Chapter 4

Basic Probability

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Why do we need to survey the probability theory in this course?

- It is a link btw. Descriptive Statistics and Inferential Statistics.
- It has real life applications that are widely used

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Example:
Intent to Purchase Study

- **M&R Electronics survey a stratified sample of households in the US to identify the following:**
- The probability of a household planning to purchase a curved screen TV in the next 12 months.
- The probability of a household purchasing the above product in the next year.
- The probability that a household who is planning to purchase a curved screen TV actually makes the purchase in the next year.

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Basic Probability Concepts

- **Probability** – the chance that an uncertain event will occur (always between 0 and 1)
- **Impossible Event** – an event that has no chance of occurring (probability = 0)
- **Certain Event** – an event that is sure to occur (probability = 1)

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Assessing Probability

There are three approaches to assessing the probability of an uncertain event:

1. ***A priori*** ***-- based on prior knowledge of the process***

$$\text{probability of occurrence} = \frac{X}{T} = \frac{\text{number of ways in which the event occurs}}{\text{total number of possible outcomes}}$$

Assuming all outcomes are equally likely

2. **Empirical**

$$\text{probability of occurrence} = \frac{\text{number of ways in which the event occurs}}{\text{total number of possible outcomes}}$$

3. **Subjective**

based on a combination of an individual's past experience, personal opinion, and analysis of a particular situation

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A *priori* probability

When randomly selecting a day from the year 2014 what is the probability the day is in January?

$$\text{Probability of Day In January} = \frac{X}{T} = \frac{\text{number of days in January}}{\text{total number of days in 2014}}$$

$$\frac{X}{T} = \frac{31 \text{ days in January}}{365 \text{ days in 2013}} = \frac{31}{365}$$

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Empirical probability

Find the probability of selecting a male taking statistics from the population described in the following table:

	Taking Stats	Not Taking Stats	Total
Male	84	145	229
Female	76	134	210
Total	160	279	439

$$\text{Probability of male taking stats} = \frac{\text{number of males taking stats}}{\text{total number of people}} = \frac{84}{439} = 0.191$$

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Subjective probability

- Subjective probability may differ from person to person
  - A media development team assigns a 60% probability of success to its new ad campaign. The chief media officer of the company is less optimistic and assigns a 40% of success to the same campaign
- The assignment of a subjective probability is based on a person's experiences, opinions, and analysis of a particular situation
- Subjective probability is useful in situations when an empirical or a priori probability cannot be computed

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Events

Each possible outcome of a variable is an event.

- **Simple event (A)**
  - An event described by a single characteristic
  - e.g., A household planning on purchasing a curved screen TV
- **Joint event (A and B)**
  - An event described by two or more characteristics
  - e.g. A household that was planning to purchase a curved screen TV actually makes the purchase.
- **Complement of an event A (denoted A')**
  - All events that are not part of event A
  - e.g., A household NOT planning on purchasing a curved TV

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Sample Space

The Sample Space is the collection of all possible events

e.g. All 6 faces of a dice:

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Organizing & Visualizing Events

- EVENT **A**:

A randomly selected day is in January

- EVENT **B**:

A randomly selected day is Wednesday

- EVENT **A and B (A ∩ B)**:

A randomly selected day is in January, **AND** it is a Wednesday

- Event **A or B (A∪B)**:

A randomly selected day is in January, **OR** it is a Wednesday

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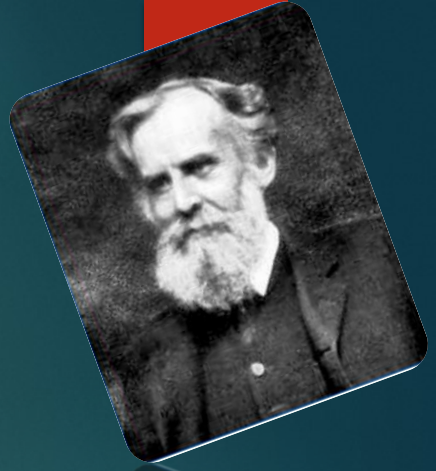
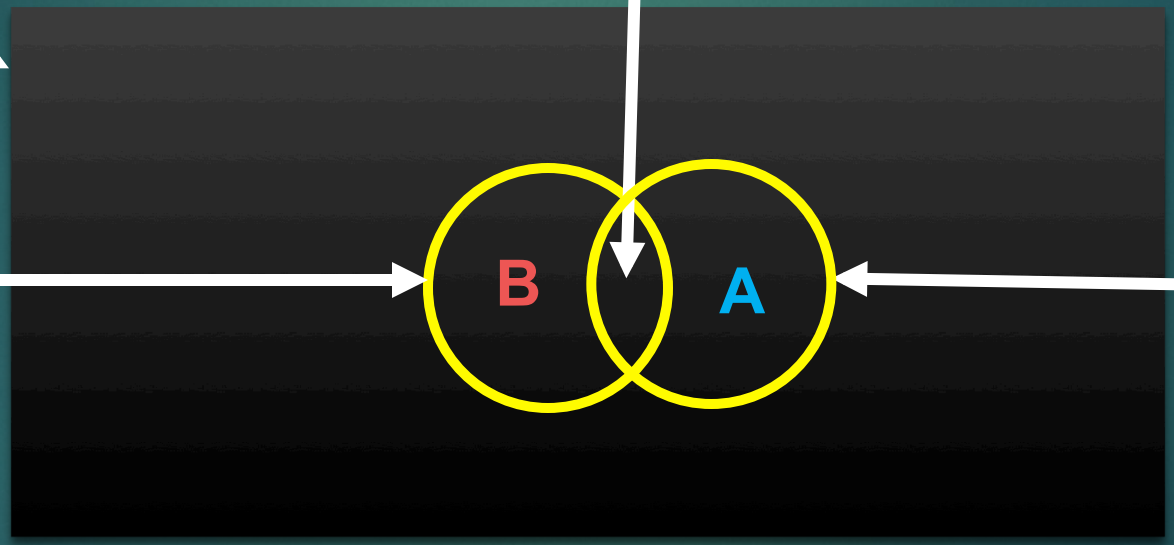
Venn Diagram



Sample Space

(All days in a year)

