1.
a)

$$Primal:$$

$$\max 275.75x_1 + 120.50x_2$$

$$s.t.,$$

$$\begin{gathered}
100.05x_1 + 60.75x_2 \leqslant 810.50 \\
5.50x_1 + 10.25x_2 \leqslant 655.80 \\
75.30x_1 + 24.84x_2 \leqslant 520..75 \\
x_1 \geqslant 0 \\
x_2 \geqslant 0
\end{gathered}$$

$$Dual:$$

$$\min 810.50w_1 + 655.80w_2 + 520w_3$$

$$s.t.,$$

$$\begin{gathered}
100.05w_1 + 5.50w_2 + 75.30w_3 \geqslant 275.75 \\
60.75w_1 + 10.25w_2 + 24.84w_3 \geqslant 120.50
\end{gathered}$$

$$\begin{gathered}
w_1 \geqslant 0 \\
w_2 \geqslant 0 \\
w_3 \geqslant 0
\end{gathered}$$

b)

b)

```python
from gurobipy import *

m = Model('')

m.setParam(GRB.Param.OutputFlag, 0)
x_hard = m.addVar(vtype=GRB.CONTINUOUS, name = 'hard_seeds')
x_serrated = m.addVar(vtype=GRB.CONTINUOUS, name = 'serrated_seeds')

m.setObjective(275.75*x_hard + 120.50*x_serrated, GRB.MAXIMIZE)

c1_1 = m.addConstr(x_hard*100.05 + x_serrated*60.75 <= 810.50, name = 'water')
c2_1 = m.addConstr(x_hard*5.50 + x_serrated*10.25 <= 655.80, name ='electricity')
c3_1 = m.addConstr(x_hard*75.30 + x_serrated*24.84 <= 520.75, name = 'gas')

m.optimize()
print(f'Total profit: {m.objVal}')
for v in m.getVars():
    if v.varName == 'hard_seeds':
        print(f'Hard seed bags produced: {v.x}')
    else:
        print(f'Serratred seed bags produced: {v.x}')

for c in m.getConstrs():
    print(f'{c.constrName} shadow price: {c.Pi}')
```

```
Total profit: 2033.233990955054
Hard seed bags produced: 5.505725067524782
Serratred seed bags produced: 4.274110403195811
water shadow price: 1.0645150636621188
electricity shadow price: 0.0
gas shadow price: 2.247613119264343
```

Comment

The dual problem provides bounds for the primal objective function. At the optimal solution, by strong duality, both the primal and the dual's objective function values meet. THe dual variables can be seen as shadow prices (maximum amount of currrency willing to spend for an increase in the respective constraint's right hand side without changing the basis), or as the rate of change of the objective function of the primal with respect of changes on the respective right hand side of the constraints (without changing the basis).

c)

$$Primal:$$

$$\max 275.75x_1 + 120.50x_2$$

$$s.t.,$$

$$\begin{aligned} 100.05x_1 + 60.75x_2 + s_1 &= 810.50 \\ 5.50x_1 + 10.25x_2 + s_2 &= 655.80 \\ 75.30x_1 + 24.84x_2 + s_3 &= 520..75 \end{aligned}$$

$$x_1 \geqslant 0$$
$$x_2 \geqslant 0$$
$$s_1 \geqslant 0$$
$$s_2 \geqslant 0$$
$$s_3 \geqslant 0$$

$$Dual:$$

$$\min 810.50w_1 + 655.80w_2 + 520w_3$$

$$s.t.,$$

$$100.05w_1 + 5.50w_2 + 75.30w_3 \geqslant 275.75$$
$$60.75w_1 + 10.25w_2 + 24.84w_3 \geqslant 120.50$$
$$w_1 \geqslant 0$$
$$w_2 \geqslant 0$$
$$w_3 \geqslant 0$$
$$w_1 \ free$$
$$w_2 \ free$$
$$w_3 \ free$$

d)

d)

```
m = Model('')

m.setParam(GRB.Param.OutputFlag, 0)

w_1 = m.addVar(vtype=GRB.CONTINUOUS, lb = -GRB.INFINITY, ub = GRB.INFINITY, name = 'water')
w_2 = m.addVar(vtype=GRB.CONTINUOUS, lb = -GRB.INFINITY, ub = GRB.INFINITY, name = 'electricity')
w_3 = m.addVar(vtype=GRB.CONTINUOUS, lb = -GRB.INFINITY, ub = GRB.INFINITY, name = 'gas')

m.setObjective(w_1*810.50+w_2*655.80+w_3*520.75, GRB.MINIMIZE)

c1_1 = m.addConstr(w_1*100.05+5.50*w_2+75.30*w_3 >= 275.75, name = 'hard_seeds')
c2_1 = m.addConstr(w_1*60.75+10.25*w_2+24.84*w_3 >= 120.50, name ='serrated_seeds')
c2_3 = m.addConstr(w_1 >= 0, name ='slack_1')
c2_4 = m.addConstr(w_2 >= 0, name ='slack_2')
c2_5 = m.addConstr(w_3 >= 0, name ='slack_3')

m.optimize()

print(f'Total profit: {m.objVal}')
```

```
Total profit: 2033.2339909550537
```

e)

