UNIVERSITY OF BIRMINGHAM | COLLEGE OF ENGINEERING AND PHYSICAL SCIENCES

Nanophotonics

Lecture 16-19:Metamaterials

Angela Demetriadou

a.demetriadou@bham.ac.uk

Room G05a, Medical Physics

Office hours: Friday 14:00-15:00

Structure of the course

<u>Part I</u>: Introduction to Light-matter interaction

- Intro: Maxwell's equations, Constitutive relations
- Optical properties of materials
- Helmholtz equations
- Evanescent waves and propagating Surface Plasmon Polaritons

<u>Part II</u>: Mie theory and Nanoplasmonics

- Mie theory: Light interaction with spherical objects
- Nanoplasmonics, plasmonic nano-antennas and their applications
- Diffraction limit and near-field imaging techniques

- Numerical techniques

<u>Part III</u>: Periodic structures

- Metamaterials and Metasurfaces
- Negative refraction and the Perfect lens
- Transformation Optics and Cloaking
- Photonic Crystals, examples and applications

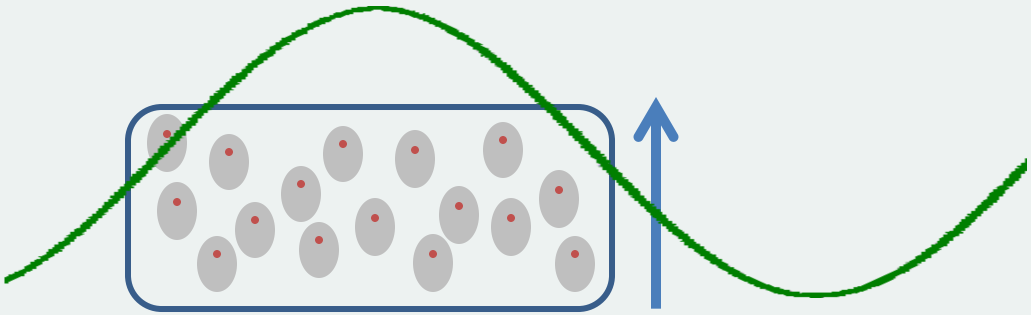
In this lecture

- What are metamaterials?
  - General concepts
  - Historical facts
  - The nanophotonics technology of the future (?)
- Electric metamaterials (Wire-mesh metamaterial)
  - An alternative route to plasmonics (?)
- Homogenization techniques
- Magnetic metamaterials (Split-Ring resonator –SRR)
- Negative refractive index
  - The first negative refracting metamaterial
  - Other designs of negative metamaterials (Mie resonances, fishnet structure)

Note: The recommended textbooks may not have a good description of metamaterials (it's a new research area). On each slide I include the relevant papers with all the info you need (they are also uploaded on Canvas).

Previously…

- **Optical properties of materials** arise from the collective response of their atoms to incident light (see lecture 2).



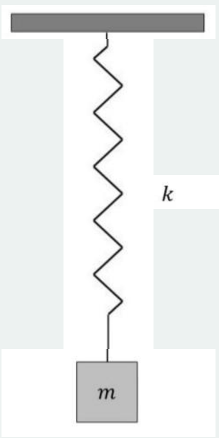
Since $\lambda \gg atoms$, we can **homogenize** the response, because atoms respond **collectively**:

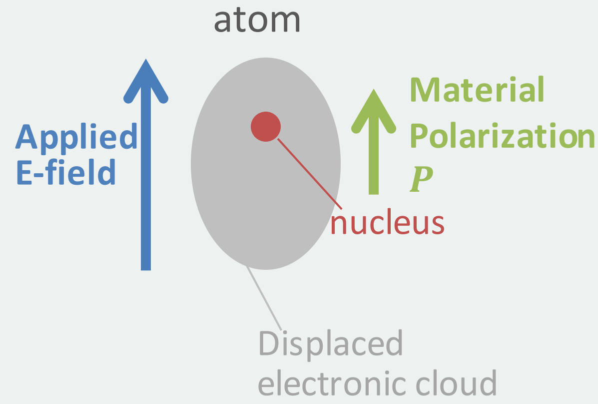
$$\boldsymbol{P} = -Nq_e\langle\boldsymbol{r}\rangle$$

where $\langle\boldsymbol{r}\rangle$ is the mean charge displacement of the N atoms

$\boldsymbol{P}$: density per volume of dipole moments

Polarized state





Electric dipole moment of each atom:

$$\boldsymbol{p} = -q_e\boldsymbol{r}$$

where $\boldsymbol{r}$ is the charge displacement

- For **dielectrics/insulators**: Lorentzian response for the electric permittivity:

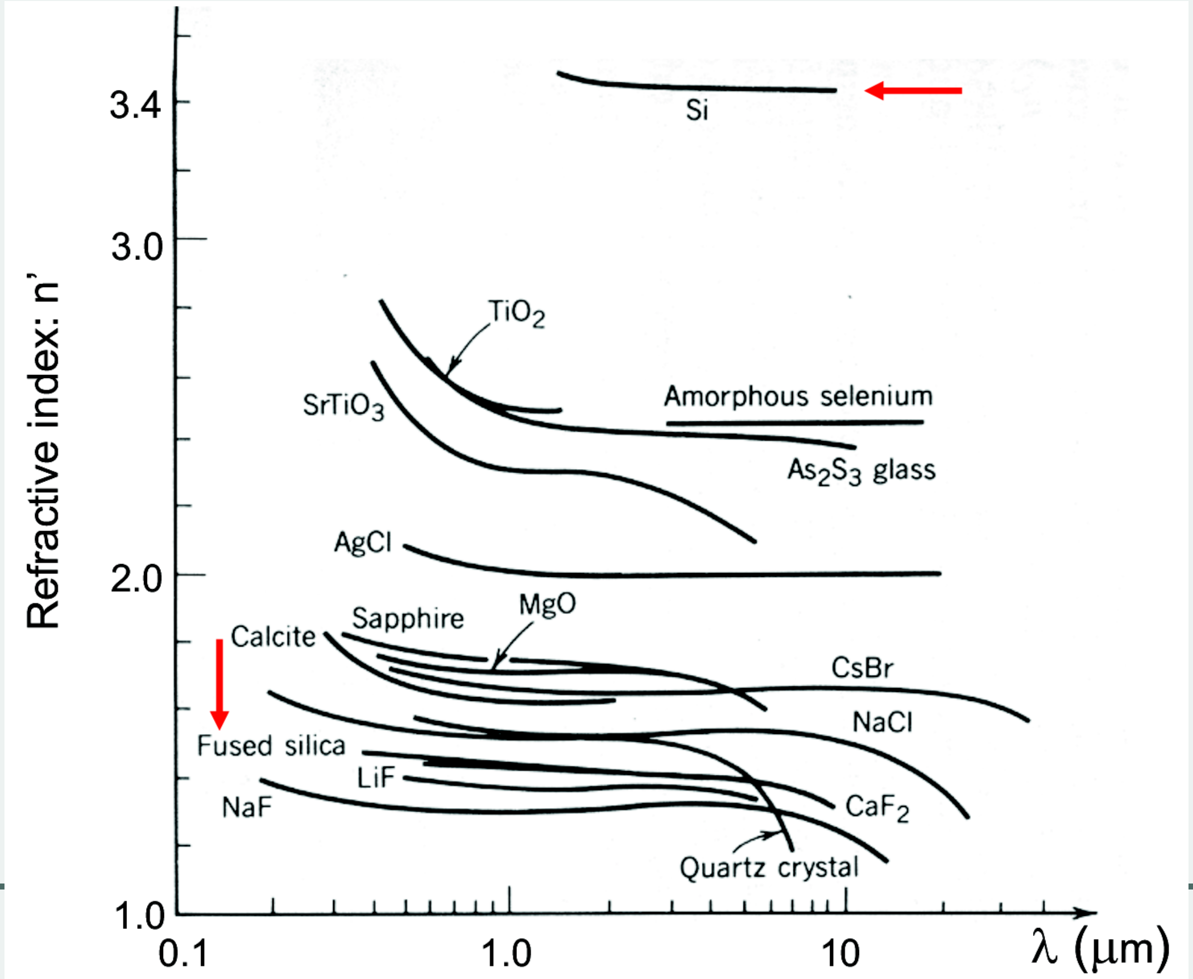
$$\boxed{\varepsilon = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\Gamma}}$$

where $\omega_p^2 = \frac{Nq_e^2}{\varepsilon_0\, m_e}$, N is the number density of the atoms, q_e the electron charge, m_e is the electron mass.

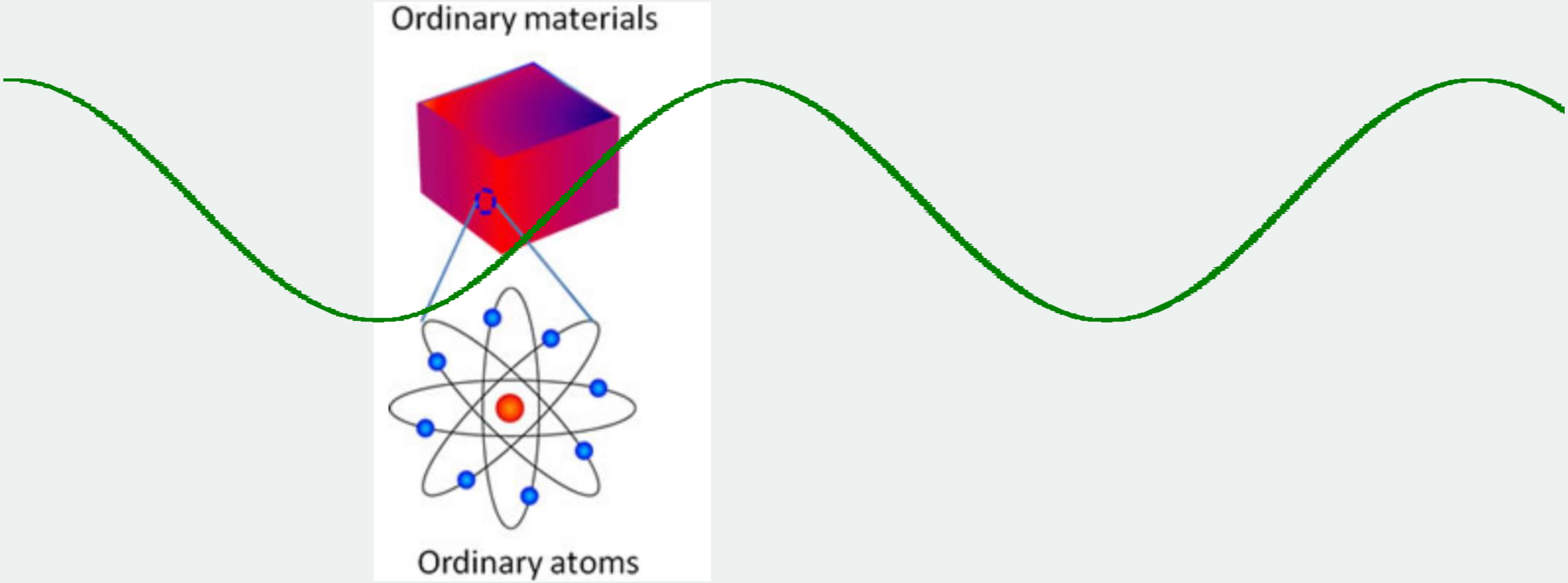
Previously…

- For **dielectrics/insulators**: Lorentzian response for the electric permittivity:

$$\boxed{\varepsilon = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\Gamma}}$$

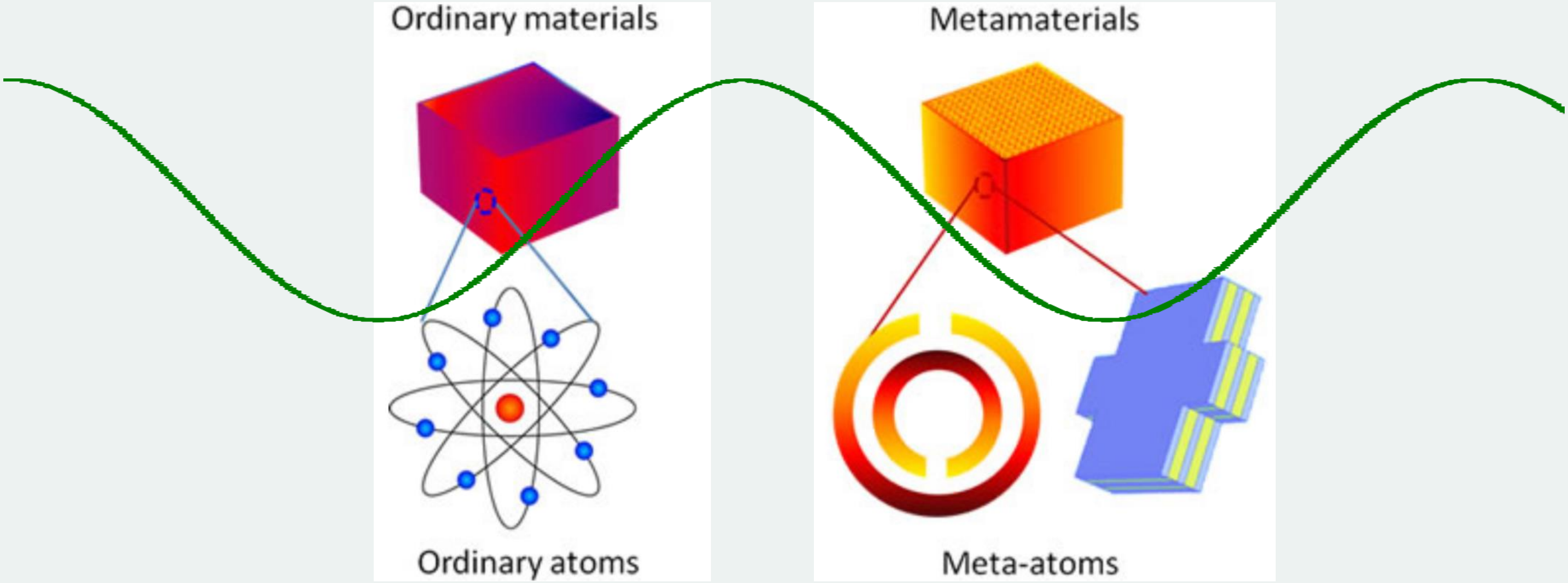


Optical properties of materials



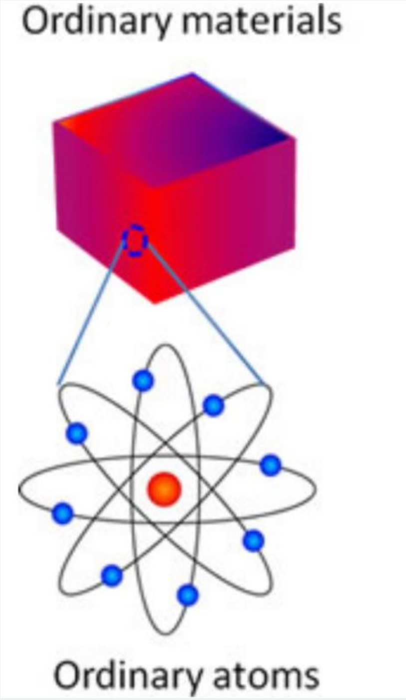
- The optical properties of material are determined from the **collective response of their atoms/molecules to incident light**.
- Because the atoms are much smaller than wavelength, many atoms together contribute to a macroscopic optical behaviour

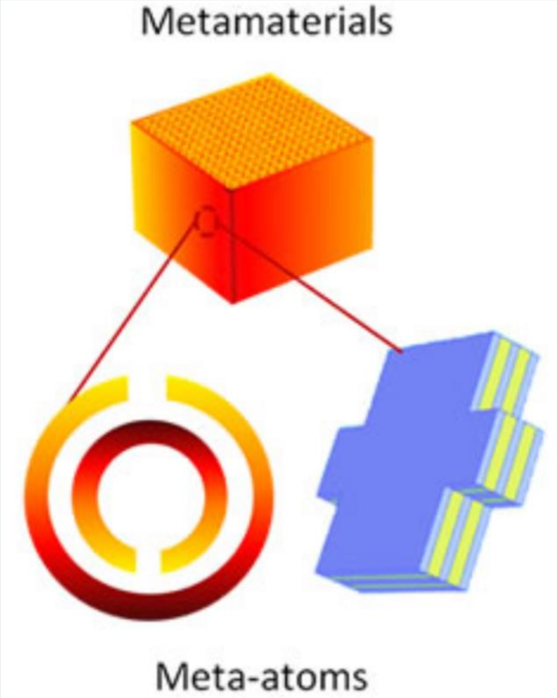
Engineering the optical properties of materials



- Can we engineer "atoms" such that the overall medium would have optical properties that are specific and unique, and cannot otherwise be found in nature?
- We cannot engineer/change actual atoms, but we can design/fabricate nano-structures that exhibit specific response. We will call them "**meta-atoms**".
- Placing many meta-atoms together, we create a new **artificial** medium whose optical properties are dependent on the meta-atoms' design (which we control): **Metamaterials**
- **Meta-atoms need to be much smaller than the wavelength** -> macrosc. optical properties

Metamaterials

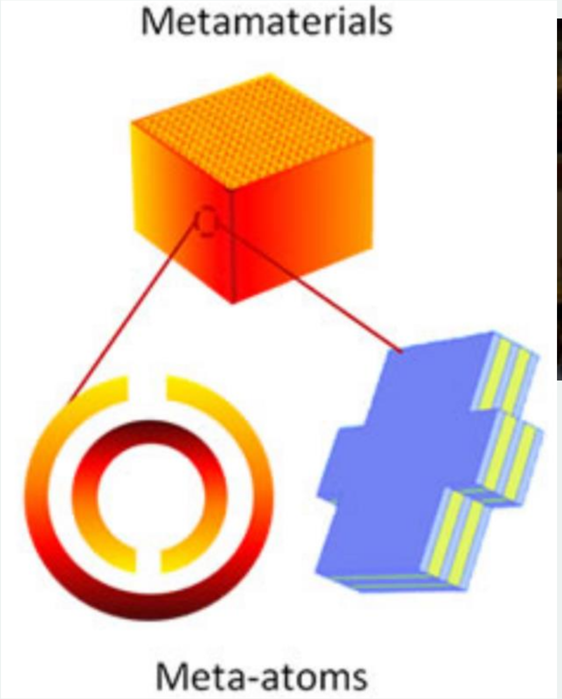




- "**Meta**" is a Greek prefix meaning: Beyond.

