# EIGENFACES

An introduction to facial recognition

# Images

- An image is a point in a *high dimensional* space
- An *n* x *m* pixel image is a point in $R^{nm}$

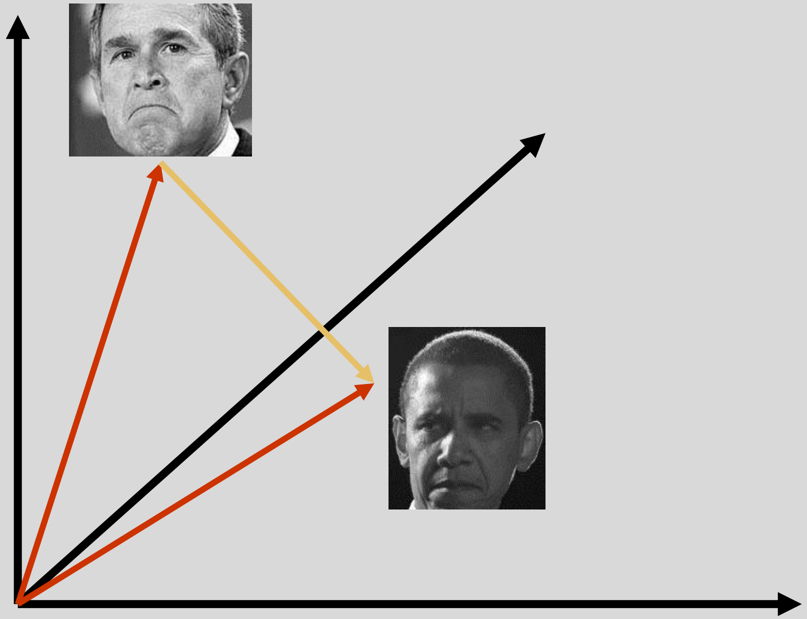
  A 192x168 black and white image is represented by 32256 dimensions

# Face Space

- If each face is a 192x168 image, then imagine a 32,256D space

  (i.e., 32,256 axes, not just 3 as depicted!)

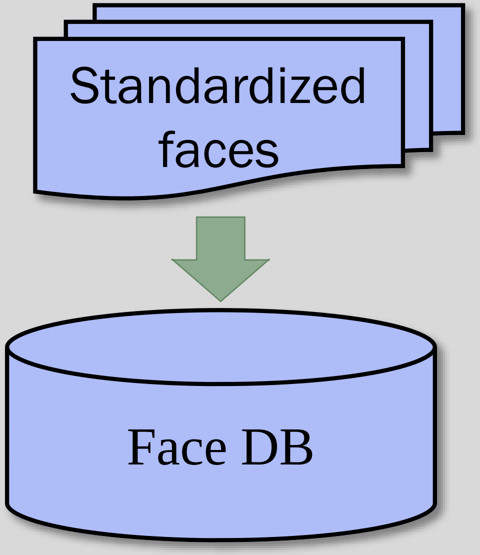
- Every face is a point in the space
- We can calculate *distances* between faces



# Intrinsic dimensionality

- However, faces all have similar features, i.e., faces, and thus face images, are highly correlated
  - *The intrinsic dimensionality of the face space is much smaller*
- PCA can be used to reduce the high dimensional space to a much lower space and still capture a large part of the information
  - For the example problem, out of the 32,256 eigenvectors, **only about ½ of 1 percent will be needed**!

# Face Recognition



# YaleB face database

http://vision.ucsd.edu/~iskwak/ExtYaleDatabase/ExtYaleB.html

# Eigenfaces

- Principal components of a data set are derived from the eigenvectors of the covariance matrix
- Principal components associated with “face” data are called eigenfaces
- Eigenfaces are the orthonormal directions in high dimensional space that can be used to represent the information from face images
- To impress your friends: *Eigenfaces are the eigenvectors of the covariance matrix of the probability distribution of the vector space of human faces*

# Eigenfaces

- YaleB face database: 16,128 B&W images of 28 human subjects under 9 poses and 64 illumination conditions, each image is 192x168
- PCA requires mean-centered data: $\mathbf{x} - \bar{\mathbf{x}}$
- The“mean face” represents face attributes that are generic and associated with all faces, e.g., an average looking face…
- The “mean face” for this database
- **The information of interest is deviation from the mean**

# Generating Eigenfaces

1. Large set of images of human faces is taken
   - Let M denote the number of images
   - Let N denote the number of pixels
2. Images are normalized to line up the eyes, mouths and other features
3. Eigenvectors of the covariance matrix of the mean-centered face image vectors are extracted
   - Let I denote the associated MxN matrix
   - Size of covariance matrix: NxN
   - If N = 192x168, then NxN= $32{,}256^2 \rightarrow$ 1,040,449,536 elements!

# Eigenfaces

- Eigenfaces are the *standardized face ingredients* derived from the statistical analysis of many images of human faces
- When ***properly weighted***, a very small subset, $k \ll nm$, of eigenfaces can be summed together to create an approximate gray-scale rendering of a human face.

