**ISyE8900 Project**

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**Abstract:** This project proposes a nonparametric approach for the modeling and forecasting of weekly interest rate spread curves by using nonlinear dimension reduction, locally linear embedding (LLE). We main focus on two spread curves: Swap Spread (LIBOR - Treasury) and Basis Spread (LIBOR - SOFR). Benchmarking on its linear dimension reduction counterparty, principle component analysis (PCA), we show the LLE based framework yields a higher out-of-sample forecast accuracy as well as a better profit and loss (PnL) profile in backtesting various systematic term structure trading strategies.

## I. Introduction

In the fixed-income trading industry, there is an increasing demand for accurately forecasting the short-term movement of spread term structure in a neat but efficient way as the low interest rate environment persists to squeeze the spread magnitude among multiple key interest rates curves. Besides, The Alternative Reference Rates Committee (ARRC) has identified the Secured Overnight Financing Rate (SOFR) as the successor rate of LIBOR, and continues supporting the launch of SOFR-based financial products in coming years. Thus, during the process of LIBOR's fallback, market participants are imperative to measure the LIBOR-SOFR spread when USD LIBOR-based activity gradually decreases until completely unusable. *(https://www.newyorkfed.org/arrc/sofr-transition)*

The recent development of statistical learning greatly inspired the interest in dimensionality reduction and predictive models with sparse and nonlinear features in the finance industry. In the past decades, the use of such methods are limited to principal component and latent factor analysis, because more complex models may not be suitable for structural analysis and parameter interpretation. However, instead of an identification problem, modeling the time variation of interest rate spread curves is a forecasting problem, in which larger out-of-sample forecast power are the main goal.

In this paper we implement and compare a variety of dimension reduction methods for the prediction one-step Swap/Basis spread changes. The research design follows the work by Chen, Deng and Huo (2008), whereby the comparison of different transform methods and modeling objective is based on out-of-sample performance including both statistical accuracy and term structure relative-value trading strategies. Forecasting time variation requires a careful approximation of an unknown encoder that maps information from high-dimensional (long spectrum of underlying maturities) to low-dimensional representations (latent drivers of the entire term structure). After obtaining such low-dimensional representation of spread curves, forecasts could be made by first predicting each new coordinate of the manifold using (regularized) ARMA family approaches and then map them back to the high-dimensional space utilizing the corresponding reconstruction method.

## II. Literature Review

Regarding the debate that how much predictability of curve movements one could capture using simply historical panel data, we confirm the superior out-of-sample forecasting performance of short-term interest rate dynamics if we model the spread curve as a whole.

Inspired by the idea of summarizing term structures by a small set of linear combinations of yields, Diebold and Li (2006) use the AR family models to obtain encouraging results for long-horizon ex-ante forecasts by reformulating the Nelson and Siegel (1987) model. Yet considering the substantial information about future curve dynamics (specifically the long-end tenors) embedded in the macroeconomic variables, researchers have also tried extract macroeconomic information as a set of latent factors, then add these exogenous variables (e.g. real activity, inflation, and fed funds rate) into the term structure modeling framework (Ang and Piazzesi, 2003; Diebold, Rudebusch and Aruoba, 2006; Cooper and Priestley, 2008; Ludvigson and Ng, 2009). Though a large part of term structure model specification has been deployed, a unified conclusion with regard to the factor selection has not been achieved yet.

Starting from late 1990s, there is a growing number of literature that explores the use of machine learning methods in asset pricing. Early attempts are Kuan and White (1994) and Yao et al. (2000), where they initially introduced the use of neural networks into the economics discipline. Some recent work further validates the advantages of shrinkage and data compression, two main categories of remedies to avoid overfitting, such as Kelly and Pruitt (2013), Freyberger et al. (2017), Messmer (2017), and Feng, Polson and Xu (2019).

Focused on the context of unsupervised learning, PCA have been applied in the fixed income market for decades, such as Steeley (1990) and Litterman and Scheinkman (1991). Practitioners usually interpret the principle components as level, slope, and curvature effects. To link the factors with more understandable economic instances in the financial markets, Duffie and Singleton (1997) propose a multi-factor model for IRS (interest rate swap) that accommodates counterparty default risk and liquidity differences between the Treasury and Swap markets. By extending this work, Liu, Longstaff and Mandell (2006) estimate a five factor model to analyze swap spreads.

There is some unavoidable limit associated with the PCA approach, for example, encoding a great amount of information regarding curve shapes into a covariance matrix. This will miss a great amount of predictive information, since the behavior of interest rates have been shown depend on the absolute level of rates. Thus, to better capture the non-linear relationship among neighbor tenors, Kondratyev (2018) proposes a neural networks based term structure algorithm for brent oil forward price and USD swap curve.

*(https://github.com/DarseZ/CurveFrcst-Using-ManifoldLrn/blob/main/papers/ANNLearnCurveDynamics.pdf)*

While term structure modeling has been well developed in the Treasury markets, to the best of our current knowledge, advances in modeling the spread term structure using nonlinear dimension reduction methods are comparatively small. Inspired by the work of Kondratyev, we believe the heterogeneity of temporal evolution will be better captured by manifold learning combined with adaptive time series forecasting.

The rest of the paper is organized as following. Section III starts from the economic meaning of four interest rate curves used in this project (LIBOR, SOFR, OIS, Treasury), then provides a structured formulation of how to build the required curves and implement nonlinear dimension reduction, finally outlines the times series forecasting architectures and trading strategies design. Sections IV describe the design of the empirical applications and the results. Section V concludes.