- **Metamaterials** are artificially engineered composites of periodic (sometimes even non-periodic) structures with exotic, unique and novel macroscopic properties (ε, μ) that are not found in Nature (i.e. that are beyond what nature provided).
- Can we use them to build the photonics technology of the future?

Historical facts



David Smith

- Metamaterials were first introduced in 1999-2000 by **Prof. Sir John Pendry** (Imperial College London).
- Pendry theoretically introduced metamaterials and clearly explained how one can demonstrate their properties experimentally.
- He is considered the father of metamaterials and has been knighted in 2004 for "his services to science".
- **Prof. David R Smith** (Duke University) together with John Pendry experimentally demonstrated negative refraction (2000, 2001) and cloaking (2006).

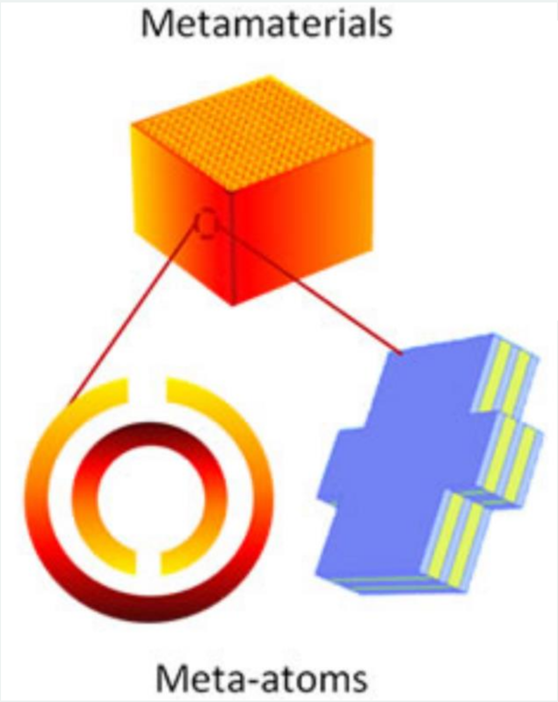
Metamaterials - terminology

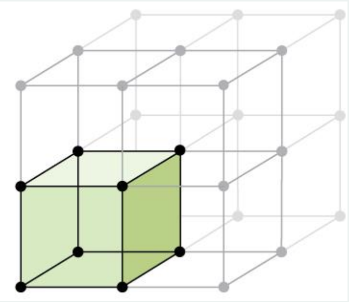
- **Meta-atom**: the sub-unit that composes a metamaterial.

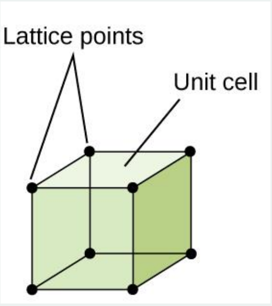
- **Unit cell**: the smallest group of atoms which has the overall symmetry of a crystal, and from which the entire lattice can be built up by repetition in three dimensions.

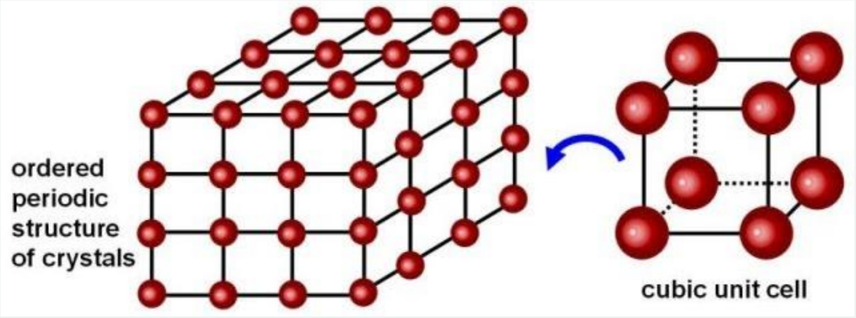
- **Lattice constant** (a): refers to the physical dimension of the unit cell in a crystal lattice

- Only **for frequencies that $\boldsymbol{\lambda \gg a}$** , we can describe such periodic structures with **macroscopic optical parameters** (ε, μ) and n. (typically we need $\lambda \geq 10a$).









Classification of EM materials

μ

$\varepsilon < 0$ and $\mu > 0$

$n = \sqrt{\varepsilon\mu} \rightarrow \ imaginary$

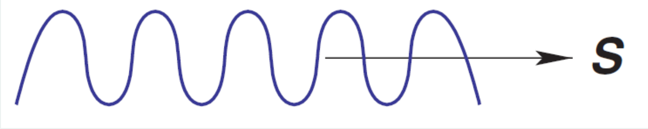
- Electrical Plasma
- Many metals (UV – optical freq.)

- Evanescent decaying waves
- Artificially:???

$\varepsilon > 0$ and $\mu > 0$

$n = \sqrt{\varepsilon\mu} > 0$

- Ordinary dielectric optical materials
- Propagating waves



ε

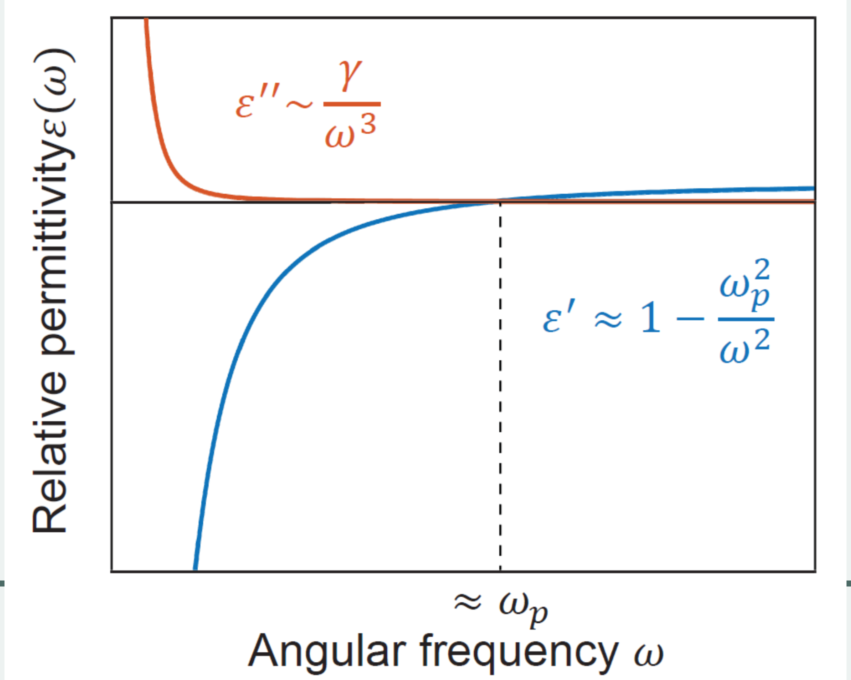
Electrical Metamaterials

- For **bulk metals**: Drude response for the electric permittivity :

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}$$

where $\omega_p^2 = \frac{N q_e^2}{\varepsilon_0 \, m_e}$

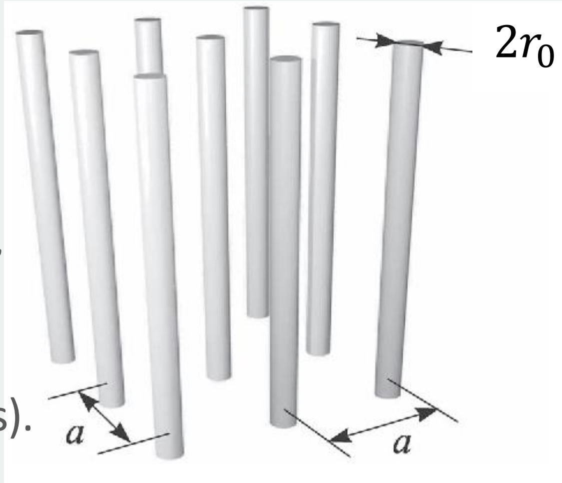
- Can we create a metamaterial with a Drude electric permittivity (ε), but with control over the value of ω_p?
- By controlling the value of ω_p, we essentially control the negative value of $\mathrm{Re}(\varepsilon)$ at a particular frequency.



Wire metamaterial

For **bulk metals**: Drude response for the electric permittivity :

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma} \qquad \text{where } \omega_p^2 = \frac{N q_e^2}{\varepsilon_0\, m_e}$$



- Can we create a metamaterial with a Drude electric permittivity, but with control over the value ω_p?
- Yes, with a **wire metamaterial** (for mathematical convenience, we'll work with metals that are PEC - Perfect Electric Conductors).

- Consider a **periodic lattice** of continuous PEC/metal wires with radius r_0 and lattice constant a.

- By forcing the electrons to move within the restricted volume occupied by the wires, we can change the number density of the electrons:

$$\boxed{N_{eff} = N \frac{\pi r_0^2}{a^2}}$$

which is the <u>**effective number density**</u> for the electrons coupled to the EM wave.

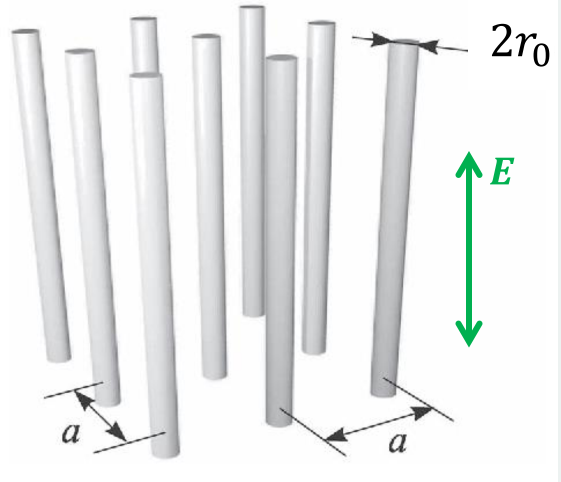
JB Pendry, *et al.*, J. Phys.: Condens. Matter, 10, 4785–4809 (1998)
JB Pendry, *et al.* Physical Review Letters, 76, 4773, (1996)

Wire metamaterial

- By restricting the electrons to move within the wires:

$$\boxed{N_{eff} = N\frac{\pi r_0^2}{a^2}}$$

which is the **effective number density** for the electrons coupled to the EM wave.



- But an E-field oscillating along the wires, induces a current j
- And any flowing current induces a magnetic field: $H(r) = \frac{j}{2\pi r}$
- To assume that there is no coupling (interaction) between neighbouring wires, we need to enforce that the magnetic field is zero at the boundary of the unit cell:

$$H(r) = \frac{j}{2\pi r}\left[1 - \frac{\pi r^2}{\pi R_c^2}\right] = \frac{j}{2\pi r}\left[1 - \frac{\pi r^2}{a^2}\right]$$

where R_c is the radius of putative cylindrical unit cell with $\pi R_c^2 = a^2 \Rightarrow R_c = a/\sqrt{\pi}$.

- The magnetic field can be expressed with the vector potential $\boldsymbol{A}(r)$: $\mu_0 \boldsymbol{H}(r) = \nabla \times \boldsymbol{A}(r)$
- And applying Stoke's theorem, $A(r = r_0)$:

$$A(r_0) = \mu_0 \frac{j}{2\pi} ln\left(\frac{a}{r_0\sqrt{\pi}}\right)$$

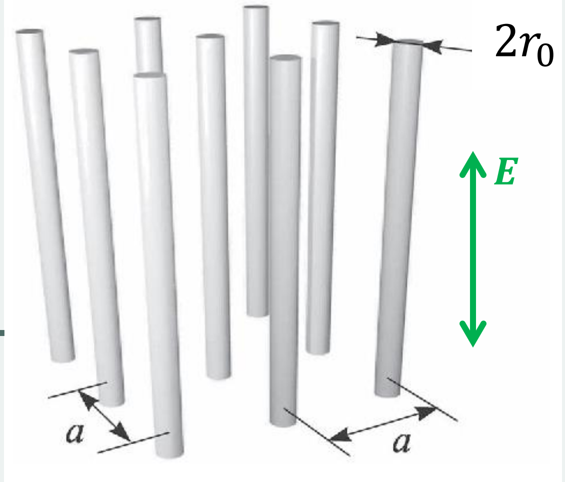
JB Pendry, *et al.*, J. Phys.: Condens. Matter, 10, 4785–4809 (1998)
JB Pendry, *et al.* Physical Review Letters, 76, 4773, (1996)

Wire metamaterial

- By restricting the electrons to move within the wires:

$$\boxed{N_{eff} = N\frac{\pi r_0^2}{a^2}}$$

which is the **effective number density** for the electrons coupled to the EM wave.



$$A = \mu_0 \frac{j}{2\pi} ln\left(\frac{a}{r_0\sqrt{\pi}}\right)$$

- The current flowing in the wire is given by: $j = Nq_e v\,\pi r_0^2$, where v is the electron velocity.
- The momentum of a moving electron: $m_e v$
- But from classical mechanics: an electron moving in a magnetic field would have an additional contribution to its momentum ($q_e A$)
- Therefore, the momentum per unit length of the wire:

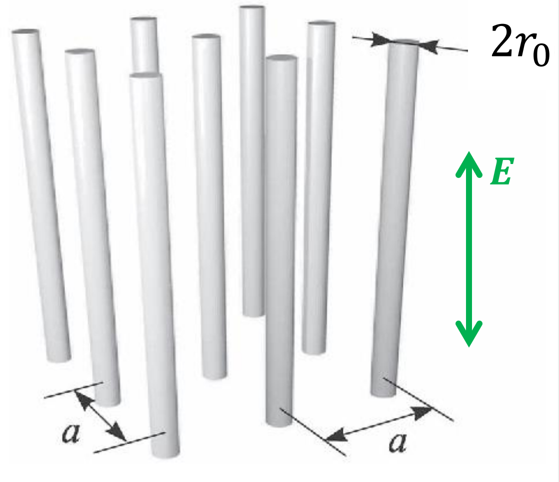
$$m_{eff}v = q_e A$$

where m_{eff} is an **effective mass** for the electron. The electrons appear heavier due to the magnetic field.

$$\boxed{m_{eff} = \mu_0 \frac{q_e^2 N\pi r_0^2}{2\pi} ln\left(\frac{a}{r_0\sqrt{\pi}}\right)}$$

JB Pendry, *et al.*, J. Phys.: Condens. Matter, 10, 4785–4809 (1998)
JB Pendry, *et al.* Physical Review Letters, 76, 4773, (1996)

Wire metamaterial



- By restricting the electrons to move within the wires:

$$\boxed{N_{eff} = N\frac{\pi r_0^2}{a^2}}$$

which is the **effective number density** for the electrons coupled to the EM wave.

- And an **effective electron mass**:

$$\boxed{m_{eff} = \mu_0 \frac{q_e^2 N\pi r_0^2}{2\pi} ln\left(\frac{a}{r_0\sqrt{\pi}}\right)}$$

- For **bulk metals**: the electric permittivity: $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}$ where $\omega_p^2 = \frac{Nq_e^2}{\varepsilon_0\, m_e}$

- For the **parallel wire metamaterial**: $\varepsilon_{zz}^{eff} = 1 - \frac{\omega_{p,eff}^2}{\omega^2 + i\omega\Gamma}$ where $\omega_{p,eff}^2 = \frac{N_{eff}\, q_e^2}{\varepsilon_0\, m_{eff}}$

$$\boxed{\omega_{p,eff}^2 = c_0^2 \frac{2\pi}{a^2 ln\left(\frac{a}{r_0\sqrt{\pi}}\right)}}$$

A metamaterial that I can tune its plasma frequency.

