Question 5

Steps for expressing the MLE are as follows:

a) Expressing the joint probability distribution as a product assuming independence
*P(X 1 ,X 2 ,...,X n ) =P(X 1 )P(X 2 ) ∗ ∗ ∗ P(X n )*

b) The joint probability distribution becomes the likelihood function L(θ)
*L(θ) = P(θ 1 )P(θ 2 ) ∗ ∗ ∗ P(θ n )*

$$L(\theta) = \prod_{i=1}^{n} P(\theta_i)$$

c) Taking the natural log of the likelihood function becomes the log likelihood function

$$l(\theta) = log(L(\theta))$$

$$l(\theta) = log(\prod_{i=0}^{n} P(\theta_i))$$

d) The value of θˆ that renders a maximum value of the log likeliehood function becomes the maximum likelihood estimator

$$l'(\hat{\theta}) = 0 = \frac{d}{d\hat{\theta}} log(\prod_{i=0}^{n} P(\hat{\theta}_i))$$

e) To see if θˆ is the max estimator (Notice that we are deriving the function twice.)

$$l''(\hat{\theta}) = \frac{d^2}{d\hat{\theta}^2} log(\prod_{i=0}^{n} P(\hat{\theta}_i)) < 0$$

1. https://newonlinecourses.science.psu.edu/stat414/node/191/ . Refer this link for further clarification. As we are finding the maximum likelihood estimator 𝜆ˆ using probability density function for the Poisson distribution:

   $$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

   Hence start with

   $$L(\lambda) = \prod_i \frac{\lambda^{x_i} e^{-\lambda}}{x!} \quad i \in (1, n)$$

   ,

   $$l(\lambda) = log(\prod_i \frac{\lambda^{x_i} e^{-\lambda}}{x!}) \quad i \in (1, n)$$

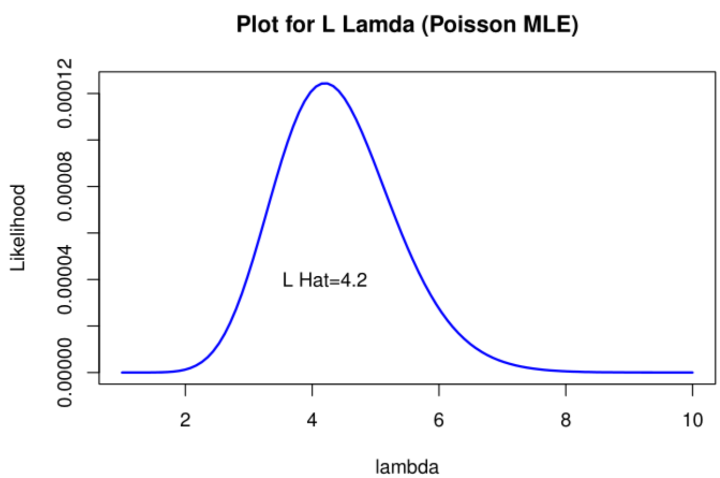
   Then obtain

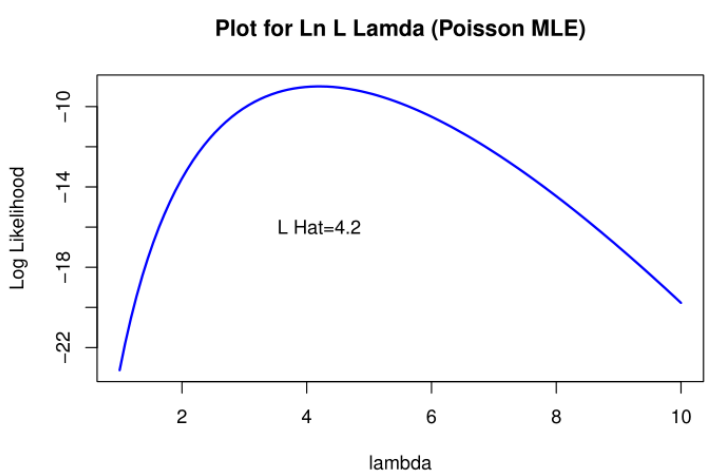
   and finally get $\hat{\lambda}$ by solving $\frac{dl(\lambda)}{d\lambda} = 0$

2. By performing a double derivate on the function *l(y)*
3. Show that this second derivative is < 0. (Note: Use to fact that *log* is a monotonically increasing function to justify your answer). Alternatively, to prove 𝜆ˆ is indeed maximum we need to show second derivative of *l*(𝜆) is negative when evaluated at 𝜆 *= mean x (bar).*
4. Show that the maximum of L(θ) is equal to the maximum of l(θ)
5. Skeleton of the function

```
myml = function(x)
{
# Create lambda vector as a set of values ranging from 0 to 2*max(x)
lambda=seq(.01,2*max(x),0.5)
# Calculate Poisson MLE
lambdahat = round(sum(x)/length(x),4)
#the likelihood function
lik <- exp(-length(x)*lambda)*(lambda^sum(x))/prod(factorial(x))
#the loglikelihood function
loglik <- -length(x)*lambda+sum(x)*log(lambda)+log(prod(factorial(x)))
return(list(mle=lambdahat…..)}
```

Expected Plots :



## Question 6

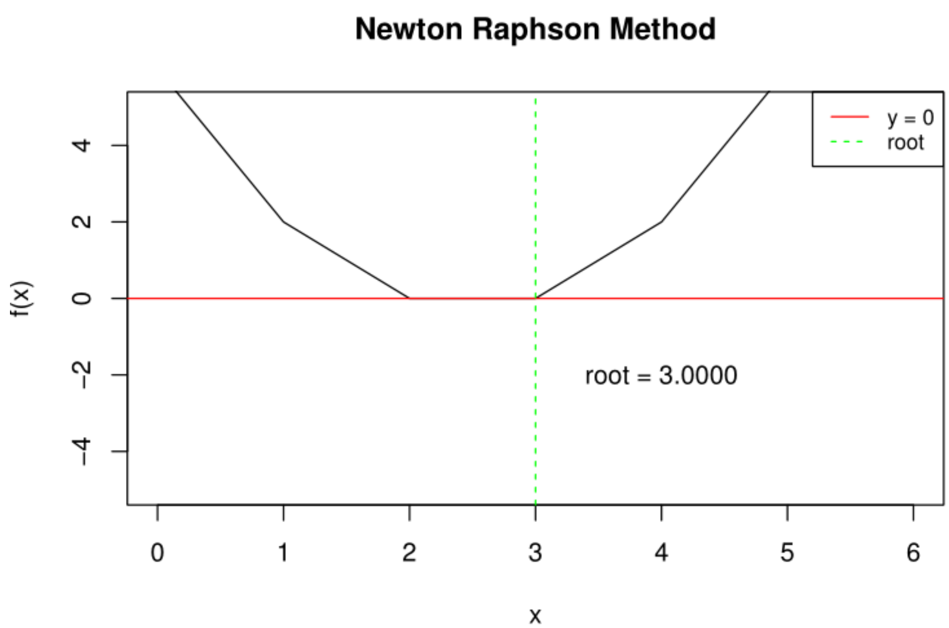
Hint: From results above we found $\hat{\lambda} = \overline{x}$ ,the sample mean. We need to prove E(x) = λ. Start with proving for a random variable Y ~ Poisson(λ), E(Y ) = λ

### Question 8

mynr() Uses the Newton-Raphson method to calculate a root. As inputs, it requires a function to be passed, a derivative to be passed, an initial guess, a tolerance, and a maximum number of iterations.

https://rpubs.com/aaronsc32/newton-raphson-method

Expected Answer:




or