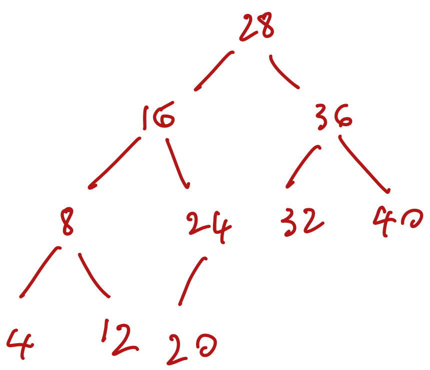
ESERCIZIO 2 **(Splay trees)**

Si descrivano le operazioni di *zig-zag*, *zig-zig* e *zig* in uno splay tree di tipo bottom-up.
Quindi si eseguano nell'ordine dato le seguenti operazioni su uno splay tree la cui configurazione iniziale è quella di un albero binario completo contenente le 10 chiavi $\{4i : 1 \le i \le 10\}$:

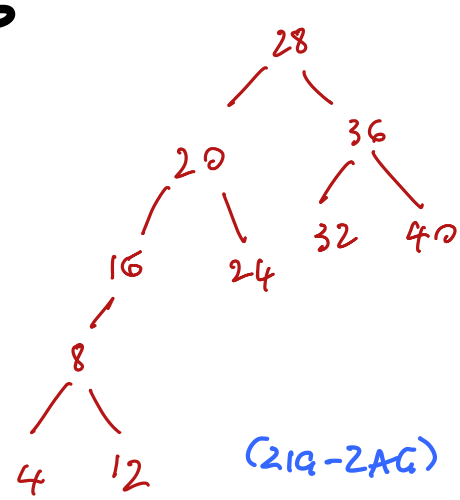
- SEARCH 20, 40
- DELETE 24
- INSERT 30

Nota bene: Si ricorda che un albero binario si dice *completo* quando tutti i suoi livelli, con al più l'eccezione dell'ultimo, sono completi e tutti i nodi nell'ultimo livello si trovano il più a sinistra possibile.

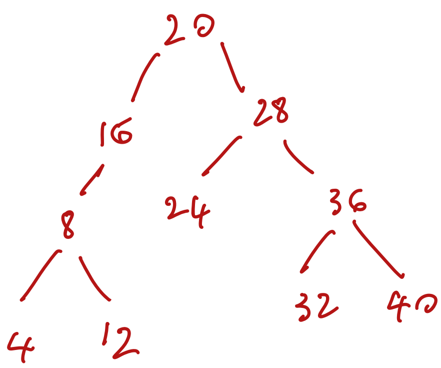
BOTTOM-UP



SEARCH(20) →

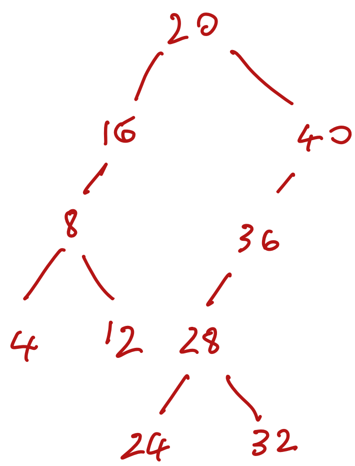


(ZIG-ZAG)

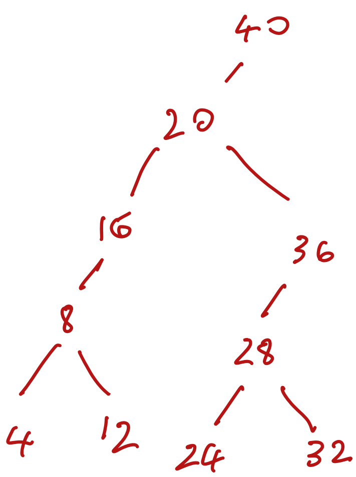


(ZIG)

