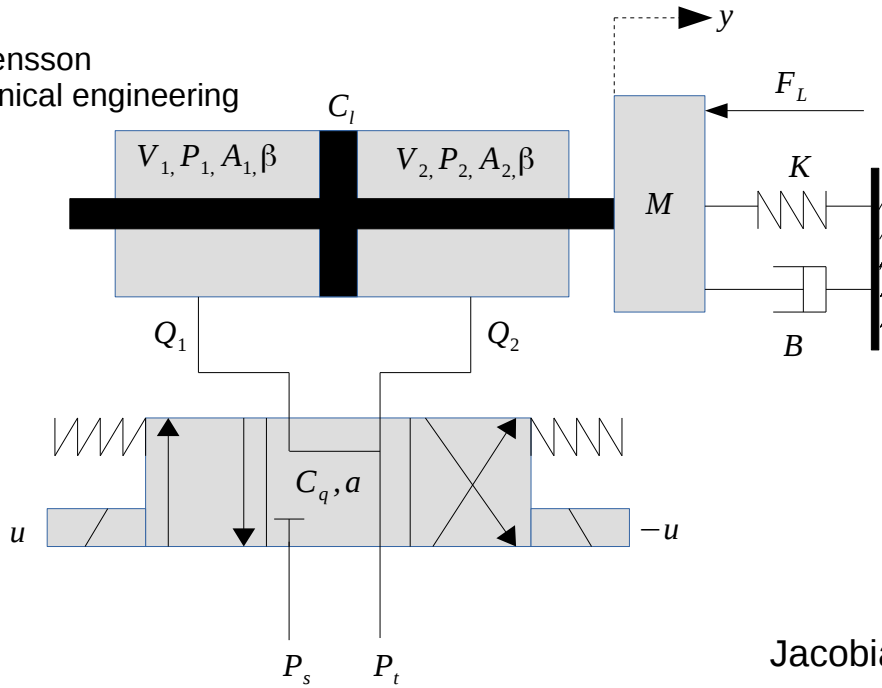By:
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Valve equation: Nonlinear - Linearized

$$Q_1 = C_q a u \sqrt{\frac{2}{\rho}(P_s - P_1)} \qquad Q_1 = q_1 u - p_1 P_1$$

$$Q_2 = C_q a u \sqrt{\frac{2}{\rho}(P_2 - P_t)} \qquad Q_2 = q_2 u + p_2 P_2$$

Acturator flow equation with valve equation:

$$Q_1 = A_1 \dot{y} + \frac{V_1}{\beta}\dot{P}_1 + C_l(P_1 - P_2) = q_1 u - p_1 P_1$$

$$Q_2 = A_2 \dot{y} - \frac{V_2}{\beta}\dot{P}_2 + C_l(P_1 - P_2) = q_2 u + p_2 P_2$$

Acturator force equation:

$$M\ddot{y} = A_1 P_1 - A_2 P_2 - K y - B\dot{y} - F_L$$

Jacobian

$$q_1 = \frac{\partial Q_1}{\partial u} = C_q a \sqrt{\frac{2}{\rho}(P_s - P_1)}$$

$$p_1 = \frac{\partial Q_1}{\partial P_1} = -\frac{C_q a u}{2\sqrt{\rho(P_s - P_1)}}$$

$$q_2 = \frac{\partial Q_1}{\partial u} = C_q a \sqrt{\frac{2}{\rho}(P_2 - P_t)}$$

$$p_2 = \frac{\partial Q_2}{\partial P_2} = \frac{C_q a u}{2\sqrt{\rho(P_2 - P_t)}}$$

State space model:

$$
\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} =
\begin{bmatrix}
-\dfrac{\beta(p_1 + C_l)}{V_1} & \dfrac{\beta C_l}{V_1} & 0 & -\dfrac{\beta A_1}{V_1} \\[2mm]
\dfrac{\beta C_l}{V_2} & -\dfrac{\beta(p_2 + C_l)}{V_2} & 0 & \dfrac{\beta A_2}{V_2} \\[2mm]
0 & 0 & 0 & 1 \\[2mm]
\dfrac{A_1}{M} & -\dfrac{A_2}{M} & -\dfrac{K}{M} & -\dfrac{B}{M}
\end{bmatrix}
\begin{bmatrix} P_1 \\ P_2 \\ y_1 \\ y_2 \end{bmatrix} +
\begin{bmatrix}
\dfrac{\beta q_1}{V_1} & 0 \\[2mm]
-\dfrac{\beta q_2}{V_2} & 0 \\[2mm]
0 & 0 \\[2mm]
0 & -\dfrac{1}{M}
\end{bmatrix}
\begin{bmatrix} u \\ F_L \end{bmatrix}
$$

$$
\begin{bmatrix} P_1 \\ P_2 \\ y \\ \dot{y} \end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} P_1 \\ P_2 \\ y_1 \\ y_2 \end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix} u \\ F_L \end{bmatrix}
$$