2

Decisions involving multiple objectives

Introduction

Many decision problems involve a number of objectives, and often these objectives conflict. For example, a local authority, faced with a decision on the route of a new road, might have to balance objectives such as minimizing cost and minimizing environmental damage. If the route incurring the lowest construction costs would also lead to the destruction of an important wildlife habitat then some judgment would have to be made about the relative importance of the objectives. Similarly, an individual choosing a house will have to balance factors such as cost, number of bedrooms, closeness to work, facilities available in the neighborhood and so on.

This chapter will examine how decisions involving multiple objectives can be analyzed. As we stated in Chapter 1, the central idea is that, by splitting the problem into small parts and focusing on each part separately, the decision maker is likely to acquire a better understanding of the problem than that which would be achieved by taking a holistic view. The unaided decision maker has 'limited information-processing capacity' (Wright[1]) and, when faced with a large and complex problem, there may be too much information for him to handle simultaneously. (We will look at some of the biases that are associated with unaided decision making in the next chapter.) It can also be argued that, by requiring a commitment of time and effort, analysis encourages the decision maker to think deeply about his problem, enabling him to develop a rationale which is explicit and defensible. After such analysis the decision maker should be better able to explain and justify why a particular option is favored.

The methodology outlined in this chapter is underpinned by a set of axioms. We will discuss these towards the end of the chapter, but, for the moment, we can regard them as a set of generally accepted propositions or 'a formalization of common sense' (Keeney[2]). If the decision maker accepts the axioms then it follows that the results of the analysis will indicate how he should behave if he is rational. The analysis is therefore normative or prescriptive; it shows which alternative should be chosen if the decision maker acts consistently with his stated preferences.

The method explained here is normally applied in situations where a particular course of action is regarded as certain (or virtually certain) to lead to a given outcome so that uncertainty is not a major concern of the analysis. (We will consider, in later chapters, techniques which are designed to be used where risk and uncertainty are central concerns of the decision maker.) Nevertheless, there are exceptions to this rule, and we will show later how the method can be adapted to problems involving risk and uncertainty.

It should be emphasized that the main role of our analysis is to enable the decision maker to gain an increased understanding of his or her decision problem. If at the end of this analysis no single best course of action has been identified, this does not mean that the analysis was worthless. Often the insights gained may suggest other approaches to the problem or lead to a greater common understanding among a heterogeneous group of decision makers. They may lead to a complete reappraisal of the nature of the problem or enable a manager to reduce a large number of alternatives to a few, which can then be put forward to higher management with arguments for and against. Because of this, the analysis is flexible. Although we present it in this chapter as a series of stages, the decision maker is always free at any point to return to an earlier stage or to change the definition of the problem. Indeed, it is likely that this will happen as a deeper understanding of the nature of the problem is gained through the analysis.

Basic terminology

Objectives and attributes

Before proceeding, we need to clarify some of the basic terms we will be using. An *objective* has been defined by Keeney and Raiffa[3] as an indication of the preferred direction of movement. Thus, when stating objectives, we use terms like 'minimize' or 'maximize'. Typical objectives might be to minimize costs or maximize market share. An *attribute* is used to measure performance in relation to an objective. For example, if we have the objective 'maximize the exposure of a television advertisement' we may use the attribute 'number of people surveyed who recall seeing the advertisement' in order to measure the degree to which the objective was achieved. Sometimes we may have to use an attribute which is not directly related to the objective. Such an attribute is referred to as a *proxy attribute*. For example, a company may use the proxy attribute 'staff turnover' to measure how well they are achieving their objective of maximizing job satisfaction for their staff.

Value and utility

For each course of action facing the decision maker we will be deriving a numerical score to measure its attractiveness to him. If the decision involves no element of risk and uncertainty we will refer to this score as the *value* of the course of action. Alternatively, where the decision involves risk and uncertainty, we will refer to this score as the *utility* of the course of action. Utility will be introduced in Chapter 5.

An office location problem

The following problem will be used to illustrate the analysis of decisions involving multiple objectives. A small printing and photocopying business must move from its existing office because the site has been acquired for redevelopment. The owner of the business is considering seven possible new offices, all of which would be rented. Details of the location of these offices and the annual rent payable are given below.

Location of office		*Annual rent* ($)
Addison Square	(A)	30 000
Bilton Village	(B)	15 000
Carlisle Walk	(C)	5 000
Denver Street	(D)	12 000
Elton Street	(E)	30 000
Filton Village	(F)	15 000
Gorton Square	(G)	10 000

While the owner would like to keep his costs as low as possible, he would also like to take other factors into account. For example, the Addison Square office is in a prestigious location close to potential customers, but it is expensive to rent. It is also an old, dark building which will not be comfortable for staff to work in. In contrast, the Bilton Village office is a new building which will provide excellent working conditions, but it is several miles from the center of town, where most potential customers are to be found. The owner is unsure how to set about making his choice, given the number of factors involved.

An overview of the analysis

The technique which we will use to analyse the office location problem is based on the Simple Multi-attribute Rating Technique (SMART), which was put forward by Edwards[4] in 1971. Because of the simplicity of both the responses required of the decision maker and the manner in which these responses are analyzed, SMART has been widely applied. The analysis involved is transparent, so the method is likely to yield an enhanced understanding of the problem and be acceptable to the decision maker who is distrustful of a mathematical 'black-box' approach. This, coupled with the relative speed by which the method can be applied, means that SMART has been found to be a useful vehicle for decision conferences (see Chapter 11), where groups of decision makers meet to consider a decision problem. The cost of this simplicity is that the method may not capture all the detail and complexities of the real problem. Nevertheless, in practice, the approach has been found to be extremely robust (see Watson and Buede[5]).

The main stages in the analysis are shown below:

Stage 1: *Identify the decision maker (or decision makers).* In our problem we will assume that this is just the business owner, but in Chapter 12 we will look at the application of SMART to problems involving groups of decision makers.

Stage 2: *Identify the alternative courses of action.* In our problem these are, of course, represented by the different offices the owner can choose.

Stage 3: *Identify the attributes which are relevant to the decision problem.* The attributes which distinguish the different offices will be factors such as rent, size and quality of working conditions. In the next section we will show how a value tree can be of use when identifying relevant attributes.

Stage 4: For each attribute, assign values to *measure the performance of the alternatives on that attribute.* For example, how well do the offices compare when considering the quality of the working conditions they offer?

Stage 5: *Determine a weight for each attribute.* This may reflect how important the attribute is to the decision maker (though we will discuss the problem of using importance weights later).

Stage 6: *For each alternative, take a weighted average of the values assigned to that alternative.* This will give us a measure of how well an office performs over all the attributes.

Stage 7: *Make a provisional decision.*

Stage 8: *Perform sensitivity analysis* to see how robust the decision is to changes in the figures supplied by the decision maker.

Constructing a value tree

Stages 1 and 2 of our analysis have already been completed: we know who the decision maker is and we have identified the courses of action open to him. The next step is to identify the attributes which the decision maker considers to be relevant to his problem. You will recall that an attribute is used to measure the performance of courses of action in relation to the objectives of the decision maker. This means that we need to arrive at a set of attributes which can be assessed on a numeric scale. However, the initial attributes elicited from the decision maker may be vague (e.g. he might say that he is looking for the office which will be 'the best for his business'), and they may therefore need to be broken down into more specific attributes before measurement can take place. A value tree can be useful here, and Figure 2.1 shows a value tree for this problem.