SEARCH(40) →

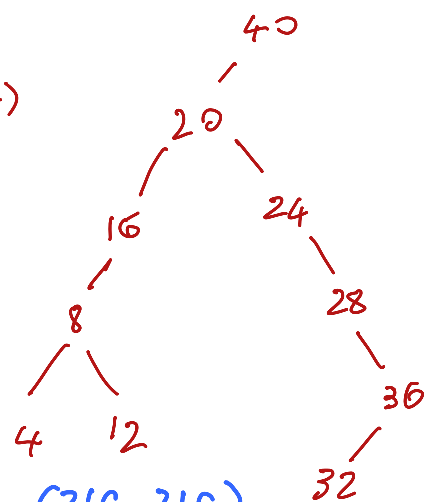


(ZIG-ZIG)

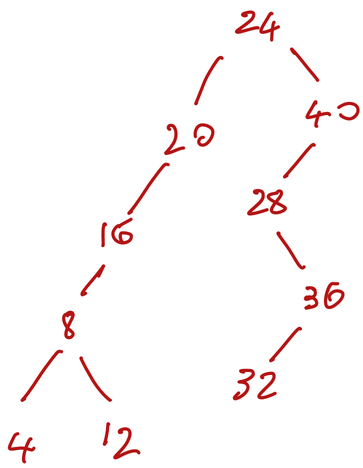


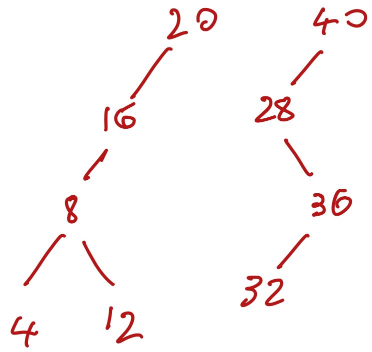
(ZIG)

DELETE(24) →

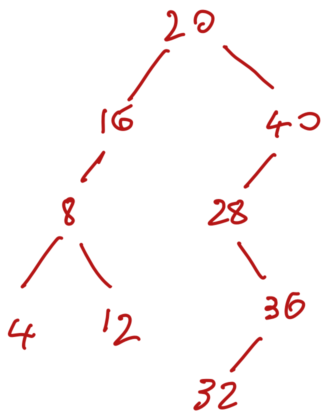


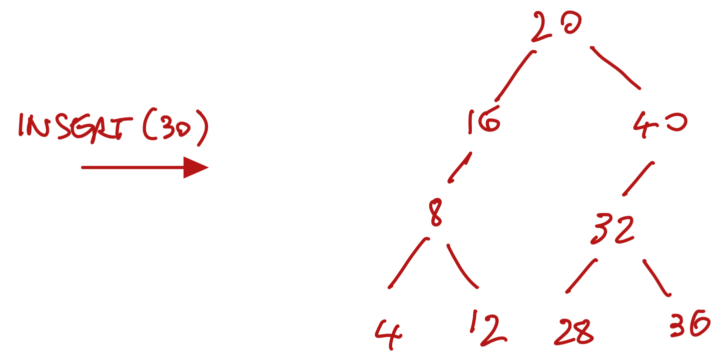
(ZIG-ZIG)



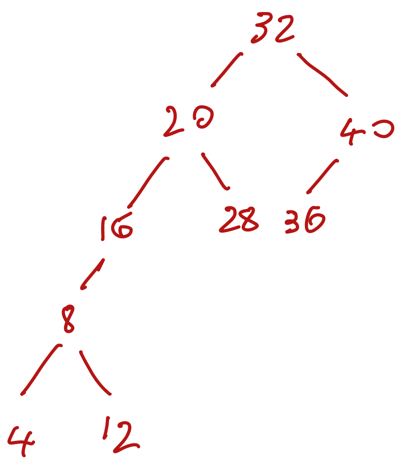


(ZIG-ZAG)

