

Types of Variables: Quantitative, Qualitative

The general mathematical form for a linear function of a single variable is

$$y = \beta_0 + \beta_1 x \quad (15$$

Lines - Nice and Easy!

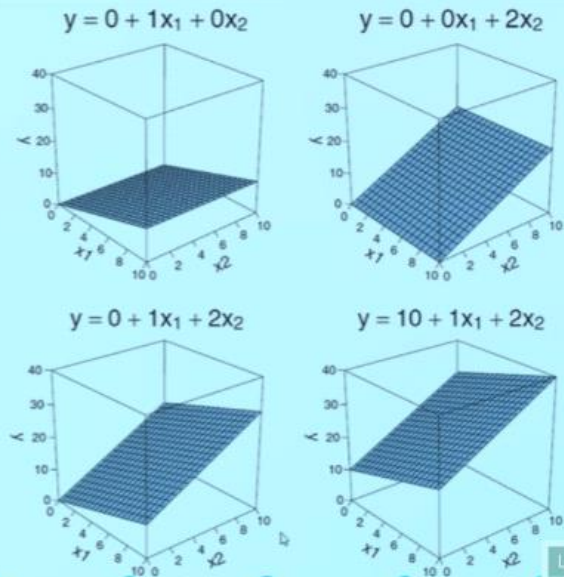
- In general, a linear combination of K predictor variables has the form:

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + \cdots + \beta_K x_K \\ &= \beta_0 + \sum_{k=1}^K \beta_k x_k \end{aligned} \quad (15.2)$$

Lines - Nice and Easy!

Figure 15.2:

Example of linear functions of two variables, x_1 and x_2 . Upper left: Only x_1 has an influence on y . Upper right: only x_2 has an influence on y . Lower left: x_1 and x_2 have an additive influence on y . Lower right: Nonzero intercept is added.

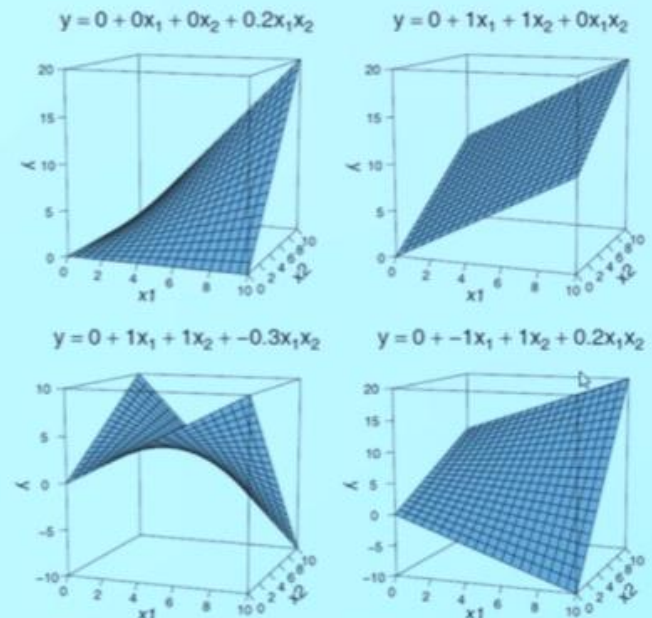


Lines - Nice and Easy!

Interaction

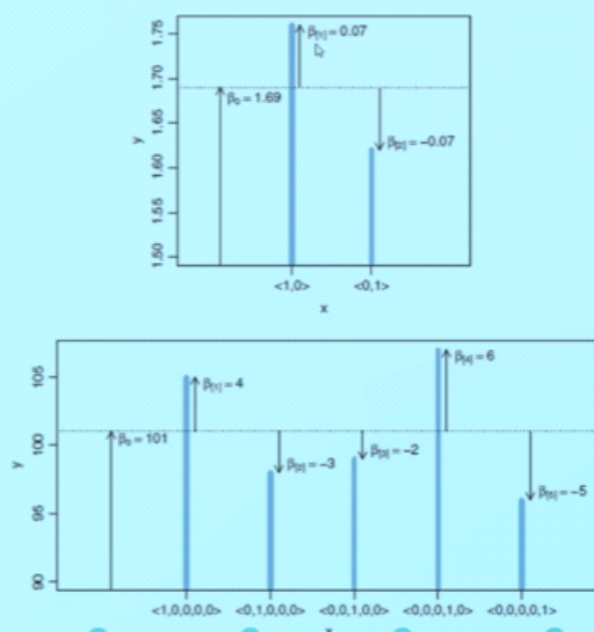
Figure 15.3:

Multiplicative interaction of two variables, x_1 and x_2 . Upper right panel shows zero interaction for comparison. Figure 18.8, p. 526, provides additional perspective and insight.



Combination of Nominal Predictors

Figure 15.4. Examples of nominal predictor (Equations 15.3 and 15.4). Upper panel shows a case with $J = 2$, lower panel shows a case with $J = 5$. In each panel, the baseline value of y is on the far left. Notice that the deflections from baseline sum to zero.



Logit vs. Logistic

- Logistic turns linear predictor between 0 and 1
- Logit turns the 0 to 1 to the linear predictor

Logistic and Logit

- Some authors, and programmers, prefer to express the connection between predictors and predicted in the opposite direction, by first transforming the predicted variable to match the linear model. In other words, you may see the link expressed either of these ways:

$$y = \text{logistic}(\ln(x))$$

$$\text{logit}(y) = \ln(x) \quad (15.15)$$

Generalized Linear Model Components

G.L.M

1. A probability/density function
2. A linear predictor

$$\eta = \beta_0 + \beta_1 x_1 + \dots + \beta_r x_r$$

$$\eta = X\beta$$

- link \rightarrow 3. A link function

$$g(\mu) = X\beta = \eta$$

G.L.M

The exponential family

$$f(y|\theta, \phi) = \exp \left\{ \frac{y\theta - b(\theta)}{a(\phi)} + c(y, \phi) \right\}$$

$$\int_{-\infty}^{\infty} f(y) dy = 1$$



$$\frac{d}{d\theta} \int_{-\infty}^{\infty} f(y) dy = \frac{d}{d\theta} 1 = 0$$