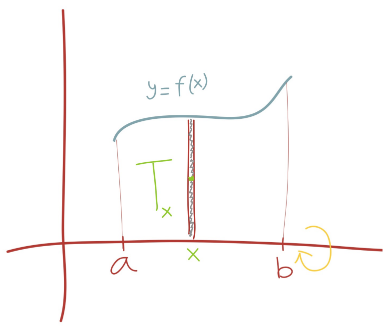
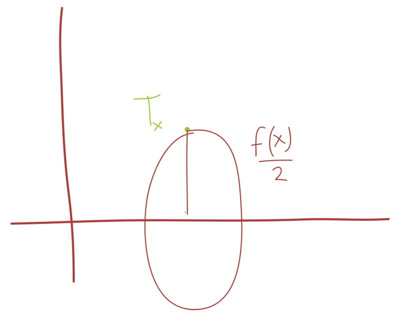
# 1 Pappos-guldins regel för rotationsvolymer



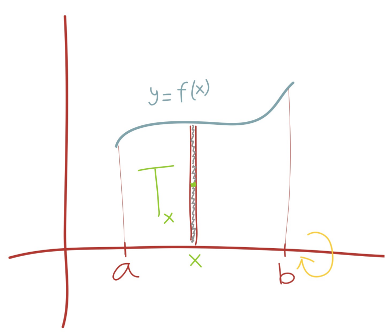



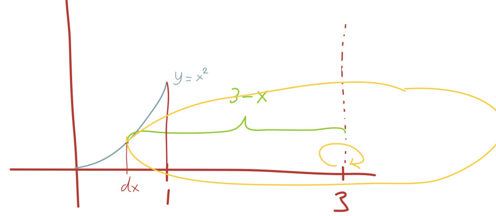

$$D = \{(x, y) : a \leq x \leq b, 0 \leq y \leq f(x)\}$$

$$A(x) = \pi f^2(x)$$

$$dV = \pi f^2(x)\ dx; T_x = (x, \frac{f(x)}{2})$$

$$dV = 2\pi \frac{f(x)}{2} f(x)\ dx$$




$$T_x = (x, \frac{f(x)}{2})$$

$$dV = 2\pi x f(x)dx$$

Pappos-guldins regel för rotationsvolym:

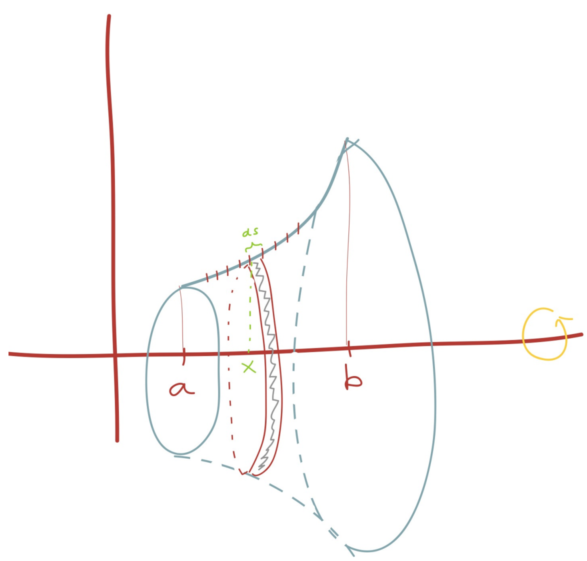
$$dV = tyngdpunktens\ vag * arean$$

## 1.1 Ex 1

Beräkna volymen av den kropp som uppkommer då

$$D = \{(x,y) : 0 \leq x \leq 1, 0 \leq y \leq x^2\}$$

roteras kring linjen $x = 3$




$$T_x = (x, \frac{x^2}{2})$$

$$dV = 2\pi(3-x)(f(x)dx)$$

$$f(x) = x^2$$

$$dV = 2\pi(3-x)(x^2dx)$$

$$V = 2\pi \int_0^1 (3-x)x^2dx = 2\pi \int_0^1 (3x^2 - x^3)dx = \cdots = \frac{3}{2}\pi$$

# 2 Rotationsarea

$$y = f(x), a \leq x \leq b, f, f' kontinuerliga$$

$$dA = 2\pi f(x)ds \implies A = \int_a^b dA = 2\pi \int_a^b f(x)ds = \left[ds = \sqrt{1 + f'(x)^2}dx\right] =$$

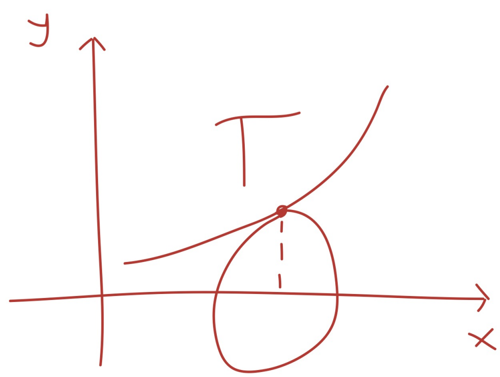
$$A = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}dx$$

Pappos-guldins regel för rotationsarea:

$$dA = 2\pi f(x)ds$$

## 2.1 Ex 2

Bestäm arean av den yta som uppkommer då $y = 2x, 1 \leq 2 \leq 2$ roteras ett varv




kring linjen $y = -2$

$$dA = 2\pi(f(x) + 2)ds = 2\pi(f(x) + 2)\sqrt{1 + f'^2(x)}dx$$

$$dA = 2\pi(2x + 2)\sqrt{1 + 2^2}dx$$

$$A = 2\pi \int_1^2 (2x + 2)\sqrt{5}dx = 4\pi\sqrt{5} \int_1^2 (x + 1)dx = 10\pi\sqrt{5}$$

