

Chapter 12 - Methods for Bayes Null Hypothesis Testing

Sunday, April 3, 2022 4:40 PM

Resource of ROPE:

https://cran.r-project.org/web/packages/bayestestR/vignettes/region_of_practical_equivalence.html

ROPE: Region of practical significance

Rejected ROPE:

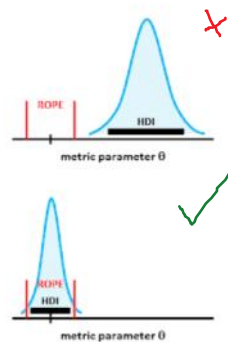
- Parameter is not credible if the ROPE does not lie within the 95% HDI
- A parameter value is declared to be not credible, or rejected, if its entire ROPE lies outside the 95% highest density interval (HDI) of the posterior distribution of that parameter.

Accepted ROPE:

- if the rope lies within the HDI
- A parameter value is declared to be accepted for practical purposes if that value's ROPE completely contains the 95% HDI of the posterior of that parameter.
- With this decision rule, a null value of a parameter can be accepted only when there is sufficient precision in the estimate of the parameter.

Decision by posterior HDI and ROPE

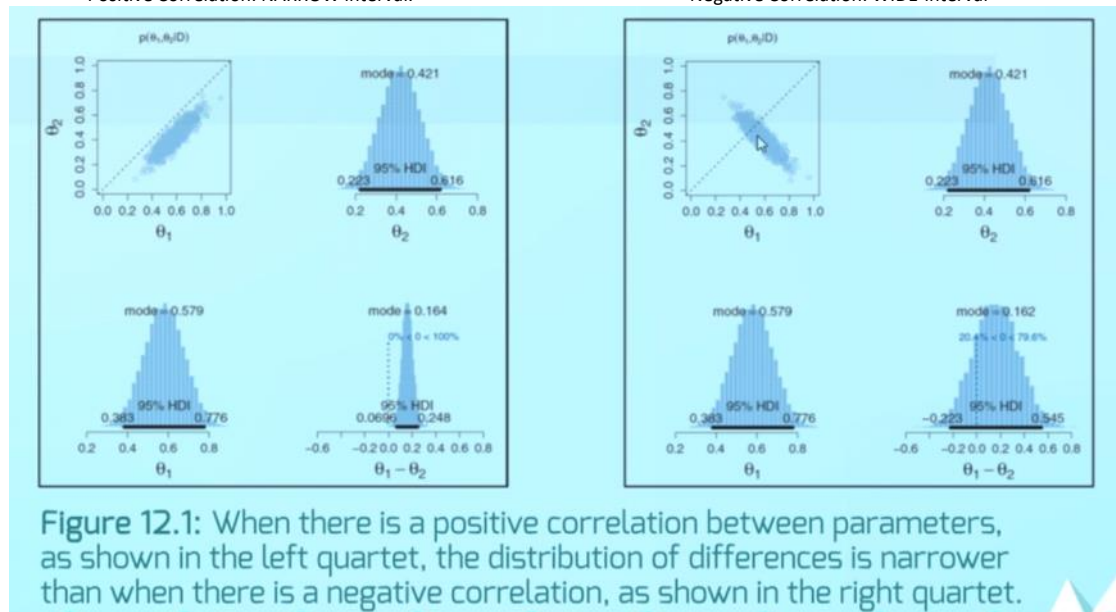
- Consider a landmark parameter value. Values that are equivalent to that landmark for practical purposes define the ROPE around that value.
 - ROPE is specified according to utilities and/or predictions of competing theories. Routinely done in frequentist *equivalence testing* and clinical *non-inferiority testing*.
- The ROPed parameter value is rejected if its ROPE excludes the 95% HDI.
 - Rejects the ROPed value, not entire ROPed interval.
- The ROPed parameter value is accepted for practical purposes if its ROPE includes the 95% HDI.
 - Accepts the ROPed value, not entire ROPed interval.
- If ROPE and 95% HDI overlap, neither accept nor reject the ROPed parameter value.



How does Negative and Positive Correlation affect the Testing?

Positive Correlation: NARROW interval.

Negative Correlation: WIDE interval.

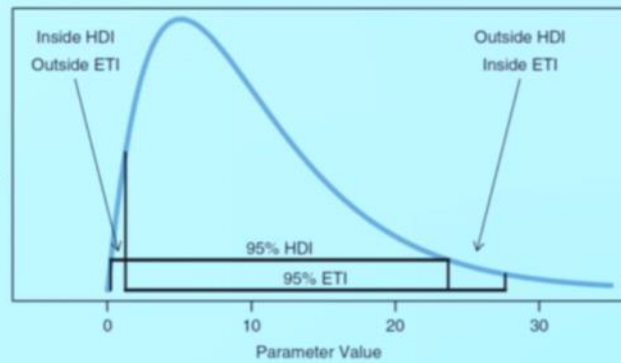


Why choose HDI for this comparison and not ETI?

