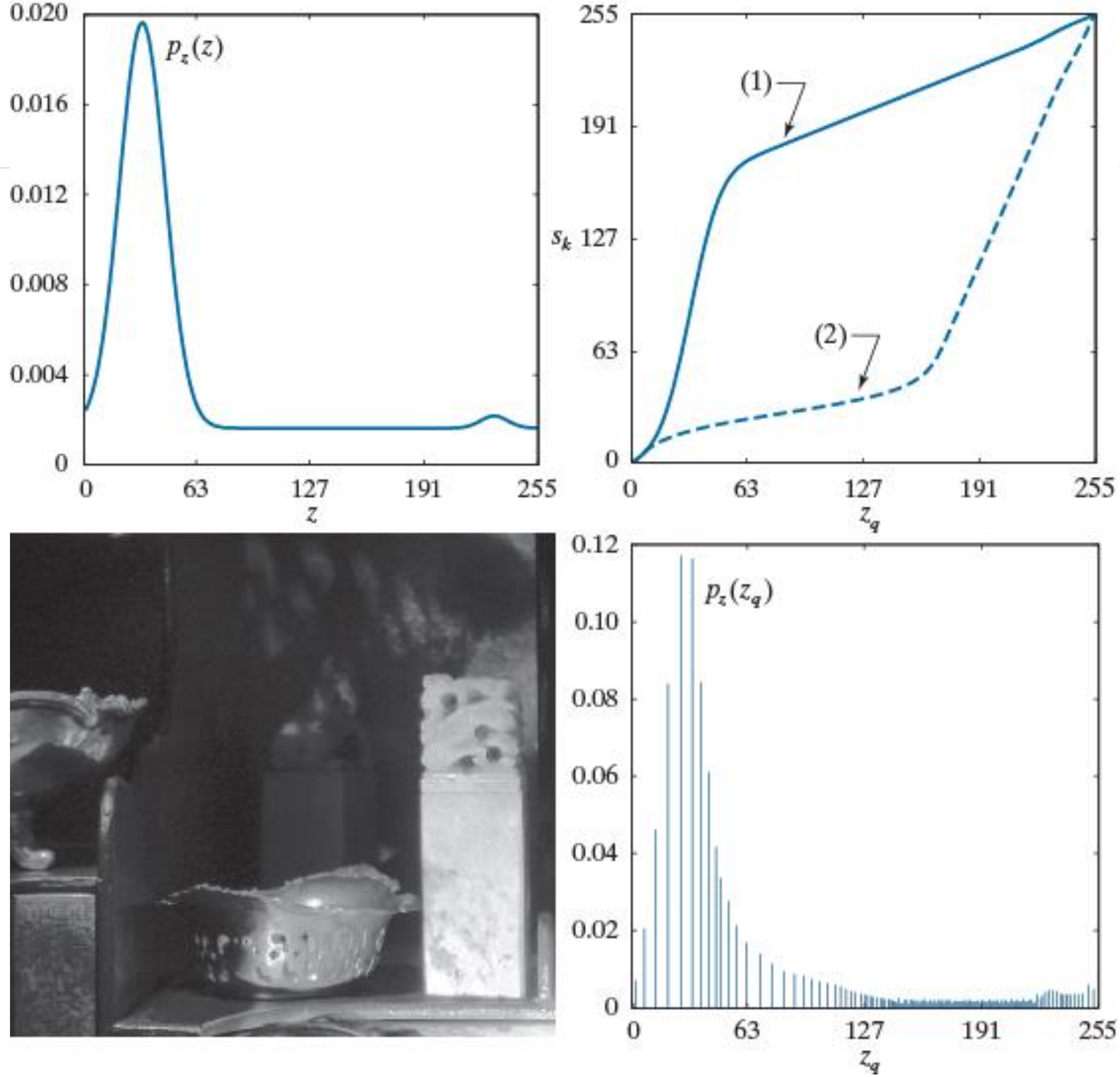
FIGURE 3.24
(a) An image, and
(b) its histogram.





