ISE 4623/5023: Deterministic Systems Models / Systems Optimization

University of Oklahoma

School of Industrial and Systems Engineering

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Individual Assignment 1 - Linear Algebra (100 points) Solution/Answers

Problem 1 (30 points)

Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 2 \end{bmatrix}$

a) $AB = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1\cdot1+2\cdot2+2\cdot4 & 1\cdot1+2\cdot3+2\cdot2 \\ 2\cdot1+1\cdot2+3\cdot4 & 2\cdot1+1\cdot3+3\cdot2 \end{bmatrix} = \begin{bmatrix} 1+4+7 & 1+6+4 \\ 2+2+12 & 2+3+6 \end{bmatrix} = \begin{bmatrix} 13 & 11 \\ 16 & 11 \end{bmatrix}$

b) $BA = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1\cdot1+1\cdot2 & 1\cdot2+1\cdot1 & 1\cdot2+1\cdot3 \\ 2\cdot1+3\cdot2 & 2\cdot2+3\cdot1 & 2\cdot2+3\cdot3 \\ 4\cdot1+2\cdot2 & 4\cdot2+2\cdot1 & 4\cdot2+2\cdot3 \end{bmatrix} = \begin{bmatrix} 1+2 & 2+1 & 2+3 \\ 2+6 & 4+3 & 4+9 \\ 4+4 & 8+2 & 8+6 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 8 & 7 & 13 \\ 8 & 10 & 14 \end{bmatrix}$

c) $-B = -1\cdot\begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -3 \\ -4 & -2 \end{bmatrix}$

d) $A+2B =$ Not possible: Different dimensions

e) $A^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 2 & 3 \end{bmatrix}$

f) $B^T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 2 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 2 \end{bmatrix}$

Problem 2 (30 points)

a) $\begin{matrix} 3x_1 + x_2 = 2 \\ -6x_1 - 2x_2 = -4 \end{matrix}$ → Infinite solutions since $R_2 = -2\cdot R_1$

b) $\begin{matrix} 2x_1 + x_2 = 4 \\ x_1 + 2x_2 = 5 \end{matrix} \rightarrow \left[\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & 2 & 5 \end{array}\right] \xrightarrow{R_1=R_1-2\cdot R_2} \left[\begin{array}{cc|c} 0 & -3 & -6 \\ 1 & 2 & 5 \end{array}\right] \xrightarrow{R_1=\frac{R_1}{-3}} \left[\begin{array}{cc|c} 0 & 1 & 2 \\ 1 & 2 & 5 \end{array}\right]$

$\xrightarrow{R_2=R_2-2\cdot R_1} \left[\begin{array}{cc|c} 0 & 1 & 2 \\ 1 & 0 & 1 \end{array}\right] \xrightarrow{R_1\leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$

c) $\begin{matrix} 2x_1 + x_2 = 4 \\ -4x_1 - 2x_2 = -5 \end{matrix}$ → No solution since $\begin{matrix} -2\cdot R_1 = -4x_1 - 2x_2 = -8 \\ \neq \\ R_2 = -4x_1 - 2x_2 = -5 \end{matrix}$

Problem 3 (40 points)

Computing an inverse matrix

Consider a 2x2 matrix:

$$\underset{2\times 2}{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The 2×2 inverse matrix is then:

$$\underset{2\times 2}{A^{-1}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Where $D = ad - bc$. D is called the determinant of the matrix.

The 3×3 matrix can be defined as:

$$\underset{3\times 3}{B} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$$

Then the inverse matrix is:

$$\underset{3\times 3}{B^{-1}} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}^{-1} = \frac{1}{det(B)}\begin{bmatrix} (ek-fh) & -(bk-ch) & (bf-ce) \\ -(dk-fg) & (ak-cg) & -(af-cd) \\ (dh-eg) & -(ah-bg) & (ae-bd) \end{bmatrix}$$

Where $det(B)$ is equal to:

$$det(B) = a(ek-fh) - b(dk-fg) + c(dh-eg)$$

Figure 1: https://rpubs.com/aaronsc32/inverse-matrices

a) $C^{-1} = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}^{-1} = \dfrac{1}{(1\cdot 2 - 4\cdot 2)} \cdot \begin{bmatrix} 2 & -2 \\ -4 & 1 \end{bmatrix} = -\dfrac{1}{6} \cdot \begin{bmatrix} 2 & -2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{3} & \dfrac{1}{3} \\ \dfrac{2}{3} & -\dfrac{1}{6} \end{bmatrix}$

b) $D^{-1} = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}^{-1} \rightarrow$ Not possible since R_2 is linearly dependent of R_1: $R_2 = 2 \cdot R_1$

c) $E^{-1} = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 2 & 6 \\ 5 & 4 & 10 \end{bmatrix}^{-1} \rightarrow$ Not possible since R_3 is linearly dependent of R_1 and R_2: $R_3 = R_1 + R_2$

d) $F^{-1} = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 2 & 6 \\ 5 & 4 & 8 \end{bmatrix}^{-1} = \dfrac{1}{1\cdot(2\cdot 8 - 4\cdot 6) - 2\cdot(4\cdot 8 - 6\cdot 5) + 4\cdot(4\cdot 4 - 2\cdot 5)} \cdot \begin{bmatrix} (2\cdot 8 - 4\cdot 6) & -(2\cdot 8 - 4\cdot 4) & (2\cdot 6 - 2\cdot 4) \\ -(4\cdot 8 - 6\cdot 5) & (1\cdot 8 - 5\cdot 4) & -(1\cdot 6 - 4\cdot 4) \\ (4\cdot 4 - 2\cdot 5) & -(1\cdot 4 - 5\cdot 2) & (1\cdot 2 - 4\cdot 2) \end{bmatrix}$

$$= \begin{bmatrix} -\frac{2}{3} & 0 & \frac{1}{3} \\ -\frac{1}{6} & -1 & \frac{5}{6} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$