Chapter 9

Fundamentals of Hypothesis Testing: One-Sample Tests

The Plant Operation Management at Oxford Cereals

Monitoring the amount that each cereal box is filled with is among the responsibilities of the Oxford Cereals plant operation managers.

- Company guideline: the average box weight should be 368 grams.
- If not, the cereal-filling process should be adjusted.

- Adjusting the production line is costly though. So you do not want to make unnecessary adjustments.
- **How should we decide whether the process should be adjusted or not?**

The Plant Operation Management at Oxford Cereals

- Essentially, we need to figure out a way to use a **sample of boxes** and say something about the average weight of boxes that are filled at our plant.
- Yet, we need to keep in mind that the sample averages may vary from sample to sample.

The Plant Operation Management at Oxford Cereals

- To do this we need to form a **hypothesis about the population mean;** i.e. the average weight box of all the boxes that are filled at our plant.

The Plant Operation Management at Oxford Cereals

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A **hypothesis** is a statement about the **population parameter.**

Yet, we use a sample to test this hypothesis.

What is a hypothesis?

- **Hypothesis**:

  An **unambiguous claim** that is made about a **population parameter**, such as mean (μ) or proportion (π).

- Example:

  Hypothesis – the mean weight for all the boxes that are filled at Oxford Cereals plants is 368 grams

How to state a hypothesis?

- We always state a **null hypothesis** along with an **alternative hypothesis**.
  - The null hypothesis states the status quo
    - Example: The mean box weight is equal to 368 grams

      H0: $\mu=368$
  - The alternative is the opposite
    - Example: The mean box weight is not equal to 368 grams

      H1: $\mu\neq368$

The Null Hypothesis, H_0

- We always begin with **the assumption that the null hypothesis is true** (Like in legal practice when we always assume that someone is innocent)
- Then we **try to reject the null** hypothesis (Like in legal practice when we try to prove that someone is not innocent)
- It may or may not be rejected.

- Tip:

The null hypothesis always contains "=", or "≤", or "≥" sign

Resource: researchskills.epigeum.com

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The Alternative Hypothesis, H_1

- The alternative is the opposite of the null hypothesis, challenging the status quo
- May or may not be proven

- Tip:

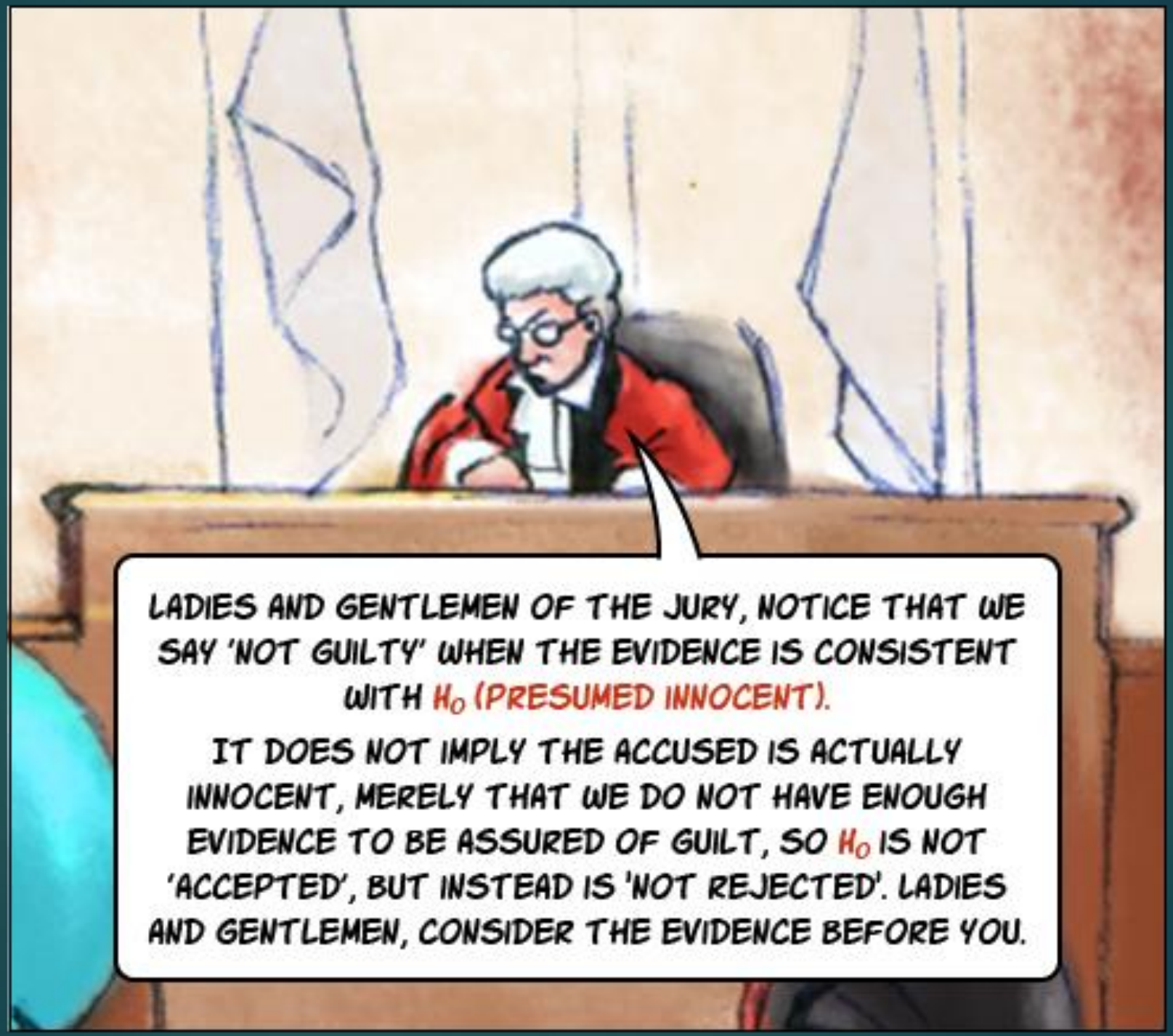
The alternative hypothesis never contains the "=", or "≤", or "≥" signs. Instead, it contains "≠", or "<" or ">".

Hypothesis testing, The Bottom Line…

- You **reject** the null hypothesis when the sample evidence suggest that **the alternative is far more likely**.

Hypothesis testing, The Bottom Line…

- You **reject** the null hypothesis when the sample evidence suggest that **the alternative is far more likely**.
- You **can never prove the null**. You are either **able to reject it** (and accept the alternative) or you are **unable to reject it**. That's all…



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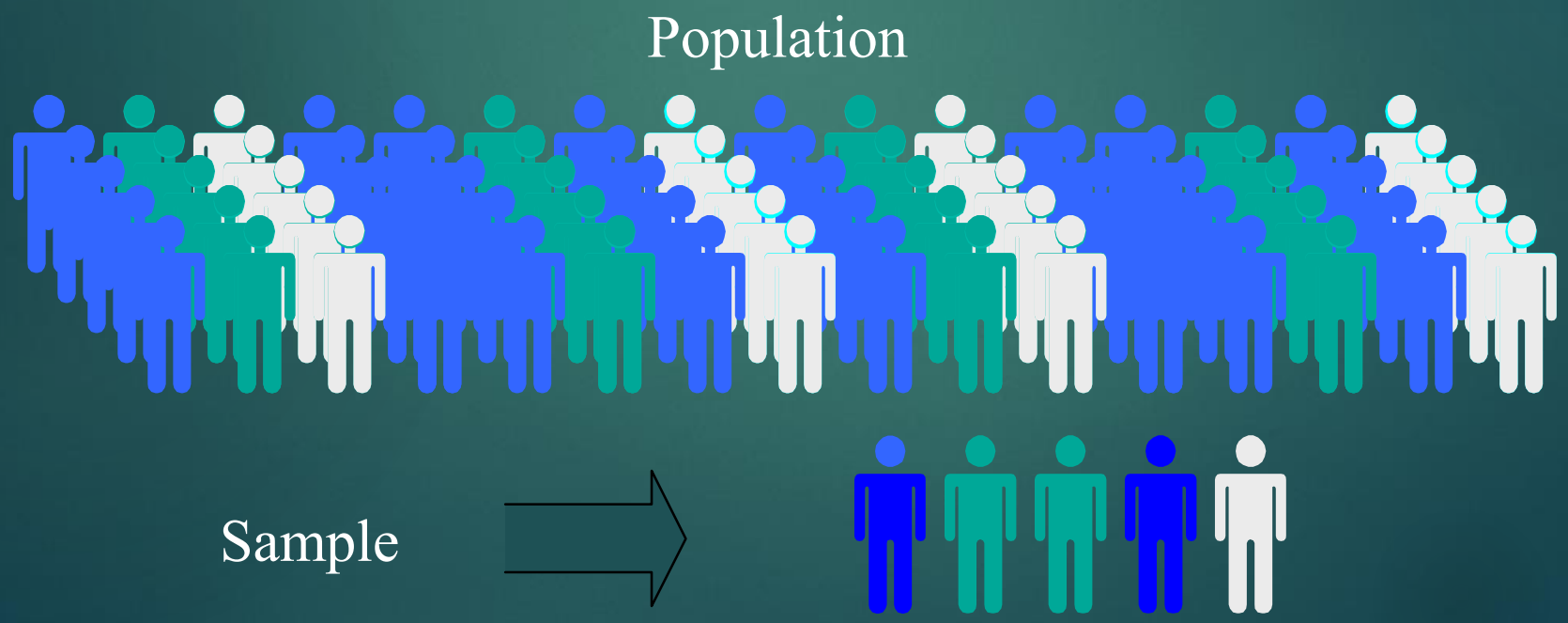
The Hypothesis Testing Process*

* In this section, we only focus on the process, not the details.

The Hypothesis Testing Process

- Claim: The population mean age is 50.
  - H_0: $\mu = 50$, H_1: $\mu \neq 50$
- Sample the population and find the sample mean.



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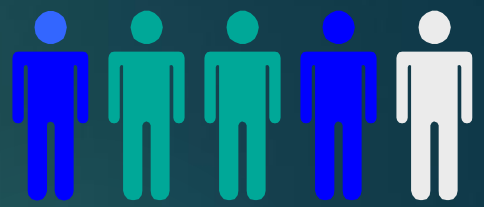

- Suppose the sample mean age was $\overline{X} = 20$.
- This is significantly lower than the claimed mean population age of 50, which is suggested by the null.

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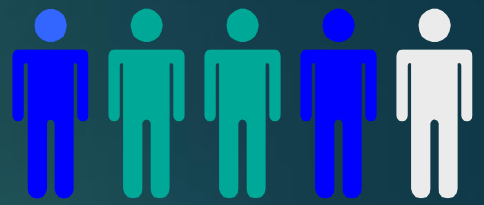

- Suppose the sample mean age was $\overline{X} = 20$.
- This is significantly lower than the claimed mean population age of 50, which is suggested by the null.
- If the null hypothesis were true, the probability of getting such a different sample mean would be very small, so you reject the null hypothesis .

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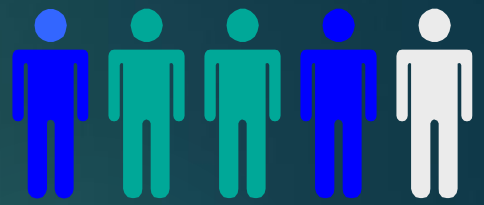
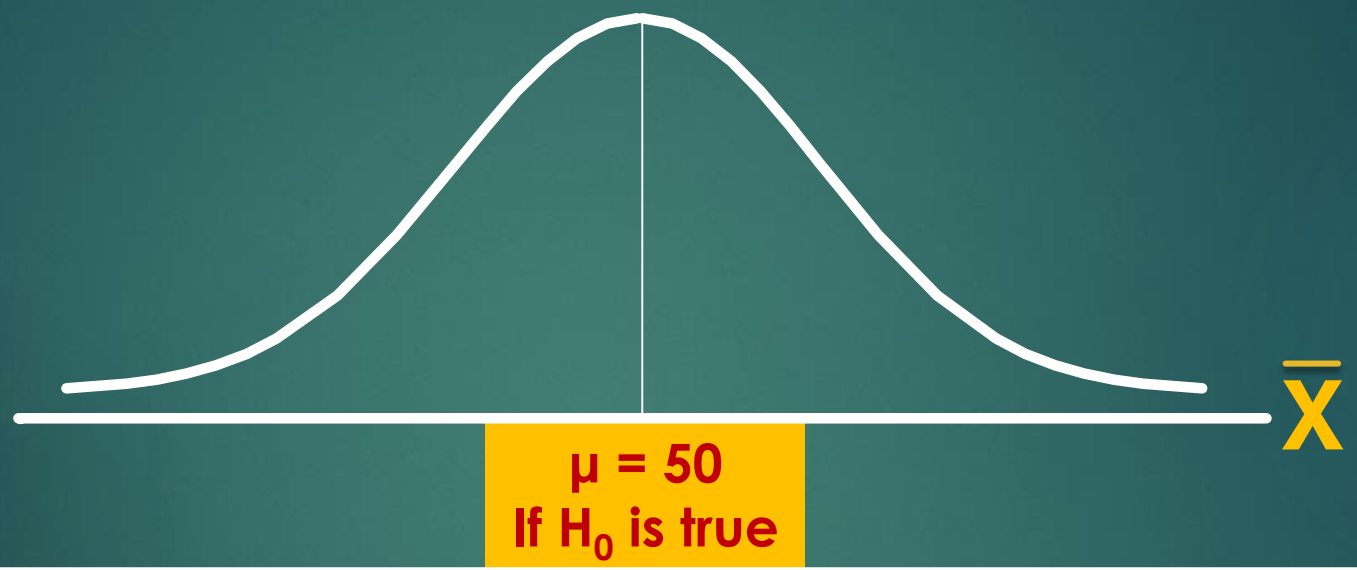


