Decision Analysis for Management Judgment

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15 The analytic hierarchy process

Introduction

The analytic hierarchy process (AHP) offers an alternative approach to SMART when a decision maker is faced with a problem involving multiple objectives. It can also be used to handle problems involving uncertainty. The method, which was developed by Thomas Saaty when he was acting as an adviser to the US government, has been very widely applied to decision problems in areas such as economics and planning, energy policy, material handling and purchasing, project selection, microcomputer selection, budget allocations and forecasting.[1] Because the AHP involves a relatively complex mathematical procedure a user-friendly computer package, EXPERT CHOICE,[2] has been developed to support the method.

The proponents of the AHP argue that it offers a number of advantages over methods like SMART, and its widespread use is evidence of its popularity among decision makers. Nevertheless, the method is not without its critics who have questioned its axiomatic basis and the extent to which it can lead to a reliable representation of a decision maker's preferences. In this chapter we illustrate the use of the AHP and then make an assessment of the method's relative strengths and limitations.

Choosing a new packaging machine

We will use the following problem to demonstrate the application of the AHP. A manager in a food processing company has to choose a new packaging machine to replace the existing one which is wearing out.

The manager has a limited budget for the purchase and has narrowed down the possible options to three: (i) the Aztec, (ii) the Barton and (iii) the Congress. However, the decision is still proving to be difficult because of the variety of attributes associated with the machines, such as the purchase price, reputation for reliability and the quality of after-sales support provided by the different manufacturers.

An overview of the AHP

Once the decision maker and the alternative courses of action have been identified, the main stages of the method are shown below:

Stage 1: *Set up the decision hierarchy.* This is similar to a value tree in SMART but, as we will see, the main difference is that the alternative courses of action also appear on the hierarchy at its lowest level.

Stage 2: *Make pairwise comparisons of attributes and alternatives.* This is used to determine the relative importance of attributes and also to compare how well the options perform on the different attributes. For example, how much more important is the initial purchase price than the cost of upgrading the machine at a later date? Is the Aztec strongly preferred to the Barton for the quality of after-sales support?

Stage 3: *Transform the comparisons into weights and check the consistency of the decision maker's comparisons.*

Stage 4: *Use the weights to obtain scores for the different options and make a provisional decision.*

Stage 5: *Perform sensitivity analysis.* This will enable the decision maker to examine how robust the provisional decision is to changes in the ratings of importance and preference.

Note that stages 3, 4 and 5 require a computer package, like EXPERT CHOICE, because of the complexity of the calculations that are involved. We next consider each of the stages in more detail.

Setting up the decision hierarchy

Figure 15.1 shows the decision hierarchy for the packaging machine problem. At the top of the tree is a statement of the general objective of the decision, in our case: 'Choose a Machine'. The 'general' attributes associated with the decision problem ('Costs' and 'Quality') are then set out below this. As shown, these attributes can be broken down into more detail at the next level. For example, within 'Quality' the manager wishes to consider the attributes 'Reliability', 'After-Sales Support', 'Speed of Delivery' and 'Customization' (i.e. the extent to which the manufacturer is able to adapt the machine for the specific requirements of the food company). If necessary, this process of breaking down attributes continues until all the essential criteria for making the decision have been specified. Finally, the alternative courses of action are added to the hierarchy, below each of the lowest-level attributes.

Making pairwise comparisons of attributes and alternatives

As we saw in Chapter 2, SMART uses swing weights to compare the 'importance' of attributes, while the performance of options is measured on a 0–100 scale. The AHP uses a fundamentally different approach.

