Problem 3

Similar to 2, I only estimate beta with bias = 0, in all the tests below, I use

- warmup of learning rate
- learning rate decay for non-adaptive methods

In [3]:

```
from hw_1_3 import *
_, beta_est = nag(X, Y_gt, 0.01, d, beta_0)
```

```
NAG ended with L_1 diff as:  1.1847692899228934
Total time: 89.13268494606018 total steps: 10001
```

In [4]:

```
gd_loss = gd(X, Y_gt, 0.01, d, beta_est)
```

```
vanilla GD ended with L_1 diff as:  42.08500024090012
Total time: 86.5514760017395 total steps: 10001
```

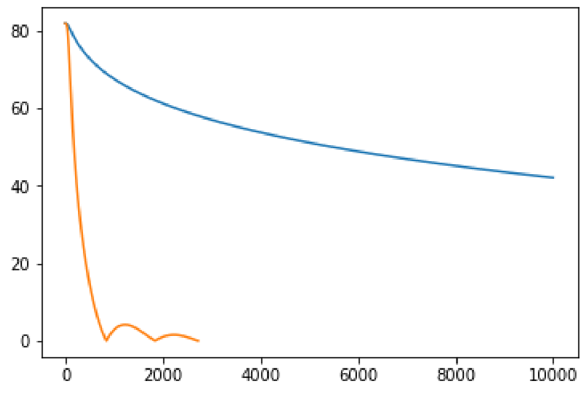
In [11]:

```
nag_loss, _ = nag(X, Y_gt, 0.01, d, beta_est)
```

```
NAG ended with L_1 diff as:  0.0009671487964988679
Total time: 24.532387495040894 total steps: 2722
```

In [12]:

```
import matplotlib.pyplot as plt
%matplotlib inline
plt.plot(gd_loss)
plt.plot(nag_loss)
plt.show()
```



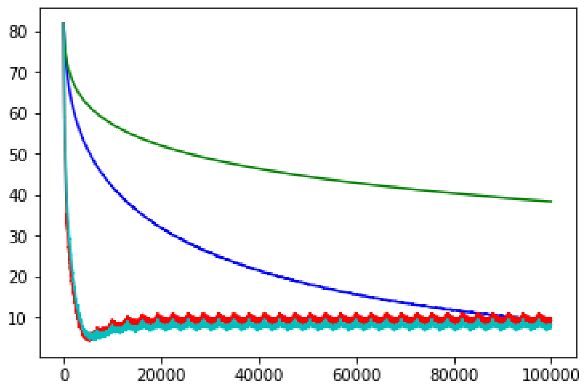
We see from above plot that

- NAG converges faster
- NAG yields better results at termination time
- L_1 loss from NAG shows disturbance
  - such disturbance goes smaller as time goes

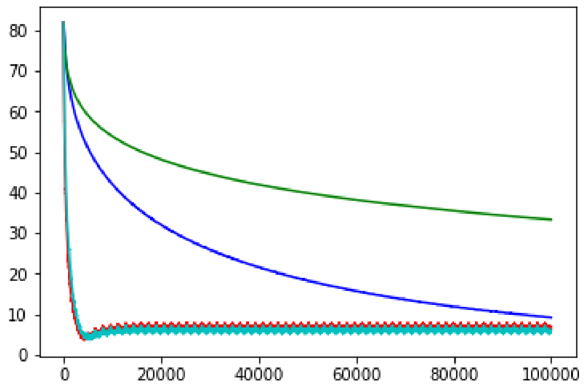
In [13]:

```
import matplotlib.pyplot as plt
%matplotlib inline
for batch_size in [32, 64, 128]:
    sgd_loss = sgd(X, Y_gt, 0.01, d, batch_size, beta_est)
    adagrad_loss = adagrad(X, Y_gt, 0.01, d, 1e-8, batch_size, beta_est)
    rmsprop_loss = rmsprop(X, Y_gt, 0.01, d, 1e-8, batch_size, beta_est)
    adam_loss = adam(X, Y_gt, 0.01, d, 0.9, 0.999, 1e-8, batch_size, beta_est)
    plt.clf()
    plt.plot(sgd_loss, color='b')
    plt.plot(adagrad_loss, color='g')
    plt.plot(rmsprop_loss, color='r')
    plt.plot(adam_loss, color='c')
    plt.show()
```

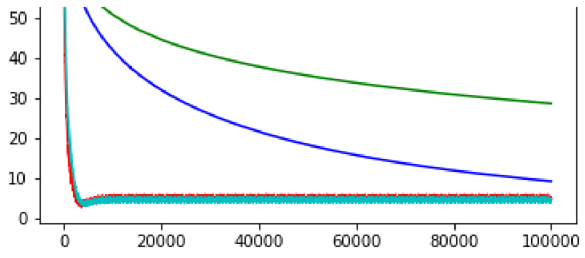
```
SGD ended with L_1 diff as:  9.089506974905682
Total time: 2.2679333686828613
AdaGrad ended with L_1 diff as:  38.38244586420935
Total time: 2.4318063259124756
RMSprop ended with L_1 diff as:  9.675100723387237
Total time: 2.524564504623413
Adam ended with L_1 diff as:  8.363492159318252
Total time: 3.2511160373687744
```



```
SGD ended with L_1 diff as:  9.119980024670259
Total time: 3.530276298522949
AdaGrad ended with L_1 diff as:  33.32072177295851
Total time: 3.88824725151062
RMSprop ended with L_1 diff as:  6.808032554722099
Total time: 4.166238307952881
Adam ended with L_1 diff as:  6.135580491122062
Total time: 5.348047733306885
```



```
SGD ended with L_1 diff as:  9.130085547588333
Total time: 4.361374616622925
AdaGrad ended with L_1 diff as:  28.583852256471978
Total time: 4.767404556274414
RMSprop ended with L_1 diff as:  4.8515965149099864
Total time: 4.812380075454712
Adam ended with L_1 diff as:  4.491760740311056
Total time: 5.9629807472229
```