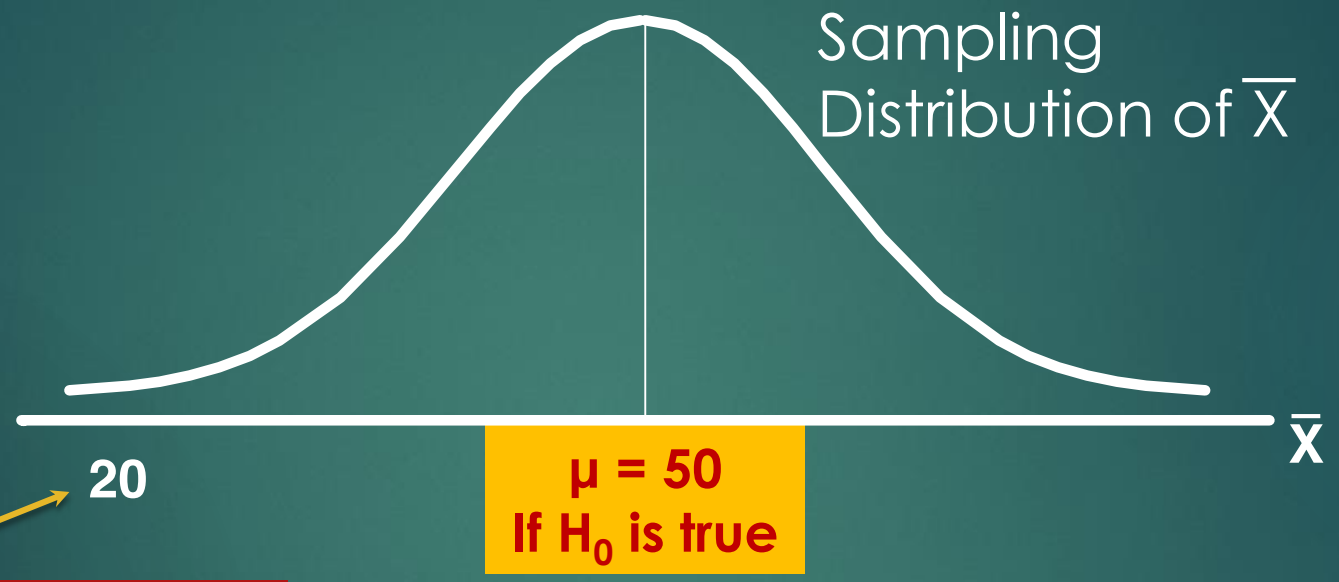
- Suppose the sample mean age was $\overline{X} = 20$.
- This is significantly lower than the claimed mean population age of 50, which is suggested by the null.
- If the null hypothesis were true, the probability of getting such a different sample mean would be very small, so you reject the null hypothesis .
  - In other words, getting a sample mean of 20 is so unlikely if the population mean was 50.
  - Thus, we conclude that the population mean must not be 50.

The Hypothesis Testing Process



The sampling distribution of the means looks like the above if the null hypothesis is true.

The Hypothesis Testing Process



Since it is unlikely that you would get a sample mean of this value …

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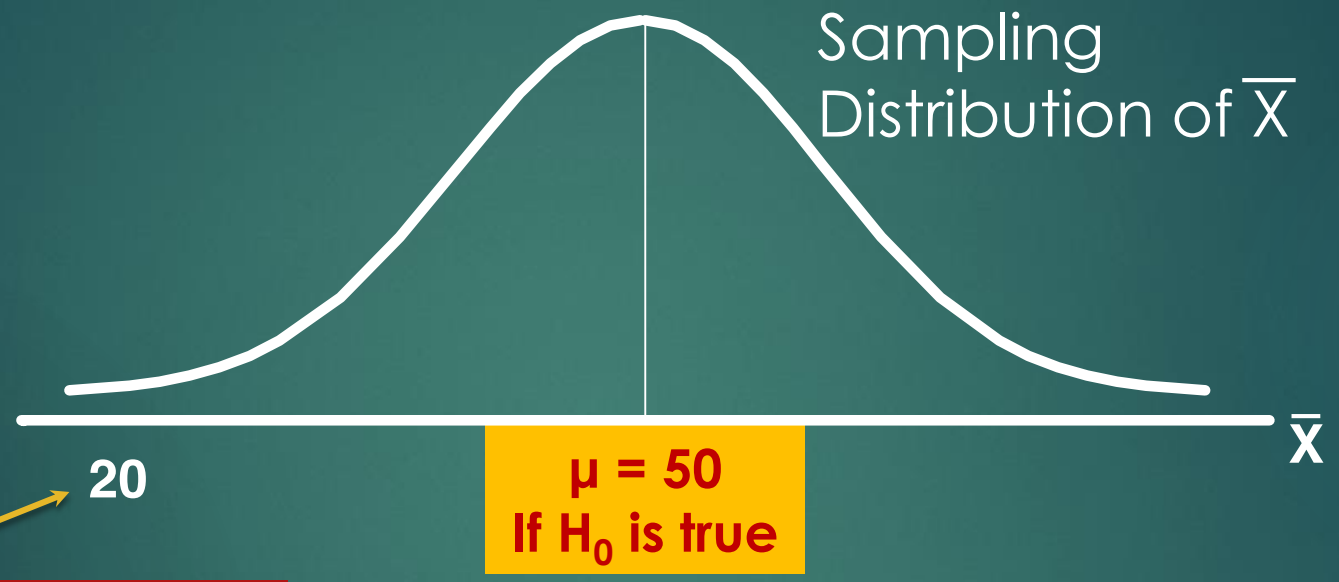
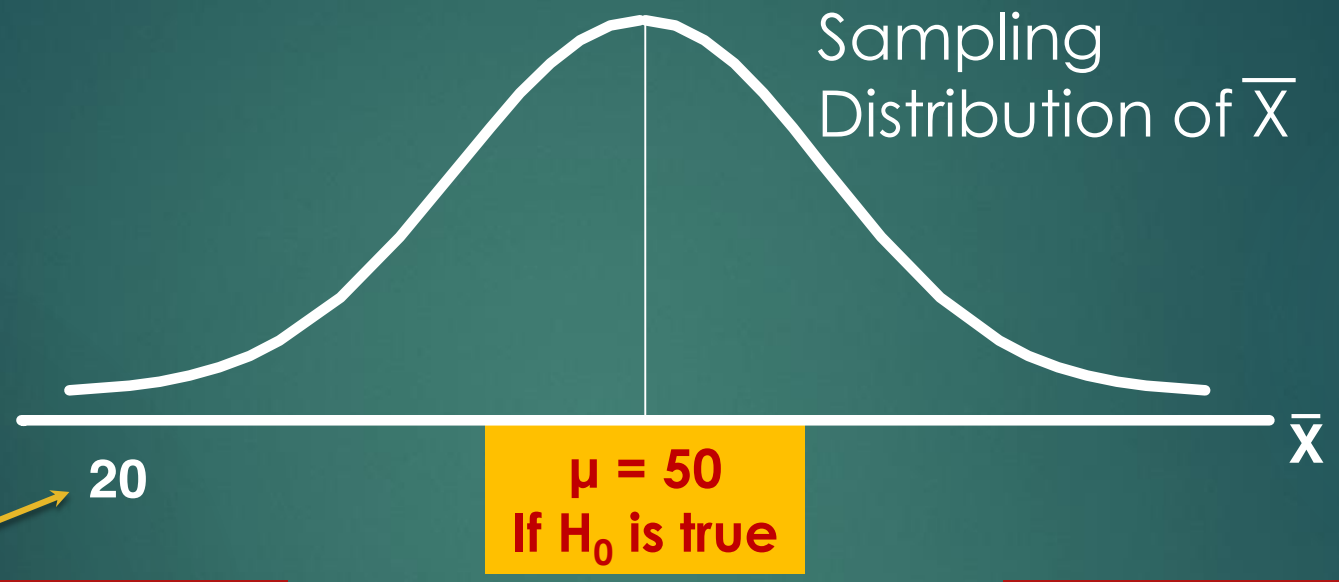


Since it is unlikely that you would get a sample mean of this value ...

... when 50 was in fact the population mean,...

The Hypothesis Testing Process



Since it is unlikely that you would get a sample mean of this value ...

... when 50 was in fact the population mean,...

... you can safely reject the null hypothesis that the mean is 50:
H0: $\mu = 50$.

The Test Statistic and Critical Values

- If the sample mean is **close to the stated population mean**, the **null hypothesis is NOT rejected**.
- If the sample mean is **far from the stated population mean**, the **null hypothesis is rejected**.

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- **How far is "far enough" to reject H_0?**

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- **How far is "far enough" to reject H_0?**
- The **Critical Value** of a **Test Statistic** creates a "line in the sand" for decision making -- it answers the question of how far is far enough.
- **Test Statistics** are defined using the Sampling Distribution of Sample Statistics, such as Sample Mean or Proportion.

The Hypothesis Testing Process

- We need to identify the **probability of getting a certain sample result if the null hypothesis is true**.
- We need to know the:
  - Sampling Distribution of Sample Statistics
  - Test Statistic
  - Critical Values

Example:
The Oxford Cereals Plant

The company guideline says that the mean of all boxes must be equal to 368 grams

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- When the sample mean is about 367.9 grams, we might be quite close to the standards set by the company

  → **WE MAY NOT BE ABLE TO REJECT THE NULL**

- When the sample mean is about 320 grams, however, we might conclude that the population mean is not equal to 368 grams

  → **WE MAY BE ABLE TO REJECT THE NULL.**

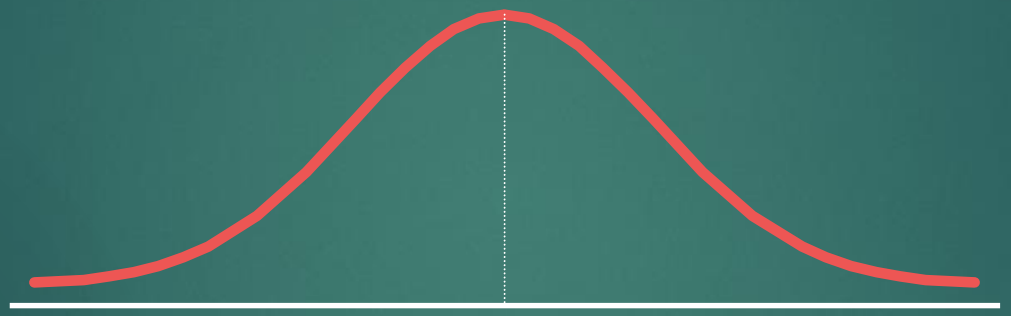
The Oxford Cereals Plant

But how can we make the final decision on whether the null hypothesis is rejected or not?

- **To determine this, we need a Test Statistic and we need its Critical Values.**

Regions of Rejection and Non Rejection

Sampling Distribution of the Sample Statistic, such as the sample mean or proportion.



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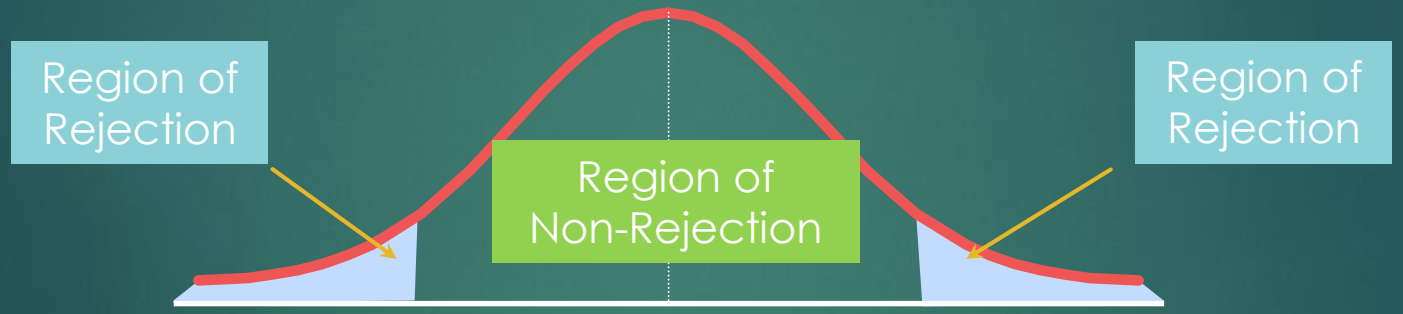

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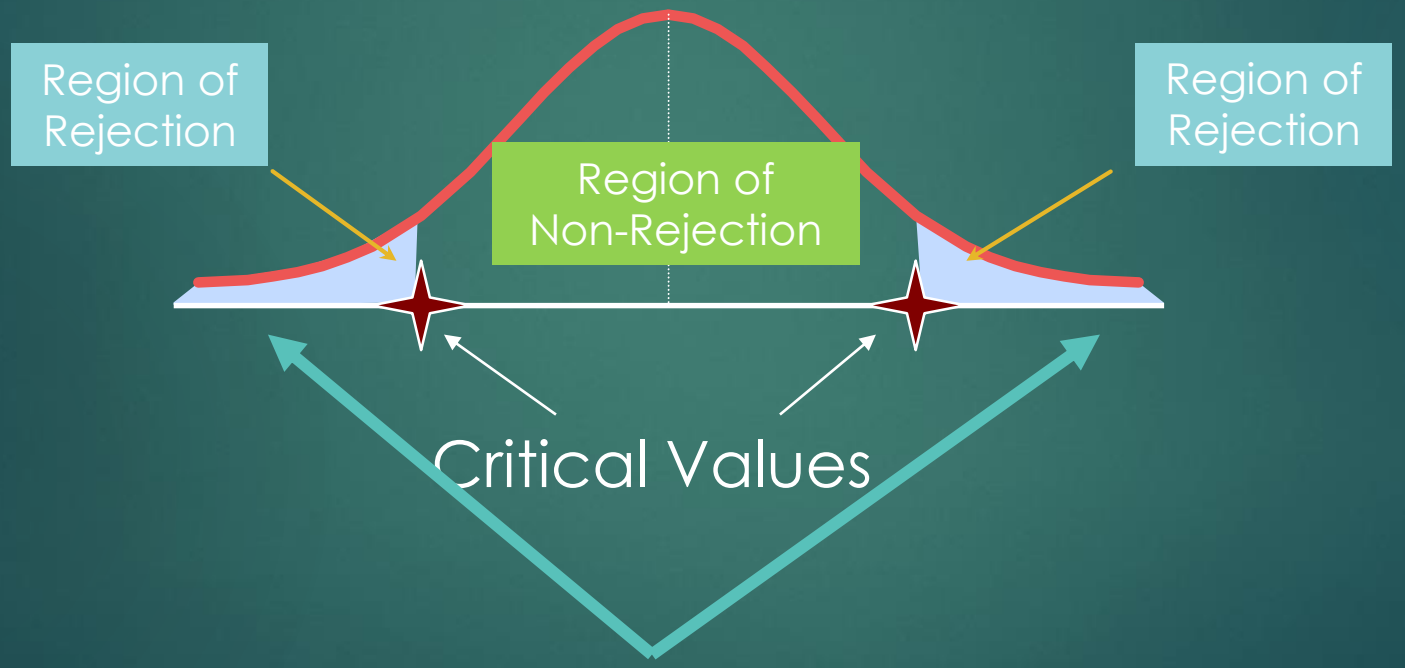

If the test statistics falls into Rejection Region, we can safely reject the null. Otherwise, we cannot.