(A∩B): Days That Are In January and Are Wednesdays



Wednesdays

January Days

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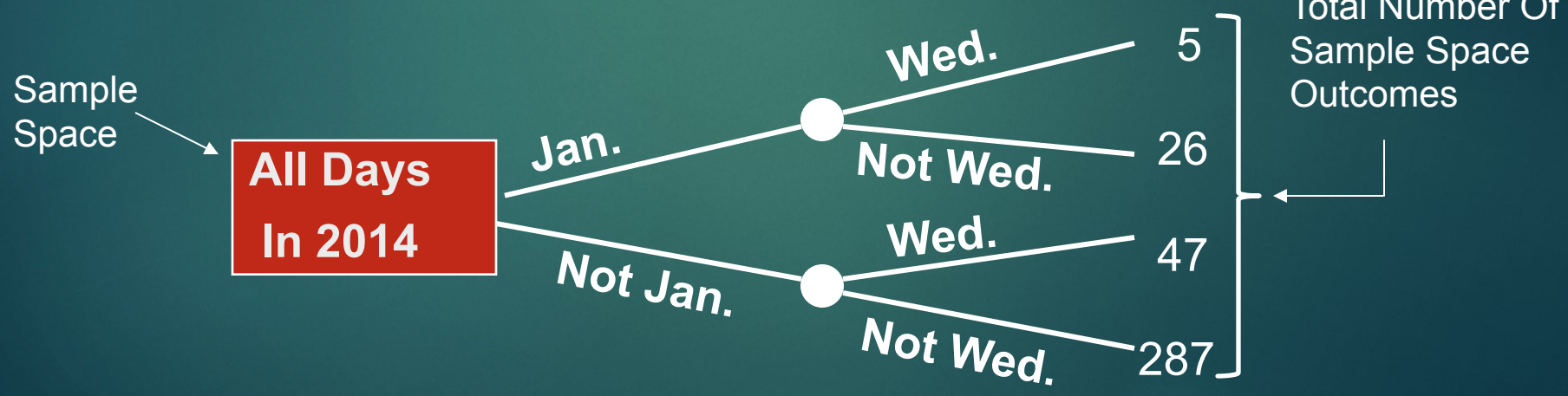
Organizing & Visualizing Events

- **Contingency Tables** -- For All Days in 2014

	Jan.	Not Jan.	Total
Wed.	5	47	52
Not Wed.	26	287	313
Total	31	334	365

Total Number Of Sample Space Outcomes

- **Decision Trees**



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Definition: Simple Probability

- **Simple Probability** refers to the probability of a simple event (an event that can be described with only one characteristics)
  - **Example:** Compute the following: P(Jan.) and P(Wed.)

	Jan.	Not Jan.	Total
Wed.	5	47	52
Not Wed.	26	287	313
Total	31	334	365

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Definition: Simple Probability

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  - **Example:** Compute the following: P(Jan.) and P(Wed.)

	Jan.	Not Jan.	Total
Wed.	5	47	52
Not Wed.	26	287	313
Total	31	334	365

P(Wed.) = 52 / 365

P(Jan.) = 31 / 365

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Definition: Joint Probability

- **Joint Probability** refers to the probability of an occurrence of two or more events (joint event).
  - Example: Compute the following:

    P(Jan. and Wed.) and P(Not Jan. and Not Wed.)

	Jan.	Not Jan.	Total
Wed.	5	47	52
Not Wed.	26	287	313
Total	31	334	365

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Definition: Joint Probability

- **Joint Probability** refers to the probability of an occurrence of two or more events (joint event).
  - Example: Compute the following:

    P(Jan. and Wed.) and P(Not Jan. and Not Wed.)

	Jan.	Not Jan.	Total
Wed.	5	47	52
Not Wed.	26	287	313
Total	31	334	365

P(Not Jan. and Not Wed.) = 287 / 365

P(Jan. and Wed.) = 5 / 365

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Mutually Exclusive Events

- Mutually exclusive events
  - Events that cannot occur simultaneously

Example: Randomly choosing a day from 2015

A = day in January; **B** = day in February

- Events A and B are mutually exclusive

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Collectively Exhaustive Events

- Collectively exhaustive events
  - One of the events must occur

Example: Randomly choose a day from 2015

A = Weekday; **B** = Weekend;

C = January; **D** = Spring;

- Events A, B, C and D are collectively exhaustive (but not mutually exclusive – a weekday can be in January or in Spring)
- Events A and B are collectively exhaustive and also mutually exclusive

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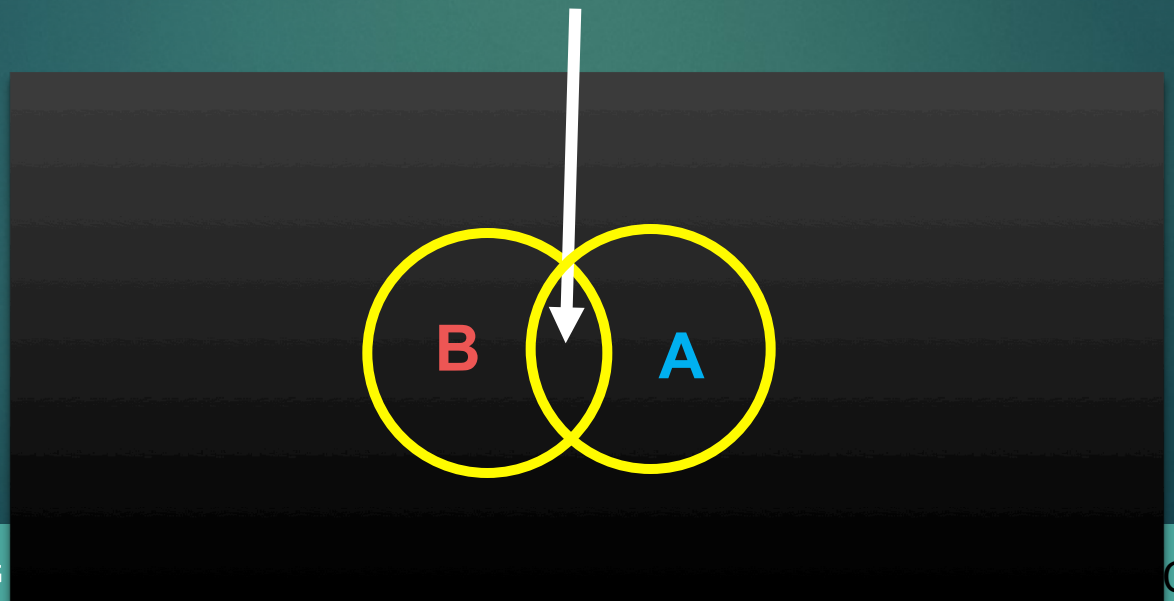
Joint and Marginal Probabilities

- The probability of a joint event, A and B:

$$P(A \text{ and } B) = \frac{\text{number of outcomes satisfying A and B}}{\text{total number of elementary outcomes}}$$

(A∩B): Days That Are In January and Are Wednesdays



Joint and Marginal Probabilities

- The probability of a joint event, A and B:

$$P(A \text{ and } B) = \frac{\text{number of outcomes satisfying A and B}}{\text{total number of elementary outcomes}}$$

- Computing a marginal (or simple) probability:

$$P(A) = P(A \text{ and } B_1) + P(A \text{ and } B_2) + \cdots + P(A \text{ and } B_k)$$

  - Where B_1, B_2, …, B_k are *k* mutually exclusive AND collectively exhaustive events (i.e., they do not happen simultaneously, but together they cover the entire sample space.

