

Systems Optimization - Assignment 4 Advanced LP

Daniel Carpenter – 113009743

1.a) Dual

$$\begin{aligned} \text{Min } z &= 810.50w_1 + 655.80w_2 + 520.75w_3 \\ \text{s.t. } &100.05w_1 + 5.50w_2 + 75.30w_3 \geq 275.75 \\ &60.75w_1 + 10.25w_2 + 24.84w_3 \geq 120.50 \\ &w_1, w_2, w_3 \geq 0 \end{aligned}$$

1.b) Dual variable explanation

In order to obtain optimal profit, you must use 1.064515 units of w_1 and 0 units of w_2 .

Dual Results explanation

$$\text{Optimal profit} = \$2,033.2339$$

The dual objective will be the same as the primal, since the dual would be the upper bound estimate. Convergence implies optimality.

Values of dual variables w_1, w_2 and w_3 will correspond with the primal's values of s_1, s_2 , and s_3 respectively.

(Python code on next page)

Snapshot of Problem 1 (b):

The screenshot displays the Spyder Python IDE interface. The main editor window shows a Python script named `un99c0.py` defining a Gurobi optimization problem. The script includes imports for `gurobi` and `math`, and defines a model with three continuous variables (`w1`, `w2`, `w3`) and one linear expression (`z`). The objective function is to maximize `z`, which is defined as `z = (818.58*w1) + (455.88*w2) + (528.75*w3)`. The model is solved using the Gurobi optimizer, and the optimal solution is printed to the console.

The right-hand pane shows the variable explorer, listing the variables and their values:

Name	Type	Size	Value
model	Model	1	Model object of gurobipy module
modelib	Model	1	Model object of gurobipy module
modelid	Model	1	Model object of gurobipy module
w1	Var	1	Var object of gurobipy module
w2	Var	1	Var object of gurobipy module
w3	Var	1	Var object of gurobipy module
z	LinExpr	1	LinExpr object of gurobipy module

The bottom-right pane shows the console output, which includes the Gurobi optimizer version (9.1.2 build v9.1.2rc0 (win64)), the thread count (10 physical cores, 20 logical processors), and the optimal solution values:

```
--- Production Variables ---
w1: 1.064515863662119
w2: 0.0

In [9]: runcell(0, 'C:/Users/daniel.carpenter/OneDrive - the Chickasaw Nation/Documents/GitHub/OU-ISA/Systems Optimization/02 - Notes and Assignments/week 09 - Advanced LP/Python - Advanced LP/Python/Problem 1.py')

SOLUTION TO PROBLEM 1
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (win64)
Thread count: 10 physical cores, 20 logical processors, using up to 20 threads
Optimize a model with 2 rows, 3 columns and 5 nonzeros
Model fingerprint: 0b44da5c
Coefficient statistics:
  Matrix range [6e+00, 1e+02]
  Objective range [5e+02, 6e+02]
  Bounds range [0e+00, 0e+00]
  RHS range [1e+02, 3e+02]
Presolve removed 0 rows and 1 columns
Presolve time: 0.00s
Presolved: 2 rows, 2 columns, 4 nonzeros

Iteration    Objective      Primal Inf.    Dual Inf.     Time
  0      0.000000e+00      0.43927e+01      0.000000e+00      0s
  2      2.032340e+03      0.000000e+00      0.000000e+00      0s

Solved in 2 iterations and 0.00 seconds
Optimal objective 2.03233991e+03

Optimal value (profit in USD Thousands): 2033.2339909558537
--- Production Variables ---
w1: 1.064515863662119
w2: 0.0
w3: 2.2476131192643427
```

1(c) Min $z =$

$$\begin{array}{rcl}
 810.50 w_1 & + & 655.80 w_2 + 520.75 w_3 \\
 100.05 w_1 & + & 75.30 w_2 + 75.30 w_3 \geq 275.75 \\
 60.75 w_1 & + & 24.84 w_2 + 24.84 w_3 \geq 120.50
 \end{array}$$

$$\begin{array}{rcl}
 w_1 & & \geq 0 \\
 & w_2 & \geq 0 \\
 & & w_3 \geq 0
 \end{array}$$

1(d) Dual variables

$$\begin{array}{rcl}
 w_1 & = & 1.06451 \dots \\
 w_2 & = & 0.00 \dots \\
 w_3 & = & 2.2476131 \dots
 \end{array}$$

Similarity to problem 1(b), the dual variables w_1 , w_2 , and w_3 correspond to the values of the primal's s_1 , s_2 and s_3 , respectively.

