Systems Optimization - Assignment 4 Advanced LP

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1. a) Dual

$$\text{Min } z = 810.50w_1 + 655.80w_2 + 520.75w_3$$

$$\begin{aligned} \text{st.}\quad & 100.05w_1 + 5.50w_2 + 75.30w_3 \geq 275.75 \\ & 60.75w_1 + 10.25w_2 + 24.84w_3 \geq 120.50 \\ & w_1,\ w_2,\ w_3 \geq 0 \end{aligned}$$

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1. b) Dual variable explanation

In order to obtain optimal profit, you must use 1.064515 units of w_1 and 0 units of w_2.

Dual Results explanation

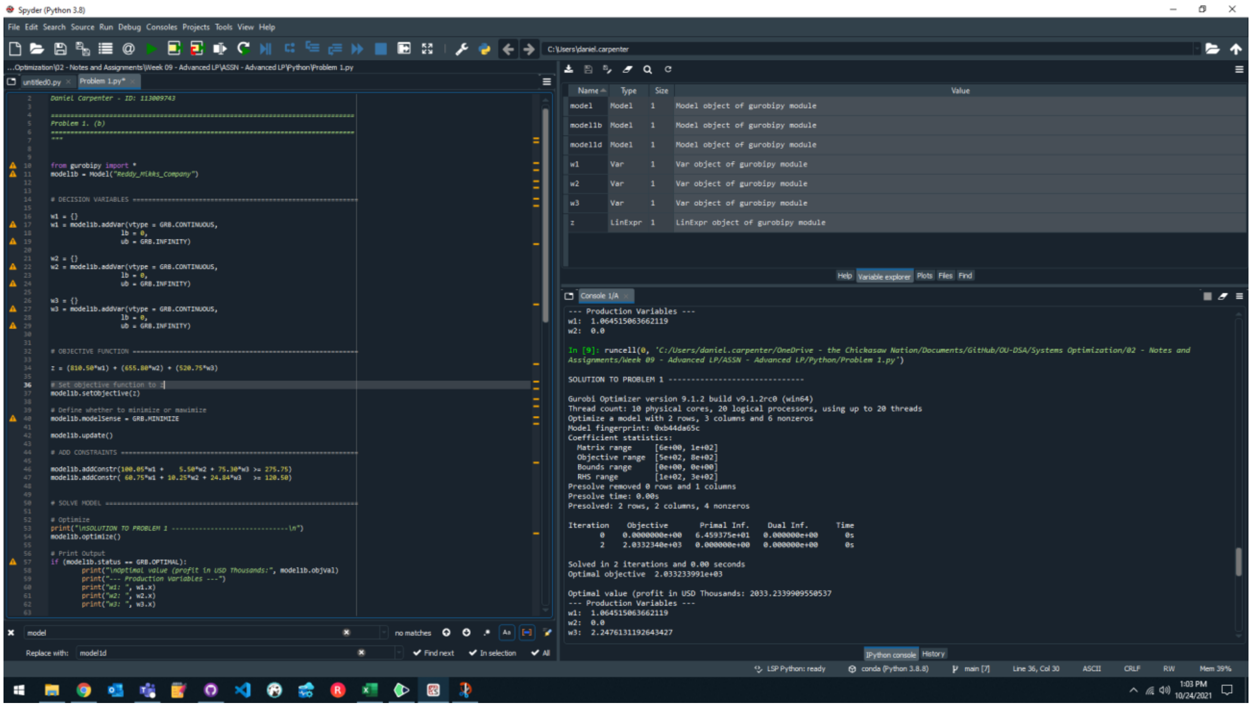
Optimal profit = \$2,033.2339

The dual objective will be the same as the primal, since the dual would be the upper bound estimate. Convergyergene implies optimality.

Values of dual variables w_1, w_2 and w_3 will correspond with the primal's values of s_1, s_2, and s_3 respectively.

