DETERMINISTIC SYSTEMS MODELS/SYSTEMS OPTIMIZATION
ISE 4623/5023
EXAM 1
Fall 2021

Last name: Carpenter

First name: Daniel

Student ID: 113009743

Section (mark with X):

___ ISE 4623
_X_ ISE 5023

Pledge: "On my honor, I have neither given nor received inappropriate assistance in the completion of this Exam."

Student signature: ______

first: Daniel

last: Carpenter

ID: 113009743

Section: ISE 5023

Signature: Daniel Carpenter

Problem 1:

a) Decision Variables

C = hours spent per week training Cardiovascular
W = hours spent per week weightlifting

b) Objective function

maximize $z = 0.6C + W$

c) Constraints

$$
\begin{aligned}
C & & \geq 2 \\
C & & \leq 6 \\
C &+ W & \leq 8 \\
C &+ 2W & \leq 13 \\
C & & \geq 0 \\
 & W & \geq 0
\end{aligned}
$$

Problem 2 (25 points)

Suppose you have the following LP model

$$\text{Minimize } z = 5x_1 + 2x_2$$

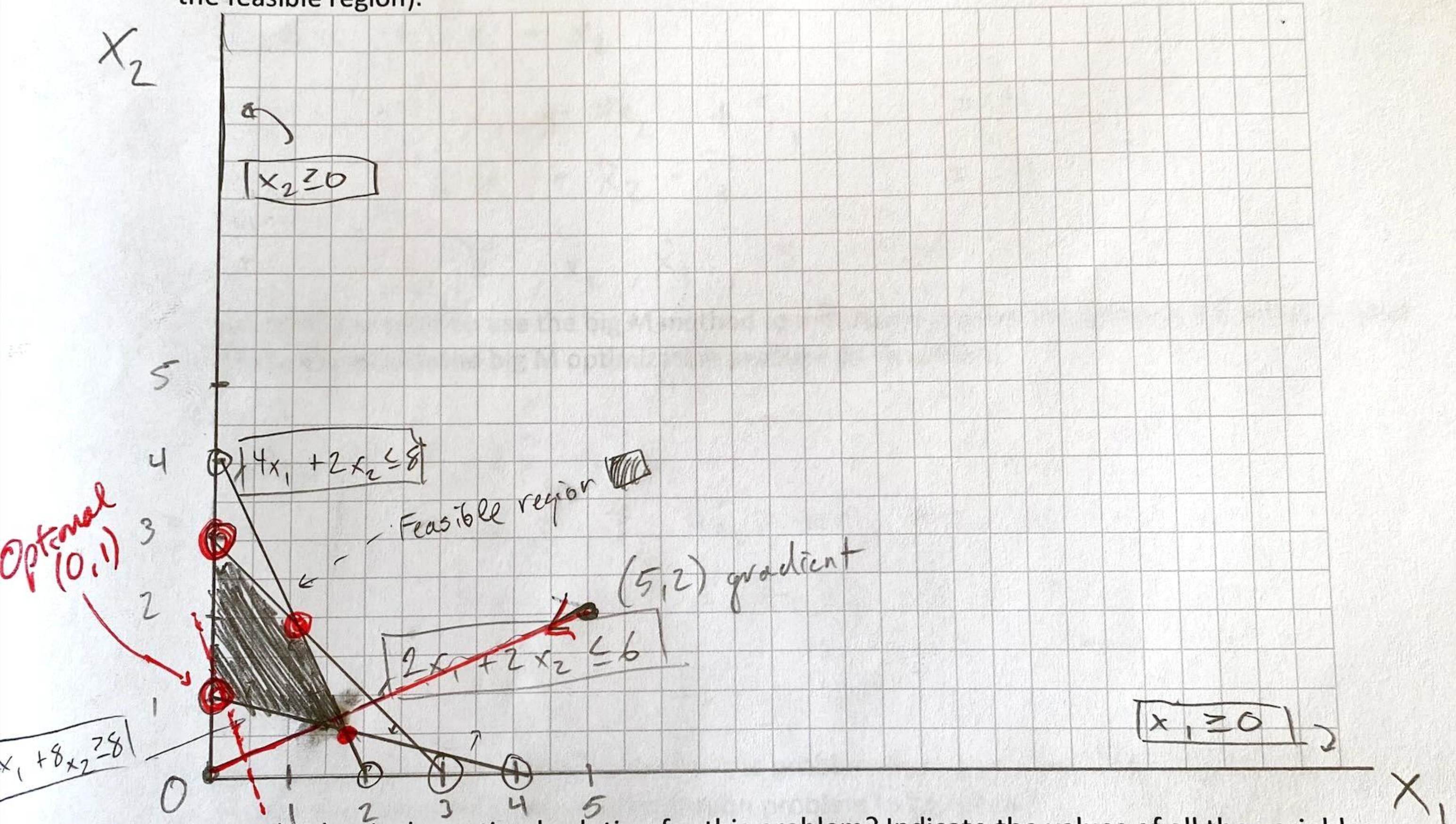
s.t.

$$2x_1 + 8x_2 \geq 8 \rightarrow x_1 = 8/2 = 4 \quad x_2 = 8/8 = 1$$
$$2x_1 + 2x_2 \leq 6 \rightarrow x_1 = 6/2 = 3 \quad x_2 = 6/2 = 3$$
$$4x_1 + 2x_2 \leq 8 \rightarrow x_1 = 8/4 = 2 \quad x_2 = 8/2 = 4$$
$$x_1, x_2 \geq 0$$

a. (5 points) What is the gradient of this problem?

$$\text{gradient} = \left(\frac{df}{dx_1}, \frac{df}{dx_2}\right) = (5, 2)$$

b. (15 points) Plot the gradient and the feasible region (clearly indicating all the constraints and "shading" the feasible region).



c. (5 points) What is the optimal solution for this problem? Indicate the values of all the variables (including the slacks) and the objective function.

