Homework 3 - Principal Component Analysis

Daniel Carpenter

August 2022

Table of contents

Packages . . . 2

1. Glass Data **2**

Get and Clean Data . . . 2

(a) Mathematics of PCA . . . 2

(b) Applications of PCA . . . 6

(c) !! . . . 7

2. Principal components for dimension reduction **7**

3. Housing data dimension reduction and exploration **8**

Check list:

- 1 (iv - v)

Packages

```
library(tidyverse) # get tidverse for piping
library(skimr)
library(knitr)
library(scales)
require(lubridate)

library(mlbench)   # Glass data
library(ggbiplot) # biplots
library(scatterplot3d)
```

1. Glass Data

Get and Clean Data

```
data(Glass)

# Remove duplicates
Glass <- Glass[!duplicated(Glass), ]
```

(a) Mathematics of PCA

`i.` Create the correlation matrix of all the numerical attributes in the `Glass` data and store the results in a new object `corMat`

```
skimmed <- skim(Glass)

# Notice one factor data, for variable `type`
skimmed$skim_type
```

```
 [1] "factor"  "numeric" "numeric" "numeric" "numeric" "numeric" "numeric"
 [8] "numeric" "numeric" "numeric"
```

```
# Get only numeric data
GlassNumeric <- Glass %>% select(where(is.numeric))
```

```r
# Create correlation matrix using only numeric data type
corMat <- cor(GlassNumeric)
```

`ii.` Compute the eigenvalues and eigenvectors of `corMat`.

Eigenvalues

```r
# prcomp(corMat)
eigenValues = eigen(corMat)$values
eigenValues
```

```
[1] 2.510152168 2.058169337 1.407484057 1.144693344 0.914768873 0.528593040
[7] 0.370262639 0.064267543 0.001608997
```

Eigenvectors

```r
eigenVectors = eigen(corMat)$vectors
eigenVectors
```

```
            [,1]        [,2]        [,3]       [,4]        [,5]        [,6]
 [1,]  0.5432231 -0.28911804 -0.08849541  0.1479796  0.07670808 -0.11455615
 [2,] -0.2676141 -0.26909913  0.36710090  0.5010669 -0.14626769  0.55790564
 [3,]  0.1093261  0.59215502 -0.02295318  0.3842440 -0.11610001 -0.30585293
 [4,] -0.4269512 -0.29636272 -0.32602906 -0.1488756 -0.01720068  0.02014091
 [5,] -0.2239232  0.15874450  0.47979931 -0.6394962 -0.01763694 -0.08850787
 [6,] -0.2156587  0.15305116 -0.66349177 -0.0733491  0.30154622  0.24107648
 [7,]  0.4924367 -0.34678973  0.01380151 -0.2743430  0.18431431  0.14957911
 [8,] -0.2516459 -0.48262056 -0.07649040  0.1299431 -0.24970936 -0.65986429
 [9,]  0.1912640  0.06089167 -0.27223834 -0.2252596 -0.87828176  0.24066617
             [,7]        [,8]        [,9]
 [1,] -0.08223530  0.75177166 -0.02568051
 [2,] -0.15419352  0.12819398  0.31188932
 [3,]  0.20691746  0.07799332  0.57732740
 [4,]  0.69982052  0.27334224  0.19041178
 [5,] -0.20945417  0.38077660  0.29747147
 [6,] -0.50515516  0.11064442  0.26075531
 [7,]  0.09984144 -0.39885229  0.57999243
 [8,] -0.35043794 -0.14497643  0.19853265
 [9,] -0.07120579  0.01650505  0.01459278
```

