ISE 4623/5023: Deterministic Systems Models / Systems Optimization

University of Oklahoma
College of Engineering
School of Industrial and Systems Engineering
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Individual Assignment 2 (100 points)

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Problem 1 - (50 points)

Seeds Inc. is a company that produces and exports bags of corn seeds. For this purpose, the company has a production plant in the state of Oklahoma, in which the company processes corn of two varieties: Hard and Serrated. To make the seed bags, the plant uses three resources: water, electricity, and gas. The following table provides the basic data of the problem:

Resource	Hard (units of resources per bag of seeds)	Serrated (units of raw resources per bag of seeds)	Maximum monthly availability (units of Liters, kWh, and cm^3, respectively)
Water	100.05	60.75	810.50
Electricity	5.50	10.25	655.80
Gas	75.30	24.84	520.75

Profit per seed bag	$275.75	$120.50

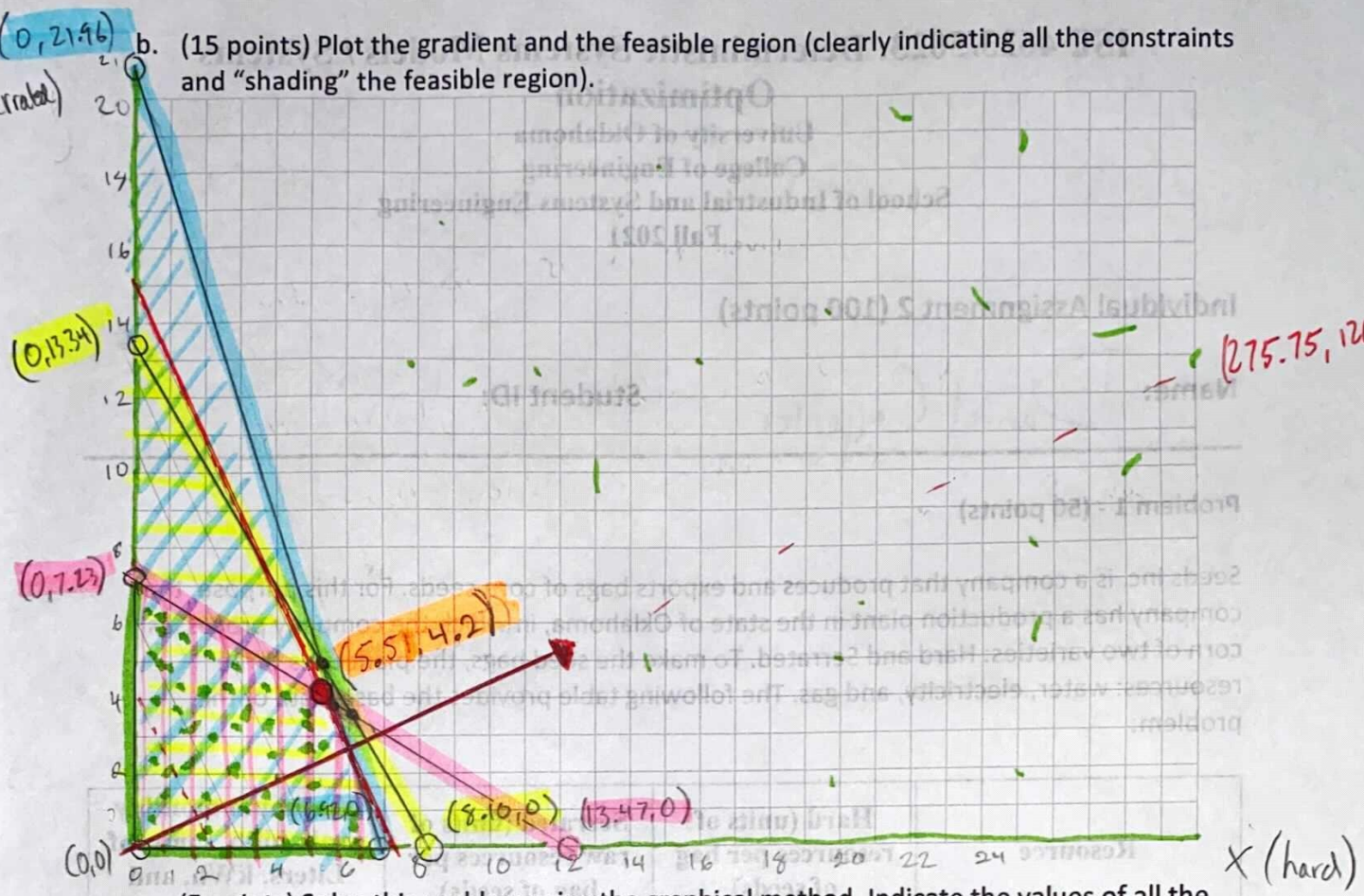
In this way, Seeds Inc. produces bags of seeds for both varieties of corn that it processes (i.e., hard corn seeds and serrated corn seeds). Given that Seeds Inc. is so famous and respected, the demand for her seed bags is always very high, so they always sell all the seed bags they produce, and they cand produce fractional numbers of seed bags (i.e., 7.22 bag of hard corn seed bags). You want to determine the production plan that retrieves Seeds Inc.'s profit. To do this, first you decide to formulate this problem as an LP model. In particular:

a. (5 points) Define the decision variables for this problem. What is its gradient?

$x \leftarrow$ optimal number of Hard corn seed bags
$y \leftarrow$ optimal number of Serrated corn seed bags.

gradient: $\left(\frac{df}{dx}, \frac{df}{dy}\right)$, or: $(275.75, 120.50)$

b. (15 points) Plot the gradient and the feasible region (clearly indicating all the constraints and "shading" the feasible region).



c. (5 points) Solve this problem using the graphical method. Indicate the values of all the variables and the objective function associated with the optimal solution.

C1) Water constraint: $100.05x + 60.75y \leq 810.50$

Solve y: $0x + 60.75y \leq 810.50$

$y \leq \frac{810.50}{60.75} \Rightarrow 13.34 \Rightarrow (0, 13.34)$

Solve x: $100.05x + 0 \leq 810.50$

$x \leq \frac{810.50}{100.05} \Rightarrow 8.10 \Rightarrow (8.10, 0)$

(2) ELECTRICITY: $5.50x + 10.25y \leq 74.09$

Solve y: $0 + 10.25y \leq 74.09 \Rightarrow \frac{74.09}{10.25} \Rightarrow 7.23 \Rightarrow (0, 7.23)$

Solve x: $5.50x + 0 \leq 74.09 \Rightarrow \frac{74.09}{5.50} \Rightarrow 13.47 \Rightarrow (13.47, 0)$

(3) GAS: $75.30x + 24.84y \leq 520.75$

Solve y: $0 + 24.84y \leq 520.75 \Rightarrow \frac{520.75}{24.84} \Rightarrow 20.96 \Rightarrow (0, 20.96)$

Solve x: $75.30x + 0 \leq 520.75 \Rightarrow \frac{520.75}{75.30} \Rightarrow 6.92 \Rightarrow (6.92, 0)$

C4/5) non-negative: $x \geq 0$

$y \geq 0$

d. **(10 points) Solve the problem using Excel Solver and indicate the values of all the variables and the objective function associated with the optimal solution. Compare your results with part c. Include a snapshot of your Excel model (Excel cells and the Solver window).**

Using Excel, I arrived at a very similar result to the graph. Obviously, excel has much more precision than the graph (by hand), but they closely resemble the final decision variables.

e. (15 points) Solve the problem using Gurobi/Python. Compare your results with parts c and d. Include a snapshot of your Gurobi/Python code and obtained results.