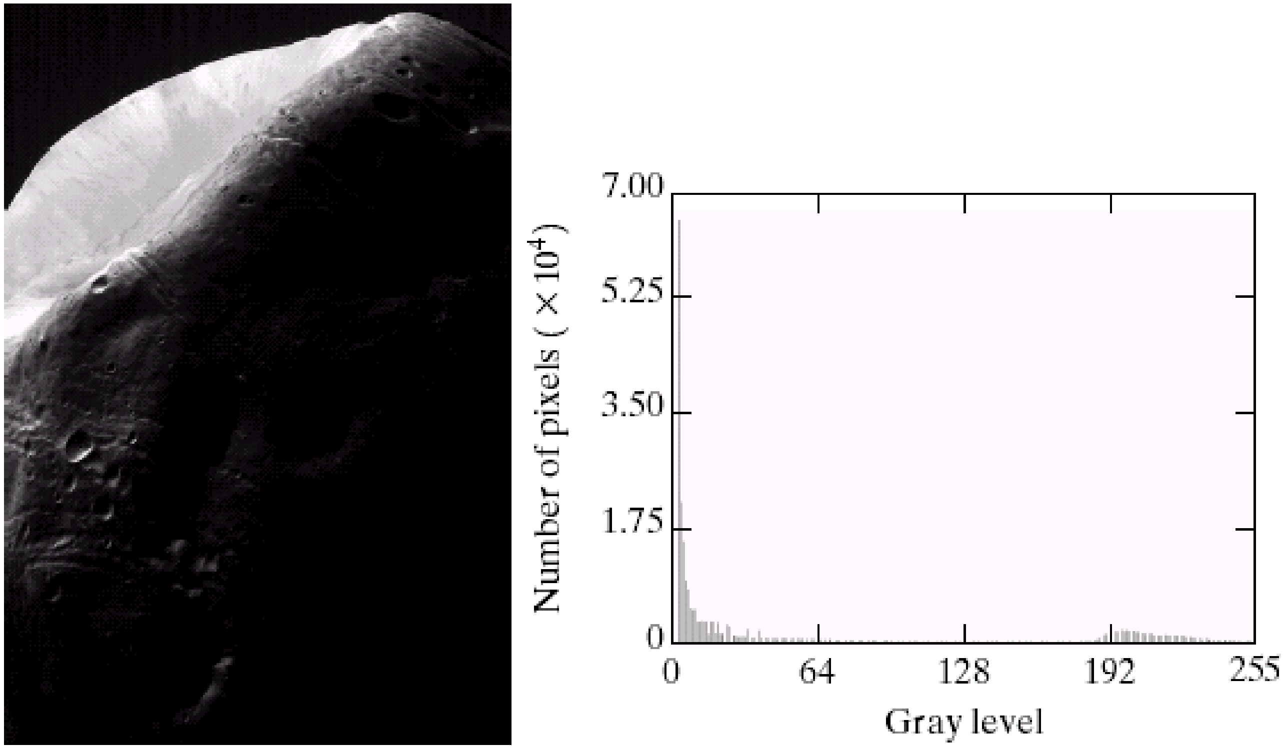
a b
c

FIGURE 3.25
(a) Histogram equalization transformation obtained using the histogram in Fig. 3.24(b). (b) Histogram equalized image. (c) Histogram of equalized image.



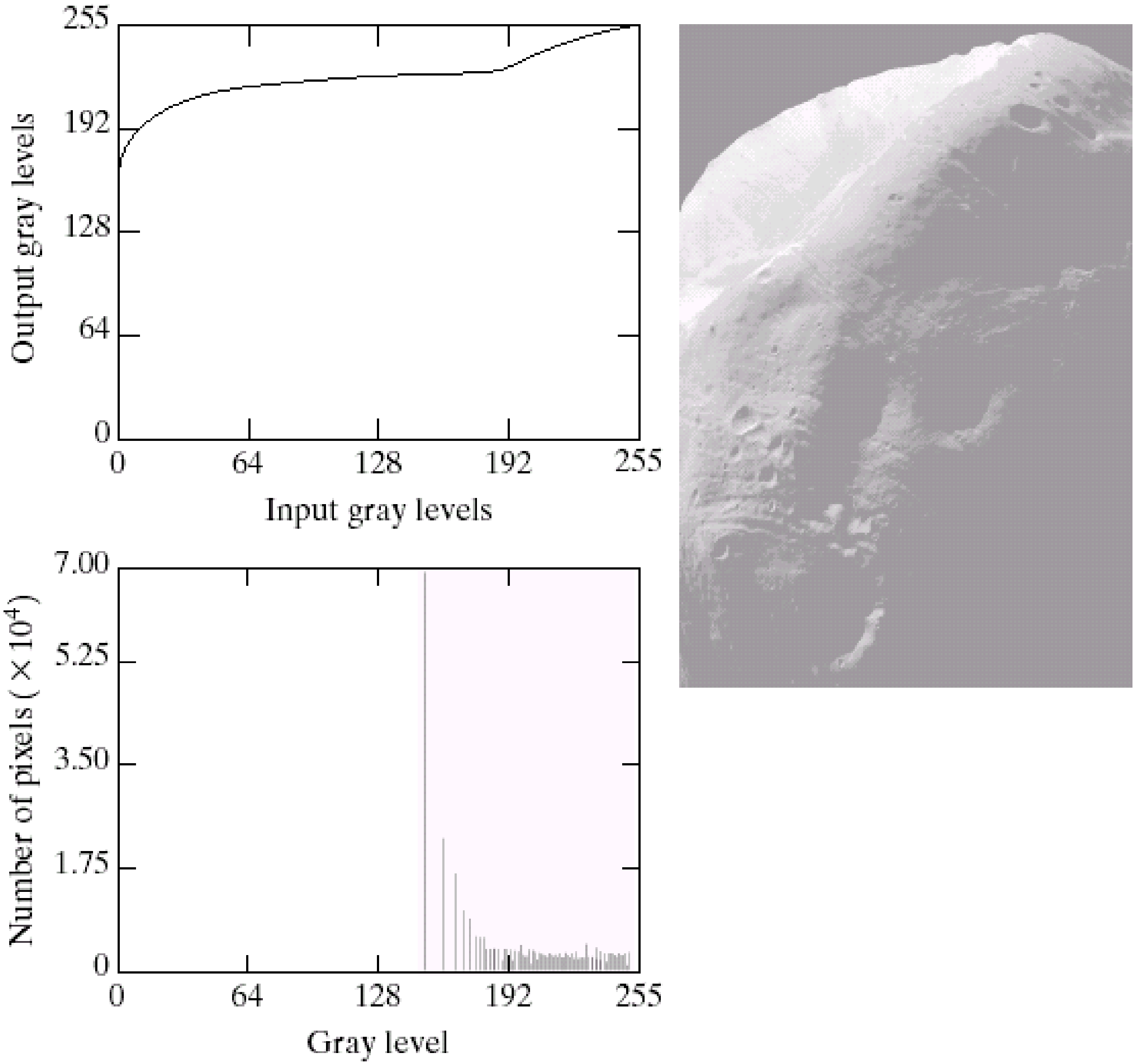
a b
c d

FIGURE 3.26 Histogram specification. (a) Specified histogram. (b) Transformation $G(z_q)$, labeled (1), and $G^{-1}(s_k)$, labeled (2). (c) Result of histogram specification. (d) Histogram of image (c).