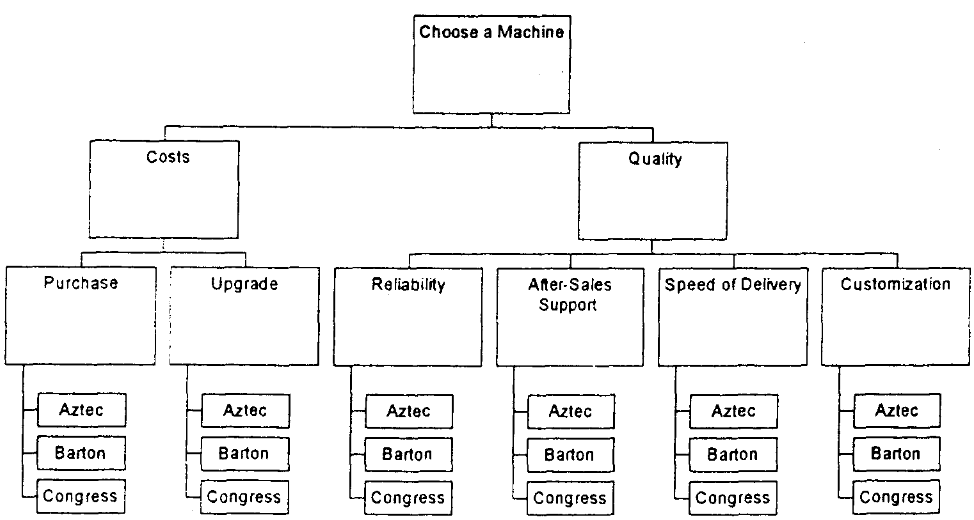

Figure 15.1 – A hierarchy for the packaging machine problem

Following each 'split' in the hierarchy the importance of each attribute is compared, in turn, with every other attribute immediately below that 'split'. Thus the importance of 'Costs' and 'Quality' are first compared. Then the four 'Quality' attributes are compared with each other for importance and so on. Note that the comparisons are pairwise so that if there are four attributes, A, B, C and D, then we need to compare the importance of A with B, A with C, A with D, B with C, B with D and, finally, C with D.

Saaty recommends that these pairwise comparisons should be carried out using verbal responses. For example, the manager is asked to consider whether 'Costs' and 'Quality' are of equal importance or whether one is more importance than the other. The manager indicates that 'Costs' are more important so he is then asked if costs are:

weakly more important (3)
strongly more important (5)
very strongly more important (7)
extremely more important (9)?

The method then converts the response to the number shown in brackets. For example, if 'Costs' are 'strongly more important' than 'Quality' then they are assumed to be five times more important. Note that intermediate responses are allowed if the decision maker prefers these (e.g. 'between weakly and strongly more important, which would be converted to a '4'), as are both graphical and direct numerical inputs on a scale from 1 'equally important' to 9.

Each set of comparisons can be represented in a table (or matrix). From the 'Costs' versus 'Quality' comparison we obtain Table 15.1.

Similarly, for the four 'Quality' attributes the manager's judgments lead to the values in Table 15.2.

The numbers in the tables represent how much more important the 'row' attribute is compared to the 'column' attribute. For example, 'Reliability' is four times more important than 'After-Sales Support'. Fractional values therefore indicate that the 'column' attributes is most

Table 15.1 – Comparing the importance of 'Costs' and 'Quality'

	Costs	Quality
Costs	1	5
Quality		1

Table 15.2 – Comparing the importance of the 'Quality' attributes

	Reliability	After-Sales Support	Speed of Delivery	Customization
Reliability	1	4	5	4
After-Sales Support		1	3	1/2
Speed of Delivery			1	1/3
Customization				1

Table 15.3 – Comparing the machines on 'Purchase Cost'

	Aztec	Barton	Congress
Aztec	1	1/3	2
Barton		1	6
Congress			1

important. For example, 'Speed of Delivery' is only 1/3 as important as 'Customization'. Note that only 1s appear on the diagonal of the tables since each attribute must have equal importance with itself. A similar table is obtained from the manager's comparison of the importance of 'Purchase' and 'Upgrade' costs.

Finally, the same process is used to compare the manager's relative *preferences* for the machines with respect to each of the lower-level attributes. For example, he will be asked to consider the purchase costs of the machines and asked whether, *in terms of purchase costs*, the Aztec and Barton are 'equally preferred'. If he indicates that the Barton is preferred he will then be asked whether it is 'weakly preferred', 'strongly preferred' or 'extremely strongly preferred' (with intermediate responses allowed). This leads to the values in Table 15.3 which shows, for example, that the Aztec is twice as preferable as the Congress on purchase cost.

This process is repeated, yielding a table for each of the lowest-level attributes to represent the manager's preferences for the machines in terms of that attribute.