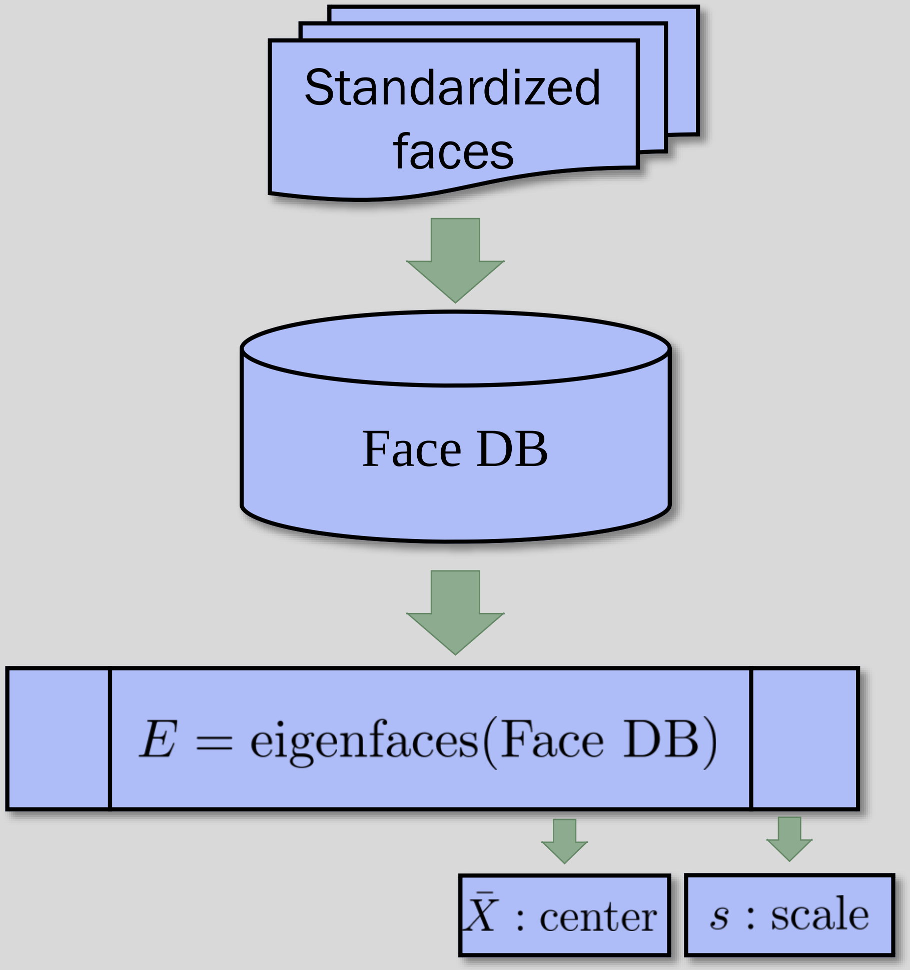
$$\mathbf{x} \approx \bar{\mathbf{x}} + a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + \ldots + a_k\mathbf{v}_k$$

# Generating Eigenfaces

Notes:

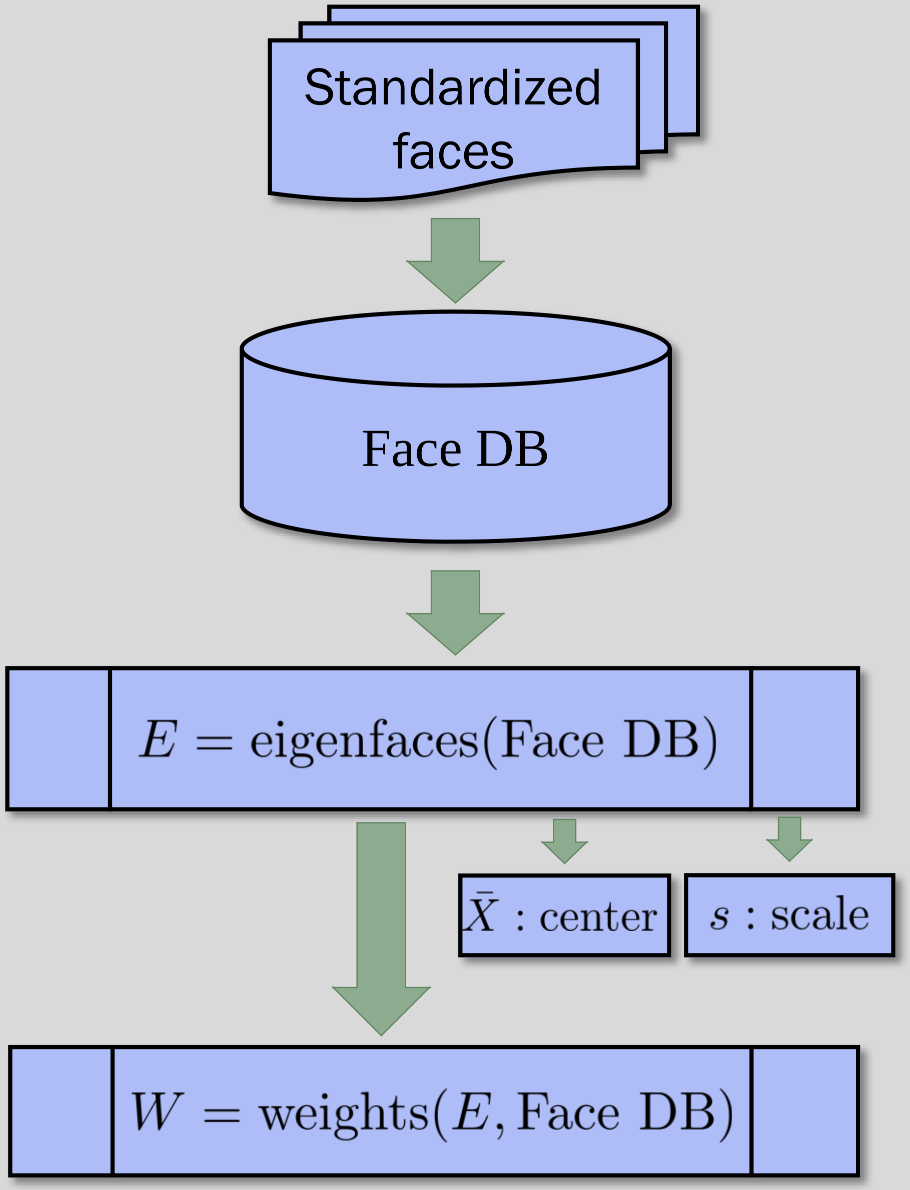
- The number of eigenvectors will be less than or equal to **min(M,N)**
- The “eigen” method in R will not work if $M < N$; however, “prcomp” in R uses a more advanced (and computationally efficient approach) called singular value decomposition which can handle both cases: $M < N$ or $N < M$