# 3 Tyngdpunkten

$$D - homogent p = 1$$

$$m(D) = A(D)$$

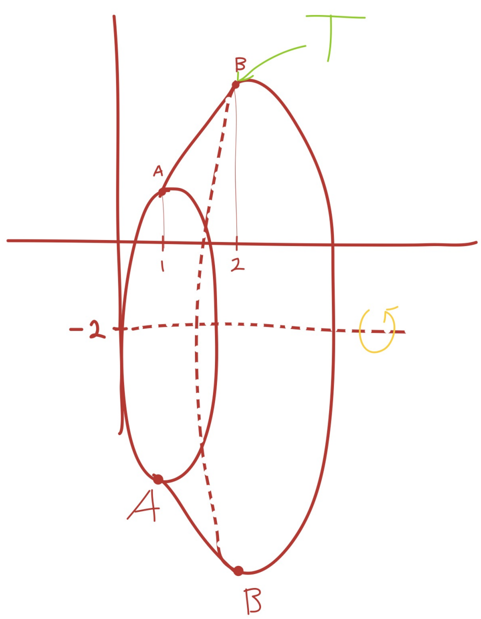
$$T = (x_T, y_T)$$

$$M_y = x_T m(D) = \int x dA \implies x_T \int dA = \int x dA$$

$$dM_y = x dA = x l(x) dx$$

$$m(D) = \int dA$$

$$x_T = \frac{\int x dA}{\int dA}$$




$$M_x = y_T \int dA = \int y dA$$

$$dM_x = y dA$$

$$y_T \int dA = \int y dA$$

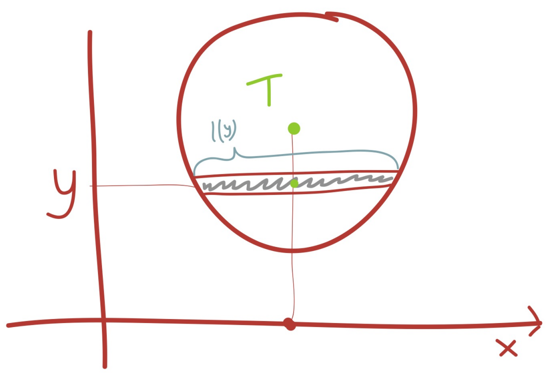
$$y_T = \frac{\int y dA}{\int dA}$$

$$dA = l(y)dy$$

$$y_T = \frac{\int y l(y) dy}{\int l(y) dy}$$

$$T = \left( \frac{\int x dA}{\int dA}, \frac{\int y dA}{\int dA} \right)$$

### 3.1




$$D = \{(x, y) : a \le x \le b, g(x) \le y \le f(x)\}$$

$$dA = (f(x) - g(x))dx$$

$$A = \int dA = \int_a^b (f(x) - g(x))dx$$

$$x_T \int_a^b (f(x)-g(x))dx = \int_a^b x dA$$

$$x_T \int_a^b (f(x)-g(x))dx = \int x(f(x)-g(x))dx$$

$$x_T = \frac{\int_a^b x(f(x)-g(x))dx}{\int_a^b (f(x)-g(x))dx}$$

$$dM_x = \frac{f(x)+g(x)}{2} dA = \frac{f(x)g(x)}{2}(f(x)-g(x))dx$$

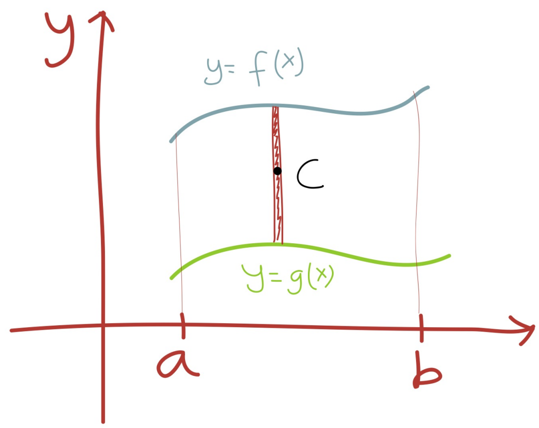
$$y_T \int dA = \int dM_x$$

$$\implies y_T \int_a^b dA = \int_a^b \frac{f(x)+g(x)}{2}(f(x)-g(x))dx$$

$$y_T \int_a^b (f(x)-g(x))dx = \frac{1}{2}\int_a^b (f^2(x)-g^2(x))dx$$

$$y_T = \frac{1}{2}\frac{\int_a^b (f^2(x)-g^2(x))dx}{\int_a^b f(x)-g(x)dx}$$

### 3.2 Ex 3




$AB = 2a$ $\Delta OAB$ likbent homogen triangel

$$T = (x_T, y_T); f(x) = \frac{a}{h}x; g(x) = -\frac{a}{h}x$$

$$x_T = \frac{\int_0^h x 2\frac{a}{h} x dx}{\int_0^h 2\frac{a}{h} x dx} =$$

$$= \frac{2\frac{a}{h}\int_0^h x^2 dx}{2\frac{a}{h}\int_0^h x dx} =$$

$$= \frac{\left[\frac{x^3}{3}\right]_0^h}{\left[\frac{x^2}{2}\right]_0^h} = \frac{\frac{1}{3}h^3}{\frac{1}{2}h^2} = \frac{2}{3}h; y_T = 0$$