What if we reach a wrong conclusion?

Possible Errors in Hypothesis Test Decision Making

- **Type I Error**
  - Rejecting a true null hypothesis
    - Analogous to concluding that someone is guilty, when she is, in fact, innocent

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- **Type I Error**
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    - **H_0: She is pregnant**

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  - It is a **False Alarm** – If you commit Type I Error, you may adjust the filling process at Oxford Cereals, while in fact you did not need to do so.

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  - We use **α** to denote the **probability of Type I Error**.
  - The probability of committing a Type I Error is called the **Level of Significance**.
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    - Traditionally, **α = 0.01**, **0.05**, or **0.10**
  - We specify the desired level of significance (like we did the desired level of confidence in confidence interval estimation) before performing the hypothesis test.
  - Once you determine **α**, you can determine the **Critical Values**.

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  - Failure to reject a false null hypothesis
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    - **H0: She is 9 months pregnant**

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    - If the hypothesized value and the actual value are far from one another (e.g. H0 : μ=1, while the population mean is actually 5), then the probability of rejecting the false null hypothesis is fairly low.

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    - Thus β ↑, when the difference btw. hypothesized and the actual parameter ↓

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Possible Hypothesis Test Outcomes		
	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0		
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Possible Errors in Hypothesis Test Decision Making

- **1-a**: the probability of not rejecting H_0 when it is true;
  a.k.a.: confidence coefficient
- **(1-a)*100%**: the confidence level of a hypothesis test
- **1-β**: is the probability of rejecting H_0 when it is false;
  a.k.a.: power of a statistical test

Type I & II Error Relationship

- Type I and Type II errors cannot happen at the same time. Why?
  - A Type I error can only occur if **H_0 is true**
  - A Type II error can only occur if **H_0 is false**

If Type I error probability (α) ↑, then

Type II error probability (β) ↓

Example:
The Oxford Cereals Plant

The company guideline says that the mean of all boxes must be equal to 368 grams

- **Type I Error:** Concluding that the mean fill is different from 368 grams, while it is in fact 368 grams;
  - i.e. Rejecting H0: μ=368, while μ is equal to 368 grams.
- If it is costly to adjust the filling process, you would like to avoid Type I Error. So, you set **a** fairly low; e.g. a=0.01

Example:
The Oxford Cereals Plant

The company guideline says that the mean of all boxes must be equal to 368 grams

- **Type II Error:** Concluding that the mean fill is not different from 368 grams, while it is different;
  - i.e. Failure to reject H0: μ=368, while μ is not equal to 368 grams

Back to The Hypothesis Testing Process

Hypothesis Testing

- STEP 1.)

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Given step 3, determine the critical values that divide the rejection and non-rejection regions

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Collect data and compute the value of the test statistic

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- STEP 5.) Collect data and compute the value of the test statistic
- STEP 6.) **If the test statistic falls into the non-rejection region, do not reject the null hypothesis H_0.**

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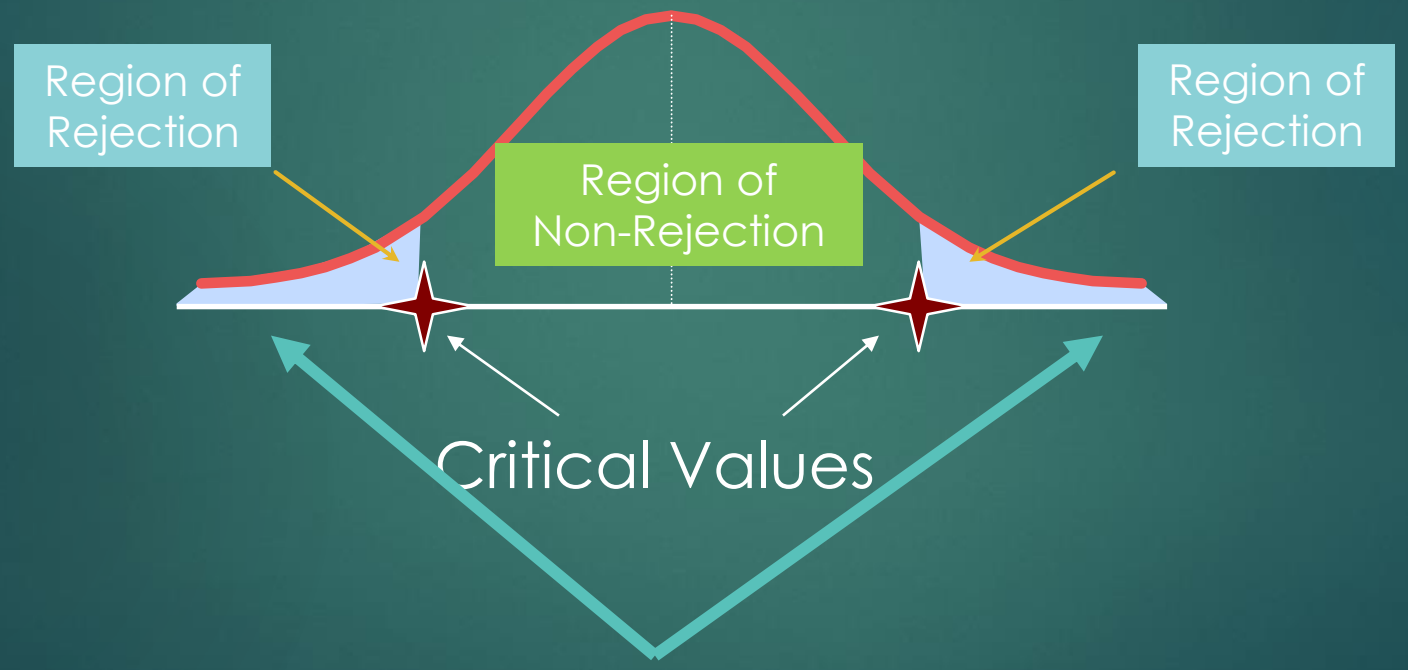
- STEP 5.) Collect data and compute the value of the test statistic
- STEP 6.) If the test statistic falls into the non-rejection region, do not reject the null hypothesis H_0. If the test statistic falls into the rejection region, reject the null hypothesis. **Express the conclusion in the context of the problem**

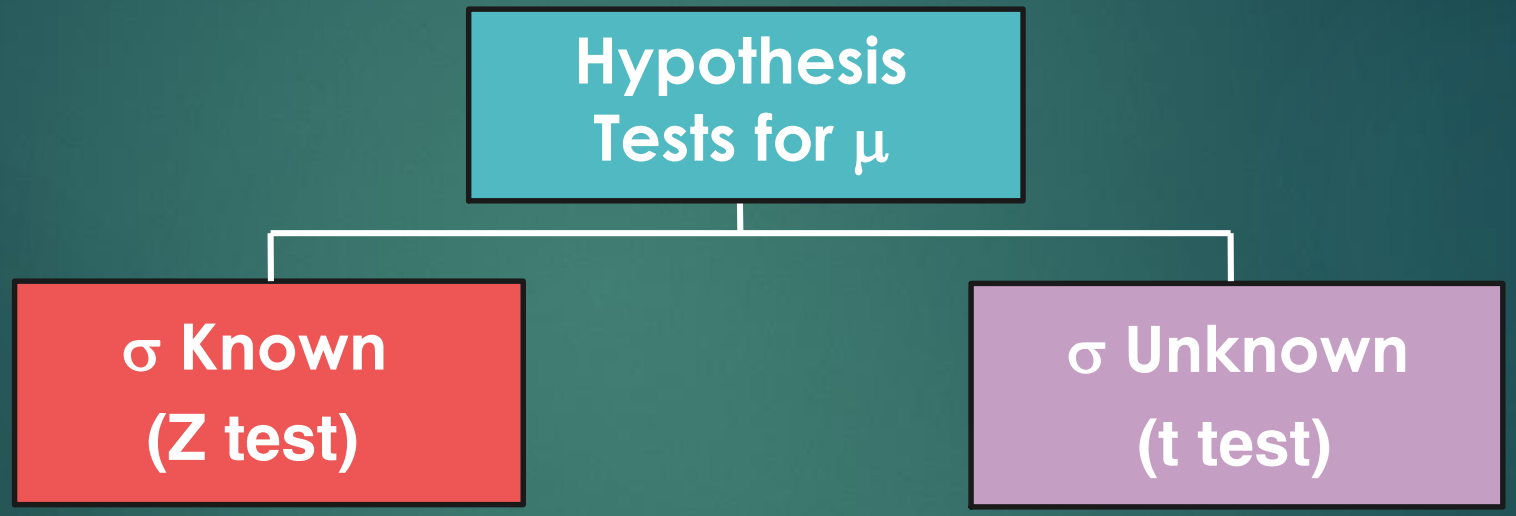
Reminder:

Sampling Distribution of the Sample Statistic, such as the sample mean or proportion.



If the test statistics falls into Rejection Region, we can safely reject the null. Otherwise, we cannot.

Hypothesis Tests for the Mean



Z Test of Hypothesis for the Mean (σ Known)

- Convert sample statistic ($\overline{X}$) to a Z_{STAT} test statistic

Hypothesis Tests for μ

- **σ Known (Z test)**
- **σ Unknown (t test)**

The test statistic is:

$$Z_{STAT} = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

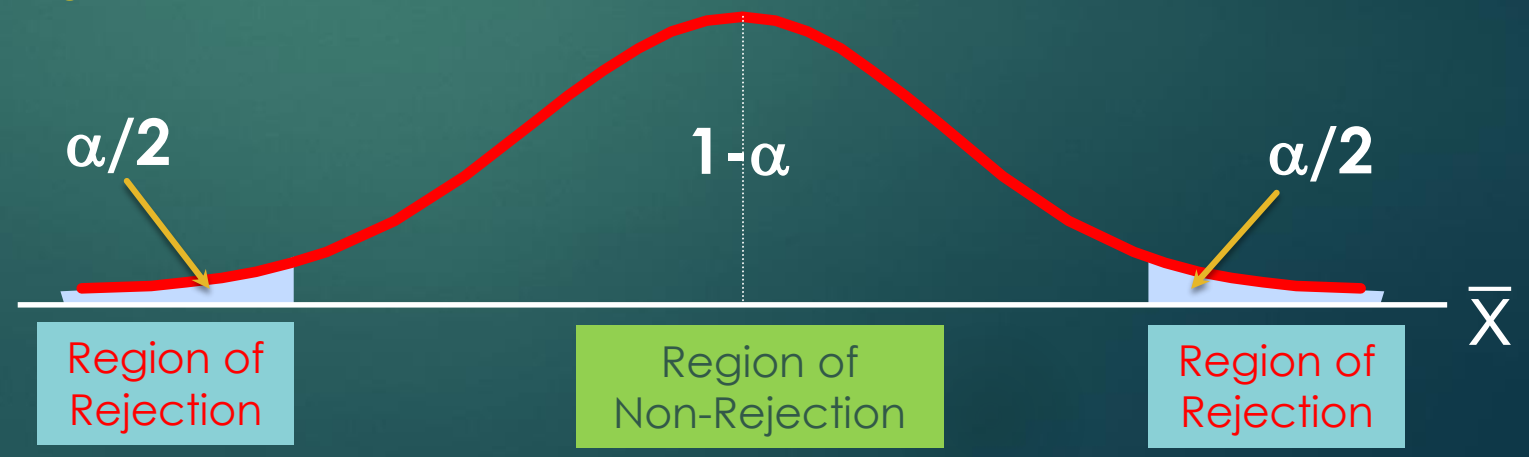
About the Z-State

- **Z-Stat** is a **Test Statistics**, which is a function of the mean and the standard deviation of **the sampling distribution of the mean** in a case where the standard deviation of population is known.
- It represents the difference between sample mean and the population mean in standard error units
- **Z-Stat** can be used when:
  - The population is normal

  or
  - The sample size is large enough (Central Limit Theorem)

Critical Value Approach to Testing

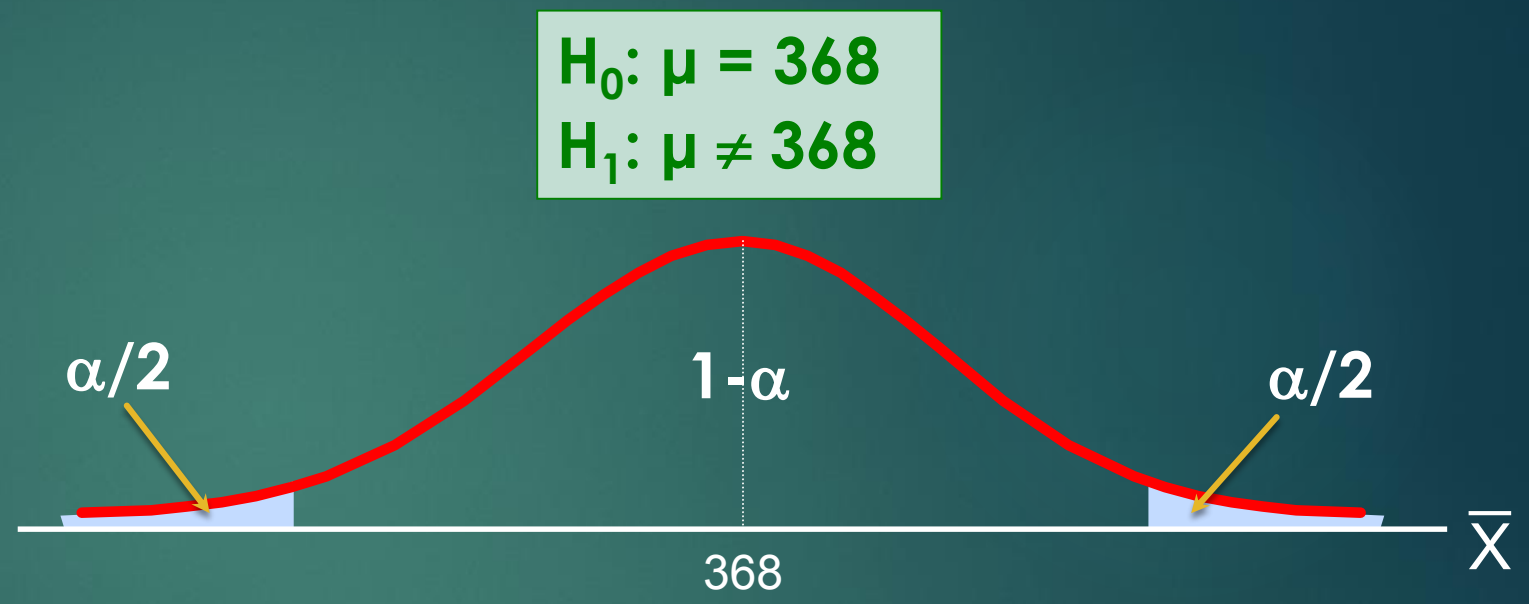
- For a two-tail test for the mean, σ known:
- Convert sample statistic ($\overline{X}$) to test statistic (Z_{STAT})
- Determine the critical Z values for a specified level of significance α from a table or computer
  - **Decision Rule:**
  - **If the test statistic falls in the rejection region, reject H_0 ;**
  - **Otherwise, do not reject H_0**