a b

(a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

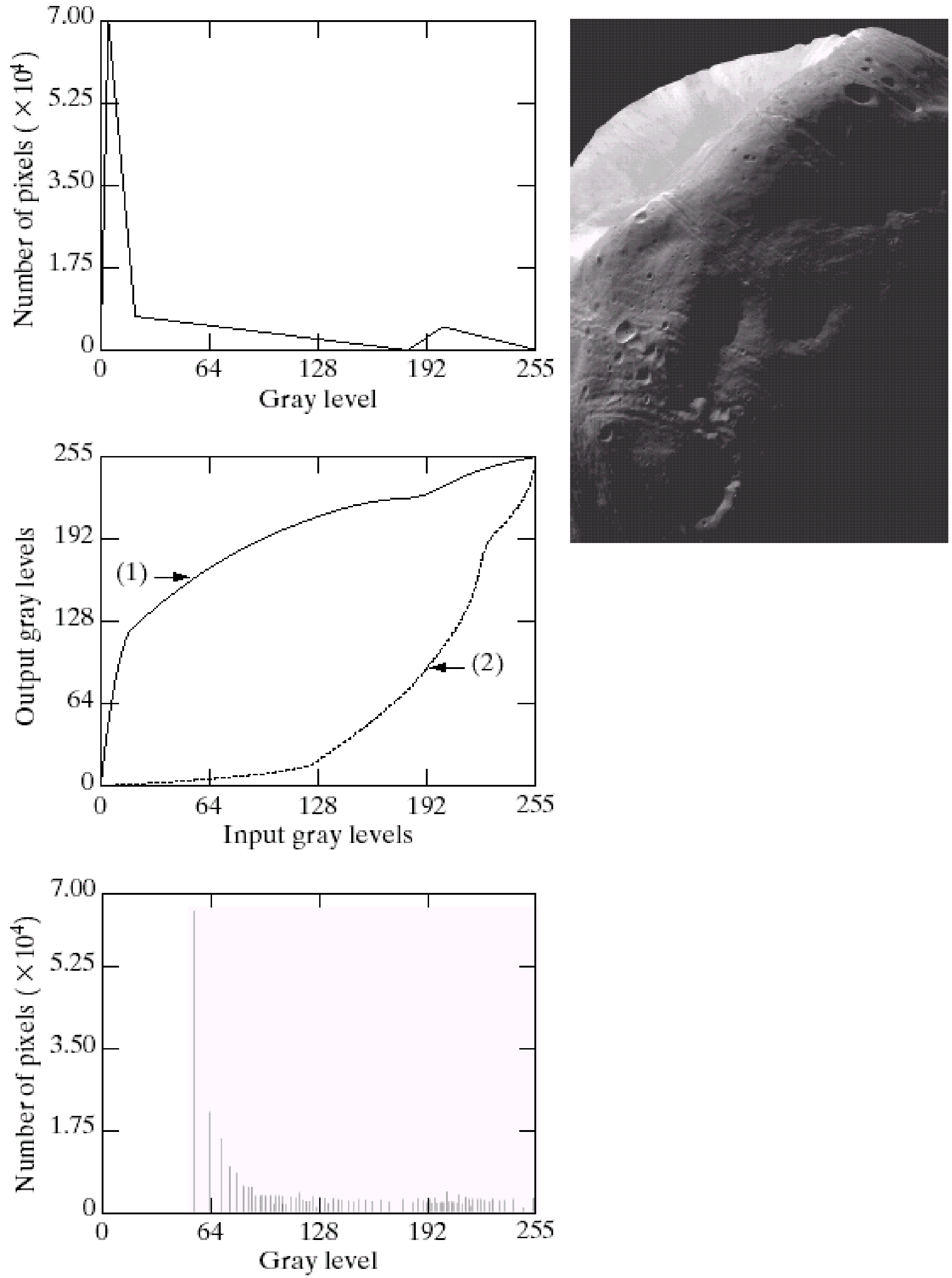


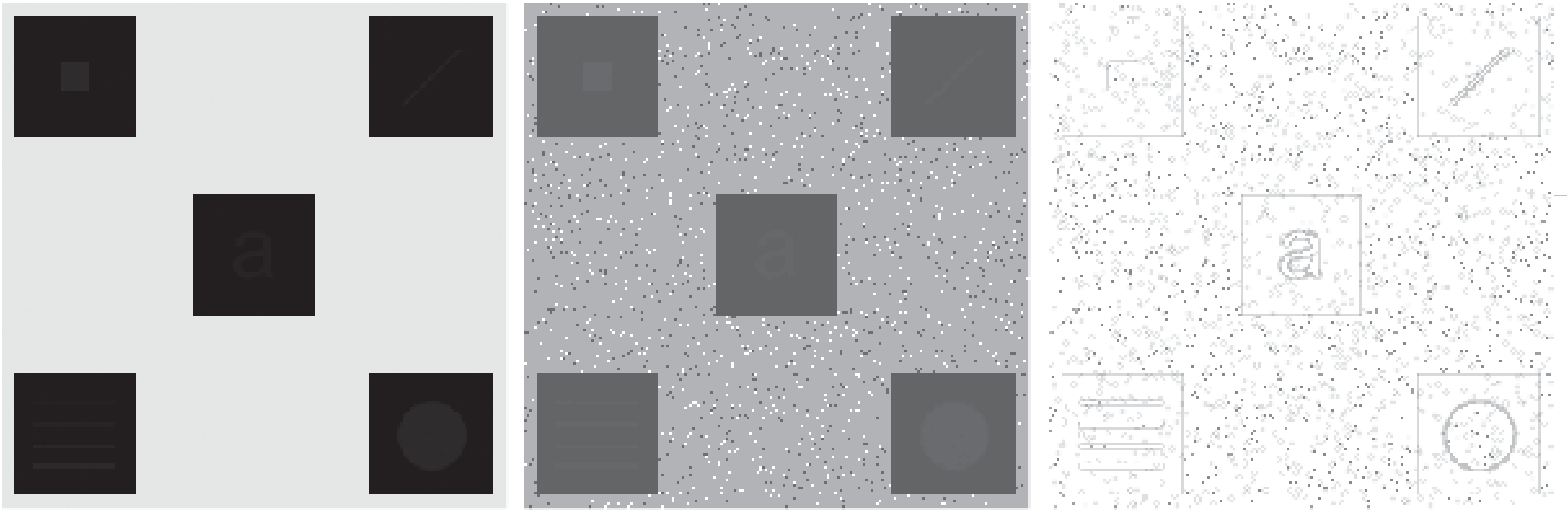
a b
c

(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

a c
b
d

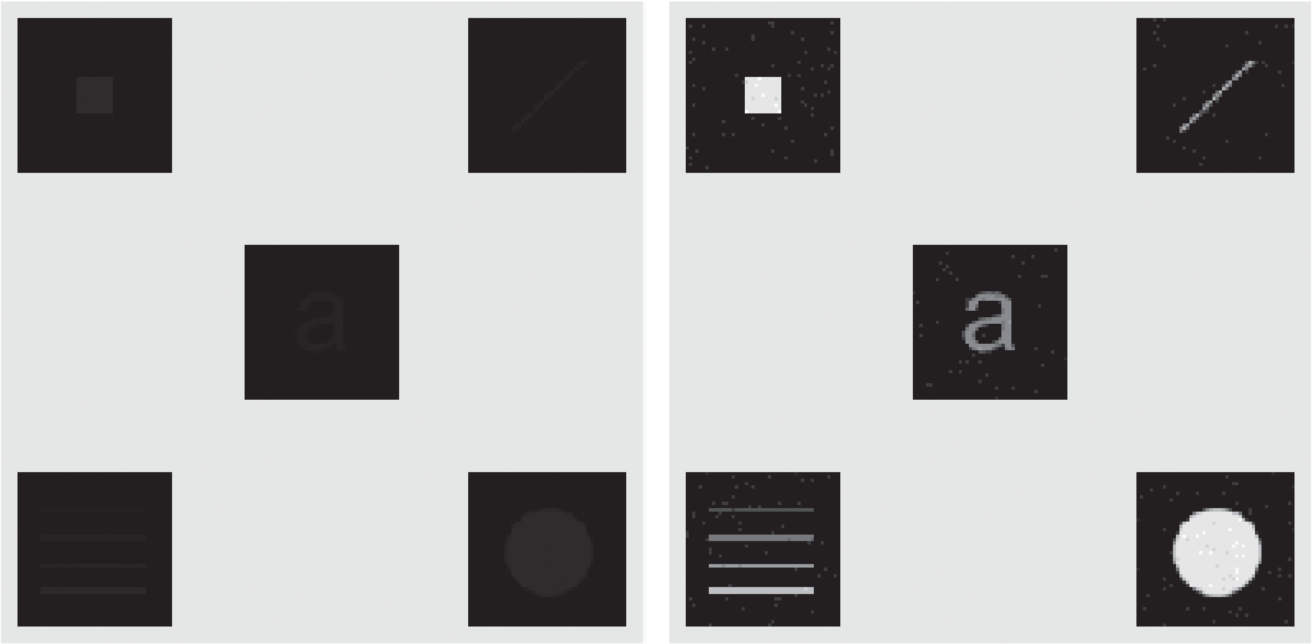
(a) Specified histogram.
