ISE 4623/5023: Deterministic Systems Models / Systems Optimization

University of Oklahoma
College of Engineering
School of Industrial and Systems Engineering
Fall 2021

Individual Assignment 3 (100 points)

Name: **Student ID:**

Note: Be explicit in the procedures of each point (i.e., show all the steps and calculations made)

Problem 1 - (25 points)

Seeds Inc. is a company that produces and exports bags of corn seeds. For this purpose, the company has a production plant in the state of Oklahoma, in which the company processes corn of two varieties: Hard and Serrated. To make the seed bags, the plant uses three resources: water, electricity, and gas. The following table provides the basic data of the problem:

Resource	**Hard (units of resources per bag of seeds)**	**Serrated (units of raw resources per bag of seeds)**	**Maximum monthly availability (units of Liters, kWh, and cm^3, respectively)**
Water	100.05	60.75	810.50
Electricity	5.50	10.25	655.80
Gas	75.30	24.84	520.75

Profit per seed bag	$275.75	$120.50

In this way, Seeds Inc. produces bags of seeds for both varieties of corn that it processes (i.e., hard corn seeds and serrated corn seeds). Given that Seeds Inc. is so famous and respected, the demand for her seed bags is always very high, so they always sell all the seed bags they produce, and they can produce fractional numbers of seed bags (i.e., 7.22 bag of hard corn seed bags). You want to determine the production plan that retrieves the greatest profit to Seeds Inc. To do this, first you decide to formulate this problem as an LP model. In particular:

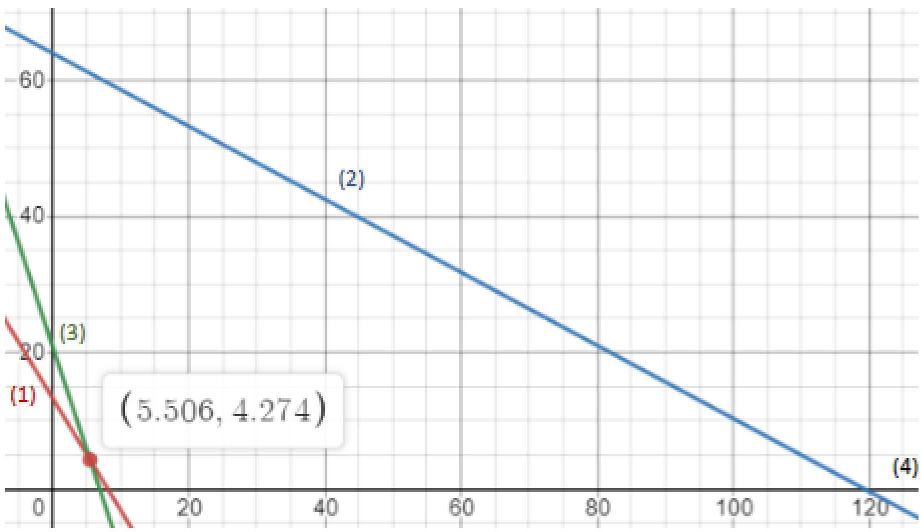
a. (5 points) Write the previous problem in its standard form.

max
$z - 275.75 \cdot x1 - 120.5 \cdot x2 = 0$
$s., t.,$
$100.05x1 + 60.75x2 + s1 = 810.50$
$5.50x1 + 10.25x2 + s2 = 655.80$
$75.30x1 + 24.84x2 + s3 = 520.75$
$x1, x2 \geq 0$

b. (5 points) How many basic solutions does this problem have? How many basic feasible solutions does this problem have? Explain your answer in detail.

Basic solutions: $\binom{n\ variables}{m\ constraints} = \binom{5}{3} = 10$

Feasible solutions by counting the feasible region constraints intersections: 6.



c. (15 points) Solve this problem using the Simplex algorithm. Show each iteration clearly (indicating the values of all the variables, reduced costs, and the objective function associated with each iteration) until optimality is achieved. How do you know you have converged to the optimal solution?