Optimal $\Rightarrow$ $x_1 = 0$

$x_2 = 1$

$z = 5(0) + 2(1)$

$= 2$

Problem 3 (35 points)

Suppose you have the following LP model

$$\text{Maximize } z = -2x_1 - x_2$$
s.t.
$$-x_1 - 4x_2 \le -4$$
$$2x_1 + x_2 - x_3 = 4$$
$$x_1, x_2 \ge 0$$
$$x_3 \le 0$$

You would like to solve it using Simplex Tableau. For this:

a. (5 points) Write the problem in standard form (so that it contains only non-negative right-hand side, non-negative variables, and equality constraints). Indicate the variables, objective function, the associated constraints, and the nature of the variables (if they are non-negative, non-positive, etc.).

Maximize $-2\hat{x}_1 - \hat{x}_2$

st. $\hat{x}_1 + 4\hat{x}_2 + S_1 = 4$

$2\hat{x}_1 + \hat{x}_2 - \hat{x}_3 = 4$

$\hat{x}_1, \hat{x}_2, \hat{x}_3, S_1 \ge 0$

b. (5 points) If you want to use the big M method to initialize and solve the problem from part a, what would be the associated big M optimization problem to be solved?

Maximize $-2\hat{x}_1 - \hat{x}_2 - Mr$

st. $\hat{x}_1 + 4\hat{x}_2 - S_1 + r = 4$

$2\hat{x}_1 + \hat{x}_2 - \hat{x}_3 = 4$

$\tilde{x}_1\ \hat{x}_2, \hat{x}_3\ S_1 \ge 0$

c. (5 points) If you want to initialize and solve the problem from part a with the two-phase method, what would be the associated phase-1 optimization problem to be solved?

Maximize $-2\hat{x} - \hat{x}_2 - r$

St $-\hat{x}_1 + 4\hat{x}_2 - S_1 + r = 4$

$2\hat{x}_1 + \hat{x}_2 - \hat{x}_3 = 4$

d. (10 points) Using Simplex Tableau solve the associated phase-1 optimization problem and complete the table below with the optimal basic solution for the phase-1 problem. Don't forget to write all the steps/iterations made, including the row operations used.

$R_2 \leftarrow R_2$

$R_1 \leftarrow R_1 - (-1) R_{2(new)}$

$R_3 \leftarrow R_3$

Make R identity

Basic	z	x_1	x_2	x_3	s1	r	Solution
z	1	1	-5	0	-1	0	4
r	6	-1	-4	0	-1	1	4
	0	2	1	-1	0	0	4

Optimal value of objective function of phase-1 problem
4

Variable	Value	Basic or non-basic?
x_1	0	non
x_2	0	non
r	4	basic
x_3	6	non
s_1	0	non

e. (10 points) Initialize the phase-2 problem (using the solution from part d) and solve it to optimality using Simplex Tableau. Complete the table below with the associated basic solution. Don't forget to write all the steps/iterations made, including the row operations used. Is this solution feasible for the problem in part a? Is this solution optimal for the problem in part a? Explain your answers in detail.

Choose x_2 since most promising.
Min ratio says R_3 choose.

$R_3 \leftarrow R_3$

$R_1 \leftarrow R_1 + 3(R_{3(new)})$

$R_2 \leftarrow R_2 + 4(R_{3(new)})$

Basic	z	x_1	x_2	x_3	s_1	Solution
z	1	7	0	-3	-1	16
	0	7	0	4	-1	20
x_2	0	2	1	-1	0	4

Optimal value of objective function of phase-2 problem
16

Variable	Value	Basic or non-basic?
x_1	0	non
x_2	4	basic
x_3	0	non
s_1	0	non

Is this basic solution feasible (with respect to the problem in part a)? Yes / No
Explanation:
This solution will fall within constraints

Is this basic solution optimal (with respect to the problem in part a)? Yes / No
Explanation:
The problem is left non optimal since still promising directions to go (negative values in Z row)

Problem 4 (20 points)

Suppose that you have the following Python/Gurobi code associated with an LP model

```
from gurobipy import *
model=Model("PepitaPerez_Company")

x1={}
x1=model.addVar(vtype=GRB.CONTINUOUS, lb=2, ub=GRB.INFINITY)

x2={}
x2=model.addVar(vtype=GRB.CONTINUOUS, lb=0, ub=6)

z=2*x1+4*x2
model.setObjective(z)
model.modelSense=GRB.MAXIMIZE
model.update()

model.addConstr(3*x1+2*x2<=12)
model.addConstr(x1+2*x2<=6)
model.addConstr((-1)*x1+x2<=1)

model.update()
model.optimize()

if model.status==GRB.OPTIMAL:
    print ("\n Optimal value (profit in USD thousands):", model.objVal)
    print ("--- Production quantities---")
    print ("x1", x1.x)
    print ("x2", x2.x)
```

a. (4 points) What are the decision variables associated with this LP model?

x_1 and x_2

b. (4 points) What is the objective function of this LP model? Do not forget to indicate if you are maximizing or minimizing it.

Max $z = 2x_1 + 4x_2$

c. (12 points) What are the constraints associated with this LP model?

$3x_1 + 2x_2 \leq 12$

$x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$