Two-Tail Tests

DCOVA

- There are two cutoff values (critical values), defining the regions of rejection…



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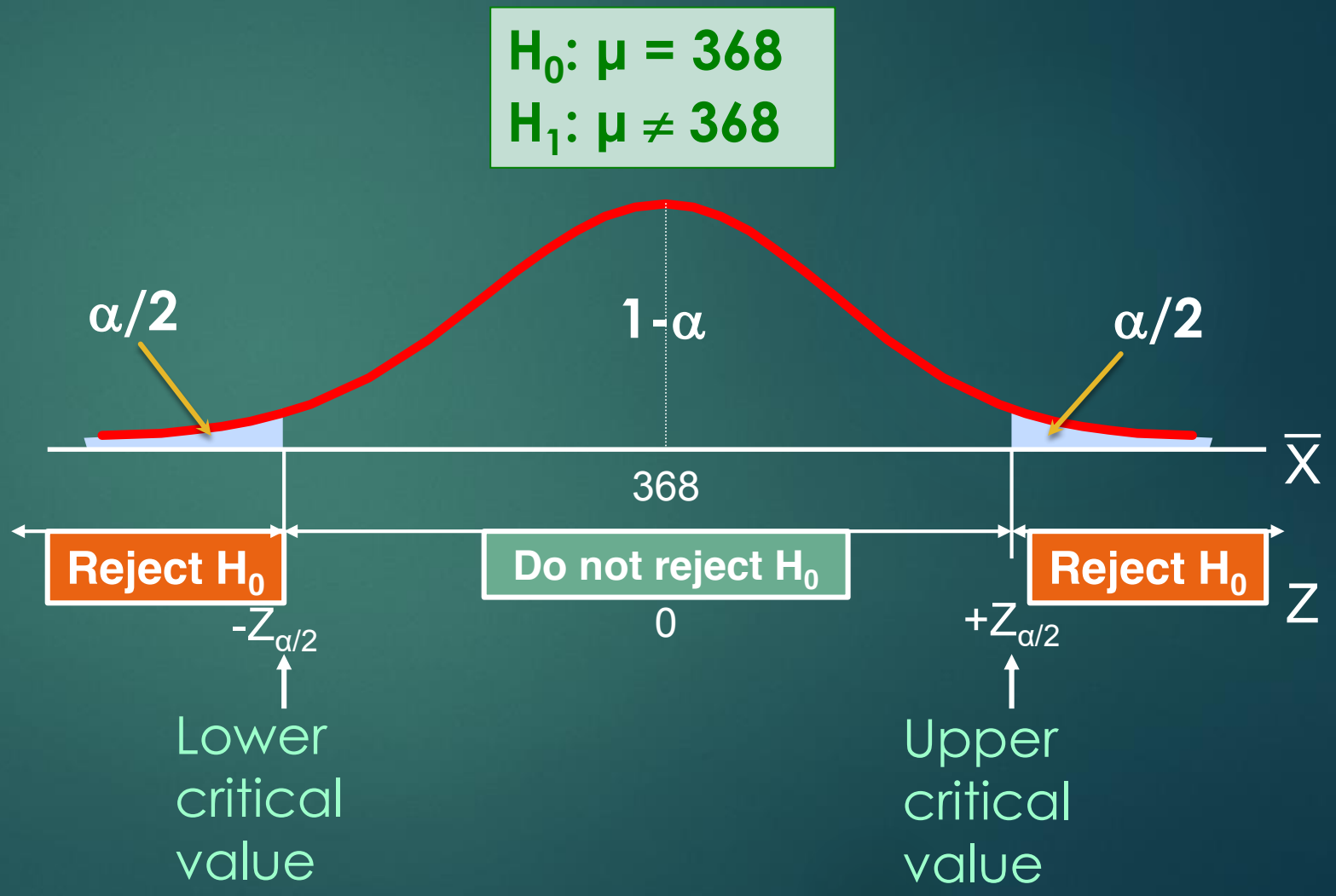


Example:
The Oxford Cereals Plant

A sample of 25 cereal boxes has a mean weight of 372.5 grams, and the population standard deviation is given as 15 grams. Can we reject the null hypothesis with 95% confidence?

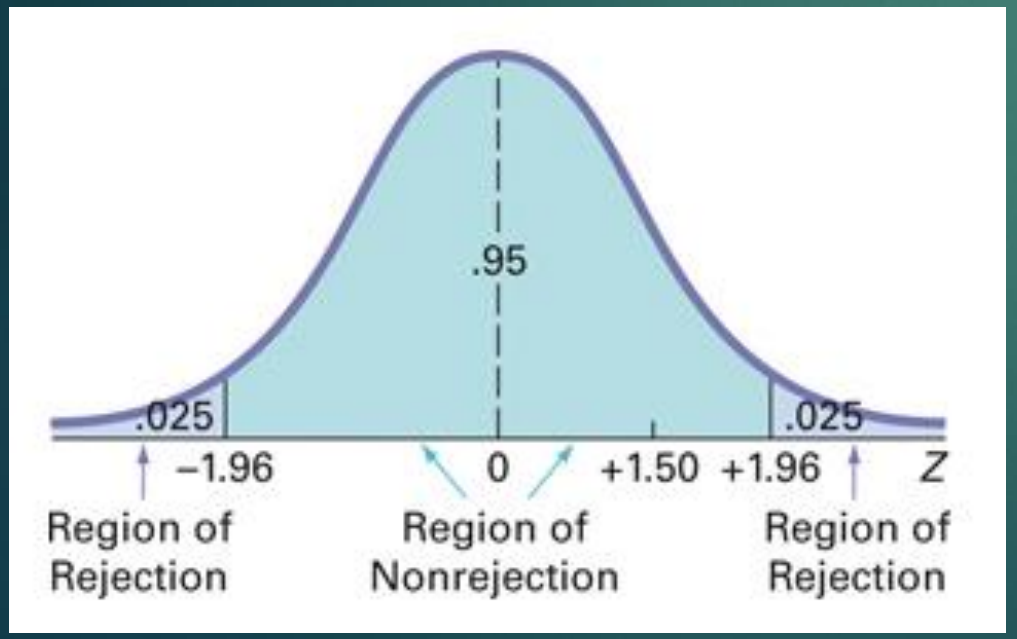
$H_0: \mu = 368$

$H_1: \mu \neq 368$

$$Z_{STAT} = \frac{\overline{X}}{\sqrt{n}}$$

Example:
The Oxford Cereals Plant

A sample of 25 cereal boxes has a mean weight of 372.5 grams, and the population standard deviation is given as 15 grams. Can we reject the null hypothesis with 95% confidence?



$H_0: \mu = 368$

$H_1: \mu \neq 368$

$$Z_{STAT} = \frac{\bar{X}}{\sqrt{n}} = \frac{372.5 \quad 368}{\frac{15}{\sqrt{25}}} = +1.5$$

WE CANNOT REJECT THE NULL HYPOTHESIS!

Hypothesis Testing Example, The 6 Steps

You are managing a production line, manufacturing bolts. You are asked to make sure that the mean diameter of the bolts is equal to 30 mm. Using a sample of 100 bolts, in which the mean diameter is 29.84 mm, test the claim that the true mean diameter of a manufactured bolt is 30mm, assuming that the standard deviation of diameter in the population is equal to 0.8 mm.

1. State the appropriate null and alternative hypotheses
   - $H_0: \mu = 30$ $H_1: \mu \neq 30$ (This is a two-tail test)

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1. State the appropriate null and alternative hypotheses
   - $H_0: \mu = 30$ $H_1: \mu \neq 30$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test

Hypothesis Testing Example, The 6 Steps

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   - σ is known (σ=0.8 mm), so we employ a Z-stat.

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Hypothesis Testing Example, The 6 Steps

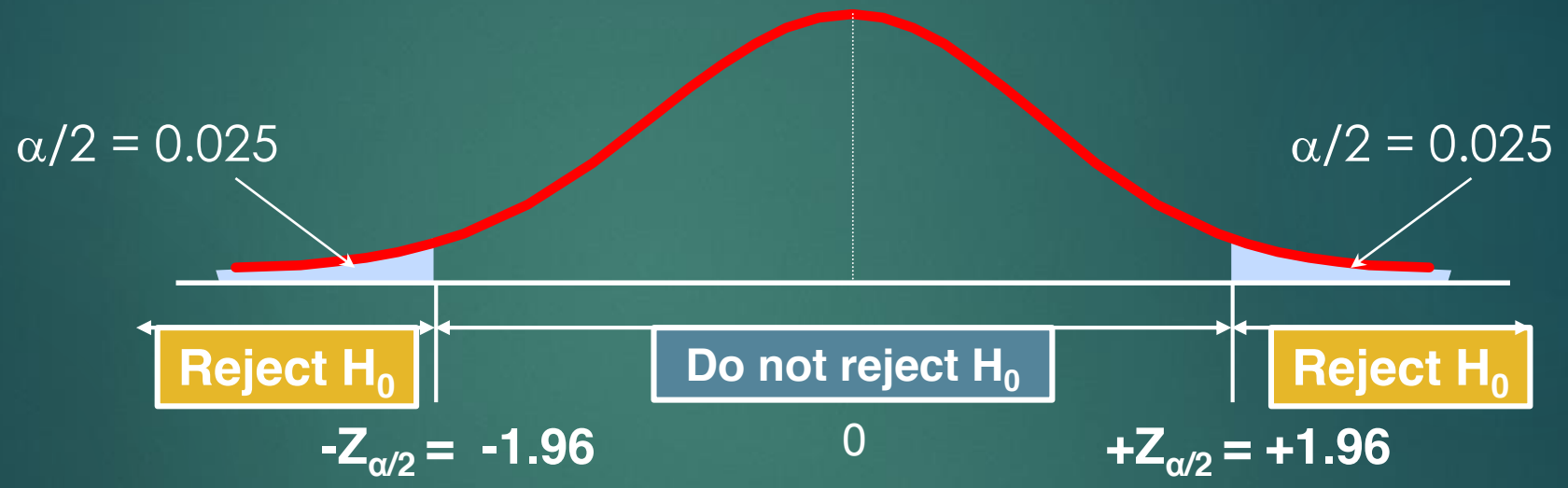
3. Determine the appropriate technique
   - σ is known (σ=0.8 mm), so we employ a Z-stat.
4. Determine the critical values
   - For α = 0.05 the critical Z values are ±1.96
5. Collect the data and compute the test statistic
   - The sample results are

     n = 100, $\overline{X}$ = 29.84 (σ = 0.8 is assumed known)

$$Z_{STAT} = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{29.84 - 30}{\frac{0.8}{\sqrt{100}}} = \frac{-0.16}{0.08} = -2.0$$

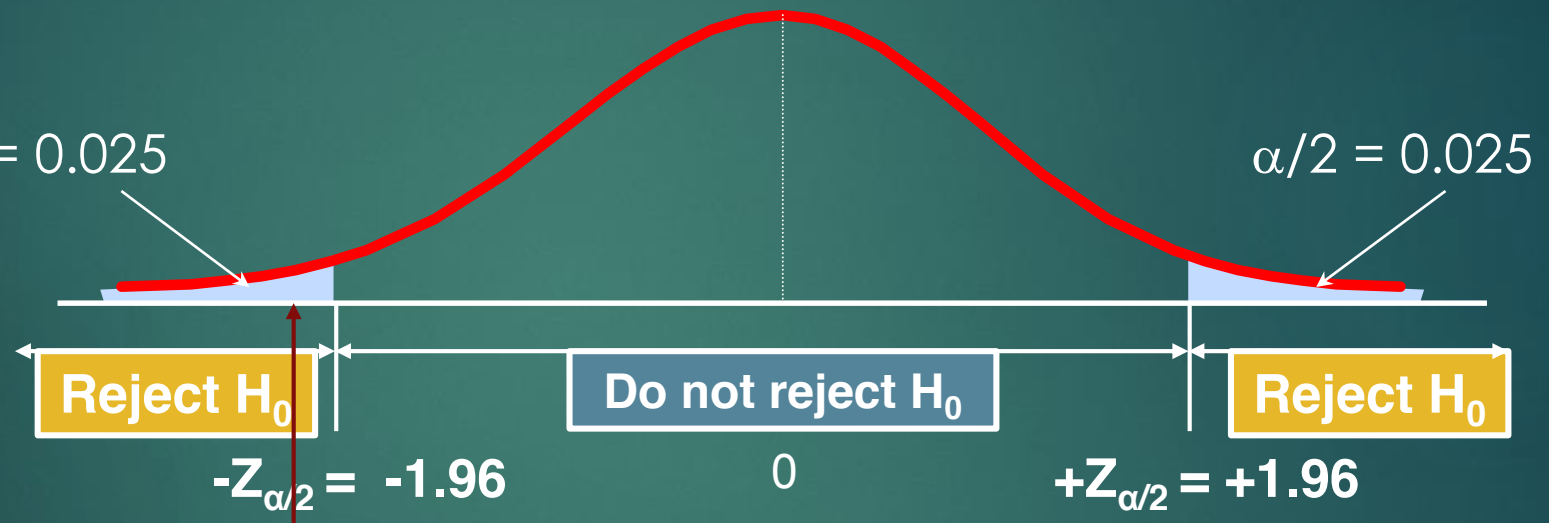
Hypothesis Testing Example, The 6 Steps

- 6. Is the test statistic in the rejection region?