(b) Curve (1) is from Eq.(3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq.(3.3-17).
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).





a b c

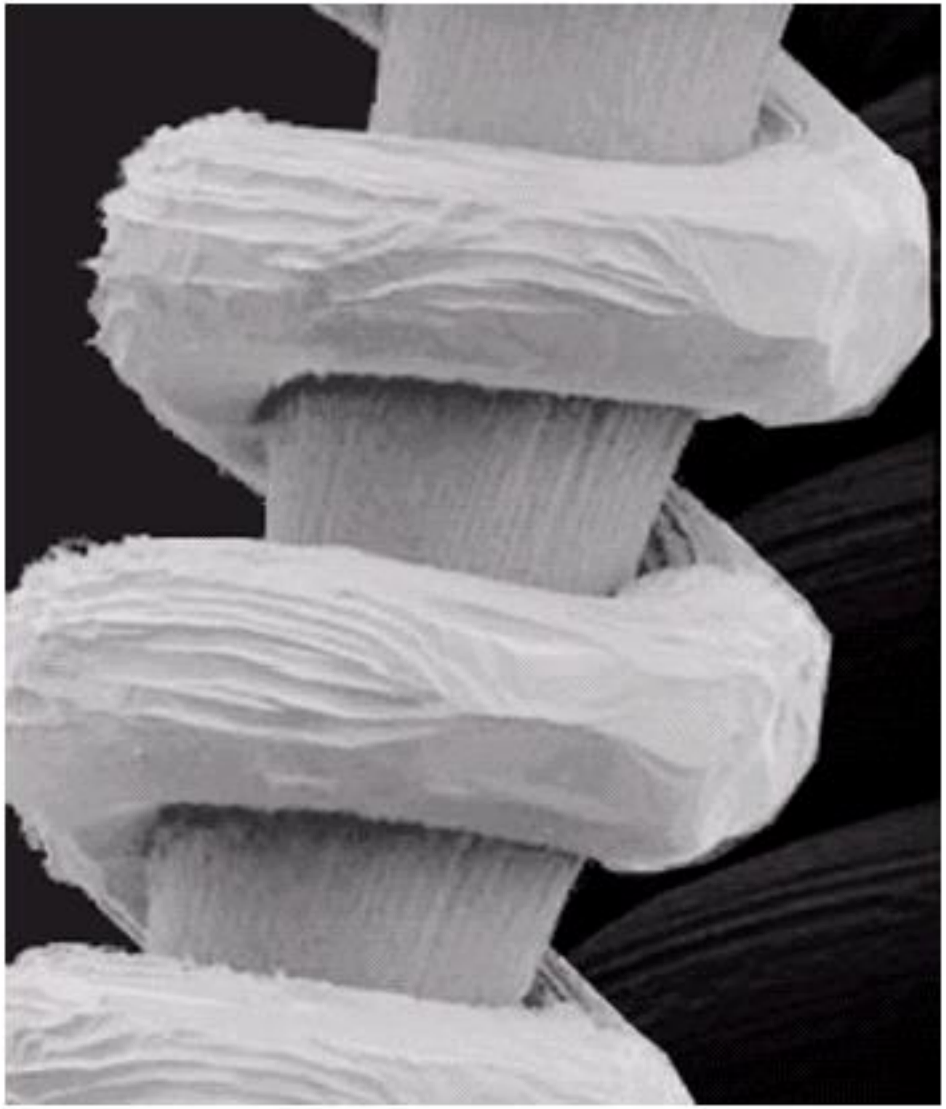
FIGURE 3.32
(a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization.

