

ESERCIZIO 1

Utilizzando i tre metodi dell'analisi ammortizzata, si determini il costo ammortizzato per operazione di una sequenza di n operazioni, ove il costo c_i dell' i -esima operazione sia dato da

$$c_i = \begin{cases} 4 \cdot i & \text{se } i \text{ è potenza esatta di } 9 \\ \frac{3}{2} & \text{altrimenti.} \end{cases}$$

METODO DELL'AGGREGAZIONE

$$T(n) = \sum_{i=1}^n c_i = \sum_{\substack{i=1 \\ i \neq 9^k}}^n c_i + \sum_{i \in 9^N} c_i$$

$$= \sum_{\substack{i=1 \\ i \neq 9^k}}^n \frac{3}{2} + \sum_{i \in 9^N} 4 \cdot i$$

$$\leq \frac{3}{2} n + 4 \cdot \sum_{j=0}^{\lfloor \lg n \rfloor} 9^j$$

$$= \frac{3}{2} n + 4 \cdot \frac{9^{\lfloor \lg n \rfloor + 1} - 1}{9 - 1}$$

$$\leq \frac{3}{2} n + \frac{1}{2} \cdot 9^{\lg n + 1}$$

$$= \frac{3}{2} n + \frac{9}{2} n$$

$$= \frac{12}{2} n$$

$$= 6n$$

$$c_i = \frac{T(n)}{n} \leq \frac{6n}{n} = 6$$

METODO DEGLI ACCANTONAMENTI

i	c_i	\hat{c}_i	
①	4	4	
2	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9$)
3	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9$)
4	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	⋮
5	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	
6	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	
7	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	
8	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9$)
• ⑨	$4 \cdot 9$	$\frac{9}{2}$	
10	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9^2$)
11	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9^2$)
12	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9^2$)
⋮	⋮	⋮	⋮
79	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9^2$)
80	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9^2$)
⑧1	$4 \cdot 9^2$	$\frac{9}{2}$	
82	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9^3$)
83	$\frac{3}{2}$	$\frac{3}{2} + \frac{9}{2}$	(su $i=9^3$)
⋮	⋮	⋮	⋮

$$81 - 9 = 72 = 8 \cdot 9$$

$$\frac{4 \cdot 9^k}{2 \cdot 8 \cdot 9} = \frac{9}{2}$$

$$i = 9^k$$

$$\frac{4 \cdot 9^k}{9^k - 9^{k-1}} = \frac{4 \cdot 9^k}{9^{k-1}(9-1)} = \frac{9}{2}$$

$$\hat{c}_i = \begin{cases} 4 & \text{se } i = 1 \\ \frac{9}{2} & \text{se } i \in 9^k \text{ e } i \neq 1 \\ 6 & \text{se } i \notin 9^k \end{cases}$$

$$\frac{q}{2} (i - q^{\lfloor \frac{q}{2} i \rfloor}) \geq 0 \quad \left| \quad \sum_{i=2}^n c_i \leq \sum_{i=1}^3 \tilde{c}_i \leq 6n \right.$$

METODO DEL POTENZIALE

$$\Phi_i = \begin{cases} 0 & \text{se } i=0 \\ \frac{q}{2} (i - q^{\lfloor \frac{q}{2} i \rfloor}) & \text{se } i>0 \end{cases}$$

$$\forall i \quad \Phi_i \geq \Phi_0 = 0, \text{ infatti}$$

$$\lfloor \frac{q}{2} i \rfloor \leq \frac{q}{2} i$$

$$q^{\lfloor \frac{q}{2} i \rfloor} \leq q^{\frac{q}{2} i} = i$$

$$\rightarrow \Phi_i = \frac{q}{2} (i - q^{\lfloor \frac{q}{2} i \rfloor}) \geq 0 = \Phi_0$$

$$\downarrow$$

$$\sum_{i=1}^n c_i \leq \sum_{i=1}^n \tilde{c}_i$$

$$i=1$$

$$\tilde{c}_1 = c_1 + \Phi_1 - \Phi_0$$

$$= 4 + 0 - 0$$

$$= 4$$

$$i \in \mathbb{N} \wedge i \neq 1$$

$$i = 9^k \rightarrow 9^{k-1} = \frac{9^k}{9} = \frac{i}{9}$$

$$\hat{c}_i = c_i + \phi_i - \phi_{i-1}$$

$$= 4 \cdot i + \frac{9}{2} (i - 9^{\lfloor \log_9 i \rfloor}) - \frac{9}{2} (i-1 - 9^{\lfloor \log_9 (i-1) \rfloor})$$

$$= 4 \cdot i + \frac{9}{2} (\cancel{i} - \cancel{i}) - \frac{9}{2} (i-1 - \frac{i}{9})$$

$$= 4 \cdot i - \underbrace{\frac{9}{2} i + \frac{i}{2}}_{-4i} + \frac{9}{2}$$

$$= \frac{9}{2}$$

$$i \notin \mathbb{N}$$

$$\hat{c}_i = c_i + \phi_i - \phi_{i-1}$$

$$= \frac{3}{2} + \frac{9}{2} (\cancel{i} - 9^{\lfloor \log_9 i \rfloor}) - \frac{9}{2} (\cancel{i-1} - 9^{\lfloor \log_9 (i-1) \rfloor})$$

$$= \frac{3}{2} + \frac{9}{2} = \frac{12}{2} = 6$$

$$\hat{c}_i = \begin{cases} 4 & \text{se } i=1 \\ \frac{9}{2} & \text{se } i \in \mathbb{N} \wedge i \neq 1 \\ 6 & \text{se } i \notin \mathbb{N} \end{cases}$$

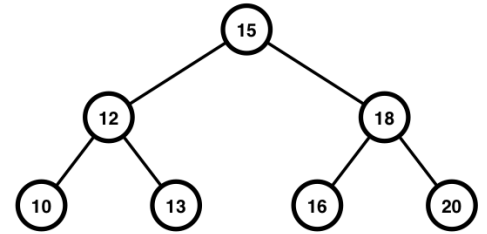
$$\sum_{i=1}^n c_i \leq \sum_{i=1}^n \hat{c}_i \leq 6n$$

ESERCIZIO 1

- (a) Si descrivano le operazioni di *zig-zag*, *zig-zig* e *zig* in uno splay tree di tipo bottom-up.

Quindi si eseguano nell'ordine dato le seguenti operazioni sullo splay tree a lato:

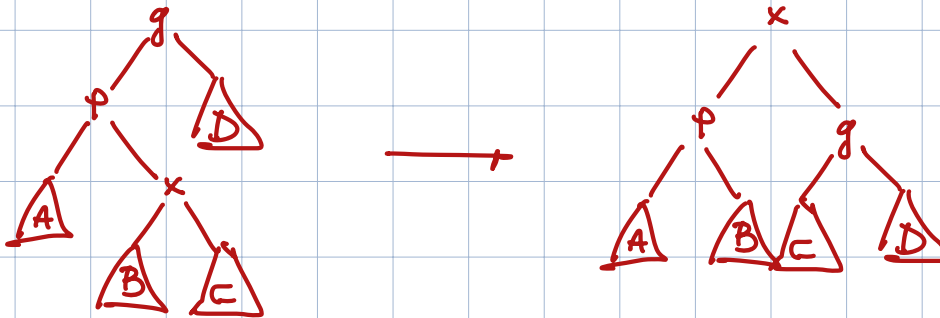
- SEARCH 20, 13
- INSERT 17
- DELETE 15



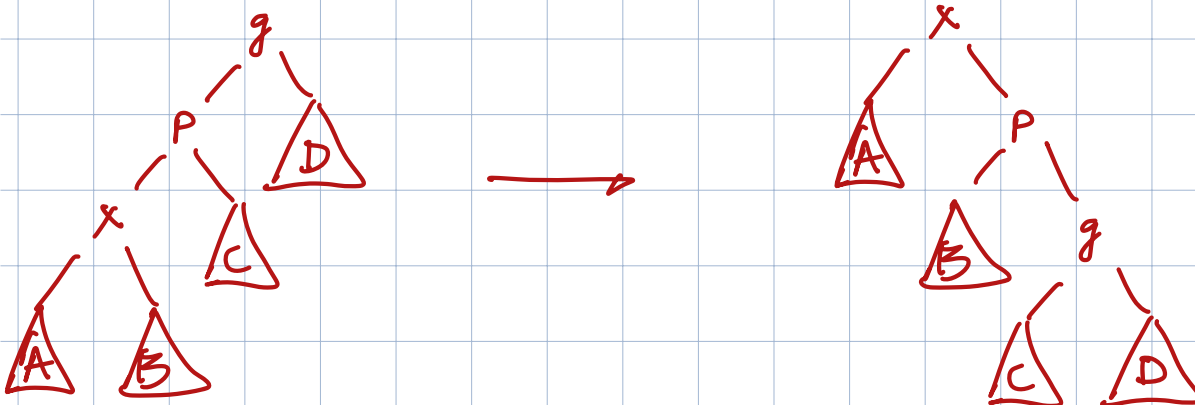
- (b) Si descrivano le operazioni di *zig-zag*, *zig-zig* e *zig*, nonché l'operazione di assemblaggio finale, in un splay tree di tipo top-down.