(Python code on next page)

Snapshot of Problem 1 (b):



1(c) Min $z = 810.50w_1 + 655.80w_2 + 520.75w_3$

s.t.

$$100.05w_1 + 75.30w_2 + 75.30w_3 \geq 275.75$$
$$60.75w_1 + 24.84w_2 + 24.84w_3 \geq 120.50$$
$$w_1 \geq 0$$
$$w_2 \geq 0$$
$$w_3 \geq 0$$

1(d) Dual Variables

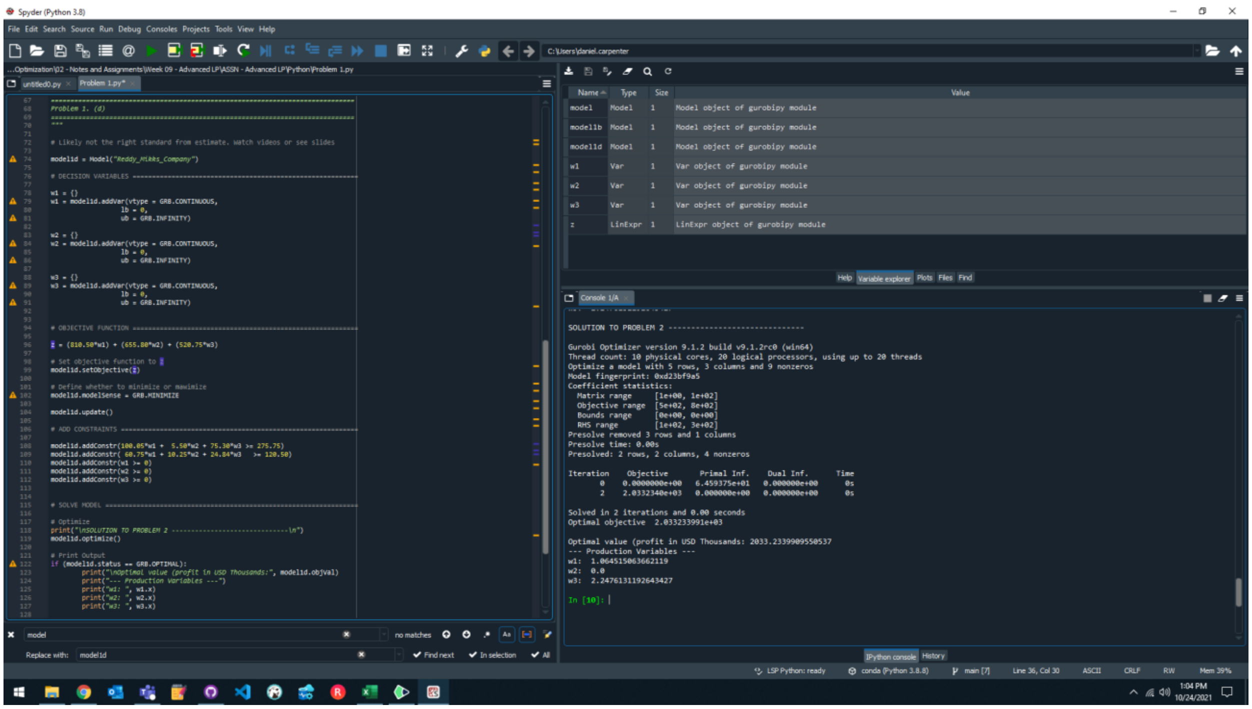
$w_1 = 1.06451\ldots$
$w_2 = 0.00\ldots$
$w_3 = 2.2476131\ldots$

Simalarily to problem 1(b), the dual variables w_1, w_2, and w_3 correspond to the values of the primal's S_1, S_2 and S_3, respectively.

The objective function's value is associated with the outcome of the primals, since it is optimal.

The results correspond exactly to part 1(b).

Snapshot of Problem 1 (d):



1(e) First, it would not be advantagious to change the allocation of electricity, since that constraint is not active. Also, this is shown that the dual is zero for w_2.

However, Increasing the maximum allowable limit of the constraints associated with water or/and gas could lead to an increased optimal while keeping the basis the same.

Profit per Resource Δ: $\left(\text{Dual} \begin{array}{l} w_1 : 1.0645151 \ldots \\ w_3 : 2.2476131 \end{array} \right)$

Marginal Δ in obj. Fun:

Water: $1.06455151 - 1.00 = 0.0645151$ per unit profit

Amt to increase: (see excel) = 463.073 units

$463.073 \times 0.0645151 = 29.87521$ total increase in obj. fun.

Marginal Δ in obj. fun.

Gas: $2.24761 - \$0.75 = \$1.497613$ per unit profit

Amt to increase: (see excel) =

$89.2515 \times \$1.497613 = \$133.6642$ Total increase in obj. fun.