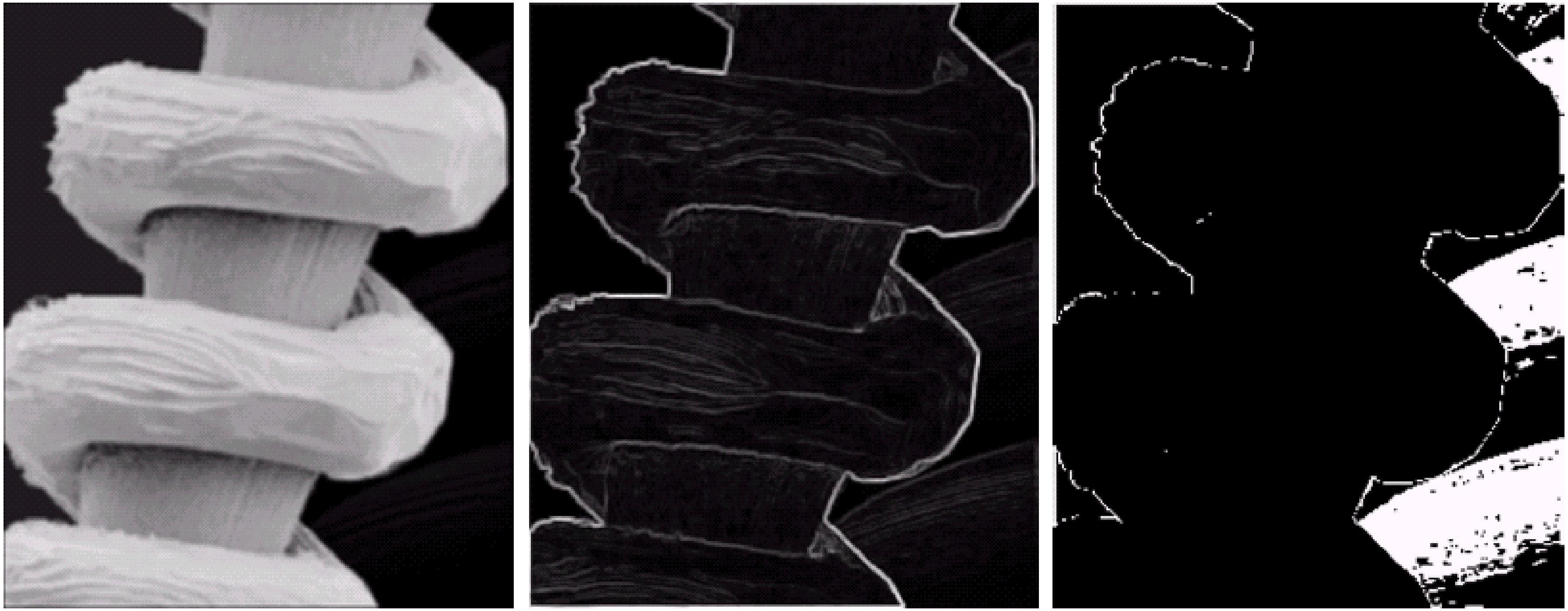


a b

FIGURE 3.33
(a) Original image. (b) Result of local enhancement based on local histogram statistics. Compare (b) with Fig. 3.32(c).

SEM image of a tungsten filament and support, magnified approximately 130X. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

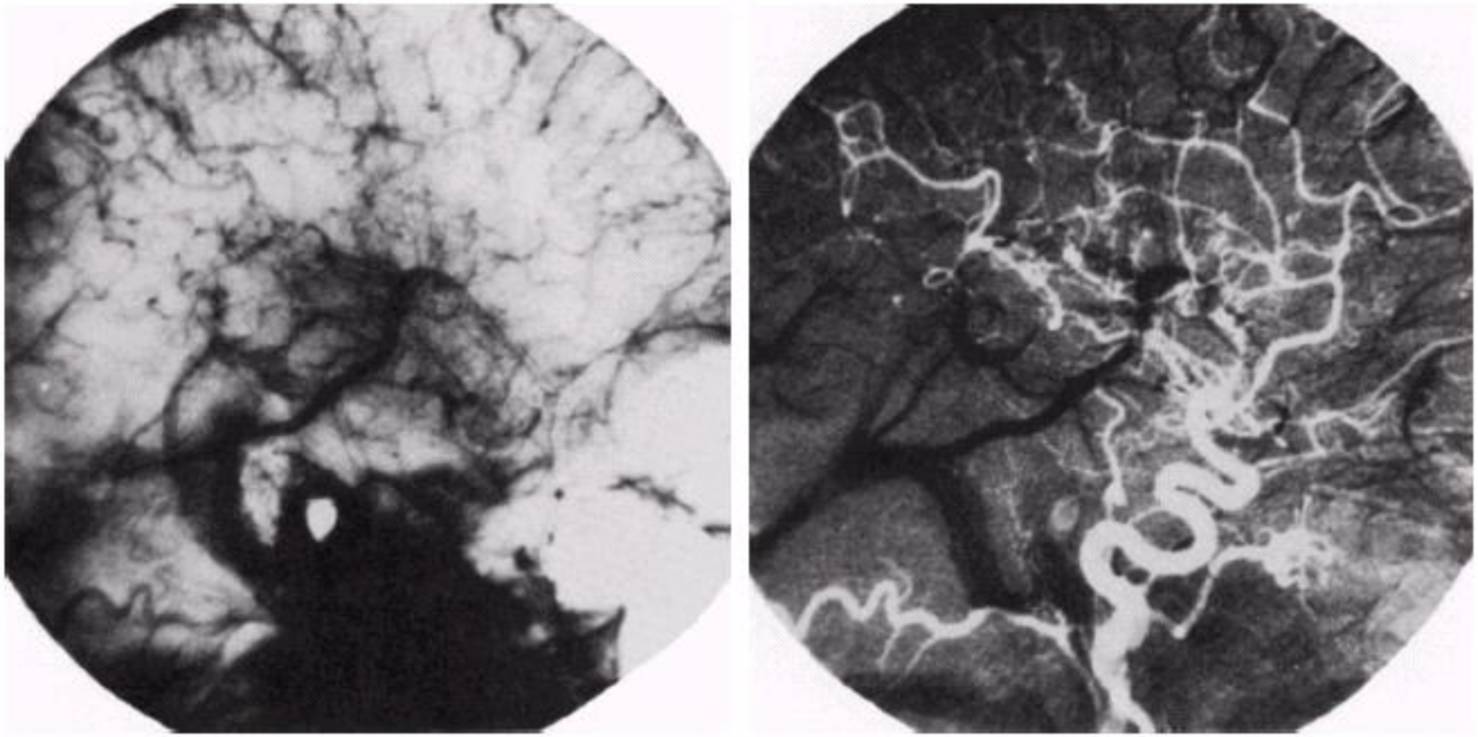




a b c

(a) Image formed from all local means obtained from Fig. 3.24 using Eq.(3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq.(3.3-22). (c) Image formed from all multiplication constats used to produce the enhanced image shown in Fig. 3.26.