(c)

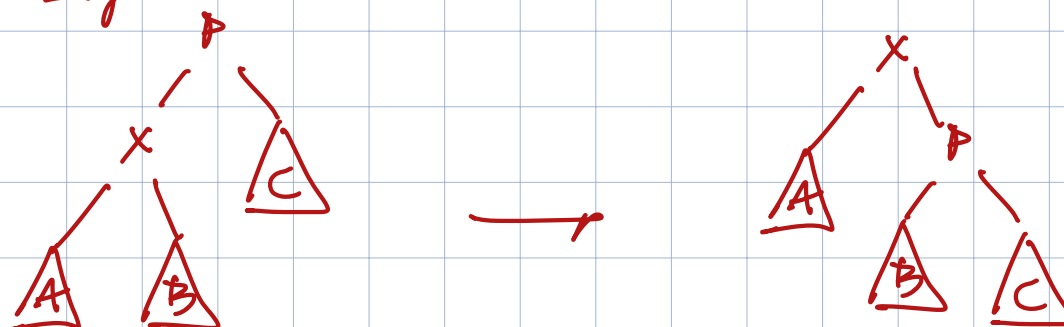
zig-zag



zig-zig

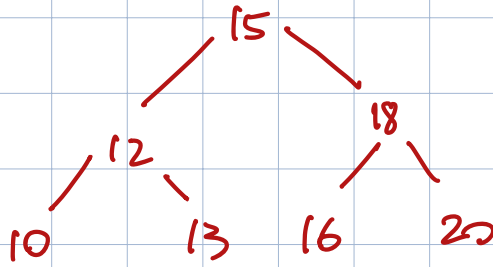


zig

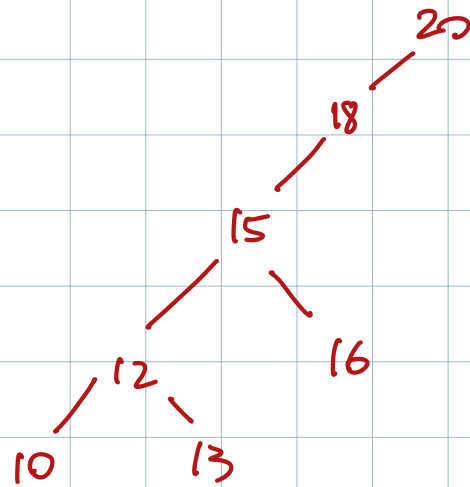


- SEARCH 20, 13
- INSERT 17
- DELETE 15

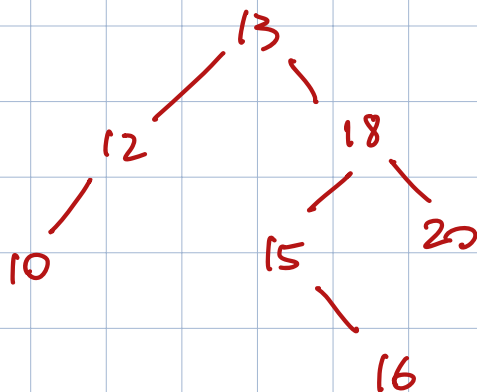
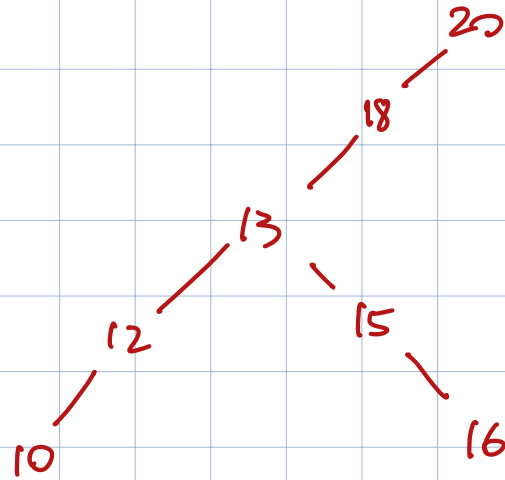
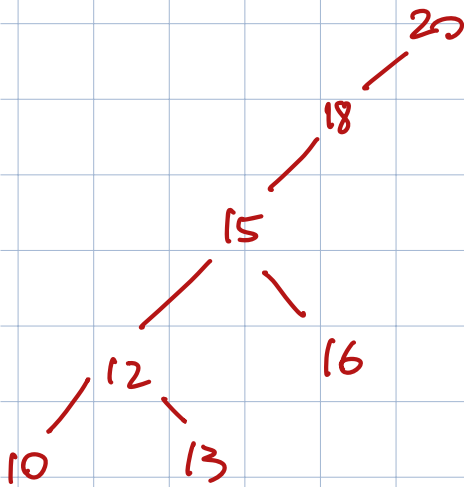
SEARCH (20)



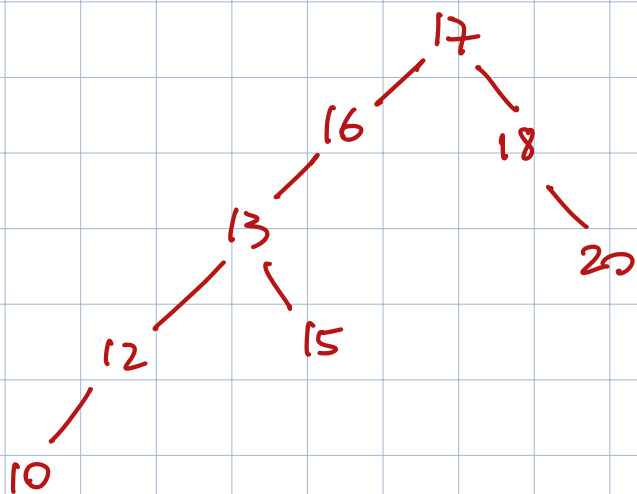
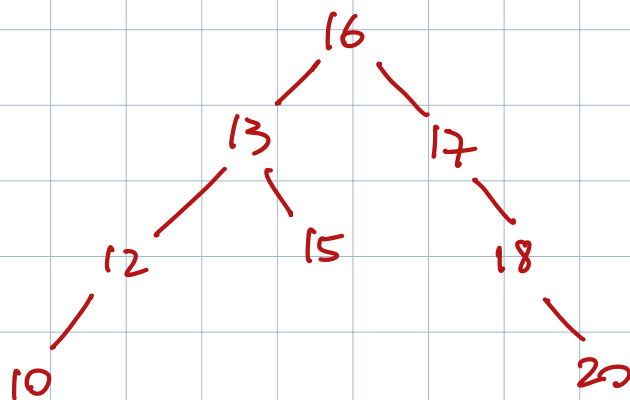
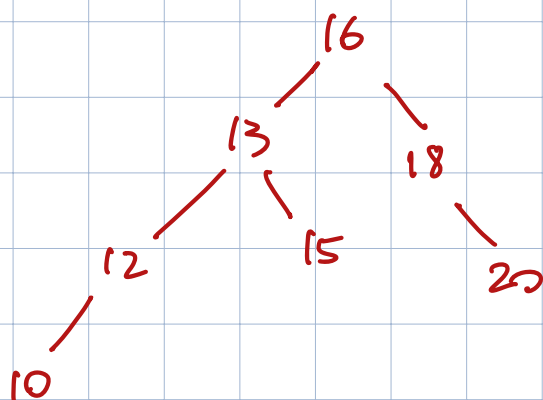
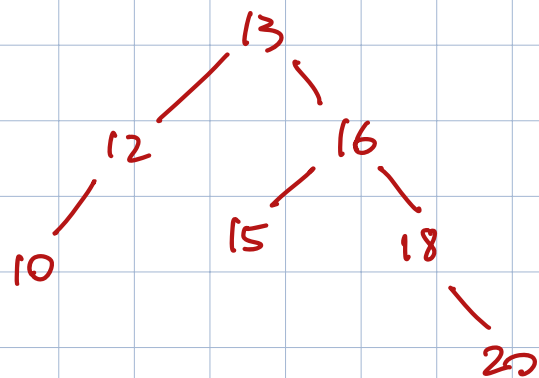
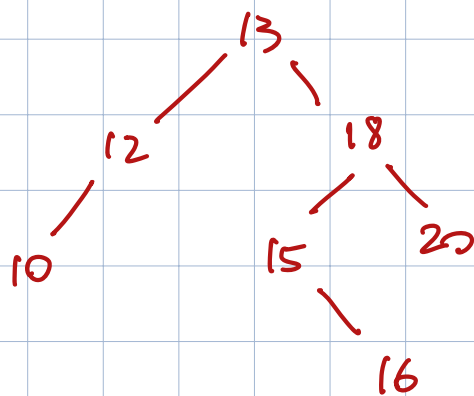
SPLAY(20)