Obtaining weights and checking consistency

After each table has been obtained the AHP converts it into a set of weights, which are then automatically normalized to sum to 1. A number of conversion methods are possible, but the AHP uses a mathematical approach based on eigenvalues (see Saaty[3] for details of this method). Because of the complexities of this method a computer package like EXPERT CHOICE is essential to carry out the calculations if any of the tables involves more than two rows or columns. Figure 15.2 shows the weights obtained from all the tables in the hierarchy. For table 15.1, where 'Costs' were considered to be five times more important than 'Quality' the derivation of the weights is clear (a 5:1 ratio yields weights of 5/6 and 1/6, i.e. 0.833 and 0.167). The derivation is less transparent for the larger tables. For example, for Table 15.2 the weights are 'Reliability': (0.569), 'After-Sales Service': (0.148), 'Speed of Delivery': (0.074), 'Customization': (0.209) suggesting that the decision maker considered 'Reliability' to be by far the most important of the 'Quality' attributes.

Along with the weights the AHP also yields an inconsistency index (this is produced automatically on EXPERT CHOICE – again see Saaty[3] for details of the method of calculation). The index is designed to alert the decision maker to any inconsistencies in the comparisons which have been made, with a value of zero indicating perfect consistency. For example, suppose a decision maker's responses imply that attribute A is twice as important as B, while B is judged to be three times as important as C. To be perfectly consistent the decision maker should judge that A is six times more important than C. Any other response will lead to a

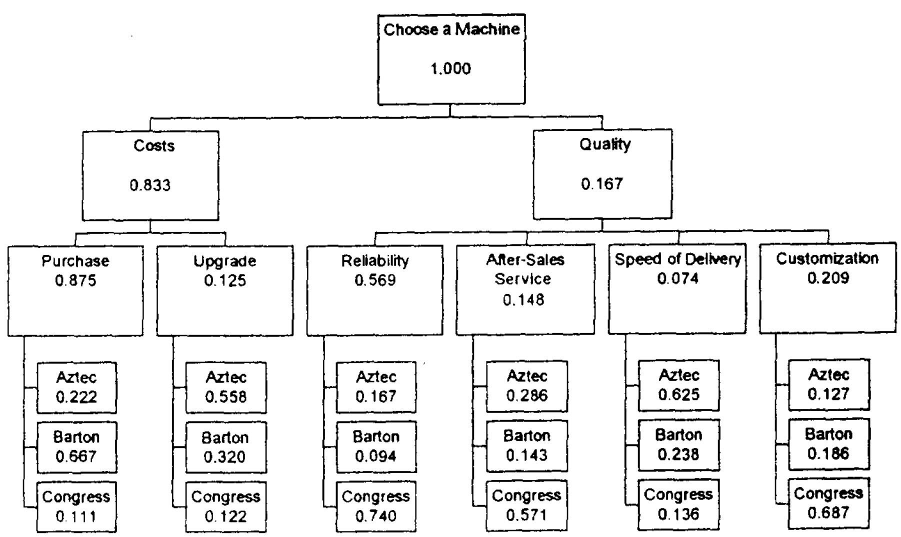

Figure 15.2 – Weights for the packaging machine problem

index of greater than zero. Saaty recommends that inconsistency should only be a concern if the index exceeds 0.1 (as a rule of thumb), in which case the comparisons should be re-examined. Obviously, there can be no inconsistency in Table 15.1, since only one comparison was made. For Tables 15.2 and 15.3, the inconsistency indices were 0.059 and 0, respectively. Values of less than 0.1 were also obtained for all of the other tables in the hierarchy. Saaty stresses, however, that minimizing inconsistency should not be the main goal of the analysis. A set of erroneous judgments about importance and preference may be perfectly consistent, but they will not lead to the 'best' decision.

Combining the weights to make a provisional decision

Although EXPERT CHOICE will automatically calculate the scores for the options it is useful to demonstrate how the score for the Aztec machine was obtained. In Figure 15.2, all of the paths that lead from the top of the hierarchy to the Aztec option are identified. All of the weights in each path are then multiplied together and the results for the different paths summed, as shown below:

$$
\begin{aligned}
\text{Score for Aztec} = {} & 0.833 \times 0.875 \times 0.222 \\
& + 0.833 \times 0.125 \times 0.558 \\
& + 0.167 \times 0.569 \times 0.167 \\
& + 0.167 \times 0.148 \times 0.286 \\
& + 0.167 \times 0.074 \times 0.625 \\
& + 0.167 \times 0.209 \times 0.127 = 0.255
\end{aligned}
$$

Note that the Aztec scores well on attributes which are considered to be relatively unimportant such as 'Upgrade Costs' (which carries only 0.125 of the 0.833 weight allocated to costs) and 'Speed of Delivery' (which carries only 0.074 of the weight allocated to 'Quality', which itself is relatively unimportant). It scores less well on the more important attributes so its overall score is relatively low. The scores for all three machines are shown below:

Aztec	0.255
Barton	0.541
Congress	0.204

This clearly suggests that the Barton should be purchased.