We start constructing the tree by addressing the attributes which represent the general concerns of the decision maker. Initially, the owner identifies two main attributes, which he decides to call 'costs' and 'benefits'. There is, of course, no restriction on the number of attributes which the decision maker can initially specify (e.g. our decision maker might have specified 'short-term costs', 'long-term costs',

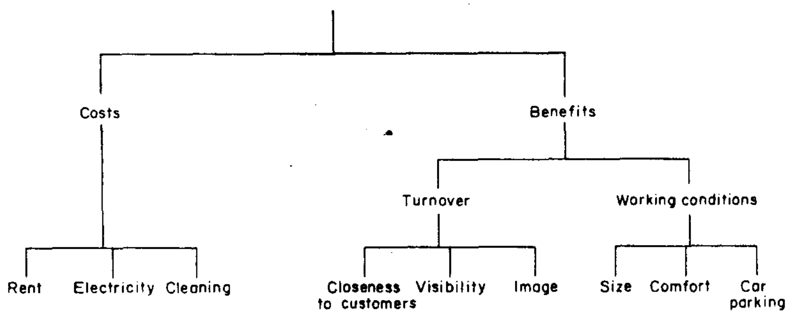

Figure 2.1 – A value tree for the office location problem

'convenience of the move' and 'benefits' as his initial attributes). Nor is there any requirement to categorize the main attributes as costs and benefits. In some applications (e.g. Wooler and Barclay[6]) 'the risk of the options' is an initial attribute. Buede and Choisser[7] describe an engineering design application for the US Defense Communications Agency, where the main attributes are 'the effectiveness of the system' (i.e. factors such as quality of performance, survivability in the face of physical attack, etc.) and 'Implementation' (i.e. manning, ease of transition from the old system, etc.).

Having established the main attributes for our business owner, we need to decompose them to a level where they can be assessed. The owner identifies three main costs that are of concern to him: the annual rent, the cost of electricity (for heating, lighting, operating equipment, etc.) and the cost of having the office regularly cleaned. Similarly, he decides that benefits can be subdivided into 'potential for improved turnover' and 'staff working conditions'. However, he thinks that he will have difficulty assessing each office's potential for improving turnover without identifying those attributes which will have an impact on turnover. He considers these attributes to be 'the closeness of the office to potential customers', 'the visibility of the site' (much business is generated from people who see the office while passing by) and 'the image of the location' (a decaying building in a back street may convey a poor image and lead to a loss of business). Similarly, the owner feels that he will be better able to compare the working conditions of the offices if he decomposes this attribute into 'size', 'comfort' and 'car parking facilities'.

Having constructed a value tree, how can we judge whether it is an accurate and useful representation of the decision maker's concerns?

Keeney and Raiffa[3] have suggested five criteria which can be used to judge the tree:

(i) *Completeness.* If the tree is complete, all the attributes which are of concern to the decision maker will have been included.
(ii) *Operationality.* This criterion is met when all the lowest-level attributes in the tree are specific enough for the decision maker to evaluate and compare them for the different options. For example, if our decision maker felt that he was unable to judge the 'image' of the locations on a numeric scale, the tree would not be operational. In this case we could attempt to further decompose image into new attributes which were capable of being assessed, or we could attempt to find a proxy attribute for image.
(iii) *Decomposability.* This criterion requires that the performance of an option on one attribute can be judged independently of its performance on other attributes. In our problem, if the owner feels unable to assess the comfort afforded by an office without also considering its size, then decomposability has not been achieved, and we will need to look again at the tree to see if we can redefine or regroup these attributes.
(iv) *Absence of redundancy.* If two attributes duplicate each other because they actually represent the same thing then one of these attributes is clearly redundant. The danger of redundancy is that it may lead to double-counting, which may cause certain objectives to have undue weight when the final decision is made. One way of identifying redundancy is to establish whether the decision would in any way be affected if a given attribute was eliminated from the tree. If the deletion of the attribute would not make any difference to the choice of the best course of action then there is no point in including it.
(v) *Minimum size.* If the tree is too large any meaningful analysis may be impossible. To ensure that this does not happen, attributes should not be decomposed beyond the level where they can be evaluated. Sometimes the size of the tree can be reduced by eliminating attributes which do not distinguish between the options. For example, if all the offices in our problem offered identical car-parking facilities then this attribute could be removed from the tree.

Sometimes it may be necessary to find compromises between these criteria. For example, to make the tree operational it may be necessary to increase its size. Often several attempts at formulating a tree may be required before an acceptable structure is arrived at. This process of modification is well described in an application reported by Brownlow and Watson,[8] where a value tree was being used in a decision problem relating to the transportation of nuclear waste. The tree went through a number of stages of development as new insights were gained into the nature of the problem.

Measuring how well the options perform on each attribute

Having identified the attributes which are of concern to the owner, the next step is to find out how well the different offices perform on each of the lowest-level attributes in the value tree. Determining the annual costs of operating the offices is relatively straightforward. The owner already knows the annual rent and he is able to obtain estimates of cleaning and electricity costs from companies which supply these services. Details of all these costs are given in Table 2.1.

At a later stage in our analysis we will need to trade off the costs against the benefits. This can be an extremely difficult judgment to make. Edwards and Newman[9] consider this kind of judgment to be 'the least secure and most uncomfortable to make' of all the judgments required in decisions involving multiple objectives. Because of this we will now ignore the costs until the end of our analysis and, for the moment, simply concentrate on the benefit attributes.

Table 2.1 – Costs associated with the seven offices

Office	Annual rent ($)	Annual cleaning costs ($)	Annual electricity costs ($)	Total cost ($)
Addison Square	30 000	3 000	2 000	35 000
Bilton Village	15 000	2 000	800	17 800
Carlisle Walk	5 000	1 000	700	6 700
Denver Street	12 000	1 000	1 100	14 100
Elton Street	30 000	2 500	2 300	34 800
Filton Village	15 000	1 000	2 600	18 600
Gorton Square	10 000	1 100	900	1 2000

In measuring these attributes our task will be made easier if we can identify variables to represent the attributes. For example, the size of an office can be represented by its floor area in square feet. Similarly, the distance of the office from the town center may provide a suitable approximation for the attribute 'distance from potential customers'. However, for other attributes such as 'image' and 'comfort' it will be more difficult to find a variable which can be quantified. Because of this, there are two alternative approaches which can be used to measure the performance of the offices on each attribute: direct rating and the use of value functions.

Direct rating

Let us first consider those attributes which cannot be represented by easily quantifiable variables, starting with the attribute 'image'. The owner is first asked to rank the locations in terms of their image from the most preferred to the least preferred. His rankings are:

1. Addison Square
2. Elton Street
3. Filton Village
4. Denver Street
5. Gorton Square
6. Bilton Village
7. Carlisle Walk

Addison Square, the best location for image, can now be given a value for image of 100 and Carlisle Walk, the location with the least appealing image, can be given a value of 0. As we explain below, any two numbers could have been used here as long as the number allocated to the most-preferred location is higher than that allocated to the least preferred. However, the use of 0 and 100 makes the judgments which follow much easier and it also simplifies the arithmetic.

The owner is now asked to rate the other locations in such a way that the space between the values he gives to the offices represents his strength of preference for one office over another in terms of image. Figure 2.2. shows the values allocated by the owner. This shows that the *improvement* in image between Carlisle Walk and Gorton Square is perceived by the owner to be twice as preferable as the improvement in image between Carlisle Walk and Bilton Village. Similarly, the

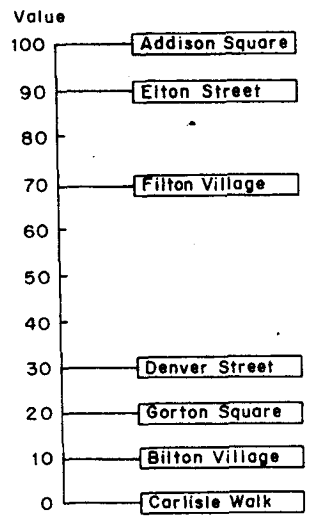

Figure 2.2 – A value scale for office image

improvement in image between Carlisle Walk and Addison Square is seen to be ten times more preferable than the improvement between Carlisle Walk and Bilton Village.

Note that it is the *interval* (or improvement) between the points in the scale which we compare. We cannot say that the image of Gorton Square is twice as preferable as that of the Bilton Village office. This is because the allocation of a zero to represent the image of Carlisle Walk was arbitrary, and we therefore have what is known as an *interval scale*, which allows only intervals between points to be compared. The Fahrenheit and Celsius temperature scales are the most well-known examples of interval scales. We cannot, for example, say that water at 80°C is twice the temperature of water at 40°C. You can verify this by converting the temperatures to degrees Fahrenheit to obtain 175°F and 104°F, respectively. Clearly, the first temperature is no longer twice the second temperature. However, we can say that an increase in temperature from 40° to 80°C is twice that of an increase from 40° to 60°C. You will find that such a comparison does apply, even if we convert the temperatures to degrees Fahrenheit.