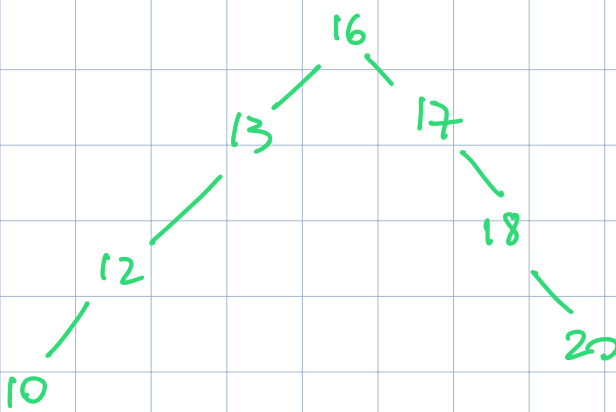
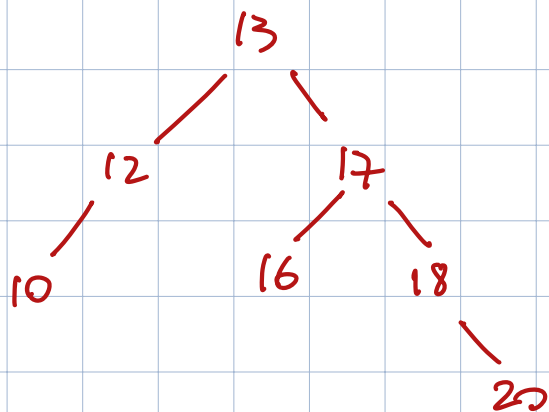
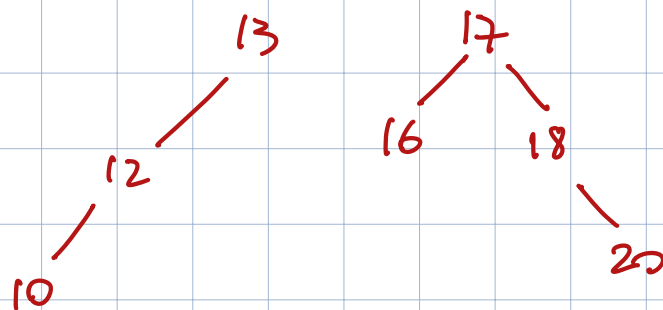
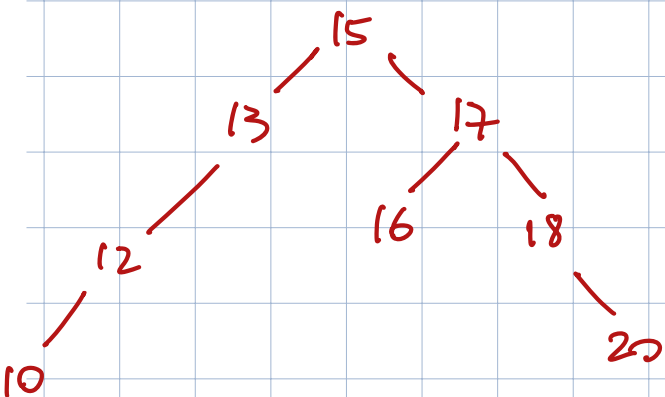
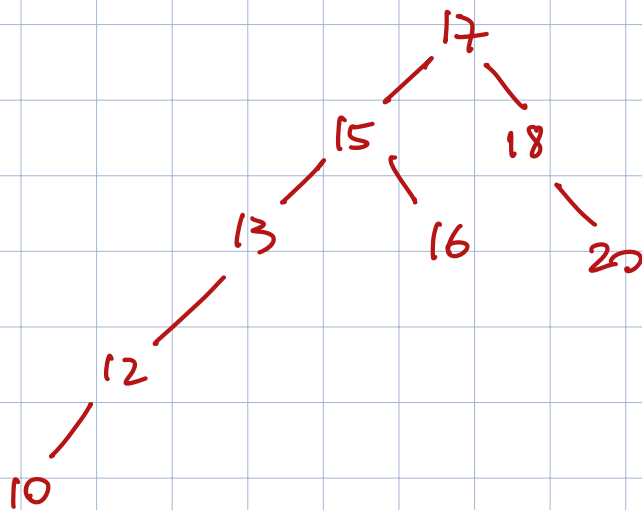
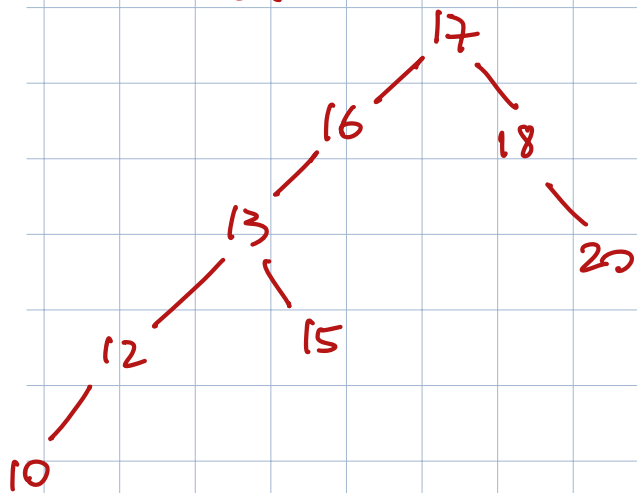
SEARCH (13)



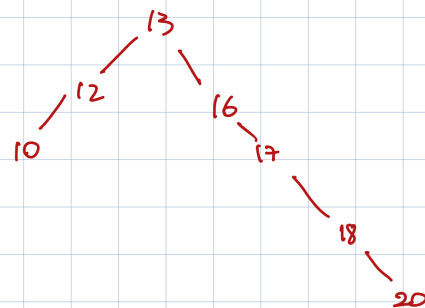
INSERT (17)



DELETE (15)



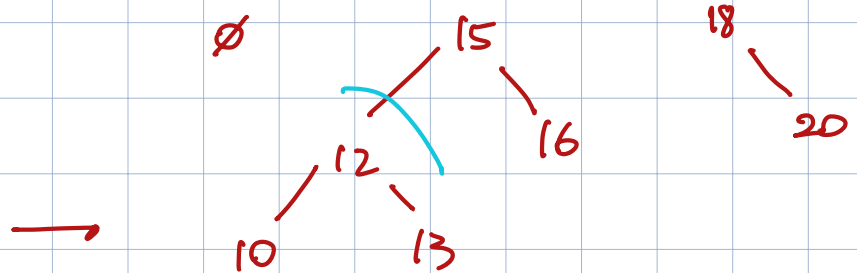
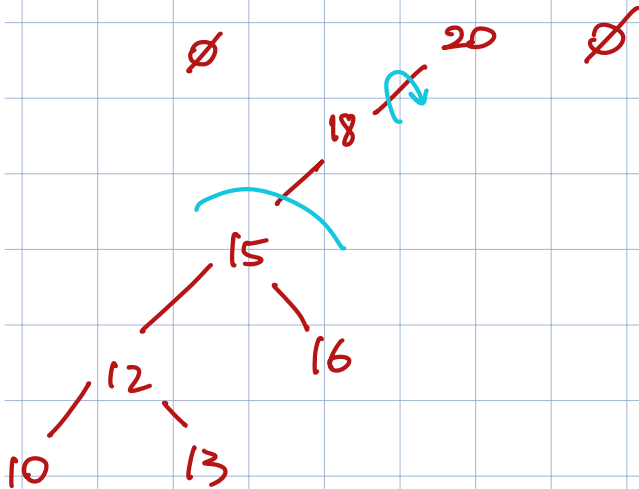
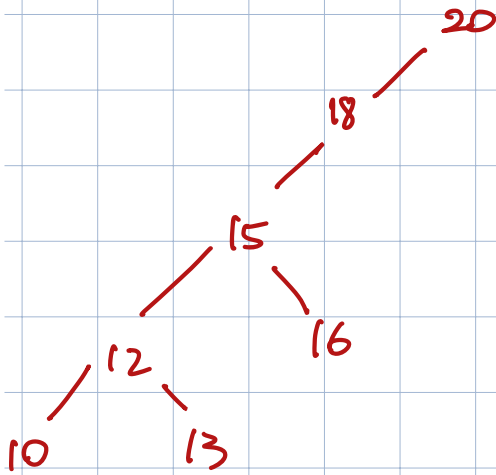
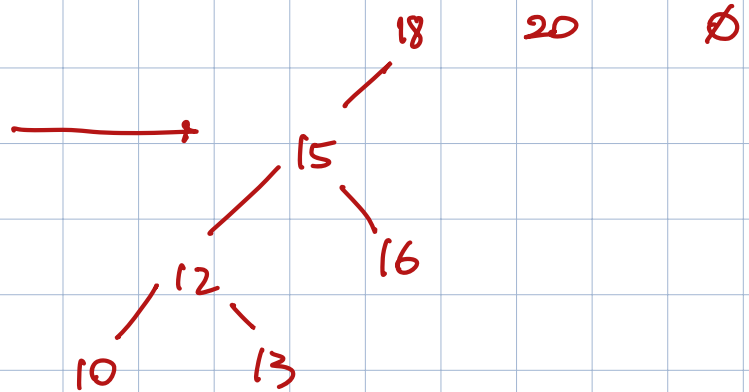
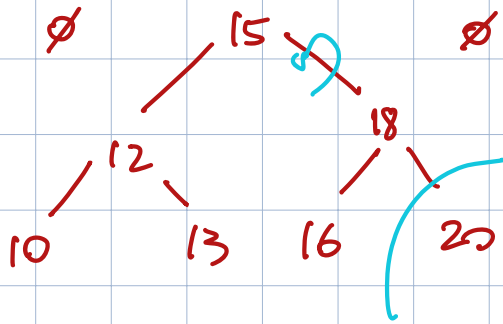
TOP-DOWN

