ASSOCIATION MINING

ISE/DSA 5103

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ASSOCIATION RULES

- Study of “what goes with what”
  - Customers who bought X also bought Y
  - What symptoms go with what diagnosis
- Transaction-based or event-based
- Also called *market basket analysis* and *affinity analysis*, *frequent pattern mining*
- Originated with study of customer transactions databases to determine associations among items purchased

WHAT IS FREQUENT PATTERN ANALYSIS?

- Frequent pattern: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- Motivation: Finding inherent regularities in data
  - What products were often purchased together?— Beer and diapers?!
  - What are the subsequent purchases after buying a PC?
  - What kinds of DNA are sensitive to this new drug?
  - Can we automatically classify web documents?

APPLICATIONS

- **Market Basket Analysis:** given a database of customer transactions, where each transaction is a set of items the goal is to find groups of items which are frequently purchased together.
- **Credit Cards/ Banking Services:** each card/account is a transaction containing the set of customer's payments
- **Medical Treatments:** each patient is represented as a transaction containing the ordered set of diseases

Bound Away
Last Train Home

Share your own customer images

List Price: $16.98
Price: **$16.98** and eligible for **FREE Super Saver Shipping** on orders over $25. See details.

Availability: Usually ships within 24 hours

Want it delivered Tomorrow? Order it in the next 4 hours and 9 minutes, and choose **One-Day S** checkout. See details.

41 used & new from $6.99

See more product details

Based on customer purchases, this is the #82 Early Adopter Product in Alternative Rock.

801x612

Buy this title for only $.01 when you get a new Amazon Visa® Card

Apply now and if you're approved instantly, **save $30** off your first purchase, earn **3% rewards**, get a **0% APR,*** and pay **no**

Amazon Visa discount:	$30.00
Applied to this item:	- $16.97
Discount remaining:	$13.03

Find out how
(Don't show again)

Customers who bought this title also bought:

- Time and Water ~ Last Train Home (Why?)
- Cold Roses ~ Ryan Adams & the Cardinals (Why?)
- Tambourine ~ Tift Merritt (Why?)
- Last Train Home ~ Last Train Home (Why?)
- True North ~ Last Train Home (Why?)
- Universal United House of Prayer ~ Buddy Miller (Why?)
- Wicked Twisted Road [ENHANCED] ~ Reckless Kelly (Why?)
- Hacienda Brothers ~ Hacienda Brothers (Why?)

Association Rule Mining (ARM)

Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

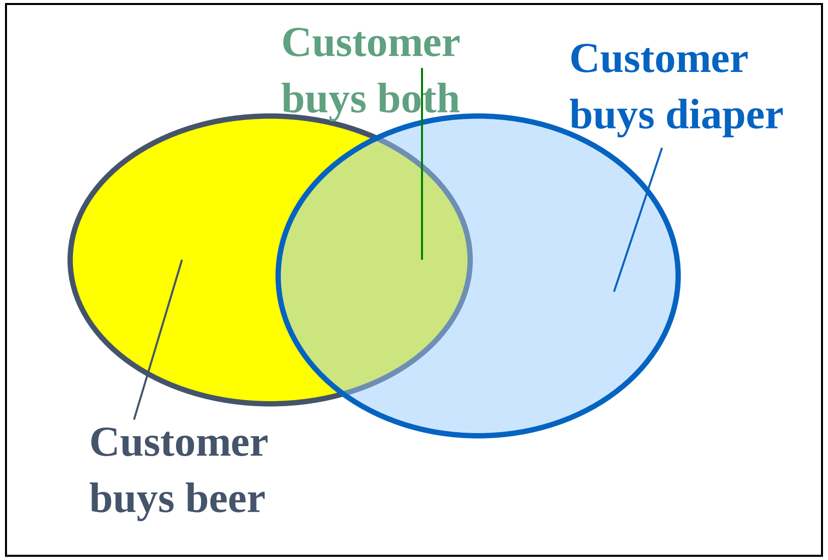
Example of Association Rules

{Diaper} $\rightarrow$ {Beer},
{Milk, Bread} $\rightarrow$ {Eggs,Coke},
{Beer, Bread} $\rightarrow$ {Milk},

Implication means co-occurrence, not causality!