The objective function's value is associated with the outcome of the primal, since it is optimal.

The results correspond exactly to part 1(b).

Snapshot of Problem 1 (d):

The screenshot displays a Jupyter Notebook titled "Problem 1.py" within the Spyder Python IDE. The notebook contains a Gurobi optimization model for a production planning problem. The model is defined with three decision variables: $w1$, $w2$, and $w3$, all of type `GUR.CONTINUOUS`. The objective function is to maximize profit, calculated as $180 \cdot w1 + 75 \cdot w2 + 100 \cdot w3$. Constraints include resource limits: $180 \cdot w1 + 40 \cdot w2 + 50 \cdot w3 \leq 275.75$, $40 \cdot w1 + 75 \cdot w2 + 10 \cdot w3 \leq 120.50$, and $w1 \leq 4$, $w2 \leq 4$, $w3 \leq 4$. The model is solved using the Gurobi Optimizer, and the optimal solution is printed.

```
67 Problem 1. (d)
68
69 """
70
71 # likely not the right standard from estimate, watch videos or see slides
72
73 modelid = Model("Ready_Make_Company")
74
75 # DECISION VARIABLES
76
77 w1 = {}
78 w1 = modelid.addVar(vtype = GUR.CONTINUOUS,
79                    lb = 0,
80                    ub = GUR.INFINITY)
81
82 w2 = {}
83 w2 = modelid.addVar(vtype = GUR.CONTINUOUS,
84                    lb = 0,
85                    ub = GUR.INFINITY)
86
87 w3 = {}
88 w3 = modelid.addVar(vtype = GUR.CONTINUOUS,
89                    lb = 0,
90                    ub = GUR.INFINITY)
91
92
93 # OBJECTIVE FUNCTION
94
95 # z = (180.00*w1) + (40.00*w2) + (100.00*w3)
96
97 # set objective function to z
98 modelid.setObjective(z)
99
100 # define whether to minimize or maximize
101 modelid.setSense = GUR.MAXIMIZE
102
103 modelid.update()
104
105 # ADD CONSTRAINTS
106
107 modelid.addConstr(180.00*w1 + 40.00*w2 + 50.00*w3 >= 275.75)
108 modelid.addConstr(40.00*w1 + 75.00*w2 + 10.00*w3 >= 120.50)
109 modelid.addConstr(w1 >= 4)
110 modelid.addConstr(w2 >= 4)
111 modelid.addConstr(w3 >= 4)
112
113 # SOLVE MODEL
114
115 # Optimize
116 print("SOLUTION TO PROBLEM 2 .....")
117 modelid.optimize()
118
119 # Print Results
120 if (modelid.status == GUR.OPTIMAL):
121     print("Optimal value (Profit in USD Thousands):", modelid.objval)
122     print("Production Variables ---")
123     print("w1:", w1.x)
124     print("w2:", w2.x)
125     print("w3:", w3.x)
```

The console output shows the Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (win64) solving the model. The optimal value (profit in USD Thousands) is 2033.233909558537. The production variables are: $w1 = 1.06451981662119$, $w2 = 0.8$, and $w3 = 2.2476131192643427$.

Name	Type	Size	Value
model	Model	1	Model object of gurobipy module
modelid	Model	1	Model object of gurobipy module
modelid	Model	1	Model object of gurobipy module
w1	Var	1	Var object of gurobipy module
w2	Var	1	Var object of gurobipy module
w3	Var	1	Var object of gurobipy module
z	LinExpr	1	LinExpr object of gurobipy module

1(e) First, it would not be advantageous to change the allocation of electricity, since that constraint is not active. Also, this is shown that the dual is zero for w_2 .

However, increasing the maximum allowable limit of the constraints associated with water or/and gas could lead to an increased optimal while keeping the basis the same.

Profit per Resource Δ : $\left(\begin{array}{l} \text{Dual } w_1 : 1.0645151 \\ w_2 : 2.2476131 \end{array} \right)$

Marginal Δ in obj. Fun:

Water: $1.0645151 - 1.00 = 0.0645151$ per unit profit

Amt to increase: (see excel) = 463.073 units

$463.073 \times 0.0645151 = 29.8752$ ^{total} increase in obj. fun.

Marginal Δ in obj. Fun.

Gas: $2.24761 - 0.75 = \$1.497613$ per unit profit

Amt to increase: (see excel) =

$89.2515 \times \$1.497613 = \133.6642 ^{Total increase} in obj. fun.

