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Assn. 1 – Linear Algebra

① $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 2 \end{bmatrix}$

1a) AB → size of AB => $(\underline{2} \times 3)(3 \times \underline{2}) = 2 \times 2$

$$AB = \begin{bmatrix} 1\times1 + 2\times2 + 2\times4 & 1\times1 + 2\times3 + 2\times2 \\ 2\times1 + 1\times2 + 3\times4 & 2\times1 + 1\times3 + 3\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+8 & 1+6+4 \\ 2+2+12 & 2+3+6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 13 & 11 \\ 16 & 11 \end{bmatrix}$$

1b) BA → size of BA → $(\underline{3} \times 2)\cdot(2 \cdot \underline{3}) = 3 \times 3$

$B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 2 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$$BA = \begin{bmatrix} 1\times1 + 1\times2 & 1\times2 + 1\times1 & 1\times2 + 1\times3 \\ 2\times1 + 3\times2 & 2\times2 + 3\times1 & 2\times2 + 3\times3 \\ 4\times1 + 2\times2 & 4\times2 + 2\times1 & 4\times2 + 2\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2 & 2+1 & 2+3 \\ 2+6 & 4+3 & 4+9 \\ 4+4 & 8+2 & 8+6 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & 3 & 5 \\ 8 & 7 & 13 \\ 8 & 10 & 14 \end{bmatrix}$$

1c) $-B = -1\cdot(B) = -1\cdot\begin{bmatrix}1 & 1\\ 2 & 3\\ 4 & 2\end{bmatrix} = \begin{bmatrix}-1 & -1\\ -2 & -3\\ -4 & -2\end{bmatrix}$

1d) $A+2B$ $\quad 2\cdot(B) = 2\cdot\begin{bmatrix}1 & 1\\ 2 & 3\\ 4 & 2\end{bmatrix} = \begin{bmatrix}2 & 2\\ 4 & 6\\ 8 & 4\end{bmatrix}$

Operation $(A+2B)$ is not possible since A is not the same size 2B. I.e. $A = 2\times3$ matrix, and $2B = 3\times2$ matrix.

1e) A^T

$A = \begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 3\end{bmatrix}$ $\quad A^T = \begin{bmatrix}1 & 2\\ 2 & 1\\ 2 & 3\end{bmatrix}$

1f) B^T

$B = \begin{bmatrix}1 & 1\\ 2 & 3\\ 4 & 2\end{bmatrix}$ $\quad B^T = \begin{bmatrix}1 & 2 & 4\\ 1 & 3 & 2\end{bmatrix}$

Problem 2a

$$\begin{aligned} 3x_1 + x_2 &= 2 \\ -6x_1 - 2x_2 &= -4 \end{aligned} \Rightarrow \left[\begin{array}{cc|c} 3 & 1 & 2 \\ -6 & -2 & -4 \end{array}\right]$$

$$R_1 \cdot \tfrac{1}{3} \rightarrow R_1 \quad \left[\begin{array}{cc|c} 1 & 1/3 & 2/3 \\ -6 & -2 & -4 \end{array}\right]$$

$$R_2 + R_1 \cdot 6 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & 1/3 & 2/3 \\ 0 & 0 & 0 \end{array}\right]$$

Solve for x_1

$$1x_1 + \tfrac{1}{3}x_2 = \tfrac{2}{3}$$

$$\boxed{x_1 = -\tfrac{1}{3}x_2 + \tfrac{2}{3}}$$

Solve for x_2

$$\tfrac{1}{3}x_2 = -x_1 + \tfrac{2}{3}$$

$$x_2 = -3x_1 + 6$$

$$x_2 = -3\left(-\tfrac{1}{3}x_2 + \tfrac{2}{3}\right) + 6$$

$$x_2 = x_2 - 6 + 6$$

$$\boxed{x_2 = x_2}$$

Therefore Solution is $\begin{pmatrix} -\tfrac{1}{3}x_2 + \tfrac{2}{3} \\ x_2 \end{pmatrix}$

$\boxed{2b}$ $\begin{aligned} 2x_1 + x_2 &= 4 \\ x_1 + 2x_2 &= 5 \end{aligned} \quad > \left[\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & 2 & 5 \end{array}\right]$

$R_1 \cdot (1/2) \rightarrow R_1 \quad \left[\begin{array}{cc|c} 1 & 1/2 & 2 \\ 1 & 2 & 5 \end{array}\right]$

$R_2 - R_1 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & 1/2 & 2 \\ 0 & 3/2 & 3 \end{array}\right]$

$R_2 \times (2/3) \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & 1/2 & 2 \\ 0 & 1 & 2 \end{array}\right]$

$R_1 - (1/2)R_2 \rightarrow R_1 \quad \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$