- Wires are parallel to the z-axis, the electric permittivity is a tensor: $\varepsilon = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varepsilon_{zz}^{eff} \end{pmatrix}$

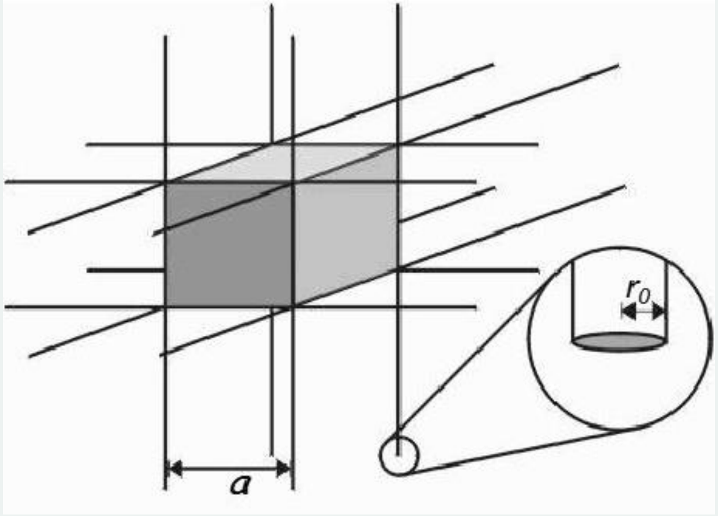
JB Pendry, *et al.*, J. Phys.: Condens. Matter, 10, 4785–4809 (1998)
JB Pendry, *et al.* Physical Review Letters, 76, 4773, (1996)

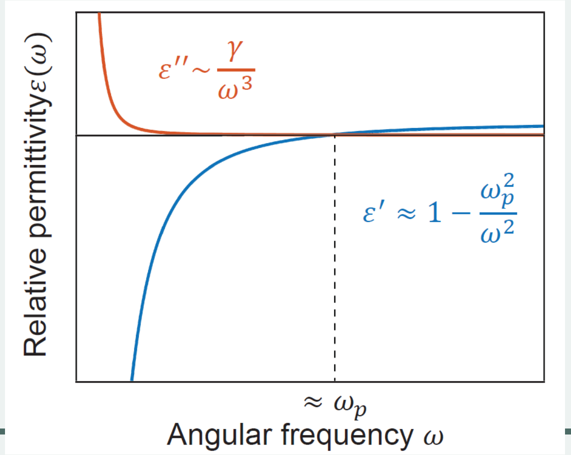
Wire-mesh metamaterial

- By placing the wires along the three orthogonal axis:
- A **wire-mesh metamaterial** with an isotropic structure:

Therefore, the **effective electric permittivity** for the wire-mesh structure is:

$$\varepsilon_{eff} = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma} \qquad \text{where} \qquad \omega_p^2 = c_0^2 \frac{2\pi}{a^2 \ln\left(\frac{a}{r_0\sqrt{\pi}}\right)}$$





- As long as $\lambda \gg a$, the ε_{eff} would have a Drude-like behaviour.
- ε_{eff} is the effective macroscopic quantity describing the wire-mesh metamaterials.
- It means, that I can essentially **replace** the wire-mesh metamaterials with a **bulk, homogeneous, isotropic medium with $\varepsilon = \varepsilon_{eff}$.**

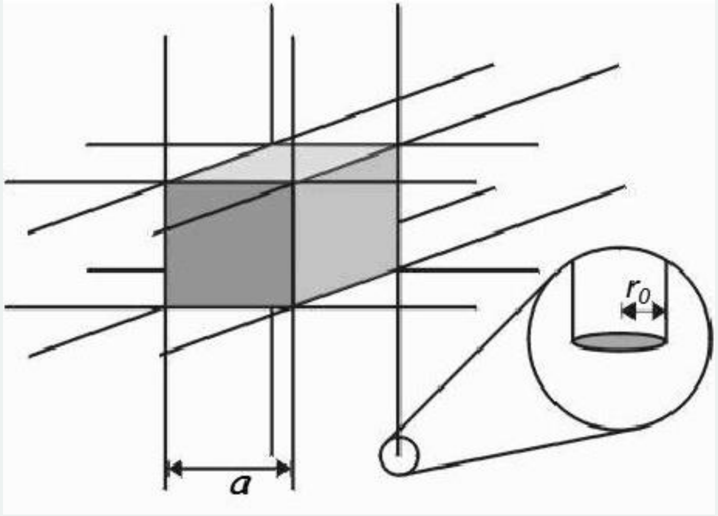
JB Pendry, *et al.*, J. Phys.: Condens. Matter, 10, 4785–4809 (1998)
JB Pendry, *et al.* Physical Review Letters, 76, 4773, (1996)

Wire-mesh metamaterial

- By placing the wires along the three orthogonal axis:
- A **wire-mesh metamaterial** with an isotropic structure:

Therefore, the **effective electric permittivity** for the wire-mesh structure is:

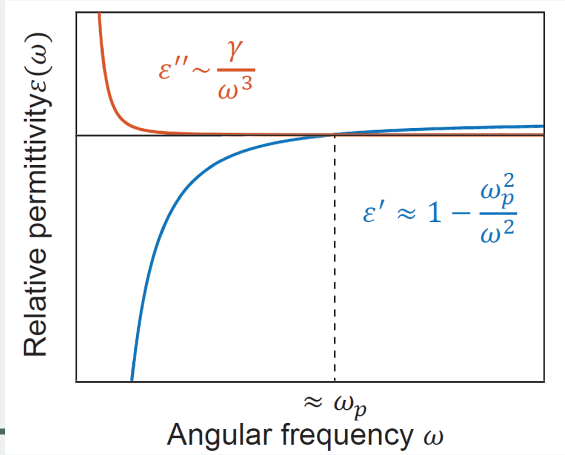
$$\varepsilon_{eff} = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma} \quad \text{where} \quad \omega_p^2 = c_0^2 \frac{2\pi}{a^2 \ln\left(\frac{a}{r_0\sqrt{\pi}}\right)}$$

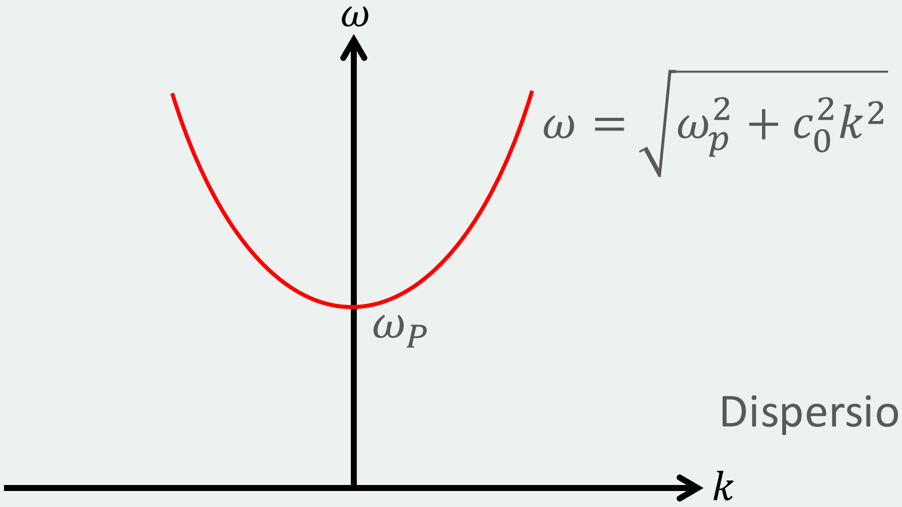


➢ Exercise: What is the dispersion relation of a wire-mesh metamaterial with $\Gamma = 0$:

$$k = \omega\sqrt{\varepsilon_{eff}\mu \;\; \varepsilon_0\mu_0} \Rightarrow c_0^2 k^2 = \omega^2\left(1 - \omega_p^2/\omega^2\right)$$

$$\omega = \sqrt{\omega_p^2 + c_0^2 k^2}$$





Dispersion diagram

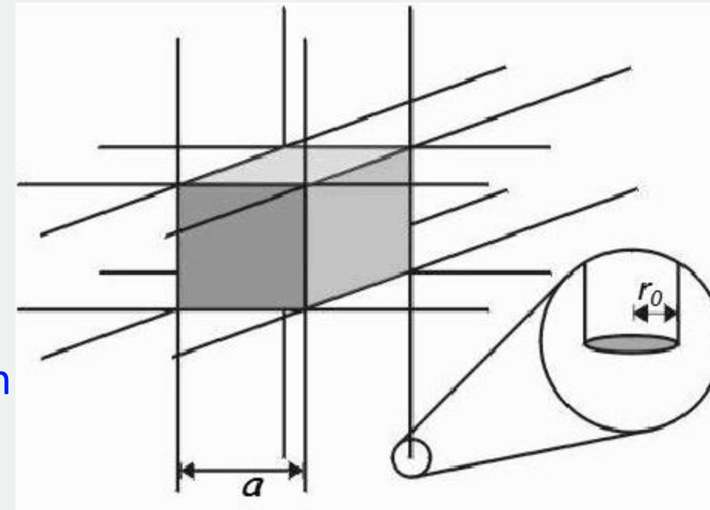
JB Pendry, *et al.*, J. Phys.: Condens. Matter, 10, 4785–4809 (1998)
JB Pendry, *et al.* Physical Review Letters, 76, 4773, (1996)

Wire-mesh metamaterial

- By placing the wires along the three orthogonal axis:
- A **wire-mesh metamaterial** with an isotropic structure:

Therefore, the **effective electric permittivity** for the wire-mesh structure is:

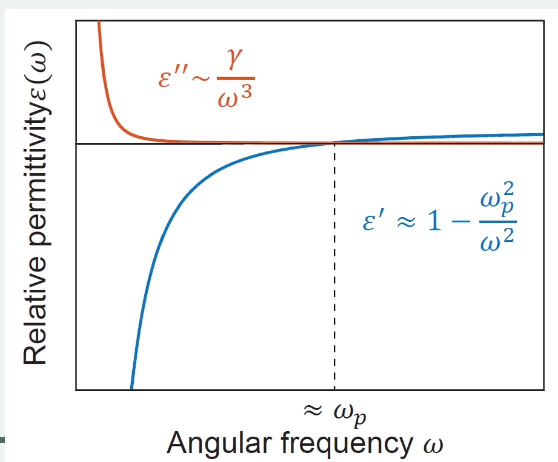
$$\varepsilon_{eff} = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma} \quad \text{where} \quad \omega_p^2 = c_0^2 \frac{2\pi}{a^2 \ln\left(\frac{a}{r_0\sqrt{\pi}}\right)}$$



➢ Exercise: What is the dispersion relation of a wire-mesh metamaterial with $\Gamma = 0$:

$$k = \omega\sqrt{\varepsilon_{eff}\mu\ \varepsilon_0\mu_0} \Rightarrow c_0^2 k^2 = \omega^2\left(1 - \omega_p^2/\omega^2\right)$$

$$\omega = \sqrt{\omega_p^2 + c_0^2 k^2}$$

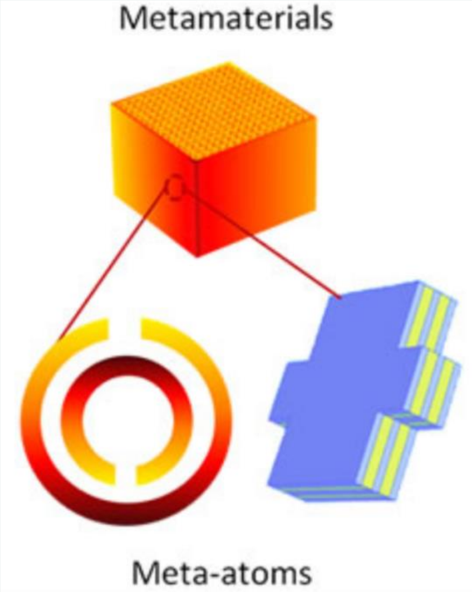


- With the **wire-mesh metamaterials**, we created an **artificial medium** (plasma) that has the same behaviour as metals.
- But now we can **engineer** the plasma frequency of the artificial medium, by simply changing (r_0, a).
- For example: a wire-mesh with $r_0 = 10nm$ and $a = 1\mu m$, has a plasma frequency: $\omega_p \approx 374.46\ THz \Rightarrow \lambda_p \approx 801nm$
- Is this an alternative route to plasmonics??

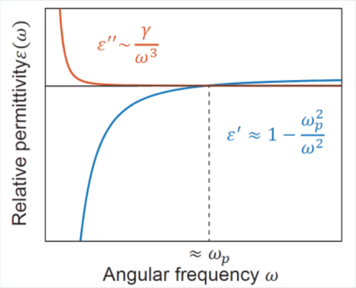
JB Pendry, *et al.*, J. Phys.: Condens. Matter, 10, 4785–4809 (1998)
JB Pendry, *et al.* Physical Review Letters, 76, 4773, (1996)

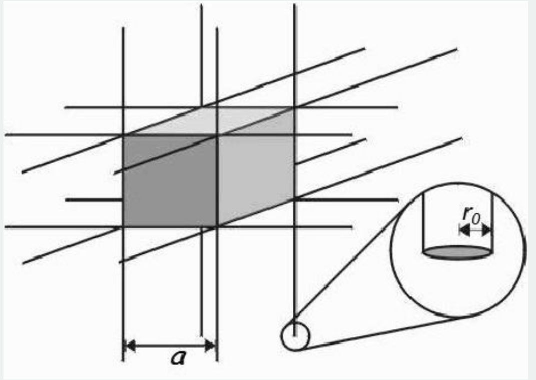
Summary: Wire metamaterials

- **Metamaterials are artificial media** that are composed of specifically engineered/designed composites (meta-atoms) arranged usually in periodic lattices.
- Metamaterials can be described with **macroscopic optical parameters when $\lambda \gg a$**.
- Metamaterials can exhibit **exotic, unique and novel macroscopic properties (ε, μ) that are not found in nature**.

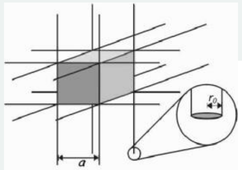


- **Parallel wires** arranged in a periodic lattice -> electric metamaterials
- **Wire-mesh metamaterials** have a Drude-like macroscopic electric permittivity, with a plasma frequency determined by the lattice constant and the radius of the wires (i.e. it can be tailored).





Classification of EM materials



μ

$\varepsilon < 0$ and $\mu > 0$

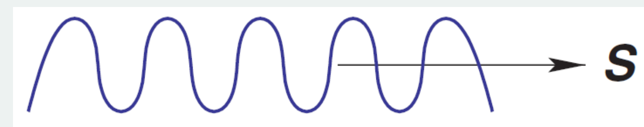
$n = \sqrt{\varepsilon\mu} \rightarrow \ imaginary$

- Electrical Plasma
- Many metals (UV – optical freq.)
- Evanescent decaying waves
- Artificially: **Wire metamaterials**

$\varepsilon > 0$ and $\mu > 0$

$n = \sqrt{\varepsilon\mu} > 0$

- Ordinary dielectric optical materials
- Propagating waves



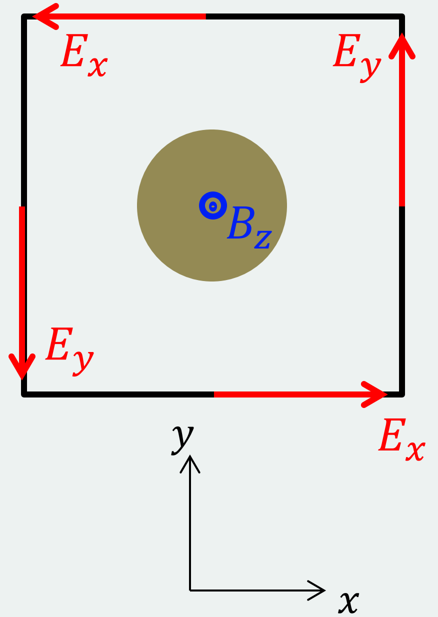
ε

$\varepsilon > 0$ and $\mu < 0$

$n = \sqrt{\varepsilon\mu} \rightarrow \ imaginary$

- Magnetic Plasma (?)
- Some natural materials (up to GHz)
- Evanescent decaying waves
- Artificially:???