Iteration 0								
basis	z	x1	x2	s1	s2	s3	Solution	
	1	-275.75	-120.5	0	0	0	0	
s1	0	100.05	60.75	1	0	0	810.5	8.10094953
s2	0	5.5	10.25	0	1	0	655.8	119.236364
s3	0	75.3	24.84	0	0	1	520.75	6.91567065
Iteration 1								
basis	z	x1	x2	s1	s2	s3	Solution	
	1	0	-29.5354582	0	0	3.66201859	1906.99618	
s1	0	0	27.7454582	1	0	-1.32868526	118.587151	4.2741104
s2	0	0	8.43565737	0	1	-0.07304117	617.763811	73.2324446
x1	0	1	0.32988048	0	0	0.01328021	6.91567065	20.9641707
Iteration 2								
basis	z	x1	x2	s1	s2	s3	Solution	
	1	0	0	1.06451506	0	2.24761312	2033.23399	Optimal
x2	0	0	1	0.03604194	0	-0.04788839	4.2741104	
s2	0	0	0	-0.30403741	1	0.33092886	581.70888	
x1	0	1	0	-0.01188953	0	0.02907766	5.50572507	

Problem 2 (75 points)

Andes Inc. is an oil company that has a refinery on the Texas coast. The refinery processes crude oil from Saudi Arabia and Venezuela, producing gasoline, diesel, and lubricants. The two crude oils differ in their chemical composition, which is why they produce different amounts of each product. A barrel of crude from Saudi Arabia produces 0.3 barrels of gasoline, 0.4 barrels of diesel, and 0.2 barrels of lubricants. On the other hand, a barrel from Venezuela produces 0.4 barrels of gasoline, 0.2 barrels of diesel, and 0.3 barrels of lubricants. The remaining 10% of the crude is lost in the refining process.

Crudes also differ in price and availability. Andes Inc. can buy up to 9,000 barrels per day from Saudi Arabia at a price of $20 per barrel. You can buy from Venezuela up to 6,000 barrels per day at a price of $ 15 per barrel.

The contracts established by Andes Inc. forces them to produce 2,000 barrels per day of gasoline, 1,500 barrels per day of diesel, and 500 barrels per day of lubricants.

You want to determine the supply plan for the crude oil that results in the least cost for Andes Inc. To do this, first you decide to formulate this problem as an LP model. In particular:

a. (15 points) Suppose that the number of barrels bought from both Saudi Arabia and Venezuela are exactly 9,000 and 6,000, respectively. However, this solution is not optimal. Find the basic solution associated with these values and use it (as an initial feasible solution) in the Simplex Algorithm to find the optimal solution to the problem.

basis	z	x1	x2	s1	s2	s3	s4	s5	Solution	
	1	-20	-15	0	0	0	0	0	0	
s1	0	0.3	0.4	-1	0	0	0	0	2000	
s2	0	0.4	0.2	0	-1	0	0	0	1500	
s3	0	0.2	0.3	0	0	-1	0	0	500	
x1	0	1	0	0	0	0	1	0	9000	
x2	0	0	1	0	0	0	0	1	6000	

basis	z	x1	x2	s1	s2	s3	s4	s5	Solution	
	1	-20	-15	0	0	0	0	0	0	
s1	0	-0.3	-0.4	1	0	0	0	0	-2000	
s2	0	-0.4	-0.2	0	1	0	0	0	-1500	
s3	0	-0.2	-0.3	0	0	1	0	0	-500	
x1	0	1	0	0	0	0	1	0	9000	
x2	0	0	1	0	0	0	0	1	6000	

basis	z	x1	x2	s1	s2	s3	s4	s5	Solution	
	1	-20	0	0	0	0	0	15	90000	
s1	0	-0.3	0	1	0	0	0	0.4	400	
s2	0	-0.4	0	0	1	0	0	0.2	-300	
s3	0	-0.2	0	0	0	1	0	0.3	1300	
x1	0	1	0	0	0	0	1	0	9000	
x2	0	0	1	0	0	0	0	1	6000	

basis	z	x1	x2	s1	s2	s3	s4	s5	Solution	
	1	0	0	0	0	0	20	15	270000	
s1	0	0	0	1	0	0	0.3	0.4	3100	10333.33
s2	0	0	0	0	1	0	0.4	0.2	3300	8250
s3	0	0	0	0	0	1	0.2	0.3	3100	15500
x1	0	1	0	0	0	0	1	0	9000	9000
x2	0	0	1	0	0	0	0	1	6000	#¡DIV/0!

basis	z	x1	x2	s1	s2	s3	s4	s5	Solution	
z	1	0	0	0	-50	0	0	5	105000	
s1	0	0	0	1	-0.75	0	0	0.25	625	2500
s2	0	0	0	0	2.5	0	1	0.5	8250	16500
s3	0	0	0	0	-0.5	1	0	0.2	1450	7250
x1	0	1	0	0	-2.5	0	0	-0.5	750	-1500
x2	0	0	1	0	0	0	0	1	6000	6000

basis	z	x1	x2	s1	s2	s3	s4	s5	Solution	
z	1	0	0	-20	-35	0	0	0	92500	
s1	0	0	0	4	-3	0	0	1	2500	
s2	0	0	0	-2	4	0	1	0	7000	
s3	0	0	0	-0.8	0.1	1	0	0	950	
x1	0	1	0	2	-4	0	0	0	2000	
x2	0	0	1	-4	3	0	0	0	3500	

b. (25 points) Find an initial basic solution using the big M initialization method.

