Algorithm and examples

Method 2. TwoPhase method

Solve the Linear programming problem using Two-Phase method calculator

Type your linear programming problem

```
.3x1 + .4x2 >= 2000
.4x1 + .2x2 >= 1500
.2x1 + .3x2 >= 500
x1 <= 9000
x2 <= 6000
and x1,x2 >= 0
```

OR

Total Variables : 2 Total Constraints : 5 Generate

Min Z = 20 x1 + 15 x2

Subject to constraints

.3	x1 + .4	x2	>=	2000
.4	x1 + .2	x2	>=	1500
.2	x1 + .3	x2	>=	500
1	x1 + 0	x2	<=	9000
0	x1 + 1	x2	<=	6000

and x1,x2 >= 0

Mode : Decimal

☐ Solve after converting Min function to Max function

Calculate : Zj-Cj

☐ Alternate Solution (if exists) ☑ Artificial Column Remove ☑ Subtraction Steps

Find Random New

Solution Help

Solution will be displayed step by step (In 6 parts)

Solution

Find solution using Two-Phase method
MIN Z = 20x1 + 15x2
subject to
.3x1 + .4x2 >= 2000
.4x1 + .2x2 >= 1500
.2x1 + .3x2 >= 500
x1 <= 9000
x2 <= 6000
and x1,x2 >= 0

Solution:
Problem is

Min $Z = 20x_1 + 15x_2$

subject to

$$0.3x_1 + 0.4x_2 \geq 2000$$
$$0.4x_1 + 0.2x_2 \geq 1500$$
$$0.2x_1 + 0.3x_2 \geq 500$$
$$x_1 \leq 9000$$

$x_2 \le 6000$

and $x_1, x_2 \ge 0$;

-->Phase-1<--

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint-1 is of type ' $\ge$ ' we should subtract surplus variable S_1 and add artificial variable A_1

2. As the constraint-2 is of type ' $\ge$ ' we should subtract surplus variable S_2 and add artificial variable A_2

3. As the constraint-3 is of type ' $\ge$ ' we should subtract surplus variable S_3 and add artificial variable A_3

4. As the constraint-4 is of type ' $\le$ ' we should add slack variable S_4

5. As the constraint-5 is of type ' $\le$ ' we should add slack variable S_5

After introducing slack,surplus,artificial variables

Min $Z = A_1 + A_2 + A_3$

subject to

$$
\begin{aligned}
0.3x_1 + 0.4x_2 - S_1 + A_1 &= 2000 \\
0.4x_1 + 0.2x_2 - S_2 + A_2 &= 1500 \\
0.2x_1 + 0.3x_2 - S_3 + A_3 &= 500 \\
x_1 + S_4 &= 9000 \\
x_2 + S_5 &= 6000
\end{aligned}
$$

and $x_1, x_2, S_1, S_2, S_3, S_4, S_5, A_1, A_2, A_3 \ge 0$

Iteration-1		C_j	0	0	0	0	0	0	0	1	1	1	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	**MinRatio** $\frac{X_B}{x_2}$
A_1	1	2000	0.3	0.4	-1	0	0	0	0	1	0	0	$\frac{2000}{0.4} = 5000$
A_2	1	1500	0.4	0.2	0	-1	0	0	0	0	1	0	$\frac{1500}{0.2} = 7500$
A_3	1	500	0.2	(0.3)	0	0	-1	0	0	0	0	1	$\frac{500}{0.3} = 1666.6667 \rightarrow$
S_4	0	9000	1	0	0	0	0	1	0	0	0	0	---
S_5	0	6000	0	1	0	0	0	0	1	0	0	0	$\frac{6000}{1} = 6000$
$Z = 4000$		**Z_j**	**0.9**	**0.9**	**-1**	**-1**	**-1**	**0**	**0**	**1**	**1**	**1**	
		$Z_j - C_j$	0.9	0.9 $\uparrow$	-1	-1	-1	0	0	0	0	0	

Positive maximum $Z_j - C_j$ is 0.9 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 1666.6667 and its row index is 3. So, the leaving basis variable is A_3.

$\therefore$ The pivot element is 0.3.