Enhanced SEM image. Compare with Fig.3.24. Note in particular the enhanced area on the right side of the image.



a b

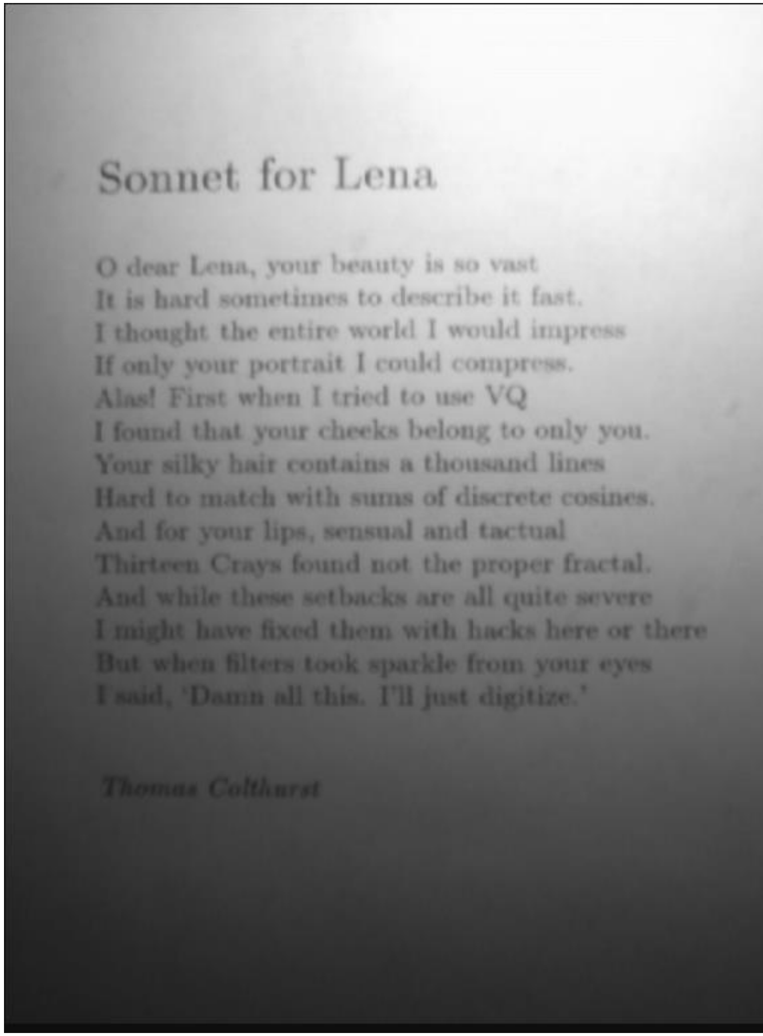
Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.

Image Subtraction

- In this case, the difference between two “similar” images is compute to highlight or enhance the differences between them:

$$g(m,n) = f_1(m,n) - f_2(m,n)$$

- It has applications in image segmentation and enhancement



Sonnet for Lena

O dear Lena, your beauty is so vast
It is hard sometimes to describe it fast.
I thought the entire world I would impress
If only your portrait I could compress.
Alas! First when I tried to use VQ
I found that your cheeks belong to only you.
Your silky hair contains a thousand lines
Hard to match with sums of discrete cosines.
And for your lips, sensual and tactual
Thirteen Crays found not the proper fractal.
And while these setbacks are all quite severe
I might have fixed them with hacks here or there
But when filters took sparkle from your eyes
I said, 'Damn all this. I'll just digitize.'