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Joint and Marginal Probabilities

Event	Event		Total
	B_1	B_2	
A_1			
A_2			
Total			

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Joint and Marginal Probabilities

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	
Total			

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Joint and Marginal Probabilities

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	
Total			

Joint Probabilities

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Joint and Marginal Probabilities

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

Marginal (Simple) Probabilities

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Joint and Marginal Probabilities

Event	Event		Total
	B_1	B_2	
A_1	$P(A_1 \text{ and } B_1)$	$P(A_1 \text{ and } B_2)$	$P(A_1)$
A_2	$P(A_2 \text{ and } B_1)$	$P(A_2 \text{ and } B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

Joint Probabilities

Marginal (Simple) Probabilities

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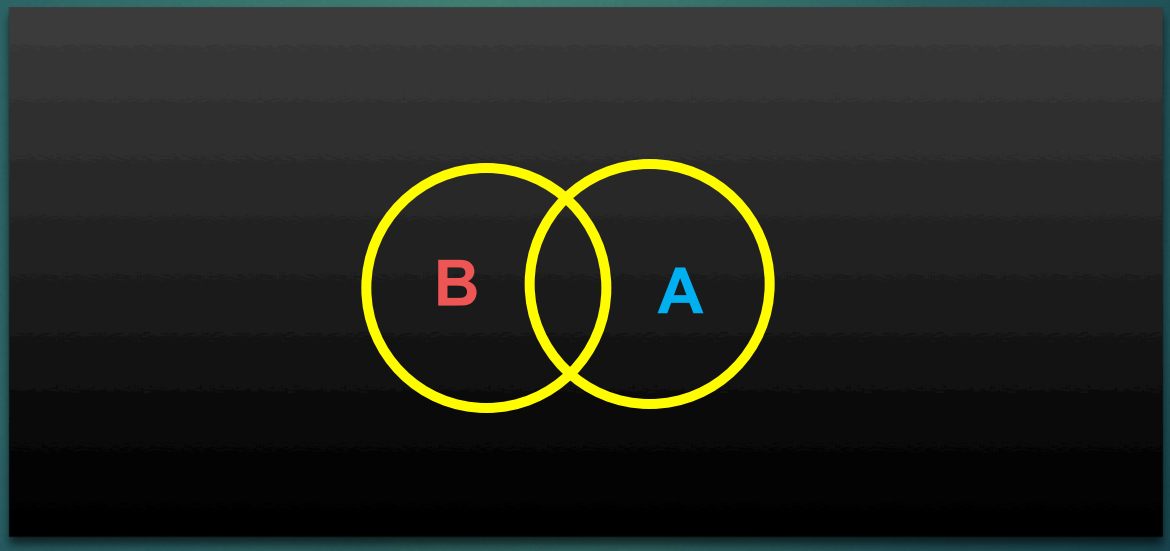
General Addition Rule

General Addition Rule:

P(A or B) = P(A) + P(B) - P(A and B)



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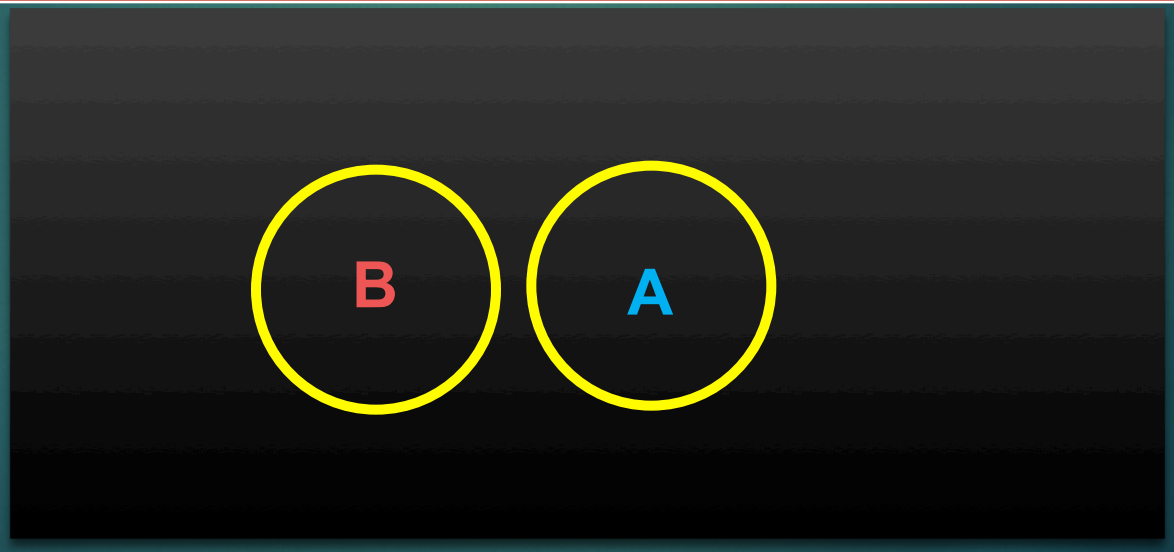
General Addition Rule

If A and B are mutually exclusive, then P(A and B) = 0, so the rule can be simplified:

P(A or B) = P(A) + P(B)

For mutually exclusive events A and B



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Example
Empirical Probability

- In response to a survey conducted by M&R Electronics (1,000 households):
  - 250 households said that they are planning to purchase a curved TV screen.
  - 300 households actually purchased a curved TV screen.
  - 200 of these 300 households that purchased the TV set were planning to purchase that product.
- **P(Planned to Purchase OR Actually Purchased)?**

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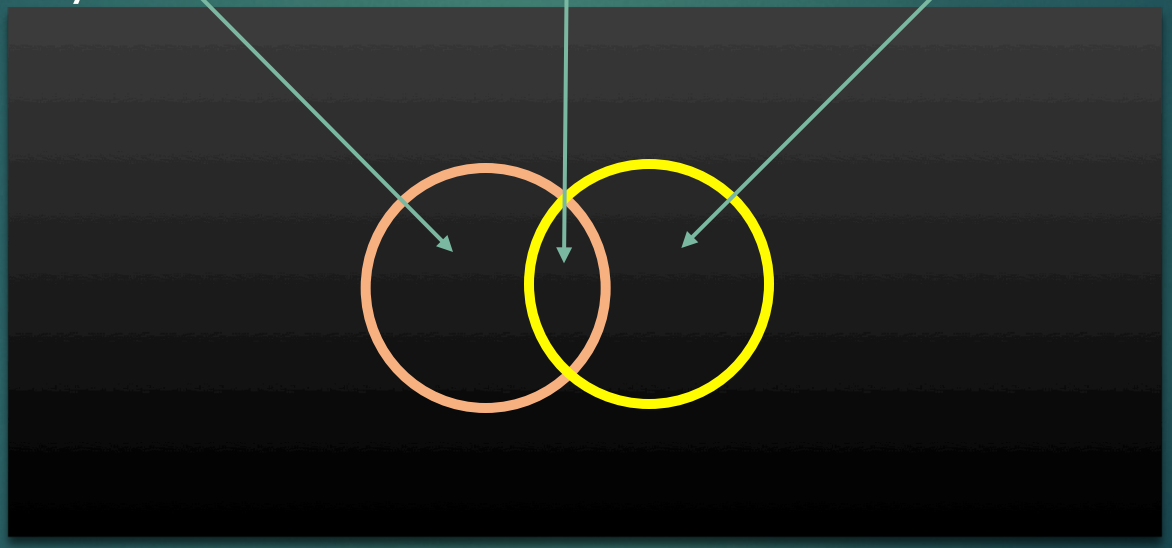
Planning to Purchase AND Actually Purchased (200 HHs)

Planning to Purchase
(250 HHs)

Actually Purchased
(300 HHs)



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Example
Empirical Probability

P(Planned to Purchase OR Actually Purchased)=

P(Planned to Purchase)

