

# Lab. 4 Bayes' Box and R.

In this lab you will need to be able to make the Bayes' Box for Binomial experiments.

## Objectives:

- Review classical methods
- Learn how the components of a Bayes' box are constructed using R. **Matrix names on cols**
- Learn how to construct a Bayes' box using an R function.
- Learn how to make graphical output from the Bayes box
- Learn how to make both point and interval estimates from a Bayes' box
- Learn how to automate the process with an all inclusive function.

## Output:

- Make an Rmd document for this lab ( ALL formulae need to be put in latex)
- At the conclusion render into the 3 file types
- Upload the 4 files to the server.

## Bayes' Rule

$$p(\theta|x) = \frac{p(\theta)p(x|\theta)}{\sum_{\theta} p(\theta)p(x|\theta)}$$

## Experiment

A biased coin is tossed

- $n = 10$  times
- $x = 3$  successes (Heads)
- Each trial is independent
- There are only two possible outcomes per trial
- $P(H) = \theta$  is constant **<- Find theta**

## Tasks:

1. Classical estimates for  $\theta$ 
  - a. What is the classical point estimate for  $\theta$ ?
  - b. Using  $\hat{\theta} \pm 1.95 \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$ 
    - i. find the classical 95% ci for  $\theta$  **<- Use theta for 95% CI**
    - ii. interpret the interval (see <http://onlinestatbook.com/2/estimation/confidence.html> )
    - iii. How many samples were used to make the interval?
    - iv. How many samples are assumed to give the interpretation?

**1. w/95% confidence, theta lies within the two limits**

**2. if we repeat overtime 95% of the time theta w within bounds**

20 values must be 1/20th (uniform)

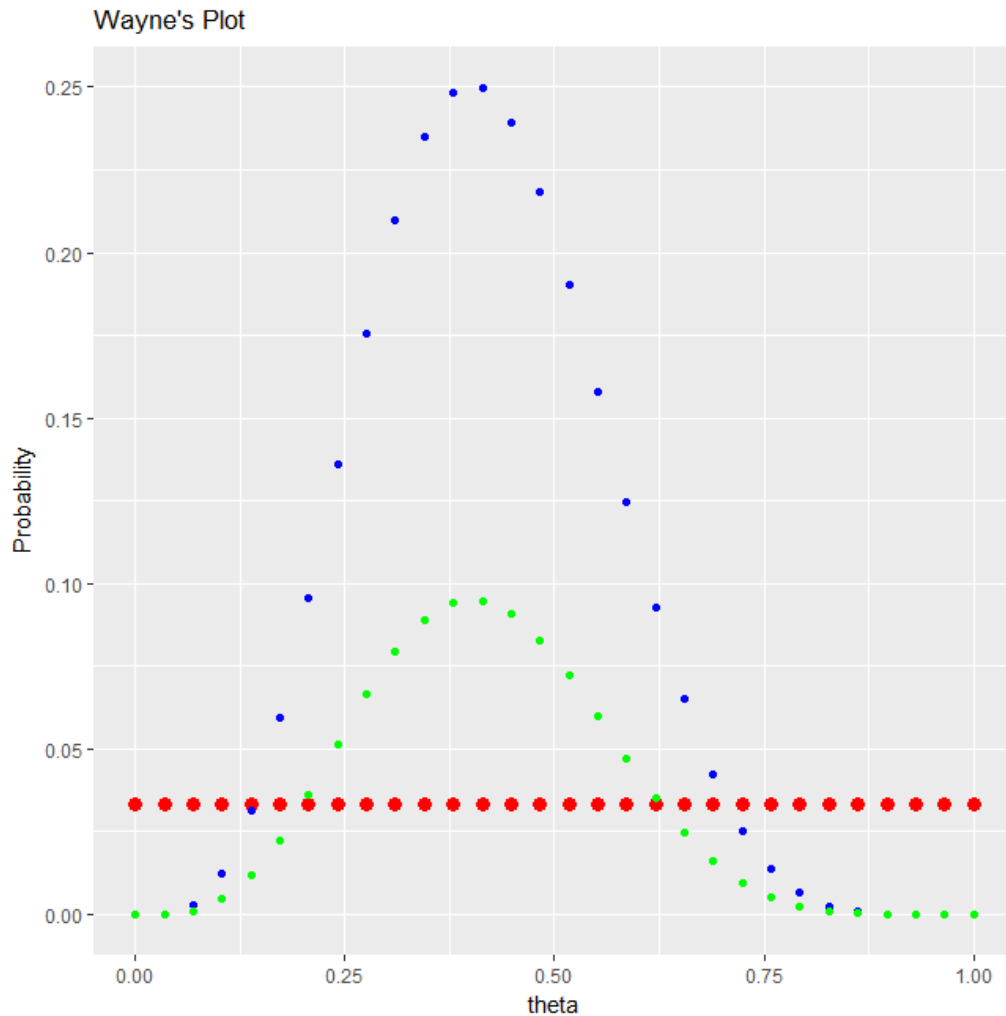
2. Bayesian methods – Bayes' rule. We will assume discrete values of  $\theta$  and a uniform prior on the values of  $\theta$

- Suppose we take  $\theta = \text{seq}(0, 1, \text{length} = 20)$  Break  $\theta$  up into 20 intervals
- Plot the prior using base R.
- Write down the formula for the likelihood in this case – you can substitute the  $n$  and  $x$  so that it is a function of  $\theta$
- Plot the likelihood over the discrete values of  $\theta$ . Place in your document.
- Now add the prior plot onto the same graph. Place in your document.
- We need the posterior in order to obtain the third and last plot – what formula will we use to obtain this? Write it down.

3. Bayes' Box – below is Bayes' box – it will need more rows!  $h = \text{numerator of bayes rule}$

theta	P(theta)	P(x theta)	h(theta)	P(theta x)
Total				

- Make a matrix in R that will hold the 5 column vectors. Show here.
- Rewrite Bayes' theorem in terms of  $h = \text{prior} * \text{lik}$ . Show formula here.
- Now make a plot similar to the one below (except it will be for the data given above) from the matrix using ggplot
  - Put your name on it
  - Make sure it has the right colors
  - Make an improvement to the plot by changing the x ticks to represent the number of  $\theta$  values.



4. Now make a function called `mybinpost()` that will do the following:
  - a. Take arguments `n`, `x`, `theta`, `alpha`
    - i. `n` = number of trials
    - ii. `x` = number of successes
    - iii. `theta` = vector of possible theta values (sequence from 0 to 1)
    - iv. `alpha/2` = tail probability (used to obtain 1-alpha Bayesian credible interval)
    - v. Assume uniform prior
  - b. The function will produce
    - i. the above plot.
    - ii. A list containing
      1. the Bayes box as a matrix
      2. the Bayes box in latex (package "xtable")
      3. the number of trials `n`
      4. the number of successes `x`
      5. the Bayesian point estimate
      6. The 95% Bayesian credible interval (BCI)
5. Now use your function and include all output into your document for the following invocations

- a. `mybinpost(n=20,x=12,theta = seq(0,1,length=50), alpha=0.05)`
- b. `mybinpost(n=50, x=25, theta=seq(0,1,length=30), alpha=0.1)`