# Face Recognition



# The first thirty eigenfaces

# Face Recognition



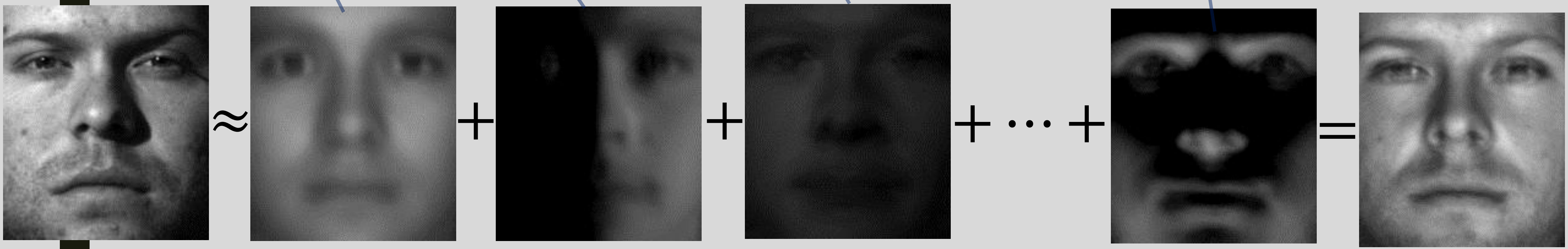
# Lower dimensional faces

The weights are the projections onto face subspace:

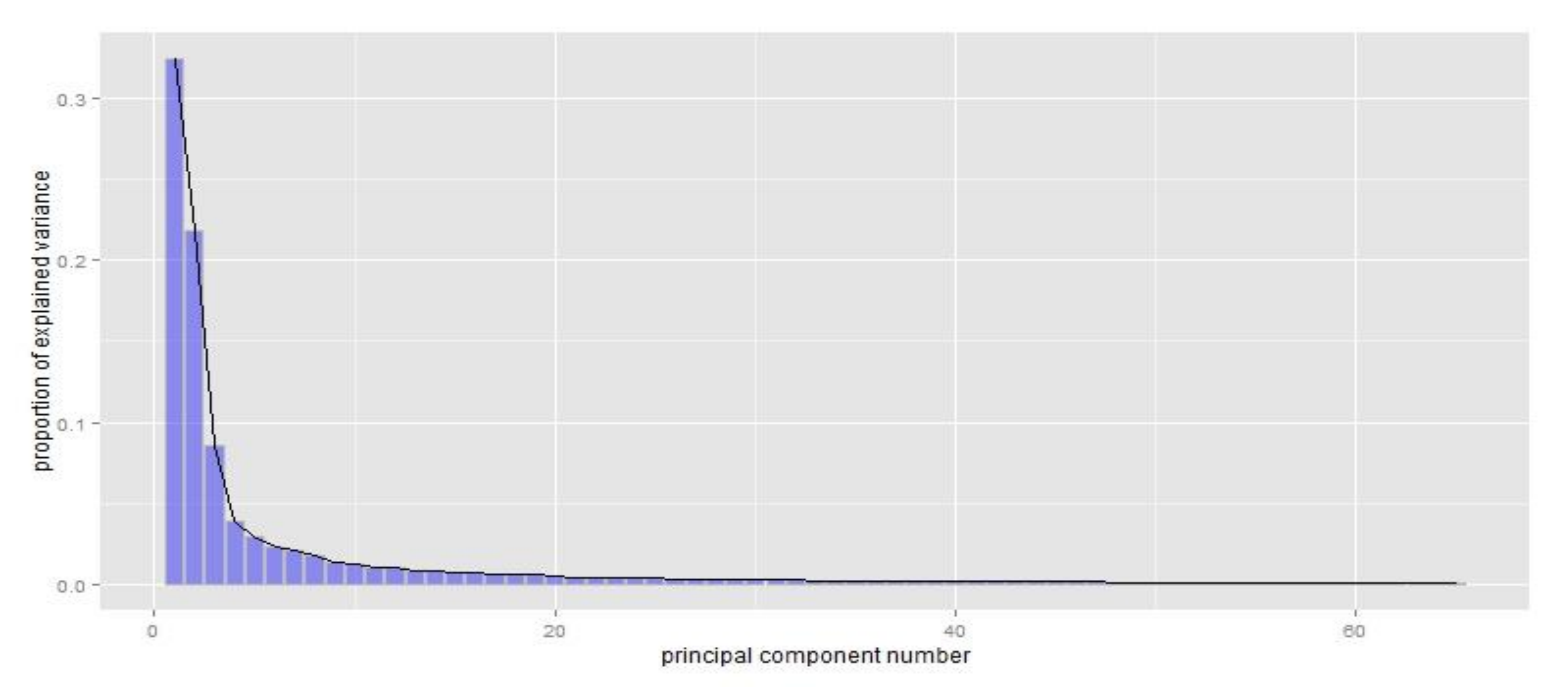
$$a_i = (\mathbf{x} - \bar{\mathbf{x}}) \cdot \mathbf{v}_i \quad \text{for } i = 1 \ldots, k$$