Basic Concepts: Frequent Patterns

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- ***itemset***: A set of one or more items
- ***k-itemset:*** $X = \{x_1, \ldots, x_k\}$
- ***(absolute) support***, or, ***support count*** of X: Frequency or occurrence of an itemset X
- ***(relative) support***, *s*, is the fraction of transactions that contains X (i.e., the probability that a transaction contains X)
- An itemset X is ***frequent*** if X's support is no less than a ***minsup*** threshold

Basic Concepts: Association Rules

Body → Consequent (Support , Confidence)

- *Body*: represents the examined data; i.e., the "IF" part = **antecedent**
- *Consequent*: represents a discovered property for the examined data; i.e., the "THEN" part

Antecedent and consequent are *disjoint* (i.e., have no items in common)

- *Support*: percentage of the records satisfying the *body* or the *consequent*
- *Confidence*: percentage of the records satisfying both the *body* and the *consequent* of those satisfying only the *body*

Basic Concepts: Association Rules

Tid	Items bought
10	Beer, Nuts, Diaper
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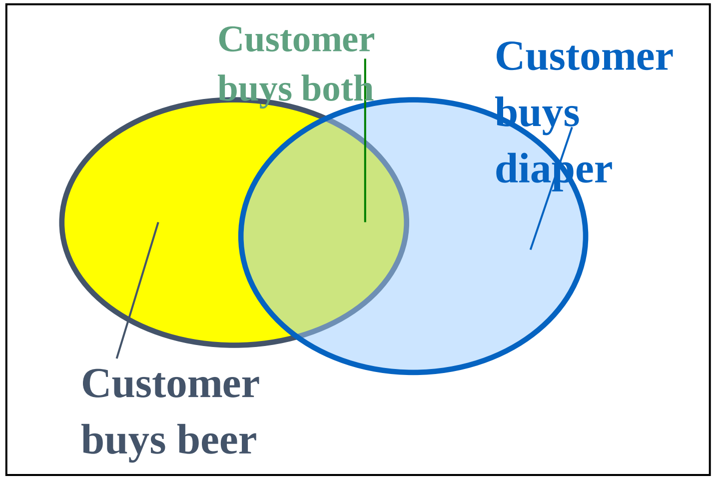

support, *s*, probability that a transaction contains X ∪ Y

confidence, *c*, conditional probability that a transaction having X also contains *Y*

Find **all** the rules *X* → *Y* with minimum support and confidence

Let min support = 50%, min confidence = 50%

Frequncy Patterns:

Beer:3, Nuts:3, Diaper:4, Eggs:3, {Beer, Diaper}:3

- Association rules: (many more!)
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

Basic Concepts: Association Rules

Tid	Items bought
10	Beer, Nuts, Diaper
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30	Beer, Diaper, Eggs
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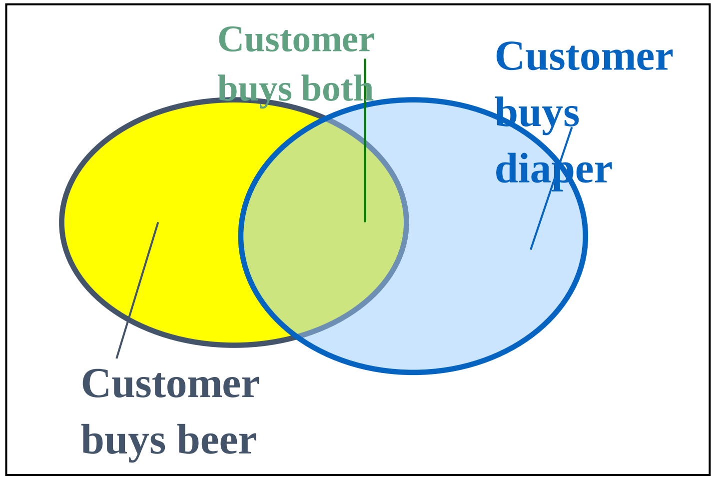
50	Nuts, Coffee, Diaper, Eggs, Milk



- Find **all** the rules $X \rightarrow Y$ with minimum support and confidence
  - ***support***, s, probability that a transaction contains $X \cup Y$

$$s = P(X \cap Y)$$

  - ***confidence***, c, conditional probability that a transaction having X also contains Y

$$c = P(Y|X) = \frac{P(X \cap Y)}{P(X)}$$

Association-rule mining task

Given a set of transactions **D**, the goal of association rule mining is to find **all** rules having