Thomas Colthurst

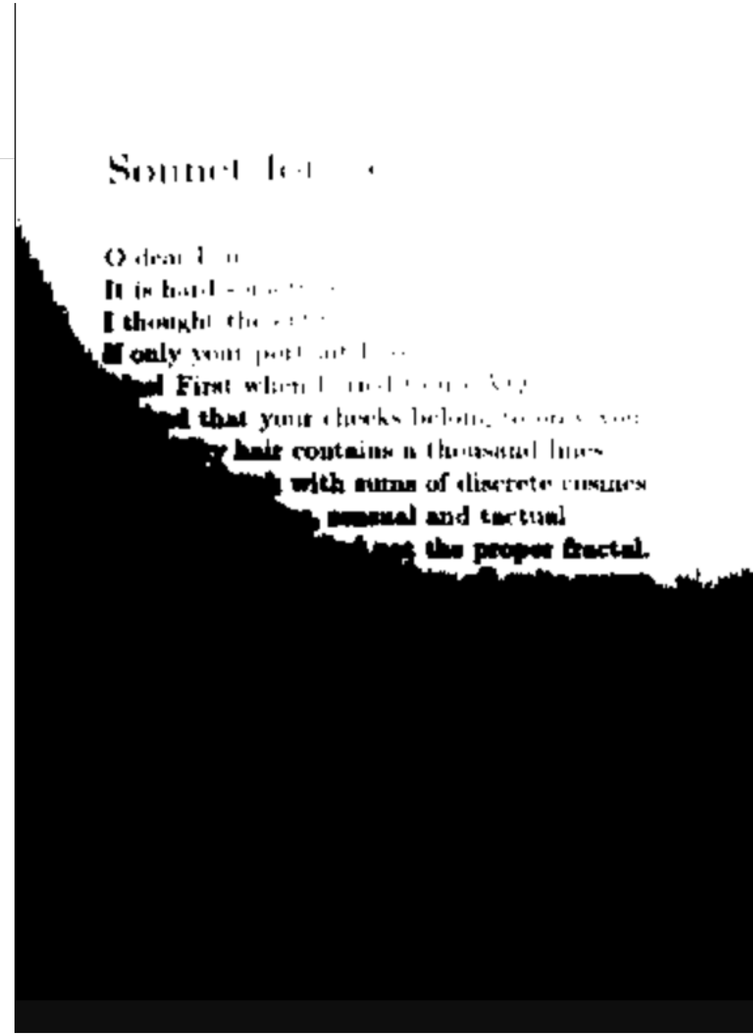


Image Subtraction

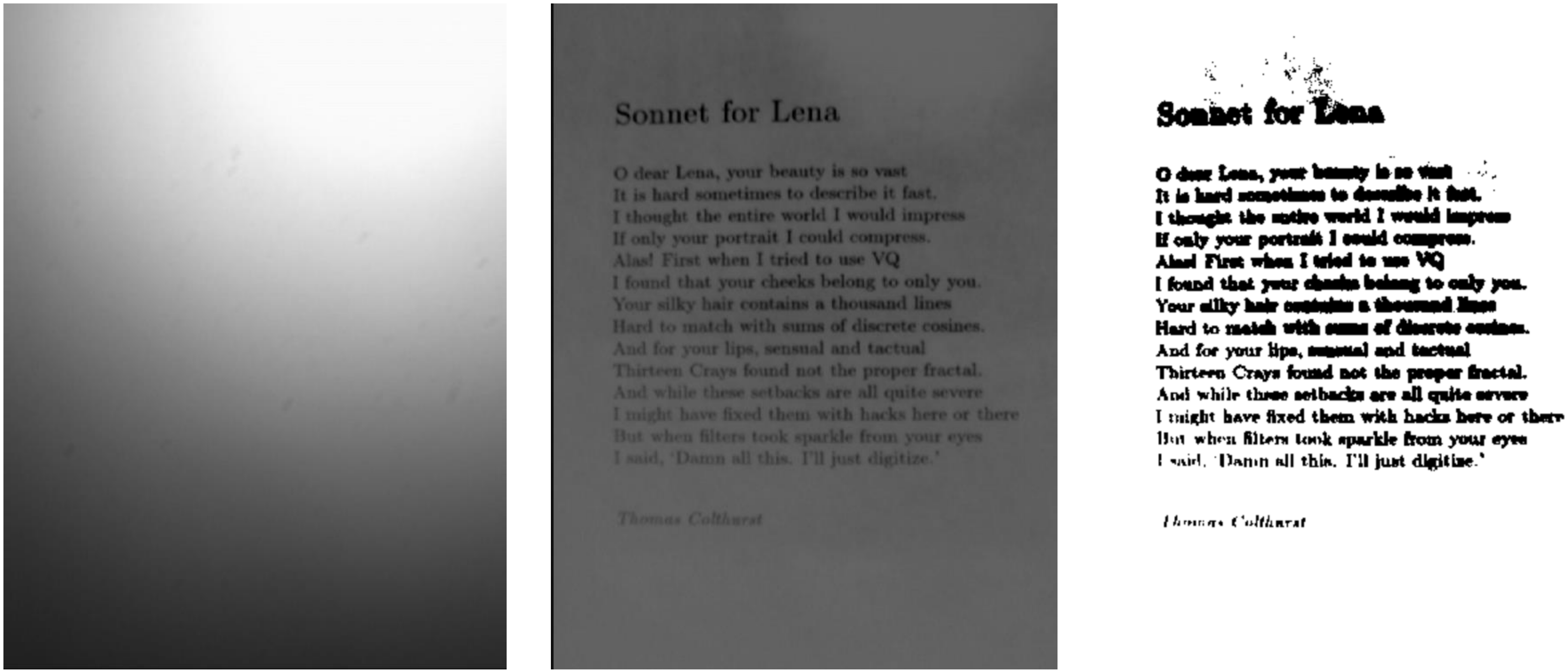


Image Averaging for noise reduction

- Noise is any random (unpredictable) phenomenon that contaminates an image.

- Noise is inherent in most practical systems:
  - Image acquisition
  - Image transmission
  - Image recording

- Noise is typically modeled as an additive process:

$$\underbrace{g(m,n)}_{\text{Noisy Image}} = \underbrace{f(m,n)}_{\text{Noise-free Image}} + \underbrace{\eta(m,n)}_{\text{Noise}}$$

- The noise $\eta(m, n)$ at each pixel (m, n) is modeled as a random variable.

- Usually, $\eta(m, n)$ has **zero-mean** and the noise values at different pixels are **uncorrelated**.

- Suppose we have M observations $\{g_i(m, n)\}$, i=1, 2, …, M, we can (partially) mitigate the effect of noise by "averaging"

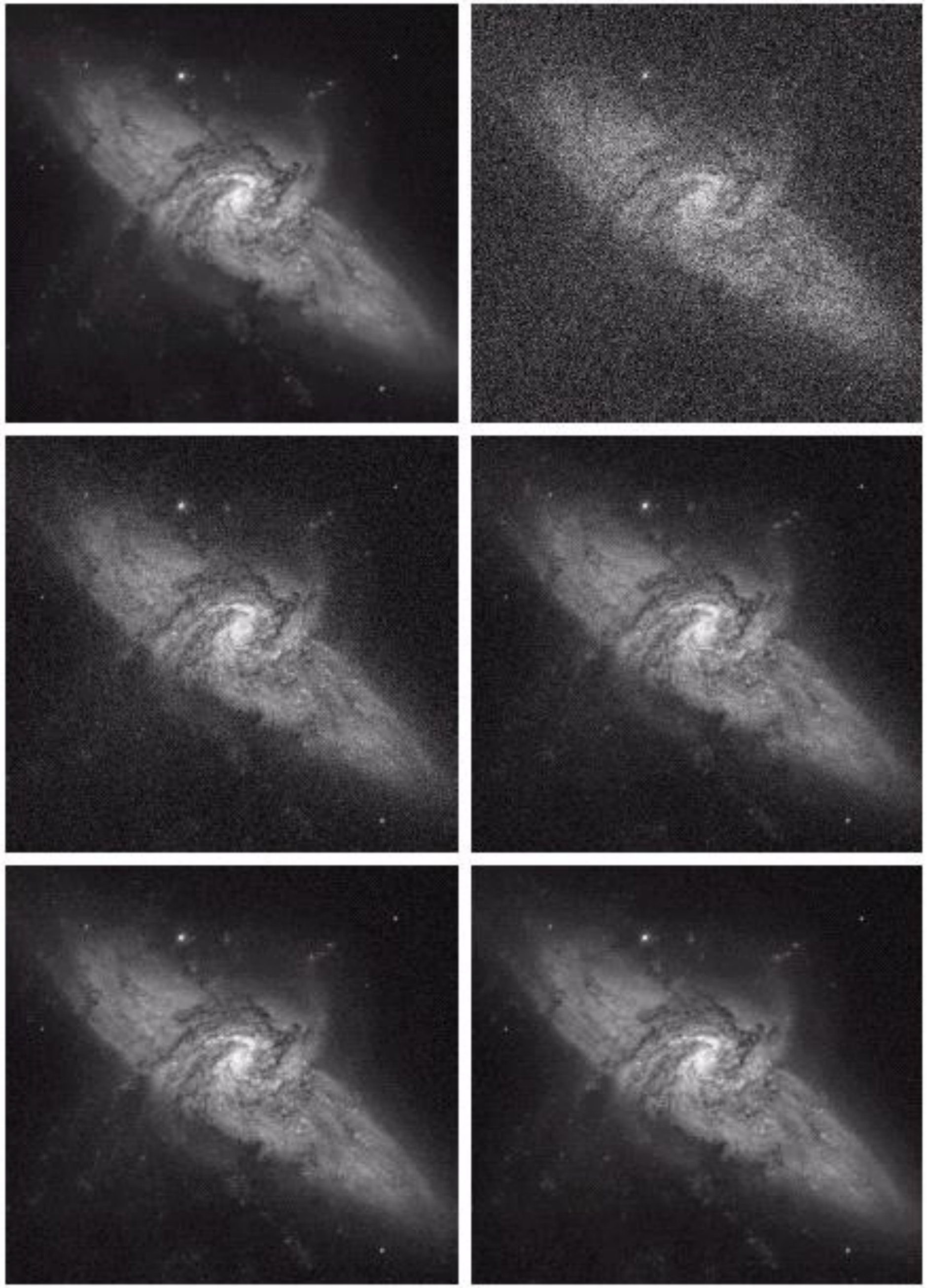
$$\overline{g}(m,n) = \frac{1}{M}\sum_{i=1}^{M} g_i(m,n)$$