Image reconstruction:

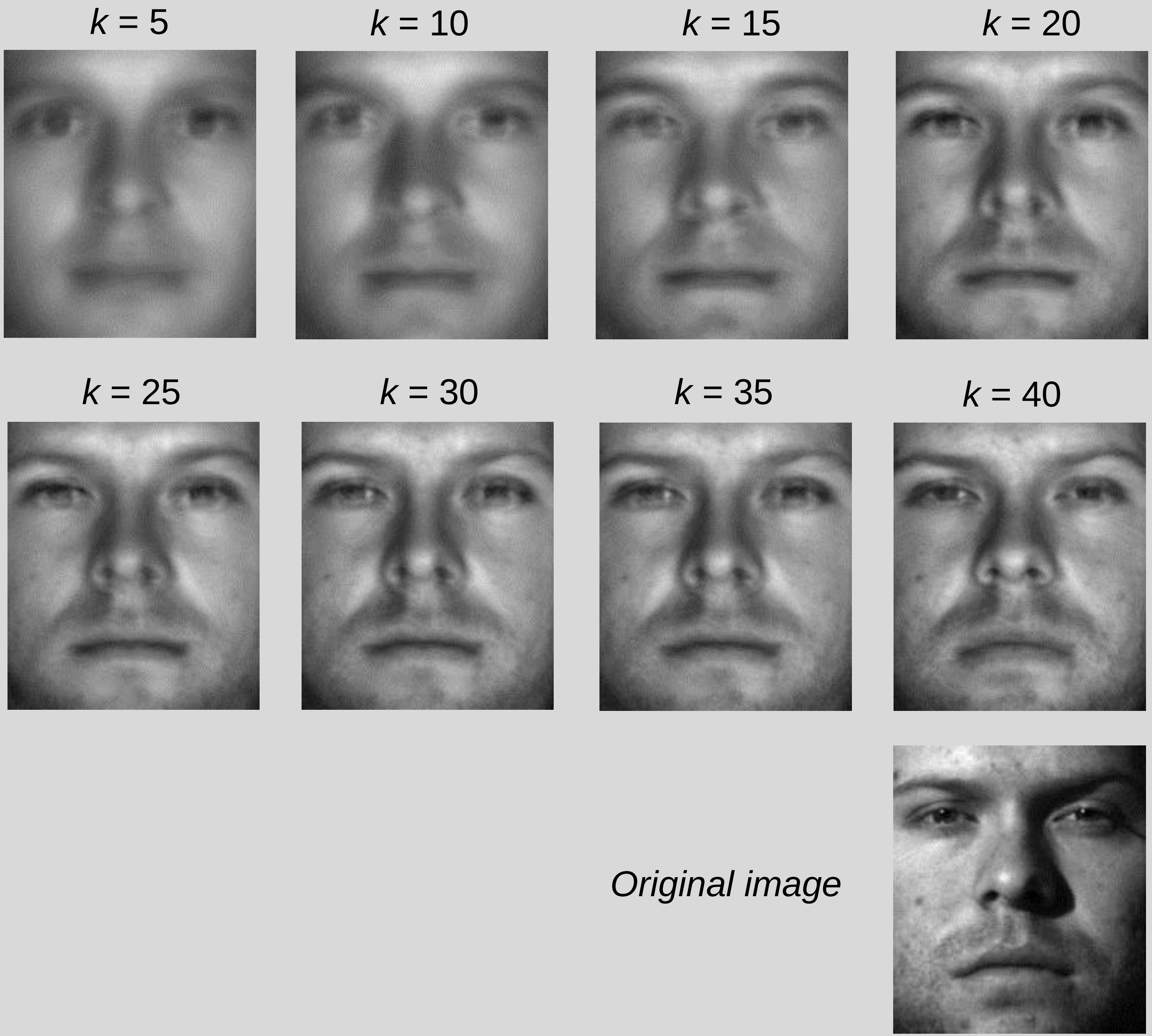
$$\mathbf{x} \approx \bar{\mathbf{x}} + \underbrace{a_1\mathbf{v}_1} + \underbrace{a_2\mathbf{v}_2} + \ldots + \underbrace{a_k\mathbf{v}_k}$$