Snapshot of Problem 1 (e): (*Please see Excel file for all calculations*)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1																			
2				1	2	3	4	5											
3				x1	x2	s1	s2	s3											
4		A	1	100.05	60.75	1	0	0				b	810.5						
5			2	5.50	10.25	0	1	0					655.8						
6			3	75.3	24.84	0	0	1					520.75						
7																			
8																			
9																			
10				x1	x2	s1	s2	s3											
11		c^T		275.75	120.50	0	0	0											
12																			
13				1	2	4													
14				x1	x2	s2													
15		B	1	100.05	60.75	0				B^1	-0.01189	0	0.029077657						
16			2	5.5	10.25	1					0.0360419	0	-0.04788839						
17			3	75.3	24.84	0					-0.304037	1	0.330928862						
18																			
19																			
20																			
21		c^T_B		275.75	120.5	0													
22																			
23																			
24																			
25											**Change Electricity to New Optimal**					**Change Gas to New Optimal**			
26		**basic**	**z**	**x1**	**x2**	**s1**	**s2**	**s3**	**solution**			**Cost Change**	**Unit per Constraint Change**	$ 1.00			**Cost Change**	**Unit per Constraint Change**	$ 0.75
27		z	1.00	-	-	1.0645151	-	2.2476131	2,033.23			0.0645							
28		x1	-	1.00	0.00	(0.01)	-	0.03	5.51		1	(0.0119)	463.0734	29.8752		-	0.0291	(189.3456)	
29		x2	-	0.00	1.00	0.04	-	(0.05)	4.27		0	0.0360	(118.5872)			-	(0.0479)	89.2515	133.6642
30		s2	-	-	0.00	(0.30)	1.00	0.33	581.71		0	(0.3040)	1,913.2806			1.0000	0.3309	(1,757.8064)	
31																			

Snapshot of Problem 1 (f): (*Please see Excel file for all calculations*)

		1	2	3	4	5	6		
		x1	x2	x3	s1	s2	s3		
A	1	100.05	60.75	***80.55***	1	0	0	b	810.5
	2	5.50	10.25	***8.35***	0	1	0		655.8
	3	75.3	24.84	***50.43***	0	0	1		520.75

	x1	x2	x3	s1	s2	s3
c^T	275.75	120.50	***170.45***	0	0	0

		1	2	5
		x1	x2	s2
B	1	100.05	60.75	0
	2	5.5	10.25	1
	3	75.3	24.84	0
c^T_B		275.75	120.5	0

B^1	-0.01189	0	0.0290777
	0.0360419	0	-0.047888
	-0.304037	1	0.3309289

basic	z	x1	x2	s1	s2	s3	solution
z	1.00	-	-	28.64	1.06	-	2,033.23
x1	-	1.00	0.00	0.51	(0.01)	-	5.51
x2	-	0.00	1.00	0.49	0.04	-	4.27
s2	-	-	0.00	0.55	(0.30)	1.00	581.71

Solution optimal since no more promising directions.

1(f) Using algebraic sensitivity analysis (see excel), the previous model's optimal equals the new model's optimal. Hence, the new constraint is not active.

2(a)