Increases

water: yes, electricity: no, gas: yes

	B				B^-1		
	100.05	60.75	0		-0.01188953	0	0.029077657
	5.5	10.25	1		0.03604194	0	-0.047888388
	75.3	24.84	0		-0.30403741	1	0.330928862
	x1	x2	s2				
Cbt	275.75	120.5	0				
				beta		beta_1	Profit = Cbt*B^-1*Beta-Beta
	xb	5.50572507		1	-0.01188953	463.07337	29.87520792
		4.2741104	+	0	0.03604194		
		581.70888		0	-0.30403741		
						beta_2	
		5.50572507	+	0	0	0	
		4.2741104		1	0		
		581.70888		0	1		
		5.50572507	+	0	0.02907766	beta_3	Profit
		4.2741104		0	-0.04788839	89.2514993	111.3513414
		581.70888		1	0.33092886		

f)

z	x1	x2	x3	s1	s2	s3	sol
1	-275.75	-120.5	-170.45	0	0	0	0
0	100.05	60.75	80.55	1	0	0	810.5
0	5.5	10.25	8.35	0	1	0	655.8
0	75.3	24.84	50.43	0	0	1	520.75
B				B^-1			
100.05	60.75	0		-0.01188953	0	0.029077657	
5.5	10.25	1		0.03604194	0	-0.047888388	
75.3	24.84	0		-0.30403741	1	0.330928862	
c	275.75	120.5	170.45	0	0	0	
cb	275.75	120.5	0				
	x1	x2	x3	s1	s2	s3	
reduced costs	0	0	28.643818	1.06451506	0	2.247613119	
	No promising direction						

2.

a)

$$Primal:$$

$$\min 20x_1 + 15x_2$$

$$s.t.,$$

$$0.3x_1 + 0.4x_2 \geqslant 2000$$
$$0.4x_1 + 0.2x_2 \geqslant 1500$$
$$0.2x_1 + 0.3x_2 \geqslant 500$$
$$x_1 \leqslant 9000$$
$$x_2 \leqslant 6000$$
$$x_1 \geqslant 0$$
$$x_2 \geqslant 0$$

$$Dual:$$

$$\max 2000w_1 + 1500w_2 + 500w_3 + 9000w_3 + 6000w_4$$

$$s.t.,$$

$$0.3w_1 + 0.4w_2 + 0.2w_3 + w_4 \leqslant 20$$
$$0.4w_1 + 0.2w_2 + 0.3w_3 + w_5 \leqslant 15$$

$$w_1 \geqslant 0$$
$$w_2 \geqslant 0$$
$$w_3 \geqslant 0$$
$$w_4 \leqslant 0$$
$$w_5 \leqslant 0$$

b)

z_dual	92500							
w1	20							
w2	35							
w3	0							
w4	0							
w5	0							
	w1	w2	w3	w4	w5	Totals		
Objective	2000	1500	500	9000	6000	92500		
x1	0.3	0.4	0.2	1	0	20	<=	20
x2	0.4	0.2	0.3	0	1	15	<=	15
	>=0	>=0	>=0	<=0	<=0			

c)

$$Primal:$$

$$\min 20x_1 + 15x_2$$

$$s.t.,$$

$$0.3x_1 + 0.4x_2 - s_1 = 2000$$
$$0.4x_1 + 0.2x_2 - s_2 = 1500$$
$$0.2x_1 + 0.3x_2 - s_3 = 500$$
$$x_1 + s_4 = 9000$$
$$x_2 + s_5 = 6000$$
$$x_1 \geqslant 0$$
$$x_2 \geqslant 0$$
$$s_1 \geqslant 0$$
$$s_2 \geqslant 0$$

$$s_3 \geqslant 0$$
$$s_4 \geqslant 0$$
$$s_5 \geqslant 0$$

$$Dual:$$

$$\max 2000w_1 + 1500w_2 + 500w_3 + 9000w_3 + 6000w_4$$

$$s.t.,$$

$$0.3w_1 + 0.4w_2 + 0.2w_3 + w_4 \leqslant 20$$
$$0.4w_1 + 0.2w_2 + 0.3w_3 + w_5 \leqslant 15$$

$$w_1 \geqslant 0$$
$$w_2 \geqslant 0$$
$$w_3 \geqslant 0$$
$$w_4 \leqslant 0$$
$$w_5 \leqslant 0$$

$$w_1 \ free$$
$$w_2 \ free$$
$$w_3 \ free$$
$$w_4 \ free$$
$$w_5 \ free$$

d)

z_dual	92500							
w1	20							
w2	35							
w3	0							
w4	0							
w5	0							
	w1	w2	w3	w4	w5	Totals		
Objective	2000	1500	500	9000	6000	92500		
x1	0.3	0.4	0.2	1	0	20	<=	20
x2	0.4	0.2	0.3	0	1	15	<=	15
s1	-1	0	0	0	0	-20	<=	0
s2	0	-1	0	0	0	-35	<=	0
s3	0	0	-1	0	0	0	<=	0
s4	0	0	0	1	0	0	<=	0
s5	0	0	0	0	1	0	<=	0
	free	free	free	free	free			

e & f)

B				B^-1							
	w1	w2			w1	w2					
	0.3	0.4			-2	4					
	0.4	0.2			4	-3					
			beta								
Xb	20	+	1	-2	10	0	-8.75	<=	saudi_change	<=	10
	35	-	0	4	8.75	0	11.25	<=	saudi_c_change	<=	30
	20	-	0	4	5	0	-5	<=	ven_change	<=	11.66667
	35	+	1	-3	11.66667	0	10	<=	ven_c_change	<=	26.66667