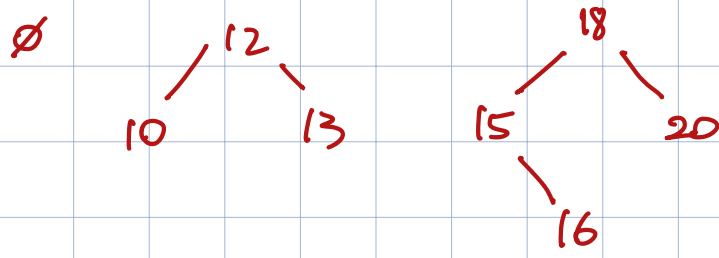


IN MODALITA' TOP-DOWN

- SEARCH 20, 13
- INSERT 17
- DELETE 15

SEARCH (20)

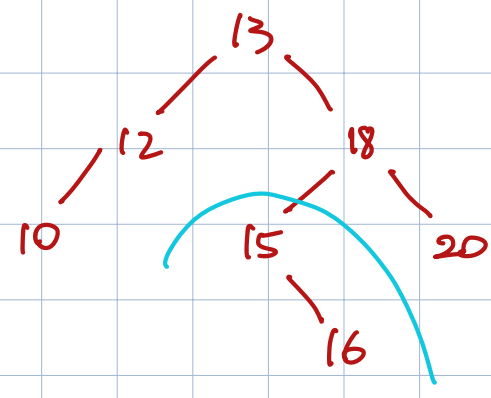




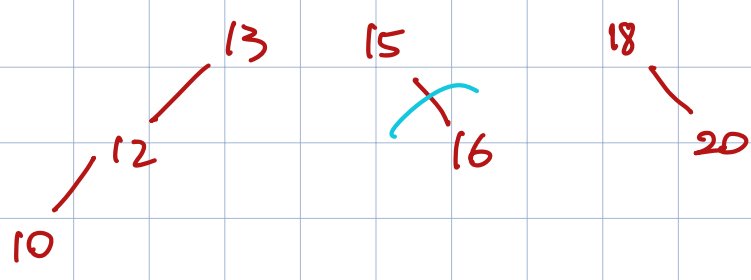
ZIG-ZAG
 →
 SEMPLIFICATA

ZIG
 →

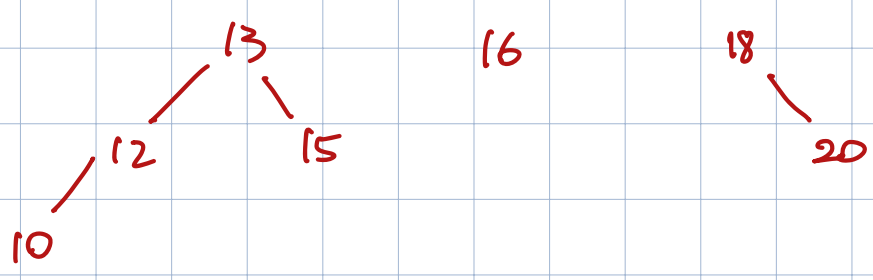
RIASSEMBLAGGIO
 →



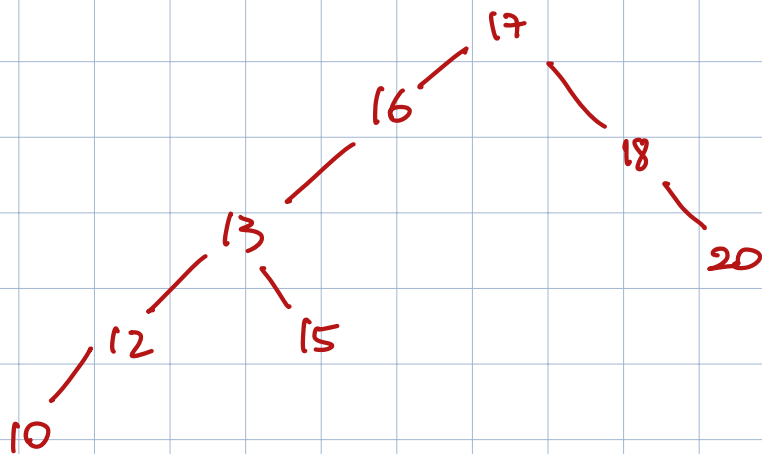
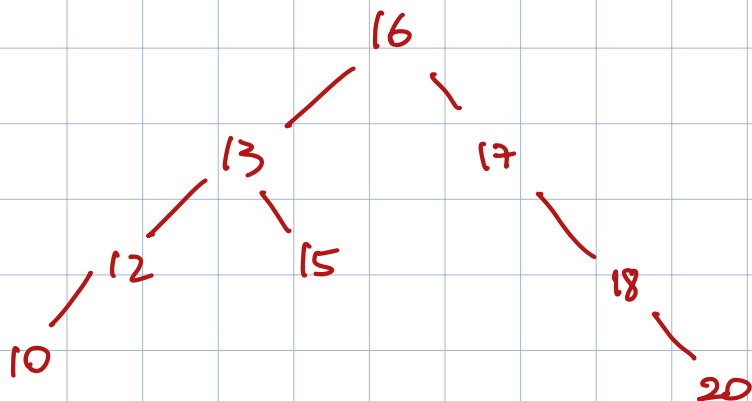
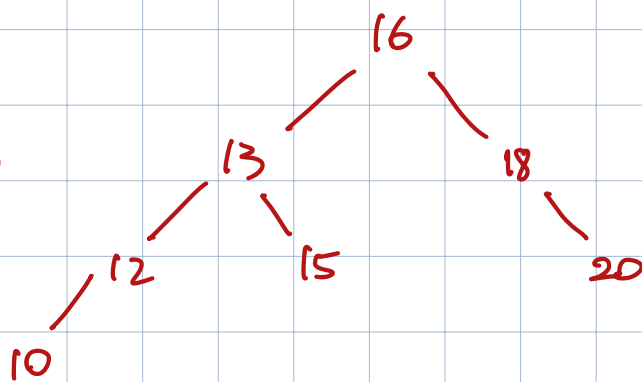
INSERT (17)



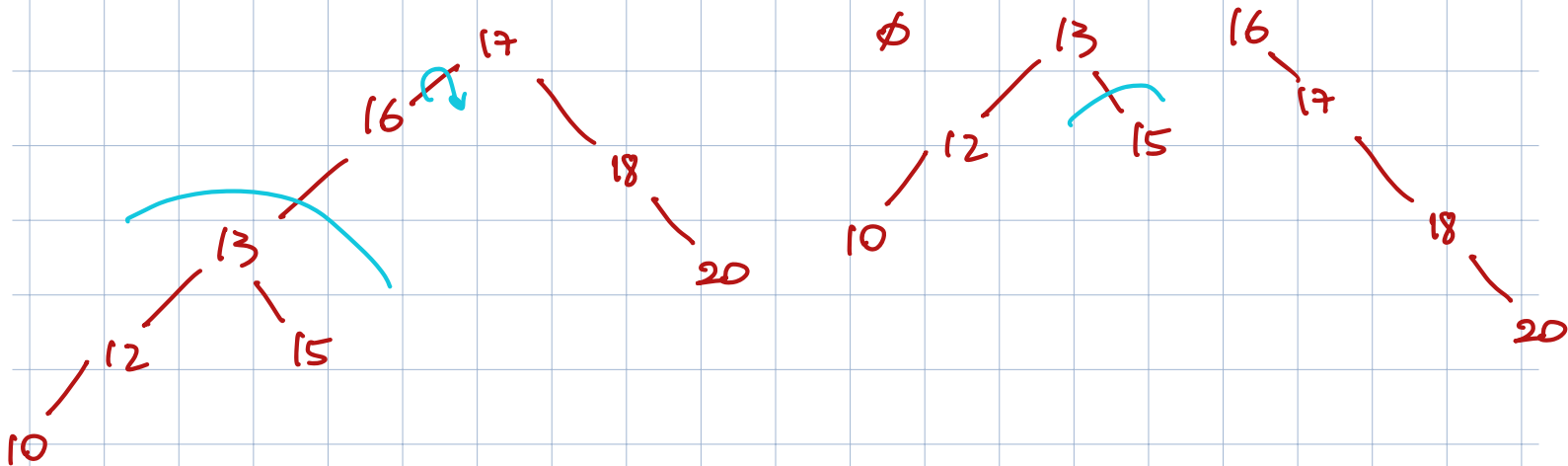
ZIG-ZAG
 (NON SEMPLIFICATA)