**III. Problem formulation and Application**

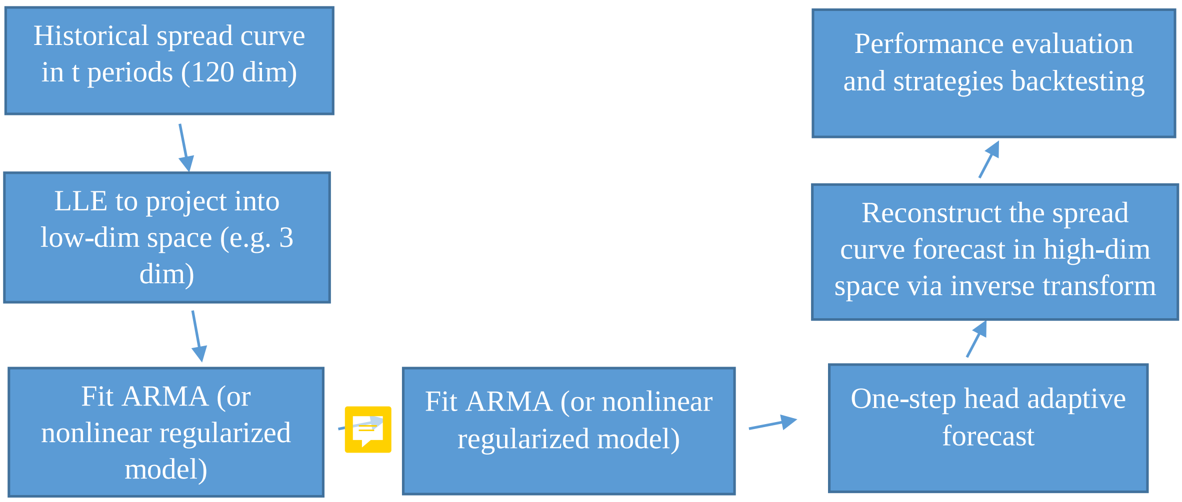



Fig. 1. Conceptual flowchart of the framework

A. Curves Building and Interest Rate Processes Simulation

From the website of Fed Reserve Bank of New York, we get the functional parameters of the Nelson-Siegel-Svensson structural models for Treasury instantaneous forward rates as described in Svensson (1994). Then, using the calibrated parameters as shown in equation (1) and (2), we could construct the zero (forward) rate curve by plugging the maturity index into the zero rate expression. (*https://www.federalreserve.gov/data/nominal-yield-curve.htm)*

$$y_i(n) = \beta_0 + \beta_1 \frac{1-\exp(-\frac{n}{\tau_1})}{\frac{n}{\tau_1}} + \beta_2 \left[\frac{1-\exp(-\frac{n}{\tau_1})}{\frac{n}{\tau_1}} - \exp\left(-\frac{n}{\tau_1}\right)\right] + \beta_3 \left[\frac{1-\exp(-\frac{n}{\tau_2})}{\frac{n}{\tau_2}} - \exp\left(-\frac{n}{\tau_2}\right)\right], \quad (1)$$

$$f_t(n,0) = \beta_0 + \beta_1 \exp(-\frac{n}{\tau_1}) + \beta_2 \frac{n}{\tau_1} \exp(-\frac{n}{\tau_1}) + \beta_3 \frac{n}{\tau_2} \exp(-\frac{n}{\tau_2}), \quad (2)$$

The data format should be daily data with continuous maturity spectrum from 1m to 30y (360 dimensions). To make it consistent with the following two swap curves, we re-sample it to get a weekly dataset with discrete maturity spectrum from 3m to 30y (120 dimensions).

For the LIBOR and SOFR curves, we directly download them from Bloomberg terminal by manually changing "As of Date" variable. If time permits, we will replicate the whole curve bootstrapping pipeline to get these curves using market instruments prices (deposits, futures/forwards, swaps).

For the preparation of accuracy and trading strategies backtesing, we take a parametric model, the Ornstein-Uhlenbeck process (equation (3) and (4)), to estimate the parameters of underlying interest rates process for Treasury, LIBOR, and SOFR respectively. With the calibrated process, we simulate a large number of realizations (e.g. 10,000).

$$\mathrm{d}S_t = \lambda(\mu - S_t)dt + \sigma dW_t, \qquad (3)$$

$$S_{(t+\delta)} = S_t e^{-\lambda\delta} + \mu(1 - e^{-\lambda\delta}) + \sigma\sqrt{\frac{1-e^{-\lambda\delta}}{2\lambda}}\,\mathrm{N}(0, 1), \qquad (4)$$

For each simulated scenario, we will obtain two panel data for the swap spread and basis spread respectively. Each of them will be in the shape of N by M, where N is the number of weeks along the calendar dates and M is the number of tenors.

The data preparation could be seen in *https://github.com/DarseZ/CurveFrcst-Using-ManifoldLrn/blob/main/CurveBuild.ipynb*

We would keep add most recent data points (dates) into the dataset as the project moved forward.