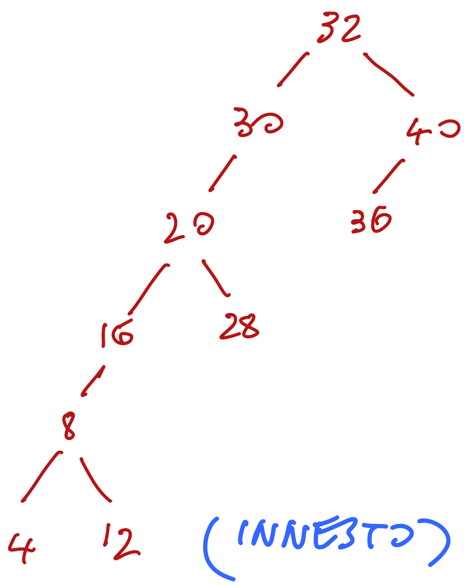




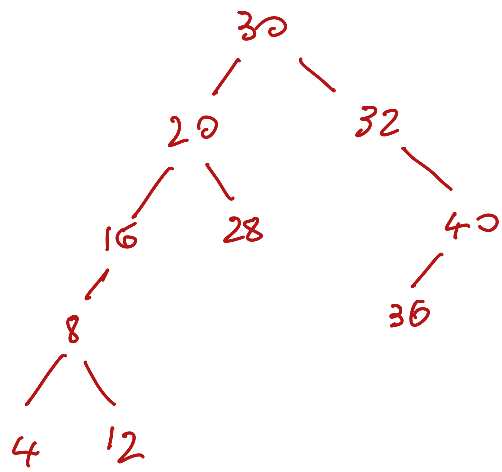
(ZIG-ZAG)



(ZIG-ZAG)



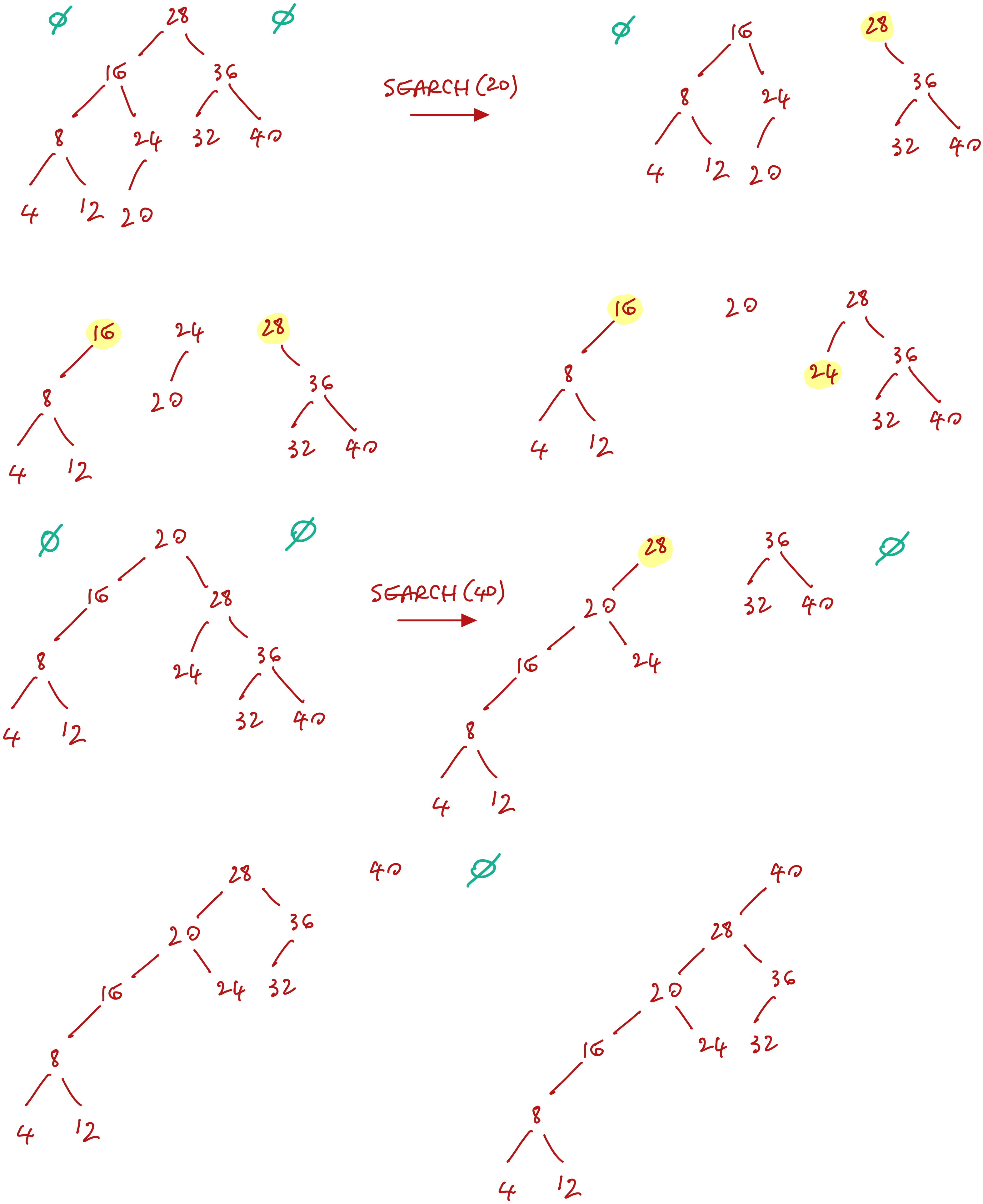
(INNESTO)