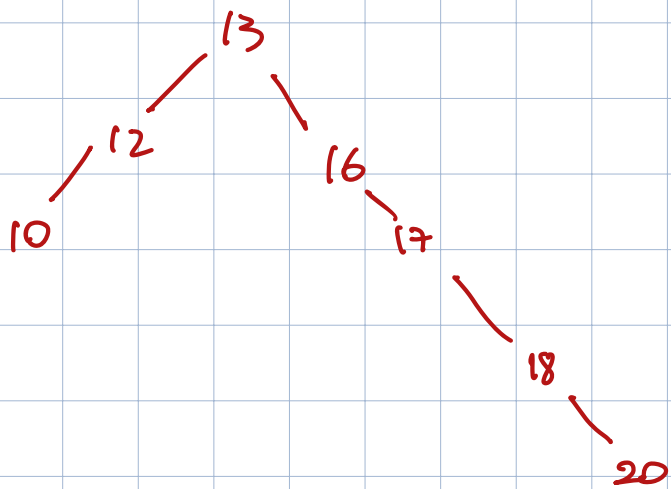
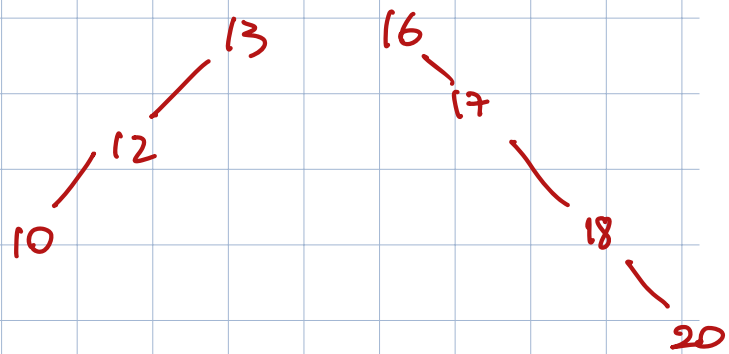
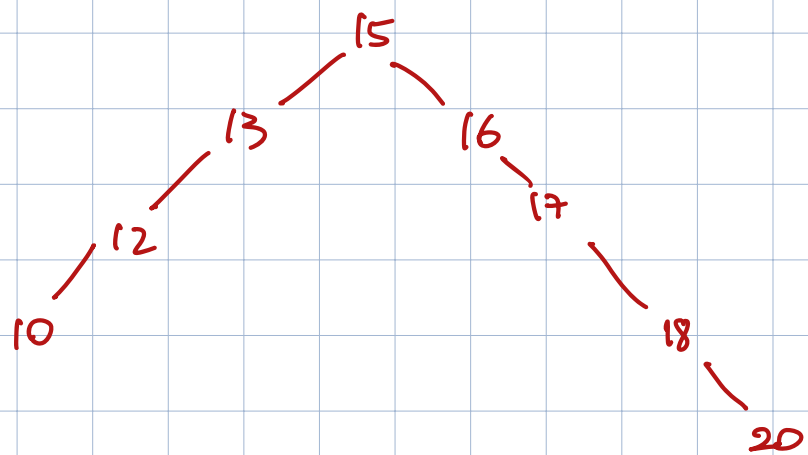
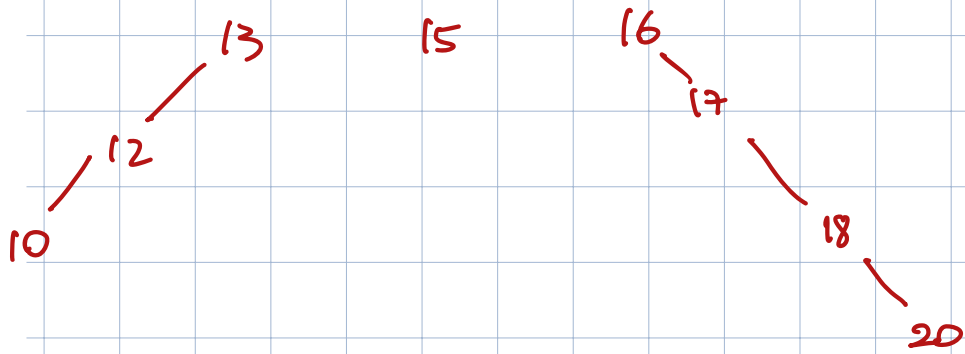


RIASSEMBLAGGIO →



DELETE (15)





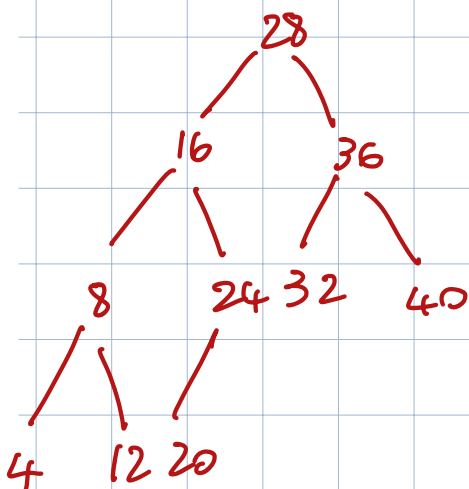
ESERCIZIO 2 (Splay trees)

Si descrivano le operazioni di *zig-zag*, *zig-zig* e *zig* in uno splay tree di tipo bottom-up.

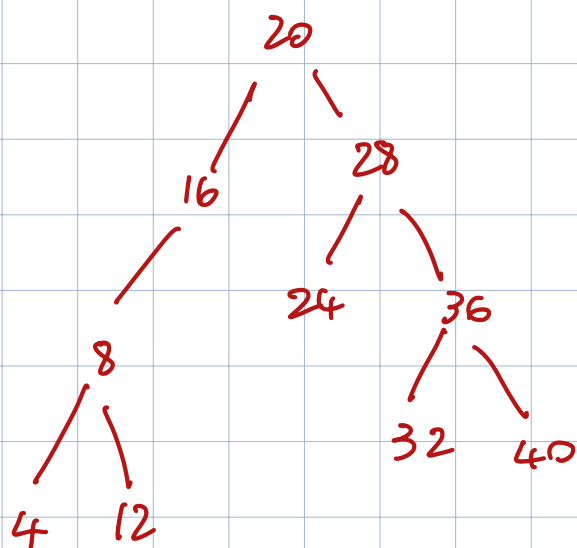
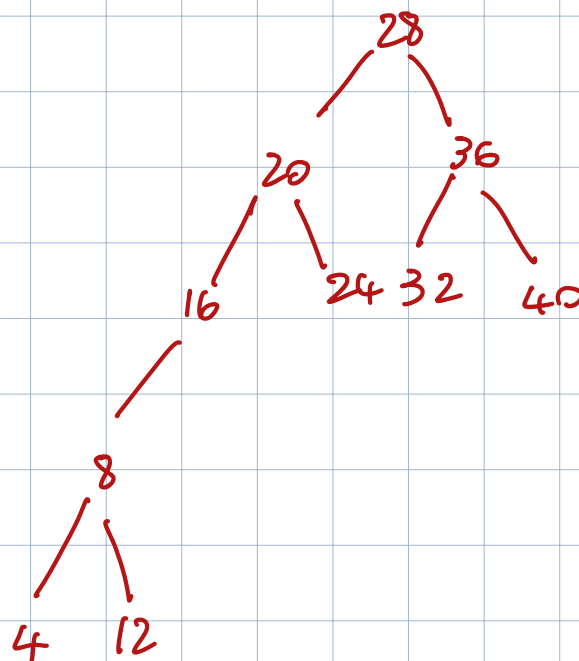
Quindi si eseguano nell'ordine dato le seguenti operazioni su uno splay tree la cui configurazione iniziale è quella di un **albero binario completo** contenente le 10 chiavi $\{4i : 1 \leq i \leq 10\}$:

- SEARCH 20, 40
- DELETE 24
- INSERT 30

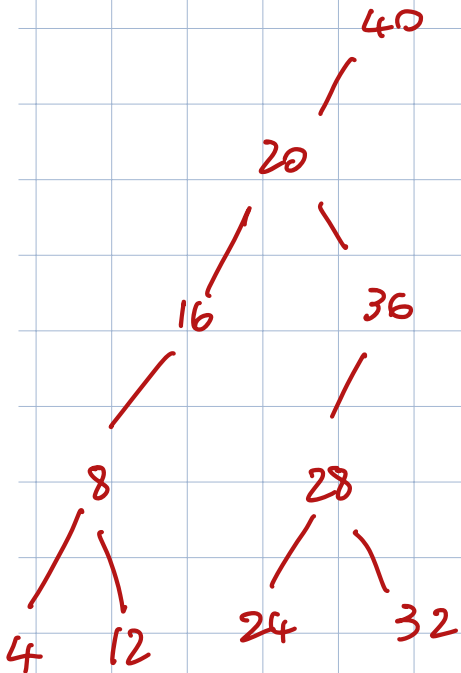
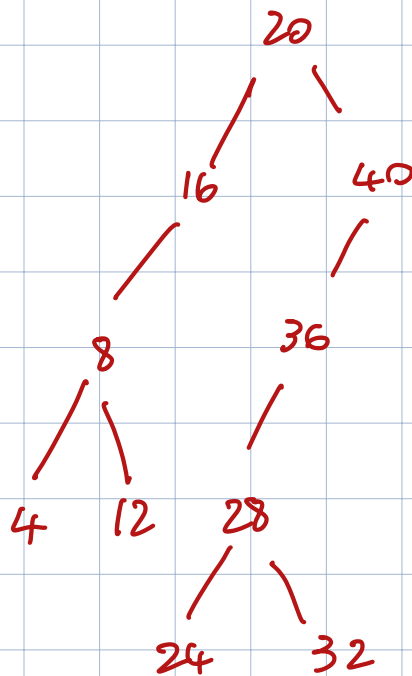
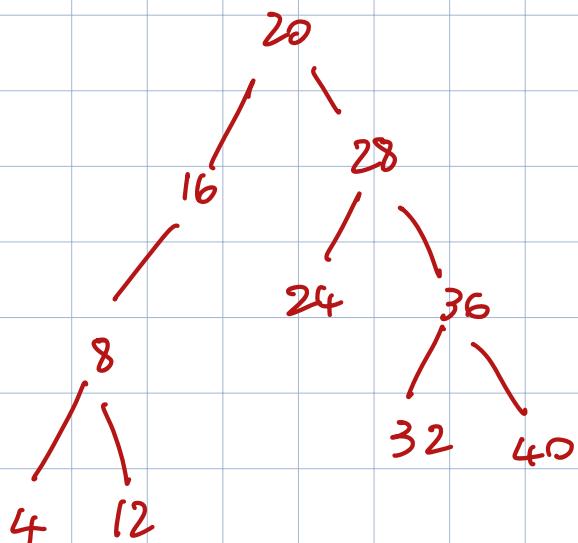
Nota bene: Si ricorda che un albero binario si dice *completo* quando tutti i suoi livelli, con al più l'eccezione dell'ultimo, sono completi e tutti i nodi nell'ultimo livello si trovano il più a sinistra possibile.



