Exam 2 Review

Deterministic Systems Models / Systems Optimization

Andrés D. González

Assistant Professor
School of Industrial and Systems Engineering, The University of Oklahoma

ISE 4623/5023: Deterministic Systems Models / Systems Optimization

The University of Oklahoma, Norman, OK, USA

Topics to be covered in Exam 2

- Module 3. Sensitivity analysis and duality
- Module 4. LP models for common structures
- Everything from previous modules (particularly in topics where the students had the lowest performance from previous exams/assignments/quizzes)

Module 3. Sensitivity analysis and duality

Including (but not limited to):

- Constructing dual problems and understand their connection with the primal
  - Meaning of dual variables and their application
  - Weak and strong duality theorems
- Sensitivity and post-optimal analysis
  - Add or remove activities/variables
  - Add or remove constraints
  - Change parameters (costs, resources, etc.)

Module 4. LP models for common structures

Including (but not limited to):

- Understanding common problems and their LP models/structures
  - Data Envelopment Analysis (DEA)
    - DMUs, inputs, outputs
    - Efficiency, reference set, etc.
    - Generalized network models
      - Transportation problem
      - Assignment problem
      - Minimum cost flow problem
        - Connection with transportation problem, assignment problem, etc.
        - Shortest path problem
        - Max flow problem
- Knowing how to adapt/modify/extend common structures to fit particular contexts
  - Handle unbalanced networks (total demand is different from total supply capacity)
  - Handle possible unmet demand or excess production
  - Adding capacities to nodes (e.g., maximum flow that can go through some nodes)
  - Adding/adapting flow constraints for different types of capacities (number of units, weight, size, etc.)

Data Envelopment Analysis (DEA) Efficiency Model

- Sets:

N = set of DMUs

I = set of *inputs*

O = set of *outputs*

- Parameters:

y_{oj} = amount of output $o \in O$ produced by DMU $j \in N$

x_{ij} = amount of input $i \in I$ produced by DMU $j \in N$

p = Individual DMU under study

- Decision variables:

λ_j = fraction of the jth DMU ($j \in N$) used to achieve efficiency (i.e., get to the efficiency frontier)

ϕ_p = Proportional increase (growth rate) of DMU under study (p)

- Objective function:

$$\max \ \phi_p$$

- Constraints:

$$\sum_{j \in N} y_{oj}\lambda_j \geq y_{op}\phi_p \qquad , \forall o \in O$$

Sum of output * weights must be more than the person of comparison

$$\sum_{j \in N} x_{ij}\lambda_j \geq x_{io} \qquad , \forall i \in I$$

Sum of inputs * weights for others in sample must be less than the person of comparison? Check notes

$$\lambda_j \geq 0 \qquad , \forall j \in N$$

$$\phi_p \ \ unrestricted$$

Transportation Problem

- Sets:

S = set of *sources* $\{1,2,\ldots,m\}$

D = set of *destinations* $\{1,2,\ldots,n\}$

- Parameters:

a_i = supply of source node $i \in S$

b_j = demand of destination node $j \in D$

c_{ij} = unit cost of flow through arc (i,j), where $i \in S$ and $j \in D$

- Decision variables:

x_{ij} = flow through arc (i,j), where $i \in S$ and $j \in D$

- Objective function:

$$\min z = \sum_{i \in S} \sum_{j \in D} c_{ij} x_{ij}$$

Flows meet the supply or demand

Change to <= if unmet demand

- Constraints:

$$\sum_{i \in S} x_{ij} = b_j \qquad , \forall j \in D$$

flow equals demand

$$\sum_{j \in D} x_{ij} = a_i \qquad , \forall i \in S$$

flow equals supply

$$x_{ij} \geq 0 \qquad , \forall i \in S, \forall j \in D$$

Assignment Problem

- Sets:

W: Set of workers $\{1,2,3,\ldots,n\}$

J: Set of jobs $\{1,2,3,\ldots,n\}$

- Parameters:

c_{ij}: cost of assigning worker $i \in W$ to job $j \in J$

- Variables:

x_{ij}: binary variable that is 1 if worker $i \in W$ is assigned to job $j \in J$, and is 0 otherwise.

- Objective function:

$$\min z = \sum_{i \in W} \sum_{j \in J} c_{ij} x_{ij}$$

- Constraints:

$$\sum_{i \in W} x_{ij} = 1 \qquad , \forall j \in J$$

$$\sum_{j \in J} x_{ij} = 1 \qquad , \forall i \in W$$

Each worker must be assigned a job

$$x_{ij} \in \{0,1\} \qquad , \forall i \in W, \ \forall j \in J$$

Minimum Cost Flow Problem (MCFP)

- Sets:

N: Set of nodes $\{1,2,3,\dots,n\}$

A: Set of arcs

- Parameters:

c_{ij}: unit cost of sending a commodity through arc $(i,j) \in A$

b_i: demand/supply of commodities in node $i \in N$

u_{ij}: maximum flow through arc $(i,j) \in A$

l_{ij}: minimum flow through arc $(i,j) \in A$

- Variables:

x_{ij}: flow through arc $(i,j) \in A$

- Objective function:

$$\min z = \sum_{(i,j)\in A} c_{ij} x_{ij}$$

- Constraints:

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = b_i \quad ,\forall i \in N$$

$$x_{ij} \le u_{ij} \quad ,\forall (i,j) \in A$$

Within upper bound

$$x_{ij} \ge l_{ij} \quad ,\forall (i,j) \in A$$

Within Lower bound

Flows meet the supply or demand

Change to <= if unmet demand

Multicommodity Minimum Cost Flow Problem (MMCFP)

- Sets:

N: Set of nodes $\{1,2,3,\dots,n\}$

A: Set of arcs

K: Set of commodities

- Parameters:

c_{ijk}: unit cost of sending a commodity $k \in K$ through arc $(i,j) \in A$

b_{ik}: demand/supply of commodity $k \in K$ in node $i \in N$

u_{ij}: maximum total flow through arc $(i,j) \in A$

l_{ij}: minimum total flow through arc $(i,j) \in A$

- Variables:

x_{ijl}: flow of commodity $l \in L$ through arc $(i,j) \in A$

- Objective function:

$$\min z = \sum_{k \in K} \sum_{(i,j) \in A} c_{ijk} x_{ijk}$$

- Constraints:

Flows meet the supply / Demand (change <= for unmet demand)

$$\sum_{j:(i,j) \epsilon A} x_{ijk} - \sum_{j:(j,i) \epsilon A} x_{jik} = b_{ik} \qquad ,\forall i \in N, \forall k \in K$$

$$\sum_{k \in K} x_{ijk} \le u_{ij} \qquad ,\forall (i,j) \in A$$

Within upper bound

$$\sum_{k \in K} x_{ijk} \ge l_{ij} \qquad ,\forall (i,j) \in A$$

Within Lower bound

THANK YOU

QUESTIONS?

Andrés D. González | andres.gonzalez@ou.edu