- support ≥ ***minsup*** threshold
- confidence ≥ ***minconf*** threshold

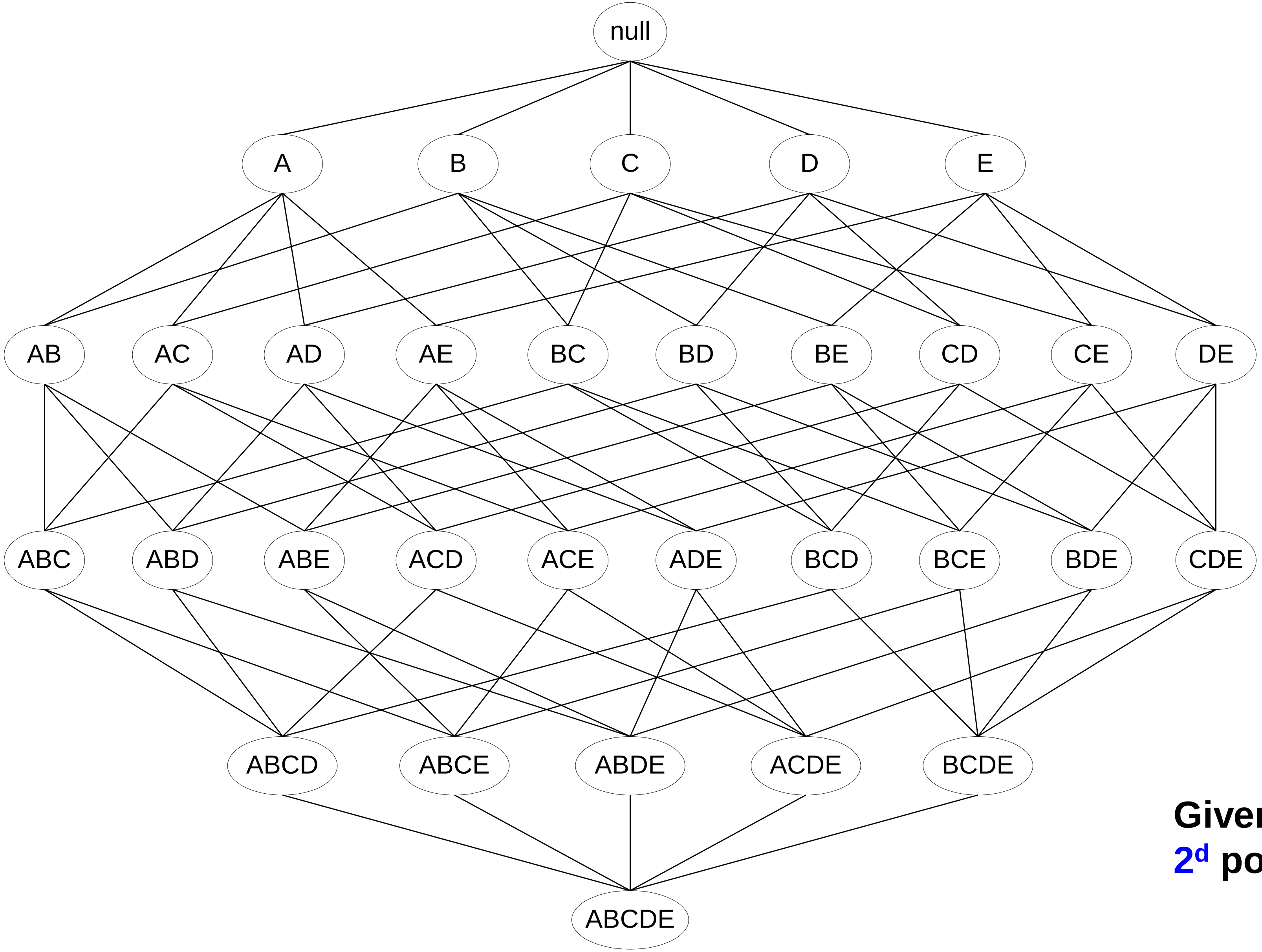
Finding frequent sets

- Notation: The input is a transaction database **D** where every transaction consists of a subset of items from some universe ***I***

- **Task:** Given a transaction database **D** and a **minsup** threshold find all frequent itemsets and the frequency of each set in this collection

- **Stated differently:** Count the number of times combinations of attributes occur in the data. If the fraction of the combination is above **minsup** report it.

How many itemsets are there?



Given d items, there are 2^d possible itemsets

When is the task sensible and feasible?