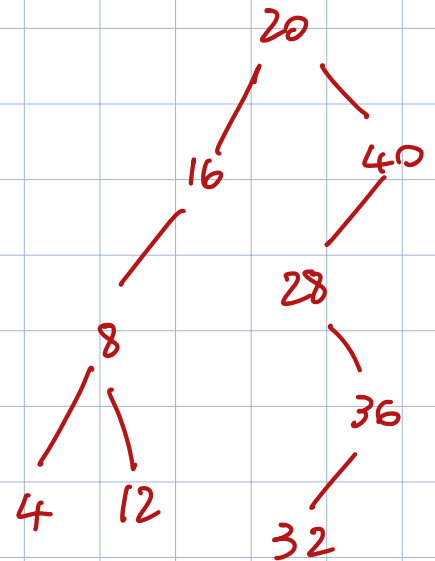
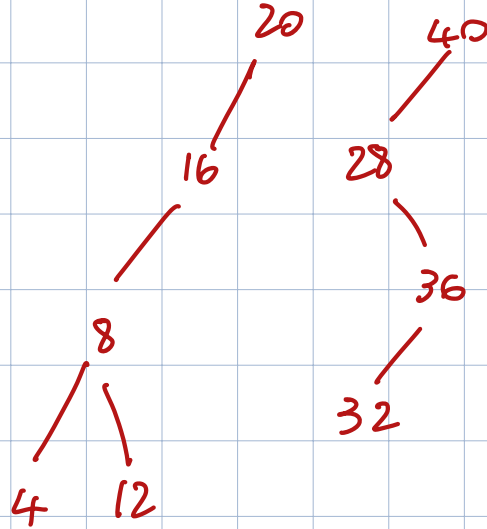
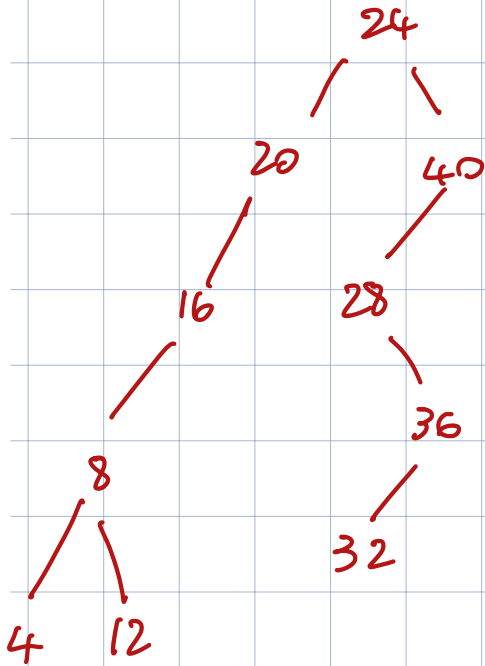
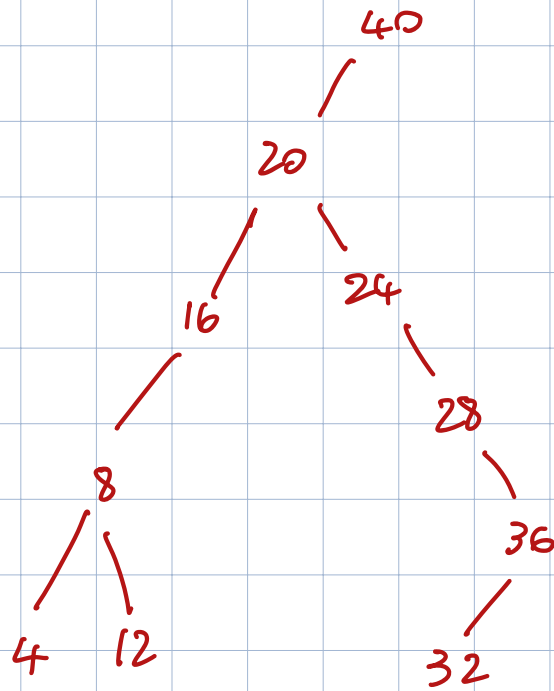
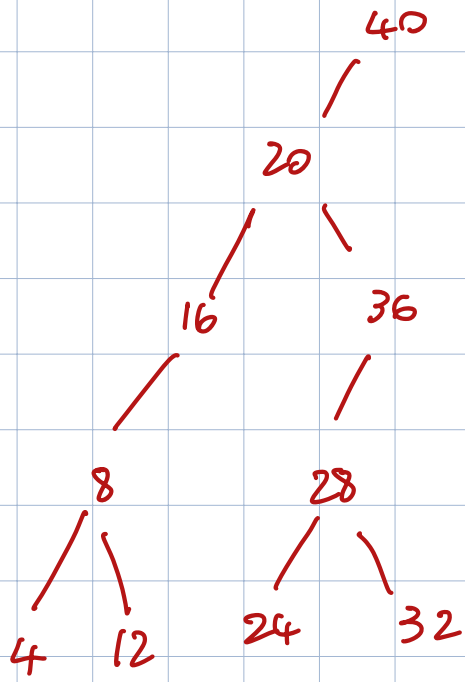
SEARCH (20)



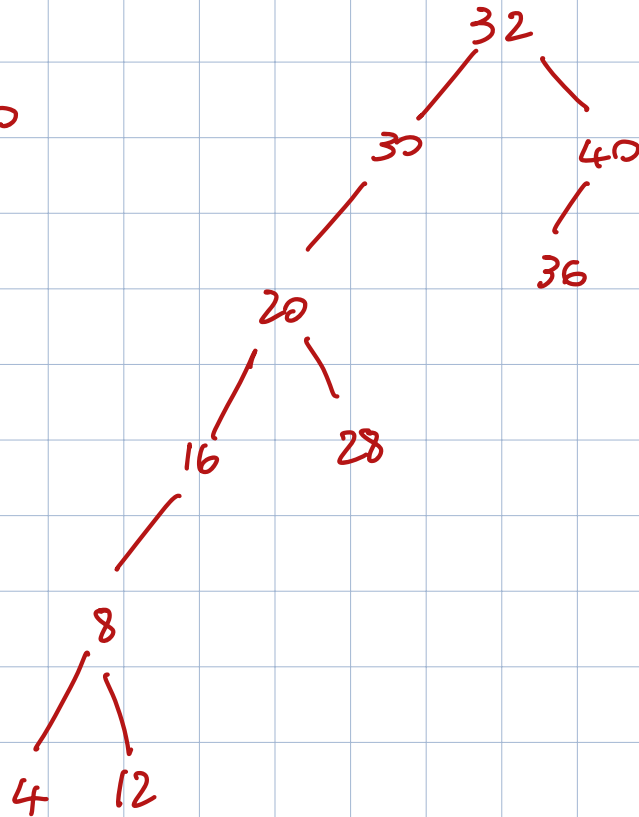
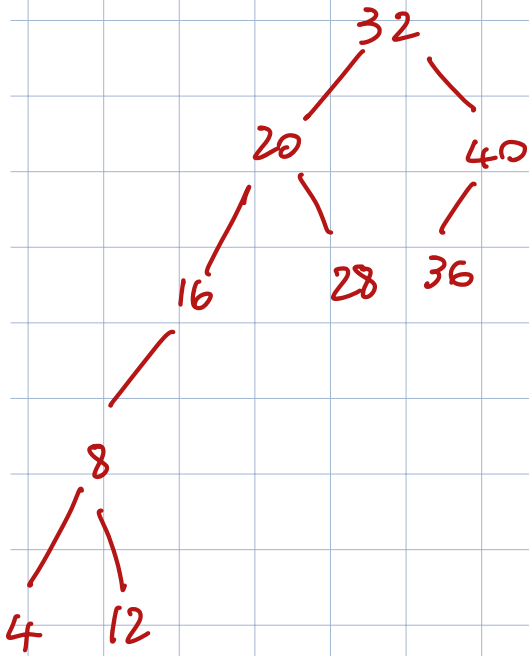
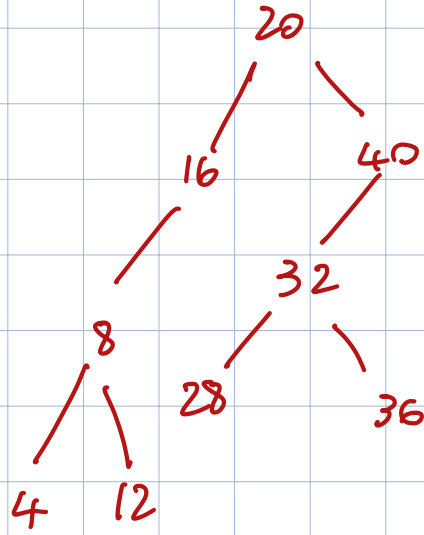
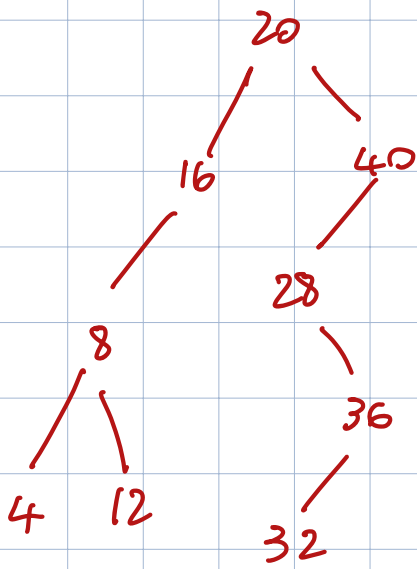
SEARCH(40)