Homogenization techniques



- To derive the effective properties of magnetic metamaterials, we will need to use homogenization techniques.
- **Homogenization techniques** tell us how to average the EM fields in a unit cell, to obtain the **effective optical parameters** of a metamater.
- Let's consider a 2D problem. A cylinder in a periodic square lattice:
- From Maxwell's equations, a oscillating magnetic field induces a circulating E-field ($\nabla \times \boldsymbol{E} = -\frac{d\boldsymbol{B}}{dt}$). For this 2D problem drawn here:

$$\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} = i\omega B_z$$

- But the values of E_x, E_y and B_z vary within the unit cell.
- The total fields per unit cell, are then given by:

$$\iint \left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) dxdy = \iint i\omega B_z \; dxdy \;\Rightarrow\; \int E_x dx \; - \int E_y dy = i\omega \iint B_z \; dxdy$$

- The **averaged** optical response (what an EM wave experiences per unit cell):

$$\frac{1}{a}\int E_x dx \; - \frac{1}{a}\int E_y dy = i\omega \frac{1}{a^2}\iint B_z \; dxdy$$

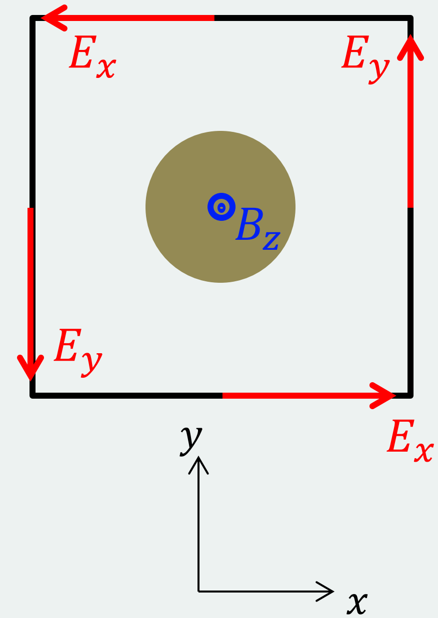
DR. Smith and JB. Pendry, Journal of the Optical Society of America B, 23, 391-403, (2006)

Homogenization techniques

- The **averaged** optical response (what an EM wave experiences per unit cell):

$$\frac{1}{a}\int E_x dx \; - \frac{1}{a}\int E_y dy = i\omega \frac{1}{a^2}\iint B_z \; dxdy$$

$$\widetilde{E_x}\left(\frac{a}{2},0,0\right) - \widetilde{E_x}\left(-\frac{a}{2},0,0\right) - \widetilde{E_y}\left(0,\frac{a}{2},0\right) + \widetilde{E_y}\left(0,-\frac{a}{2},0\right)$$
$$= i\omega \frac{1}{a}\widetilde{B_z}(0,0,0)$$



- To average $\boldsymbol{E}$ and $\boldsymbol{H}$ fields, we need line integrals
- To average $\boldsymbol{B}$ and $\boldsymbol{D}$ fields, we need surface integrals

Array of (empty) cylinders (1)

- Consider an array of metal (PEC) empty cylinders.
- An incident magnetic field (H_0) along the cylinders induces a **magnetic field inside** the cylinders:

$$H_{inside} = H_0 + j\left(1 - \frac{\pi r_0^2}{a^2}\right)$$

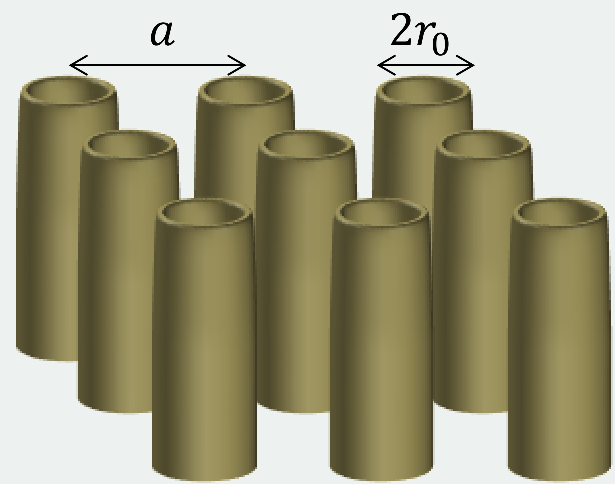
The term (j): is the field caused directly by the current

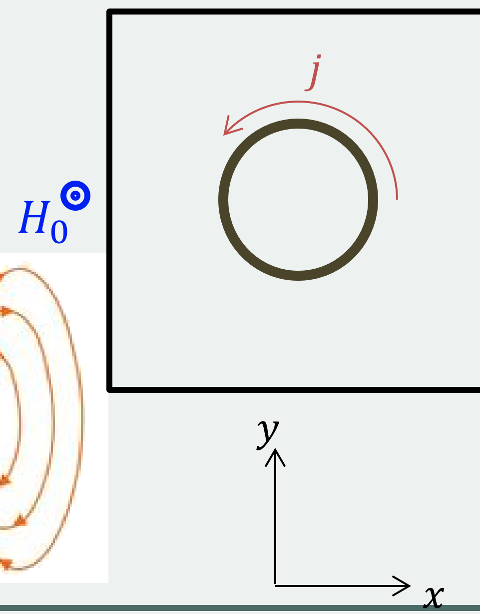
The term $\left(j\frac{\pi r_0^2}{a^2}\right)$: the depolarizing fields at the remote ends of the cylinders.

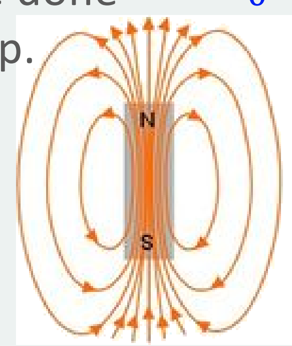
- We need to find j:
- Any current flowing in a loop carries an **electromotive force (emf)** - an electromagnetic work that would be done on an electric charge if it travels once around the loop.

$$\mathcal{E} = \oint_C \boldsymbol{E}\, dl = \iint_S \nabla \times \boldsymbol{E}\, dS = -\iint_S \frac{\partial \boldsymbol{B}}{\partial t} dS$$

$$\mathcal{E} = i\omega\mu_0 \iint_S H_{inside}\, dS = i\omega\mu_0 \pi r_0^2 H_{inside}$$







Note: The H-field inside the cylinder is uniform, since we are considering infinitely long cylinders with $r_0 \ll \lambda$.
JB. Pendry, AJ. Holden, DJ. Robbins, WL. Stewart, IEEE Trans. on Microwave theory and Techn., 47, 2075, (1999)

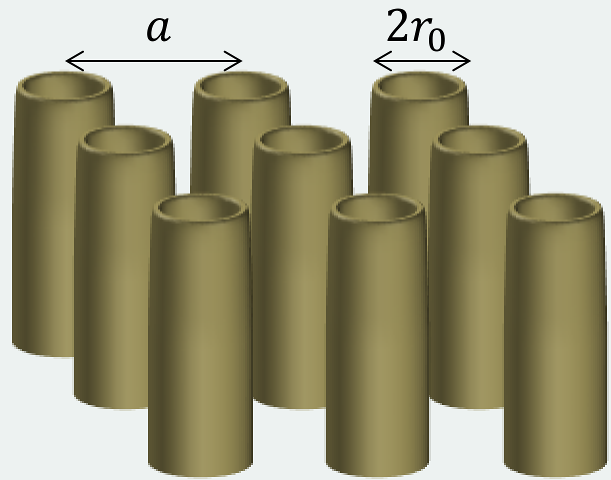
Array of (empty) cylinders (2)

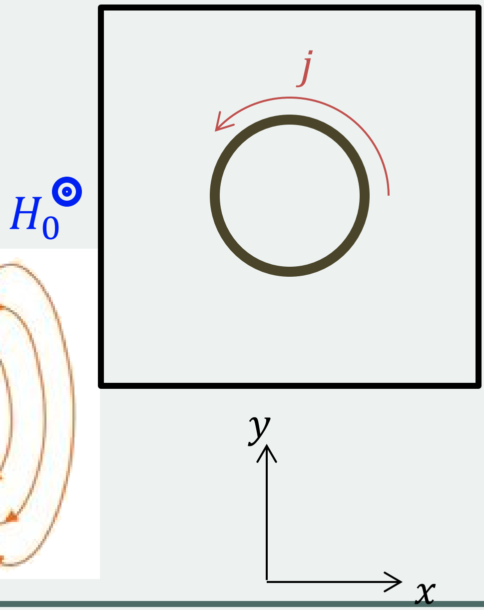
- Magnetic field inside cyl.: $H_{inside} = H_0 + j\left(1 - \frac{\pi r_0^2}{a^2}\right)$
- Electromotive force (emf): $\mathcal{E} = i\omega\mu_0\pi r_0^2 H_{inside}$
- The emf needs to be balanced by the Ohmic losses in the metal. Ohmic losses $= 2\pi r_0 \rho j$ (ρ: metal resistivity)

$$i\omega\mu_0\pi r_0^2 H_{inside} = 2\pi r_0 \rho j$$

Solving the above equation, we can find the current j induced by the incident field H_0:

$$\boxed{j = -\frac{H_0}{\left[1 - \frac{\pi r_0^2}{a^2}\right] + i\,\frac{2\rho}{\omega r_0 \mu_0}}}$$

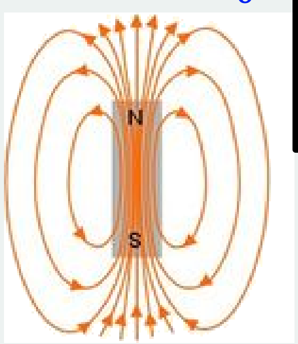




- The effective magnetic permeability μ_{eff}:

$$\mu_{eff} = \frac{B_{ave}}{\mu_0 H_{ave}}$$

- $B_{ave} = \mu_0 H_0$ (when integrating over the unit cell, all the induced magnetic fields cancel each other out).



Note: The H-field inside the cylinder is uniform, since we are considering infinitely long cylinders with $r_0 \ll \lambda$.
JB. Pendry, AJ. Holden, DJ. Robbins, WL. Stewart, IEEE Trans. on Microwave theory and Techn., 47, 2075, (1999)

Array of (empty) cylinders (3)

- The current j induced by the incident field H_0:

$$\boxed{j = -\frac{H_0}{\left[1 - \frac{\pi r_0^2}{a^2}\right] + i\,\frac{2\rho}{\omega r_0 \mu_0}}}$$

- The effective magnetic permeability $\mu_{eff} = \frac{B_{ave}}{\mu_0 H_{ave}}$
- $B_{ave} = \mu_0 H_0$
- To obtain H_{ave}, we average the H-field over a line lying entirely outside the cylinders (see homogeniz. tech.):

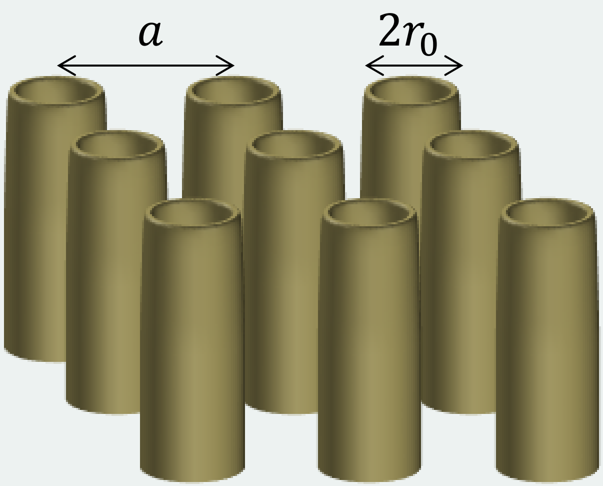
$$H_{ave} = H_{outside} = H_0 - j\,\frac{\pi r_0^2}{a^2}$$

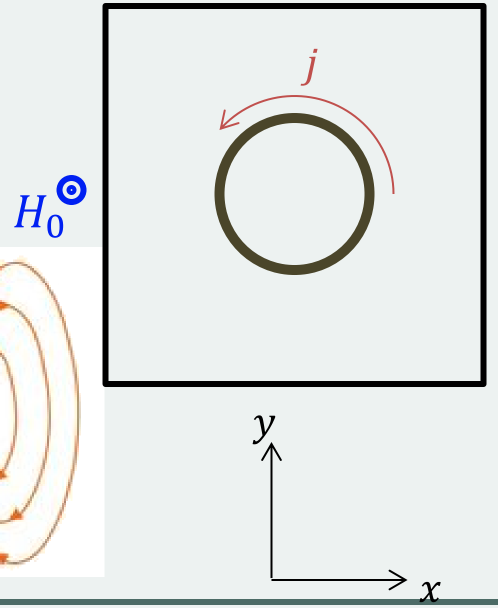
(integrate over the depolarizing fields only).

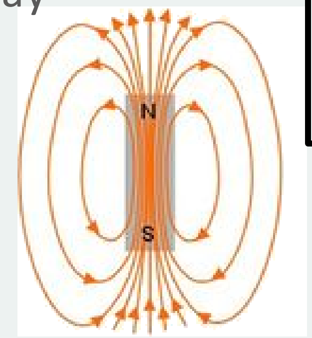
- Hence, the **effective magnetic permeability** of an array of empty cylinders:

$$\mu_{eff} = 1 - \frac{\pi r_0^2}{a^2}\left[\frac{1}{1 + i\,\frac{2\rho}{\omega r_0 \mu_0}}\right]$$

- I can tune μ_{eff} with (r_0, a), but weak response.







Note: The H-field inside the cylinder is uniform, since we are considering infinitely long cylinders with $r_0 \ll \lambda$.

JB. Pendry, AJ. Holden, DJ. Robbins, WL. Stewart, IEEE Trans. on Microwave theory and Techn., 47, 2075, (1999)

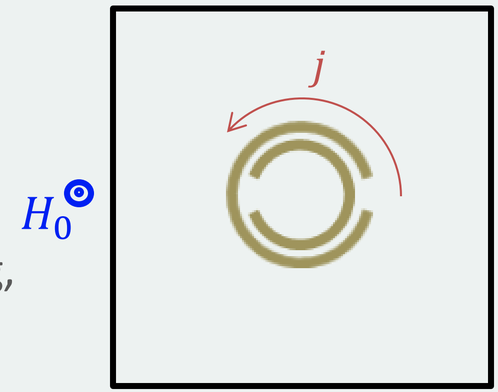
Split-Ring Resonators (SRRs)-1

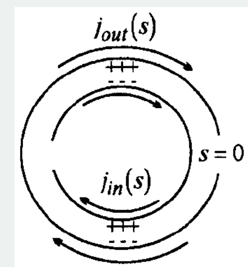
- The array of empty metal cylinders gives a magnetic response, but a weak one.
- To increase the effective magnetic response, we need to add **capacitive elements** in the design of the structures.
- The gaps prevent the current from flowing around any one ring, but the high capacitance, enables the current to flow.
- The field inside SRR: $H_{inside} = H_0 + j\left(1 - \frac{\pi r_0^2}{a^2}\right)$
- The electromotive force (emf): $\mathcal{E} = i\omega\mu_0\pi r_0^2 H_{inside}$
- But now the emf needs to balanced by Ohmic losses and the Ohmic drop in the potential due to the capacitance:

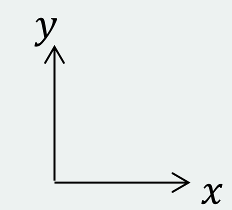
$$i\omega\mu_0\pi r_0^2 H_{inside} = 2\pi r_0 \rho j - \frac{j}{i\omega C}$$

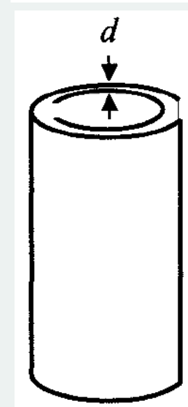
where $C = \frac{\pi r_0 \varepsilon_0}{2d}$.

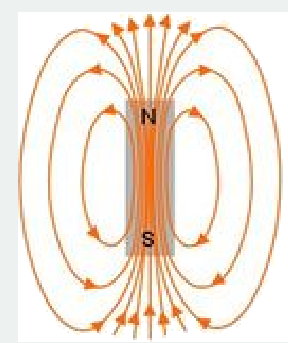
Note: Ohmic drop in potential due to the capacitance: $j = C\frac{\partial V}{\partial t} = -i\omega C\, V \Rightarrow V = -\frac{j}{i\omega C}$ and we have two capacitors in series.