Snapshot of Problem 1 (e): *(Please see Excel file for all calculations)*

[illegible]

Snapshot of Problem 1 (f): (Please see Excel file for all calculations)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2				1	2	3	4	5	6							
3				x1	x2	x3	s1	s2	s3							
4		A	1	100.05	60.75	80.55	1	0	0				b	810.5		
5			2	5.50	10.25	8.35	0	1	0					655.8		
6			3	75.3	24.84	50.43	0	0	1					520.75		
7																
8																
9																
10				x1	x2	x3	s1	s2	s3							
11		c^T		275.75	120.50	170.45	0	0	0							
12																
13				1	2	5										
14				x1	x2	s2										
15		B	1	100.05	60.75	0				B^1	-0.01189	0	0.0290777			
16			2	5.5	10.25	1					0.0360419	0	-0.047888			
17			3	75.3	24.84	0					-0.304037	1	0.3309289			
18																
19																
20																
21		c^T_B		275.75	120.5	0										
22																
23																
24																
25																
26		basic	z	x1	x2	s1	s2	s3	solution							
27		z	1.00	-	-	28.64	1.06	-	2,033.23							Solution optimal since no more promising directions.
28		x1	-	1.00	0.00	0.51	(0.01)	-	5.51							
29		x2	-	0.00	1.00	0.49	0.04	-	4.27							
30		s2	-	-	0.00	0.55	(0.30)	1.00	581.71							
31																

1(f) Using algebraic sensitivity analysis (see excel), the previous model's optimal equals the new model's optimal. Hence, the new constraint is not active.

2(a)

$$\begin{aligned} \text{Max } & 2000w_1 + 1500w_2 + 500w_3 - 9000w_4 - 6000w_5 \\ \text{s.t. } & 0.3w_1 + 0.4w_2 + 0.2w_3 - w_4 \leq 20 \\ & 0.4w_1 + 0.2w_2 + 0.3w_3 - w_5 \leq 15 \end{aligned}$$

2(b)

Dual variables associate with primal variables

s_1, s_2, s_3, s_4 , and s_5 .

$$w_1 = 20$$

$$w_2 = 35$$

$$w_3 = 0$$

$$w_4 = 0$$

$$w_5 = 0$$

Duals

no opportunity/not active

Dual objective value associates directly with the outcome of primal, which makes sense because dual and primal should converge.

Snapshot of Problem 2 (b): (Please see Excel file for all calculations)

The screenshot displays an Excel spreadsheet and the Solver Parameters dialog box for a linear programming problem.

Excel Spreadsheet Data:

	w1	w2	w3	w4	w5	Solution
Max	2000	1500	500	-9000	-6000	92,500
s.t.	0.3	0.4	0.2	-1.0	0.0	20.0
	0.4	0.2	0.3	0.0	-1.0	15.0

Solver Parameters Dialog Box:

- Set Objective:** \$H\$3
- To:** ☒ Max ☐ Min ☐ Value Of: 0
- By Changing Variable Cells:** \$C\$3:\$G\$3
- Subject to the Constraints:** \$H\$6:\$H\$7 <= \$J\$6:\$J\$7
- ☒ Make Unconstrained Variables Non-Negative
- Select a Solving Method:** Simplex LP
- Solving Method:** Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

$$\begin{aligned}
 \text{Min } z &= 2000w_1 + 1500w_2 + 500w_3 + 9000w_4 + 6000w_5 \\
 \text{s.t.} \quad &0.3w_1 + 0.4w_2 + 0.2w_3 + w_4 \geq 20 \\
 &0.4w_1 + 0.2w_2 + 0.3w_3 + w_5 \geq 15 \\
 &-w_1 \geq 0 \\
 &-w_2 \geq 0 \\
 &-w_3 \geq 0 \\
 &w_4 \geq 0 \\
 &w_5 \geq 0
 \end{aligned}$$

2(d) Dual Vars associate with slack variables of primal.
 Objective function output relates exactly to primal output. Please see next explanation for examples.

Snapshot of Problem 2 (d): (Please see Excel file for all calculations)

The screenshot displays an Excel spreadsheet for a linear programming problem and the Solver Parameters dialog box.