Hypothesis Testing Example, The 6 Steps

- 6. Is the test statistic in the rejection region?



Reject H_0

if $Z_{STAT} < -1.96$

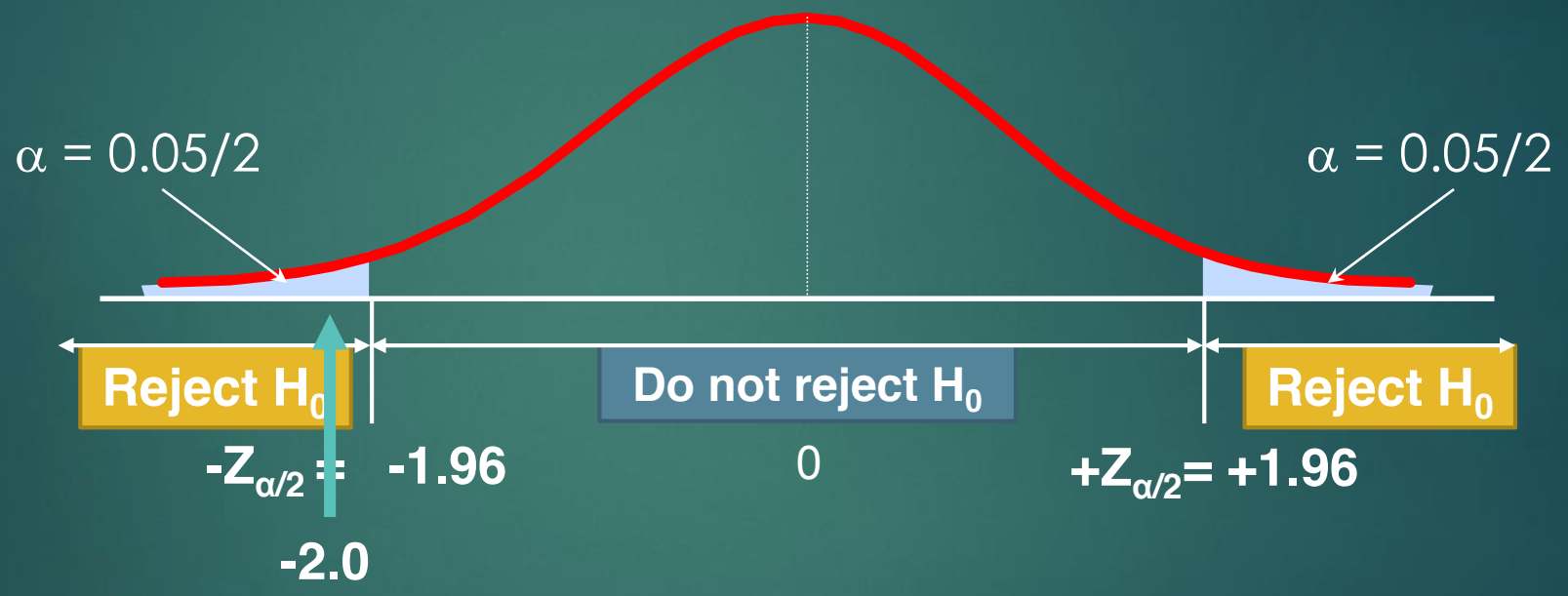
or $Z_{STAT} > 1.96$;

otherwise do not reject H_0

Here, $Z_{STAT} = -2.0 < -1.96$, so the test statistic is in the rejection region

Hypothesis Testing Example, The 6 Steps

6 (continued). Reach a decision and interpret the result



Since Z_{STAT} = -2.0 < -1.96, we can safely reject the null hypothesis and conclude there is sufficient evidence that the mean diameter of manufactured bolts in this plant is not equal to 30 mm.

MAKE SURE THAT YOU SOLVE EXAMPLE 9.3 ON PP. 316 AND 317.

Now let's talk about p-value, shall we?!

They dry toothbrush

- How likely is it that your son's toothbrush is dry when she just brushed his teeth?
- Highly unlikely, isn't it?
- So, if you find that his toothbrush are dry, can't you safely reject the null hypothesis that he just brushed his teeth?
- I bet you can!

The p-value

- The **Z-Stat** is a **test statistic**
- **Assuming that the null hypothesis is true**, the Z-Stat gives us the **deviation from the hypothesized population mean** in terms of standard deviation

$$Z_{STAT} = \frac{\overline{X}}{\sqrt{n}}$$

- Example:

Z-Stat=1.5 → The sample mean is located **one and a half standard deviation** from the hypothesized population mean.

The p-value

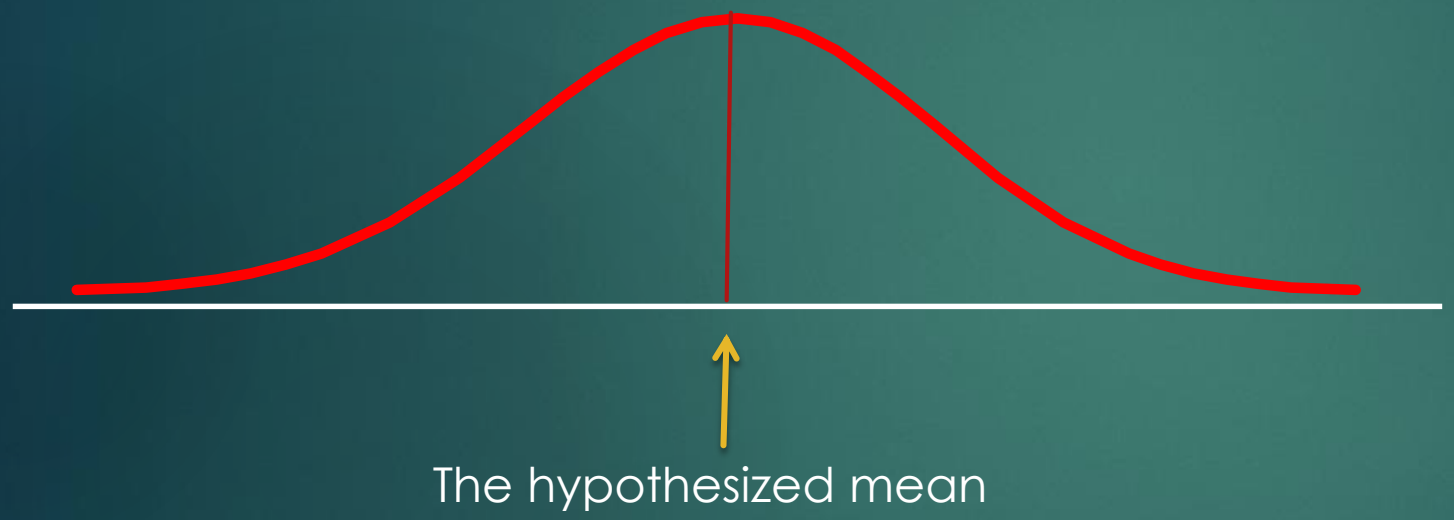
We need to consider the following:

- Given the sampling distribution of the means, the p-value is the **probability** of getting a test statistic (in this case a Z-Stat) equal or more extreme than the test statistic that we got from our sample.
- When this probability is less than the desired probability for type I error (a: the probability of rejecting a true null), then we can safely reject the null hypothesis.

Thus: WHEN THE p-VALUE IS LOW, THE NULL MUST GO!

The p-value, Two-tail Test

Z-Stat=1.5 → The sample mean is located **one and a half standard deviation** from the hypothesized population mean.



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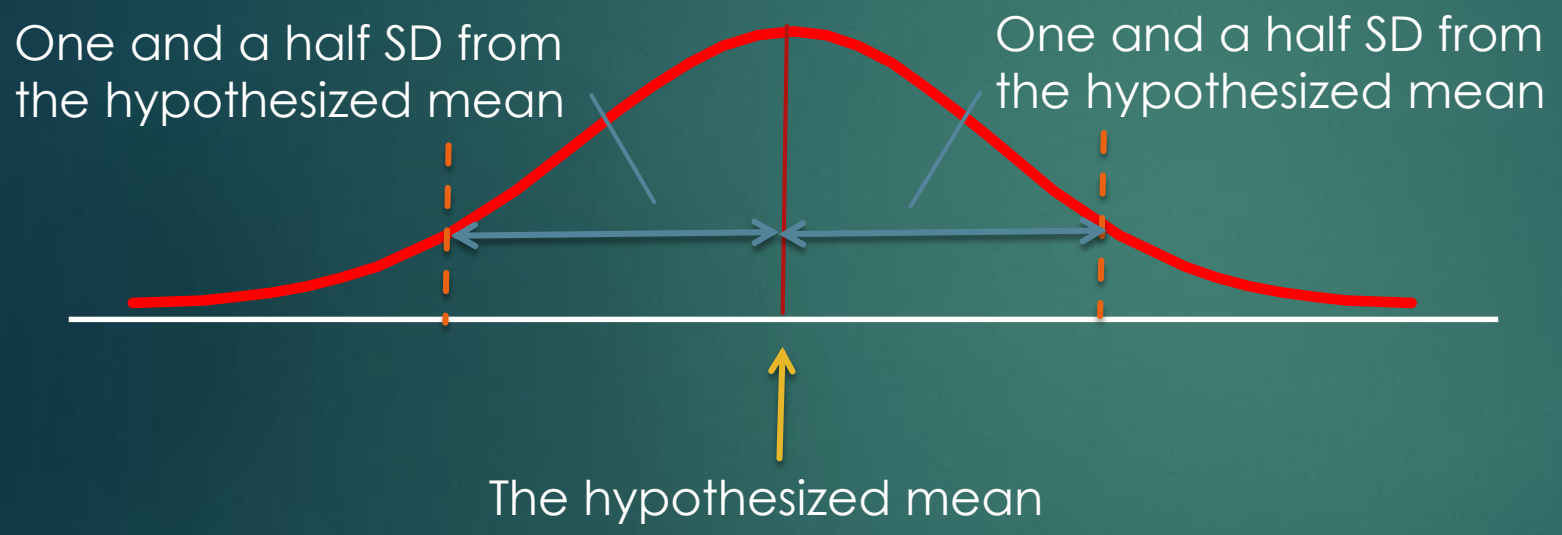

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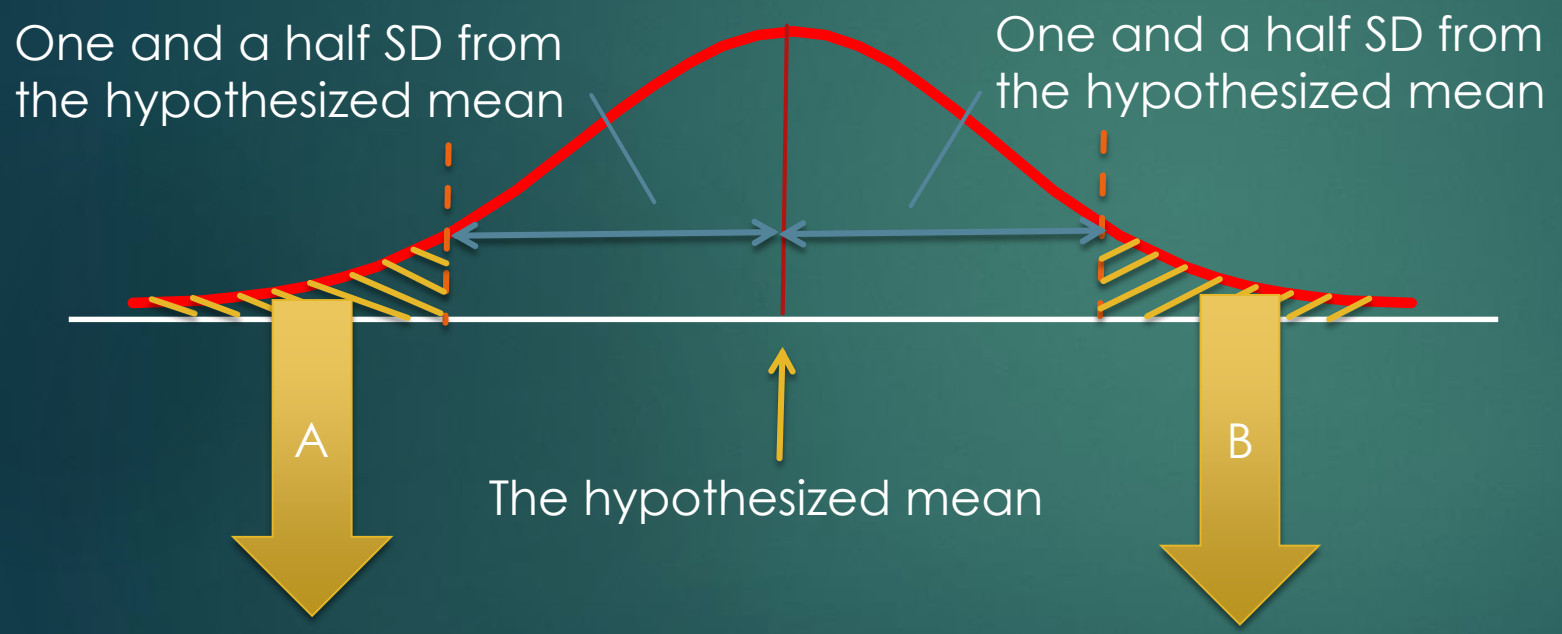


Given the sampling distribution of the means, the p-value is the **probability** of getting a test statistic (in this case a Z-Stat) equal or more extreme than the test statistic that we got from our sample.

p-value=A+B

The p-value

We need to compare the p-value to a

- p-value < a → If the null hypothesis were true, the probability of getting such extreme deviations from the hypothesized mean would have been quite low. So, we can safely reject the null

Remember: WHEN THE p-VALUE IS LOW, THE NULL MUST GO!

