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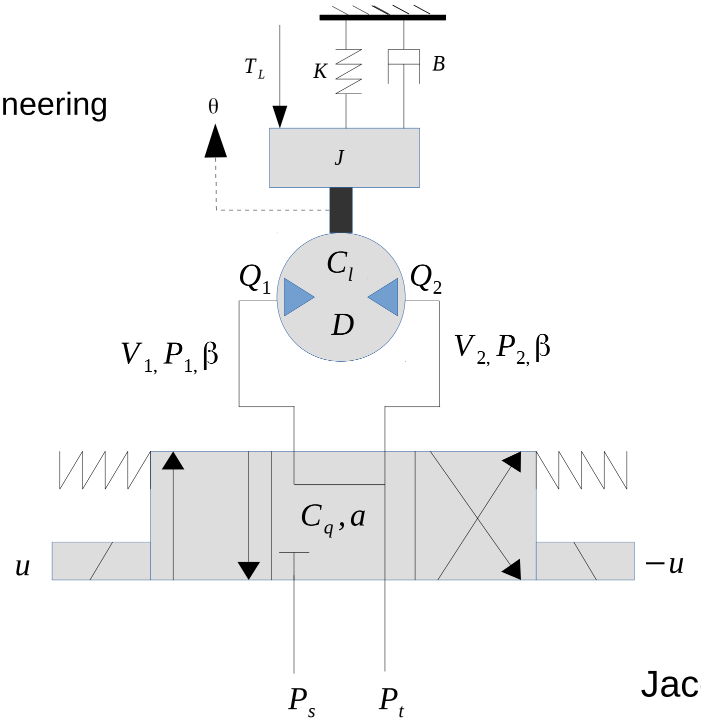

Valve equation: Nonlinear - Linearized

$$Q_1 = C_q a u \sqrt{\frac{2}{\rho}(P_s - P_1)} \qquad Q_1 = q_1 u - p_1 P_1$$

$$Q_2 = C_q a u \sqrt{\frac{2}{\rho}(P_2 - P_t)} \qquad Q_2 = q_2 u + p_2 P_2$$

Jacobian

$$q_1 = \frac{\partial Q_1}{\partial u} = C_q a \sqrt{\frac{2}{\rho}(P_s - P_1)}$$

$$p_1 = \frac{\partial Q_1}{\partial P_1} = -\frac{C_q a u}{2\sqrt{\rho(P_s - P_1)}}$$

$$q_2 = \frac{\partial Q_1}{\partial u} = C_q a \sqrt{\frac{2}{\rho}(P_2 - P_t)}$$

$$p_2 = \frac{\partial Q_2}{\partial P_2} = \frac{C_q a u}{2\sqrt{\rho(P_2 - P_t)}}$$

Acturator flow equation with valve equation:

$$Q_1 = D\dot{\theta} + \frac{V_1}{\beta}\dot{P}_1 + C_l(P_1 - P_2) = q_1 u - p_1 P_1$$

$$Q_2 = D\dot{\theta} - \frac{V_2}{\beta}\dot{P}_2 + C_l(P_1 - P_2) = q_2 u + p_2 P_2$$

Acturator force equation:

$$J\ddot{\theta} = D P_1 - D P_2 - K\theta - B\dot{\theta} - T_L$$

State space model:

$$\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -\frac{\beta(p_1 + C_l)}{V_1} & \frac{\beta C_l}{V_1} & 0 & -\frac{\beta D}{V_1} \\ \frac{\beta C_l}{V_2} & -\frac{\beta(p_2 + C_l)}{V_2} & 0 & \frac{\beta D}{V_2} \\ 0 & 0 & 0 & 1 \\ \frac{D}{J} & -\frac{D}{J} & -\frac{K}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} \frac{\beta q_1}{V_1} & 0 \\ -\frac{\beta q_2}{V_2} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} u \\ T_L \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ T_L \end{bmatrix}$$