+

P(Actually Purchased)

-

P(Planned to Purchase AND Actually Purchased)=

250/1000+300/1000-200/1000=

(250+300-200)/1000=

350/1000=0.35

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Example

PREFERENCE	GENDER: Female	GENDER: Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

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PREFERENCE	GENDER: Female	Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

What is the probability that a randomly selected individual

- Prefers Pepsi
- Is male AND prefers Pepsi
- Is male OR prefers Pepsi

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PREFERENCE	GENDER Female	Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

What is the probability that a randomly selected individual

- Prefers Pepsi:

  P(Pepsi)=175/500=0.35

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PREFERENCE	GENDER Female	Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

What is the probability that a randomly selected individual

- Prefers Pepsi:

  P(Pepsi)=175/500=0.35

- Is male AND prefers Pepsi

  P(Pepsi ∩ Male)=80/500=0.16

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PREFERENCE	GENDER Female	Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

What is the probability that a randomly selected individual

- Prefers Pepsi:

  P(Pepsi)=175/500=0.35

- Is male AND prefers Pepsi

  P(Pepsi ∩ Male)=80/500=0.16

- Is male OR prefers Pepsi

  P(Pepsi ∪ Male)=(220+175-80)/500=0.63

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PREFERENCE	GENDER Female	Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

P(Pepsi ∪ Male)=

P(Pepsi)+P(Male)-P(Pepsi ∩ Male)=
220/500+175/500-80/500=
(220+175-80)/500=0.63

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Conditional Probabilities

- A conditional probability is the probability of one event, given that another event has occurred:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

The conditional probability of A given that B has occurred

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

The conditional probability of B given that A has occurred

Where P(A and B) = joint probability of A and B
P(A) = marginal or simple probability of A
P(B) = marginal or simple probability of B

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Conditional Probability

In a sample of used cars, 70% have working air conditioning (AC) and 40% have a working GPS. 20% of the cars have both.

- What is the probability that a car has a GPS, given that it has AC ?

We would like to find: $P(GPS \mid AC)$

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Conditional Probability

	GPS	No GPS	Total
AC	0.2		0.7
No AC			
Total	0.4		

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Conditional Probability

	GPS	No GPS	Total
AC	0.2	0.5	0.7
No AC	0.2		
Total	0.4		

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Conditional Probability

	GPS	No GPS	Total
AC	0.2	0.5	0.7
No AC	0.2		0.3
Total	0.4	0.6	1.0

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Conditional Probability

	GPS	No GPS	Total
AC	0.2	0.5	0.7
No AC	0.2	0.1	0.3
Total	0.4	0.6	1.0

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Conditional Probability

	GPS	No GPS	Total
AC	0.2	0.5	0.7
No AC	0.2	0.1	0.3
Total	0.4	0.6	1.0

$$P(GPS \mid AC) = \frac{P(GPS \text{ and } AC)}{P(AC)} = \frac{0.2}{0.7} = 0.2857$$

- Given AC, we only consider the top row (70% of the cars). Of these, 20% have a GPS. 20% of 70% is about 28.57%.

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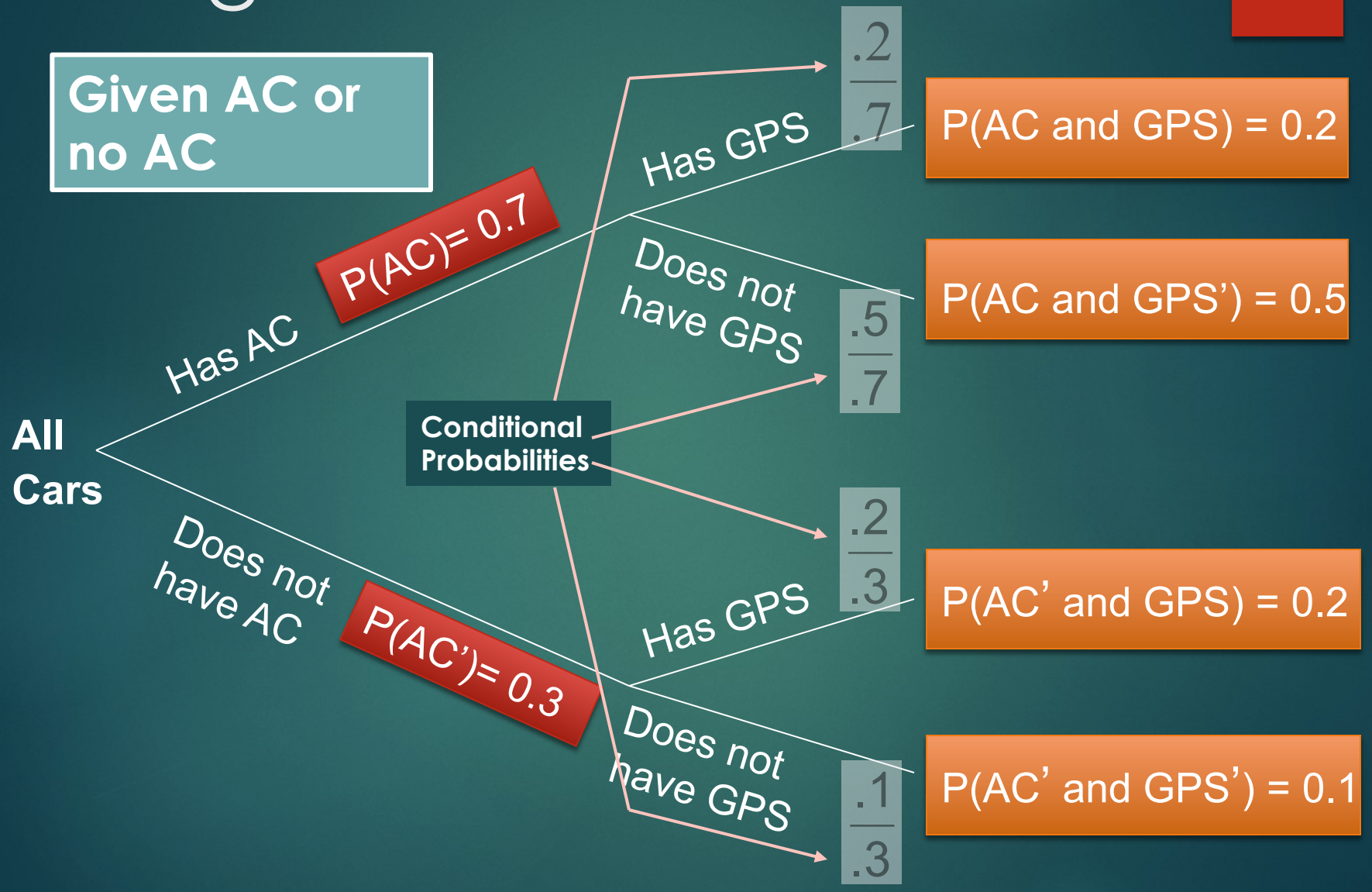
Using Decision Trees

