Power spectrum of log-normal transformed fields in redshift-space

A Slosar

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The log-normal transformation that `CoLoRe` does is to

1. Take the linear field in real space and smooth it $P = P_L(k)e^{-k^2r^2}$
2. Sample from the linear field
3. Transform the liner field according to transformation

$$\rho_{\rm ln}(\mathbf{x}) = \exp\left[b\delta(\mathbf{x})\right] \tag{1}$$

4. Move galaxies around based on linear velocities.

The mean density of the transformed fied is given by

$$\bar{\rho}_{\rm ln} = \int \frac{1}{\sqrt{2\pi\sigma^2}} e^{b\delta - \frac{\delta^2}{2\sigma^2}} d\delta = \exp\frac{\sigma^2 b^2}{2}, \tag{2}$$

where σ^2 is the field variance (that is why smoothing matters!) and the transformed overdensity is given by

$$\delta_{\rm ln} = \frac{\rho_{\rm ln}}{\bar{\rho}_{\rm ln}} - 1 \tag{3}$$

Based on this prescription, the transformed field in redshift-space is, on large scales

$$\delta_{{\rm ln},s}(\mathbf{k}) = \delta_{\rm ln}(\mathbf{k}) + \beta\delta(\mathbf{k})\mu^2, \tag{4}$$

where $\beta = f/b$ is the usual Kaiser RSD distortion parameter. Note that the second δ multiplying Kaiser term is linear. This might seem a bit counter-intuitive, but can be shown to be using peak-background split argument. It also likely breaks down an smaller scales, but this is not crucial for us.

We have that the auto power-spectrum is given by

$$P_s(\mathbf{k}) = P_{\ln\ln}(\mathbf{k}) + 2P_{\ln L}\beta(\mathbf{k})\mu^2 + \beta^2 P_{LL}(\mathbf{k})\mu^4 \tag{5}$$

Missing terms can be calculated via Fourier transform of the correlation functions.

$$\begin{aligned} \xi_{\ln L}(r) &= \int d\delta_1 \int d\delta_2 G(\delta_1, \delta_2|\sigma^2, \xi(r))\delta_{\ln}(\delta_1)\delta_2 && (6) \\ \xi_{\ln\ln}(r) &= \int d\delta_1 \int d\delta_2 G(\delta_1, \delta_2|\sigma^2, \xi(r))\delta_{\ln}(\delta_1)\delta_{\ln}(\delta_2), && (7) \end{aligned}$$

where Gaussian G describes correlations between two points in the Gaussian field. Doing the math, one gets, see the maple script `lntrans.mw`:

$$\begin{aligned} \xi_{\ln L}(r) &= b\xi(r) && (8) \\ \xi_{\ln\ln}(r) &= e^{b^2\xi(r)} - 1 && (9) \end{aligned}$$

So, to make these predictions, one needs to logFFT to ξ, apply the above transforms and logFFT back. Interesting the $L\ln$ terms behaves exactly as if no transformation was done!