We conclude from above plot that

- for convergence speed: AdaGrad < SGD < RMSprop = Adam
    - only in this specific setting
    - Adam is slightly better than RMSprop
- AdaGrad does suffer from gradient vanishing
- generally speaking, all algorithms performs better under larger batch size
    - except for SGD, which hits the limit of 9.1x
- smaller batch size causes disturbance in L_1 loss after convergence of RMSprop and Adam
    - with batch size grows, such disturbance gets smaller
- the gap between final result of SGD and (Adam or RMSprop) goes larger for larger batch size

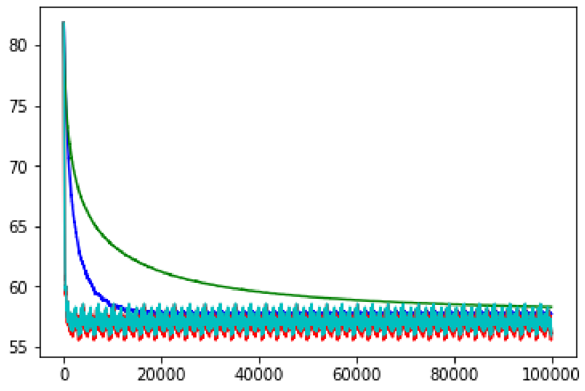
In [14]:

```python
np.random.seed(1234)
sparse_rate = 0.3
M = np.random.uniform(size=(n,d)) < sparse_rate
X[M] = 0.
```

In [15]:

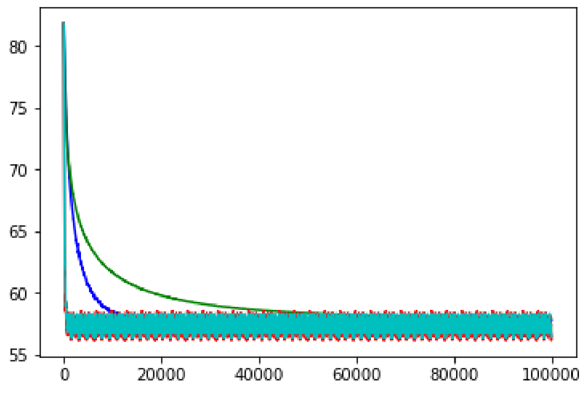
```python
import matplotlib.pyplot as plt
%matplotlib inline
for batch_size in [32, 64, 128]:
    sgd_loss = sgd(X, Y_gt, 0.01, d, batch_size, beta_est)
    adagrad_loss = adagrad(X, Y_gt, 0.01, d, 1e-8, batch_size, beta_est)
    rmsprop_loss = rmsprop(X, Y_gt, 0.01, d, 1e-8, batch_size, beta_est)
    adam_loss = adam(X, Y_gt, 0.01, d, 0.9, 0.999, 1e-8, batch_size, beta_est)
    plt.clf()
    plt.plot(sgd_loss, color='b')
    plt.plot(adagrad_loss, color='g')
    plt.plot(rmsprop_loss, color='r')
    plt.plot(adam_loss, color='c')
    plt.show()
```

```
SGD ended with L_1 diff as:  57.74325215623408
Total time: 2.2330245971679688
AdaGrad ended with L_1 diff as:  58.30687856300753
Total time: 2.4275107383728027
RMSprop ended with L_1 diff as:  56.09351403571297
Total time: 2.569098472595215
Adam ended with L_1 diff as:  56.477997851590615
Total time: 3.358017683029175
```

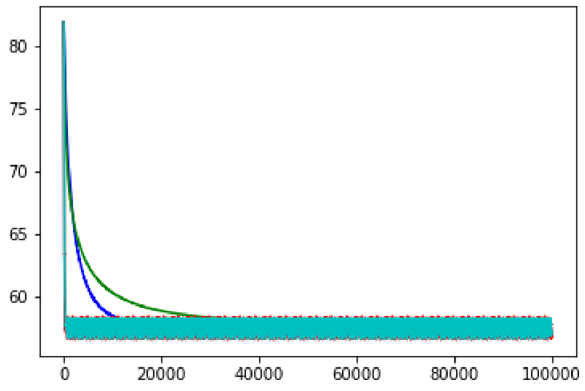


```
SGD ended with L_1 diff as:  57.78443471597452
Total time: 3.466726303100586
```

```
AdaGrad ended with L_1 diff as:  57.95036400912456
Total time: 3.9653944969177246
RMSprop ended with L_1 diff as:  56.533170866174935
Total time: 4.007282972335815
Adam ended with L_1 diff as:  56.664281028993386
Total time: 5.384597063064575
```



```
SGD ended with L_1 diff as:  57.797680800123366
Total time: 4.19179105758667
AdaGrad ended with L_1 diff as:  57.83287014075975
Total time: 4.435136318206787
RMSprop ended with L_1 diff as:  56.886631457256904
Total time: 4.495975494384766
Adam ended with L_1 diff as:  56.93494475625021
Total time: 6.0927064418792725
```



We can also derive conclusions similar to (2), besides

- variation in loss gets larger in sparse cases
- all algorithm hits a limit sooner or later
    - such limit is irrelivant to batch size, thus an inherent limit in this problem

```
In [ ]:
```