Therefore, $x_1 = 1$
$x_2 = 2$

2c

$$\begin{aligned} 2x_1 + x_2 &= 4 \\ -4x_1 - 2x_2 &= 5 \end{aligned} \quad \rangle \quad \left[\begin{array}{cc|c} 2 & 1 & 4 \\ -4 & -2 & 5 \end{array}\right]$$

$$R_1 \cdot (1/2) \to R_1 \quad \left[\begin{array}{cc|c} 1 & 1/2 & 2 \\ -4 & -2 & 5 \end{array}\right]$$

$$R_2 + 4(R_1) \to R_2 \quad \left[\begin{array}{cc|c} 1 & 1/2 & 2 \\ 0 & 0 & 13 \end{array}\right]$$

There is no solution. There is nothing we can do to the second equation to Satisfy the equality.

i.e. no value of x_1 or x_2 can meet the criteria of $\{0x_1 + 0x_2 = 13\}$.

3a

$$C = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$C^{-1} = \frac{1}{(1\cdot 2)-(4\cdot 2)} \begin{bmatrix} 2 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \frac{1}{2-8} \begin{bmatrix} & \\ & \end{bmatrix}$$

$$= \frac{1}{-6} \begin{bmatrix} 2 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2/6 & 2/6 \\ 4/6 & -1/6 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} -1/3 & 1/3 \\ 2/3 & -1/6 \end{bmatrix}$$

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$$D = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$$

$$D^{-1} = \frac{1}{(2\cdot 8)-(4\cdot 4)} \begin{bmatrix} 8 & -4 \\ -4 & 2 \end{bmatrix}$$

$$= \frac{1}{16-16} \begin{bmatrix} 8 & -4 \\ -4 & 2 \end{bmatrix}$$

$$= \frac{1}{0} \begin{bmatrix} 8 & -4 \\ -4 & 2 \end{bmatrix}$$

Since $\det(D) = (2\cdot 8)-(4\cdot 4) = 0$,

The matrix is not invertible

3c

$$E = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 2 & 6 \\ 5 & 4 & 10 \end{bmatrix}$$

Using triangles rule:

$$\text{Det}(E) = 1 \cdot 2 \cdot 10 + 2 \cdot 6 \cdot 5 + 4 \cdot 4 \cdot 4$$
$$- 5 \cdot 2 \cdot 4 - 4 \cdot 6 \cdot 1 - 4 \cdot 2 \cdot 10$$
$$= 20 + 60 + 64 - (40 + 24 + 80)$$
$$= 144 - 144$$
$$= 0$$

Since Det(E) = 0, The matrix is not invertible.

3d

$$F = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 2 & 6 \\ 5 & 4 & 8 \end{bmatrix}$$

$$\det(F) = 1\cdot2\cdot8 + 2\cdot6\cdot5 + 4\cdot4\cdot4 - 5\cdot2\cdot4 - 4\cdot2\cdot8 - 4\cdot6\cdot1$$

$$= 16 + 60 + 64 - (40 + 64 + 24)$$

$$= 140 - 128$$

$\det(F) = 12$, which $\det(F) > 0 \rightarrow$ invertible

$\rightarrow$ Next page shows inverse calcs.

3d continued

$$F = \begin{bmatrix} 1 & 2 & 4 \\ 4 & 2 & 6 \\ 5 & 4 & 8 \end{bmatrix}$$

$$F^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 4 & 2 & 6 & 0 & 1 & 0 \\ 5 & 4 & 8 & 0 & 0 & 1 \end{array}\right] \quad R_2 - 4R_1 \to R_2 \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & -6 & -10 & -4 & 1 & 0 \\ 5 & 4 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$R_2\left(-\tfrac{1}{6}\right) \to R_2 \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & 5/3 & 2/3 & -1/6 & 0 \\ 5 & 4 & 8 & 0 & 0 & 1 \end{array}\right] \quad R_3 - 5R_1 \to R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & 5/3 & 2/3 & -1/6 & 0 \\ 0 & -6 & -12 & -5 & 0 & 1 \end{array}\right]$$

$$R_3 + R_2(6) \to R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & 5/3 & 2/3 & -1/6 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array}\right] \quad R_3\left(-\tfrac{1}{2}\right) \to R_3 \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & 5/3 & 2/3 & -1/6 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

$$R_2 - \tfrac{5}{3}(R_3) \to R_2 \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/6 & -1 & 5/6 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right] \quad R_1 - R_2(2) \to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 4/3 & 2 & -5/3 \\ 0 & 1 & 0 & -1/6 & -1 & 5/6 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

$$R_1 - R_3(4) \to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/3 & 0 & 1/3 \\ 0 & 1 & 0 & -1/6 & -1 & 5/6 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

Therefore inverse of F $\Rightarrow F^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/3 & 0 & 1/3 \\ 0 & 1 & 0 & -1/6 & -1 & 5/6 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$