Given AC or no AC



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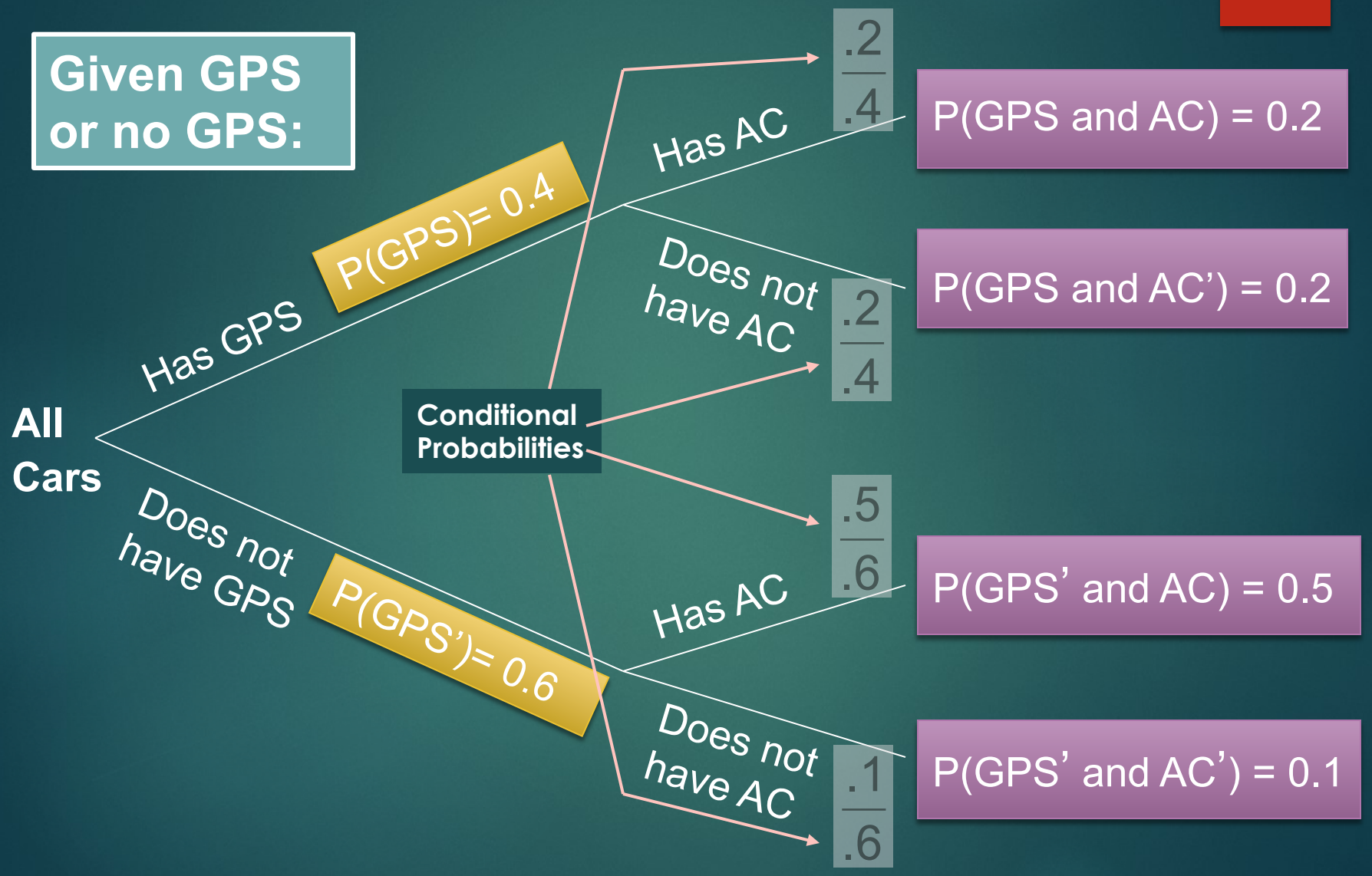
Using Decision Trees

Given GPS or no GPS:



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Independence

- Two events are independent if and only if:

$$P(A \mid B) = P(A)$$

- Events A and B are independent when the probability of one event is not affected by the fact that the other event has occurred

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Multiplication Rules

- Multiplication rule for two events A and B:

$$P(A \text{ and } B) = P(A \mid B)P(B)$$

Note: If A and B are independent, then $P(A \mid B) = P(A)$ and the multiplication rule simplifies to

$$P(A \text{ and } B) = P(A)P(B)$$

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PREFERENCE	GENDER Female	Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

- Given that a randomly selected person is a man, what is the probability that he prefers Pepsi?
- What about a woman?

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PREFERENCE	GENDER: Female	GENDER: Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

Given that a randomly selected person is a **man**, what is the probability that he prefers Pepsi?

P(Pepsi | Male)= P(Pepsi ∩ Male)/P(Male)= (80/500)/(220/500)= 80/220=0.36

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PREFERENCE	GENDER: Female	GENDER: Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

Given that a randomly selected person is a **woman**, what is the probability that he prefers Pepsi?

P(Pepsi | Female)= P(Pepsi ∩ Female)/P(Female)= (95/500)/(280/500)= 95/280=0.34

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PREFERENCE	GENDER: Female	GENDER: Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

Is preference for soda drinks independent of gender?

Reminder: If A and B are independent:

$$P(A \mid B) = P(A)$$

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PREFERENCE	GENDER: Female	GENDER: Male	Total
Coke	120	95	215
Pepsi	95	80	175
Neither/Unsure	65	45	110
Total	280	220	500

Source: Data extracted from "Public Policy Polling" Report 2013, **bit.ly/YKXfzN**.

P(Pepsi)=175/500=0.35
P(Pepsi | Male)=0.36
P(Pepsi | Female)=0.34

Thus, P(Pepsi | GENDER) ≠ P(Pepsi)
In this case, preferences depend upon gender

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- Assignments:

  Homework for Section 4.1 and 4.2

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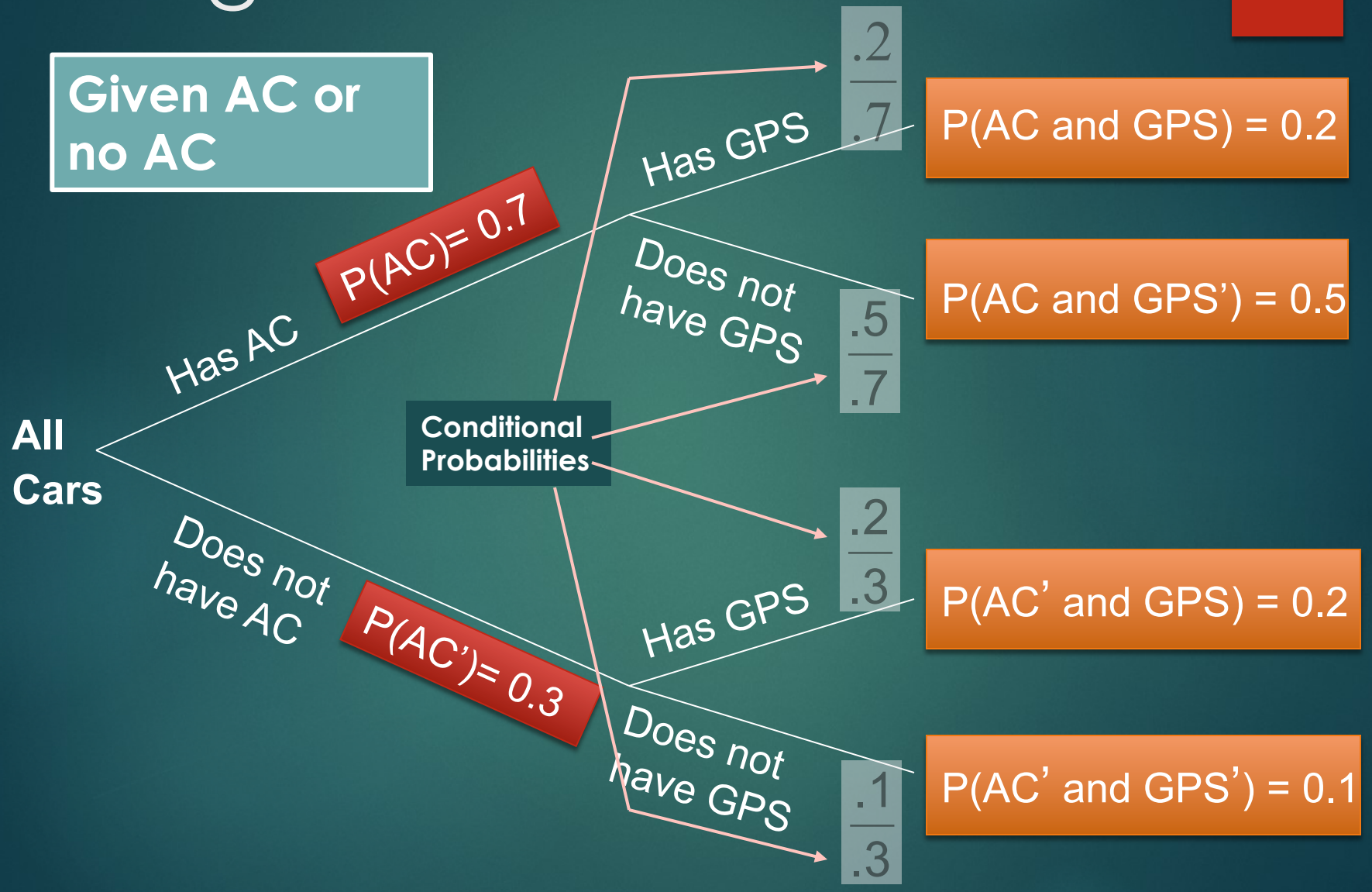
Using Decision Trees

Given AC or no AC



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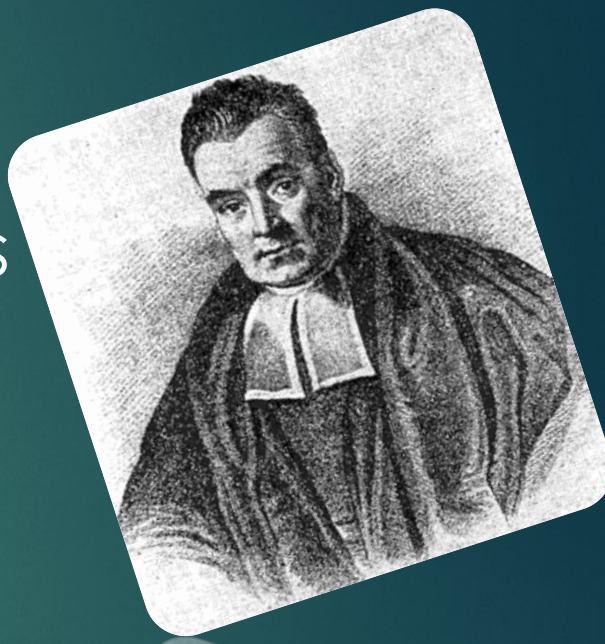
Bayes' Theorem

- Bayes' Theorem is used to revise previously calculated probabilities based on new information.
- It is an extension of conditional probability.



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Bayes' Theorem

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

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Bayes' Theorem

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B).P(B)$$

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Bayes' Theorem

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B).P(B)$$

$$P(B|A) = \frac{P(A|B).P(B)}{P(A)}$$

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Bayes' Theorem

Let us now consider a set of ***k*** events that are **mutually exclusive** and **collectively exhaustive.**

$B_1, B_2, B_3, \ldots, B_k$

A reminder about Marginal Probability:

$$P(A) = P(A \cap B1) + P(A \cap B2) + \cdots + P(A \cap Bk)$$

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Bayes' Theorem

Let us now consider a set of ***k*** events that are **mutually exclusive** and **collectively exhaustive.**

$B_1, B_2, B_3, \ldots, B_k$

A reminder about Marginal Probability:

$$P(A) = P(A \cap B1) + P(A \cap B2) + \cdots + P(A \cap Bk)$$

Also, we know that for, say, B_i:

$$P(A \cap Bi) = P(A|Bi).P(Bi)$$

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Bayes' Theorem

Let us now consider a set of ***k*** events that are **mutually exclusive** and **collectively exhaustive.**

$B_1, B_2, B_3, \ldots, B_k$

A reminder about Marginal Probability:

$$P(A) = P(A \cap B1) + P(A \cap B2) + \cdots + P(A \cap Bk)$$

Also, we know that for, say, B_i:

$$P(A \cap Bi) = P(A|Bi).P(Bi)$$

Thus:

$$P(A) = P(A|B_1).P(B_1) + P(A|B_2).P(B_2) + \ldots + P(A|B_k).P(B_k)$$

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Bayes' Theorem

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)}$$

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A|B_1).P(B_1) + P(A|B_2).P(B_2) + \ldots + P(A|B_k).P(B_k)}$$

Everything is, therefore, stated in the SIMPLE or CONDITIONAL PROBABILITY terms. No more joint probabilities.

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Bayes' Rule Applications

M&R Electronics

- Historically, **40%** of new TV sets have been **successful** in the market, and **60%** have been **unsuccessful**.

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Bayes' Rule Applications

M&R Electronics

- Historically, **40%** of new TV sets have been **successful** in the market, and **60%** have been **unsuccessful**.
- Historically, **80%** of **successful** TVs received **positive reviews**, and **30%** of **unsuccessful** TVs received **positive reviews.**

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Bayes' Rule Applications

M&R Electronics

- Historically, **40%** of new TV sets have been **successful** in the market, and **60%** have been **unsuccessful**.
- Historically, **80%** of **successful** TVs received **positive reviews**, and **30%** of **unsuccessful** TVs received **positive reviews.**
- The new M&R Curved Screen TV has received a **positive review**.

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Bayes' Rule Applications

M&R Electronics

- Historically, **40%** of new TV sets have been **successful** in the market, and **60%** have been **unsuccessful**.
- Historically, **80%** of **successful** TVs received **positive reviews**, and **30%** of **unsuccessful** TVs received **positive reviews**.
- The new M&R Curved Screen TV has received a **positive review**.
- What is the probability that this TV set is **successful** in the market?

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Bayes' Rule Applications
M&R Electronics

- What is the probability that this TV set is **successful** in the market now that it has received a **positive review**?

P(S|Pos)=?

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Bayes' Rule Applications
M&R Electronics

- Historically, **40%** of new TV sets have been **successful** in the market, and **60%** have been **unsuccessful**.