Excel Spreadsheet Data:

	w1	w2	w3	w4	w5	Solution
Max	2,000	1,500	500	9,000	6,000	92,500
s.t.	0.3	0.4	0.2	1.0	0.0	20.0
	0.4	0.2	0.3	0.0	1.0	15.0
	-1	0	0	0	0	-20.0
	0	-1	0	0	0	-35.0
	0	0	-1	0	0	0.0
	0	0	0	1	0	0.0
	0	0	0	0	1	0.0

Solver Parameters Dialog Box:

- Set Objective: \$H\$3
- To: ☒ Max ☐ Min ☐ Value Of: 0
- By Changing Variable Cells: \$C\$3:\$G\$3
- Subject to the Constraints: \$H\$6:\$H\$12 <= \$I\$6:\$I\$12
- ☒ Make Unconstrained Variables Non-Negative
- Select a Solving Method: Simplex LP
- Solving Method: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Snapshot of Problem 2 (e): (Please see Excel file for all calculations)

1. While keeping the supply fixed and the basis the same for optimality, the range of prices are the following:

i. High Price: Saudi: **\$30.00**, and Venezuela: \$15.00, total cost: **\$112,500.00**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2				1	2	3	4	5	6	7					
3				x1	x2	s1	s2	s3	s4	s5					
4		A	1	0.30	0.40	-1	0	0	0	0			b	2,000	
5			2	0.40	0.20	0	-1	0	0	0				1,500	
6			3	0.20	0.30	0	0	-1	0	0				500	
7			4	1.00	-	0	0	0	1	0				9,000	
8			5	-	1.00	0	0	0	0	1				6,000	
9															
10															
11															
12				x1	x2	s1	s2	s3	s4	s5					
13		c^T		30.00	15.00	0	0	0	0	0					
14															
15				5	1	2	6	7							
16				s3	x1	x2	s4	s5							
17		B	1	0.00	0.30	0.40	0.00	0.00		B^1	0.80	-0.10	-1.00	0.00	0.00
18			2	0.00	0.40	0.20	0.00	0.00			-2.00	4.00	0.00	0.00	0.00
19			3	-1.00	0.20	0.30	0.00	0.00			4.00	-3.00	0.00	0.00	0.00
20			4	0.00	1.00	0.00	1.00	0.00			2.00	-4.00	0.00	1.00	0.00
21			5	0.00	0.00	1.00	0.00	1.00			-4.00	3.00	0.00	0.00	1.00
22															
23			c^T_B	0	30	15	0	0							
24															
25															
26															
27															
28		basic	z	x1	x2	s1	s2	s3	s4	s5					
29		z	1.00	-	-	-	(75.00)	-	-	-					
30		s3	-	0.00	0.00	(0.80)	0.10	1.00	-	-					
31		x1	-	1.00	-	2.00	(4.00)	-	-	-					
32		x2	-	-	1.00	(4.00)	3.00	-	-	-					
33		s4	-	-	-	(2.00)	4.00	-	1.00	-					
34		s5	-	-	-	4.00	(3.00)	-	-	1.00					
35															

ii.

iii. Low Price: Saudi: **\$11.25**, and Venezuela: \$15.00, total cost: **\$75,000.00**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2				1	2	3	4	5	6	7					
3				x1	x2	s1	s2	s3	s4	s5					
4		A	1	0.30	0.40	-1	0	0	0	0			b	2,000	
5			2	0.40	0.20	0	-1	0	0	0				1,500	
6			3	0.20	0.30	0	0	-1	0	0				500	
7			4	1.00	-	0	0	0	1	0				9,000	
8			5	-	1.00	0	0	0	0	1				6,000	
9															
10															
11															
12				x1	x2	s1	s2	s3	s4	s5					
13		c^T		11.25	15.00	0	0	0	0	0					
14															
15				5	1	2	6	7							
16				s3	x1	x2	s4	s5							
17		B	1	0.00	0.30	0.40	0.00	0.00		B^1	0.80	-0.10	-1.00	0.00	0.00
18			2	0.00	0.40	0.20	0.00	0.00			-2.00	4.00	0.00	0.00	0.00
19			3	-1.00	0.20	0.30	0.00	0.00			4.00	-3.00	0.00	0.00	0.00
20			4	0.00	1.00	0.00	1.00	0.00			2.00	-4.00	0.00	1.00	0.00
21			5	0.00	0.00	1.00	0.00	1.00			-4.00	3.00	0.00	0.00	1.00
22															
23			c^T_B	0	11.25	15	0	0							
24															
25															
26															
27															
28		basic	z	x1	x2	s1	s2	s3	s4	s5					
29		z	1.00	0.00	0.00	(37.50)	(0.00)	-	-	-					
30		s3	-	0.00	0.00	(0.80)	0.10	1.00	-	-					
31		x1	-	1.00	-	2.00	(4.00)	-	-	-					
32		x2	-	-	1.00	(4.00)	3.00	-	-	-					
33		s4	-	-	-	(2.00)	4.00	-	1.00	-					
34		s5	-	-	-	4.00	(3.00)	-	-	1.00					
35															

iv.