- If **minsup = 0**, then all subsets of ***I*** will be frequent and thus the size of the collection will be very large
- This summary is very large (maybe larger than the original input) and thus not interesting
- The task of finding all frequent sets is interesting typically only for relatively large values of **minsup**
  - *It is also probably only useful with minsup relatively large.*

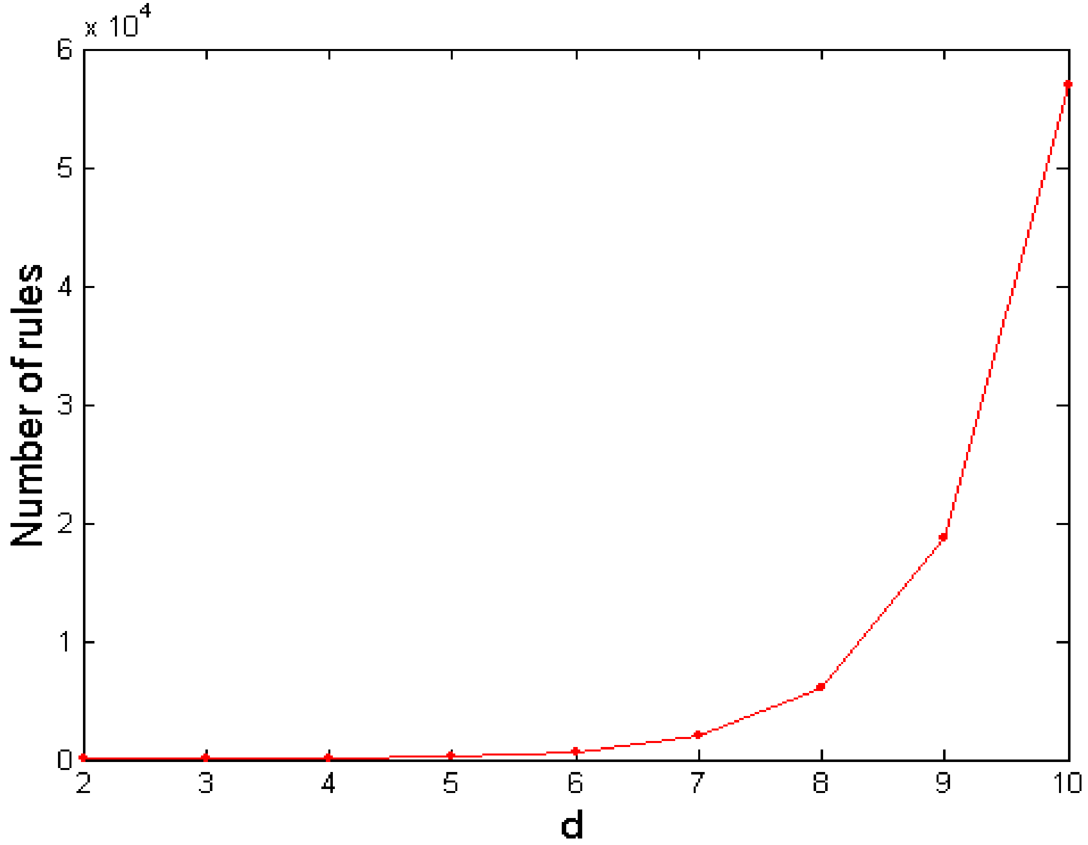
Brute-force algorithm for ARM

- List all possible association rules
- Compute the support and confidence for each rule
- Prune rules that fail the *minsup* and *minconf* thresholds

→ **Computationally prohibitive!**

How many association rules are there?

- Given **d** unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1} \left[\binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \right]$$