The p-value

We need to compare the p-value to a

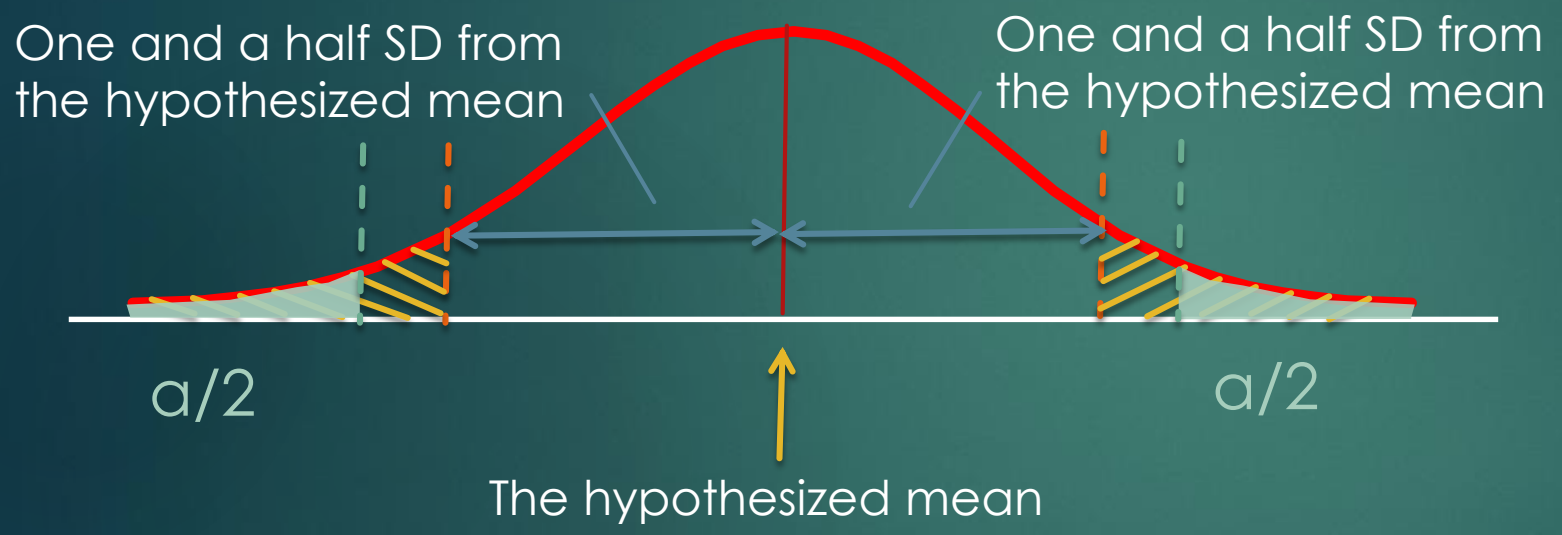
- p-value < a → If the null hypothesis were true, the probability of getting such extreme deviations from the hypothesized mean would have been quite low. So, we can safely reject the null

Remember: WHEN THE p-VALUE IS LOW, THE NULL MUST GO!

- p-value ≥ a → If the null hypothesis were true, getting such deviations from the hypothesized mean is not highly unlikely. So, we cannot reject the null

The p-value, Two-tail Test

Z-Stat=1.5 → The sample mean is located **one and a half standard deviation** from the hypothesized population mean.



p-value >a → Cannot reject the null

The p-value

The p-value is about the data, not the hypothesis

- Given the data in the sample, we can say something about the likelihood of getting extreme deviations from the hypothesized mean.
- If unlikely (p-value<a), we can reject the null
  - Assuming that your son has just brushed his teeth, it is unlikely that his toothbrush is dry. Once you find it's dry, you can reject the null.
- If likely (p-value≥a), we cannot reject the null

The 5 Step p-value approach to Hypothesis Testing

Step 1.)

State the null hypothesis, H_0 and the alternative hypothesis, H_1

The 5 Step p-value approach to Hypothesis Testing

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Choose the level of significance, α, and the sample size, n

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Determine the appropriate test statistic and sampling distribution

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Step 1.) State the null hypothesis, H_0 and the alternative hypothesis, H_1

Step 2.) Choose the level of significance, α, and the sample size, n

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Step 4.)

Collect data and compute the value of the test statistic and the p-value

The 5 Step p-value approach to Hypothesis Testing

Step 1.) State the null hypothesis, H_0 and the alternative hypothesis, H_1

Step 2.) Choose the level of significance, α, and the sample size, n

Step 3.) Determine the appropriate test statistic and sampling distribution

Step 4.) Collect data and compute the value of the test statistic and the p-value

Step 5.)

If the p-value is < a then reject H_0, otherwise do not reject H_0. State the conclusion in the context of the problem

p-value Hypothesis Testing Example

Test the claim that the true mean diameter of a manufactured bolt is 30mm.
(Assume $\sigma = 0.8$)

1. State the appropriate null and alternative hypotheses
   - $H_0: \mu = 30$ $H_1: \mu \neq 30$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test

p-value Hypothesis Testing Example

3. Determine the appropriate technique
   - σ is assumed known so this is a Z test.
4. Collect the data, compute the test statistic and the p-value
   - Suppose the sample results are

     n = 100, $\overline{X}$ = 29.84 (σ = 0.8 is assumed known)

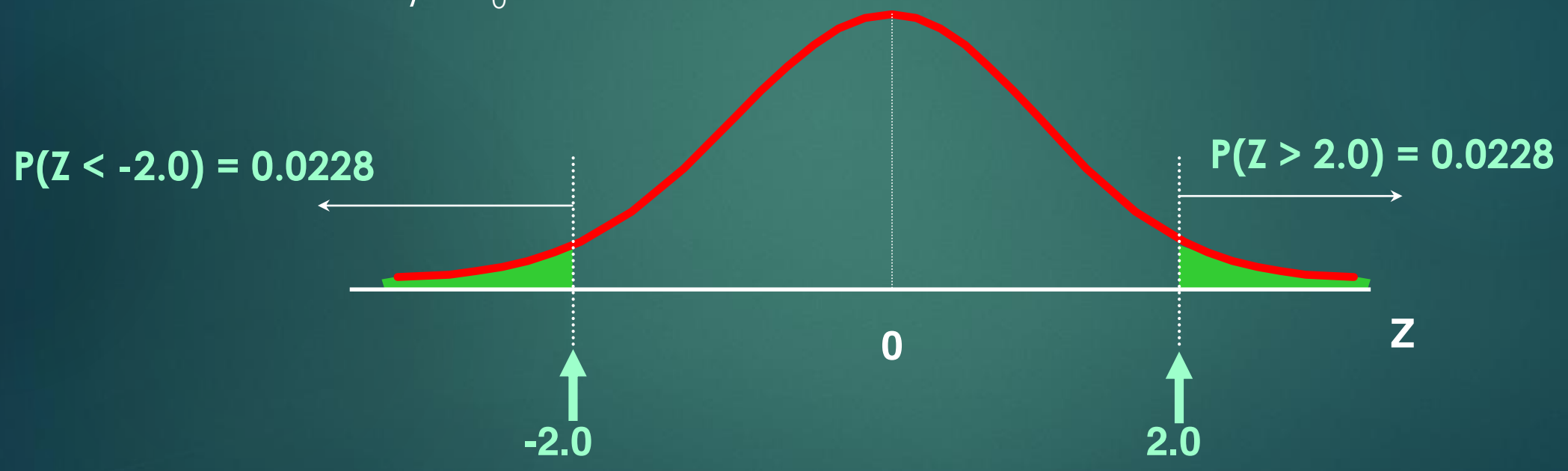
   So the test statistic is:

$$Z_{STAT} = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{29.84 - 30}{\frac{0.8}{\sqrt{100}}} = \frac{-0.16}{0.08} = -2.0$$

p-Value Hypothesis Testing Example: Calculating the p-value

4. (continued) Calculate the p-value.

- How likely is it to get a Z_{STAT} of -2 (or something further from the mean (0), in either direction) if H_0 is true?



p-value = 0.0228 + 0.0228 = 0.0456

p-value Hypothesis Testing Example

- 5. Is the p-value < a?
  - Since p-value = 0.0456 < a = 0.05, reject H_0
- 5. (continued) State the managerial conclusion in the context of the situation.
  - There is sufficient evidence to conclude the average diameter of a manufactured bolt is not equal to 30mm.

Example: How to make use of Excel to test H0	
Null Hypothesis: μ =	368
Level of Significance: a =	0.05
Population Standard Deviation:	15
Sample Size	25
Sample Mean	372.5
Intermediate Calculations	
Standard Error of the Mean	3
Z Test Statistic	1.5
Two-Tail Test	
Lower Critical Value	-1.9600
Upper Critical Value	1.9600
p-Value	0.1336
Do not reject the null hypothesis	

σ is given → Use Z-Stat

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$$Z_{STAT} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

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=NORM.S.INV(0.025)

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=NORM.S.INV(0.025)

=2 * (1 - NORM.S.DIST(1.5,TRUE))

What does all this remind me of?

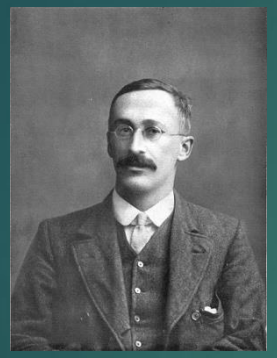
Connection Between Two Tail Tests and Confidence Intervals

- For $\overline{X} = 29.84$, $\sigma = 0.8$ and $n = 100$, the 95% confidence interval is:

$$29.84 - (1.96)\frac{0.8}{\sqrt{100}} \quad \text{to} \quad 29.84 + (1.96)\frac{0.8}{\sqrt{100}}$$

Thus: $29.6832 \le \mu \le 29.9968$

- Since **this interval does not contain the hypothesized mean (30)**, we reject the null hypothesis at $\alpha = 0.05$



What if population standard deviation is unknown?

Hypothesis Testing: σ Unknown

- If the population standard deviation is unknown, you instead **use the sample standard deviation S**.
- Because of this change, you **use the t distribution** instead of the Z distribution **to test the null hypothesis about the mean**.

Hypothesis Testing: σ Unknown

- If the population standard deviation is unknown, you instead **use the sample standard deviation S**.
- Because of this change, you **use the t distribution** instead of the Z distribution **to test the null hypothesis about the mean**.

NOTE:

- When using the t distribution you must assume the population you are sampling from follows a normal distribution or it is fairly symmetric and the sample size is large enough.

But here’s the good news:
All other steps, concepts, and conclusions are the same.

t Test of Hypothesis for the Mean (σ Unknown)

- Convert sample statistic ($\overline{X}$) to a t_{STAT} test statistic

Hypothesis Tests for μ

- **σ Known (Z test)**
- **σ Unknown (t test)**

The test statistic is:

$$t_{STAT} = \frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}}$$

Example: Two-Tail Test (σ Unknown)

The average cost of a hotel room in New York is said to be \$168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in an $\overline{X}$ of \$172.50 and an S of \$15.40. Test the appropriate hypotheses at $\alpha = 0.05$.

(Assume the population distribution is normal)

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$H_0: \mu = 168$

$H_1: \mu \neq 168$

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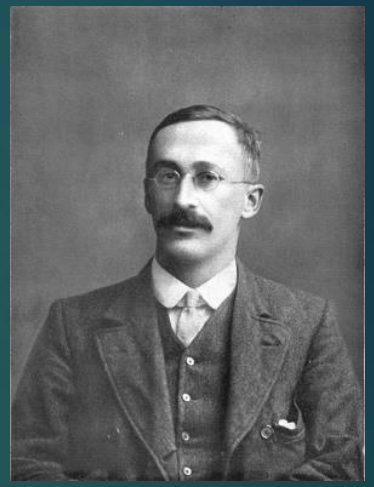
$H_0: \mu = 168$

$H_1: \mu \neq 168$



Example Solution: Two-Tail t Test

$H_0: \mu = 168$

$H_1: \mu \neq 168$

- $\alpha = 0.05$
- $n = 25$, $df = 25-1=24$
- σ is unknown, so use a t statistic
- Critical Value:

  $\pm t_{24,0.025} = \pm 2.0639$

Example Solution: Two-Tail t Test

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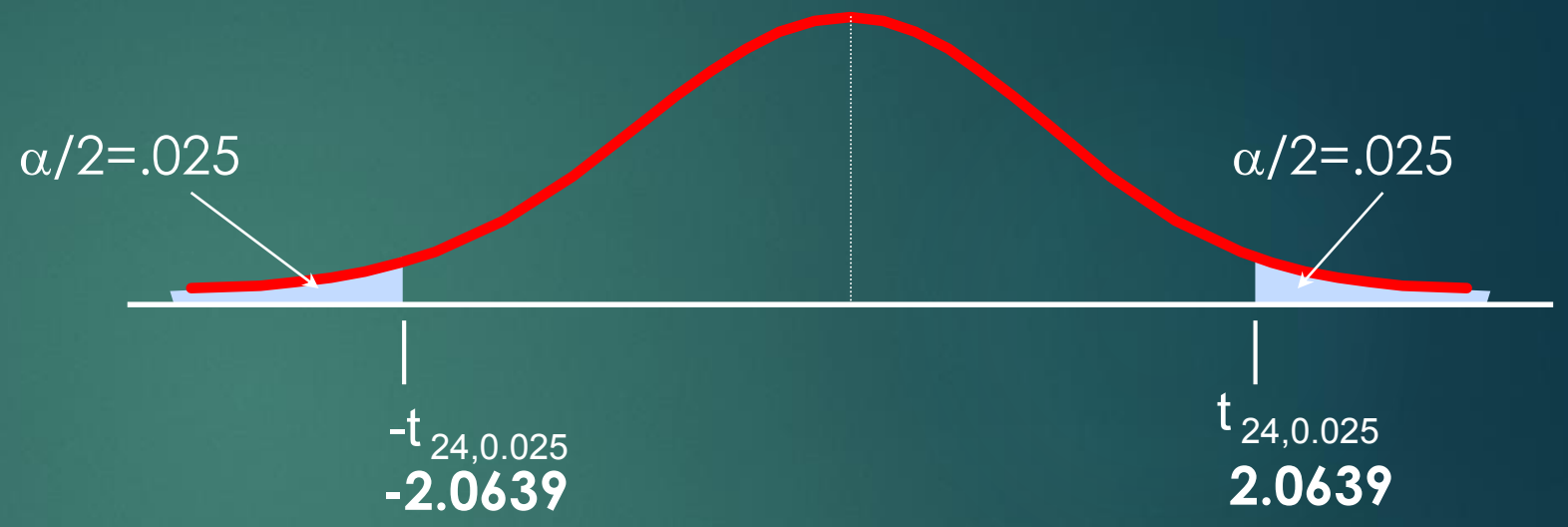

