ESERCIZIO 1

$5^{\mathbb{N}} = \{ 5^m : m \in \mathbb{N} \}$

Utilizzando i tre metodi dell'analisi ammortizzata, si determini il costo ammortizzato per operazione di una sequenza di n operazioni, ove il costo effetivo c_i dell'i-esima operazione sia dato da

$$c_i = \begin{cases} 6 \cdot i & \text{se } i \text{ è potenza esatta di } 5 \\ 3 & \text{altrimenti} . \end{cases}$$

$5^{\mathbb{N}}$

$c_i = O(n)$

$\to O(n^2)$

METODO DELL'AGGREGAZIONE

$$T(n) = \sum_{i=1}^{n} c_i = \sum_{\substack{i=1 \\ i \in 5^{\mathbb{N}}}}^{n} 6 \cdot i + \sum_{\substack{i=1 \\ i \notin 5^{\mathbb{N}}}}^{n} 3$$

$$6 \cdot \sum_{\substack{i=1 \\ i \in 5^{\mathbb{N}}}}^{n} i =$$

$$\leq 6 \cdot \sum_{j=0}^{\lfloor \log_5 n \rfloor} 5^j + 3n$$

$$6 \cdot \left(1 + 5 + 5^2 + \ldots + 5^{\lfloor \log_5 n \rfloor} \right)$$

$$= 6 \cdot \frac{5^{\lfloor \log_5 n \rfloor + 1} - 1}{5 - 1} + 3n$$

$$\leq \overset{3}{\cancel{6}} \cdot \frac{5^{\log_5 n + 1} \cancel{-1}}{\cancel{4}\, 2} + 3n$$

$$5^{\log_5 n + 1} = 5^{\log_5 n} \cdot 5^1 = n \cdot 5$$

$$= \frac{15}{2} n + 3n$$

$$= \frac{21}{2} n$$

$$\hat{c}_i = \frac{T(n)}{n} \leq \frac{21}{2} n \cdot \frac{1}{n} = \frac{21}{2}$$

METODO DEL POTENZIALE

$$c_i = \begin{cases} 6 \cdot i & \text{se } i \text{ è potenza esatta di } 5 \\ 3 & \text{altrimenti} . \end{cases}$$

1	6
2	3
3	3
4	3
5	$6 \cdot 5$
6	3
7	3
8	3
$i \to$ 9	3
$\vdots$	$\vdots$
23	3
24	3
25	$6 \cdot 5^2$
26	3
27	3
$\vdots$	$\vdots$

5^k	$6 \cdot 5^k$
5^k+1	3
5^k+2	3
$\vdots$	
$5^{k+1}-1$	3
5^{k+1}	$6 \cdot 5^{k+1}$
$\vdots$	$\vdots$

$$\frac{6 \cdot 5^{k+1}}{5^{k+1} - 5^k}$$

$$= \frac{6 \cdot 5}{5-1} = \frac{6 \cdot 5}{4} = \frac{3 \cdot 5}{2}$$

$$= \frac{15}{2}$$

$$\phi(i) \equiv \phi_i \equiv \phi(D_i)$$

$$3 + \frac{15}{2} = \frac{21}{2}$$

$$\phi(i) = \begin{cases} \frac{15}{2}\left(i - 5^{\lfloor \log_5 i \rfloor}\right) & \text{se } i > 0 \\ 0 & \text{se } i = 0 \end{cases}$$

$$\phi(i) \geq \phi(0) \ \forall i \longleftrightarrow \phi(i) \geq 0 \quad \forall i$$

$$i \geq 5^{\lfloor \log_5 i \rfloor}$$

$$\phi(i) = \frac{15}{2}\left(i - 5^{\lfloor \log_5 i \rfloor}\right) \geq 0 = \phi(0)$$

$i = 1$

$$\hat{c}_i = c_i + \phi(i) - \phi(i-1)$$
$$= 6 + 0 - 0$$
$$= 6$$

$i = 1$

$i \in 5^{\mathbb{N}} \wedge i \neq 1$

$i \notin 5^{\mathbb{N}}$

$i \notin 5^{\mathbb{N}}$

$i \notin 5 \rightarrow \lfloor \log_5 i \rfloor = \lfloor \log_5 (i-1) \rfloor$

$$\hat{c}_i = c_i + \phi(i) - \phi(i-1)$$

$$= 3 + \frac{15}{2}\left(\cancel{i} - \cancel{5^{\lfloor \log_5 i \rfloor}}\right) - \frac{15}{2}\left((\cancel{i} - 1) - \cancel{5^{\lfloor \log_5 (i-1) \rfloor}}\right)$$