HDI is a narrower estimate and will allow for better comparison

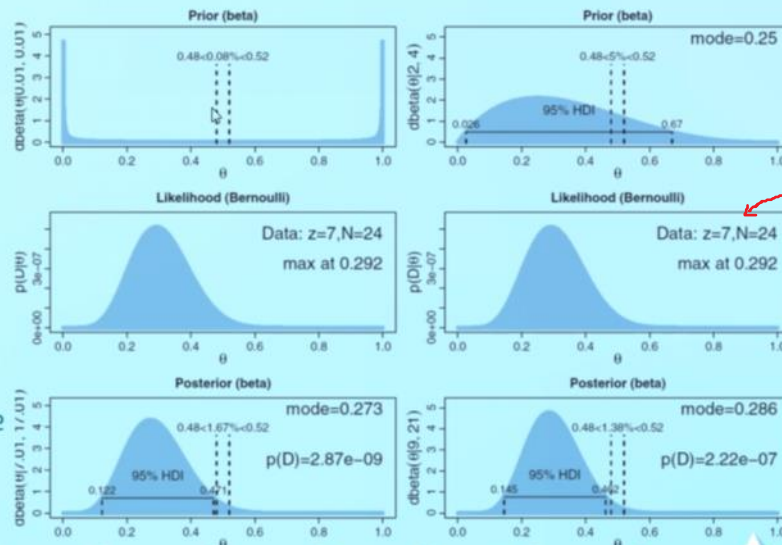
Why HDI, not ETI?

Figure 12.2: A skewed distribution has different 95% highest density interval (HDI) than 95% equal-tailed interval (ETI).



See the effects of the ROPE

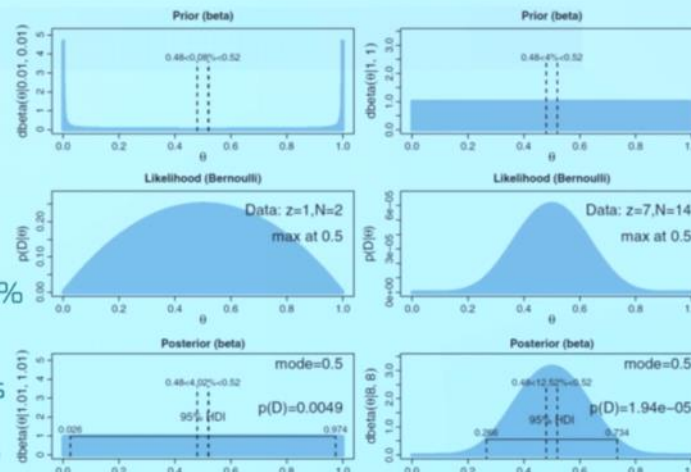
Figure 12.3: Left column, Haldane prior. Right column: Mildly informed prior. Vertical dashed lines mark a ROPE from 0.48 to 0.52. Annotation above the dashed lines indicates the percentage of the distribution within the ROPE.



7 out of 24 accepted

Bayes factors can accept null with poor precision (bot good)

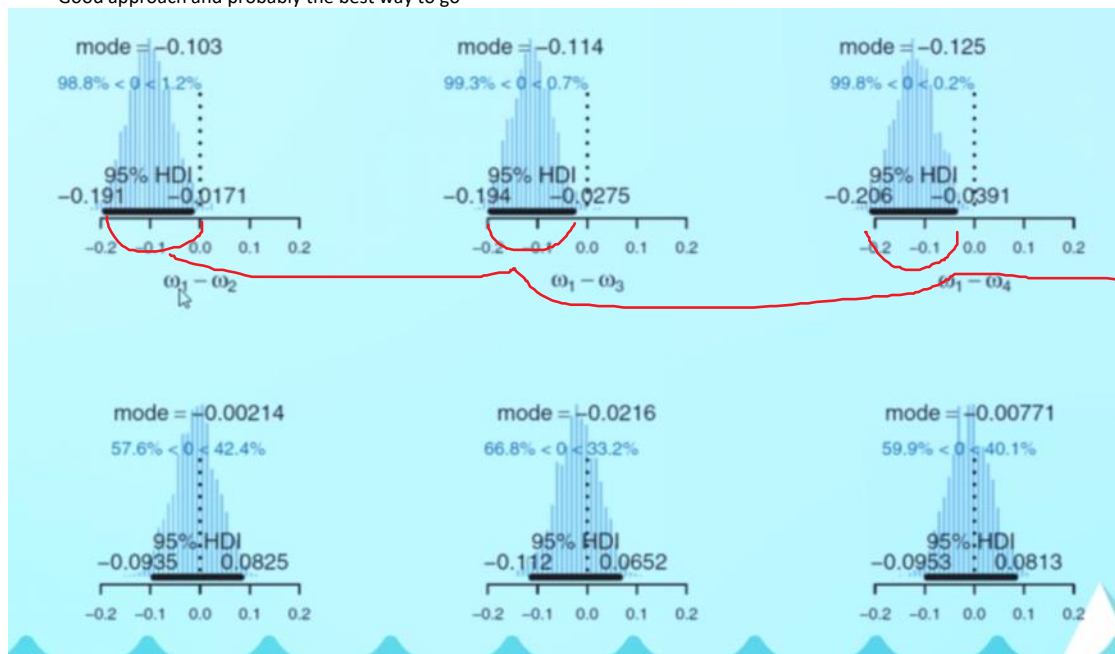
Figure 12.4: Bayes' factor (Model Comparison) can accept the null even with low precision on estimate. Left column: Haldane prior. Bayes' factor is 51.0 in favor of null, but 95% HD extends from 0.026 to 0.974 (!). Right column: Uniform prior. Bayes' factor is 3.14 in favor of null, but the 95% HDI extends from 0.266 to 0.734 (!).