- In this case, we can show that

$$E[\overline{g}(m,n)] = f(m,n)$$

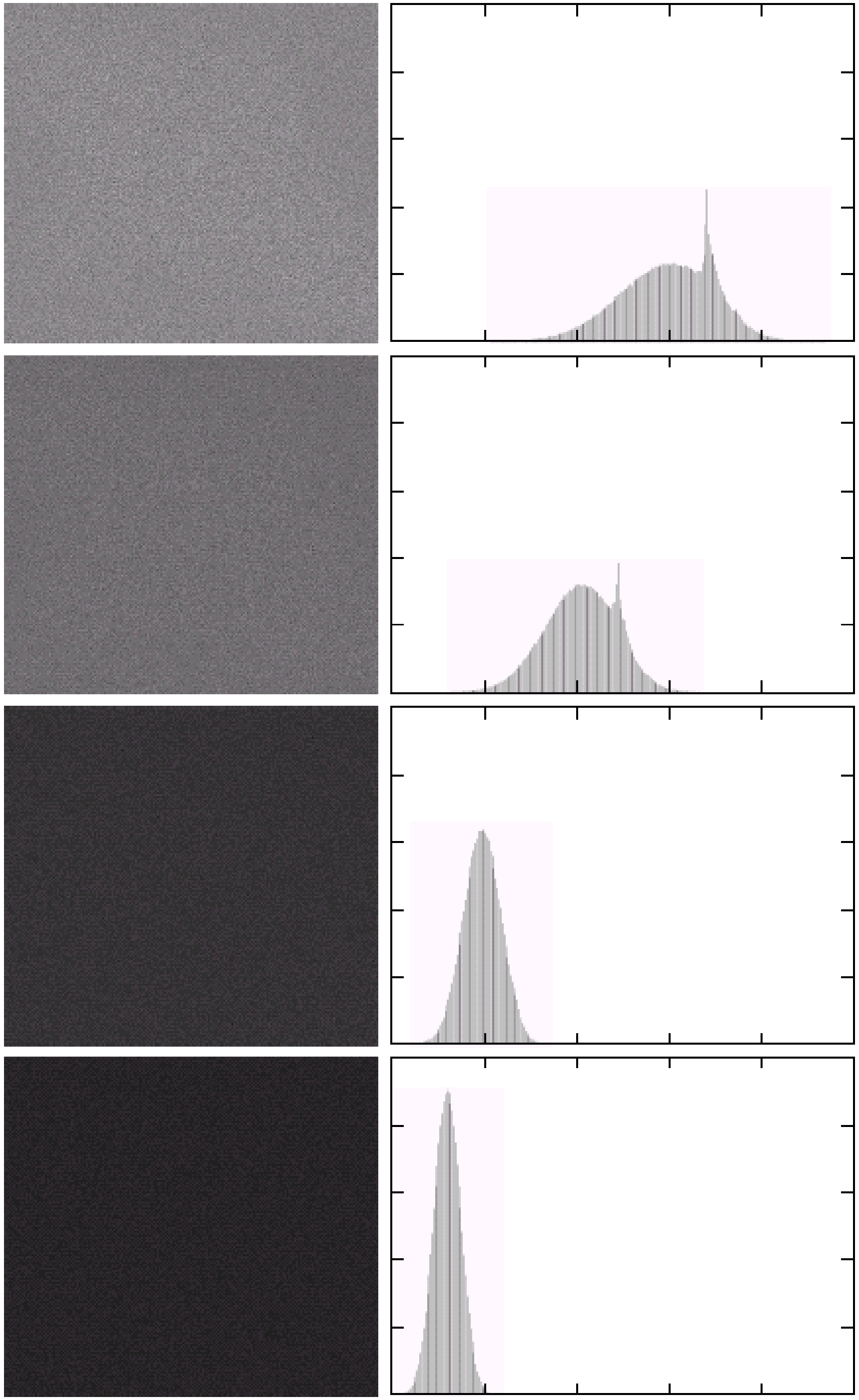
$$Var[\overline{g}(m,n)] = \frac{1}{M} Var[\eta(m,n)]$$

- Therefore, as the number of observations increases ($M \to \infty$), the effect of noise tends to zero.



a b
c d
e f

(a) Image of Galaxy Pair NGC3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)-(f) Results of averaging K = 8, 16, 64, and 128 noisy images. (Original image courtesy of NASA).



a b

(a) From top to bottom: Difference images between Fig.3.30 (a) and the four images in Figs.3.30(c) through (f), respectively. (b) Corresponding histograms.