Having established an initial set of values for image, these should be checked to see if they consistently represent the preferences of the decision maker. We can achieve this by asking him, for example, if he is happy that the improvement in image between Elton Street and Addison Square is roughly as preferable as the improvement in image between Gorton Square and Denver Street. Similarly, is he happy that the improvement in image between Carlisle Walk and Denver Street is less preferable than that between Denver Street and Elton Street? The answers to these questions may lead to a revision of the values. Of course, if the owner finds it very difficult to make these sorts of judgments we may need to return to the value tree and see if we can break image down into more measurable attributes. Nevertheless, it should be emphasized that the numbers allocated by the owner to the different offices do not need to be precise. As we will see later, the choice of a course of action is generally fairly robust, and it often requires quite substantial changes in the figures supplied by the decision maker before another option is preferred.

This procedure for obtaining values can be repeated for the other less easily quantified attributes. The values allocated by the owner for the attributes 'comfort', 'visibility' and 'car-parking facilities' are shown in Table 2.2 (see page 32).

Value functions

Let us now consider the benefit attributes which can be represented by easily quantified variables. First, we need to measure the owner's relative strength of preference for offices of different sizes. The floor area of the offices is shown below.

		Floor area (ft²)
Addison Square	(A)	1000
Bilton Village	(B)	550
Carlisle Walk	(C)	400
Denver Street	(D)	800
Elton Street	(E)	1500
Filton Village	(F)	400
Gorton Square	(G)	700

Now it may be that an increase in area from 500 ft² to 1000 ft² is very attractive to the owner, because this would considerably improve working conditions. However, the improvements to be gained from an increase from 1000 ft² to 1500 ft² might be marginal and make this increase less attractive. Because of this, we need to translate the floor areas into values. This can be achieved as follows.

The owner judges that the larger the office, the more attractive it is. The largest office, Elton Street, has an area of 1500 ft^2 so we can give 1500 ft^2 a value of 100. In mathematical notation we can say that:

$$\nu(1500) = 100$$

where $\nu(1500)$ means 'the value of 1500 ft^2'. Similarly, the smallest offices (Carlisle Walk and Filton Village) both have areas of 400 ft^2 so we can attach a value of 0 to this area, i.e. $\nu(400) = 0$.

We now need to find the value of the office areas which fall between the most-preferred and least-preferred areas. We could ask the owner to directly rate the areas of the offices under consideration using the methods of the previous section. However, because areas involving rather awkward numbers are involved, it may be easier to derive a value function. This will enable us to estimate the values of any office area between the most and least preferred. There are several methods which can be used to elicit a value function, but one of the most widely applied is *bisection*.

This method requires the owner to identify an office area whose value is halfway between the least-preferred area (400 ft^2) and the most-preferred area (1500 ft^2). Note that this area does not necessarily have to correspond to that of one of the offices under consideration. We are simply trying to elicit the owner's preferences for office areas in general, and having obtained this information we can then use it to assess his preference for the specific office areas which are available to him. Initially, the owner suggests that the midpoint area would be 1000 ft^2. This implies that an increase in area from 400 ft^2 to 1000 ft^2 is just as attractive as an increase from 1000 ft^2 to 1500 ft^2. However, after some thought he rejects this value. The increases from smaller areas will, he reasons, reduce overcrowding and so be much more attractive than increases from larger areas which would only lead to minor improvements. He is then offered other candidates for the midpoint position (for example, 900 ft^2 and 600 ft^2), but rejects these values as well. Finally, he agrees that 700 ft^2 has the midpoint value, so $\nu(700) = 50$.

Having identified the midpoint value, the decision maker is now asked to identify the 'quarter points'. The first of these will be the office area, which has a value halfway between the least-preferred area (400 ft^2) and the midpoint area (700 ft^2). He decides that this is 500 ft^2, so $\nu(500) = 25$. Similarly, we ask him to identify an area which has a value halfway between the midpoint area (700 ft^2) and the best area (1500 ft^2). He judges this to be 1000 ft^2, which implies that $\nu(1000) = 75$. We now have the values for five floor areas and this enables us to plot the value

function for office size, which is shown in Figure 2.3. This value function can be used to estimate the values for the actual areas of the offices under consideration. For example, the Bilton Village office has an area of 550 ft^2 and the curve suggests that the value of this area is about 30.

A similar method can be applied to the attribute 'closeness to customers'. This attribute has been represented by the variable 'distance from town center' and the value function is shown in Figure 2.4. Note that the greater the distance from the town center, the lower the value will be. The curve also suggests that a move from 0 to 2 miles from the town center is far more damaging to business than a move from 6 to 8

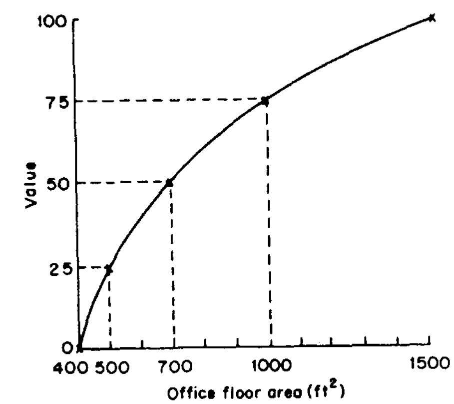

Figure 2.3 – Constructing a value function for office floor area

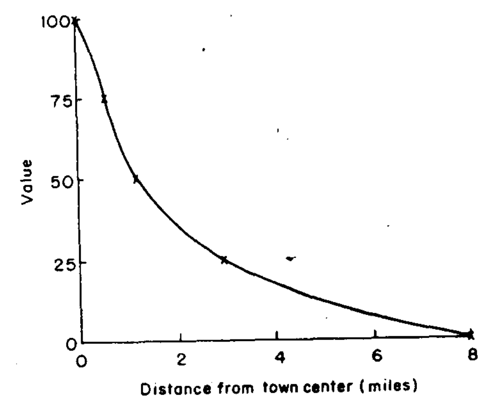

Figure 2.4 – A value function for distance from customers

miles. The values identified for the seven offices in terms of 'office area' and 'closeness to customers' are shown in Table 2.2 (see page 32).

Determining the weights of the attributes

In the previous section we derived values to measure how well each office performed on each attribute. For example, we found that Addison Square was the best office for image and closeness to customers, but it was the least-preferred office for providing comfortable working conditions for staff. Clearly, in order to make a decision the owner now needs to combine the values for the different attributes in order to gain a view of the overall benefits which each office has to offer.

An intuitively appealing way of achieving this is to attach weights to each of the attributes that reflect their importance to the decision maker. For example, he might consider office floor area to be more important than distance from customers and therefore give a weight of 5 to floor area and only 1 to distance from customers. Unfortunately, this approach can lead to serious problems. To see this, consider the following decision.

Suppose that the choice is between just two offices, X and Y, and that we are evaluating these on only two attributes: office area and distance from customers. The table below shows how the offices perform on these two attributes:

Office	*Floor area*	*Distance from customers*
X	400 ft^2	0 miles
Y	402 ft^2	15 miles

The values assigned to the offices would therefore be:

Office	*Floor area*	*Distance from customers*
X	0	100
Y	100	0
Weights	5	1

If we assume that the weights of 5 and 1 apply then the aggregate value for each office can be obtained by multiplying each value by its weight and summing the results, as shown below.

Office X: Aggregate value = $(5 \times 0) + (1 \times 100) = 100$
Office Y: Aggregate value = $(5 \times 100) + (1 \times 0) = 500$

According to this, the decision maker should choose office Y. This is because it has the largest floor area and therefore performed best on the attribute which was considered to be most important. However, a look at the original figures reveals that Y's floor area exceeds X's by only 2 square feet! Indeed, the weights imply that the decision maker is prepared to sacrifice moving 15 miles closer to his customers to gain just two extra square feet of office space, which seems an unlikely proposition.

