ISE 4623/5023: Deterministic Systems Models / Systems Optimization

University of Oklahoma
School of Industrial and Systems Engineering
Fall 2021

Individual assignment 6 (100 points + 30 points for graduate students/extra credit)

NOTE: For all problems, you need to upload a PDF file of the solution, along with support files of any software used (Excel, Gurobi/Python, etc).

PLEDGE:

"On my honor, I affirm that I have neither given nor received inappropriate aid in the completion of this exercise."

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Problem 1 (40 points): Transportation problem

Tiara company has two plants in different locations around the country where they produce drilling bits. Their sales partner has three distribution centers where they ship these drilling bits to their various customers. The plants can produce a given number of bits per week and the expected demand for each distribution center is also known. There is a shipping cost from each plant to each warehouse. Which plant should produce and ship how many bits to which distribution centers to meet the demand at each location with minimum cost? The following Table shows the transportation cost from each plant to each distribution center as well as the total demand of each distribution center and the total supply of each plant.

Table 1. Transportation costs, per pound, from production plants to warehouses

From/To	Center 1	Center 2	Center 3	Plants' supply
Plant 1	$25	$85	$25	350
Plant 2	$50	$35	$95	300
Centers' demand	100	350	200	

a. **(15 points)** Formulate the objective function and the necessary constraints using the given information.

b. **(15 points)** What is the transportation strategy (number of each product shipped from each plant to each center) that minimizes the total cost? To do that solve the problem using Excel or Gurobi. Show your results clearly and discuss.

c. **(10 points)** Reformulate the problem for the case that each plant loses 30% of their production (due to and unforeseen disruption). Solve the problem using Excel or Gurobi and discuss your results in detail.

Problem 2 (60 points mandatory for everyone + 30 points for graduate students/extra credit): Minimum cost flow problem

- **Part 1 (60 points): Single commodity supply chain transportation**

Transportation plays a significant role in supply chains and their efficiency. Reducing the transportation costs between the manufacturers, warehouses, and retailers helps to provide more affordable products prices for customers. A major manufacturing company, MC Inc., produces its star product, Product A, to be sold at five retailers across the state. First, the products are manufactured in one of three plants, then distributed to three warehouses for storage, and finally are distributed to each of the retailers. Each retailer sends a separate order to the manufacture's head office, which is then dispatched from the appropriate warehouse to the retailer. The company has collected data regarding weekly orders and production and would like to find a way to minimize the costs of the entire operation.

The objective in this supply chain transportation problem is to minimize the total cost of transportation and unmet demand. The following Tables provide information related to the transportation costs from plants to warehouses, and from warehouses to retailers (Tables 2 and 3).

Table 2. Transportation costs, per pound, from production plants to warehouses

	Unit shipping cost (per pound)		
From/To	Warehouse 1	Warehouse 2	Warehouse 3
Plant 1	$25	$85	$25
Plant 2	$50	$35	$95
Plant 3	$50	$40	$55

Table 3. Transportation costs, per pound, from warehouses to retailers

	Unit shipping cost (per pound)				
From/To	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5
Warehouse A	$75	$50	$60	$75	$30
Warehouse B	$85	$15	$85	$85	$90
Warehouse C	$90	$85	$35	$35	$95

One of the most important obstacles for the company is the work force performance. The number of workers in Plant 1, Plant 2, and Plan 3 are 90, 120, 80, respectively. Also, assume that each employee works 48 hours per week. Furthermore, Table 4 illustrates the number of units that can be produced by a worker every hour.

Table 4. Average number of units produced per person per hour

Production facility	Product A (units/hour-person)
Plant 1	0.5
Plant 2	0.5
Plant 3	0.2

The following table shows the retailers weekly demand from each type of products.

Demand of Retailer

Table 5. Weekly orders (in number of units) made by the retailers on the first week of October

Product request	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5
Product A	175	120	140	100	160

Tip: generally, in a network there are three different types of nodes: (i) supply nodes, (ii) transshipment nodes, and (iii) demand nodes. Note that, commodities arrive and depart the transshipment nodes, however, in demand nodes commodities are going to be consumed by the nodes. For these nodes, limited capacities may cause unmet demands, which should be accounted for (of course, given that you want to satisfy the demands as much as possible, you would need to penalize any unsupplied demand).

There is also a maximum capacity for sending material from plants to warehouses and from warehouses to retailers. Meaning that only certain weight of products can be sent through each link. Tables 6 and 7 show the necessary information regarding this limitation. Keep in mind that one unit of Product A weighs one pound.

Upper Bound Flow

Table 6. Maximum weekly weight (of products) that can be sent from each plant to each warehouse

	Maximum weekly amount of product shipped (in pounds)		
From/To	Warehouse 1	Warehouse 2	Warehouse 3
Plant 1	120	150	170
Plant 2	150	160	180
Plant 3	150	170	180

Table 7. Maximum weekly weight (of products) that can be sent from each warehouse to each retailer

	Unit shipping cost (per pound)				
From/To	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5
Warehouse A	160	190	110	180	150
Warehouse B	170	190	150	140	120
Warehouse C	140	160	180	120	100

a. **(30 points)** Using the provided information, formulate an LP model to minimize the transportation costs (and unmet demands) for the described supply chain. Clearly indicate sets, parameters, variables, objective function, and constraints used.

b. **(30 points)** What is the transportation strategy (number of units of Product A shipped from each plant to each warehouse, and from each warehouse to each retailer) that minimizes the total cost? Also, report the total unmet demand (if any) for each retailer. To do that, solve the LP model using Excel or Gurobi. Show your results clearly and discuss.

- **Part 2 (20 points): (Bonus for undergraduates/mandatory for graduates) Multi commodity supply chain transportation**

To expand their market share, the company decided to fabricate a second product, Product B. To fabricate both Product A and Product B, all plants were adapted accordingly. The workers' performance related to the Product B is illustrated in Table 8.

Table 8. Average number of units produced per person per hour

Production facility	Product B (units/hour-person)
Plant 1	0.1
Plant 2	0.1
Plant 3	0.4

Weekly orders for Product B required by the retailers are provided in Table 9.

Table 9. Weekly orders (in number of units) made by the retailers on the first week of October

Product request	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5
Product B	100	150	110	300	230

Keep in mind that one unit of Product B weighs 3 pounds.

c. **(10 points)** Considering this multicommodity supply chain (which includes both Products A and B), reformulate the problem to minimize the transportation and unmet demand cost (if any). Clearly indicate sets, parameters, variables, objective function, and constraints used.

d. **(10 points)** Solve the LP model using Excel or Gurobi to find the optimal transportation strategy (number of each product shipped from each plant to each warehouse, and from each warehouse to each retailer) with respect to total cost minimization. Also, report the total unmet demand (if any) for each retailer. Show your results clearly and discuss.

- **Part 3 (10 points) (Bonus for undergraduates/ mandatory for graduates) Capacitated nodes**

There is also limitation for handling material by warehouses, where there is a maximum weight of products can be handled by each warehouse, as shown in Table 10.

Table 10: Maximum total weight that can be handled in each warehouses (in pounds)

Warehouse 1	Warehouse 2	Warehouse 3
660	700	1000

e. **(5 points)** Formulate the necessary constraint(s) that would have to be added to the LP model in Problem 2c to address the material handling capacity of each warehouse.

f. **(5 points)** Solve this new LP model using Excel or Gurobi to find the optimal transportation strategy (number of each product shipped from each plant to each warehouse, and from each warehouse to each retailer) with respect to total cost minimization. Also, report the total unmet demand (if any) for each retailer. Show your results clearly and discuss.