Power spectrum of log-normal transformed or clipped fields in redshift-space

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The log-normal transformation that `CoLoRe` does is to

1. Take the linear field in real space and smooth it $P = P_L(k)e^{-k^2r^2}$
2. Sample from the linear field
3. Transform the liner field according to transformation

$$\rho_{\rm ln}(\mathbf{x}) = \exp\left[b\delta(\mathbf{x})\right] \tag{1}$$

   or

$$\rho_{\rm clipped}(\mathbf{x}) = \begin{cases} 1 + b\delta(\mathbf{x}) & \text{if } b\delta(\mathbf{x}) > -1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

4. Move galaxies around based on linear velocities.

Probability of δ is given by

$$P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\delta^2}{2\sigma^2}\right] \tag{3}$$

The mean density of the transformed fied is given by

$$\bar{\rho}_{\rm ln} = \int \exp\left[b\delta(\mathbf{x})\right] P(\delta) d\delta = \exp\frac{\sigma^2 b^2}{2}, \tag{4}$$

or

$$\bar{\rho}_{\rm clipped} = \int_{-b^{-1}}^{\infty} (1 + b\delta)P(\delta) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{1}{\sqrt{2}b\sigma}\right) + \sqrt{2}b\sigma\pi^{-1/2}e^{(2b^2\sigma^2)^{-1}}\right] \tag{5}$$

where σ^2 is the field variance (that is why smoothing matters!) and the transformed overdensity is given by

$$\delta_{\rm ln} = \frac{\rho_{\rm ln}}{\bar{\rho}_{\rm ln}} - 1 \tag{6}$$

Based on this prescription, the transformed field in redshift-space is, on large scales

$$\delta_{{\rm ln},s}(\mathbf{k}) = \delta_{\rm ln}(\mathbf{k}) + \delta(\mathbf{k}) f \mu^2, \tag{7}$$

where $f = d\ln g/d\ln a$ is the logarithmic growth. Situation for clipping is identical. Note that the second δ multiplying Kaiser term is linear. This might seem a bit counter-intuitive, but can be shown to be using peak-background split argument. It also likely breaks down an smaller scales, but this is not crucial for us.

We have that the auto power-spectrum is given by

$$P_s(\mathbf{k}) = P_{\rm ln\,ln}(\mathbf{k}) + 2P_{{\rm ln}\,L}(\mathbf{k}) f\mu^2 + P_{LL}(\mathbf{k}) f^2 \mu^4 \tag{8}$$

Missing terms can be calculated via Fourier transform of the correlation functions.

$$\begin{aligned} \xi_{{\rm ln}\,L}(r) &= \int d\delta_1 \int d\delta_2 G(\delta_1, \delta_2 | \sigma^2, \xi(r)) \delta_{\rm ln}(\delta_1) \delta_2 & (9) \\ \xi_{\rm ln\,ln}(r) &= \int d\delta_1 \int d\delta_2 G(\delta_1, \delta_2 | \sigma^2, \xi(r)) \delta_{\rm ln}(\delta_1) \delta_{\rm ln}(\delta_2), & (10) \end{aligned}$$

where Gaussian G describes correlations between two points in the Gaussian field. Doing the math, one gets, see the maple script `lntrans.mw`:

$$\begin{aligned} \xi_{{\rm ln}\,L}(r) &= b\xi(r) & (11) \\ \xi_{\rm ln\,ln}(r) &= e^{b^2\xi(r)} - 1 & (12) \end{aligned}$$

for lognormal and

$$\begin{aligned} \xi_{{\rm clipped},L}(r) &= \frac{C}{C + \sqrt{2} b\sigma e^{(2b^2\sigma^2)^{-1}}} b\xi & (13) \\ \xi_{\rm clipped,clipped}(r) &= \text{maple doesn't do the integral,} & (14) \\ C &= \sqrt{\pi}\left(1 + {\rm erf}\left((\sqrt{2}{\rm b}\sigma)^{-1}\right)\right) & (15) \end{aligned}$$

So, to make these predictions, one needs to logFFT to ξ, apply the above transforms and logFFT back. Interesting the $L \ln$ terms behaves exactly as if no transformation was done!

Note however, knowing the three power spectra allows one to make predictions in Fourier space. To do the same in real space would require to go multipoles and transform $\ell = 0, 2, 4$ separately.

This has now been implemented into `CoLoRe`.