a) The results of Gurobi compare similar to how excel compares to the graphs

b) Gurobi yield one additional decimal place accuracy, which holds regardless of the amount precision tailored in Excel. However, these differences by no means materially impact the result (i.e., the economic significance between both have the same effect)

Problem 2 (50 points)

Andes Inc. is an oil company that has a refinery on the Texas coast. The refinery processes crude oil from Saudi Arabia and Venezuela, producing gasoline, diesel, and lubricants. The two crude oils differ in their chemical composition, which is why they produce different amounts of each product. A barrel of crude from Saudi Arabia produces 0.3 barrels of gasoline, 0.4 barrels of diesel, and 0.2 barrels of lubricants. On the other hand, a barrel from Venezuela produces 0.4 barrels of gasoline, 0.2 barrels of diesel, and 0.3 barrels of lubricants. The remaining 10% of the crude is lost in the refining process.

Crudes also differ in price and availability. Andes Inc. can buy up to 9,000 barrels per day from Saudi Arabia at a price of $20 per barrel. You can buy from Venezuela up to 6,000 barrels per day at a price of $ 15 per barrel.

The contracts established by Andes Inc. forces them to produce 2,000 barrels per day of gasoline, 1,500 barrels per day of diesel, and 500 barrels per day of lubricants.

You want to determine the supply plan for the crude oil that results in the least cost for Andes Inc. To do this, first you decide to formulate this problem as an LP model. In particular:

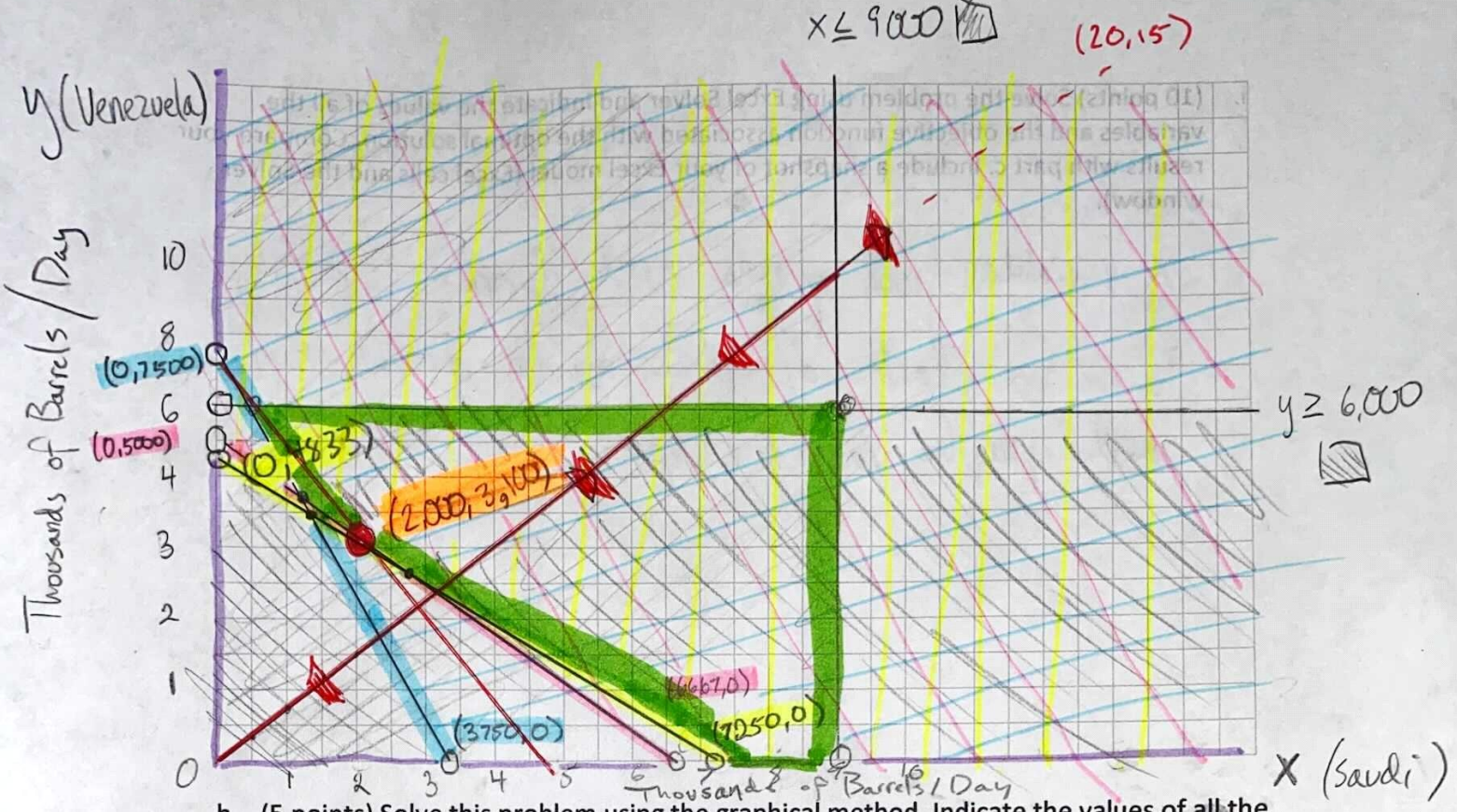
f. (5 points) Define the decision variables for this problem. What is its gradient?