iii. Use `prcomp` to compute the principal components of the `Glass` attributes (make sure to use the scale option).

```
# Using only numeric data
pc.glass <- prcomp(GlassNumeric, scale = TRUE)
pc.glass
```

```
Standard deviations (1, .., p=9):
[1] 1.58434597 1.43463213 1.18637433 1.06990343 0.95643550 0.72704404 0.60849210
[8] 0.25351044 0.04011231

Rotation (n x k) = (9 x 9):
          PC1         PC2         PC3        PC4         PC5         PC6
RI -0.5432231  0.28911804 -0.08849541 -0.1479796  0.07670808 -0.11455615
Na  0.2676141  0.26909913  0.36710090 -0.5010669 -0.14626769  0.55790564
Mg -0.1093261 -0.59215502 -0.02295318 -0.3842440 -0.11610001 -0.30585293
Al  0.4269512  0.29636272 -0.32602906  0.1488756 -0.01720068  0.02014091
Si  0.2239232 -0.15874450  0.47979931  0.6394962 -0.01763694 -0.08850787
K   0.2156587 -0.15305116 -0.66349177  0.0733491  0.30154622  0.24107648
Ca -0.4924367  0.34678973  0.01380151  0.2743430  0.18431431  0.14957911
Ba  0.2516459  0.48262056 -0.07649040 -0.1299431 -0.24970936 -0.65986429
Fe -0.1912640 -0.06089167 -0.27223834  0.2252596 -0.87828176  0.24066617
           PC7         PC8         PC9
RI -0.08223530 -0.75177166 -0.02568051
Na -0.15419352 -0.12819398  0.31188932
Mg  0.20691746 -0.07799332  0.57732740
Al  0.69982052 -0.27334224  0.19041178
Si -0.20945417 -0.38077660  0.29747147
K  -0.50515516 -0.11064442  0.26075531
Ca  0.09984144  0.39885229  0.57999243
Ba -0.35043794  0.14497643  0.19853265
Fe -0.07120579 -0.01650505  0.01459278
```

iv. Compare the results from (ii) and (iii) - Are they the same? Different? Why?

- The eigenvalues differ
- The eigenvectors are the same in absolute value, but the signs are the opposite within each value of the vectors
- Why do they differ? Past `ii` uses the correlation matrix; the principal component analysis (`ii`) uses the covariance matrix, which is a scaled, or *normalized*, version of the correlation matrix.

v. Using R demonstrate that principal components 1 and 2 from (iii) are orthogonal. (Hint: the inner product between two vectors is useful in determining the angle between the two vectors)

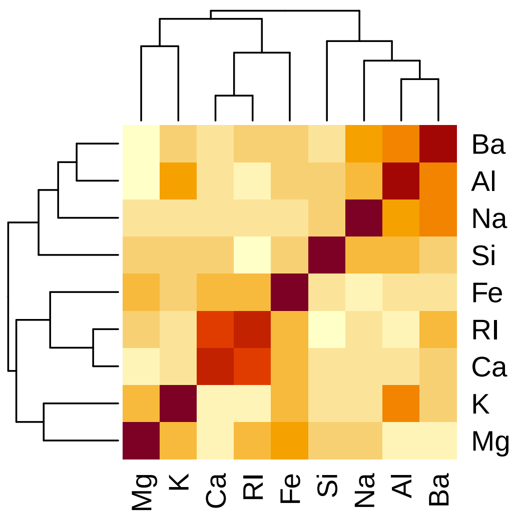
```
PC1.glass <- pc.glass$x[,1]
PC2.glass <- pc.glass$x[,2]

angle <- acos( sum(PC1.glass*PC2.glass) / ( sqrt(sum(PC1.glass * PC1.glass)) * sqrt(sum(PC

angle
```

```
[1] 1.570796
```

(b) Applications of PCA

i. Create a visualization of the corMat correlation matrix (i.e., a heatmap or variant) If you are interested and have time, consider the corrplot package for very nice options, https://cran.r-project.org/web/packages/corrplot/vignettes/corrplot-intro.html.

```
heatmap(corMat)
```



ii. Provide visualizations of the principal component analysis results from the Glass data. Consider incorporating the glass type to group and color your biplot.

iii. Provide an interpretation of the first two prinicpal components the Glass data.

iv. Based on the PCA results, do you believe that you can effectively reduce the dimension of the data? If so, to what degree? If not, why?

(c) !!

2. Principal components for dimension reduction

3. Housing data dimension reduction and exploration