The problem with importance weights is that they may not take into account the *range* between the least- and most-preferred options on each attribute (see von Winterfeldt and Edwards[10]). If the options perform very similarly on a particular attribute, so that the range between worst and best is small, then this attribute is unlikely to be important *in the decision*, even though the decision maker may consider it to be an important attribute *per se*. Taking this to extremes, suppose that both offices had a floor area of 400 square feet. In this case, the weight attached to floor area should be zero because this attribute has no importance in discriminating between the two offices.

Fortunately, this problem can be avoided by using *swing weights*. These are derived by asking the decision maker to compare a change (or swing) from the least-preferred to the most-preferred value on one attribute to a similar change in another attribute. The simplest approach is to proceed as follows. Consider the lowest-level attributes on the 'Benefits' branch of the value tree (Figure 2.1). The owner is asked to imagine a hypothetical office with all these attributes at their least-preferred levels, that is, an office which is the greatest distance (i.e. 8 miles) from the town center, has the worst position for visibility and has the worst image, the smallest size and so on. Then he is asked, if just one of these attributes could be moved to its best level, which would he choose? The owner selects 'closeness to customers'. After this change has been made, he is asked which attribute he would next choose to move to its best level, and so on until all the attributes have been ranked. The owner's rankings are:

1. Closeness to customers
2. Visibility
3. Image
4. Size
5. Comfort
6. Car-parking facilities

We can now give 'closeness to customers' a weight of 100. The other weights are assessed as follows. The owner is asked to compare a swing

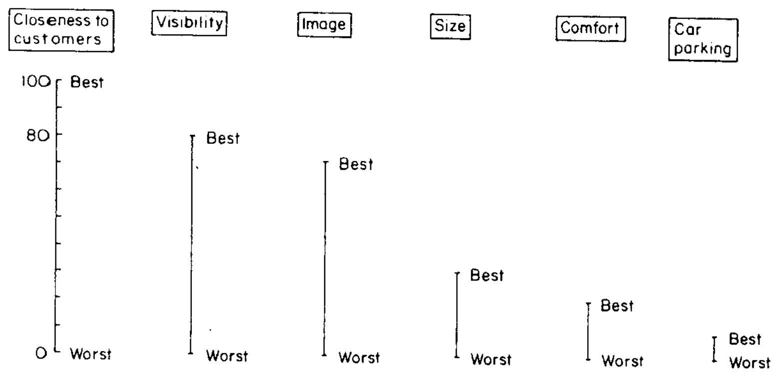

Figure 2.5 – Derivation of swing weights. For example, a swing from the worst to the best location for visibility is considered to be 80% as important as a swing from the worst to the best location for closeness to customers

from the least visible location to the most visible, with a swing from the most distant location from customers to the closest location. After some thought, he decides that the swing in 'visibility' is 80% as important as the swing in 'closeness to customers' so visibility is given a weight of 80. Similarly, a swing from the worst 'image' to the best is considered to be 70% as important as a swing from the worst to the best location for 'closeness to customers', so 'image' is assigned a weight of 70. The procedure is repeated for all the other lower-level attributes and Figure 2.5 illustrates the results. As shown below, the six weights obtained sum to 310, and it is conventional to 'normalize' them so that they add up to 100 (this will make later stages of the analysis easier to understand). Normalization is achieved by simply dividing each weight by the sum of the weights (310) and multiplying by 100.

Attribute	*Original weights*	*Normalized weights* (to nearest whole number)
Closeness to customers	100	32
Visibility	80	26
Image	70	23
Size	30	10
Comfort	20	6
Car-parking facilities	10	3
	310	100

The weights for the higher-level attributes in the value tree, 'turnover' and 'working conditions', are now found by summing the appropriate lower-level weights, so the weight for turnover is 81 (i.e. 32+26+23) and the weight for working conditions is 19 (i.e. 10+6+3).

Aggregating the benefits using the additive model

We now have (1) a measure of how well each office performs on each attribute and (2) weights which enable us to compare the values allocated to one attribute with the values allocated to the others. This means that we are now in a position to find out how well each office performs overall by combining the six value scores allocated to that office.

To do this, we will assume that the additive model is appropriate. As we show below, this simply involves adding an office's weighted value scores together to obtain a measure of the overall benefits which that office has to offer. The additive model is by far the most widely used, but it is not suitable for all circumstances. In particular, the model is inappropriate where there is an interaction between the values associated with some of the attributes. For example, when choosing a house, an attractive architecture and a pleasant garden may complement each other, leading to a combined value which is greater than the sum of the individual values. We will examine the limitations of the additive model later.

The calculations which the additive model involves are shown below for Addison Square. Each value is multiplied by the weight attached to that attribute. The resulting products are then summed and divided by 100 to obtain the overall value of benefits at that location.

Attribute	*Addison Square values*	*Weight*	*Value × weight*
Closeness to customers	100	32	3200
Visibility	60	26	1560
Image	100	23	2300
Size	75	10	750
Comfort	0	6	0
Car-parking facilities	90	3	270
			8080

Table 2.2 – Values and weights for the office location problem

Attribute	Weight	Office						
		A	B	C	D	E	F	G
Closeness	32	100	20	80	70	40	0	60
Visibility	26	60	80	70	50	60	0	100
Image	23	100	10	0	30	90	70	20
Size	10	75	30	0	55	100	0	50
Comfort	6	0	100	10	30	60	80	50
Car parking	3	90	30	100	90	70	0	80
Aggregate Benefits		80.8	39.4	47.4	52.3	64.8	20.9	60.2

Therefore the aggregate value for Addison Square is 8080/100, i.e. 80.8. Table 2.2 gives a summary of the values obtained for all the offices and their aggregate values. It can be seen that Addison Square has the highest value for benefits and Filton Village the lowest. However, so far we have ignored the costs associated with the offices and the next section shows how these can be taken into account.

Trading benefits against costs

You will recall that until now we have ignored the costs of the offices because of the difficulties which decision makers often have in making judgments about the trade-off between costs and benefits. If our decision maker had not found this to be a problem then we could have treated cost as just another attribute. We could therefore have allocated values to the various costs, with a value of 100 being given to the office which had the lowest costs and 0 to the office with the highest. Weights could then have been derived to compare swings from the least-preferred to the most-preferred level of benefits with swings from the worst to the best cost. This would have enabled the value for cost to have been included when we used the additive model to derive an overall value to measure the attractiveness of the different offices. The office achieving the highest overall value would have been the one the owner should choose. This approach has been used in problems where decision makers experienced no difficulty in assigning weights to all the attributes.

However, because our owner does have difficulties in judging the cost–benefit trade-off, we can proceed as follows. In Figure 2.6 the aggregate value of benefits has been plotted against the annual cost for each of the offices. Note that the cost scale has been 'turned round', so that the lower (and therefore more preferable) costs are to the right. This is to make this graph comparable with ones we will meet later in the book. Clearly, the higher an office appears on the benefits scale and the further to the right on the cost scale, the more attractive it will be. If we compare Addison Square (A) with Elton Street (E) it can be seen that, while both have similar costs, Addison Square has higher benefits. It would not therefore be worth considering Elton Street, and this office is said to be *dominated* by Addison Square. Similarly Gorton Square (G) not only has lower costs but also higher benefits compared with Bilton Village (B), Denver Street (D) and Filton Village (F). Therefore, B, D, and F are also dominated offices. Thus the only locations which are worth considering are Addison Square (A), Gorton Square (G) and Carlisle Walk (C). These non-dominated offices are said to lie on the *efficient frontier*.

The choice between the three offices on the efficient frontier will depend on the relative weight the owner attaches to costs and benefits. If he is much more concerned about benefits, then Addison Square will be his choice. Alternatively, if he is more concerned to keep his costs low, then he should choose Carlisle Walk. Gorton Square would be an intermediate choice. It costs $5300 more per year than Carlisle Walk, but offers slightly higher benefits.

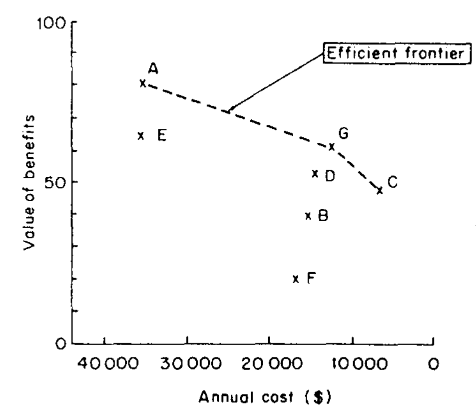

Figure 2.6 – A plot of benefits against costs for the seven offices

This information may be sufficient for the owner to make a choice. At the very least, it should illuminate his understanding of the decision problem. He may be surprised that Bilton Village has fared so badly or that Carlisle Walk has done so well, and he may wish to check back through the data he has supplied to see why this has happened.