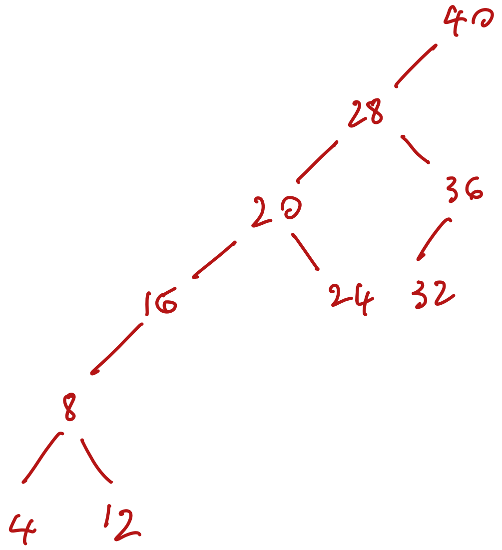
(ROTAZIONE FINALE)

- Search 20, 40
- Delete 24
- Insert 30

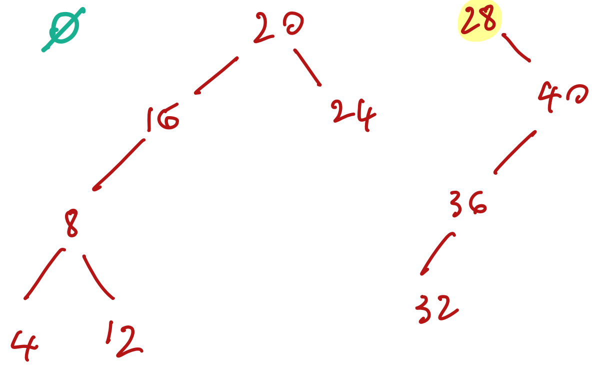
TOP-DOWN

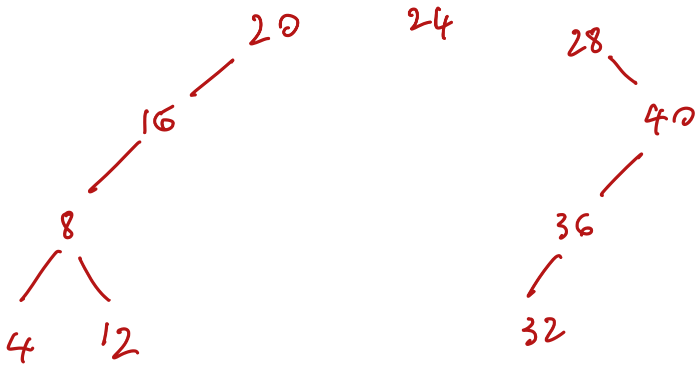


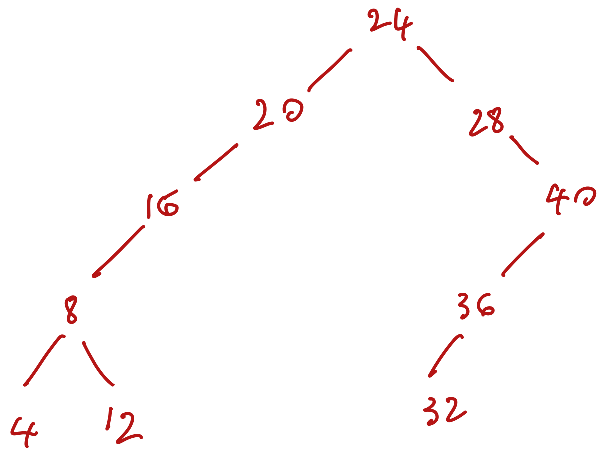
∅ 28 ∅
16 36
8 24 32 40
4 12 20
SEARCH(20)
∅ 16 28
8 24 36
4 12 20 32 40
16 24 28
8 20 36
4 12 32 40
16 20 28
8 24 36
4 12 32 40
∅ 20 ∅
16 28
8 24 36
4 12 32 40
SEARCH(40)
28 36 ∅
20 32 40
16 24
8
4 12
28 40 ∅
20 36
16 24 32
8
4 12
40
28
20 36
16 24 32
8
4 12