Sensitivity analysis

As in any decision model it is important to examine how sensitive the preferred course of action is to changes in the judgments made by the decision maker. Many of these judgments will be 'rough and ready' and the decision maker may be unsure about exactly what judgments to input. EXPERT CHOICE has a number of facilities for carrying out sensitivity analysis. In dynamic sensitivity analysis a bar chart shows the weights attached to attributes at a particular level in the hierarchy. By changing the lengths of these bars, the effect on the scores of the alternative courses of action can be examined. Other graphs allow decision makers to examine the amount of change that can be made to an attribute's weight before the preferred course of action changes.

Strengths and criticisms of the AHP

It can be seen that the AHP is fundamentally different to SMART in many respects. We next consider the relative strengths of the AHP and then look at the main criticisms which have been made of the technique.

The relative strengths of the AHP

(1) *Formal structuring of problem.* Like SMART and other decision analysis techniques the AHP provides a formal structure to problems. This allows complex problems to be decomposed into sets of simpler judgments and provides a documented rationale for the choice of a particular option.

(2) *Simplicity of pairwise comparisons.* The use of pairwise comparisons means that the decision maker can focus, in turn, on each small part of the problem. Only two attributes or options have to be considered at any one time so that the decision maker's judgmental task is simplified. Verbal comparisons are also likely to be preferred by decision makers who have difficulty in expressing their judgments numerically.

(3) *Redundancy allows consistency to be checked.* The AHP requires more comparisons to be made by the decision maker than are needed to establish a set of weights. For example, if a decision maker indicates

that attribute A is twice as important as B, and B, in turn, is four times as important as C, then it can be inferred that A is eight times more important than C. However, by also asking the decision maker to compare A with C it is possible to check the consistency of the judgments. It is considered to be good practice in decision analysis to obtain an input to a decision model by asking for it in several ways and then asking the decision maker to reflect on any inconsistencies in the judgments put forward. In the AHP this is carried out automatically.

(4) *Versatility*. The wide range of applications of the AHP is evidence of its versatility. In addition to judgments about importance and preference, the AHP also allows judgments about the relative likelihood of events to be made. This has allowed it to be applied to problems involving uncertainty and also to be used in forecasting.[4,5,6,7] AHP models have also been used to construct scenarios by taking into account the likely behavior and relative importance of key actors and their interaction with political, technological, environmental, economic and social factors (Saaty,[3] page 130).

Criticisms of the AHP

(1) *Conversion from verbal to numeric scale*. Decision makers using the verbal method of comparison will have their judgments automatically converted to the numeric scale, but the correspondence between the two scales is based on untested assumptions. If you indicate that A is weakly more important than B the AHP will assume that you consider A to be three times more important, but this may not be the case. In particular, several authors have argued that a multiplicative factor of 5 is too high to express the notion of 'strong' preference.[7]

(2) *Inconsistencies imposed by 1 to 9 scale*. In some problems the restriction of pairwise comparisons to a 1 to 9 scale is bound to force inconsistencies on the decision maker. For example, if A is considered to be 5 times more important than B, and B is 5 times more important than C, then to be consistent A should be judged to be 25 times more important than C, but this is not possible.

(3) *Meaningfulness of responses to questions*. Unlike SMART, weights are elicited in the AHP without reference to the scales on which attributes are measured. For example, a person using SMART to choose a house might be asked to compare the value of reducing the daily journey to work from 80 miles to 10 miles with the value of

increasing the number bedrooms in the house from two to four. Implicit in this type of comparison is the notion of a trade-off or exchange: 70 fewer miles may be only half as valuable as two extra bedrooms. AHP questions, which simply ask for the relative importance of attributes without reference to their scales, are therefore less well defined (if they are meaningful at all). This fuzziness may mean that the questions are interpreted in different, and possibly erroneous, ways by decision makers.[8,9]