$$\text{Max } 2000w_1 + 1500w_2 + 500w_3 - 9000w_4 - 6000w_5$$

$$\text{St} \quad 0.3w_1 + 0.4w_2 + 0.2w_3 - w_4 \le 20$$

$$0.4w_1 + 0.2w_2 + 0.3w_3 - w_5 \le 15$$

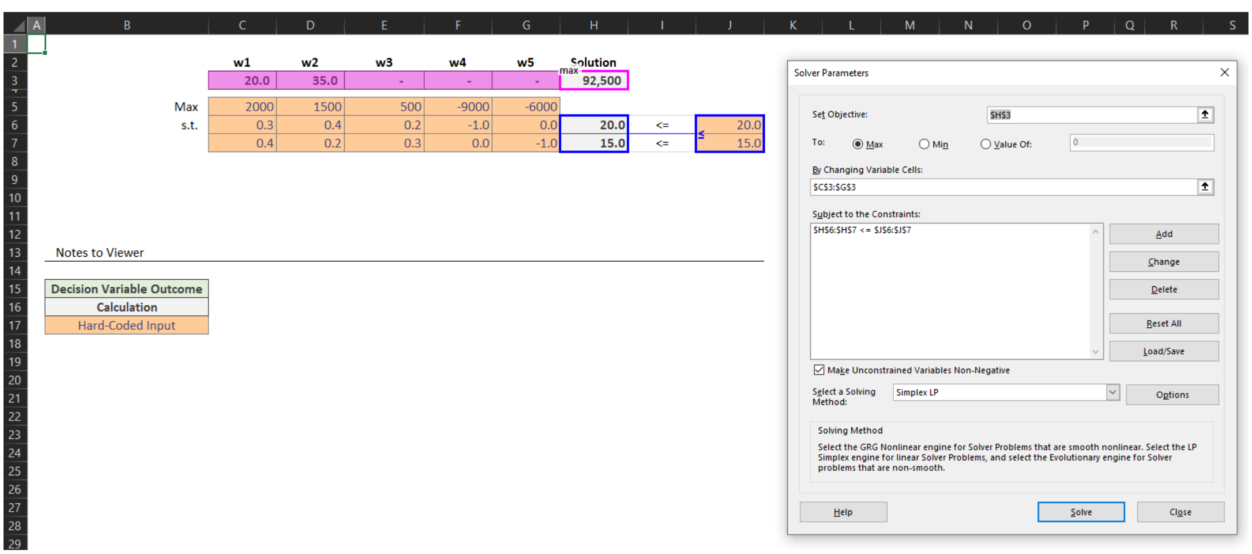
2(b) Dual variables associate with primal variables

$s_1, s_2, s_3, s_4,$ and $s_5.$

$w_1 = 20$
$w_2 = 35$ Duals
$w_3 = 0$
$w_4 = 0$
$w_5 = 0$ no opportunity/not active

Dual objective value associates directly with the outcome of primal, which makes sense because dual and primal should converge.

Snapshot of Problem 2 (b): (*Please see Excel file for all calculations*)



2(c)

$$\begin{aligned}
\text{Min } z &= 2000w_1 + 1500w_2 + 500w_3 + 9000w_4 + 6000w_5 \\
\text{s.t. } & 0.3w_1 + 0.4w_2 + 0.2w_3 + w_4 \geq 20 \\
& 0.4w_1 + 0.2w_2 + 0.3w_3 + w_5 \geq 15 \\
& -w_1 \geq 0 \\
& -w_2 \geq 0 \\
& -w_3 \geq 0 \\
& w_4 \geq 0 \\
& w_5 \geq 0
\end{aligned}$$

2(d) Dual Vars associate with slack variables of primal.

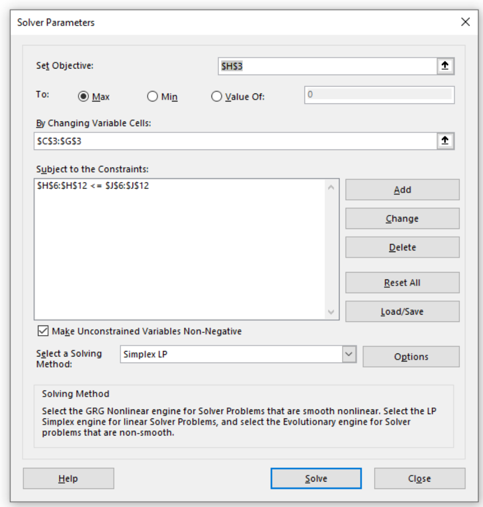
Objective function output relates exactly to primal output. Please see next explanation for examples.

Snapshot of Problem 2 (d): (*Please see Excel file for all calculations*)

	w1	w2	w3	w4	w5	Solution		
	20.0	35.0	-	-	-	max 92,500		
Max	2,000	1,500	500	9,000	6,000			
s.t.	0.3	0.4	0.2	1.0	0.0	20.0	<=	20.0
	0.4	0.2	0.3	0.0	1.0	15.0	<=	15.0
	-1	0	0	0	0	-20.0	<=	0.0
	0	-1	0	0	0	-35.0	<=	0.0
	0	0	-1	0	0	0.0	<=	0.0
	0	0	0	1	0	0.0	<=	0.0
	0	0	0	0	1	0.0	<=	0.0

Notes to Viewer

Decision Variable Outcome
Calculation
Hard-Coded Input



Snapshot of Problem 2 (e): (*Please see Excel file for all calculations*)