Two Approaches to Estimation about if groups are equal:

1. Estimation Approach

- Measure an effect of difference in two groups
- Good approach and probably the best way to go



Difference between groups

2. Model Selection Approach

- JAGS uses this approach
- Shows if the two groups are different from each other

JAGS Model - Using Psuedo Priors

The data structure has one row per subject, with the number of trials (words) for subject s denoted $nTr10fSubj[s]$, the number correctly recalled for subject s denoted $nCorrOfSubj[s]$, and the condition of subject s denoted $CondOfSubj[s]$. The model specification begins with saying that each subject has an individual ability $\theta[s]$ from a condition-specific beta distribution:

```
model {
  for ( s in 1:nSubj ) {
    nCorrOfSubj[s] ~ dbin( theta[s] , nTr10fSubj[s] )
    theta[s] ~ dbeta( aBeta[CondOfSubj[s]] , bBeta[CondOfSubj[s]] )
  }
}
```

The shape parameters of the beta distribution are then re-written in terms of the mode and concentration. Model 1 uses condition-specific $\omega[j]$, while Model 2 uses the same ω_0 for all conditions. The JAGS function `equals(md1Idx,...)` is used to select the appropriate model for index `md1Idx`:

```
for ( j in 1:nCond ) {
  # Use omega[j] for model index 1, omega0 for model index 2:
  aBeta[j] <- ( equals(md1Idx,1)*omega[j]
               + equals(md1Idx,2)*omega0 ) * (kappa[j]-2)+1
  bBeta[j] <- ( 1 - ( equals(md1Idx,1)*omega[j]
                     + equals(md1Idx,2)*omega0 ) ) * (kappa[j]-2)+1
  omega[j] ~ dbeta( a[j,md1Idx] , b[j,md1Idx] )
}
omega0 ~ dbeta( a0[md1Idx] , b0[md1Idx] )
```

The priors on the concentration parameters are then specified:

```
for ( j in 1:nCond ) {  
  kappa[j] <- kappaMinusTwo[j] + 2  
  kappaMinusTwo[j] ~ dgamma( 2.618 , 0.0809 ) # mode 20 , sd 20  
}
```

How the Pseudo Priors were Chosen

```
# Constants for prior and pseudoprior:  
aP <- 1  
bP <- 1  
# a0[model] and b0[model]  
a0[1] <- 0.48*500      # pseudo  
b0[1] <- (1-0.48)*500  # pseudo  
a0[2] <- aP            # true  
b0[2] <- bP            # true  
# a[condition,model] and b[condition,model]  
a[1,1] <- aP           # true  
a[2,1] <- aP           # true  
a[3,1] <- aP           # true  
a[4,1] <- aP           # true  
b[1,1] <- bP           # true  
b[2,1] <- bP           # true  
b[3,1] <- bP           # true  
b[4,1] <- bP           # true  
a[1,2] <- 0.40*125     # pseudo  
a[2,2] <- 0.50*125     # pseudo  
a[3,2] <- 0.51*125     # pseudo  
a[4,2] <- 0.52*125     # pseudo  
b[1,2] <- (1-0.40)*125 # pseudo  
b[2,2] <- (1-0.50)*125 # pseudo  
b[3,2] <- (1-0.51)*125 # pseudo  
b[4,2] <- (1-0.52)*125 # pseudo  
# Prior on model index:  
mdlIdx ~ dcat( modelProb[ ] )  
modelProb[1] <- 0.5  
modelProb[2] <- 0.5
```