Snapshot of Problem 2 (f): (Please see Excel file for all calculations)

2. While keeping the supply fixed and the basis the same for optimality, the range of prices are the following:

a. High Price: Saudi: \$20.00, and Venezuela: **\$26.67**, total cost: **\$133,333.33**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2				1	2	3	4	5	6	7						
3				x1	x2	s1	s2	s3	s4	s5						
4		A	1	0.30	0.40	-1	0	0	0	0			b	2,000		
5			2	0.40	0.20	0	-1	0	0	0				1,500		
6			3	0.20	0.30	0	0	-1	0	0				500		
7			4	1.00	-	0	0	0	1	0				9,000		
8			5	-	1.00	0	0	0	0	1				6,000		
9																
10																
11																
12				x1	x2	s1	s2	s3	s4	s5						
13		c^T		20.00	26.67	0	0	0	0	0						
14				5	1	2	6	7								
15				s3	x1	x2	s4	s5								
16		B	1	0.00	0.30	0.40	0.00	0.00		B^1	0.80	-0.10	-1.00	0.00	0.00	
17			2	0.00	0.40	0.20	0.00	0.00			-2.00	4.00	0.00	0.00	0.00	
18			3	-1.00	0.20	0.30	0.00	0.00			4.00	-3.00	0.00	0.00	0.00	
19			4	0.00	1.00	0.00	1.00	0.00			2.00	-4.00	0.00	1.00	0.00	
20			5	0.00	0.00	1.00	0.00	1.00			-4.00	3.00	0.00	0.00	1.00	
21																
22		c^T_B		0	20	26.666667	0	0								
23																
24																
25																
26																
27																
28		basic	z	x1	x2	s1	s2	s3	s4	s5						solution (Total Cost)
29		z	1.00	0.00	0.00	(66.67)	(0.00)	-	-	-						133,333.33
30		s3	-	0.00	0.00	(0.80)	0.10	1.00	-	-						950.00
31		x1	-	1.00	-	2.00	(4.00)	-	-	-						2,000.00
32		x2	-	-	1.00	(4.00)	3.00	-	-	-						3,500.00
33		s4	-	-	-	(2.00)	4.00	-	1.00	-						7,000.00
34		s5	-	-	-	4.00	(3.00)	-	-	1.00						2,500.00
35																

b.

c. Low Price: Saudi: \$20.00, and Venezuela: **\$10.00**, total cost: **\$75,000.00**

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2				1	2	3	4	5	6	7						
3				x1	x2	s1	s2	s3	s4	s5						
4		A	1	0.30	0.40	-1	0	0	0	0			b	2,000		
5			2	0.40	0.20	0	-1	0	0	0				1,500		
6			3	0.20	0.30	0	0	-1	0	0				500		
7			4	1.00	-	0	0	0	1	0				9,000		
8			5	-	1.00	0	0	0	0	1				6,000		
9																
10																
11																
12				x1	x2	s1	s2	s3	s4	s5						
13		c^T		20.00	10.00	0	0	0	0	0						
14				5	1	2	6	7								
15				s3	x1	x2	s4	s5								
16		B	1	0.00	0.30	0.40	0.00	0.00		B^1	0.80	-0.10	-1.00	0.00	0.00	
17			2	0.00	0.40	0.20	0.00	0.00			-2.00	4.00	0.00	0.00	0.00	
18			3	-1.00	0.20	0.30	0.00	0.00			4.00	-3.00	0.00	0.00	0.00	
19			4	0.00	1.00	0.00	1.00	0.00			2.00	-4.00	0.00	1.00	0.00	
20			5	0.00	0.00	1.00	0.00	1.00			-4.00	3.00	0.00	0.00	1.00	
21																
22		c^T_B		0	20	10	0	0								
23																
24																
25																
26																
27																
28		basic	z	x1	x2	s1	s2	s3	s4	s5						solution (Total Cost)
29		z	1.00	-	-	-	(50.00)	-	-	-						75,000.00
30		s3	-	0.00	0.00	(0.80)	0.10	1.00	-	-						950.00
31		x1	-	1.00	-	2.00	(4.00)	-	-	-						2,000.00
32		x2	-	-	1.00	(4.00)	3.00	-	-	-						3,500.00
33		s4	-	-	-	(2.00)	4.00	-	1.00	-						7,000.00
34		s5	-	-	-	4.00	(3.00)	-	-	1.00						2,500.00
35																

d.