Build the identity column for each artificial varibale

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution	
	1	-20	-15	0	0	0	0	0	-270000	-270000	-270000	0	
	0	0.3	0.4	-1	0	0	0	0	1	0	0	2000	
	0	0.4	0.2	0	-1	0	0	0	0	1	0	1500	
	0	0.2	0.3	0	0	-1	0	0	0	0	1	500	
	0	1	0	0	0	0	1	0	0	0	0	9000	
	0	0	1	0	0	0	0	1	0	0	0	6000	

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution	
	1	80980	107985	-270000	0	0	0	0	0	-270000	-270000	5.4E+08	
	0	0.3	0.4	-1	0	0	0	0	1	0	0	2000	
	0	0.4	0.2	0	-1	0	0	0	0	1	0	1500	
	0	0.2	0.3	0	0	-1	0	0	0	0	1	500	
	0	1	0	0	0	0	1	0	0	0	0	9000	
	0	0	1	0	0	0	0	1	0	0	0	6000	

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution	
z	1	188980	161985	-270000	-270000	0	0	0	0	0	-270000	9.45E+08	
r1	0	0.3	0.4	-1	0	0	0	0	1	0	0	2000	
r2	0	0.4	0.2	0	-1	0	0	0	0	1	0	1500	
r3	0	0.2	0.3	0	0	-1	0	0	0	0	1	500	
s4	0	1	0	0	0	0	1	0	0	0	0	9000	
s5	0	0	1	0	0	0	0	1	0	0	0	6000	

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution	
z	1	242980	242985	-270000	-270000	-270000	0	0	0	0	0	1.08E+09	
r1	0	0.3	0.4	-1	0	0	0	0	1	0	0	2000	5000
r2	0	0.4	0.2	0	-1	0	0	0	0	1	0	1500	7500
r3	0	0.2	0.3	0	0	-1	0	0	0	0	1	500	1666.667
s4	0	1	0	0	0	0	1	0	0	0	0	9000	#¡DIV/0!
s5	0	0	1	0	0	0	0	1	0	0	0	6000	6000

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution	
z	1	80990	0	-270000	-270000	539950	0	0	0	0	-809950	6.75E+08	
r1	0	0.033333	0	-1	0	1.333333	0	0	1	0	-1.33333	1333.333	1000
r2	0	0.266667	0	0	-1	0.666667	0	0	0	1	-0.66667	1166.667	1750
x2	0	0.666667	1	0	0	-3.33333	0	0	0	0	3.333333	1666.667	-500
s4	0	1	0	0	0	0	1	0	0	0	0	9000	#¡DIV/0!
s5	0	-0.66667	0	0	0	3.333333	0	1	0	0	-3.33333	4333.333	1300

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution	
z	1	67491.25	0	134962.5	-270000	0	0	0	-404963	0	-270000	1.35E+08	
s3	0	0.025	0	-0.75	0	1	0	0	0.75	0	-1	1000	-1333.33
r2	0	0.25	0	0.5	-1	0	0	0	-0.5	1	0	500	1000
x2	0	0.75	1	-2.5	0	0	0	0	2.5	0	0	5000	-2000
s4	0	1	0	0	0	0	1	0	0	0	0	9000	#¡DIV/0!
s5	0	-0.75	0	2.5	0	0	0	1	-2.5	0	0	1000	400

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution	
z	1	107980	0	0	-270000	0	0	-53985	-270000	0	-270000	81090000	
s3	0	-0.2	0	0	0	1	0	0.3	0	0	-1	1300	-6500
r2	0	0.4	0	0	-1	0	0	-0.2	0	1	0	300	750
x2	0	0	1	0	0	0	0	1	0	0	0	6000	#¡DIV/0!
s4	0	1	0	0	0	0	1	0	0	0	0	9000	9000
s1	0	-0.3	0	1	0	0	0	0.4	-1	0	0	400	-1333.33