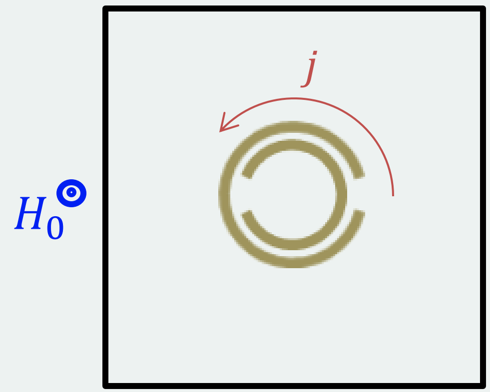




Note: The H-field inside the cylinder is uniform, since we are considering infinitely long cylinders with $r_0 \ll \lambda$.
JB. Pendry, AJ. Holden, DJ. Robbins, WL. Stewart, IEEE Trans. on Microwave theory and Techn., 47, 2075, (1999)

Split-Ring Resonators (SRRs)-2

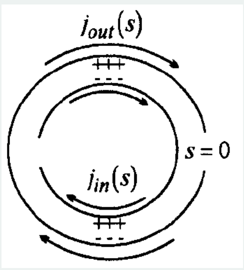
$$i\omega\mu_0\pi r_0^2 H_{inside} = 2\pi r_0\rho j - \frac{j}{i\omega C} \qquad \text{where } C = \frac{\pi r_0 \varepsilon_0}{2d}$$

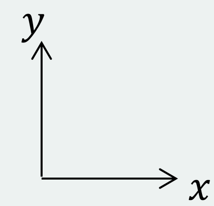


- Following the same derivation as before, the current j induced by the incident field H_0:

$$j = -\frac{H_0}{\left[1 - \frac{\pi r_0^2}{a^2}\right] + i\frac{2\rho}{\omega r_0 \mu_0} - \frac{1}{\omega^2 \mu_0 \pi r_0^2 C}}$$

- Hence, the **effective magnetic permeability** of an SRRs array :

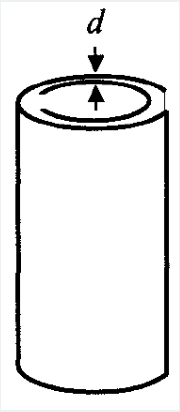


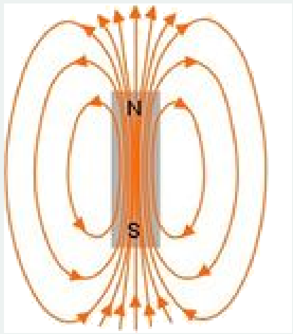


$$\mu_{eff} = \frac{B_{ave}}{\mu_0 H_{ave}} = \frac{\mu_0 H_0}{\mu_0\left[H_0 - \frac{\pi r_0^2}{a^2} j\right]}$$

$$\mu_{eff} = 1 - \frac{\pi r_0^2}{a^2}\left[\frac{1}{1 + i\frac{2\rho}{\omega r_0 \mu_0} - \frac{1}{\omega^2 \mu_0 \pi r_0^2 C}}\right]$$

$$\boxed{\therefore \mu_{eff} = 1 - \frac{\frac{\pi r_0^2}{a^2}\omega^2}{\omega^2 + i\frac{2\rho}{r_0\mu_0}\omega - \frac{2dc_0^2}{\pi^2 r_0^3}} = 1 - \frac{F\omega^2}{\omega^2 + i\Gamma\omega - \omega_0^2}}$$





<u>Note</u>: The H-field inside the cylinder is uniform, since we are considering infinitely long cylinders with $r_0 \ll \lambda$.

JB. Pendry, AJ. Holden, DJ. Robbins, WL. Stewart, IEEE Trans. on Microwave theory and Techn., 47, 2075, (1999)

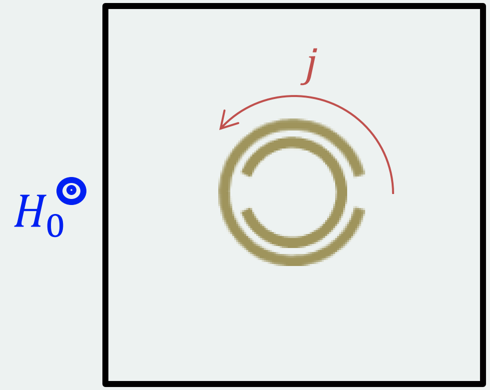
Split-Ring Resonators (SRRs)-3

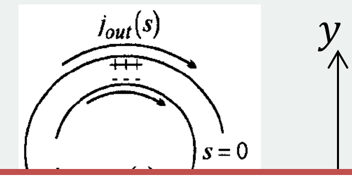
- Hence, the **effective magnetic permeability** of an SRRs array :

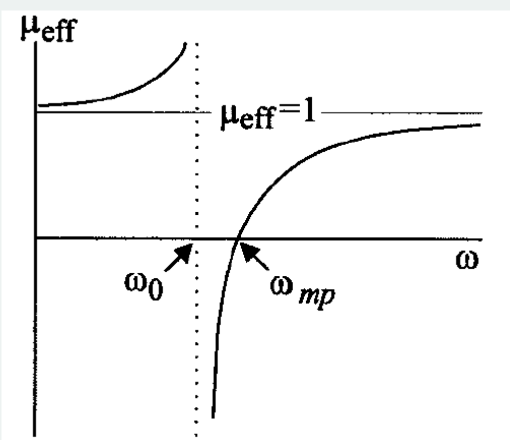
$$\boxed{\therefore\ \mu_{eff} = 1 - \frac{\frac{\pi r_0^2}{a^2}\omega^2}{\omega^2 + i\frac{2\rho}{r_0\mu_0}\omega - \frac{2dc_0^2}{\pi^2 r_0^3}} = 1 - \frac{F\omega^2}{\omega^2 + i\Gamma\omega - \omega_0^2}}$$

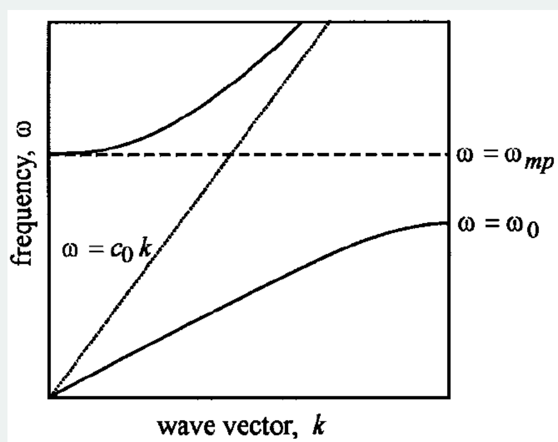
- The **resonant frequency**: $\omega_0 = c_0\sqrt{\frac{2d}{\pi^2 r_0^3}}$
- The "**magnetic plasma frequency**" $\left(\mu_{eff}(\omega = \omega_{mp}) = 0\right)$:

$$\omega_{mp} = \frac{\omega_0}{\sqrt{1-\pi r_0^2/a^2}} = c_0\sqrt{\frac{2d}{\pi^2 r_0^3(1-\pi r_0^2/a^2)}}$$









- The homogeneous (effective/overall) response of an array of SRRs is **magnetic**, with a **Lorentz**-like resonance.
- I can tune μ_{eff} by changing (r_0, a, d)
- This is valid only for frequencies where $\lambda \gg a$.

Note: The H-field inside the cylinder is uniform, since we are considering infinitely long cylinders with $r_0 \ll \lambda$.

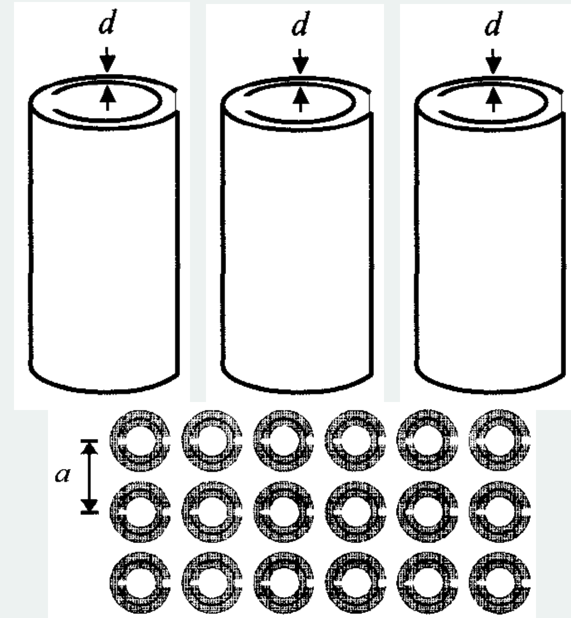
JB. Pendry, AJ. Holden, DJ. Robbins, WL. Stewart, IEEE Trans. on Microwave theory and Techn., 47, 2075, (1999)

Isotropic magnetic materials

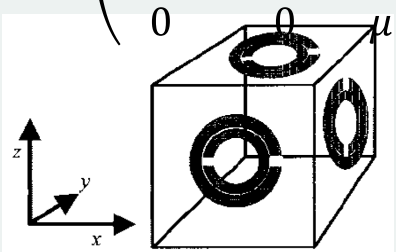
- Similarly to the parallel wire media, since we have infinitely long SRRs, the derived effective magnetic permeability (μ_{eff}) is valid only for incident H_0 along the z-axis.
- Hence, the effective magnetic permeability is a **tensor**:

$$\mu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mu^{eff} \end{pmatrix}$$

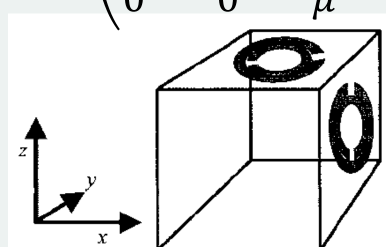
- The same applies if the SRRs were not infinite cylinders, but had a small finite thickness.
- To create an **isotropic magnetic medium**, we need to orient the SRRs along the three orthogonal axes.



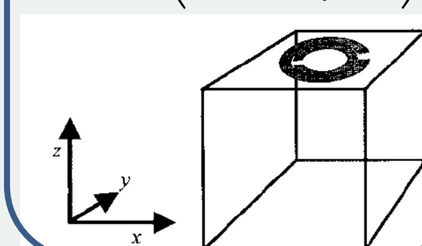
$$\mu = \begin{pmatrix} \mu^{eff} & 0 & 0 \\ 0 & \mu^{eff} & 0 \\ 0 & 0 & \mu^{eff} \end{pmatrix}$$



$$\mu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mu^{eff} & 0 \\ 0 & 0 & \mu^{eff} \end{pmatrix}$$



$$\mu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mu^{eff} \end{pmatrix}$$



JB. Pendry, AJ. Holden, DJ. Robbins, WL. Stewart, IEEE Trans. on Microwave theory and Techn., 47, 2075, (1999)

Swiss-Roll Resonators (1)

- An array of Swiss-Roll resonators behaves similarly to SRRs.
- The electromotive force (emf): $\mathcal{E} = i\omega\mu_0\pi r_0^2(N-1)H_{inside}$
- The emf per turn needs to balanced by Ohmic losses and the Ohmic drop in the potential due to the capacitance:

$$i\omega\mu_0\pi r_0^2\left[H_0 + \left(1 - \frac{\pi r_0^2}{a^2}\right)(N-1)j\right] = 2\pi r_0\rho j - \frac{j}{i\omega C}$$

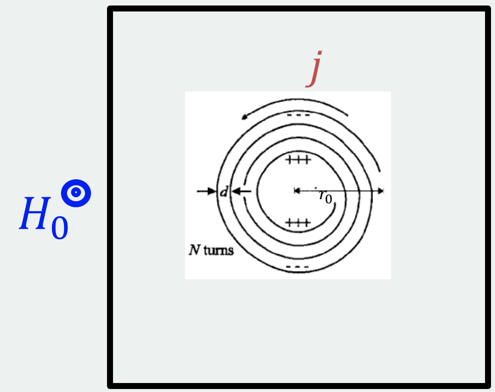
where $C = \frac{2\pi r_0\varepsilon_0}{d}$.

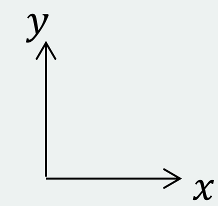
- **The effective magnetic permeability** of a Swiss-Roll array:

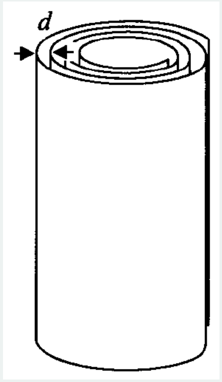
$$\boxed{\therefore\ \mu_{eff} = 1 - \frac{(\pi r_0^2/a^2)\omega^2}{\omega^2 + i\frac{2\rho}{r_0\mu_0(N-1)}\omega - \frac{2d\,c_0^2}{\pi^2\,r_0^3(N-1)}}}$$

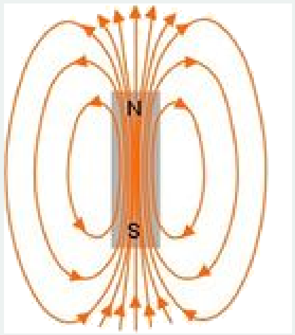
- The **resonant frequency**: $\omega_0 = c_0\sqrt{\frac{2d}{\pi^2\,r_0^3(N-1)}}$
- The "**magnetic plasma frequency**" ($\mu_{eff}(\omega = \omega_{mp}) = 0$):

$$\omega_{mp} = \frac{\omega_0}{\sqrt{1-\pi r_0^2/a^2}} = c_0\sqrt{\frac{2d}{\pi^2\,r_0^3(1-\pi r_0^2/a^2)(N-1)}}$$









Note: The H-field inside the cylinder is uniform, since we are considering infinitely long cylinders with $r_0 \ll \lambda$.
JB. Pendry, AJ. Holden, DJ. Robbins, WL. Stewart, IEEE Trans. on Microwave theory and Techn., 47, 2075, (1999)

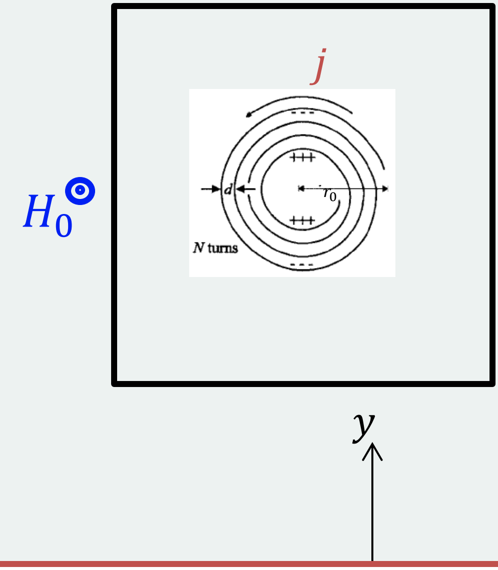
Swiss-Roll Resonators (2)

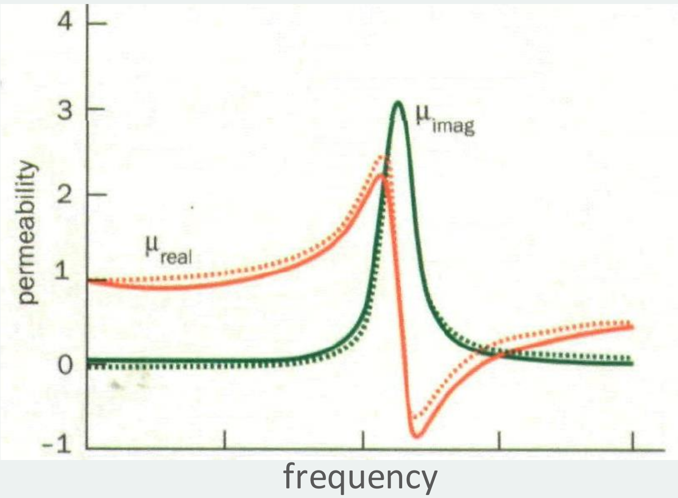
- **The effective magnetic permeability** of a Swiss-Roll array:

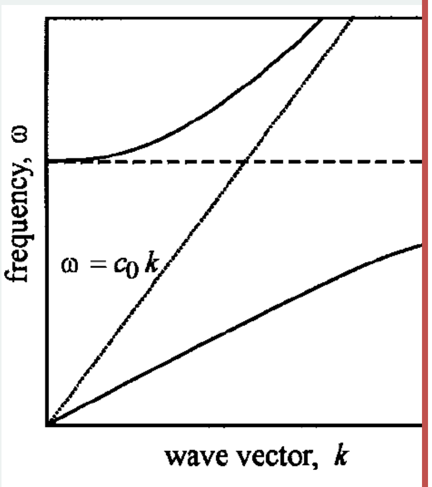
$$\boxed{\therefore\ \mu_{eff} = 1 - \frac{(\pi r_0^2/a^2)\omega^2}{\omega^2 + i\frac{2\rho}{r_0\mu_0(N-1)}\omega - \frac{2d\,c_0^2}{\pi^2 r_0^3(N-1)}}}$$

- The **resonant frequency**: $\omega_0 = c_0\sqrt{\frac{2d}{\pi^2 r_0^3(N-1)}}$
- The "**magnetic plasma frequency**" ($\mu_{eff}(\omega = \omega_{mp}) = 0$):

$$\omega_{mp} = \frac{\omega_0}{\sqrt{1-\pi r_0^2/a^2}} = c_0\sqrt{\frac{2d}{\pi^2 r_0^3(1-\pi r_0^2/a^2)(N-1)}}$$





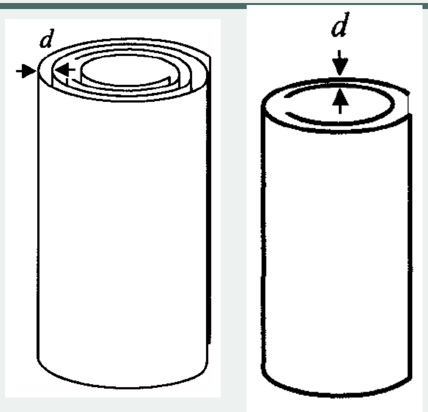


- The homogeneous (effective/overall) response of an array of SRRs is **magnetic**, with a **Lorentz**-like resonance.
- I can tune μ_{eff} by changing (r_0, a, d, N)
- This is valid only for frequencies where $\lambda \gg a$.