$$= 3 + \frac{15}{2}$$

$$= \frac{21}{2}$$

$i \in 5^{\mathbb{N}}$ & $i \neq 1$

$i \in 5^{\mathbb{N}} \longrightarrow \lfloor \log_5 i \rfloor = \log_5 i$

$\wedge \quad \lfloor \log_5 (i-1) \rfloor = \log_5 i - 1 = \log_5 \frac{i}{5}$

$$\hat{c}_i = c_i + \phi(i) - \phi(i-1)$$

$$= 6 \cdot i + \frac{15}{2}\left(\cancel{i} - 5^{\lfloor \log_5 i \rfloor}\right) - \frac{15}{2}\left((\cancel{i} - 1) - 5^{\lfloor \log_5 (i-1) \rfloor}\right)$$

$$= 6i - \frac{15}{2} \cdot i + \frac{15}{2} + \frac{\overset{3}{\cancel{15}}}{2} \cdot \frac{i}{\cancel{5}}$$

$$= \left(6 - \frac{15}{2} + \frac{3}{2}\right) i + \frac{15}{2}$$

$$= \frac{12 - 15 + 3}{2} i + \frac{15}{2}$$

$$= \frac{15}{2}$$

$$\hat{c}_i = \begin{cases} 6 & i = 1 \\ \frac{21}{2} & i \notin 5^{\mathbb{N}} \\ \frac{15}{2} & i \in 5^{\mathbb{N}} \wedge i \neq 1 \end{cases}$$

$O(m)$

$$\sum_{i=1}^{m} c_i \leq \sum_{i=1}^{m} \hat{c}_i \leq \frac{21}{2} m$$

METODO DEGLI ACCANTONAMENTI

$$\hat{c}_i = \begin{cases} 6 & i = 1 \\ \frac{21}{2} & i \notin 5^{\mathbb{N}} \\ \frac{15}{2} & i \in 5^{\mathbb{N}} \wedge i \neq 1 \end{cases}$$

$$c_i = \begin{cases} 6 \cdot i & \text{se } i \text{ è potenza esatta di } 5 \\ 3 & \text{altrimenti}\,. \end{cases}$$

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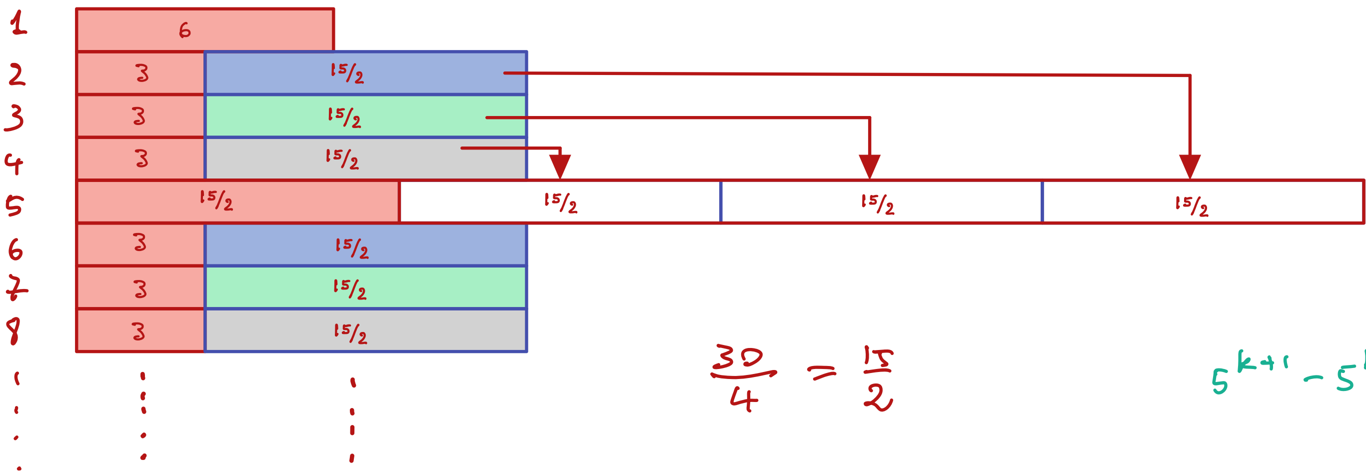
$3 + \frac{15}{2} = \frac{21}{2}$



$\frac{30}{4} = \frac{15}{2}$

$5^{k+1} - 5^{k}$