Entering $= x_2$, Departing $= A_3$, Key Element $= 0.3$

+ R_3(new) $= R_3$(old) $\div 0.3$

+ R_1(new) $= R_1$(old) - $0.4 R_3$(new)

+ R_2(new) $= R_2$(old) - $0.2 R_3$(new)

+ R_4(new) = R_4(old)

+ R_5(new) = R_5(old) - R_3(new)

Iteration-2		C_j	0	0	0	0	0	0	0	1	1	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**A_1**	**A_2**	**MinRatio** $\frac{X_B}{S_3}$
A_1	1	1333.3333	0.0333	0	-1	0	**(1.3333)**	0	0	1	0	$\frac{1333.3333}{1.3333} = 1000 \rightarrow$
A_2	1	1166.6667	0.2667	0	0	-1	0.6667	0	0	0	1	$\frac{1166.6667}{0.6667} = 1750$
x_2	0	1666.6667	0.6667	1	0	0	-3.3333	0	0	0	0	---
S_4	0	9000	1	0	0	0	0	1	0	0	0	---
S_5	0	4333.3333	-0.6667	0	0	0	3.3333	0	1	0	0	$\frac{4333.3333}{3.3333} = 1300$
Z = 2500		**Z_j**	**0.3**	**0**	**-1**	**-1**	**2**	**0**	**0**	**1**	**1**	
		$Z_j - C_j$	0.3	0	-1	-1	2 ↑	0	0	0	0	

Positive maximum $Z_j - C_j$ is 2 and its column index is 5. So, the entering variable is S_3.

Minimum ratio is 1000 and its row index is 1. So, the leaving basis variable is A_1.

∴ The pivot element is 1.3333.

Entering $= S_3$, Departing $= A_1$, Key Element $= 1.3333$

+ R_1(new) = R_1(old) ÷ 1.3333

+ R_2(new) = R_2(old) - $0.6667 R_1$(new)

+ R_3(new) = R_3(old) + $3.3333 R_1$(new)

+ R_4(new) = R_4(old)

+ R_5(new) = R_5(old) - $3.3333 R_1$(new)

Iteration-3		C_j	0	0	0	0	0	0	0	1	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**A_2**	**MinRatio** $\frac{X_B}{S_1}$
S_3	0	1000	0.025	0	-0.75	0	1	0	0	0	---
A_2	1	500	0.25	0	0.5	-1	0	0	0	1	$\frac{500}{0.5} = 1000$
x_2	0	5000	0.75	1	-2.5	0	0	0	0	0	---
S_4	0	9000	1	0	0	0	0	1	0	0	---

S_5	0	1000	-0.75	0	**(2.5)**	0	0	0	1	0	$\frac{1000}{2.5} = 400 \rightarrow$
$Z = 500$		Z_j	**0.25**	**0**	**0.5**	**-1**	**0**	**0**	**0**	**1**	
		$Z_j - C_j$	0.25	0	0.5 ↑	-1	0	0	0	0	

Positive maximum $Z_j - C_j$ is 0.5 and its column index is 3. So, the entering variable is S_1.

Minimum ratio is 400 and its row index is 5. So, the leaving basis variable is S_5.

∴ The pivot element is 2.5.

Entering $= S_1$, Departing $= S_5$, Key Element $= 2.5$

+ R_5(new) $= R_5$(old) $\div 2.5$

+ R_1(new) $= R_1$(old) + $0.75 R_5$(new)

+ R_2(new) $= R_2$(old) - $0.5 R_5$(new)

+ R_3(new) $= R_3$(old) + $2.5 R_5$(new)

+ R_4(new) $= R_4$(old)

Iteration-4		C_j	0	0	0	0	0	0	0	1	
B	**C_B**	**X_B**	**x_1**	**x_2**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**A_2**	**MinRatio** $\frac{X_B}{x_1}$
S_3	0	1300	-0.2	0	0	0	1	0	0.3	0	---
A_2	1	300	**(0.4)**	0	0	-1	0	0	-0.2	1	$\frac{300}{0.4} = 750 \rightarrow$
x_2	0	6000	0	1	0	0	0	0	1	0	---
S_4	0	9000	1	0	0	0	0	1	0	0	$\frac{9000}{1} = 9000$
S_1	0	400	-0.3	0	1	0	0	0	0.4	0	---
$Z = 300$		Z_j	**0.4**	**0**	**0**	**-1**	**0**	**0**	**-0.2**	**1**	
		$Z_j - C_j$	0.4 ↑	0	0	-1	0	0	-0.2	0	

Positive maximum $Z_j - C_j$ is 0.4 and its column index is 1. So, the entering variable is x_1.