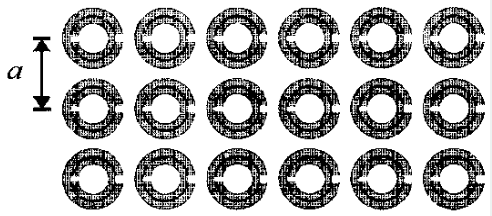
Note: The H-field inside the cylinder is uniform, since we are considering infinitely long cylinders with $r_0 \ll \lambda$.
JB. Pendry, AJ. Holden, DJ. Robbins, WL. Stewart, IEEE Trans. on Microwave theory and Techn., 47, 2075, (1999)

Summary: Magnetic metamaterials

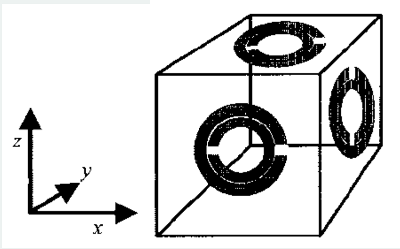
- Metamaterials can be described with **macroscopic optical parameters when $\lambda \gg a$**.
- Metamaterials can exhibit **exotic, unique and novel macroscopic properties (ε, μ) that are not found in nature**.

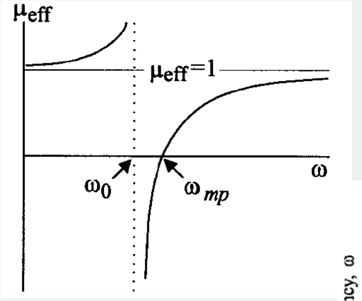


- **Homogenization techniques** dictate that to average $(\boldsymbol{D}, \boldsymbol{B})$ fields we need to integrate over an area, and to average $(\boldsymbol{E}, \boldsymbol{H})$ we need to integrate over a line.

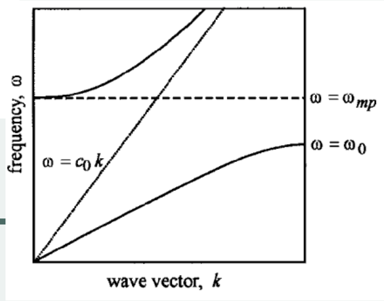


- **Split-Ring resonators** (SRRs) and **Swiss-Roll metamaterials** exhibit a macroscopic magnetic permeability that is Lorentz-like resonant.

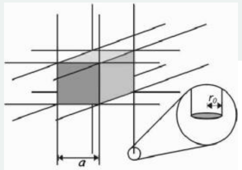




- **Magnetic metamaterials** allow us to tune μ by simply changing the geometry of the structure.



Classification of EM materials



μ

$\varepsilon < 0$ and $\mu > 0$

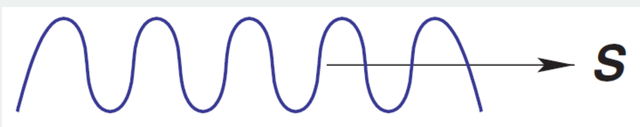
$n = \sqrt{\varepsilon\mu} \rightarrow \; imaginary$

- Electrical Plasma
- Many metals (UV – optical freq.)
- Evanescent decaying waves
- Artificially: **Wire metamaterials**

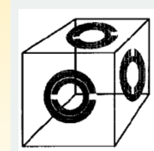
$\varepsilon > 0$ and $\mu > 0$

$n = \sqrt{\varepsilon\mu} > 0$

- Ordinary dielectric optical materials
- Propagating waves



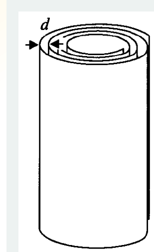
ε



$\varepsilon > 0$ and $\mu < 0$

$n = \sqrt{\varepsilon\mu} \rightarrow \; imaginary$

- Magnetic Plasma (?)
- Some natural materials (up to GHz)
- Evanescent decaying waves
- Artificially: **Magnetic metamaterials** (SRRs, Swiss-Rolls, …)

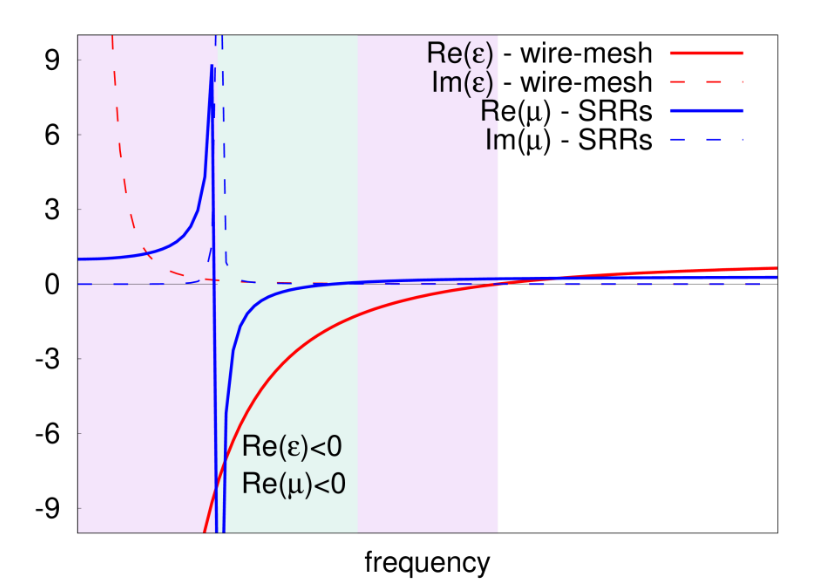


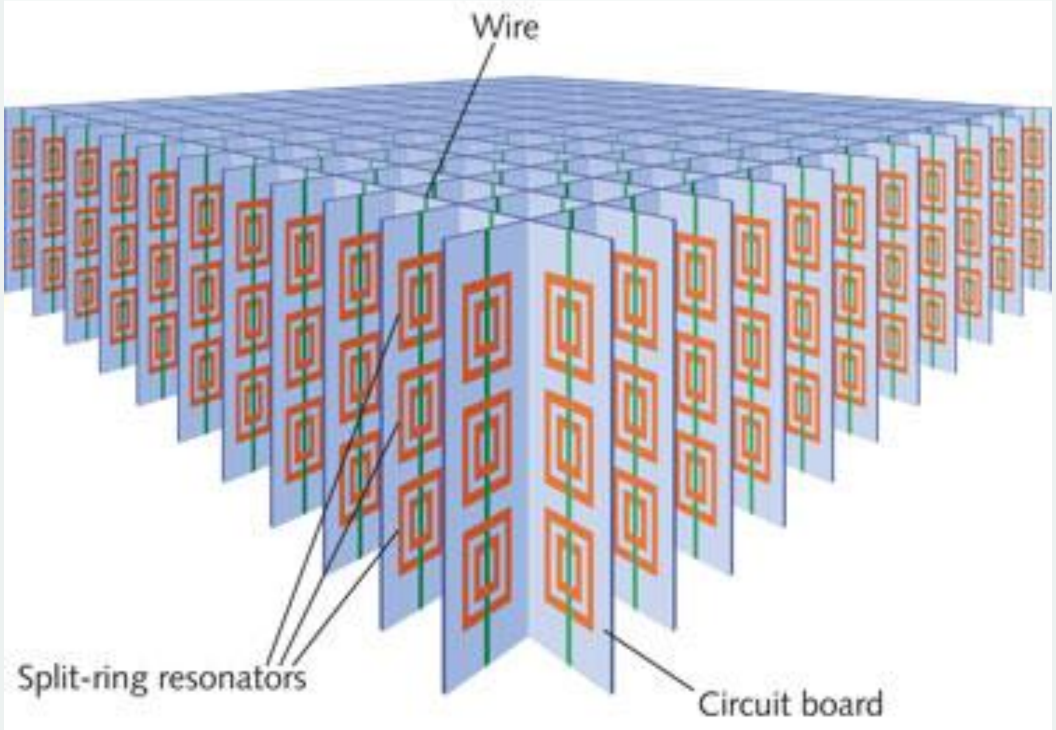
?

Double negative metamaterials

- To create a double negative medium, we can combine a wire-mesh metamaterial ($Re(\varepsilon)$) and a magnetic metamaterial ($Re(\mu)$).
- The **double negative metamaterial** exists only for the frequency regime that both:

$$Re(\varepsilon) < 0 \quad \text{and} \quad Re(\mu) < 0$$





- What is the **refractive index** of a double negative metamaterial?

$$\mathrm{Re}(n) < 0$$

Negative refractive index

- <u>Exercise</u>: I have a double negative metamaterial that at frequency ω_1 has an electric permittivity: $\varepsilon = -1 + \frac{1}{3}i$ and a magnetic permeability: $\mu = -9 + 3i$. What is the refractive index of this medium?

$$n = \sqrt{\varepsilon\mu} = \sqrt{(9-1) + i(-3-3)} = \sqrt{8-6i} = x + iy$$

To find the square root of a complex number: $n = x + iy \Rightarrow \ n^2 = (x+iy)^2$ and find the values of x and y.

$$n^2 = (x+iy)^2 = 8 - 6i$$
$$(x^2 - y^2) + 2ixy = 8 - 6i$$

$$\Rightarrow \quad x^2 - y^2 = 8, \quad 2xy = -6$$

$$|n^2| = |8 - 6i\,| = \sqrt{64+36} = \sqrt{100} = 10$$
$$|n^2| = |n|^2 = |x+iy|^2 = x^2 + y^2$$

$$\Rightarrow \quad x^2 + y^2 = 10$$

Solving the above equations to find x and y:

$$2x^2 = 18 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$$
$$y^2 = 1 \Rightarrow y = \pm 1$$

Since $2xy = -6$, then x and y have opposite signs.

Two possible solutions:

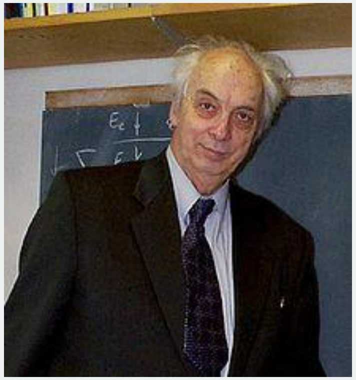
$\cancel{n = 3 - 1i}$

<u>Violates causality</u>: $Im(n) < 0$ implies that the EM wave acquires energy as it propagates though the medium.

$\boxed{n = -3 + 1i}$ Only sensible solution

Negative refractive index

- What is the **refractive index** of a double negative metamaterial? $\mathrm{Re}(n) < 0$
- **Veselago in 1968** realized theoretically that when a medium has both $\mathrm{Re}(\varepsilon)$ and $\mathrm{Re}(\mu)$ simultaneously negative, the real part of the refractive index is negative ($\mathrm{Re}(n) < 0$).
- "It could be admitted that substances with negative ε and μ have some properties different from those of substances with positive ε and μ." [Veselago]



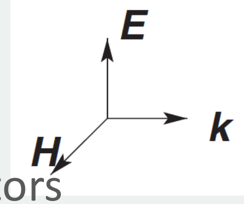
Viktor Veselago

- Consider Maxwell's equations and EM waves of the form: $e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$

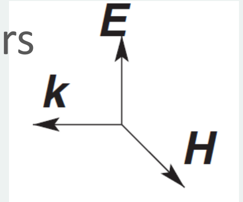
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad \text{and} \quad \nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\boldsymbol{k} \times \boldsymbol{E} = \omega\mu\mu_0\boldsymbol{H} \quad \text{and} \quad \boldsymbol{k} \times \boldsymbol{H} = -\omega\varepsilon\varepsilon_0\boldsymbol{E}$$

- $(\boldsymbol{E}, \boldsymbol{H})$ fields are different when both ε, μ are positive or negative.
- $\mathrm{Re}(\varepsilon) > 0$ and $\mathrm{Re}(\mu) > 0$, right-hand rule for the orthogonality of $(\boldsymbol{k}, \boldsymbol{E}, \boldsymbol{H})$ vectors



- $\mathrm{Re}(\varepsilon) < 0$ and $\mathrm{Re}(\mu) < 0$, left-hand rule for the orthogonality of $(\boldsymbol{k}, \boldsymbol{E}, \boldsymbol{H})$ vectors

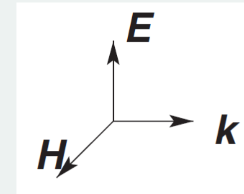


- Sometimes negative refracting media are called "left-handed media".

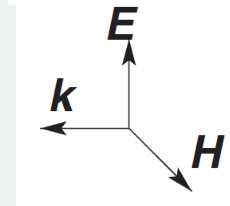
V. Veselago, Soviet Physics Uspekhi, 10, 509, (1968) - Note that this paper is old and may use different notations.

Negative refractive index

- $\mathrm{Re}(\varepsilon) > 0$ and $\mathrm{Re}(\mu) > 0$, right-hand rule for the orthogonality of $(\boldsymbol{k}, \boldsymbol{E}, \boldsymbol{H})$ vectors



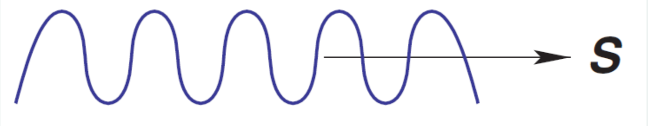
- $\mathrm{Re}(\varepsilon) < 0$ and $\mathrm{Re}(\mu) < 0$, left-hand rule for the orthogonality of $(\boldsymbol{k}, \boldsymbol{E}, \boldsymbol{H})$ vectors



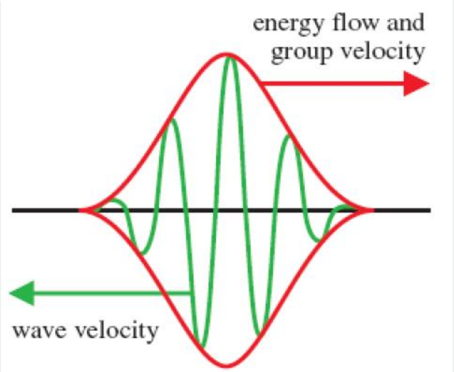
- The direction of $\boldsymbol{k} = n\boldsymbol{k_0}$ (wavevector) determines the direction of the **phase velocity** ($e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$).

- The Poynting vector tells the direction of energy propagation (**group velocity**):

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$$



- For positive media: $\boldsymbol{S} \parallel \boldsymbol{k}$ point at the same direction
- For negative media: $\boldsymbol{S}, \boldsymbol{k}$ point at opposite directions.



- For negative refractive index, **the group and phase velocities have opposite directions**.

V. Veselago, Soviet Physics Uspekhi, 10, 509, (1968) - Note that this paper is old and may use different notations.

Negative refractive index

Alternative approach:

- The refractive index is related to the electric permittivity and magnetic permeability as:

$$n = \pm\sqrt{\varepsilon\mu}$$

- When $\mathrm{Re}(\varepsilon) > 0$ and $\mathrm{Re}(\mu) > 0$, it is sensible to take the positive branch of square root.
- When $\mathrm{Re}(\varepsilon) < 0$ and $\mathrm{Re}(\mu) < 0$, it is sensible to take the negative branch of square root.