$$= 3^d - 2^{d+1} + 1$$

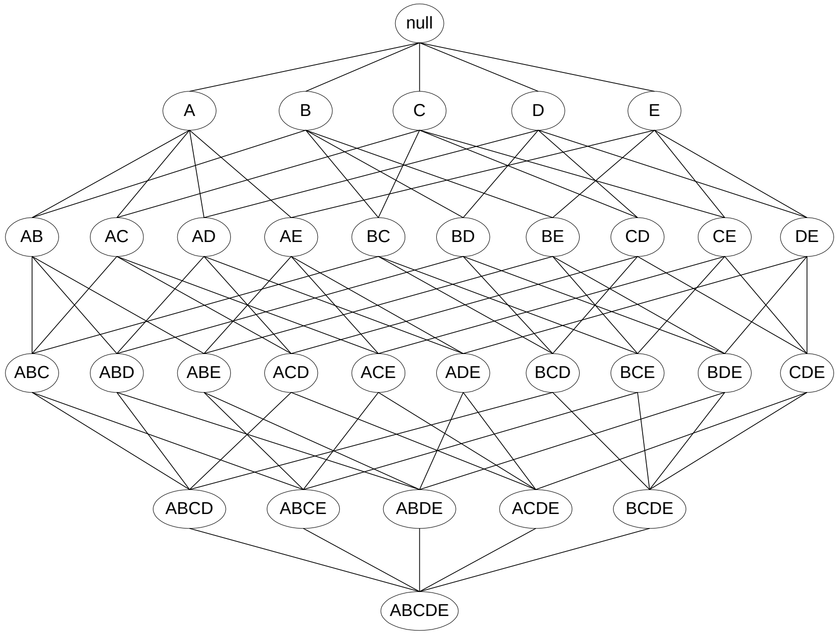
If d=6, R = 602 rules
If d=10, R = 57,002 rules
If d=20, R = 3,484,687,250 rules

Speeding-up the brute-force algorithm

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
  - Use vertical-partitioning of the data to apply the mining algorithms
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

Reduce the number of candidates

- **Apriori principle (main observation):**
  - If an itemset is frequent, then all of its subsets must also be frequent



- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - The support of an itemset ***never exceeds*** the support of its subsets
  - This is known as the ***anti-monotone*** property of support

Example

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

s(Bread) = 4/5
s(Bread, Beer) = 3/5

s(Bread) > s(Bread, Beer)