# Choosing the Dimension k

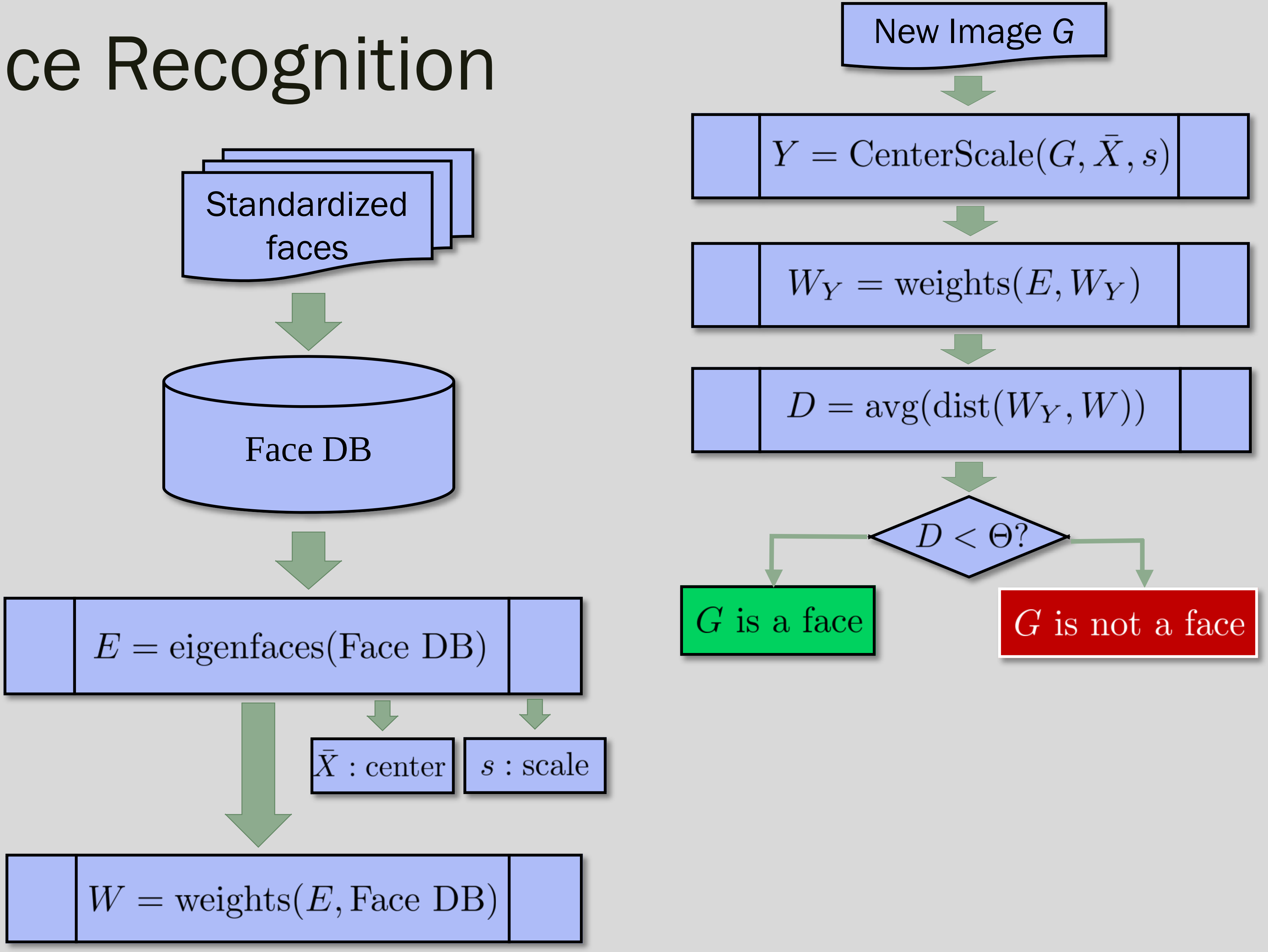



- How many eigenfaces to use?
- Look at the decay of the eigenvalues



k = 5
k = 10
k = 15
k = 20
k = 25
k = 30
k = 35
k = 40
Original image

# Face Recognition



# Mahalanobis Distance

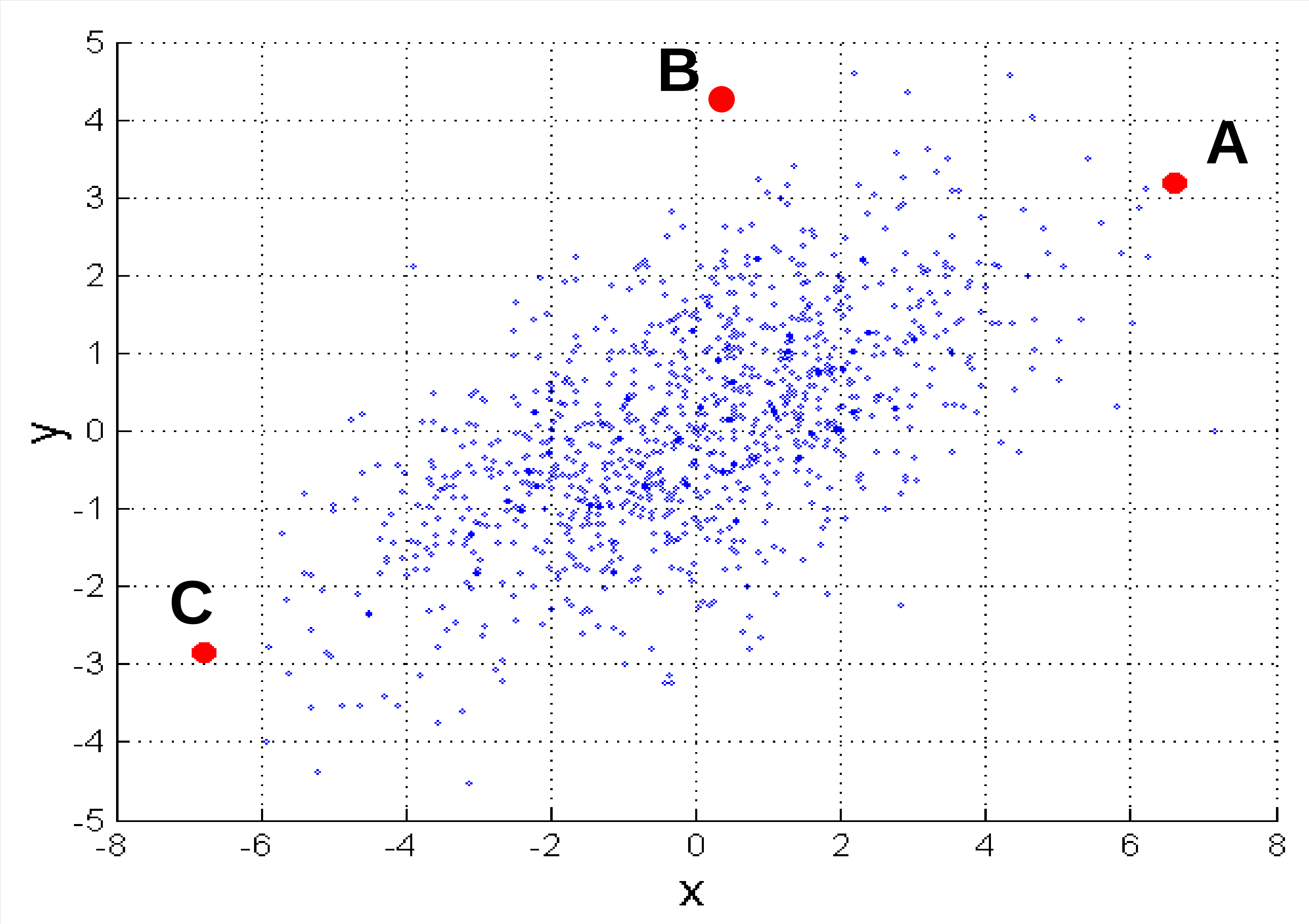
- **Mahalanobis $D^2$** distance is a multidimensional version of a z-score.
- It measures the distance of a case from the centroid (multidimensional mean) of a distribution, given the covariance (multidimensional variance) of the distribution.