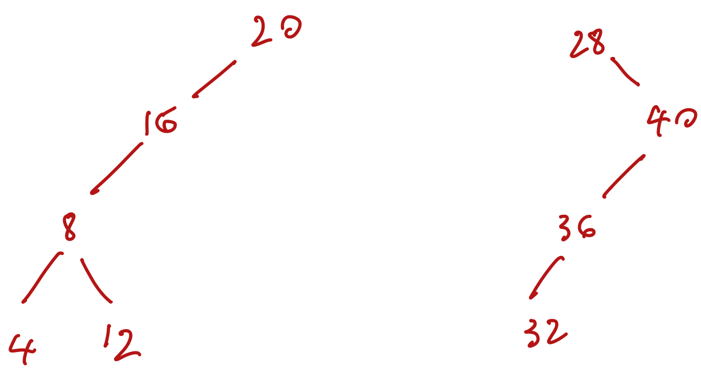


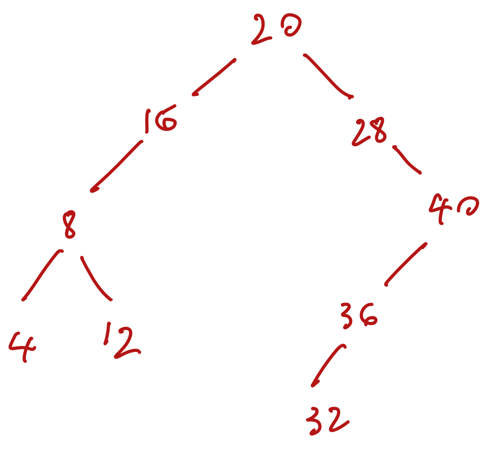
DELETE(24)