(4) *New alternatives can reverse the rank of existing alternatives.* This issue, which is related to the last point, has attracted much attention. Suppose that you are using the AHP to choose a location for a new sales office and the weights you obtained from the method give the following order of preference: 1. Albuquerque, 2. Boston, 3. Chicago. However, before making the decision you discover that a site in Denver is also worth considering so you repeat the AHP to include this new option. Even though you leave the relative importance of the attributes unchanged, the new analysis gives the following rankings: 1. Boston, 2. Albuquerque, 3. Denver, 4. Chicago, so the rank of Albuquerque and Boston has been reversed, which does not seem to be intuitively reasonable. Belton and Gear[10] showed that this arises from the way in which the AHP normalizes the weights to sum to 1. They went on to show that this is consistent with a definition of weights which is at variance with that used in SMART (see above). Most decision makers, they argued, would consider the SMART definition to be the reasonable one.

(5) *Number of comparisons required may be large.* While the redundancy built into the AHP is an advantage, it may also require a large number of judgments from the decision maker. Consider, for example, the office location problem in Chapter 2, which involved 7 alternatives and 7 attributes (if we simplify the problem to include 'Total Costs' and only lower-level benefit attributes). This would involve 168 pairwise comparisons of importance or preference. In a study by Olson *et al.*[9] this requirement to answer a large number of questions reduced the attraction of the AHP in the eyes of potential users, even though the questions themselves were considered to be easy.

(6) *The axioms of the method.* As we saw in Chapter 2, SMART is well founded on a set of axioms, that is, a set of rules which are intended to provide the basis for rational decision making. Dyer[11] has argued that the clarity and intuitive meaning of these axioms allows their appeal, as rules for rational behavior to be debated and empirically

tested. In contrast, he argues, the axioms of the AHP[3] are not founded on testable descriptions of rational behavior (see Harker and Vargas[12] for a reply to Dyer's paper).

Conclusion

The AHP is a versatile decision aid which can handle problems involving both multiple objectives and uncertainty. It is popular with many decision makers who find the questions it poses easy to answer and the EXPERT CHOICE software user friendly. Its applications have led to a huge number of published papers. Nevertheless, the method has also attracted much controversy from people who have questioned its underlying axioms and the extent to which the questions which it poses can lead to meaningful responses from decision makers. Indeed, it has been argued that the apparent simplicity of the questions belies a lack of clarity in their definition and may lead to superficial and erroneous judgments.[13] Critics have also questioned the extent to which an AHP model can faithfully represent a decision maker's preferences given the numerical representations of these judgments and the mathematical processes which are applied to them.

We should, of course, not forget that the purpose of any decision aid is to provide insights and understanding, rather than to prescribe a 'correct' solution. Often the *process* of attempting to structure the problem is more useful in achieving these aims than the numeric output of the model. Nevertheless, this process is still best served when the analytic method poses unambiguous questions and bases its suggested solutions on testable axioms and an accurate translation of the decision maker's judgments. Whether the AHP is the best technique to support this process is a question which is bound to continue to attract debate and controversy.

Exercises

(1) A decision maker is using the AHP to choose a new car. The table below shows his assessment of the relative importance of three

attributes associated with the decision. Are these judgments perfectly consistent?

	Cost	*Style*	*Comfort*
Cost	1	2	4
Style		1	2
Comfort			1

(2) A manager is using the AHP to decide which of three computer maintenance firms (Compex, Debug and EAC) should be awarded a three-year contract. Her preferences for the companies on the basis of their reputation for reliability are given below. Are the manager's judgments perfectly consistent?

	Compex	*Debug*	*EAC*
Compex	1	1/2	2
Debug		1	6
EAC			1

(3) For the hierarchy shown in Figures 15.1 and 15.2 show how the score for the Barton machine was obtained and explain why this machine obtained the highest score.

(4) One of the criticisms of the AHP is that the introduction of new alternatives can change the ranking of existing alternatives. Under what circumstances, if any, is this likely to be reasonable?

(5) A manager is hoping to appoint a new assistant and decides to use the AHP to rank the applicants for the job. Then, as a check, she decides to repeat the process using SMART. She is surprised to find that the ranking of the applicants derived from the AHP differs significantly from the ranking suggested by the SMART analysis. Discuss why these differences might have arisen.

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