Minimum ratio is 750 and its row index is 2. So, the leaving basis variable is A_2.

∴ The pivot element is 0.4.

Entering $= x_1$, Departing $= A_2$, Key Element $= 0.4$

+ R_2(new) $= R_2$(old) $\div 0.4$

+ R_1(new) $= R_1$(old) + $0.2 R_2$(new)

✚ R_3(new) = R_3(old)

✚ R_4(new) = R_4(old) - R_2(new)

✚ R_5(new) = R_5(old) + $0.3R_2$(new)

Iteration-5		C_j	0	0	0	0	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	S_5	**MinRatio**
S_3	0	1450	0	0	0	-0.5	1	0	0.2	
x_1	0	750	1	0	0	-2.5	0	0	-0.5	
x_2	0	6000	0	1	0	0	0	0	1	
S_4	0	8250	0	0	0	2.5	0	1	0.5	
S_1	0	625	0	0	1	-0.75	0	0	0.25	
Z = 0		Z_j	**0**	**0**	**0**	**0**	**0**	**0**	**0**	
		$Z_j - C_j$	0	0	0	0	0	0	0	

Since all $Z_j - C_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 750, x_2 = 6000$

Min $Z = 0$

-->Phase-2<--

we eliminate the artificial variables and change the objective function for the original,
Min $Z = 20x_1 + 15x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4 + 0S_5$

Iteration-1		C_j	20	15	0	0	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	S_5	**MinRatio** $\frac{X_B}{S_5}$
S_3	0	1450	0	0	0	-0.5	1	0	0.2	$\frac{1450}{0.2} = 7250$
x_1	20	750	1	0	0	-2.5	0	0	-0.5	---
x_2	15	6000	0	1	0	0	0	0	1	$\frac{6000}{1} = 6000$
S_4	0	8250	0	0	0	2.5	0	1	0.5	$\frac{8250}{0.5} = 16500$
S_1	0	625	0	0	1	-0.75	0	0	**(0.25)**	$\frac{625}{0.25} = 2500 \rightarrow$
Z = 105000		Z_j	**20**	**15**	**0**	**-50**	**0**	**0**	**5**	
		$Z_j - C_j$	0	0	0	-50	0	0	$5 \uparrow$	

Positive maximum $Z_j - C_j$ is 5 and its column index is 7. So, the entering variable is S_5.

Minimum ratio is 2500 and its row index is 5. So, the leaving basis variable is S_1.

∴ The pivot element is 0.25.

Entering $= S_5$, Departing $= S_1$, Key Element $= 0.25$

+ R_5(new) = R_5(old) $\div 0.25$

+ R_1(new) = R_1(old) - $0.2R_5$(new)

+ R_2(new) = R_2(old) + $0.5R_5$(new)

+ R_3(new) = R_3(old) - R_5(new)

+ R_4(new) = R_4(old) - $0.5R_5$(new)

Iteration-2		C_j	20	15	0	0	0	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	S_3	S_4	S_5	**MinRatio**
S_3	0	950	0	0	-0.8	0.1	1	0	0	
x_1	20	2000	1	0	2	-4	0	0	0	
x_2	15	3500	0	1	-4	3	0	0	0	
S_4	0	7000	0	0	-2	4	0	1	0	
S_5	0	2500	0	0	4	-3	0	0	1	
$Z = 92500$		Z_j	**20**	**15**	**-20**	**-35**	**0**	**0**	**0**	
		$Z_j - C_j$	0	0	-20	-35	0	0	0	

Since all $Z_j - C_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 2000, x_2 = 3500$

Min $Z = 92500$