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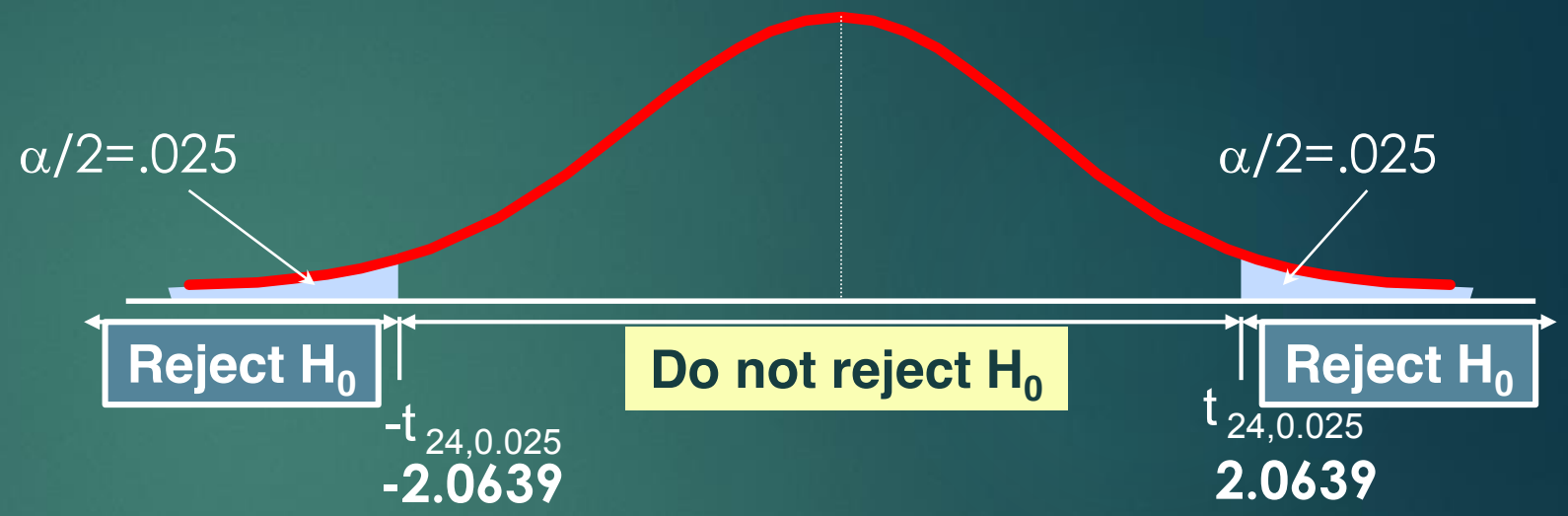

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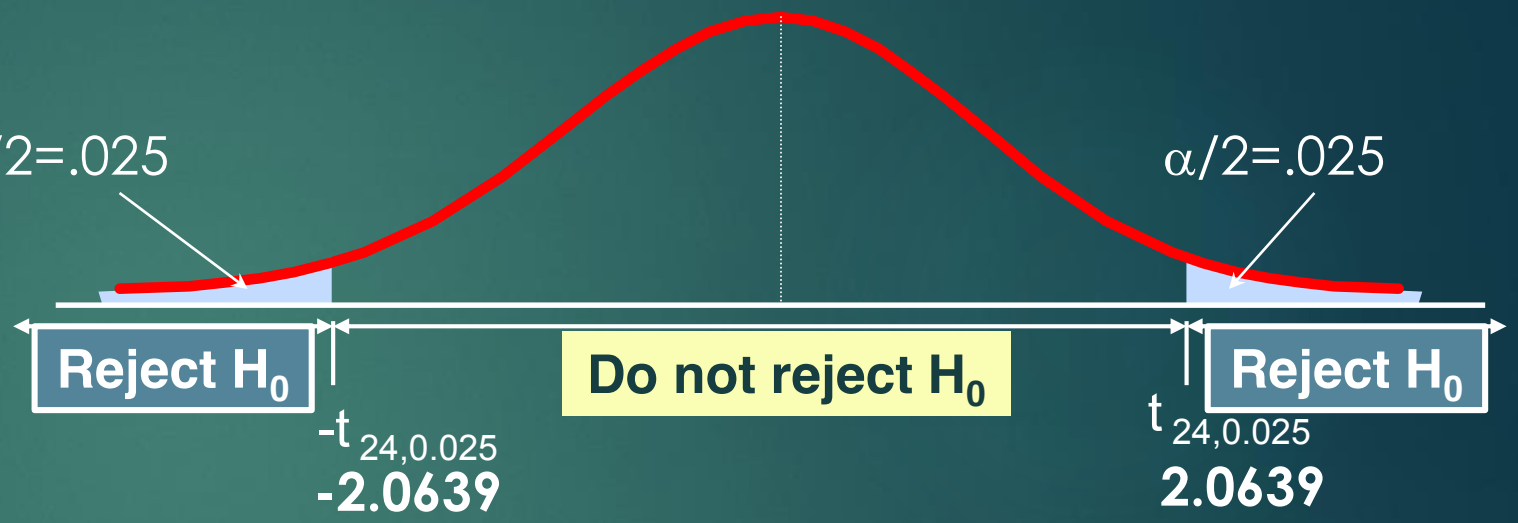

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

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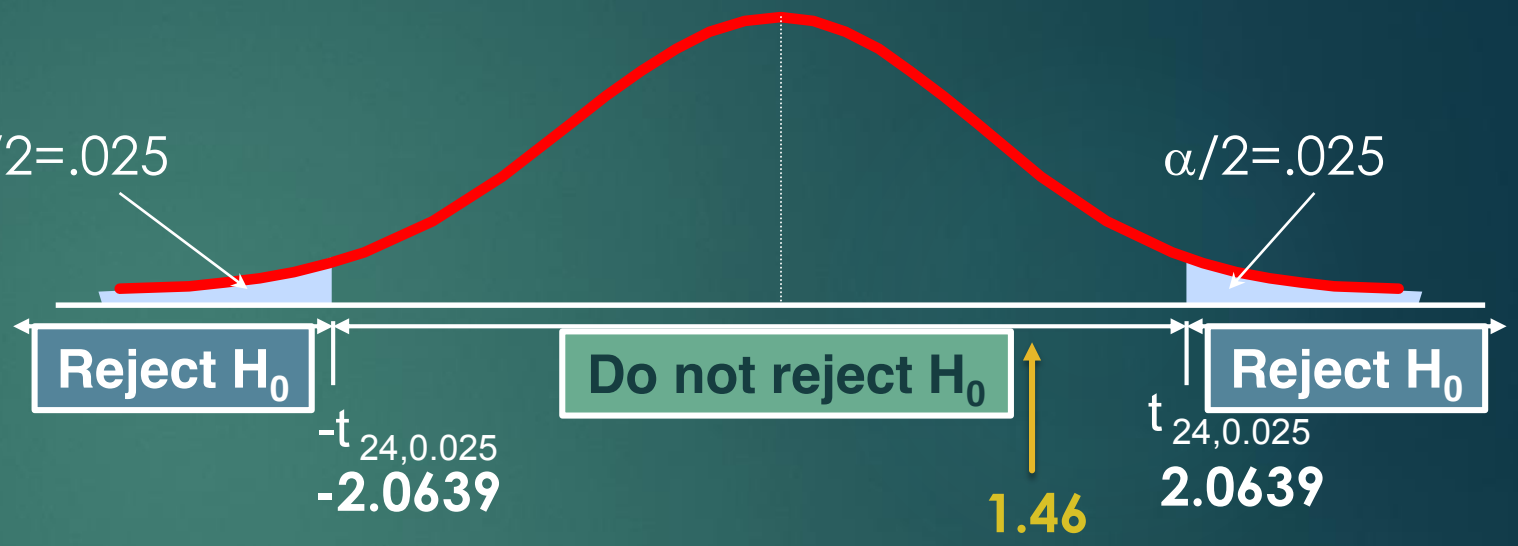

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Do not reject H_0: insufficient evidence that true mean cost is different from $168

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Null Hypothesis μ=	120
Level of Significance a=	0.05
Sample Size	12
Sample Mean	112.85
Sample Standard Deviation	20.8
Intermediate Calculations	
Standard Error of the Mean	6.0044
Degrees of Freedom	11
t Test Statistic	-1.1908
Two-Tail Test	
Lower Critical Value	-2.2010
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p-Value	0.2588
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S is given → Use t-Stat

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$SD_{SampDistMean}=S/\sqrt{n}$

df=n-1=12-1

$$T_{STAT} = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

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=T.INV.2T(0.05,11)

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=T.INV.2T(0.05,11)

=T.DIST.2T(1.1908,11)

ASSIGNMENTS:

Homework for Sec. 9.1 and 9.2

EXCEL SHEETS:

Refer to the D2L, and look for: 11-Ch09-Hyp_Tests_TwoTailTests_Zstat_Tstat

One-tail test

- We employ **one-tail tests** when the alternative hypothesis is focused on a particular direction.
  - Example:
  - Let's consider the average time required for a drive-through service at McDonalds:
  - **Two-tail test** -- H0: $\mu=188.83$ (seconds); H1: $\mu\neq188.83$ (seconds)
  - **One-tail test** -- H0: $\mu\geq188.83$ (seconds); H1: $\mu<188.83$ (seconds)

The Critical Value Approach

The Critical Value Approach

- STEP 1.) State the null hypothesis, H_0 and the alternative hypothesis, H_1

  H_0: $\mu \geq 188.83$ (seconds); H_1: $\mu < 188.83$ (seconds)

The Critical Value Approach

- STEP 1.) State the null hypothesis, H_0 and the alternative hypothesis, H_1

  H_0: $\mu \geq 188.83$ (seconds); H_1: $\mu < 188.83$ (seconds)

- STEP 2.) Choose the level of significance, α, and the sample size, n

  $\alpha = 0.05$ and $n = 25$ stores

The Critical Value Approach

- STEP 1.) State the null hypothesis, H_0 and the alternative hypothesis, H_1

  H_0: $\mu \geq 188.83$ (seconds); H_1: $\mu < 188.83$ (seconds)

- STEP 2.) Choose the level of significance, α, and the sample size, n

  α=0.05 and n=25 stores

- STEP 3.) Determine the appropriate test statistic based on the sampling distribution

  Since we do not know about the population standard deviation (i.e. σ is unknown), we are going to employ the t-Stat.

  Since the sample size is small, however, we need to assume that the population of drive-through time is normally distributed.

The Critical Value Approach

- STEP 4.) Given step 3, determine the critical values that divide the rejection and non-rejection regions

The rejection region is contained in the **lower tail** of the sampling distribution of the means (Student's t Distribution).

How did we find the t-critical-value?

You may use Excel for this: = **T.INV(Probability,df)=T.INV(0.05,24)=-1.7109**

The Critical Value Approach

- STEP 5.) Collect data and compute the value of the test statistic

$$n = 25, \bar{X} = 170.8\ (sec.), S = 21.3\ (sec.)$$

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{170.8 - 188.83}{\frac{21.3}{\sqrt{25}}} = -4.2324$$

The Critical Value Approach

- STEP 5.) Collect data and compute the value of the test statistic

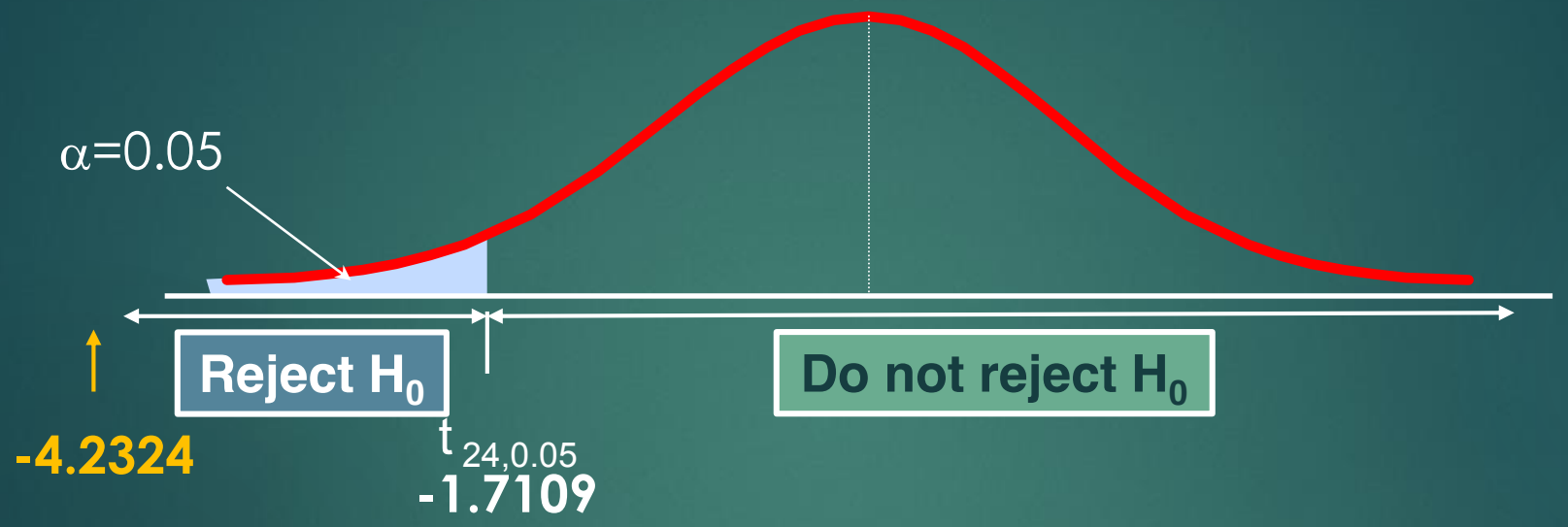
$$n = 25, \bar{X} = 170.8\ (sec.), S = 21.3\ (sec.)$$

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{170.8 - 188.83}{\frac{21.3}{\sqrt{25}}} = -4.2324$$

- STEP 6.) If the test statistic falls into the non-rejection region, do not reject the null hypothesis H_0. If the test statistic falls into the rejection region, reject the null hypothesis.