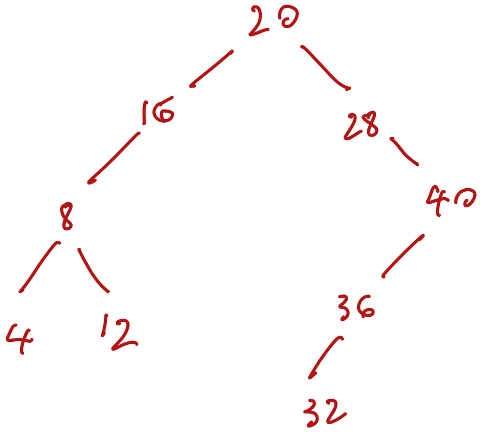




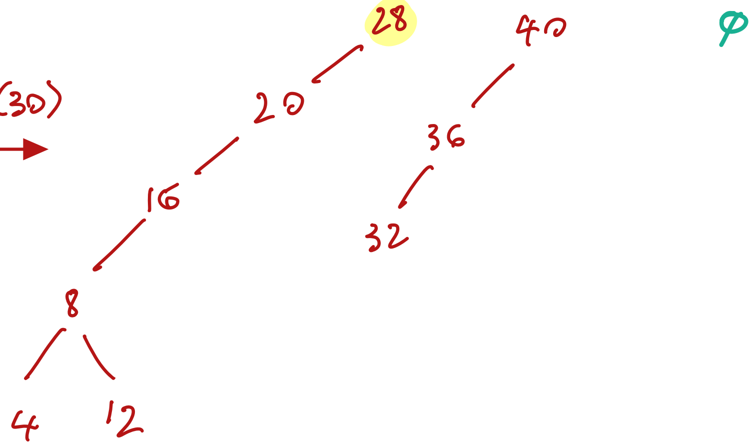


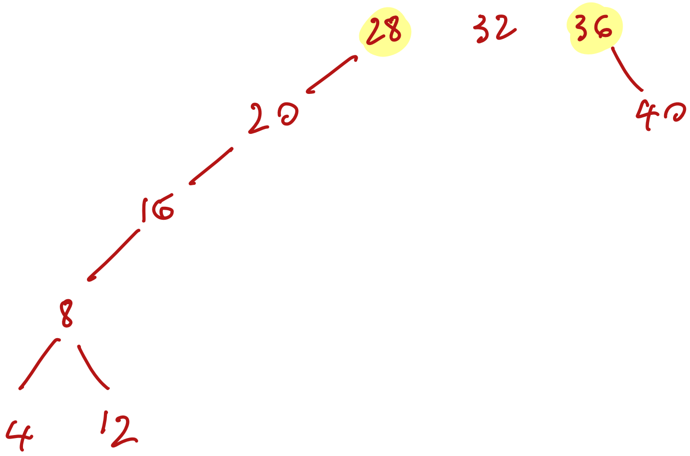


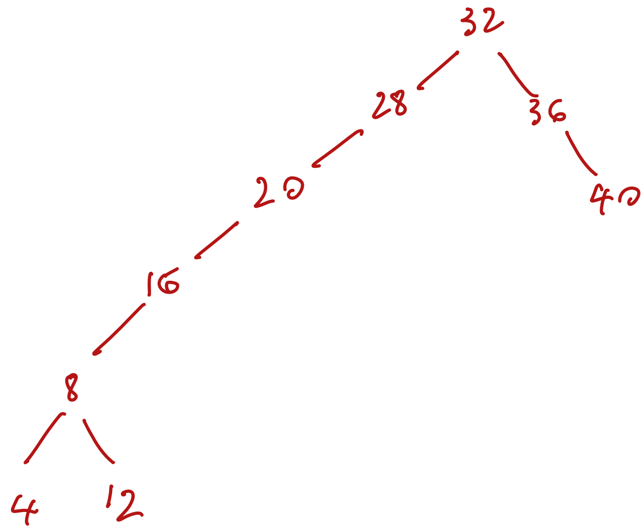




INSERT(30)




28
32
36
20
40
16
8
4
12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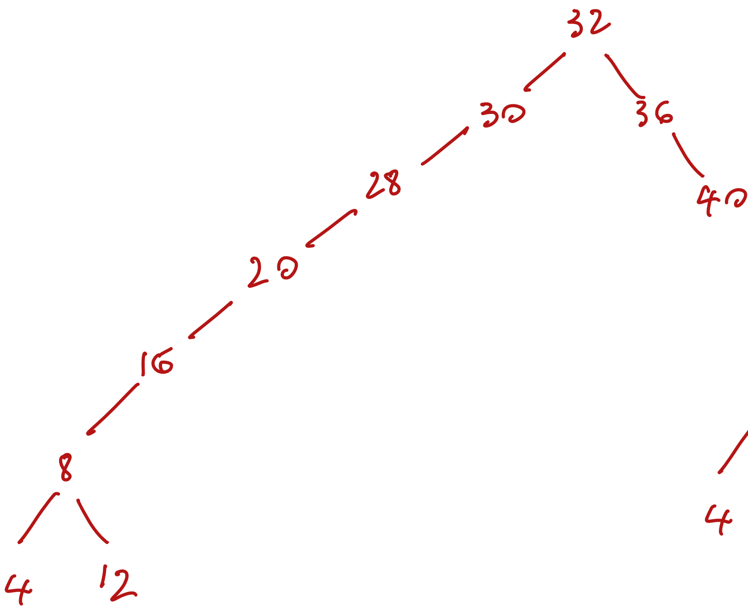
12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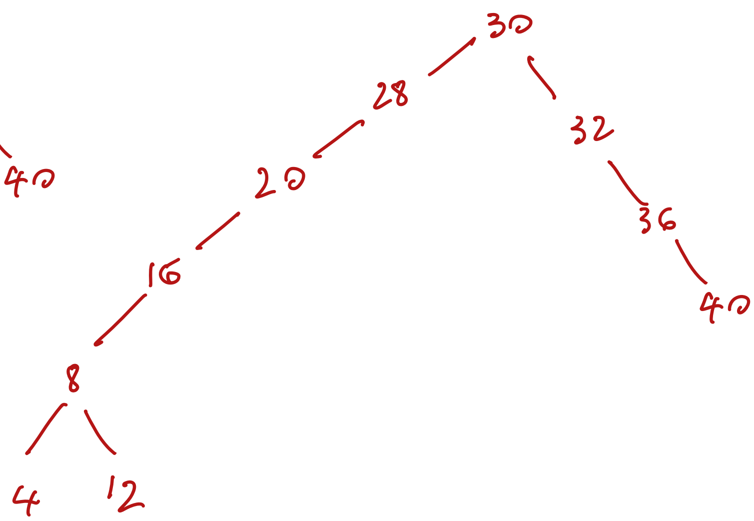
12


