

Longest Narrow Path Polyominoes

75th AACTM Meeting Talk

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February 7, 2026

Outline

- 1 Definitions and Main Problem
- 2 Lower Bounds
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Polyominoes

Definition (Solomon Golomb 1966)

An n -omino is a set of n rook-wise connected unit squares.

(Alternative) Definition

A shape formed by a set of unit squares such that each unit square is orthogonal to another unit square.



Problem Statement

- For some $n \in \mathbb{N}$, using a set of one of each of the n -ominoes what is the longest possible "path", interior hole, which is only one unit square wide at any position and is solidly surrounded (diagonals are filled in)?

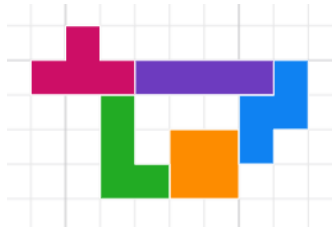


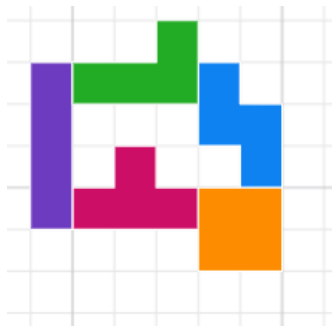
Figure: A length 4 path where $n = 4$.

Known Lower Bounds

- monominoes, dominoes, and triominoes are impossible.
- Kadon Enterprises asked about hexominoes, Aad van de Wetering gave a construction for 146 in 2004.
- Aad van de Wetering 2004, Eric Harshbarger 2005, and Dan Edlefsen 2006 each found a unique 36 path length for pentominoes

Tetrominoes Lower Bound

- There are no apparent results online, probably due to the "simplicity" of this version.
- Here is a construction for length of 6, which is probably the best you can do.



Main Result

- Tetrominoes: lower bound 6, upper bound 8
- Pentominoes: lower bound 36, upper bound 38
- Hexominoes: lower bound 146, upper bound 162

Hexomino Lower Bound

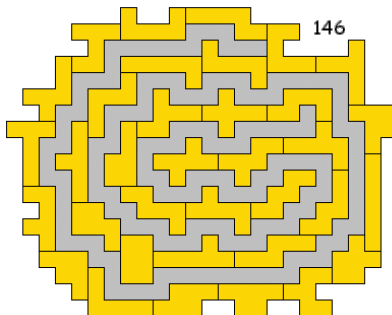


Figure: Aad van de Wetering 146 length hexomino construction

Main Result

- Tetrominoes with Pentominoes: lower bound 44, upper bound 54
- Tetrominoes with Hexominoes*: lower bound 146, upper bound 180
- Pentominoes with Hexominoes*: lower bound 146, upper bound 218
- all three*: lower bound 146, 236

Proof

Trivial proof outline:

Step 1

Find the total perimeter of the desired set of polyominoes.

Step 2

Consider the average number of polyominoes a polyomino will border: 2.

Subtract these border segments twice (Handshake Lemma) from the perimeter to obtain T

Trivial upper bound: $\frac{T-2}{2}$

Proof

Proof outline:

Step 1

Note the area of the polyominoes and add that to the known lower bound length to obtain the minimum area A of the longest narrow path configuration.

Step 2

By the Isoperimetric Theorem for rectangles with fixed integer perimeters, the square and m by $m + 1$ rectangle are the shapes with maximum area.

Proof

Proof continue:

Thus, we find the smallest m such that A fits in a m by m or m by $m + 1$ rectangle. Thus, the minimum exterior perimeter of the configuration of the longest narrow path for by the polyominoes is $4m$ or $4m + 4$.

Step 3

Upper bound: $\frac{T-4m\pm 2}{2}$

Limitations of the Proof

- The proof works for any set of polyominoes.
- With improved lower bound, the upper bound can improve
- Once the lower and upper bound become close, like for pentominoes, the proof can not narrow down the gap any more
- the proof does not address any constraints due to the shape of the polyominoes beyond area and perimeter

Open Problems

- ❶ Q: Using pentominoes, can you construct a path such that the resulting configuration fits in a 10 by 10 square with no more than one pair of pentominoes having a border of length 2 with the rest of the pairs having borders of at most 1?
- ❷ Q: Using pentominoes can you construct a path that fits in a 10 by 11 rectangle with every pair of pentominoes sharing at most a border of length 1?
- ❸ Q: What are lower bounds for multiple sets of polyominoes? (Only two sets of pentominoes have a nontrivial lower bound that is known)

Pentomino Lower Bound

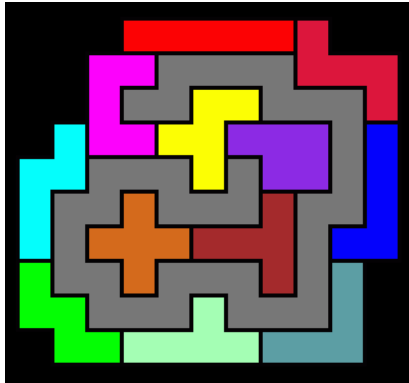


Figure: Aad van de Wetering 36 length pentomino construction

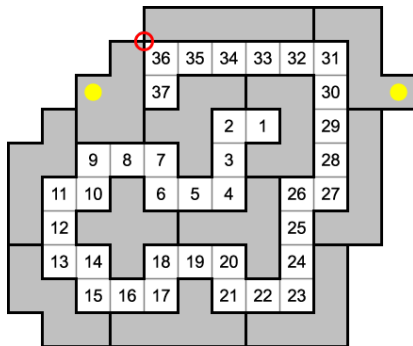


Figure: Erich Harsbarger in 2018 found a "near miss" length 37

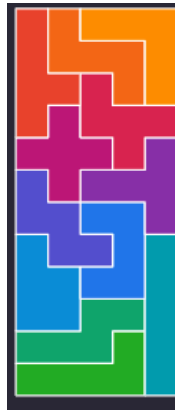
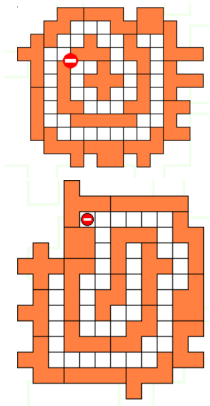
Variations

- 1 V: Polyominoes use squares, what about using other regular polygons?
- 2 V: The problem for $n > 8$ polyominoes is ill defined, and heptominoes are also possibly ill define due to having holes. How would you generalize polyominoes with holes?
- 3 V: What about volume paths of polycubes, or 3D paths of polyominoids?
- 4 V: Allow touching diagonally or not require the path to be border at corners. (this variation for pentominoes have shown up in *The Journal of Recreational Mathematics*)

Outreach

- Jeffrey Musyt (Slippery Rock University) at JMM gave the idea of a Math Circle in a Box
- I want to adjust this idea to use at Destination STEM

Circle in an Envelope



Questions?

Thank you for your time.

And thanks to Eric Harshbarger for maintaining much of the previously known results and introducing the problem to me.