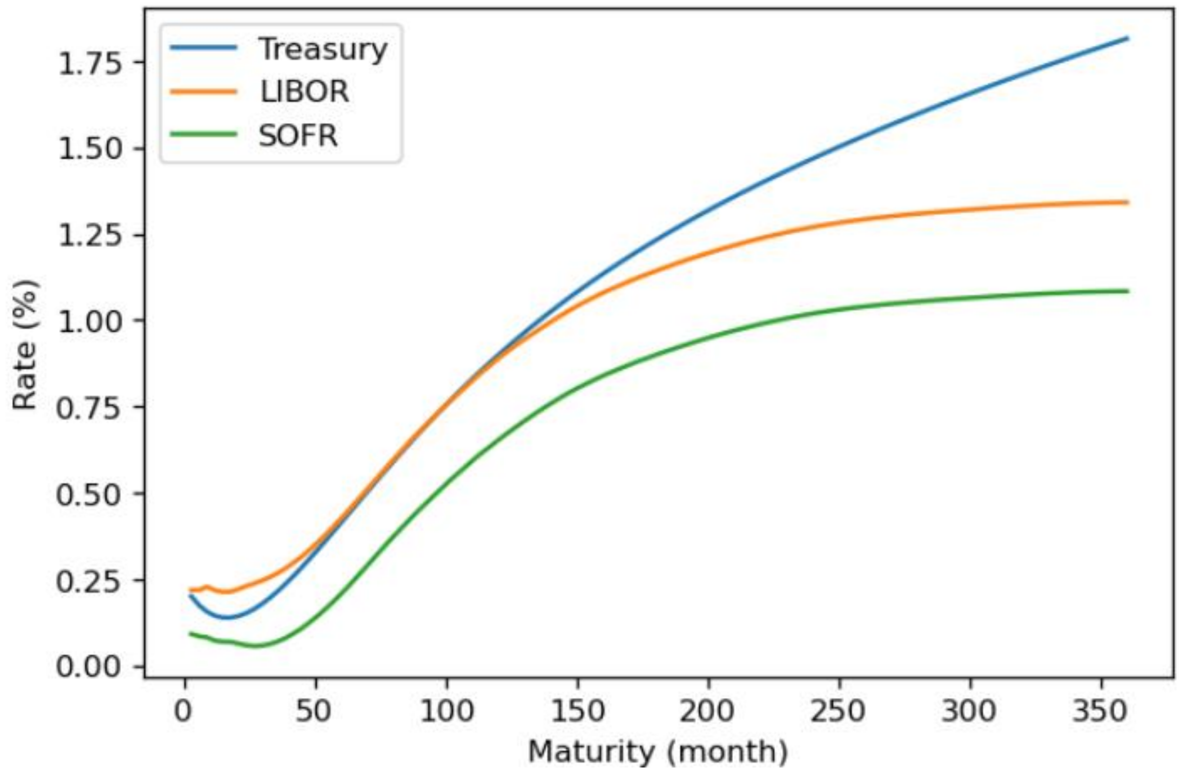



Fig. 2. Original interest rate curves snapshot on 2020-10-30

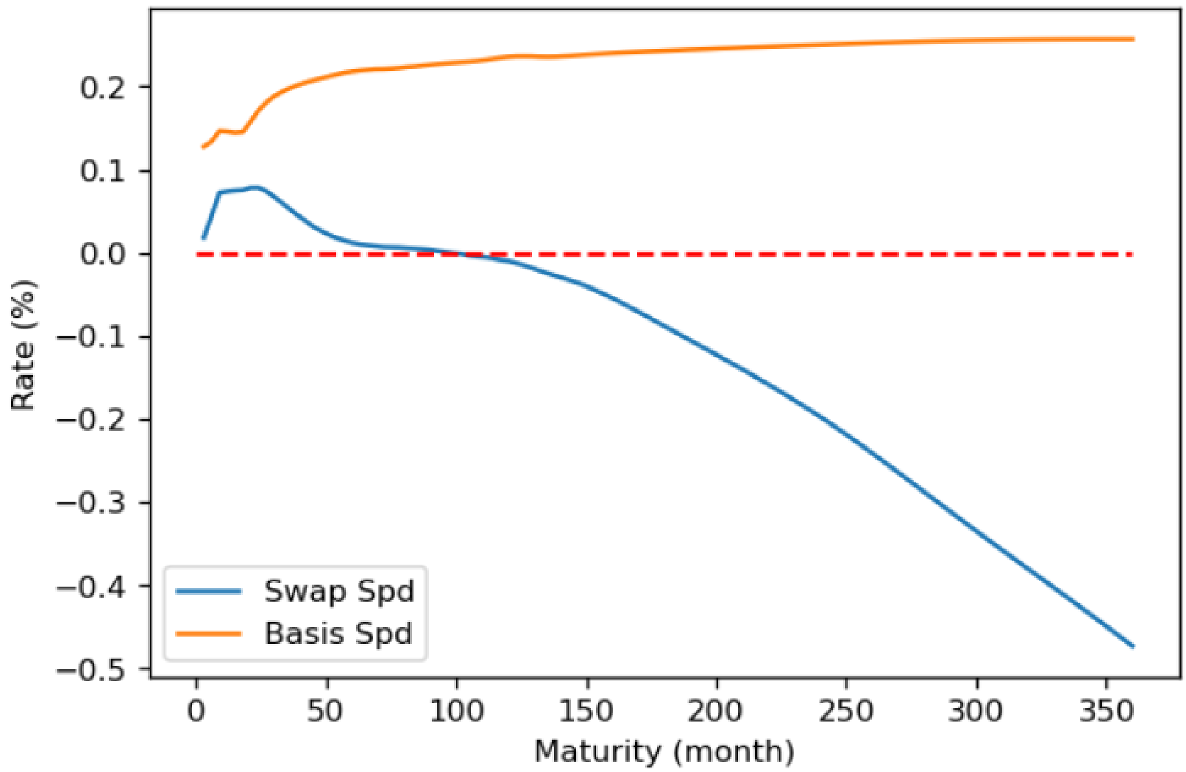



Fig. 3. Interest rate spread curves snapshot on 2020-10-30

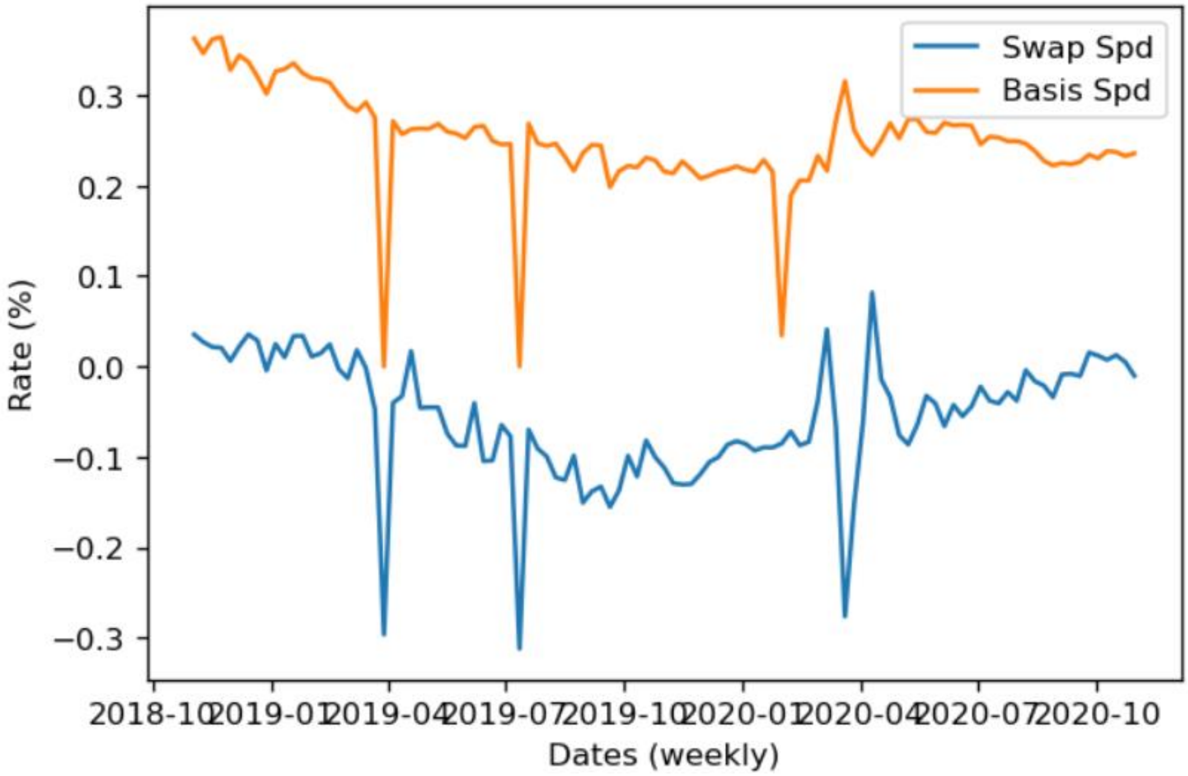



Fig. 4. Interest rate spread curves time series (10year tenor)

B. Nonlinear Dimension Reduction

Starting from baseline classical methods, PCA, which is a popular technique for explaining curve dynamics, but there are two underlying strong assumptions which may limit the predictive power for datasets with strong nonlinearity: (1) The new orthogonal basis, which is a linear combination of the original basis, usually is not able to capture the most interesting part of nonlinear behavior; (2) Mean and variance are sufficient statistics. Therefore, if the probability distribution is not Gaussian, all other necessary high-order statistics will be lost.

The second approach is Locally Linear Embedding, which is a classical manifold searching method. Given a set of D-dimensional data points $x_1, x_2, \ldots, x_n$, we try to find the embedded d-dimensional feature vectors $y_1, y_2, \ldots, y_n$. The main steps are as following.

(1) Identify nearest neighbors based on some distance metric for each data point $x_i$, where $N_i$ denote the set of indices of the nearest neighbors for this data point.

(2) Find the optimal local convex combination of the nearest neighbors to represent each data point. That is, we are optimizing function (5) to compute the weights.

$$E(w) = \sum_{i=1}^{N} \left\| x_i - \sum_{j \in N_i} W_{ij} x_j \right\|^2, \text{ s.t. } \sum_{j \in N_i} W_{ij} = 1 \quad (5)$$