30
28
32
20
36
16
40
8
4
12

S, T

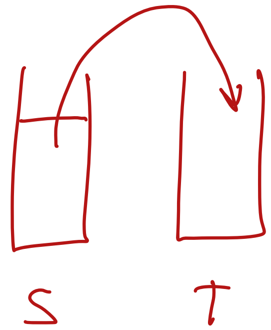
$\phi(S_i, T_i) = 2|S_i|$

$\phi(S_0, T_0) = 2|S_0| = 0$,

$\underline{\phi(S_i, T_i) \geq 0 = \phi(S_0, T_0)}$

$\hat{c}_{ENQUEUE} = c_{ENQUEUE} + \Delta\phi$

$= 1 + 2 = \underline{3}$



DEQUEUE SENZA TRAVASO

$\hat{c}_{DEQUEUE} = c_{DEQUEUE} + \Delta\phi$

$= 1 + 0 = 1$

DEQUEUE CON TRAVASO

$\hat{c}_{DEQUEUE} = c_{DEQUEUE} + \Delta\phi$

$= 2|S| + 1 - 2|S| = 1$

$\sum c_i \leq \sum \hat{c}_i = 3 \cdot \#\text{ENQUEUE} + \#\text{DEQUEUE}$

7) SI PROGETTI UNA STRUTTURA DATI PER SUPPORTARE LE SEGUENTI OPERAZIONI PER UN INSIEME DINAMICO S DI INTERI:

INSERT(S, x) - INSERISCE x IN S

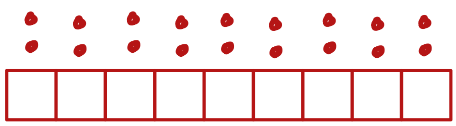
DELETE-LARGER-HALF(S) - CANCELLA GLI $\left\lceil \frac{|S|}{2} \right\rceil$ ELEMENTI PIÙ GRANDI DA S

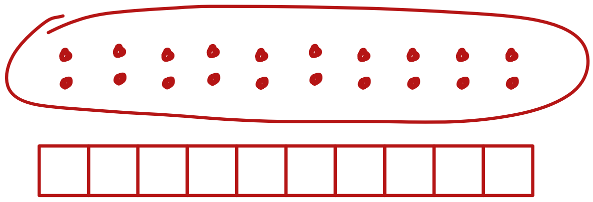
SI SPIEGHI COME IMPLEMENTARE QUESTA STRUTTURA DATI IN MODO CHE QUALSIASI SEQUENZA DI m OPERAZIONI VENGA ESEGUITA NEL TEMPO $O(m)$.

$\boxed{m = 2^k} \longrightarrow 2n$

$$\boxed{\begin{aligned} \hat{c}_{INSERT} &= 3 \\ \hat{c}_{DELETE} &= 0 \end{aligned}}$$

✓





$$2 \cdot \left\lceil \frac{|S|}{2} \right\rceil \geq 2 \frac{|S|}{2} = \underline{|S|}$$

$$\boxed{\begin{aligned} &\Phi(S) = 2|S| \\ &\Phi(S_0) = 0, \quad \Phi(S) \geq 0 = \Phi(S_0) \end{aligned}}$$

$$\boxed{\begin{aligned} \hat{c}_{INSERT} &= c_{INSERT} + \Delta\Phi \\ &= 1 + 2 = 3 \end{aligned}}$$

$$\boxed{\begin{aligned} \hat{c}_{DELETE} &= c_{DELETE} + \Delta\Phi \\ &= |S| - 2 \cdot \left\lceil \frac{|S|}{2} \right\rceil \\ &\leq |S| - 2 \cdot \frac{|S|}{2} = 0 \end{aligned}}$$