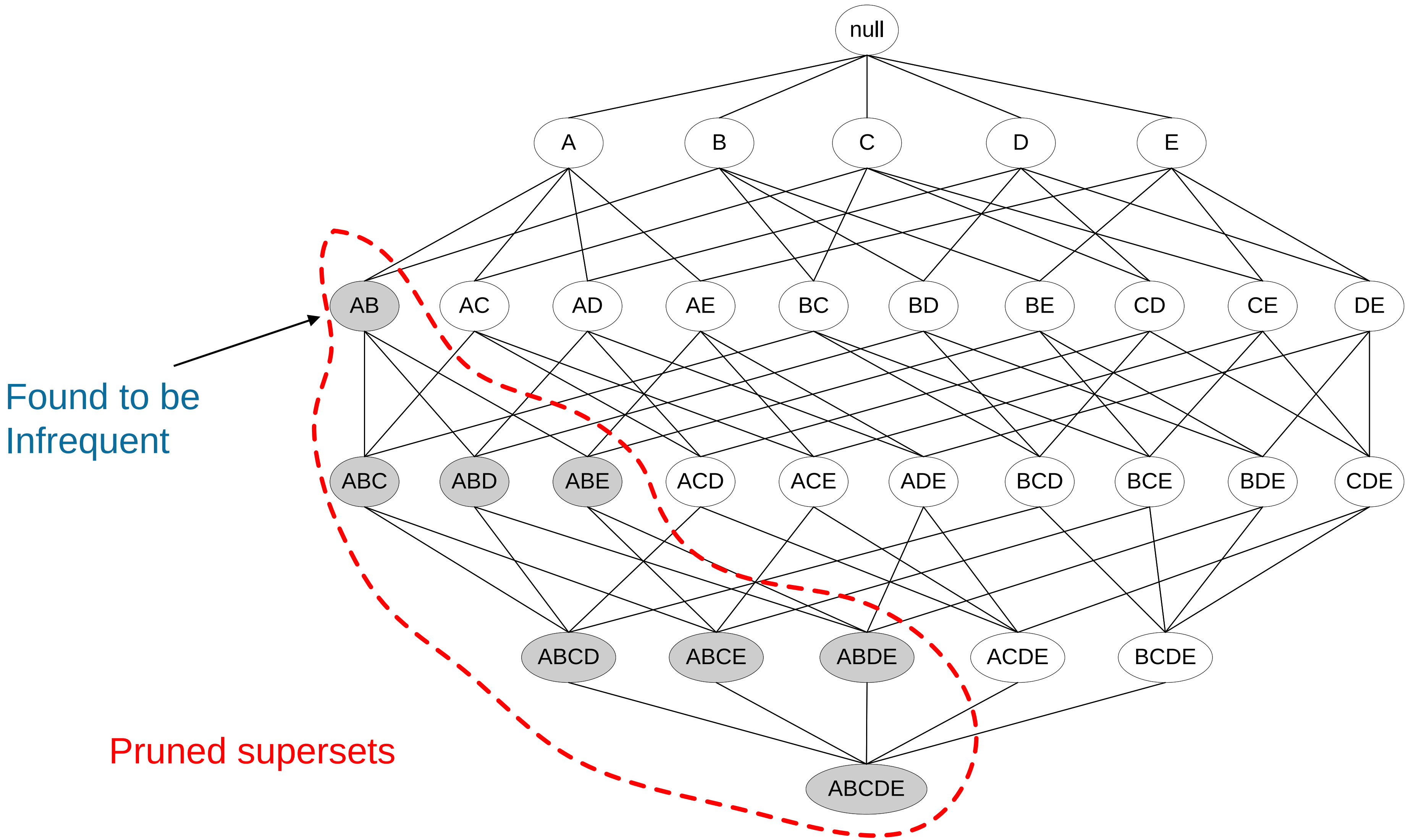
s(Milk) = 4/5
s(Bread, Milk) = 3/5

s(Milk) > s(Bread, Milk)

s(Diaper, Beer) = 3/5
s(Diaper, Beer, Coke) = 1/5

s(Diaper, Beer) > s(Diaper, Beer, Coke)

Illustrating the Apriori principle



Illustrating the Apriori principle

minsup = 3/5

Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Pairs (2-itemsets)

No need to generate candidates involving Coke or Eggs!

Itemset	Count
{Bread,Milk}	**3**
{Bread,Beer}	**2**
{Bread,Diaper}	**3**
{Milk,Beer}	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Triplets (3-itemsets)

No need to generate candidates involving {Bread, Beer} or {Milk,Diaper}!

Itemset	Count
{Bread,Milk,Diaper}	**3**

If every subset is considered,
$^6C_1 + {}^6C_2 + {}^6C_3 = 41$
With support-based pruning,
6 + 6 + 1 = 13

Exploiting the Apriori principle

1. Find frequent 1-items and put them to L_k (**k=1**)
2. Use L_k to generate a collection of *candidate* itemsets C_{k+1} with size (**k+1**)
3. Scan the database to find which itemsets in C_{k+1} are frequent and put them into L_{k+1}
4. If L_{k+1} is not empty
   - **k=k+1**
   - Goto step 2

If there is any itemset which is infrequent, its superset should not be generated/tested!

R. Agrawal, R. Srikant: "Fast Algorithms for Mining Association Rules", *Proc. of the 20th Int'l Conference on Very Large Databases*, 1994.