1. While keeping the supply fixed and the basis the same for optimality, the range of prices are the following:
   i. High Price: Saudi: **$30.00**, and Venezuela: $15.00, total cost: **$112,500.00**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
2				1	2	3	4	5	6	7					
3				x1	x2	s1	s2	s3	s4	s5					
4		A	1	0.30	0.40	-1	0	0	0	0		b	2,000		
5			2	0.40	0.20	0	-1	0	0	0			1,500		
6			3	0.20	0.30	0	0	-1	0	0			500		
7			4	1.00	-	0	0	0	1	0			9,000		
8			5	-	1.00	0	0	0	0	1			6,000		
12				x1	x2	s1	s2	s3	s4	s5					
13		c^T		30.00	15.00	0	0	0	0	0					
15				5	1	2	6	7							
16				s3	x1	x2	s4	s5							
17		B	1	0.00	0.30	0.40	0.00	0.00		B^1	0.80	-0.10	-1.00	0.00	0.00
18			2	0.00	0.40	0.20	0.00	0.00			-2.00	4.00	0.00	0.00	0.00
19			3	-1.00	0.20	0.30	0.00	0.00			4.00	-3.00	0.00	0.00	0.00
20			4	0.00	1.00	0.00	1.00	0.00			2.00	-4.00	0.00	1.00	0.00
21			5	0.00	0.00	1.00	0.00	1.00			-4.00	3.00	0.00	0.00	1.00
23		c^T_B		0	30	15	0	0							
27											solution				
28		basic	z	x1	x2	s1	s2	s3	s4	s5	(Total Cost)				
29		z	1.00	-	-	-	(75.00)	-	-	-	112,500.00				
30		s3	-	0.00	0.00	(0.80)	0.10	1.00	-	-	950.00				
31		x1	-	1.00	-	2.00	(4.00)	-	-	-	2,000.00				
32		x2	-	-	1.00	(4.00)	3.00	-	-	-	3,500.00				
33		s4	-	-	-	(2.00)	4.00	-	1.00	-	7,000.00				
34		s5	-	-	-	4.00	(3.00)	-	-	1.00	2,500.00				

   ii.

   iii. Low Price: Saudi: **$11.25**, and Venezuela: $15.00, total cost: **$75,000.00**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
2				1	2	3	4	5	6	7					
3				x1	x2	s1	s2	s3	s4	s5					
4		A	1	0.30	0.40	-1	0	0	0	0		b	2,000		
5			2	0.40	0.20	0	-1	0	0	0			1,500		
6			3	0.20	0.30	0	0	-1	0	0			500		
7			4	1.00	-	0	0	0	1	0			9,000		
8			5	-	1.00	0	0	0	0	1			6,000		
12				x1	x2	s1	s2	s3	s4	s5					
13		c^T		11.25	15.00	0	0	0	0	0					
15				5	1	2	6	7							
16				s3	x1	x2	s4	s5							
17		B	1	0.00	0.30	0.40	0.00	0.00		B^1	0.80	-0.10	-1.00	0.00	0.00
18			2	0.00	0.40	0.20	0.00	0.00			-2.00	4.00	0.00	0.00	0.00
19			3	-1.00	0.20	0.30	0.00	0.00			4.00	-3.00	0.00	0.00	0.00
20			4	0.00	1.00	0.00	1.00	0.00			2.00	-4.00	0.00	1.00	0.00
21			5	0.00	0.00	1.00	0.00	1.00			-4.00	3.00	0.00	0.00	1.00
23		c^T_B		0	11.25	15	0	0							
27											solution				
28		basic	z	x1	x2	s1	s2	s3	s4	s5	(Total Cost)				
29		z	1.00	0.00	0.00	(37.50)	(0.00)	-	-	-	75,000.00				
30		s3	-	0.00	0.00	(0.80)	0.10	1.00	-	-	950.00				
31		x1	-	1.00	-	2.00	(4.00)	-	-	-	2,000.00				
32		x2	-	-	1.00	(4.00)	3.00	-	-	-	3,500.00				
33		s4	-	-	-	(2.00)	4.00	-	1.00	-	7,000.00				
34		s5	-	-	-	4.00	(3.00)	-	-	1.00	2,500.00				

   iv.