-4.2324<-1.7109 → We can safely reject the null hypothesis

The Critical Value Approach



- **Conclusion**: For the entire population of stores, there is enough evidence to suggest that the mean time required for drive-through services at McDonald's is less than 188,83 seconds.

The p-Value Approach

The p-Value Approach

- STEP 1.) State the null hypothesis, H_0 and the alternative hypothesis, H_1

  H_0: $\mu \geq 188.83$ (seconds); H_1: $\mu < 188.83$ (seconds)

- STEP 2.) Choose the level of significance, α, and the sample size, n

  α=0.05 and n=25 stores

- STEP 3.) Determine the appropriate test statistic based on the sampling distribution

  Since we do not know about the population standard deviation (i.e. σ is unknown), we are going to employ the t-Stat.

  Since the sample size is small, however, we need to assume that the population of drive-through time is normally distributed.

The p-Value Approach

- STEP 4.) Collect data and compute the value of the test statistic and p-value

$$n = 25, \bar{X} = 170.8\ (sec.), S = 21.3\ (sec.)$$

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- Compute the p-value: **P(t<-4,2324)**

P(t<-4,2324)=T.DIST(-4.2324,24,TRUE)=0.0001

The p-Value Approach

- STEP 4.) Collect data and compute the value of the test statistic and p-value

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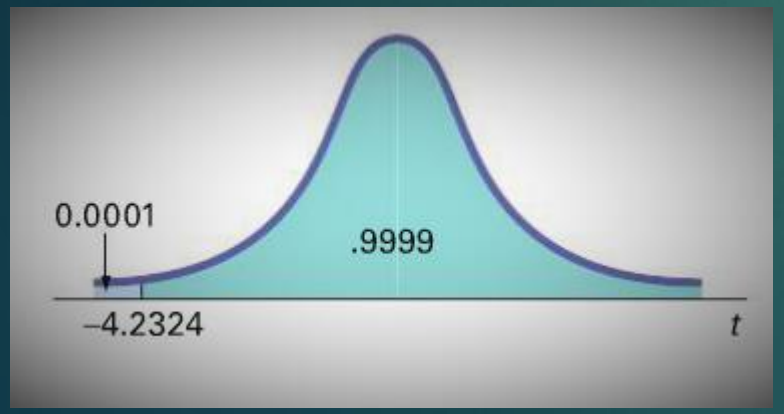
- Compute the p-value: **P(t<-4,2324)**



The p-value of 0.0001 is less than the desired probability of Type I Error (a=0.05): The p-value is low, the null must go.

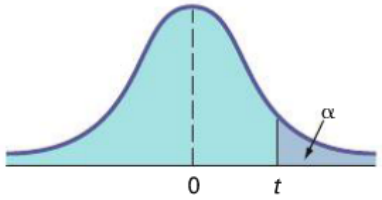
Conclusion: For the entire population of stores, there is enough evidence to suggest that the mean time required for drive-through services at McDonald's is less than 188,83 seconds.

NOTE: How to Identify the Critical Values Using the Student's t Distribution Table and Excel Functions

TABLE E.3

Critical Values of t

For a particular number of degrees of freedom, entry represents the critical value of t corresponding to the cumulative probability $(1 - \alpha)$ and a specified upper-tail area (α).



Degrees of Freedom	Cumulative Probabilities					
	0.75	0.90	0.95	0.975	0.99	0.995
	Upper-Tail Areas					
	0.25	0.10	0.05	0.025	0.01	0.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
12	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
13	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123
14	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467

The Excel Function:

Two-tail Test Critical Values
=T.INV.2T(probability, df)

One-tail Test Critical Values
=T.INV(probability, df)

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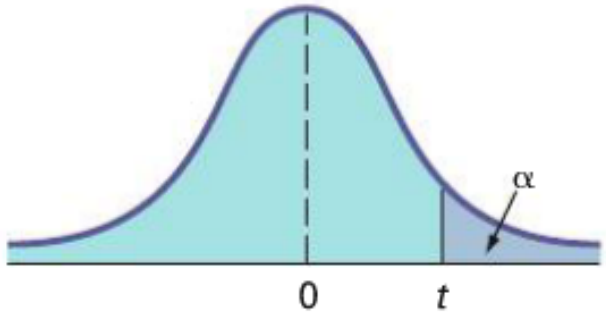

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The critical value for a **two-tail tests** with confidence level of **95%** and sample size of **15**

a=0.05 and df=14

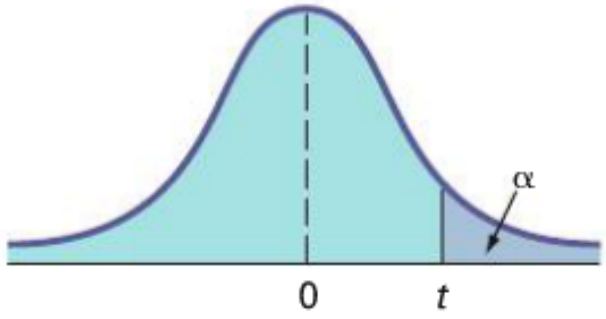
t 0.025,14= 2.1448
-t 0.025,14=-2.1448

The Excel Function:
=T.INV.2T(probability, df)
=T.INV.2T(0.05, 14)
And
=-T.INV.2T(0.05,14)

TABLE E.3

Critical Values of t

For a particular number of degrees of freedom, entry represents the critical value of t corresponding to the cumulative probability $(1 - \alpha)$ and a specified upper-tail area (α).



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15	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467

The critical value for a **one-tail tests** with confidence level of **95%** and sample size of **15**

a=0.05 and df=14

t 0.05,14= 1.7613
-t 0.05,14=-1.7613

The Excel Function:
=T.INV(probability, df)
=T.INV(0.05, 14)
And
=-T.INV(0.05,14)

EXERCISE:

Example 9.5 on PP. 330-331

ASSIGNMENTS:

Homework for Sec. 9.3

Hypothesis Tests for Proportions

- Reminder:
  - The proportion of the population in the category of interest is denoted by π
  - Sample proportion in the category of interest is denoted by *p*

$$p = \frac{X}{n} = \frac{\text{number in category of interest in sample}}{\text{sample size}}$$

  - Under some conditions, p can be approximated by a normal distribution with:

$$\mu_p = \pi \qquad \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

Hypothesis Tests for Proportions

Hypothesis Tests for π

nπ ≥ 5 and n(1-π) ≥ 5

The test statistic is:

$$Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

nπ < 5 or n(1-π) < 5

Not discussed in this chapter

Z Test for Proportion in Terms of Number in Category of Interest

- An equivalent form to the Z-Stat, but in terms of the number in the category of interest, X:

$$Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{n}{n} \times \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

$$\Rightarrow \quad Z_{STAT} = \frac{X - n\pi}{\sqrt{n\pi(1-\pi)}}$$

Example: Z Test for Proportion

A print advertisement agency claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 mails were sent and 25 responses were received. Test their claim at the $\alpha = 0.05$ significance level.

Example: Z Test for Proportion

A print advertisement agency claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 mails were sent and 25 responses were received. Test their claim at the $\alpha = 0.05$ significance level.

Is the sampling dist. of sample proportions approximately normal?

$n\pi = (500)(.08) = 40$

$n(1-\pi) = (500)(.92) = 460$

The Critical Value Approach

$H_0: \pi = 0.08$
$H_1: \pi \neq 0.08$

The Critical Value Approach

$H_0: \pi = 0.08$
$H_1: \pi \neq 0.08$

$\alpha = 0.05 \rightarrow$ **Critical Values: ± 1.96**

n = 500, Sample Proportion: p = 25/500=0.05

The Critical Value Approach

$H_0: \pi = 0.08$
$H_1: \pi \neq 0.08$

$\alpha = 0.05 \rightarrow$ Critical Values: ± 1.96

n = 500, Sample Proportion: p = 25/500=0.05

$$Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{.05 - .08}{\sqrt{\frac{.08(1-.08)}{500}}} = -2.47$$

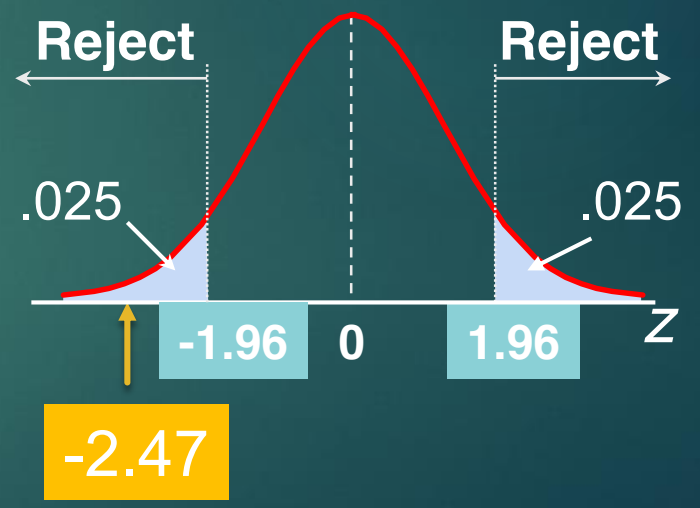
The Critical Value Approach

$H_0: \pi = 0.08$
$H_1: \pi \neq 0.08$

$\alpha = 0.05 \rightarrow$ Critical Values: ± 1.96

n = 500, Sample Proportion: p = 25/500=0.05

$$Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{.05 - .08}{\sqrt{\frac{.08(1-.08)}{500}}} = -2.47$$



Reject H_0 at $\alpha = 0.05$

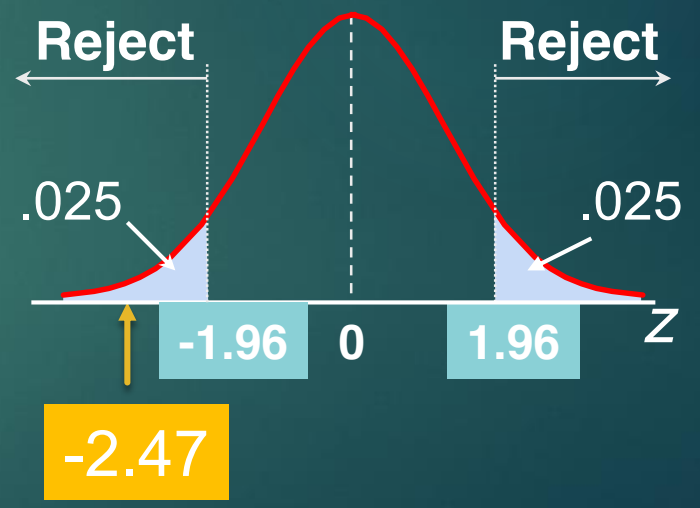
The Critical Value Approach

$H_0: \pi = 0.08$
$H_1: \pi \neq 0.08$

$\alpha = 0.05 \rightarrow$ **Critical Values: ± 1.96**

n = 500, Sample Proportion: p = 25/500=0.05

$$Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{.05 - .08}{\sqrt{\frac{.08(1-.08)}{500}}} = -2.47$$



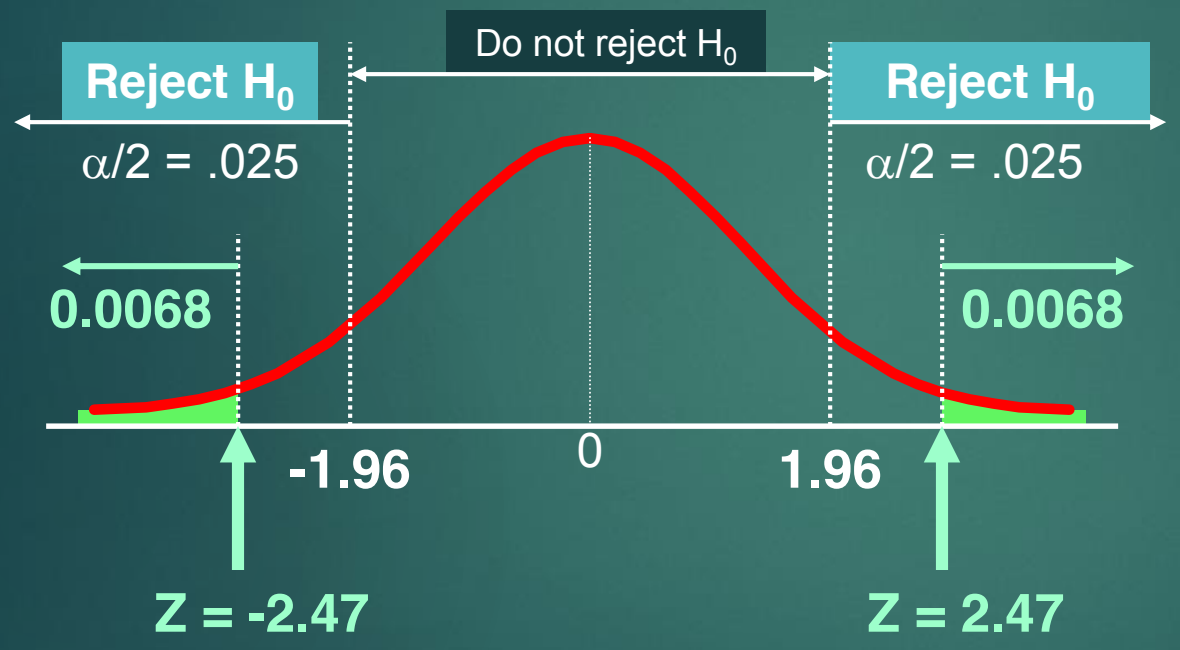
Reject H_0 at $\alpha = 0.05$

Conclusion:

There is sufficient evidence to reject the company's claim of 8% response rate.

The p-Value Approach

Calculate the p-value and compare to α



p-value = 0.0136:

$$P(Z \le -2.47) + P(Z \ge 2.47)$$
$$= 2(0.0068) = \boxed{0.0136}$$

Reject H_0 since p-value = 0.0136 < α = 0.05

EXERCISE:

Example 9.6 on PP. 334-335

ASSIGNMENTS:

Homework for Sec. 9.4