$x \leftarrow$ barrels of Saudi Arabian crude oil

$y \leftarrow$ barrels of Venezuelan crude oil

gradient $\leftarrow \left(\frac{df}{dx}, \frac{df}{dy}\right) \Rightarrow (20, 15)$

g. (15 points) Plot the gradient and the feasible region (clearly indicating all the constraints and "shading" the feasible region).



h. (5 points) Solve this problem using the graphical method. Indicate the values of all the variables and the objective function associated with the optimal solution.

(1) Gasoline: $0.3x + 0.4y \geq 2{,}000$

solve x: $0.3x + 0 \geq 2{,}000$

$x \geq \frac{2{,}000}{0.3} = 6{,}667 \rightarrow (6{,}667, 0)$

solve y: $0 + 0.4y \geq 2000$

$y \geq \frac{2000}{0.4} = 5{,}000 \rightarrow (0, 5000)$

(2) Diesel: $0.4x + 0.2y \geq 1{,}500$

solve x: $0.4x + 0 \geq 1500$

$x \geq \frac{1500}{0.4} = 3{,}750 \Rightarrow (3{,}750, 0)$

solve y: $0 + 0.2y \geq 1500$

$y \geq \frac{1500}{0.2} = 7{,}500 \rightarrow (0, 7500)$

(3) Lubricants: $0.2x + 0.3y \geq 500$

solve x: $0.2x + 0 \geq 500$

$x \geq \frac{500}{0.2} = 7{,}250 \rightarrow (7{,}250, 0)$

solve y: $0 + 0.3y \geq 500$

$y \geq \frac{500}{0.3} = 4{,}833 \rightarrow (0, 4833)$

(4/5) Floor: $x \geq 0$, $y \geq 0$ | (6/7) Ceiling: $x \leq 9{,}000$ $y \leq 6000$

i. **(10 points) Solve the problem using Excel Solver and indicate the values of all the variables and the objective function associated with the optimal solution. Compare your results with part c. Include a snapshot of your Excel model (Excel cells and the Solver window).**

X variables from graph and Excel exact.
Due to lack of precision in graph, Excel and graph varied slightly.

j. (15 points) Solve the problem using Gurobi/Python. Compare your results with parts c and d. Include a snapshot of your Gurobi/Python code and obtained results.

Excel and Gurobi/python matched exactly.

Graph in-precision issue same as excel, which makes sense given Gurobi and Excel should match, but could vary with a hand drawn graph.