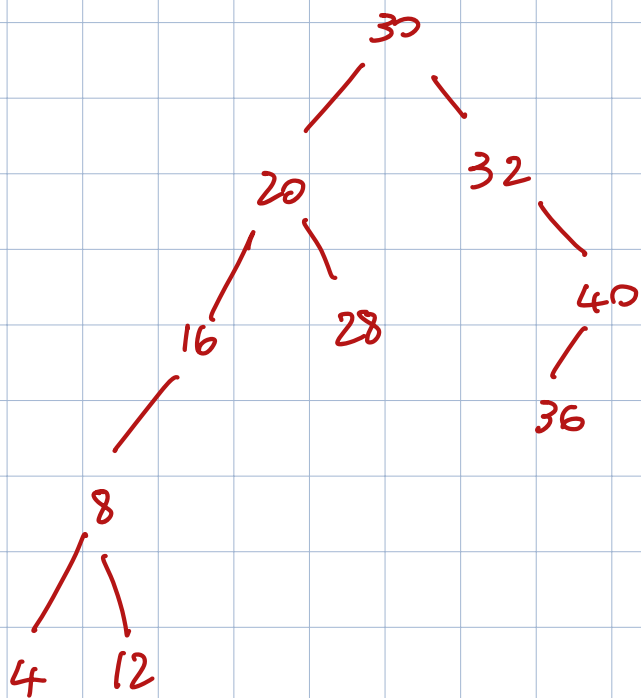


DELETE(24)

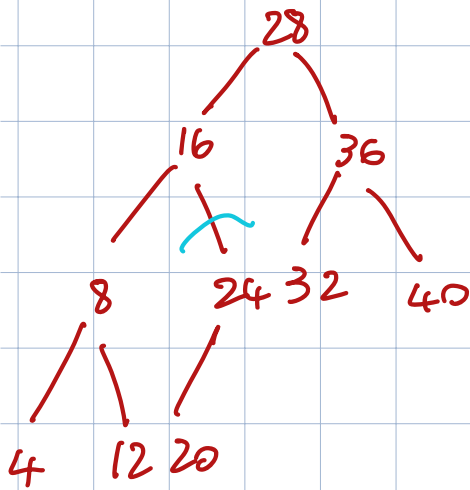


INSERT (30)

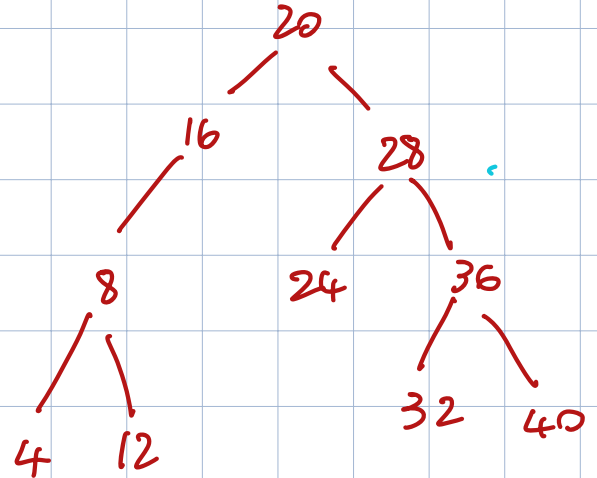
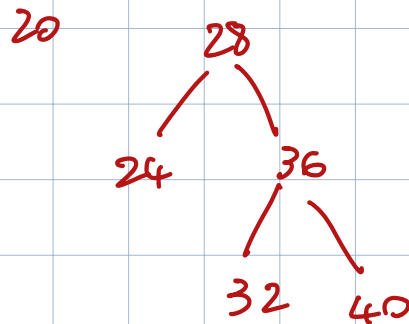
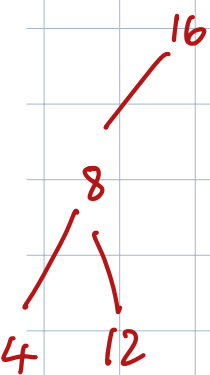
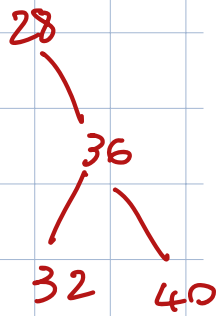
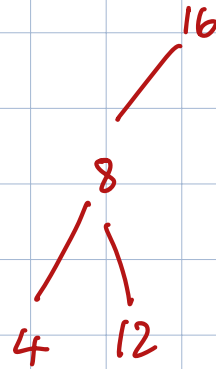




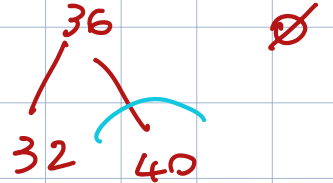
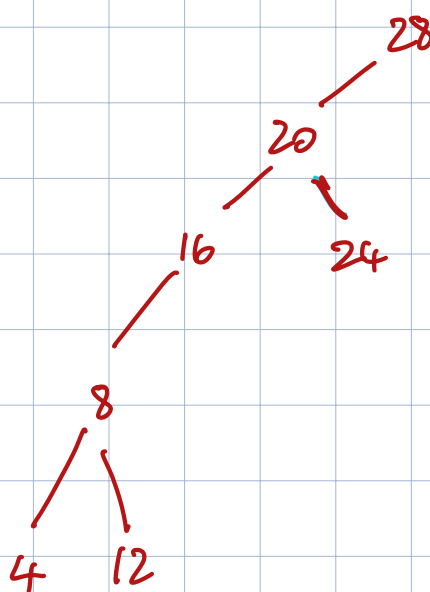
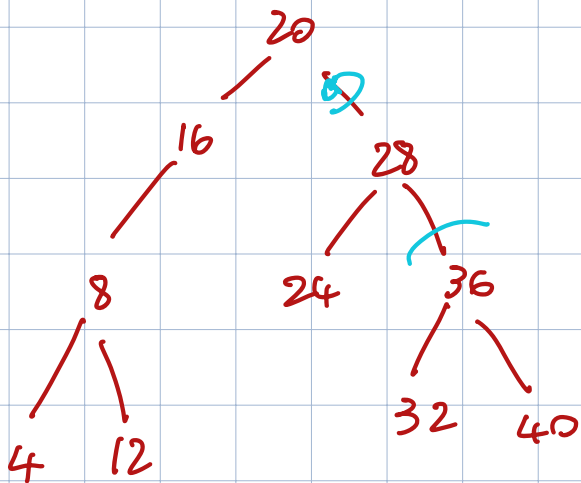
MODALITA' TOP-DOWN

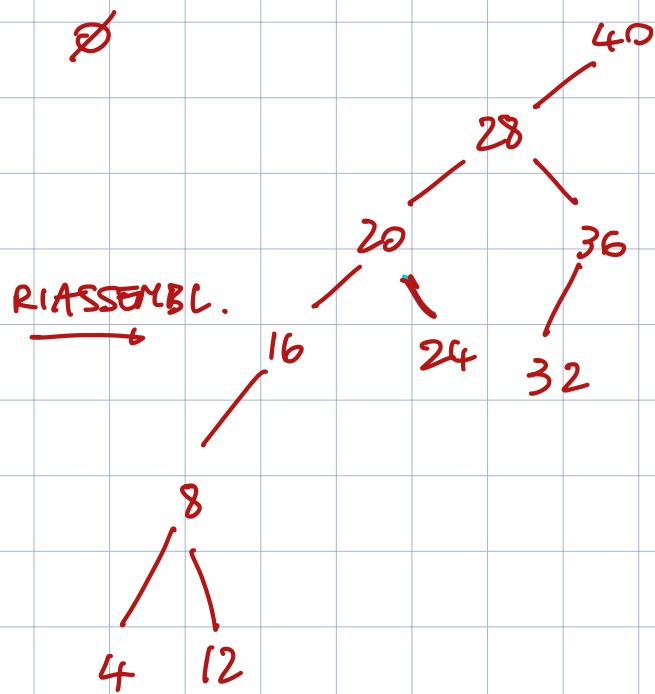
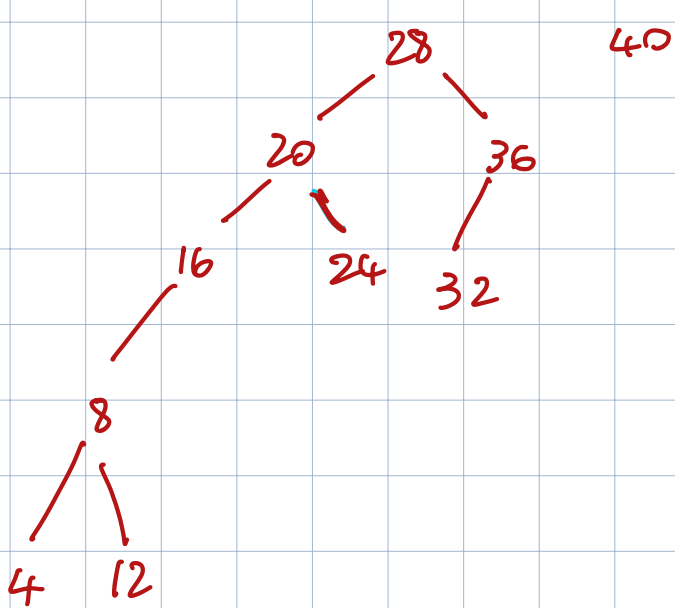