(3) Find the low-dimensional feature vectors $y_i$, which have the optimal local convex representations with the given $W_{ij}$. That is, we try to compute $y_i$ by minimizing the following cost function (6).

$$\phi(y) = \sum_{i=1}^{N} \left\| y_i - \sum_{j \in N_i} W_{ij} y_j \right\|^2 \qquad (6)$$

For the third and fourth dimension reduction approach, we would like to implement Multidimensional Scaling (MDS) and t-distributed Stochastic Neighbor Embedding (t-SNE), which belong to the category of semi-classical methods.

Compared to the direct eigen-analysis of the N data points themselves in PCA, MDS selects influential dimensions by the eigen-analysis of the $N^2$ data points of a pairwise distance matrix. The goal is to preserve the pairwise distances as best as possible after mapping to the low-dimensional space. t-SNE converts similarities between data points to joint probabilities and tries to minimize the K-L divergence between the joint probabilities of the low-dimensional embedding and the high-dimensional data.

The dimension reduction part could be seen in *https://github.com/DarseZ/CurveFrcst-Using-ManifoldLrn/blob/main/DmnsRdct_StateFrcst.ipynb*

As we try to model non-linear relationship among different tenors, the low-dim representations may not be that interpretable as the parametric framework or the baseline PCA approach. This is the cost of improving prediction power.

C. Time Series Forecasting and Inverse Transform

Our objective is to predict the most likely curve transformation given its observed shape at a particular moment in time. Our method converts original spread curves into several main drivers in low dimensional space by manifold learning. After conversion, we employ both the ARMA family models and hessian regularized nonlinear models as main time series forecasting tools.