P(S)=0.4

P(S')=0.6

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Bayes' Rule Applications
M&R Electronics

- Historically, **80%** of **successful** TVs received **positive reviews**, and **30%** of **unsuccessful** TVs received **positive reviews**.

P(Pos|S)=0.8

P(Pos|S')=0.3

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Bayes' Rule Applications

M&R Electronics

$P(S)=0.4$

$P(S')=0.6$

$P(Pos|S)=0.8$

$P(Pos|S')=0.3$

$P(S|Pos)=?$

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Bayes' Rule Applications

M&R Electronics

P(S)=0.4

P(S')=0.6

P(Pos | S)=0.8

P(Pos | S')=0.3

P(S | Pos)=?

$$P(S|Pos) = \frac{P(Pos|S)P(S)}{P(Pos)}$$

$$= \frac{P(Pos|S)P(S)}{P(Pos|S)P(S) + P(Pos|S')P(S')}$$

$$= \frac{0.8 \times 0.4}{0.8 \times 0.4 + 0.3 \times 0.6} = \frac{0.32}{0.5} = 0.64$$

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Bayes' Rule Applications

TV Viewing Habits

- A survey suggest that among couples, **60%** of the time the **husbands** are watching TV during prime time.
- When the husbands are watching TV, **40%** of the time **the wife is also watching TV**.
- When the husbands are not watching TV, **30%** of the time **the wife is watching TV**.
  - The probability that if the wife is watching TV, the husband is also watching TV.

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Bayes' Rule Applications

TV Viewing Habits

- A survey suggest that among couples, **60%** of the time the **husbands** are watching TV during prime time.

P(H)=0.6

P(H')=0.4

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Bayes' Rule Applications

TV Viewing Habits

- When the husbands are watching TV, **40%** of the time **the wife is also watching TV**.

$P(W|H)=0.4$

$P(W'|H)=0.6$

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Bayes' Rule Applications

TV Viewing Habits

- When the husbands are not watching TV, **30%** of the time **the wife is watching TV**.

P(W | H')=0.3

P(W' | H')=0.7

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Bayes' Rule Applications

TV Viewing Habits

- The probability that if the wife is watching TV, the husband is also watching TV.

  $P(H \mid W)=?$

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Bayes' Rule Applications

TV Viewing Habits

P(H)=0.6

P(H')=0.4

P(W | H)=0.4

P(W' | H)=0.6

P(W | H')=0.3

P(W' | H')=0.7

P(H | W)=?

$$P(H|W) = \frac{P(W|H)P(H)}{P(W|H)P(H) + P(W|H')P(H')}$$

$$P(H|W) = \frac{0.4 \times 0.6}{0.4 \times 0.6 + 0.3 \times 0.4} = 0.66$$

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Bayes' Rule Applications

TV Viewing Habits

- Find the probability that the wife is watching TV.

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Bayes' Rule Applications

TV Viewing Habits

- Find the probability that the wife is watching TV.

$$P(W)=P(W \cap H)+P(W \cap H')$$
$$=P(W \mid H)P(H)+P(W \mid H')P(H')$$

$$P(W \mid H)=0.4;\ P(H)=0.6;\ P(W \mid H')=0.3;\ P(H')=0.4$$

$$P(W)=0.4x0.6+0.3x0.4=0.24+0.12=0.36$$

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Bayes' Rule

- It is possible to **revise** probabilities after receiving **new information** when we apply the Bayes' Rule.
  - **Prior Probabilities:**

    Subjectively determined probabilities that are available to us initially
  - **New Information:**

    Additional information about the event that are obtained from, say, tests or reports
  - **Posterior Probabilities:**

    The revised probabilities using the Prior Probabilities and New Information

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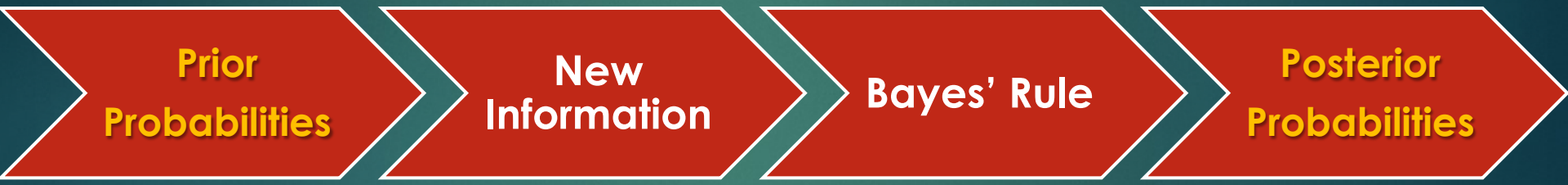
Bayes' Rule



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Bayes' Rule Applications

Medical Diagnosis

- The probability that a person has a certain disease is 3%.
- For those who have the disease, tests give a positive result 9 times out of 10.
- For those who do not have the disease, tests give a positive result 2 times out of 100.
- What is the probability that the disease is present when a test has given positive result?

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Bayes' Rule Applications
Medical Diagnosis

- The probability that a person has a certain disease is 3%. **(PRIOR)**
- For those who have the disease, tests give a positive result 9 times out of 10. **(PRIOR)**
- For those who do not have the disease, tests give a positive result 2 times out of 100. **(PRIOR)**
- What is the probability that the disease is present **(POSTERIOR)** when a test has given positive result **(NEW INFORMATION)**?

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Bayes' Rule Applications

Medical Diagnosis

- The probability that a person has a certain disease is 3%.

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P(D')=0.97

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Bayes' Rule Applications

Medical Diagnosis

- For those who have the disease, tests give a positive result 9 times out of 10.

  P(Pos | D)=0.90

- For those who do not have the disease, tests give a positive result 2 times out of 100.

  P(Pos | D')=0.02

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Bayes' Rule Applications
Medical Diagnosis

- What is the probability that the disease is present when a test has given positive result?

P(D | Pos)=?

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Bayes' Rule Applications

Medical Diagnosis

Prior Probabilities:

- **P(D)=0.03**
- **P(D')=0.97**
- **P(Pos | D)=0.90**
- **P(Pos | D')=0.02**

Posterior Probability:

- **P(D | Pos)=?**

$$P(D|Pos) = \frac{P(D \cap Pos)}{P(Pos)}$$

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Bayes' Rule Applications

Medical Diagnosis

P(D)=0.03
P(D')=0.97
P(Pos | D)=0.90
P(Pos | D')=0.02
P(D | Pos)=?

$$P(D|Pos) = \frac{P(D \cap Pos)}{P(Pos)}$$

$$P(D \cap Pos) = P(Pos|D)P(D)$$

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Bayes' Rule Applications

Medical Diagnosis

P(D)=0.03
P(D')=0.97
P(Pos | D)=0.90
P(Pos | D')=0.02
P(D | Pos)=?

$$P(D|Pos) = \frac{P(D \cap Pos)}{P(Pos)}$$

$$P(D \cap Pos) = P(Pos|D)P(D)$$

$$P(Pos) = P(Pos|D)P(D) + P(Pos|D')P(D')$$

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Bayes' Rule Applications

Medical Diagnosis

$$P(D|Pos) = \frac{P(D \cap Pos)}{P(Pos)}$$

P(D)=0.03
P(D')=0.97
P(Pos | D)=0.90
P(Pos | D')=0.02
P(D | Pos)=?

