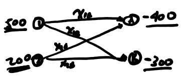


Nodes → Vertices
Arcs → Links
↓
Directed
undirected

Sets → S : {1, 2}
D : {A, B}

Parameters →
$u_{1A} = 300$
$u_{1B} = 300$
$u_{2A} = 400$
$u_{2B} = 400$
} capacity of each arc

$c_{1A} = 10$
$c_{1B} = 2$
$c_{2A} = 1$
$c_{2B} = 5$
} unit flow cost of each arc

$b_1 = 500$
$b_2 = 200$
$b_A = -400$
$b_B = -300$
} Demand/supply of each node

a_1	500
a_2	200
b_A	400
b_B	300

Variables

x_{1A}
x_{1B}
x_{2A}
x_{2B}
} Flows per each arc

Obj. function

$$\text{Min } c_{1A}x_{1A} + c_{1B}x_{1B} + c_{2A}x_{2A} + c_{2B}x_{2B}$$

constraints

$$\begin{cases} x_{1A} + x_{1B} \le b_1 \\ x_{2A} + x_{2B} \le b_2 \end{cases}$$ supply capacity for each supply node

$$\begin{cases} x_{1A} + x_{2A} \ge (-b_A) \\ x_{1B} + x_{2B} \ge (-b_B) \end{cases}$$ Demand satisfaction for each demand node

$x_{1A} \le u_{1A}$
$x_{2A} \le u_{2A}$
$x_{1B} \le u_{1B}$
$x_{2B} \le u_{2B}$
} capacity constraint for each arc

$x_{1A}, x_{2A}, x_{1B}, x_{2B} \ge 0$

Transportation problem

Sets → S : supply nodes
D : demand nodes

Parameters
u_{ij} : flow capacity of arc from node $i \in S$ to node $j \in D$
c_{ij} : unit flow cost arc from node $i \in S$ to node $j \in D$
a_i : supply of supply node $i \in S$
b_j : demand of demand node $j \in D$

variables:

x_{ij} : flow through arc from node $i \in S$ to node $j \in D$

obj. function

$$\min z = \sum_{i \in S} \sum_{j \in D} c_{ij} x_{ij}$$

constraints

equality if network is balanced

$$\sum_{j \in D} x_{ij} \boxed{\le} a_i, \quad \forall i \in S$$

$$\sum_{i \in S} x_{ij} \boxed{\ge} b_j, \quad \forall j \in D$$

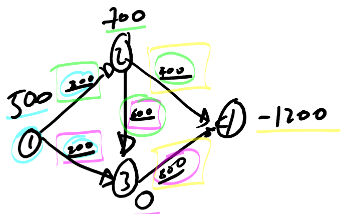
This is not part of the basic transportation problem

$$\boxed{x_{ij} \le u_{ij}, \quad \forall i \in S, \forall j \in D}$$

$$x_{ij} \ge 0, \quad \forall i \in S, \forall j \in D$$

→ Multicommodity

Minimum Cost Flow problem



Sets

N: nodes {1,2,3,4}

A: arcs {(1,2),(2,3),(1,3),(2,4),(3,4)}

L: commodities {A, B}

Parameters

$c_{ij\ell}$: unit cost per flow through arc $(i,j)\in A$ and commodity $\ell \in L$

$b_{i\ell}$: demand/supply of commodities in node $i \in N$ and commodity $\ell \in L$

u_{ij} : maximum flow through arc $(i,j) \in A$

l_{ij} minimum flow through arc $(i,j) \in A$

Variables

$x_{ij\ell}$ flow through arc $(i,j) \in A$ and commodity $\ell \in L$

Obj. Function

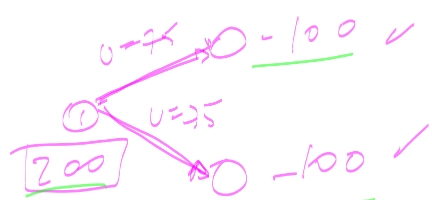
$$\min\ z = \sum_{\ell \in L}\sum_{(i,j)\in A} c_{ij\ell}\, x_{ij\ell}$$

constraints

$$\sum_{\ell \in L} x_{ij\ell} \le u_{ij} \quad \forall (i,j)\in A$$

$$\sum_{\ell \in L} x_{ij\ell} \ge l_{ij} \quad \forall (i,j)\in A$$

$$\underbrace{\sum_{j:(i,j)\in A} x_{ij\ell}}_{\text{leaving } i} - \underbrace{\sum_{k:(k,i)\in A} x_{ki\ell}}_{\text{entering } i} = b_{i\ell}, \ \forall i \in N, \ \forall \ell \in L$$



$$\left(\sum_{j:(i,j)\in A} x_{ij} - \sum_{k:(k,i)\in A} x_{ki}\right) = b_i + \boxed{s_i^- - s_i^+}$$

$$(75+75) - (0) = 200 + 0 - 50$$

$$\sum_{j:(3,j)\in A} x_{3j} = x_{34} \qquad \boxed{i=3}$$

A : arcs $\{(1,2), \boxed{(2,3)}, \boxed{(1,3)}, (2,4), \boxed{(3,4)}\}$

$$(0) - (75) = -100 + 25 + 0$$

$$(0) - (75) = -100 + 25 + 0$$

$$\sum_{k:(k,3)\in A} x_{k3} = x_{23} + x_{13}$$

$\text{Min} \sum c_{ij} x_{ij} +$

$k:(k,3)\in A$

$$X_{34} - (X_{23} - X_{13}) = b_3$$

$$\text{Min} \sum_{(i,j)\in A} C_{ij} X_{ij} +$$

V_l = volume of commodity $l \in L$

$b_{wl} = 0$ ✓ → supply/demand of node i and commodity $l \in L$

R_w → Max cap of the warehouse

$$\sum_{l\in L} \sum_{i:(i,w)\in A} V_l X_{iwl} \le R_w$$