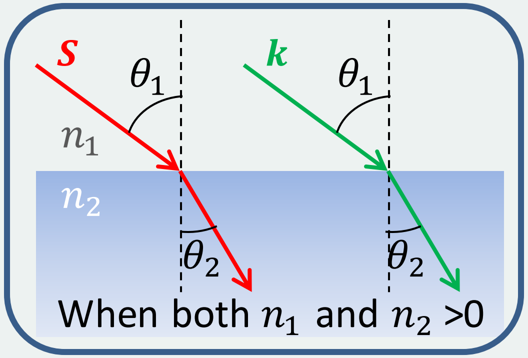
Snell's law:

- Determines the angle of refraction for rays propagating from one medium to another:

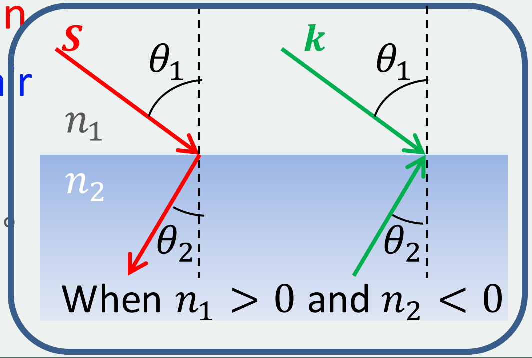
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- When both n_1 and n_2 are positive, then θ_2 is also positive.
- When $n_1 > 0$ and $n_2 < 0$, then θ_2 is negative:

Negative refraction



Negative refraction



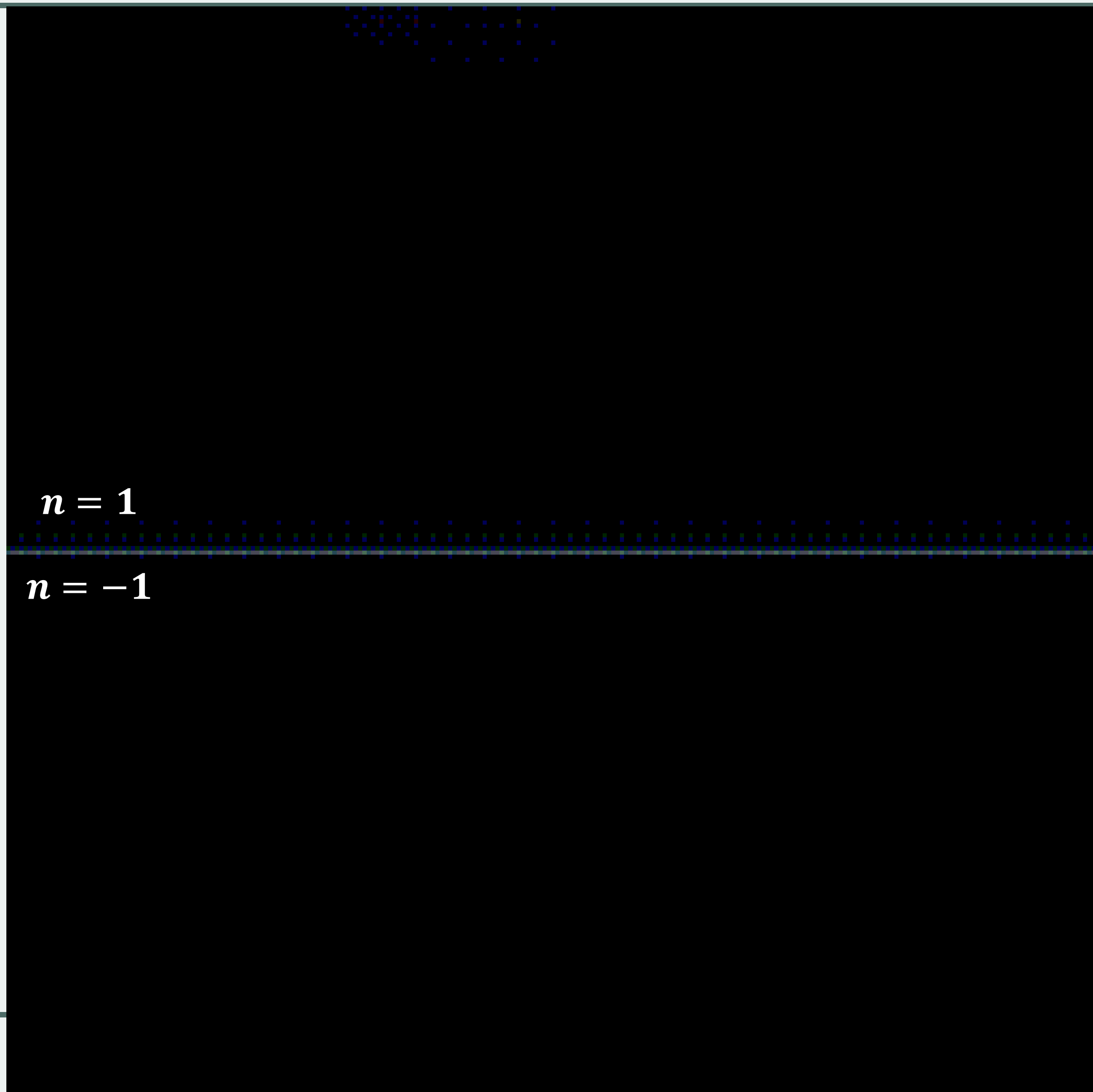
➢ Example: Find the angle of refraction of an EM wave incident from air ($n_1 = 1$) on a flat surface of a negative medium with $n_2 = -1$. The angle of incidence is $\theta_1 = 30°$.

$k_{x1} = k_{x2} \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \sin 30° = -\sin\theta_2 \Rightarrow \theta_2 = -30°$

and $k_{2z}z = n_2 k_0 \cos\theta_2 = -k_0 \cos\theta_1$

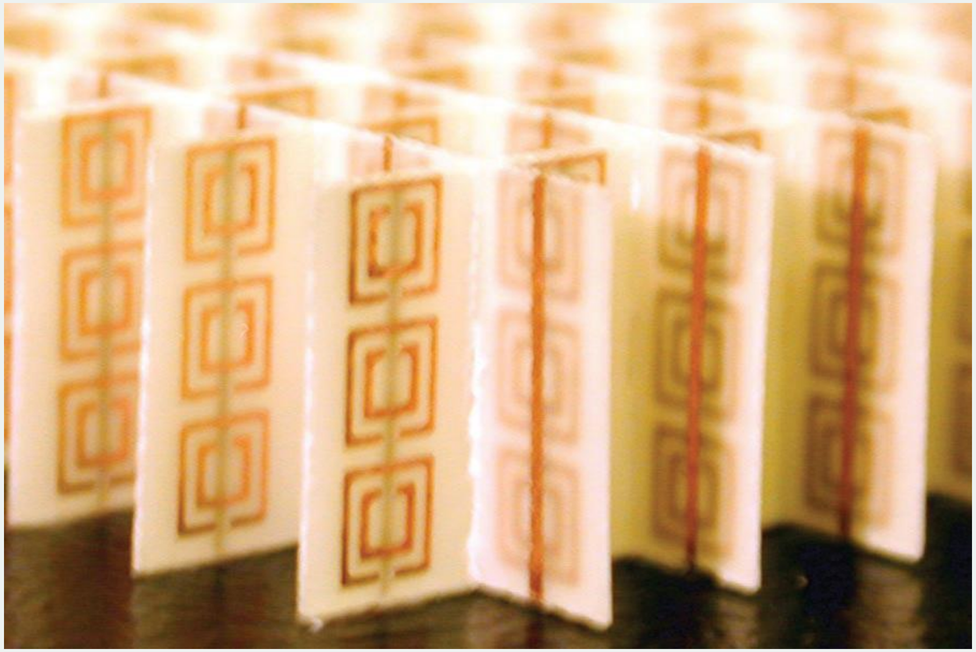
V. Veselago, Soviet Physics Uspekhi, 10, 509, (1968) - Note that this paper is old and may use different notations.

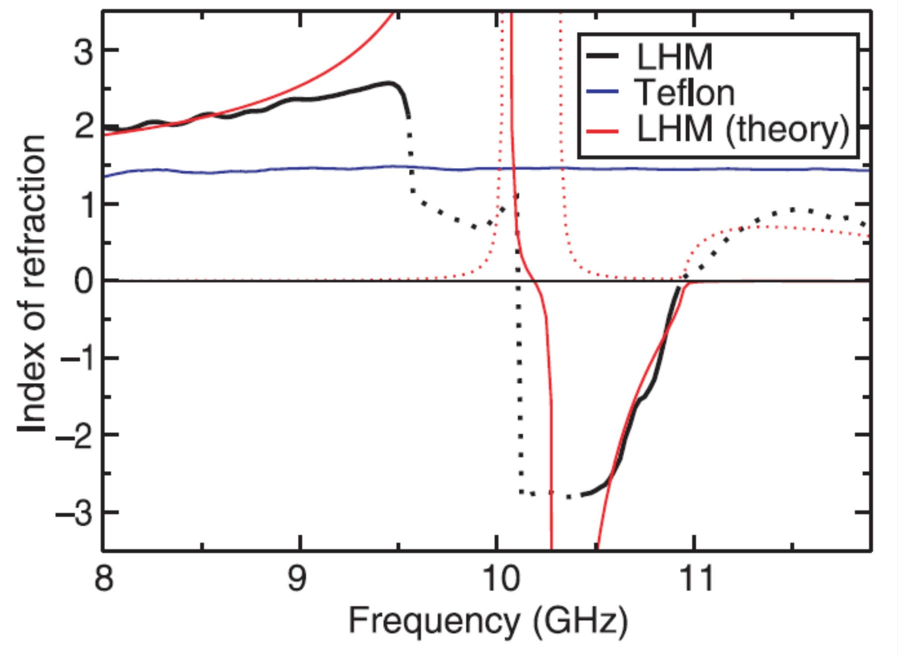
Negative refractive index



Negative refractive index – first experiment

- Difficult to realize negative refraction at optical frequencies, so first experiment was done at microwave frequencies (GHz).

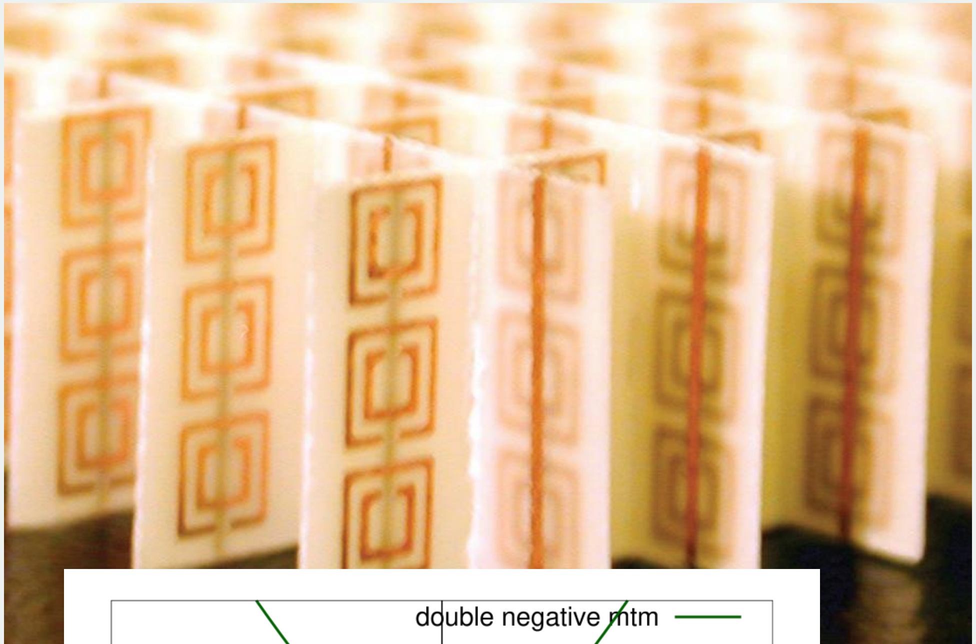


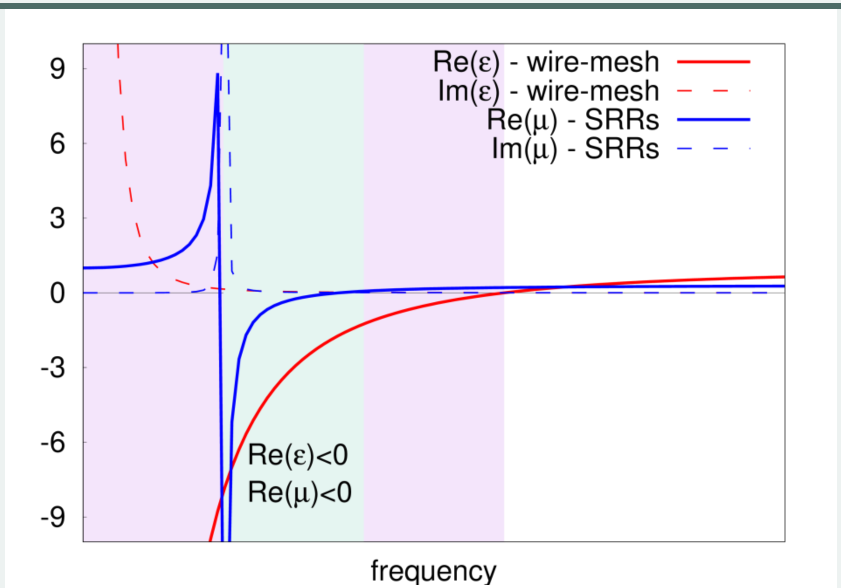


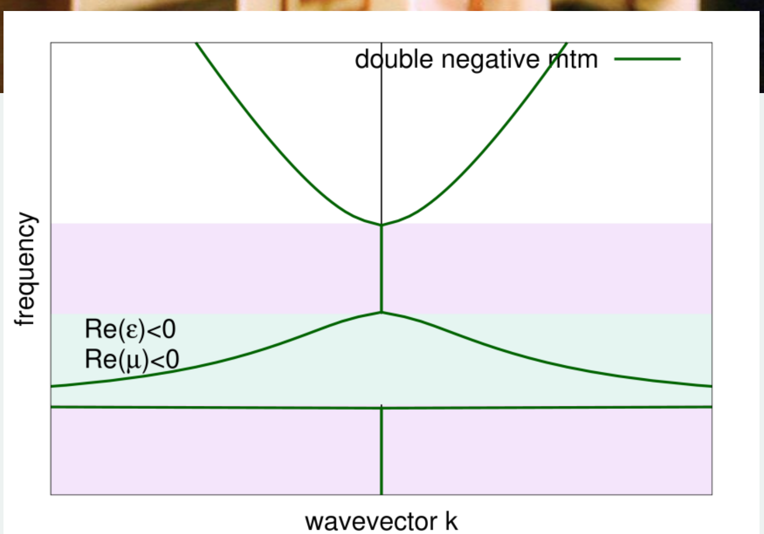
Index of refraction versus frequency. The blue curve corresponds to data from the Teflon sample, and the black curve is for the LHM data. The dotted portions of the LHM curve indicate regions where the index is expected to be either outside our limit of detection ($|n| > 3$) or dominated by the imaginary component and therefore could not be reliably determined experimentally. The solid red curve is the real component, and the dotted red curve is the imaginary component of the theoretical expression for the refractive index.

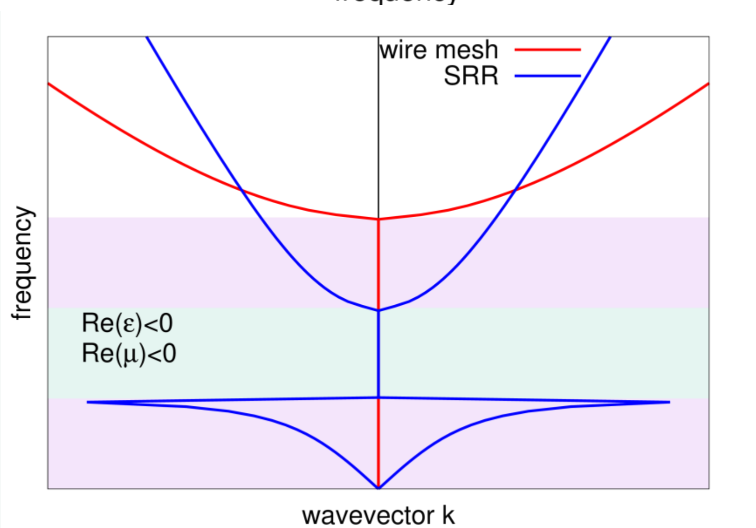
RA. Selby, DR. Smith, S. Schultz, "Experimental verification of a negative index of refraction", Science, 292, 77, (2001)

Negative refractive index – first experiment



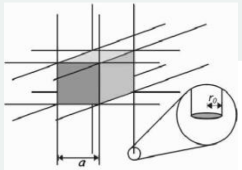






RA. Selby, DR. Smith, S. Schultz, "Experimental verification of a negative index of refraction", Science, 292, 77, (2001)

Classification of EM materials



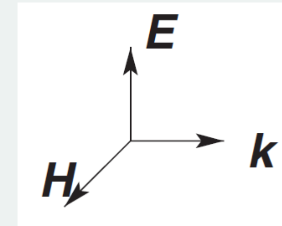
$\varepsilon < 0$ and $\mu > 0$

$n = \sqrt{\varepsilon\mu} \rightarrow imaginary$

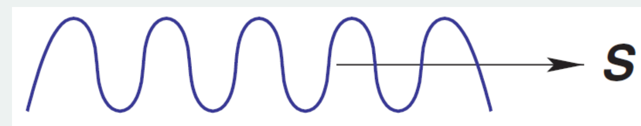
- Electrical Plasma
- Many metals (UV – optical freq.)
- Evanescent decaying waves
- Artificially: **Wire metamaterials**

$\varepsilon > 0$ and $\mu > 0$

$n = \sqrt{\varepsilon\mu} > 0$

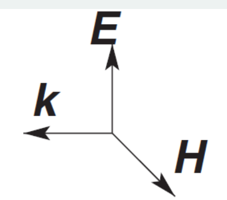


- Ordinary dielectric optical materials
- Propagating waves



μ

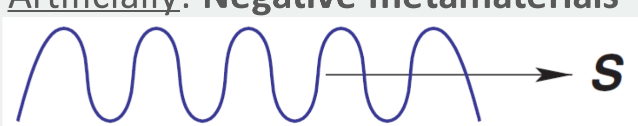
ε



$Re(\varepsilon) < 0$ and $\mathrm{Re}(\mu) < 0$

$Re(n) = \sqrt{\varepsilon\mu} < 0$

- Negative refractive index
- Propagating waves with k, S anti-parallel
- “Left-handed propagating media”
- Artificially: **Negative metamaterials**