However, it is possible that the decision maker still feels unable to choose between the three offices on the efficient frontier and thinks that a more formal approach would help him. If this is the case then the following procedure suggested by Edwards and Newman[9] can be used.

Consider first a move from Carlisle Walk (C) to Gorton Square (G). This would lead to an increase in the value of benefits from 47.4 to 60.2, an increase of 12.8. However, it would also lead to an increase in costs of $5300. Therefore each one-point increase in the value of benefits would cost him $5300/12.8, which is $414. Similarly, a move from Gorton Square (G) to Addison Square (A) would increase the value of benefits by 20.6 points at an extra cost of $23 000. This would therefore cost $23 000/20.6, which is $1117 for each extra benefit value point. So if an extra value point is worth less than $414 to the owner he should choose Carlisle Walk. If it is worth between $414 and $1117, he should choose Gorton Square, and if it is worth paying more than $1117 for each extra value point he should choose Addison Square.

Now we need to determine how much each extra value point is worth to the owner. This can be achieved by selecting a lower-level attribute from the value tree which the owner will find fairly easy to evaluate in monetary terms. The owner suggests that this is 'image'. He is then asked what extra annual cost he would be prepared to incur for a move from the office with the worst image to one with the best. He answers that it would be worth paying an extra $15 000. This means that he considers that it would be worth paying $15 000 for a 100-point increase in the value of image. Now the weight of image is 23% of the total weight allocated to the attributes. So an increase of 100 points on the image scale would increase the aggregate value of benefits by 23 points. Therefore the owner is prepared to pay $15 000 to gain 23 points in the value of aggregate benefits. This implies that he is prepared to pay $15 000/23 or $652 per point. On this basis he should choose the Gorton Square office.

Of course, the data we have been dealing with are far less precise than the above analysis might have implied, and it is unlikely that the owner will be 100% confident about the figures which he has put forward. Before making a firm recommendation therefore we should explore the effect of changes in these figures. This topic is covered in the next section.

Sensitivity analysis

Sensitivity analysis is used to examine how robust the choice of an alternative is to changes in the figures used in the analysis. The owner is a little worried about the weight of turnover (i.e. 81) relative to working conditions (i.e. 19) and he would like to know what would happen if this weight was changed. Figure 2.7 shows how the value of benefits for the different offices varies with changes in the weight placed on turnover. For example, if turnover had a weight of zero this would imply that the three turnover attributes would also have zero weights, so the weights for the six lowest-level benefit attributes would now be: closeness to customers 0, visibility 0, image 0, size 30, comfort 20, car parking 10. These normalize to 0, 0, 0, 50, 33.3, and 16.7, respectively,

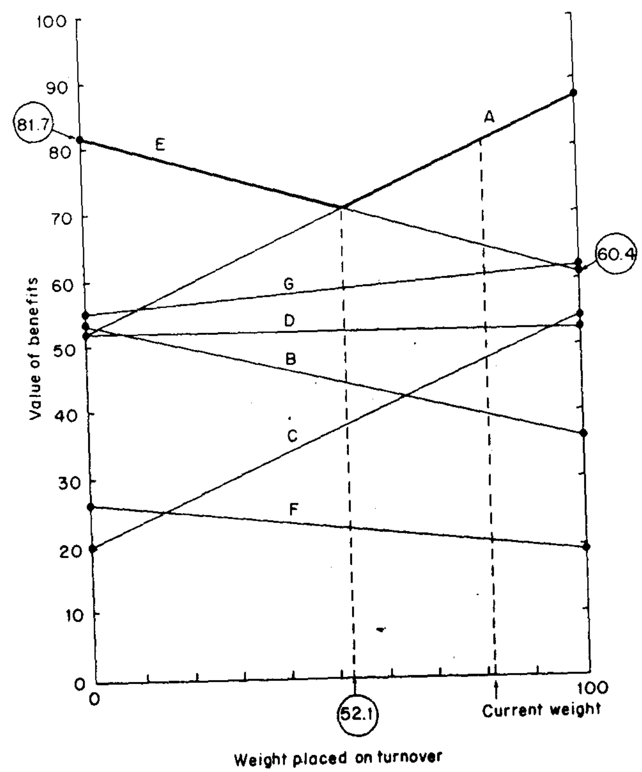

Figure 2.7 – Sensitivity analysis for weight placed on turnover

which would mean that Elton Street (E), for example, would have benefits with a value of 81.7. At the other extreme, if turnover had a weight of 100 (and therefore working conditions a weight of zero) the value of benefits for Elton Street would have been 60.4. The line joining these points shows the value of benefits for Elton Street for turnover weights between 0 and 100.

It can be seen that Elton Street gives the highest value of benefits as long as the weight placed on turnover is less than 52.1. If the weight is above this figure then Addison Square (A) has the highest value of benefits. Since the owner assigned a weight of 81 to turnover, it will take a fairly large change in this weight before Elton Street is worth considering, and the owner can be reasonably confident that Addison Square should appear on the efficient frontier.

Figure 2.7 also shows that no change in the weight attached to turnover will make the other offices achieve the highest value for benefits. Filton Village (F), in particular, scores badly on any weight. If we consider the other two offices on the efficient frontier we see that Gorton Square (G) always has higher-valued benefits than Carlisle Walk (C).

Similar analysis could be carried out on the lower-level weights. For example, the owner may wish to explore the effect of varying the weights attached to 'closeness to customers' and 'visibility' while keeping the weight attached to 'image' constant. Carrying out sensitivity analysis should contribute to the decision maker's understanding of his problem and it may lead him to reconsider some of the figures he has supplied. In many cases sensitivity analysis also shows that the data supplied do not need to be precise. As we saw above, large changes in these figures are often required before one option becomes more attractive than another: a phenomenon referred to as 'flat maxima' by von Winterfeldt and Edwards.[10]

Theoretical considerations

The axioms of the method

In our analysis of the office location problem we implicitly made a number of assumptions about the decision maker's preferences. These assumptions, which are listed below, can be regarded as the axioms of the method. They represent a set of postulates which may be regarded

as reasonable. If the decision maker accepts these axioms, and if he is rational (i.e. if he behaves consistently in relation to the axioms), then he should also accept the preference rankings indicated by the method. Let us now consider the axioms:

(1) *Decidability*: We assumed that the owner was able to decide which of two options he preferred. For example, we assumed that he could state whether the improvement in image between Carlisle Walk and Gorton Square was greater than the improvement between Carlisle Walk and Bilton Village. It may have been that the owner was very unsure about making this comparison or he may have refused to make it at all.

(2) *Transitivity*: The owner preferred the image of Addison Square to Bilton Village (i.e. A to B). He also preferred the image of Bilton Village to Carlisle Walk (i.e. B to C). If transitivity applies then the owner must therefore also prefer the image of Addison Square to Carlisle Walk (i.e. A to C).

(3) *Summation*: This implies that if the owner prefers A to B and B to C, then the strength of preference of A over C must be greater than the strength of preference of A over B (or B over C).

(4) *Solvability*: This assumption was necessary for the bisection method of obtaining a value function. Here the owner was asked to identify a distance from the center of town which had a value halfway between the worst and best distances. It was implicitly assumed that such a distance existed. In some circumstances there may be 'gaps' in the values which an attribute can assume. For example, the existence of a zone of planning restrictions between the center of the town and certain possible locations might mean that siting an office at a distance which has a value halfway between the worst and best distances is not a possibility which the decision maker can envisage.

(5) *Finite upper and lower bounds for value*: In assessing values we had to assume that the best option was not so wonderful and the worst option was not so awful that values of plus and minus infinity would be assigned to these options.