Snapshot of Problem 2 (f): (*Please see Excel file for all calculations*)

2. While keeping the supply fixed and the basis the same for optimality, the range of prices are the following:
   a. High Price: Saudi: $20.00, and Venezuela: **$26.67**, total cost: **$133,333.33**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2				1	2	3	4	5	6	7						
3				x1	x2	s1	s2	s3	s4	s5						
4		A	1	0.30	0.40	-1	0	0	0	0		b	2,000			
5			2	0.40	0.20	0	-1	0	0	0			1,500			
6			3	0.20	0.30	0	0	-1	0	0			500			
7			4	1.00	-	0	0	0	1	0			9,000			
8			5	-	1.00	0	0	0	0	1			6,000			
9																
10																
11																
12				x1	x2	s1	s2	s3	s4	s5						
13		c^T		20.00	26.67	0	0	0	0	0						
14																
15				5	1	2	6	7								
16				s3	x1	x2	s4	s5								
17		B	1	0.00	0.30	0.40	0.00	0.00		B^1	0.80	-0.10	-1.00	0.00	0.00	
18			2	0.00	0.40	0.20	0.00	0.00			-2.00	4.00	0.00	0.00	0.00	
19			3	-1.00	0.20	0.30	0.00	0.00			4.00	-3.00	0.00	0.00	0.00	
20			4	0.00	1.00	0.00	1.00	0.00			2.00	-4.00	0.00	1.00	0.00	
21			5	0.00	0.00	1.00	0.00	1.00			-4.00	3.00	0.00	0.00	1.00	
22																
23		c^T_B		0	20	26.6666667	0	0								
24																
25																
26																
27											solution					
28		basic	z	x1	x2	s1	s2	s3	s4	s5	(Total Cost)					
29		z	1.00	0.00	0.00	(66.67)	(0.00)	-	-	-	133,333.33					
30		s3	-	0.00	0.00	(0.80)	0.10	1.00	-	-	950.00					
31		x1	-	1.00	-	2.00	(4.00)	-	-	-	2,000.00					
32		x2	-	-	1.00	(4.00)	3.00	-	-	-	3,500.00					
33		s4	-	-	-	(2.00)	4.00	-	1.00	-	7,000.00					
34		s5	-	-	-	4.00	(3.00)	-	-	1.00	2,500.00					
35																

   b.
   c. Low Price: Saudi: $20.00, and Venezuela: **$10.00**, total cost: **$75,000.00**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2				1	2	3	4	5	6	7					
3				x1	x2	s1	s2	s3	s4	s5					
4		A	1	0.30	0.40	-1	0	0	0	0		b	2,000		
5			2	0.40	0.20	0	-1	0	0	0			1,500		
6			3	0.20	0.30	0	0	-1	0	0			500		
7			4	1.00	-	0	0	0	1	0			9,000		
8			5	-	1.00	0	0	0	0	1			6,000		
9															
10															
11															
12				x1	x2	s1	s2	s3	s4	s5					
13		c^T		20.00	10.00	0	0	0	0	0					
14															
15				5	1	2	6	7							
16				s3	x1	x2	s4	s5							
17		B	1	0.00	0.30	0.40	0.00	0.00		B^1	0.80	-0.10	-1.00	0.00	0.00
18			2	0.00	0.40	0.20	0.00	0.00			-2.00	4.00	0.00	0.00	0.00
19			3	-1.00	0.20	0.30	0.00	0.00			4.00	-3.00	0.00	0.00	0.00
20			4	0.00	1.00	0.00	1.00	0.00			2.00	-4.00	0.00	1.00	0.00
21			5	0.00	0.00	1.00	0.00	1.00			-4.00	3.00	0.00	0.00	1.00
22															
23		c^T_B		0	20	10	0	0							
24															
25															
26															
27											solution				
28		basic	z	x1	x2	s1	s2	s3	s4	s5	(Total Cost)				
29		z	1.00	-	-	-	(50.00)	-	-	-	75,000.00				
30		s3	-	0.00	0.00	(0.80)	0.10	1.00	-	-	950.00				
31		x1	-	1.00	-	2.00	(4.00)	-	-	-	2,000.00				
32		x2	-	-	1.00	(4.00)	3.00	-	-	-	3,500.00				
33		s4	-	-	-	(2.00)	4.00	-	1.00	-	7,000.00				
34		s5	-	-	-	4.00	(3.00)	-	-	1.00	2,500.00				
35															

   d.