$\varepsilon > 0$ and $\mu < 0$

$n = \sqrt{\varepsilon\mu} \rightarrow imaginary$

- Magnetic Plasma (?)
- Some natural materials (up to GHz)
- Evanescent decaying waves
- Artificially: **Magnetic metamaterials** (SRRs, Swiss-Rolls, …)

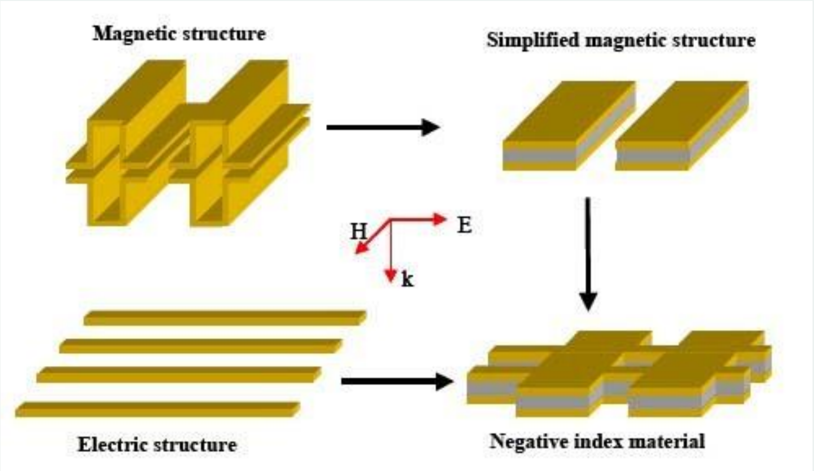
Other metamaterial designs

- The metamaterial designs we've seen so far, are the first ones proposed in 1999-2000. Since then there have been countless other designs, each one unique in design and performance.
- Here are some quick examples:

<u>Other negative metamaterials:</u>

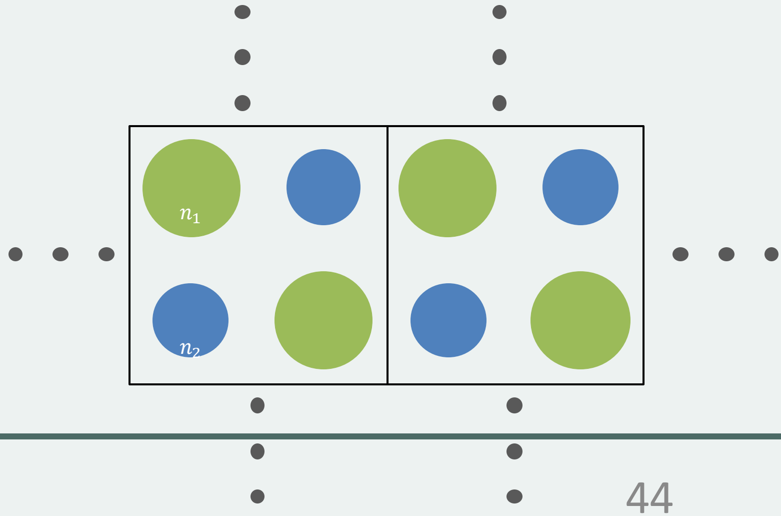
- **Fishnet structure**:

Parallel wires provide the $Re(\varepsilon) < 0$
Flattened parallel wires provide enough capacitance to create a resonant μ: $Re(\mu) < 0$



- **Array of dielectric spheres supporting Mie modes:**

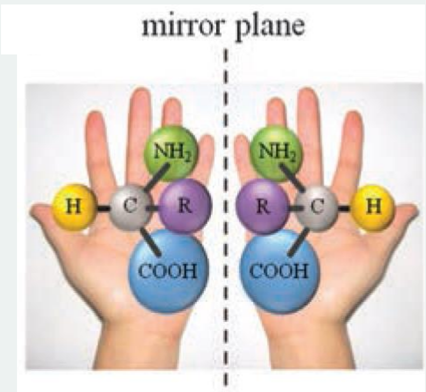
The electric and magnetic Mie modes produce Lorentz-like resonances.
We can use two different dielectric spheres, and tune the electric resonance of sphere1 with the magnetic resonance of shere2, such that $Re(\varepsilon) < 0$ and $Re(\mu) < 0$ at the same frequency regime.

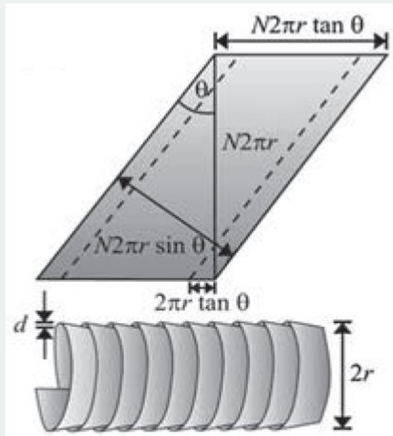


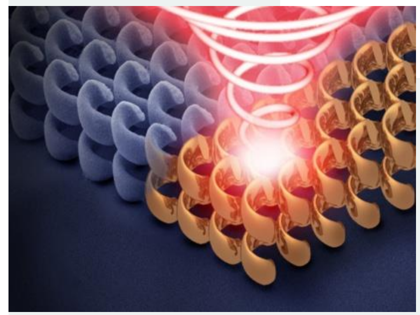
Other metamaterial designs

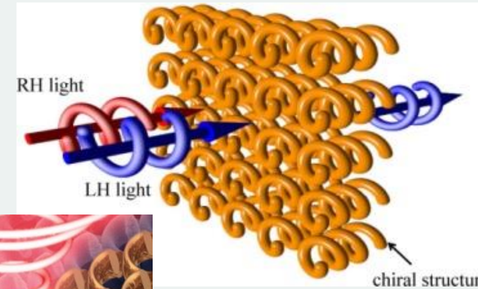
Chiral metamaterials:

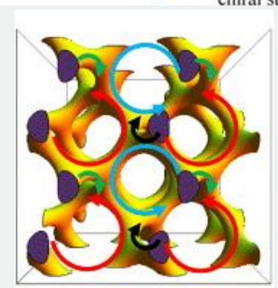
- Helical wires, Chiral Swiss Rolls, gyroid structure…

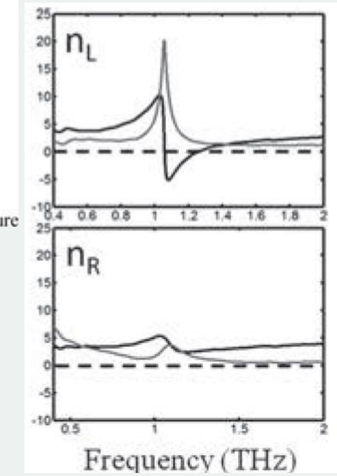




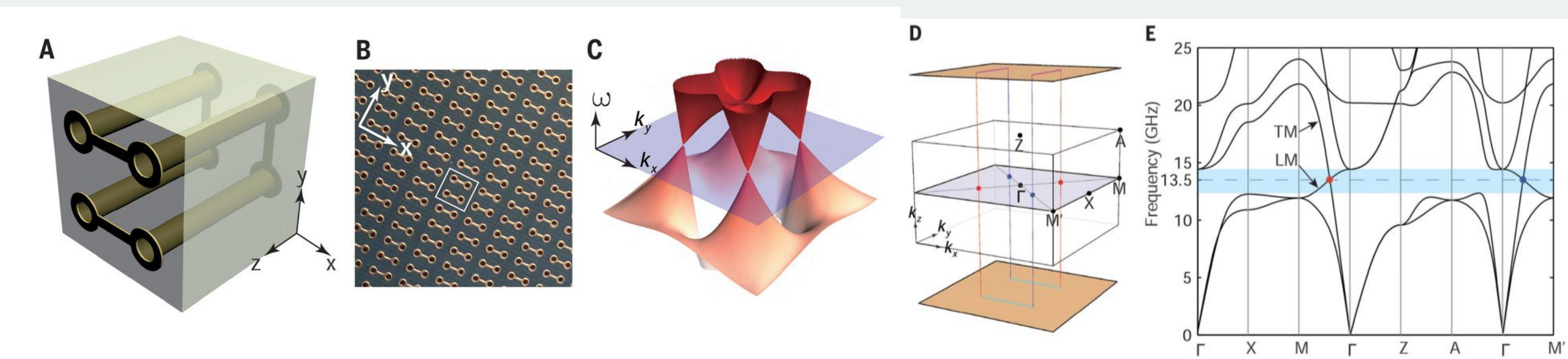








Topological metamaterials (Prof. Shuang Zhang, UoB)

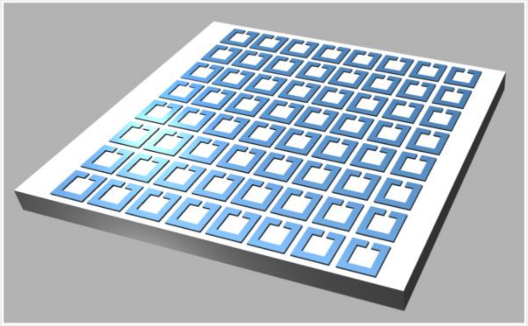
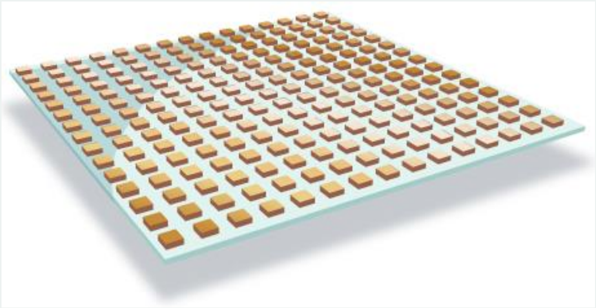
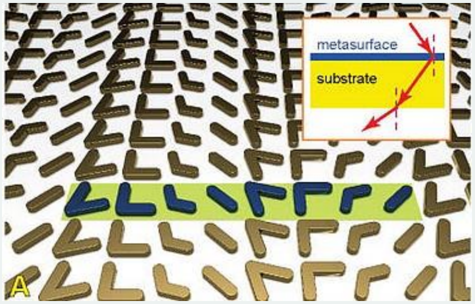


Metasurfaces

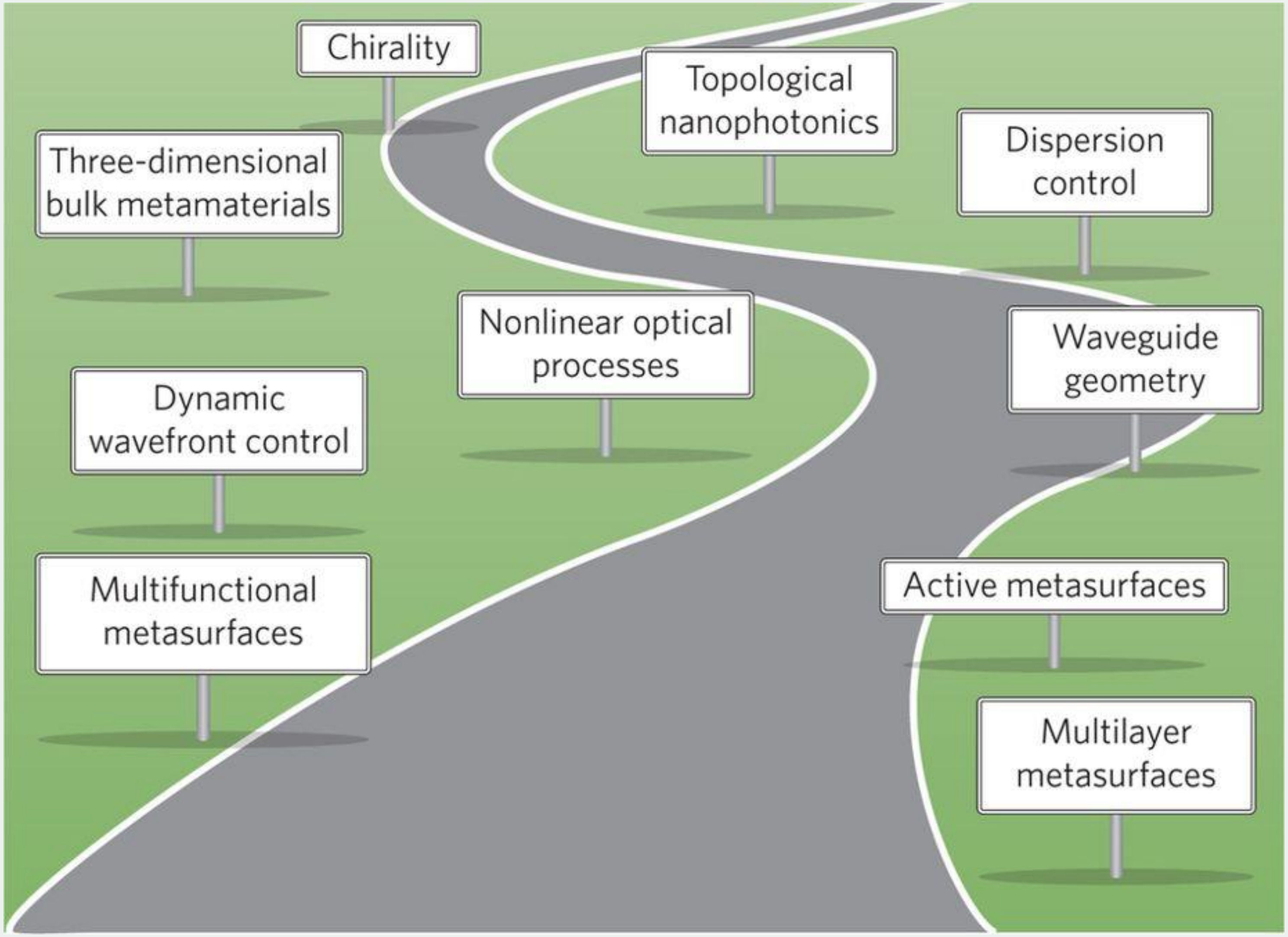
- Metasurfaces are 2D metamaterial structures.
- Metasurfaces are the metamaterial equivalent of mono-atomic materials (i.e. graphene)

- Metasurfaces are laterally (xy-axes) periodically repeated, but only one unit-cell thick (z-axis).
- Therefore, we cannot assign a macroscopic property along the z-axis.

- Metasurfaces are used extensively in the microwave antenna community, when a large effect (significant impact) is desired without using bulky devices.

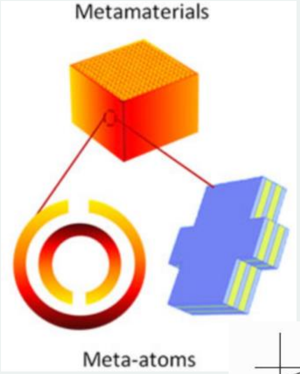




Possible road-map to the future

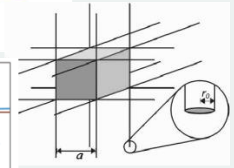


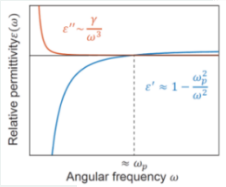
Summary

- **Metamaterials are artificial media,** composed of specifically designed composites (meta-atoms) arranged usually in periodic lattices.
- For $\lambda \gg a$ can be described with **macroscopic optical parameters.**
- Metamaterials can exhibit **exotic, unique and novel macroscopic properties (ε, μ) that are not found in nature**.

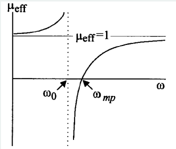


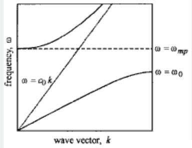
- **Electric metamaterials**: One can design metamaterials that behave as "diluted" plasmas, but now we tune the plasma frequency (wire-mesh).

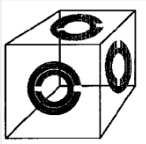




- **Magnetic metamaterials**: Split-ring resonators, Swiss-rolls (need a capacitive element to obtain an effective magnetic response).



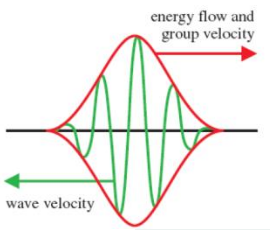




- **Negative refractive metamaterials** = **double negative** metamaterials

Causality dictates that double negative media need to have $Re(n)$

- **Negative refraction**: **Energy and wave propagation** in a negative medium propagate at opposite directions.



- For negative refraction, **Snell's law** is obeyed, and hence refraction angle from a positive to negative medium (and vice-versa) is negative.