If time permits, we will try a perturbation test (add a specific tenor into the input of the neutral net), then visualize how the entire curve will be impacted by the exogenous impulse after time interval $\Delta t$. Specifically, for each dimension reduction based approach, the procedure is as follows: (1) estimate reduced representations and loadings for each tenor using original curve, (2) re-estimate the reduced representations using perturbed curve, (3) compute the inverse transform using original loadings and new reduced representations.

Because the (inverse) manifold learning algorithm is not an injective function, we may cannot come up with a general approach. Three potential solutions: (1) nonparametric regression, by Z. Zhang and H. Zha, "Principal manifolds and nonlinear dimension reduction via tangent space alignment,"; (2) inverse manifold learning (encoding and decoding) by the scholars in Xihu University (2020). (3) simple LLE reconstruction, by Jin Chen, Shijie Deng, and Xiaoming Huo (2004).

By taking the third option, suppose low-dimensional feature vectors $y_1, y_2, \dots, y_n$ have been obtained through LLE in the previous subsection and we have a new prediction $y_{n+1}$, we could reconstruct $x_{n+1}$ using the following steps.

(1) Identify nearest neighbors based on some distance metric for each data point $y_{n+1+}$, where $N_{n+1}$ denote the set of indices of the nearest neighbors for this data point.
(2) The weights of the local optimal convex combination $w_j$ are obtained by optimizing function (7).

$$\mathrm{E(w)} = \left\| y_{n+1} - \sum_{j \in N_{n+1}} W_j x_j \right\|^2 \text{ , s.t. } \sum_{j \in N_{n+1}} W_j = 1 \qquad (7)$$

(3) The data point in high-dimensional space should be $\widehat{x_{n+1}} = \sum_{j \in N_{n+1}} W_j X_j$ .

D. Performance Evaluation and Systematic Trading Strategies

We define a statistical measure and an economic measure to evaluate the forecasting performance. For the statistical measure, regarding the relative difference of spread values between two adjacent time steps could be classified into non-negative (positive) and negative categories, we calculate the accuracy rate. For the economic measure, we backtest absolute value strategies using single asset (tenor) and three systematic relative-value strategies using multiple assets (tenors) based on forecasting.

Specifically, among the relative-value strategies, the level trading signal is defined as $0.33 * 2\mathrm{yr} + 0.33 * 5\mathrm{yr} + 0.33 * 10\mathrm{yr}$: If we predict the signal is going to increase at next period (average level increases), then float leg payment will increase, we will build a float leg receiver position.

The fly trading signal will be $-1.0 * 2\mathrm{yr} + 1.0 * 10\mathrm{yr}$: If we predict the signal is going to increase at next period (curve slope increases), then float leg payment of 10yr will increase relative to the 2yr, we will build a float leg receiver position of 10yr and a float leg payer position of 2yr.

The butterfly trading signal will be $0.5 * 2\mathrm{yr} - 1.0 * 5\mathrm{yt} + 0.5 * 10\mathrm{yr}$: If we predict the signal is going to increase at next period (curve curvature increases), then float leg payment of 10yr and 2yr will increase relative to the 5yr, we will build a float leg receiver position of 10yr and 2yr and a float leg payer position of 5yr.

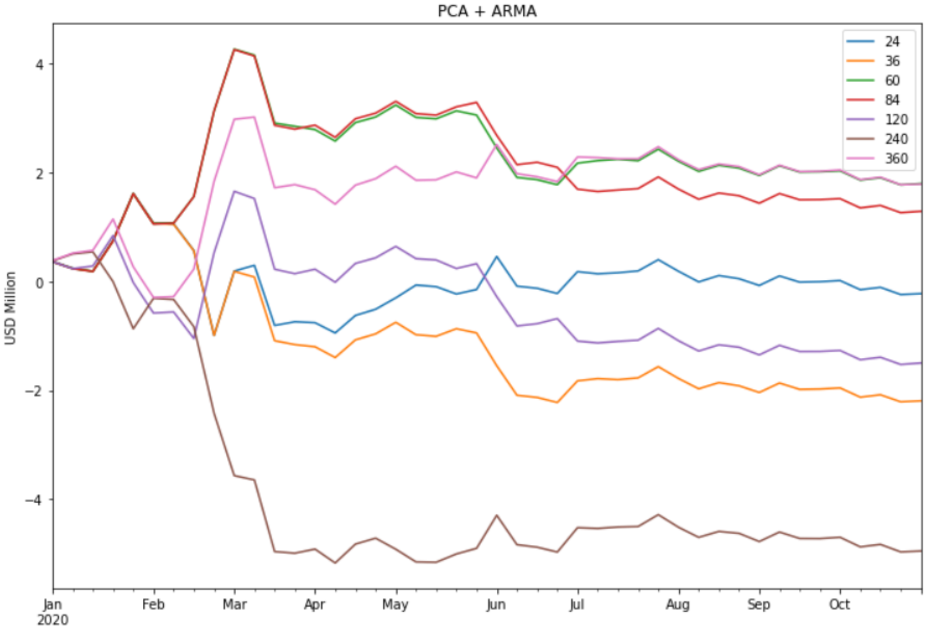
By holding the corresponding swap portfolio for one-period (a week) suggested by the trading signal, we will get three cumulative PnL plot for each given model specification. This will help us identify the difference between different algorithms in predicting specific patterns of curve dynamics.