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution	
z	1	0	0	0	-50	0	0	5	-270000	-269950	-270000	105000	
s3	0	0	0	0	-0.5	1	0	0.2	0	0.5	-1	1450	7250
x1	0	1	0	0	-2.5	0	0	-0.5	0	2.5	0	750	-1500
x2	0	0	1	0	0	0	0	1	0	0	0	6000	6000
s4	0	0	0	0	2.5	0	1	0.5	0	-2.5	0	8250	16500
s1	0	0	0	1	-0.75	0	0	0.25	-1	0.75	0	625	2500

c. (25 points) Find an initial basic solution using the two-phase initialization method.

First Phase

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution
	1	0.9	0.9	-1	-1	-1	0	0	0	0	0	
r1	0	0.3	0.4	-1	0	0	0	0	1	0	0	2000
r2	0	0.4	0.2	0	-1	0	0	0	0	1	0	1500
r3	0	0.2	0.3	0	0	-1	0	0	0	0	1	500
s4	0	1	0	0	0	0	1	0	0	0	0	9000
s5	0	0	1	0	0	0	0	1	0	0	0	6000

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution
	1	0.3	0	-1	-1	2	0	0	0	0	-3	
r1	0	0.033333	0	-1	0	1.333333	0	0	1	0	-1.33333	1333.333
r2	0	0.266667	0	0	-1	0.666667	0	0	0	1	-0.66667	1166.667
x2	0	0.666667	1	0	0	-3.33333	0	0	0	0	3.333333	1666.667
s4	0	1	0	0	0	0	1	0	0	0	0	9000
s5	0	-0.66667	0	0	0	3.333333	0	1	0	0	-3.33333	4333.333

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution
	1	0.25	0	0.5	-1	0	0	0	-1.5	0	-1	
s3	0	0.025	0	-0.75	0	1	0	0	0.75	0	-1	1000
r2	0	0.25	0	0.5	-1	0	0	0	-0.5	1	0	500
x2	0	0.75	1	-2.5	0	0	0	0	2.5	0	0	5000
s4	0	1	0	0	0	0	1	0	0	0	0	9000
s5	0	-0.75	0	2.5	0	0	0	1	-2.5	0	0	1000

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution
	1	0.4	0	0	-1	0	0	-0.2	-1	0	-1	
s3	0	-0.2	0	0	0	1	0	0.3	0	0	-1	1300
r2	0	0.4	0	0	-1	0	0	-0.2	0	1	0	300
x2	0	0	1	0	0	0	0	1	0	0	0	6000
s4	0	1	0	0	0	0	1	0	0	0	0	9000
s1	0	-0.3	0	1	0	0	0	0.4	-1	0	0	400

basis	z	x1	x2	s1	s2	s3	s4	s5	r1	r2	r3	Solution
	1	0	0	0	0	0	0	0	-1	-1	-1	
s3	0	0	0	0	-0.5	1	0	0.2	0	0.5	-1	1450
x1	0	1	0	0	-2.5	0	0	-0.5	0	2.5	0	750
x2	0	0	1	0	0	0	0	1	0	0	0	6000
s4	0	0	0	0	2.5	0	1	0.5	0	-2.5	0	8250
s1	0	0	0	1	-0.75	0	0	0.25	-1	0.75	0	625

d. (10 points) Starting from one of the initial basic solutions found in either b) or c), solve this problem using the Simplex algorithm. Indicate the values of all the variables and the objective function associated with the optimal solution.

Considering the calculated initial solution using the first phase of two-phase method.

Second Phase

basis	z	x1	x2	s1	s2	s3	s4	s5	Solution
	1	0	0	0	50	0	0	-5	
s3	0	0	0	0	-0.5	1	0	0.2	1450
x1	0	1	0	0	-2.5	0	0	-0.5	750
x2	0	0	1	0	0	0	0	1	6000
s4	0	0	0	0	2.5	0	1	0.5	8250
s1	0	0	0	1	-0.75	0	0	0.25	625

basis	z	x1	x2	s1	s2	s3	s4	s5	Solution
	1	0	0	20	35	0	0	0	
s3	0	0	0	-0.8	0.1	1	0	0	950
x1	0	1	0	2	-4	0	0	0	2000
x2	0	0	1	-4	3	0	0	0	3500
s4	0	0	0	-2	4	0	1	0	7000
s5	0	0	0	4	-3	0	0	1	2500