Assumptions made when aggregating values

In our analysis we used the additive model to aggregate the values for the different attributes. As we pointed out, the use of this model is not appropriate where there is an interaction between the scores on the attributes. In technical terms, in order to apply the model we need to assume that *mutual preference independence* exists between the attributes.

To demonstrate preference independence let us suppose that our office location problem only involves two attributes: 'distance from customers' and 'office size'. Our decision maker is now offered two offices, X and Y. These are both the same size (1000 ft^2) but X is closer to customers, as shown below:

Office	*Distance from customers*	*Office floor area*
X	3 miles	1000 ft^2
Y	5 miles	1000 ft^2

Not surprisingly, the decision maker prefers X to Y. Now suppose that we change the size of both offices to 400 ft^2. If, as is likely, the decision maker still prefers X to Y his preference for a distance of 3 miles over a distance of 5 miles has clearly been unaffected by the change in office size. This might remain true if we change the size of both offices to any other possible floor area. If this is the case, we can say that 'distance from customers' is preference independent of 'office size' because the preference for one distance over another does not depend on the size of the offices.

If we also found that 'size of office' is preference independent of 'distance from customers' then we can say that the two attributes are *mutually preference independent*. Note that mutual preference independence does not automatically follow. When choosing a holiday destination, you may prefer a warmer climate to a cooler one, irrespective of whether or not the hotel has an open-air or indoor swimming pool. However, your preference between hotels with open-air or indoor swimming pools will probably depend on whether the local climate is warm or cool.

To see what can happen when the additive model is applied to a problem where mutual preference independence does not exist, consider the following problem. Suppose now that our office location decision depends only on the attributes 'image' and 'visibility' and the owner has allocated weights of 40 and 60 to these two attributes. Two new offices, P and Q, are being compared and the values assigned to the offices for each of these attributes are shown below (0 = worst, 100 = best).

Office	*Visibility*	*Image*
P	0	100
Q	100	0

Using the additive model, the aggregate value of benefits for P will be:

$$(40\times0) + (60\times100) = 6000, \text{ i.e. 60 after dividing by 100}$$

while the aggregate value of benefits for Q will be:

$$(40\times100) + (60\times0) = 4000, \text{ i.e. 40 after dividing by 100}$$

This clearly suggests that the decision maker should choose office P. However, it may be that he considers image only to be of value if the office is highly visible. Office P's good image is, he thinks, virtually worthless because it is not on a highly visible location and he might therefore prefer office Q. Thus, if image is not preference independent of visibility, the additive model will not correctly represent the owner's preferences.

How can the absence of mutual preference independence be identified? The most obvious way in which this will reveal itself is in the use of phrases like 'this depends on . . .' when the decision maker responds to questions. For example, when asked to assign a value to the 'image' of an office, our decision maker might well have said 'that depends on how visible the office is'.

If mutual preference independence does not exist it is usually possible to return to the value tree and redefine the attributes so that a set of attributes which are mutually preference independent can be identified. For example, perhaps visibility and image could be replaced by a single attribute 'ability to attract casual customers'.

In the occasional problems where this is not possible, other models are available which can handle the interaction between the attributes. The most well known of these is the multiplicative model. Consider again the case of the house purchase decision where the quality of the architecture and attractiveness of the garden complemented each other. If we let $V(A)$ = the value of the architecture of a given house and $V(G)$ = a value for the attractiveness of the garden then we might find that the following represented the overall value of the house:

$$\text{Value} = 0.6V(A) + 0.3V(G) + 0.1V(A)V(G)$$

The numbers in the above expression represent the weights (note that they sum to 1) and the last expression, which involves multiplying the values together, represents the interaction between architecture and garden. Because the multiplicative model is not widely used we will not consider it in detail. Longer discussions can be found in Bodily[11] and von Winterfeldt and Edwards.[10]

Conflicts between intuitive and analytic results

It may be that, if the decision maker had viewed the problem holistically, then he would have ranked his preferences for the offices in a very different order from that obtained through our analysis. This could be because the problem was too large and complex for him to handle as a whole so that his true preferences were not reflected in his holistic judgments. An analogy can be made with an attempt to answer a mathematical problem by using mental arithmetic rather than a calculator. This view is supported by research which suggests that the correlation of preference rankings derived from holistic judgments with those derived from SMART-type analyses decreases as the number of attributes in the problem gets larger. In other words, the larger the problem, the less reliable holistic judgments may be (see von Winterfeldt and Edwards[10] for a summary of this research).

Alternatively, discrepancies between holistic and analytic results may result when the axioms are not acceptable to the decision maker. It is possible that the decision maker could argue the case for a different set of sensible axioms. As long as he behaved consistently with these axioms, we could not argue that his rejection of the results of our analysis was irrational.

Nevertheless, any conflict between holistic and analytic rankings should be examined, since it may suggest that an important element of the problem has not been captured by the analysis. For example, an important attribute may have been left out or the interaction between two attributes may not have been taken into account. We can, of course, never be certain that a decision model is a faithful representation of the decision maker's preferences. In a computer model of a traffic system, for example, the model's validity can be assessed by comparing its predictions with the behavior of the real system, but here we are attempting to model the decision maker's beliefs and attitudes for which there is no physical analog. This begs the question: at what point do we decide that a decision model is adequate so that further refinements and revisions are not worth carrying out?

One approach to this question is to use Phillip's[12] concept of a *requisite decision model*. We will examine this idea in more detail in Chapter 11 in the context of group decision making, but briefly, a model is considered to be requisite when it provides the decision maker with enough guidance and insight to decide upon a course of action. Thus at the point where the decision maker knows what to do next a requisite model has been achieved. Phillips argues that:

> the modeling process uses the sense of unease among the problem owners about the results of the current model as a signal that further modeling may be needed, or that intuition may be wrong. If exploration of the discrepancy between holistic judgment and model results shows the model to be at fault, then the model is not requisite – it is not yet sufficient to solve the problem. The model can be considered requisite only when no new intuitions emerge about the problem.

Thus the requisite modeling process does not attempt to obtain an exact representation of the decision maker's beliefs and preferences, or to prescribe an optimal solution to his problem. However, by exploiting the conflicts between the results of the analysis and his intuitive judgments it will help him to resolve conflicts and inconsistencies in his thinking. As a deeper understanding of the problem is obtained the model will be revised and the discrepancy between the analytical and intuitive judgments will be reduced. Eventually, the decision maker will find that the model provides enough guidance for him to reach a decision.

Simplifying SMART: SMARTER

One of the main attractions of SMART is its relative simplicity. Nevertheless, the assessment of value functions and swing weights can still be difficult tasks and the resulting decision model may therefore be an inaccurate reflection of the decision maker's true preferences. Because of this, Ward Edwards and F. Hutton Barron[13] have argued for 'the strategy of heroic approximation'. Underlying this strategy is the idea that, while a very simple decision-making model may only approximate the real decision problem, it is less likely to involve errors in the values elicited from a decision maker because of the simpler judgments it involves. Consistent with this strategy, Edwards and Barron have suggested a simplified form of SMART which they call SMARTER (SMART Exploiting Ranks).

SMARTER differs from SMART in two ways. First, value functions are normally assumed to be linear. Thus the assessment of a value function for office floor area, for example, would involve giving 400 square feet a value of 0 and 1500 square feet a value of 100, as before, and then simply drawing a straight line, rather than a curve, between these two points in Figure 2.3. Clearly, this approximation becomes more inaccurate as the curvature of the 'true' value function increases so, to guard against poor approximations, Edwards and Barron recommend that preliminary checks should be made.

In the office location problems we would ask the decision maker to think about small increases in office floor area. Specifically, would a 100 square feet increase in floor area be more attractive if it fell near the bottom of the scale (e.g. 400 to 500 square feet), in the middle (e.g. 1000 to 1100 square feet) or near the top (e.g. 1400 to 1500 square feet), or would it not matter where the increase occurred? If it does not matter, then a linear approximation can be used. Suppose, however, that the decision maker says that the increase at the lower end of the scale is most appealing, while an increase at the top end of the scale would be least useful. We could then ask how much more desirable is the improvement at the bottom compared to the top. As a rule of thumb, if the ratio is less than 2:1, then Edwards and Barron suggest that the linear approximation is probably safe, otherwise we should fall back on methods like bisection to obtain the value function.