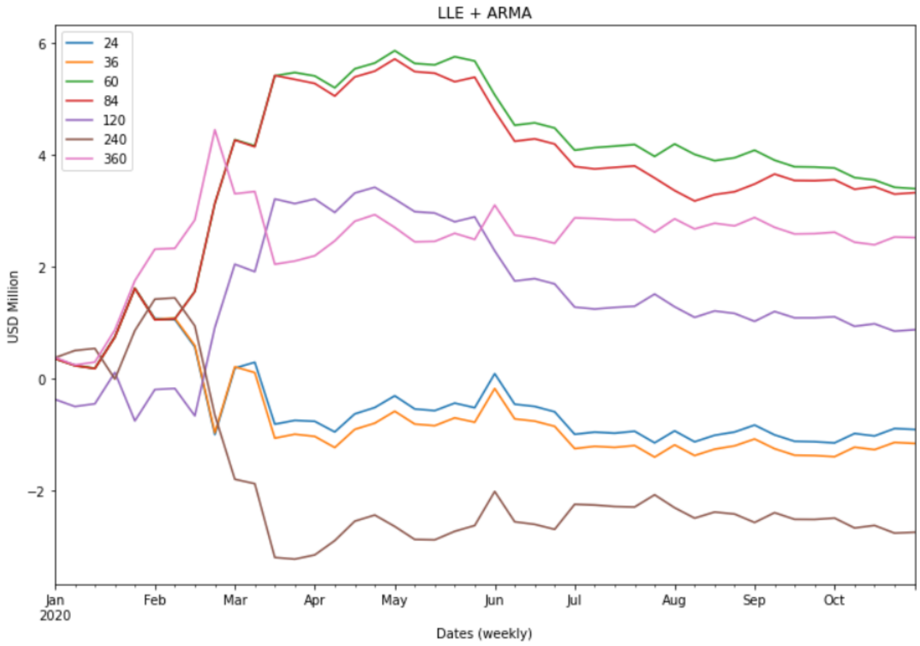
## IV. Results

A. Single Tenor Forecasting and Trading

For swap spread curve:

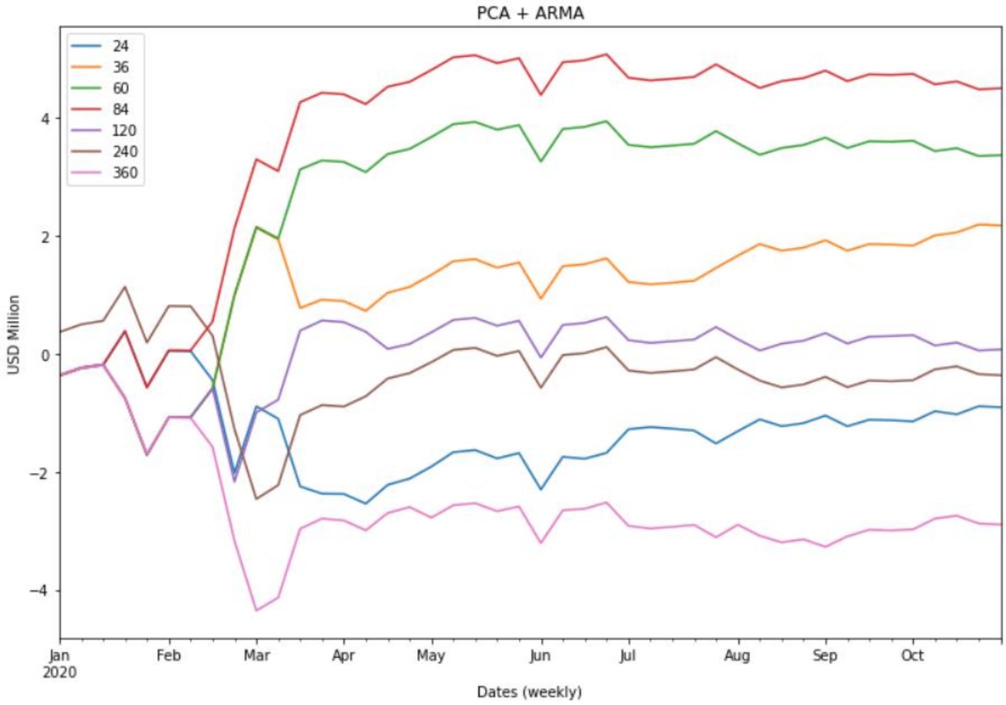
| Tenor (month) | 24 | 36 | 60 | 84 | 120 | 240 | 360 |
|---|---|---|---|---|---|---|---|
| PCA+ARMA | 56.8% | 59.1% | 54.5% | 63.6% | 65.9% | 50% | 54.5% |
| LLE+ARMA | 63.6% | 65.9% | 61.4% | 61.4% | 68.2% | 56.8% | 72.7% |