ESERCIZIO 1

Utilizzando i tre metodi dell'analisi ammortizzata, si determini il costo ammortizzato per operazione di una sequenza di n operazioni, ove il costo c_i dell'i-esima operazione sia dato da

$$c_i = \begin{cases} 3 \cdot i & \text{se } i \text{ è potenza esatta di } 4 \\ 5 & \text{altrimenti}\,. \end{cases}$$

<u>AGGREGAZIONE</u>

$$T(n) = \sum_{i=1}^{n} c_i = \sum_{\substack{i=1 \\ i \in 4^{\mathbb{N}}}}^{n} 3 \cdot i + \sum_{\substack{i=1 \\ i \notin 4^{\mathbb{N}}}}^{n} 5$$

$$\leq 3 \sum_{j=0}^{\lfloor \log_4 n \rfloor} 4^j + 5n$$

$$= \cancel{3}\, \frac{4^{\lfloor \log_4 n \rfloor + 1} - 1}{\cancel{4 - 1}} + 5n$$

$$< 4^{\log_4 n + 1} + 5n$$

$$= 4n + 5n$$

$$= 9n$$

$$\boxed{\hat{c}_i = \frac{T(n)}{n} < 9}$$

ACCANTONAMENTI

i	c_i	$\hat{c}_i$
1	3	3
2	5	5 + 4
3	5	5 + 4
4	$3 \cdot 4$	4
5	5	5 + 4
6	5	5 + 4
7	5	5 + 4
8	5	5 + 4
9	5	5 + 4
10	5	5 + 4
11	5	5 + 4
12	5	5 + 4
13	5	5 + 4
14	5	5 + 4
15	5	5 + 4
16	$3 \cdot 16$	4
$\vdots$	$\vdots$	

$$12 \cdot 4 = 3 \cdot 4 \cdot 4 = 3 \cdot 16$$

$$\hat{c}_i = \begin{cases} 3 & \text{se } i = 1 \\ 4 & \text{se } i \in 4^{\mathbb{N}} \\ 9 & \text{se } i \notin 4^{\mathbb{N}} \end{cases}$$

POTENZIALE

$$\Phi_i = \begin{cases} 0 & \text{se } i = 0 \\ 4\left(i - 4^{\lfloor \log_4 i \rfloor}\right) & \text{se } i > 0 \end{cases}$$

$$\boxed{\begin{aligned} \hat{c}_1 &= c_1 + \Delta\Phi \\ &= 3 + 0 \\ &= 3 \end{aligned}}$$

$i \notin 4^{\mathbb{N}} \longrightarrow \lfloor \log_4 i \rfloor = \lfloor \log_4 (i-1) \rfloor$

$$\begin{aligned} \hat{c}_i &= c_i + \Delta\Phi \\ &= 5 + 4\left(\cancel{i} - \cancel{4^{\lfloor \log_4 i \rfloor}}\right) - 4\left(\cancel{i} - 1 - \cancel{4^{\lfloor \log_4 (i-1) \rfloor}}\right) \\ &= 5 + 4 \\ &= 9 \end{aligned}$$

$$i \in 4^{\mathbb{N}} \longrightarrow \quad 4^{\lfloor \log_4 i \rfloor} = i \;, \quad 4^{\lfloor \log_4 (i-1) \rfloor} = \frac{i}{4}$$

$$\begin{aligned}
\hat{c}_i &= c_i + \Delta\Phi \\
&= 3 \cdot i + 4\left(\cancel{i} - 4^{\lfloor \log_4 i \rfloor}\right) - 4\left(\cancel{i} - 1 - 4^{\lfloor \log_4 (i-1) \rfloor}\right) \\
&= \cancel{3 \cdot i} - \cancel{4i} + 4 + \cancel{4 \cdot \frac{i}{4}} \\
&= 4
\end{aligned}$$

RIASSUMENDO:

$$\boxed{\hat{c}_i = \begin{cases} 3 & \text{se } i = 1 \\ 4 & \text{se } i \in 4^{\mathbb{N}} \\ 9 & \text{se } i \notin 4^{\mathbb{N}} \end{cases}}$$