The second difference between SMART and SMARTER relates to the elicitation of the swing weights. Recall that in the office location problem the decision maker was asked to compare and evaluate swings from the worst to the best positions on the different attributes. For example, a swing from the worst to the best position for 'office visibility' was considered to be 80% as important as a swing from the worst to the best position for 'closeness to customers'. In SMARTER we still have to compare swings, but the process is made easier by simply asking the decision maker to *rank* the swings in order of importance, rather than asking for a number to represent the relative importance. SMARTER then uses what are known as 'rank order centroid', or ROC, weights to convert these rankings into a set of approximate weights.

While a set of equations, or tables, is needed to obtain the ROC weights the basic idea is easy to understand. Suppose that the office location decision had involved just two attributes – 'closeness to customers' and 'visibility' – and that the decision maker had considered the swing in 'closeness to customers' to be more important than the swing in 'visibility'. We know that, after normalization, the two weights will sum to 100. Since the swing in 'closeness to customers' is more important, its normalized weight must fall between just over 50 and almost 100. This suggests an approximate weight of 75, and this is indeed what the ROC equations would give us. Clearly, the ROC weight for 'visibility' would be 25.

Table 2.3 shows the ROC weights for decision problems involving up

Table 2.3 – Rank order centroid (ROC) weights

	Number of attributes					
Rank	2	3	4	5	6	7
1	75.0	61.1	52.1	45.7	40.8	37.0
2	25.0	27.8	27.1	25.7	24.2	22.8
3		11.1	14.6	15.7	15.8	15.6
4			6.3	9.0	10.3	10.9
5				4.0	6.1	7.3
6					2.8	4.4
7						2.0

to seven attributes (see Edwards and Barron for more details). In the 'original' office location problem, the decision maker would have simply ranked the importance of the swings for the six attributes, as shown below. This would have yielded the ROC weights indicated and these could then have been used to obtain the aggregate benefits of the offices in the normal way (the original normalized SMART weights are also shown below for comparison).

Rank of swing	*Attribute*	*ROC weight*	*SMART weight*
1	Closeness to customers	40.8	32.0
2	Visibility	24.2	26.0
3	Image	15.8	23.0
4	Size	10.3	10.0
5	Comfort	6.1	6.0
6	Car parking	2.8	3.0
		100.0	100.0

How good are the ROC weights as approximations to the weights which might have been obtained in SMART? Edwards and Barron report the results of extensive simulations which suggested that SMART and SMARTER will agree on which option has the highest aggregate benefits in 75–87% of cases. Even when they did not agree, the options identified as having the highest aggregate benefits tended to have very similar scores, suggesting that an option which was 'not too bad' was being picked by SMARTER.

All of this suggests that SMARTER is a technique which is well worth employing. However, we should note a couple of reservations about the method. First, in problems where it has been necessary to separate costs from benefits you might obtain a different efficient frontier if you use

SMARTER rather than SMART. This means we should be very careful before we exclude dominated options from further consideration. In particular, if you were to employ the method suggested by Edwards and Newman for selecting an option from the efficient frontier then SMART and SMARTER may well suggest that different options should be chosen. This is because the assessment of the worth of a value point to the decision maker is based on the normalized weights and differences between the SMART and ROC weights can lead to large discrepancies in this assessment.

These discrepancies become less important if we recall that the main purpose of a decision analysis model is not to tell us what to do in a mechanistic fashion, but to yield insights and understanding about the problem in hand. However, this raises another concern about SMARTER, which Edwards and Barron acknowledge. By simplifying the decision maker's judgmental task, we may be encouraging only a superficial consideration of the problem and hence precluding the very insights which we hope to obtain. Analysts sometimes find that these insights only emerge when the decision maker is forced to grapple with more demanding judgmental tasks which require deeper thinking about the issues. Despite these concerns, we think that SMARTER is a method which could prove to be extremely effective in many decision contexts.

Summary

In this chapter we have looked at a method for analyzing decision problems where each alternative had several attributes associated with it. This meant that the performance of each alternative had to be measured on each attribute and then the attributes themselves had to be 'weighed against' each other before a decision could be made. The central idea was that, by splitting the problem into small parts and focusing on each part separately, the decision maker was likely to acquire a better understanding of his problem than he would have achieved by taking a holistic view. We saw that the method required the decision maker to quantify his strengths of preferences. While this may not have been an easy process, we found that the figures put forward did not need to be exact, though we did try to ensure that they were consistent.

The decision problem presented in this chapter was designed to be

amenable to hand calculations. This would, however, be an extremely tedious way of approaching larger problems, and for these it would be necessary to use a computer. Packages available include HIVIEW (sold by Krysalis) and V·I·S·A (sold by Visual Thinking International). These have all the features which are needed to carry out SMART-type analysis, including a facility which allows the user to construct and modify value trees on-screen.

We stated in the Introduction that this method is normally applied where risk and uncertainty are not major concerns of the decision maker. However, it is possible to apply the method even in these circumstances by treating risk as an attribute. Wooler and Barclay[6] describe such an application involving a strike-prone production facility. (The analysis involved a group of managers in a decision conference.) A value tree was used to decompose 'risk' into lower-level attributes such as 'risk of strikes' and 'public relations risks', and the various strategies were scored in terms of their performance on these attributes using direct rating (for example, the least risky option was allocated the highest value). A second part of the value tree dealt with the benefits of the strategies and these were similarly scored. A graph such as Figure 2.7 was then used to display the aggregate risk of strategies against their aggregate benefits (rather than costs against benefits). We will consider a similar approach to risk in the context of a group decision problem in Chapter 12. However, a number of techniques have been specially designed to handle decisions involving a large element of risk and uncertainty, and we will consider these methods in the following chapters.

Exercises

(1) Formulate a value tree to identify the attributes which are of concern to you when choosing a vacation.

(2) You need a word-processing package for the personal computer in your office. Because your employer will pay for the package you are not concerned about the cost, but you would like a package which is as easy to use as possible and which also has a wide range of functions such as a thesaurus, spell checker and graphics. After discussing the matter with a friend who is something of an expert in this field, you identify seven potential packages and allocate values to them to reflect their ease of use and available facilities. These values are shown below (0 = worst, 100 = best).

Package	Ease of use	Facilities available
Super Quill	100	30
Easywrite	90	70
Wordright	50	20
Lexico	0	40
Ultraword	20	100
Keywrite	40	0
Fastwrite	85	55

(a) Plot each package's value for 'ease of use' and 'facilities available' on a graph and hence determine the packages which lie on the efficient frontier.

(b) Suppose that you judge that a switch from a package with the least facilities available to one with the most facilities is only 60% as attractive as a switch from a package which is the least easy to use to one which is the most easy to use. Assuming that mutual preference independence exists between the two attributes, which package should you choose?

(c) After some reflection you realize that the extra facilities available on a package will be of little value to you if they are going to be difficult to use. What does this imply about your method of analysis in (b)?

(3) A chemical company is expanding its operations and a disused woollen mill is to be converted into a processing plant. Four companies have submitted designs for the equipment which will be installed in the mill and a choice has to be made between them. The manager of the chemical company has identified three attributes which he considers to be important in the decision: 'cost', 'environmental impact' and 'reliability'. He has assessed how well each design performs on each attribute by allocating values on a scale from 0 (the worst design) to 100 (the best). These values are shown below, together with the costs which will be incurred if a design is chosen.

		Benefits	
Design	Cost ($)	Environmental impact	Reliability
A	90 000	20	100
B	110 000	70	0
C	170 000	100	90
D	60 000	0	50

(a) The manager is having difficulty in allocating weights to the two benefit attributes. Assuming that the two weights sum to 100 and that mutual preference independence exists between the attributes, perform a sensitivity analysis to show how the design offering the highest value for aggregate benefits will vary depending upon the weight which has been allocated to 'environmental impact'.

(b) Eventually, the manager decides to allocate 'environmental impact' a weight of 30 and 'reliability' a weight of 70. By plotting the benefits and costs of the designs on a graph, identify the designs which lie on the efficient frontier.