$$P(D \cap Pos) = P(Pos|D)P(D)$$

$$P(Pos) = P(Pos|D)P(D) + P(Pos|D')P(D')$$

$$P(D|Pos) = \frac{P(Pos|D)P(D)}{P(Pos|D)P(D) + P(Pos|D')P(D')}$$

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Bayes' Rule Applications

Medical Diagnosis

P(D)=0.03
P(D')=0.97
P(Pos | D)=0.90
P(Pos | D')=0.02
P(D | Pos)=?

$$P(D|Pos) = \frac{P(Pos|D)P(D)}{P(Pos|D)P(D) + P(Pos|D')P(D')}$$

$$P(D|Pos) = \frac{0.90 \times 0.03}{0.90 \times 0.03 + 0.02 \times 0.97} = 0.58$$

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Counting Rules

- In many cases, there are a large number of possible outcomes.
- Counting rules can be used in these cases to help compute probabilities.

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Counting Rules

- Rules for counting the number of possible outcomes

- Counting Rule 1:
  - If any one of **k** different mutually exclusive and collectively exhaustive events can occur on each of **n** trials, the number of possible outcomes is equal to $\mathbf{k^n}$

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Counting Rules

- Rules for counting the number of possible outcomes

- Counting Rule 1:
  - If any one of **k** different mutually exclusive and collectively exhaustive events can occur on each of **n** trials, the number of possible outcomes is equal to $\mathbf{k^n}$
  - Example:
    - If you roll a fair die **3** times then there are $\mathbf{6^3 = 216}$ possible outcomes

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Counting Rules

- Counting Rule 2:
  - If there are $\mathbf{k_1}$ events on the first trial, $\mathbf{k_2}$ events on the second trial, … and $\mathbf{k_n}$ events on the $\mathbf{n^{th}}$ trial, the number of possible outcomes is:

    $$(k_1)(k_2) \cdots (k_n)$$

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Counting Rules

- Counting Rule 2:
  - If there are $\mathbf{k_1}$ events on the first trial, $\mathbf{k_2}$ events on the second trial, … and $\mathbf{k_n}$ events on the $\mathbf{n^{th}}$ trial, the number of possible outcomes is:
    $\mathbf{(k_1)(k_2) \cdots (k_n)}$
  - Example:
    - You want to **go to a park**, **eat at a restaurant**, and **see a movie**. There are **3 parks**, **4 restaurants**, and **6 movie** choices. How many different possible combinations are there?
    - Answer: **3**x**4**x**6** = **72** different possibilities

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Counting Rules

- Counting Rule 3:
  - The number of ways that **n** items can be arranged in order is

$$n! = (n)(n-1)\cdots(1)$$

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Counting Rules

- Counting Rule 3:
  - The number of ways that n items can be arranged in order is

    $n! = (n)(n - 1)\cdots(1)$

  - Example:
    - You have seven books to put on a bookshelf. How many different ways can these books be placed on the shelf?
    - Answer: $7! = (7)(6)(5)(4)(3)(2)(1) = 5040$ different possibilities

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Counting Rules

- Counting Rule 4:
  - Permutations: The number of ways of arranging **x** objects selected from **n** objects in order is

$$ {}_{n}P_{x} = \frac{n!}{(n-X)!} $$

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Counting Rules

- Counting Rule 4:
  - Permutations: The number of ways of arranging **x** objects selected from **n** objects **in order** is

$$ {}_{n}P_{x} = \frac{n!}{(n-X)!} $$

  - Example:
    - You have **seven** books and are going to put **three** on a bookshelf. How many different ways can the books be ordered on the bookshelf?
    - Answer: $ {}_{n}P_{x} = \frac{n!}{(n-X)!} = \frac{7!}{(7-3)!} = \frac{5040}{24} = 210 $

Thus, there are 210 different possibilities

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Counting Rules

- Counting Rule 5:
  - Combinations: The number of ways of selecting **x** objects from **n** objects, **irrespective of order**, is

$$ {}_nC_x = \frac{n!}{X!(n-X)!} $$

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Counting Rules

- Counting Rule 5:
  - Combinations: The number of ways of selecting **x** objects from **n** objects, **irrespective of order**, is

$$ {}_nC_x = \frac{n!}{X!(n-X)!} $$

  - Example:
    - You have **seven** books and are going to select **three** of them to read. How many different combinations are there, *ignoring the order* in which they are selected?
    - Answer: $ {}_nC_x = \frac{n!}{X!(n-X)!} = \frac{7!}{3!(7-3)!} = \frac{5040}{(6)(24)} = 35 $

      35 different possibilities

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Counting Rules
Example:

- There are five teams in MLB NL Central: STL, PIT, CHC, MIL, and CIN.
  - How many **orders** of finish are there for these **five teams**?
  - Among these **five teams**, how many **orders** of finish are possible for the first **three teams**?
  - Among these **five teams**, how many **combinations** of **three teams** can we choose?

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Counting Rules
Example:

- There are five teams in MLB NL Central: STL, PIT, CHC, MIL, and CIN.
  - How many **orders** of finish are there for these **five teams**?

5				

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Counting Rules
Example:

- There are five teams in MLB NL Central: STL, PIT, CHC, MIL, and CIN.
  - How many **orders** of finish are there for these **five teams**?

5	**4**			

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Counting Rules
Example:

- There are five teams in MLB NL Central: STL, PIT, CHC, MIL, and CIN.
  - How many **orders** of finish are there for these **five teams**?

5	**4**	**3**	**2**	**1**

  - N= **5** x **4** x **3** x **2** x **1**=**120** orders

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Counting Rules
Example:

- There are five teams in MLB NL Central: STL, PIT, CHC, MIL, and CIN.
  - Among these **five teams**, how many **orders** of finish are possible for the first **three teams**?

5		

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Counting Rules
Example:

- There are five teams in MLB NL Central: STL, PIT, CHC, MIL, and CIN.
  - Among these **five teams**, how many **orders** of finish are possible for the first **three teams**?

5	**4**	**3**

  - $_n\mathbf{P}_x = n!/(n-x)! = 5!/(5-3)! = 5!/2! = 120/2 = 60$

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Counting Rules
Example:

- There are five teams in MLB NL Central: STL, PIT, CHC, MIL, and CIN.
  - Among these **five teams**, how many **combinations** of **three teams** can we choose?

5	**4**	**3**

  - Is **[MIL, CHC, STL]** any different from **[MIL, STL, CHC]**? It is not. The order does not matter.

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Counting Rules
Example:

- There are five teams in MLB NL Central: STL, PIT, CHC, MIL, and CIN.
  - Among these **five teams**, how many **combinations** of **three teams** can we choose?
  - Now that **[MIL, CHC, STL]** is not different from **[MIL, STL, CHC]**, we need to correct for this.
  - 3! possible arrangement of options (**3** **2** **1**) are irrelevant here
- Thus:

$$_{x}C_{n} = n!/[x!(n-x)!] = 5!/(3!(5-3)!) = 120/12 = 10$$

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- Assignments:

  Homework for Section 4.3

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