CS 4513 – Dr. Le Gruenwald
Solutions for Practice Homework Assignment 5

Problem 7.1:

A decomposition $\{R1, R2\}$ is a lossless-join decomposition if
$R1 \cap R2 \rightarrow R1$ or $R1 \cap R2 \rightarrow R2$.
Let $R1 = (A, B, C)$ and $R2 = (A, D, E)$, then $R1 \cap R2 = A$.
Since A is a candidate key (see the answer for Problem 7.6 below), $R1 \cap R2 \rightarrow R1$.

Problem 7.2:

The nontrivial functional dependencies are: $A \rightarrow B$ and $C \rightarrow B$, and a dependency they logically imply: $AC \rightarrow B$.
There are19 trivial functional dependencies of the form $a \rightarrow b$, where $b \subseteq a$.
C does not functionally determine A because the first and third tuples have the same C but different A values. The same tuples also show B does not functionally determine A. Likewise, A does not functionally determine C because the first two tuples have the same A value and different C values. The same tuples also show B does not functionally determine C.

Problem 7.3:

Let $Pk(r)$ denote the primary key attribute of relation r .

• The functional dependencies $Pk(student) \rightarrow Pk(instructor)$ and $Pk(instructor) \rightarrow Pk(student)$ indicate a one-to-one relationship because any two tuples with the same value for student must have the same value for instructor, and any two tuples agreeing on instructor must have the same value for student.

• The functional dependency $Pk(student) \rightarrow Pk(instructor)$ indicates a many-to-one relationship since any student value which is repeated will have the same instructor value, but many student values may have the same instructor value.

Problem 7.6:

Note: this solution shows only the non-trivial members of F+.

Starting with $A \rightarrow BC$, we can conclude: $A \rightarrow B$ and $A \rightarrow C$.

Since $A \rightarrow B$ and $B \rightarrow D$, $A \rightarrow D$	(decomposition, transitive)
Since $A \rightarrow CD$ and $CD \rightarrow E$, $A \rightarrow E$	(union, decomposition, transitive)
Since $A \rightarrow A$, we have	(reflexive)

$A \rightarrow ABCDE$ from the above steps (union)
Since $E \rightarrow A$, $E \rightarrow ABCDE$ (transitive)
Since $CD \rightarrow E$, $CD \rightarrow ABCDE$ (transitive)
Since $B \rightarrow D$ and $BC \rightarrow CD$, $BC \rightarrow ABCDE$ (augmentative, transitive)
Also, $C \rightarrow C$, $D \rightarrow D$, $BD \rightarrow D$, etc.
Therefore, any functional dependency with A, E, BC, or CD on the left hand side of the arrow is in $F+$, no matter which other attributes appear in the FD. Allow * to represent any set of attributes in R, then $F+$ is
$BD \rightarrow B$, $BD \rightarrow D$, $C \rightarrow C$, $D \rightarrow D$, $BD \rightarrow BD$, $B \rightarrow D$,
$B \rightarrow B$, $B \rightarrow BD$, and all FDs of the form $A* \rightarrow \mathrm{a}$, $BC* \rightarrow \mathrm{a}$,
$CD* \rightarrow \mathrm{a}$, $E* \rightarrow \mathrm{a}$ where a is any subset of $\{A, B, C, D, E\}$.
The candidate keys are A, BC, CD, and E.

Problem 7.13:

The dependency $B \rightarrow D$ is not preserved.
$F1$, the restriction of F to (A, B, C), is $A \rightarrow ABC$, $A \rightarrow AB$, $A \rightarrow AC$, $A \rightarrow BC$,
$A \rightarrow B$, $A \rightarrow C$, $A \rightarrow A$, $B \rightarrow B$, $C \rightarrow C$, $AB \rightarrow AC$, $AB \rightarrow ABC$,
$AB \rightarrow BC$, $AB \rightarrow AB$, $AB \rightarrow A$, $AB \rightarrow B$, $AB \rightarrow C$, AC (same as AB), BC (same as AB), ABC (same as AB).
$F2$, the restriction of F to (C, D, E), is $A \rightarrow ADE$, $A \rightarrow AD$, $A \rightarrow AE$, $A \rightarrow DE$, $A \rightarrow A$,
$A \rightarrow D$, $A \rightarrow E$, $D \rightarrow D$, E (same as A), AD, AE, DE, ADE (same as A).
$(F1 \cup F2)+$ is easily seen not to contain $B \rightarrow D$ since the only FD in $F1 \cup F2$ with B as the left side is $B \rightarrow B$, a trivial FD.
Thus $B \rightarrow D$ is not preserved. Also note that $CD \rightarrow ABCDE$ is not preserved.