(c) The manager also decides that if he was offered a hypothetical design which had the lowest reliability and the worst environmental impact he would be prepared to pay $120 000 to convert that design to one which had the best impact on the environment but which still had the lowest level of reliability. Which design should the manager choose?

(4) A British company has won an important contract to supply components regularly to Poland. Four methods of transport are being considered: (i) air, (ii) sea, (iii) road and ferry and (iv) rail and ferry. The company's distribution manager has identified four relevant attributes for the decision: Punctuality, Safety of Cargo, Convenience and Costs. She has also allocated weights of 30 to punctuality, 60 to safety of cargo and 10 to convenience.

The manager then rated the performance of each form of transport on the different attributes. The values she assigned are shown below together with the estimated annual cost of using each form of transport.

		Benefits		
Form of transport	Punctuality	Safety	Convenience	Costs ($)
Air	100	70	60	150 000
Sea	0	60	80	90 000
Road and Ferry	60	0	100	40 000
Rail and Ferry	70	100	0	70 000

(a) Determine the form of transport which has the highest valued overall benefits, assuming that mutual preference independence exists between the attributes.

(b) For each form of transport, plot the value of overall benefits against costs and hence identify the forms of transport which lie on the efficient frontier.

(c) If the manager would be prepared to pay $70 000 per year to move from the least safe to the most safe form of transport, determine which alternative she should select.

(5) A local authority has to decide on the location of a new waste-disposal facility and five sites are currently being considered; Inston Common, Jones Wood, Peterton, Red Beach and Treehome Valley. In order to help them to choose between the sites the managers involved in the decision arranged for a decision analyst to attend one of their meetings. He first got the managers to consider the factors which they thought were relevant to the decision and, after some debate, four factors were identified:

(i) The visual impact of the site on the local scenery (for example, a site at Treehome Valley would be visible from a nearby beauty spot);

(ii) The ease with which waste could be transported to the site (for example, Red Beach is only two miles from the main town in the area and is close to a main highway while Inston Common is in a remote spot and its use would lead to a major increase in the volume of transport using the minor roads in the area);

(iii) The risk that the use of the site would lead to contamination of the local environment (e.g. because of leakages of chemicals into watercourses);

(iv) The cost of developing the site.

The decision analyst then asked the managers to assign scores to the sites to show how well they performed on each of the first three attributes. The scores they eventually agreed are shown below, together with the estimated cost of developing each site. Note that 0 represents the worst and 100 the best score on an attribute. In the case of risk, therefore, a score of 100 means that a site is the least risky.

	Benefits			*Costs*
Site	*Visual impact*	*Ease of transport*	*Risk*	($ million)
Inston Common	100	0	60	35
Jones Wood	20	70	100	25
Peterton	80	40	0	17
Red Beach	20	100	30	12
Treehome Valley	0	70	60	20

The decision analyst then asked the managers to imagine a site which had the worst visual impact, the most difficult transport requirements and the highest level of risk. He then asked them, if they had a chance of switching from this site to one which had just one of the benefits at its best value, which would they choose? The managers agreed that they would move to a site offering the least risk of contamination. A move to a site with the best visual impact was considered to be 80% as preferable as this, while a move to one with the most convenient transport facilities was 70% as preferable.

(a) Can we conclude from the values which were assigned to the different sites for visual impact that, in terms of visual impact, the Inston Common site is five times preferable to Red Beach? If not, what can we infer from the figures?

(b) An alternative way of allocating weights to the three benefit attributes would have involved asking the managers to allocate a score reflecting the importance of each attribute. For example, they might have judged that risk was five times more important and visual impact three times more important than ease of transport, so that weights of 5, 3 and 1 would have been attached to the attributes. What are the dangers of this approach?

(c) Assuming that mutual preference independence exists between the attributes, determine the value of aggregate benefits for each site.

(d) Plot the aggregate benefits and costs of each site on a graph and hence identify the sites which lie on the efficient frontier.

(e) Although a weight of 80 was finally agreed for visual impact, this was only after much debate and some managers still felt that a weight of 65 should have been used while others thought that 95 would have been more appropriate. Perform a sensitivity analysis on the weight assigned to visual impact to examine its effect on the aggregate benefits of the sites and interpret your results.

(6) As an experiment, a charity decides to use the Simple Multi-attribute Rating Technique (SMART) to determine a short list from the seven applicants who have applied for the post of Regional Officer for the Western region. The main criteria which will be used to compare candidates are: the salary they would expect (SALARY) (they have stated this on the application form), their experience of charity work (CHARITY EXP), their managerial experience (MANAGEMENT EXP), their educational qualifications (EDUCATION), their apparent commitment to the charity's work (COMMITMENT) (as gleaned from the application form) and the quality of the ideas they put forward on the form (IDEAS).

(a) When a value tree was used to identify the above attributes there was some doubt about whether the attributes IDEAS and COMMITMENT met Keeney and Raiffa's criterion of decomposability. Explain what this means and why the concerns might be justified.

(b) The personnel manager ranked all the criteria, except salary, in order of importance and then assigned weights as follows:

COMMITMENT	100
MANAGEMENT EXP	70
IDEAS	65
CHARITY EXP	55
EDUCATION	10

(Note that, on the basis of the application form, all of the candidates appeared to be equally committed to the charity.)

Discuss whether the method the personnel manager used to assess these weights is appropriate.

(c) Candidate A's scores for the non-monetary attributes are given below:

COMMITMENT	100
MANAGEMENT EXP	10
IDEAS	65
CHARITY EXP	0
EDUCATION	10

Using the weights given in part (b) show how the personnel manager obtained a score of 50 (subject to rounding) for this candidate.

(d) The aggregate scores for all the candidates are given below, together with their expected salaries. Assuming that the personnel manager's model is appropriate, determine the candidates who appear on the efficient frontier and explain the significance of this.

Candidate	*Aggregate score*	*Expected salary*
A	50	$46 000
B	31	$40 000
C	75	$42 000
D	90	$60 000
E	20	$54 000
F	62	$52 000
G	49	$42 000

(e) The candidate with the least management experience has only been working in management for two years, while the most experienced candidate has 10 years' experience. If the personnel manager reckons that the charity would be prepared to pay $8000 for the eight years' extra management experience, all else remaining equal, determine which candidate she should recommend for the appointment prior to the interview. State any assumptions you have made.

(7) The owner of a small business is unhappy with the service she has been receiving from her bank and has decided to move her account to a rival bank. Her decision on which bank to choose will be based not only on the estimated annual bank charges which each bank will levy, but also on the following 'benefit attributes':

(a) the proximity of the local branch,
(b) whether the local branch has a small business adviser,
(c) the maximum automatic loan allowed,
(d) whether a telephone banking facility is offered.

The alternative banks are listed below, together with their estimated annual costs and the scores the business owner has allocated for each of the 'benefit attributes'.

Bank	*Estimated annual charge $*	*Proximity*	*Small business adviser*	*Max loan*	*Telephone facility*
Central	3000	0	100	40	0
Northern	5000	100	100	80	0
Direct	2000	70	0	100	100
Royal	1000	30	0	0	100
Marks	4000	90	100	20	0

The business owner is then asked to imagine that she has her account with a hypothetical bank which had the lowest scores on all of the 'benefit attributes'. She is then asked to imagine that each attribute could be switched to its best possible value and asked to rank the attractiveness of these possible switches. Her ranks are given as:

Rank	*Switch*
1	Lowest maximum loan facility to highest
2	No telephone banking facility to provision of this facility
3	Non-availability of small business adviser to availability
4	Least close branch to closest branch

(a) SMARTER has been used to obtain scores to represent the aggregate benefits of the banks and these are given below:

Bank	*Aggregate score*
Central	35.4
Northern	62.5
Direct	83.5
Royal	29.0
Marks	30.6

Show how the score of 35.4 for the Central bank was determined.

(b) Explain why it was appropriate to consider swings (or switches) from the worst to best positions on each attribute when determining the weights of the attributes.

(c) By taking into account the estimated annual charges of the banks, determine which banks lie on the efficient frontier. Explain the significance of the efficient frontier.

(d) SMARTER is based on the 'principle of heroic approximation'. Explain how this principle applies to your analysis of the businesswoman's problem and discuss whether it is likely to be appropriate.

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