$$D^2 = (x - \bar{x})^T C^{-1} (x - \bar{x})$$

Inverse of the covariance matrix

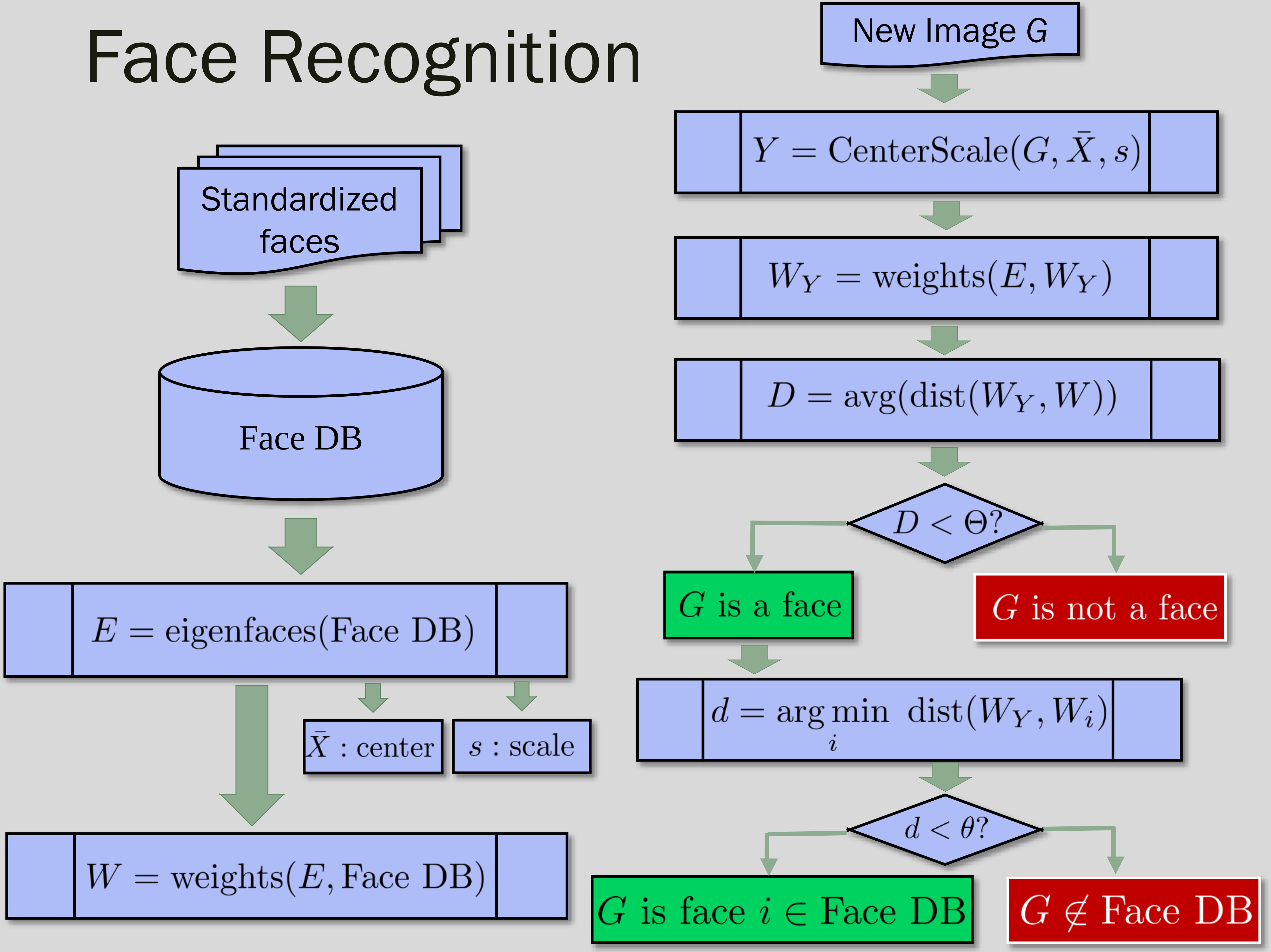
Note: When the covariance matrix is the identity Matrix, Mahalanobis distance = Euclidean distance squared.

# Mahalanobis Distance

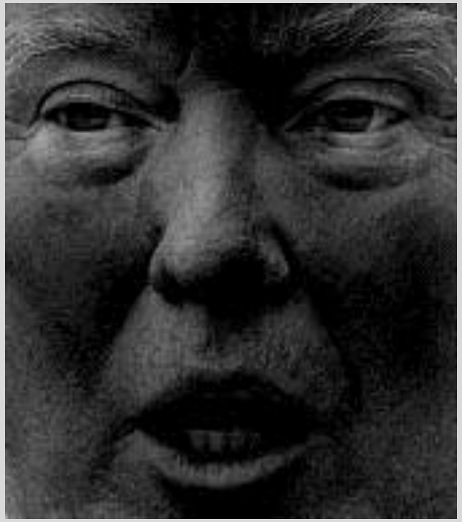



What is distance between A and B, A and C ?

# Face Recognition



# Examples

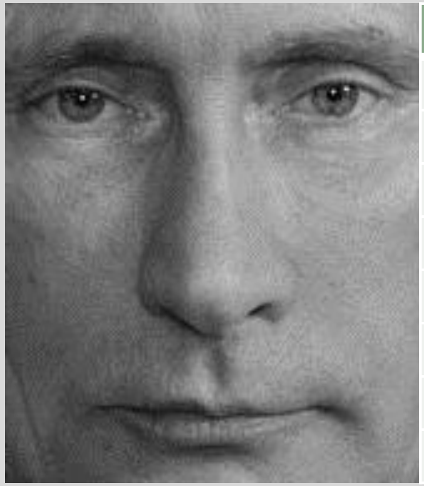


| |
|---|
| 32.14 |
| -146.53 |
| 13.05 |
| -12.62 |
| 32.72 |
| -65.65 |
| -27.31 |
| 3.37 |
| 40.45 |
| 46.86 |
| -29.58 |
| 24.91 |
| -40.86 |
| -13.56 |
| -21.38 |
| 8.5 |
| 16.09 |
| 11.81 |
| -4.54 |
| -6.84 |
| 3.08 |
| 29.1 |
| 1.7 |
| -22.5 |
| -14.41 |
| -1.96 |
| -6.75 |
| -9.03 |
| 19.31 |
| -1.63 |
| -7.07 |
| 14.36 |
| 8.09 |
| 5.31 |
| -9.98 |