ZIG-ZAG

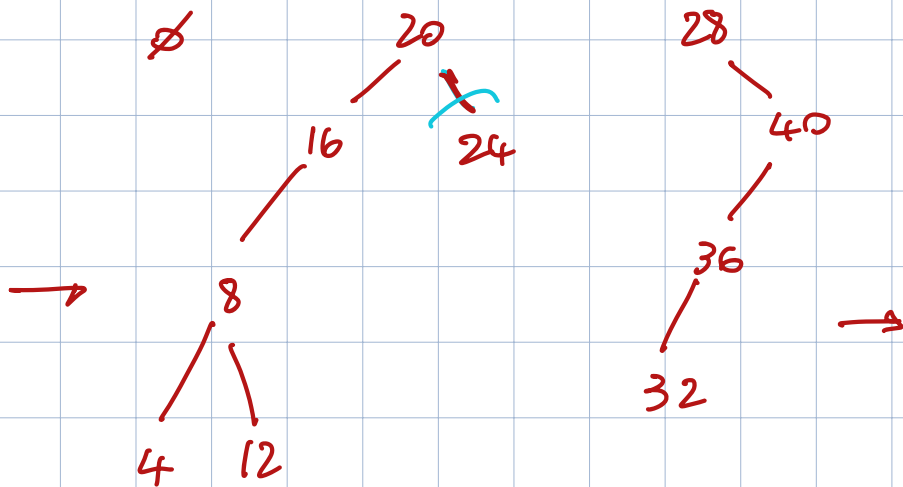
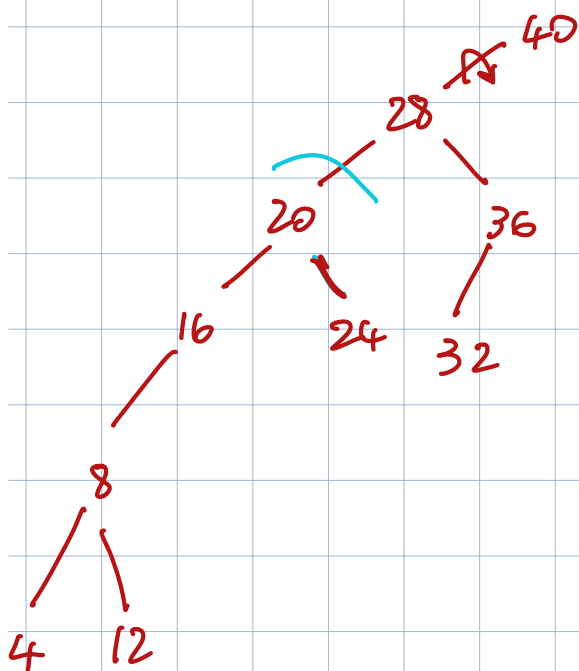


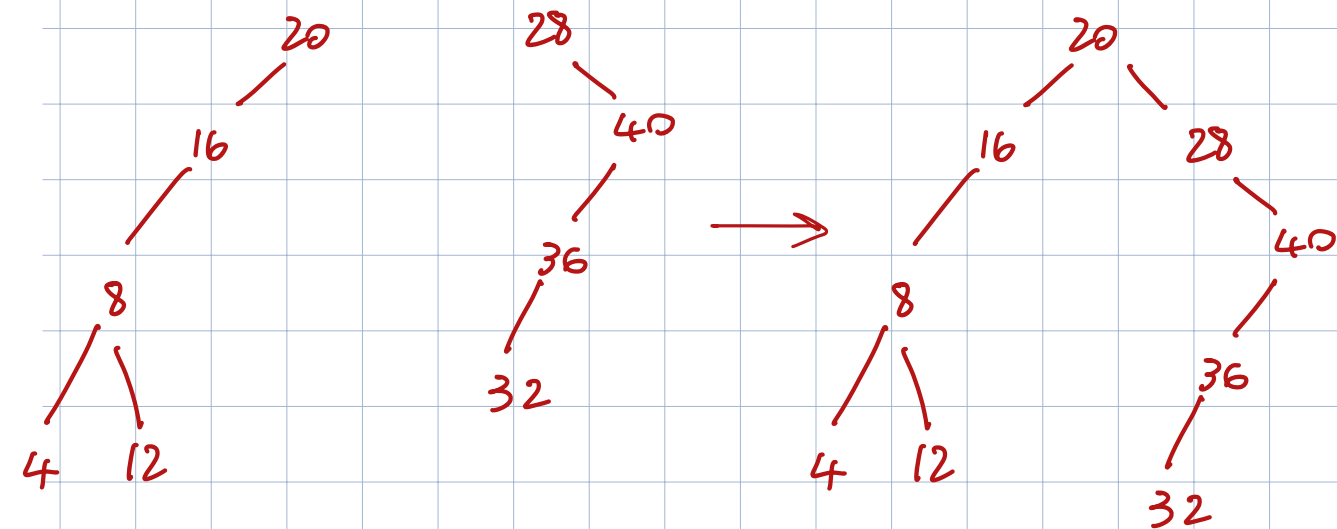
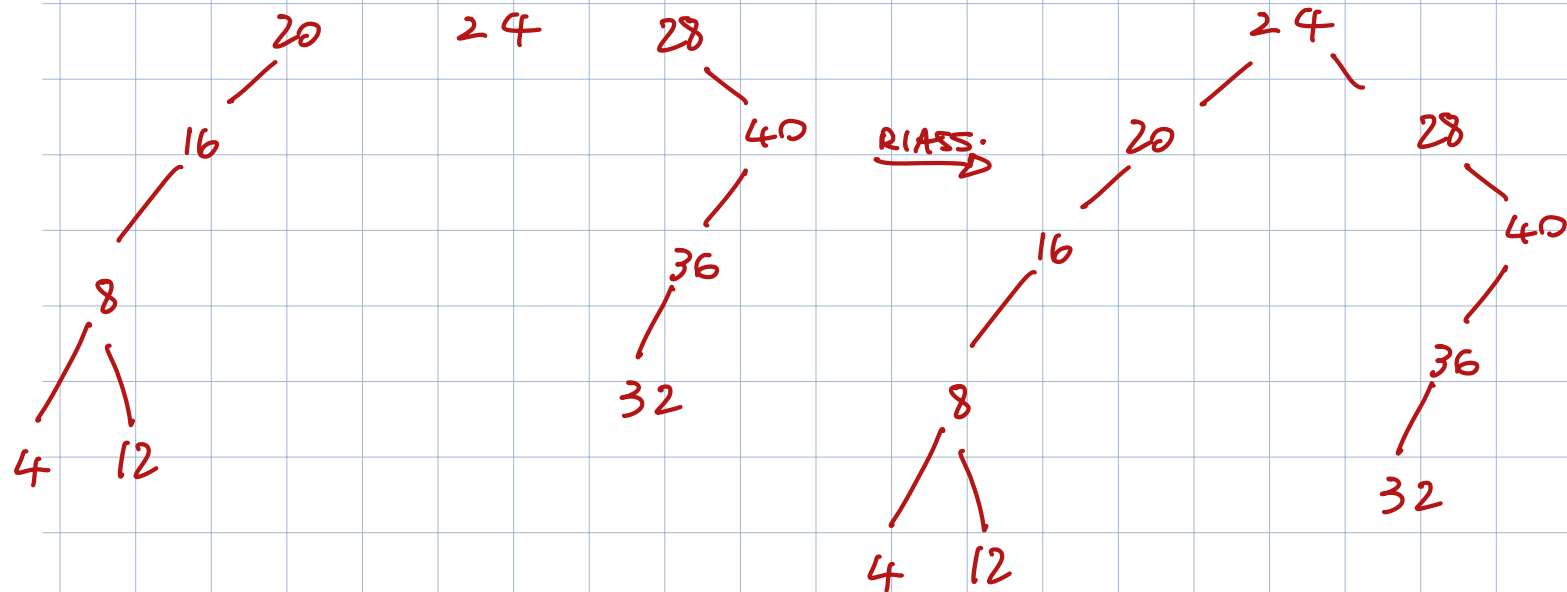
SEARCH(40)





DELETE (24)





INSERT (30)