The Apriori Algorithm—An Example

minsup = 2/4

Database

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

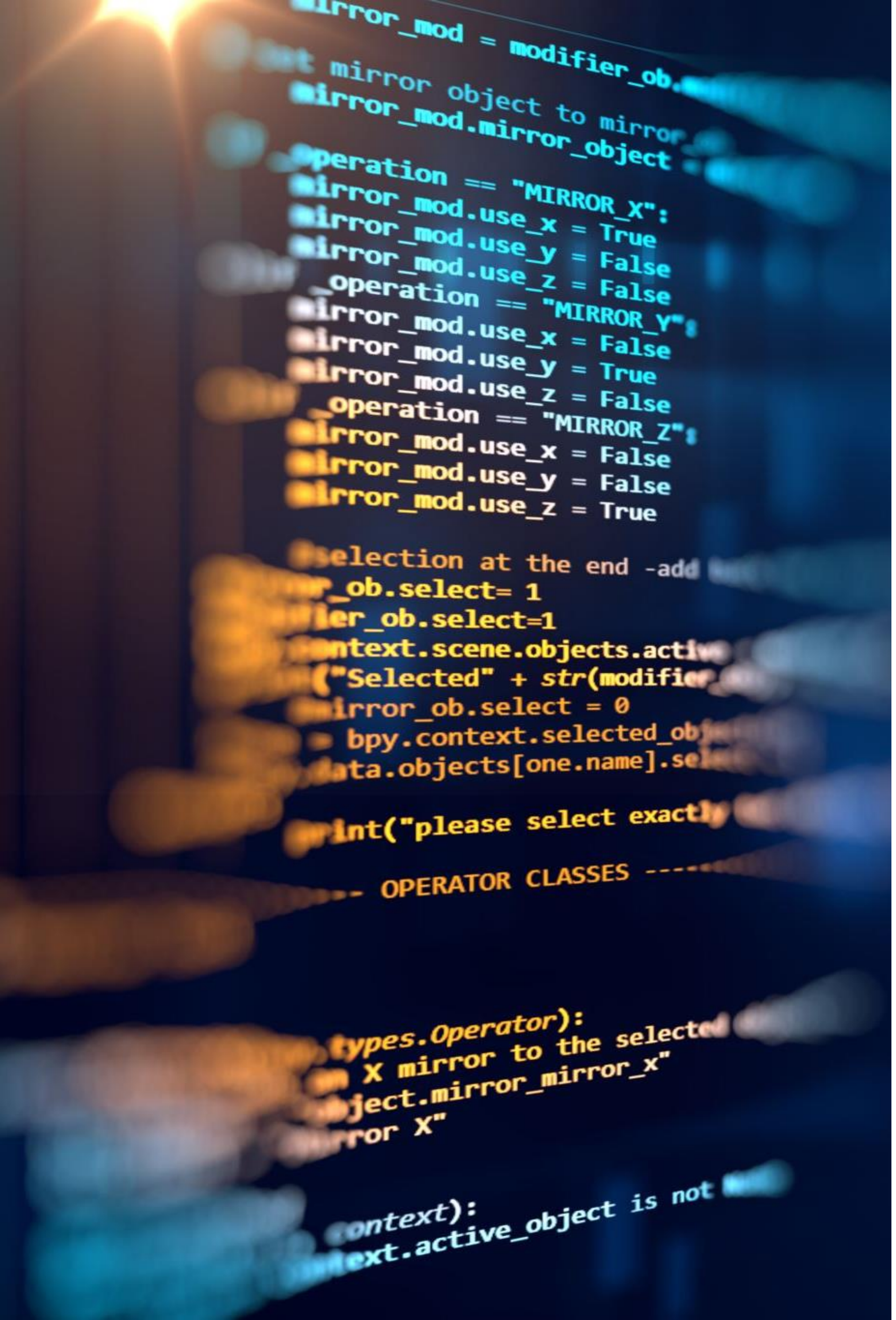
C_3

Itemset
{B, C, E}

3rd scan

L_3

Itemset	sup
{B, C, E}	2



Apriori algorithm

- Much faster than the Brute-force algorithm
  - It avoids checking all elements in the lattice
- The running time is in the worst case **$O(2^d)$**
  - Pruning really prunes in practice
- It makes multiple passes over the dataset
  - One pass for every level **k**
- Multiple passes over the dataset is inefficient when we have thousands of candidates and millions of transactions

Association Rules

Tid	Items bought
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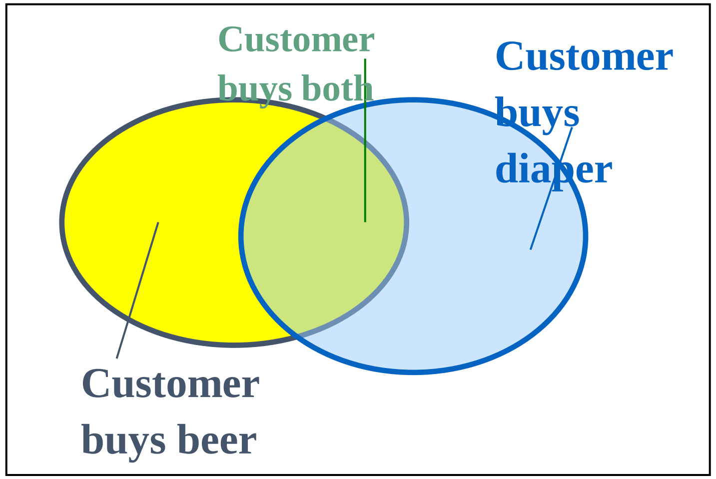

Rule: X→ Y

support, ***s(X→ Y)***, probability that transaction contains X ∪ Y

confidence, ***c(X→ Y)***,, conditional probability that a transaction having X also contains *Y*

coverage: *support of LHS, i.e.,* ***s(X)***

lift: *ratio of observed support to the expected support if the items were independent*

rule	support	confidence	coverage	lift	count
{grapes,mustard} => {onions}	0.000508	0.833333	0.00061	26.87158	5

See the file ARM.R for examples!