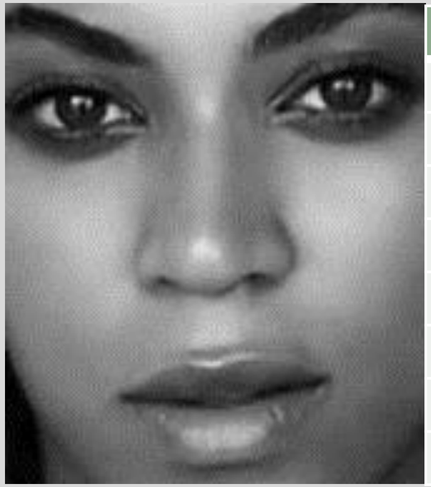
# Examples

| Trump |
| ---: |
| 32.14 |
| -146.53 |
| 13.05 |
| -12.62 |
| 32.72 |
| -65.65 |
| -27.31 |
| ... |

| Clinton |
| ---: |
| -25.15 |
| -147.83 |
| -31.92 |
| 16.32 |
| 24.3 |
| -42.75 |
| -15.83 |
| ... |



| Putin |
| ---: |
| -114.46 |
| -49.03 |
| -1.2 |
| -10.97 |
| 24.85 |
| -31.78 |
| -48.91 |
| ... |



| Knowles |
| ---: |
| 2.23 |
| -81.38 |
| 40.09 |
| -28.02 |
| 20.27 |
| -5.16 |
| 15.61 |
| ... |

| Hardy |
| ---: |
| -12.37 |
| -139.77 |
| 28.46 |
| 11.63 |
| -17.96 |
| -15.01 |
| 2.71 |
| ... |

| Kardashian |
| ---: |
| -28.4 |
| -150.75 |
| -1.17 |
| -14.82 |
| 28.69 |
| -28.13 |
| -27.63 |
| ... |

# Average distance from Face Space

99.1

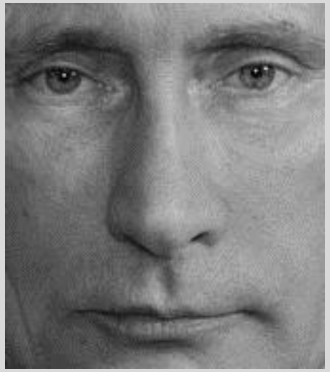
57.5

238.0

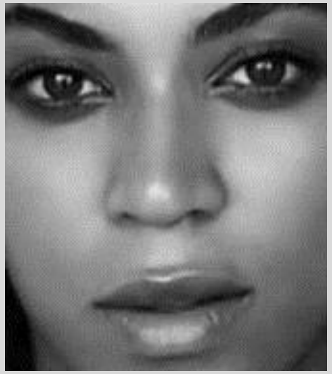
74.1

80.7

395.7



72.3



70.4

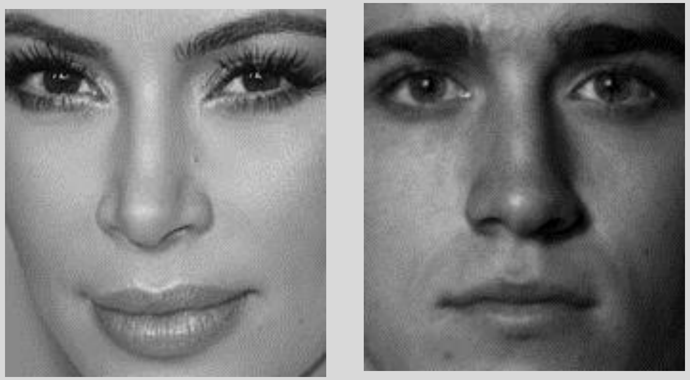
237.8

# Closest Match

50.8

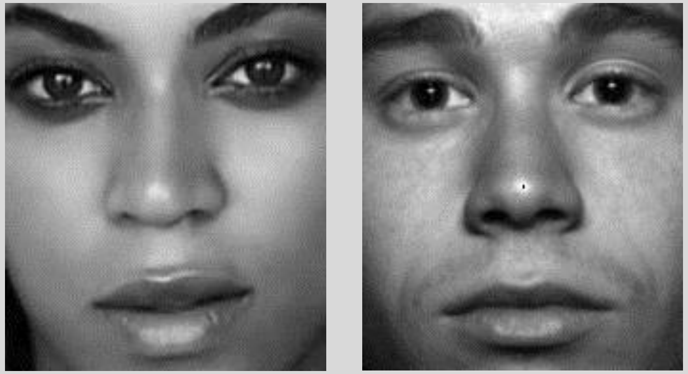
28.4

33.7



39.8

31.9



30.3