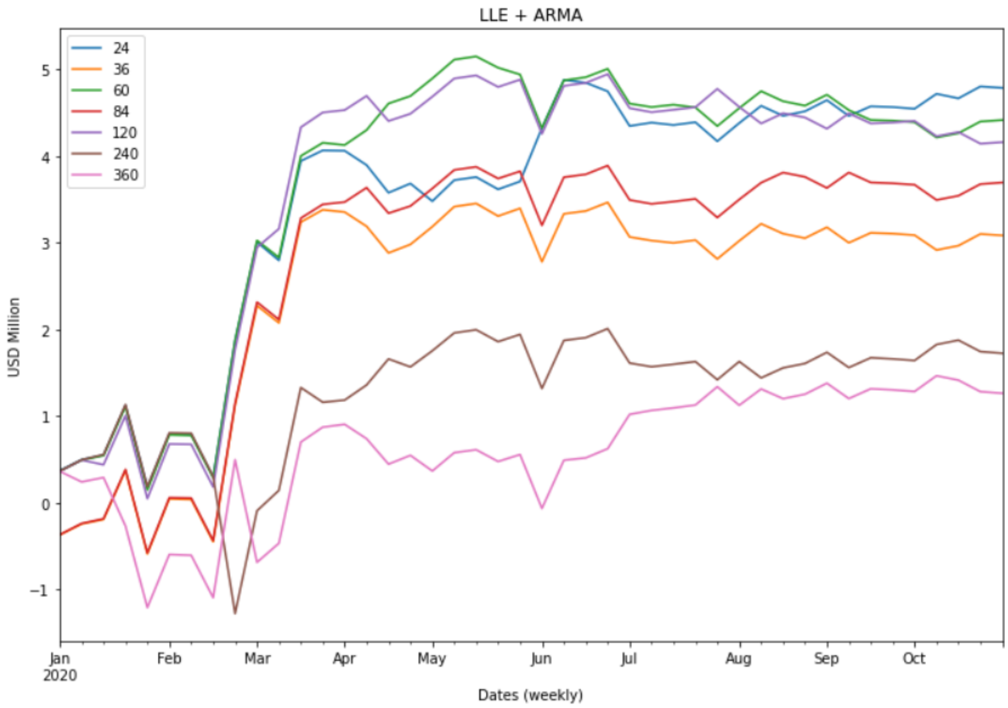



For basis spread curve:

| Tenor (month) | 24 | 36 | 60 | 84 | 120 | 240 | 360 |
|---|---|---|---|---|---|---|---|
| PCA+ARMA | 54.5% | 52.3% | 45.5% | 54.5% | 61.4% | 61.4% | 68.2% |
| LLE+ARMA | 50% | 50% | 52.3% | 47.7% | 54.5% | 65.9% | 54.5% |

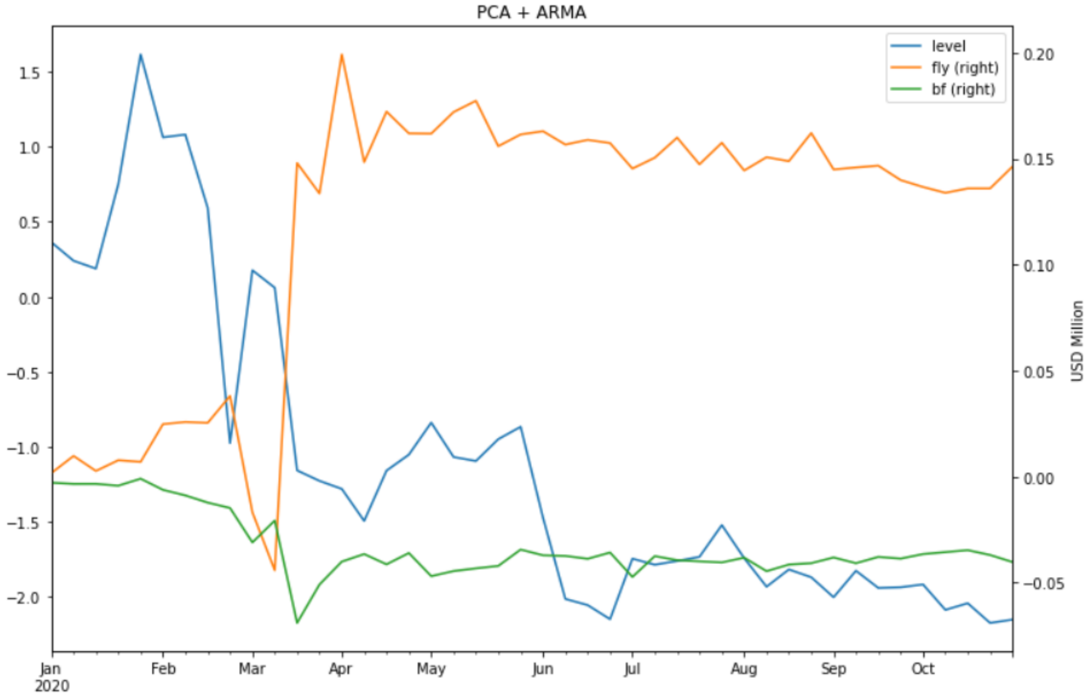


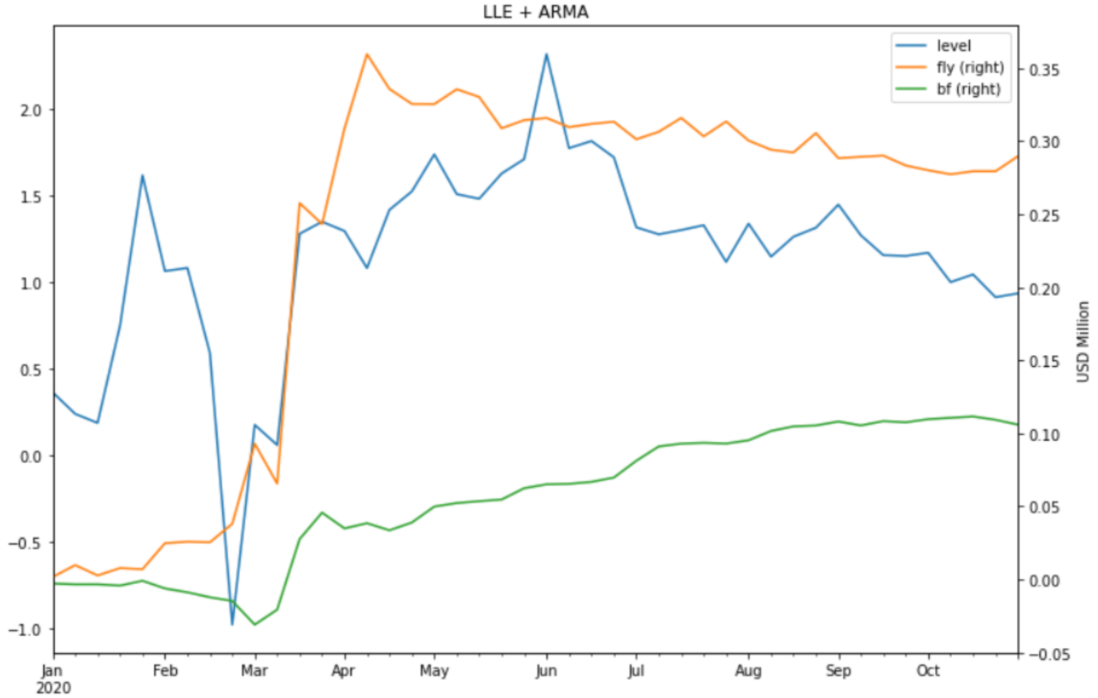




B. Multiple Tenor Forecasting and Trading

For swap spread curve:

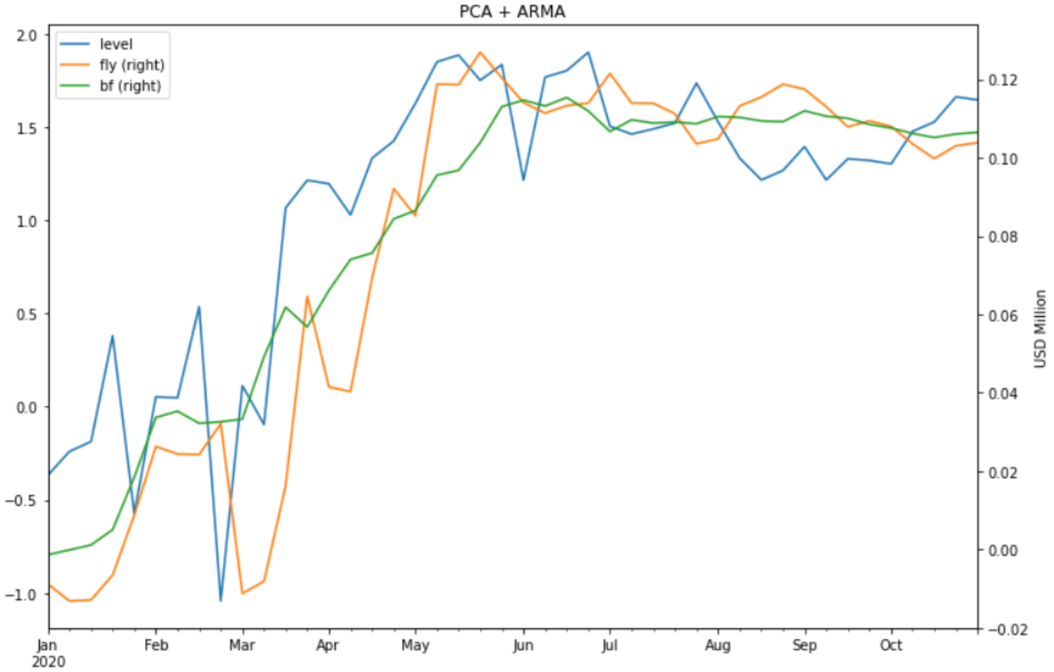
| Relative-value positions | level | fly | butterfly |
|---|---|---|---|
| PCA+ARMA | 61.4% | 52.3% | 47.7% |
| LLE+ARMA | 68.2% | 52.3% | 65.9% |

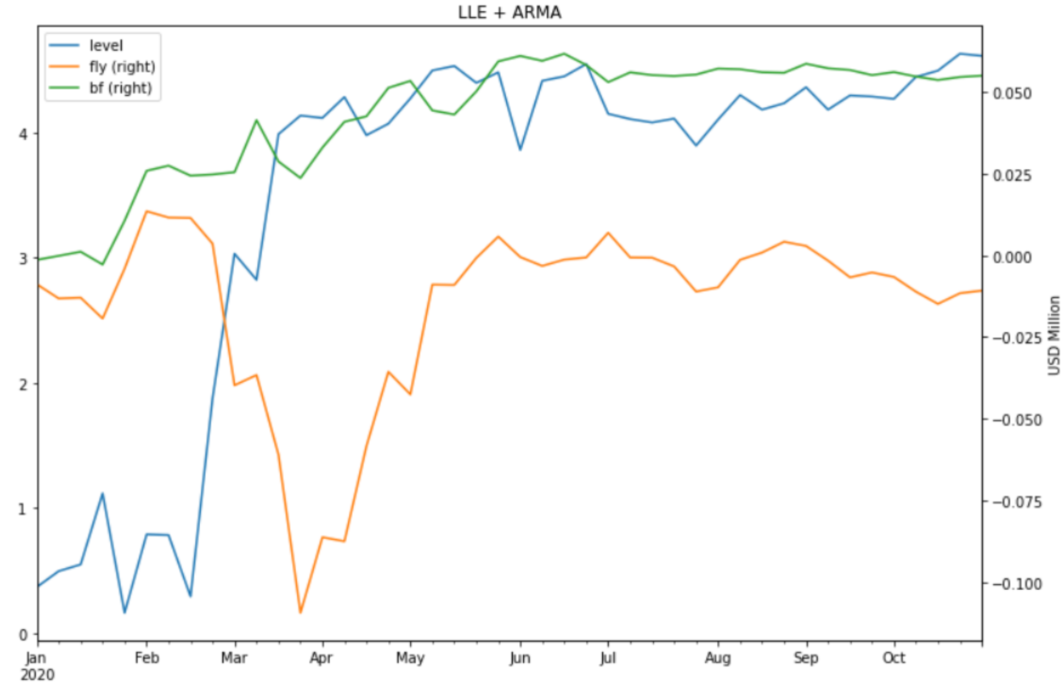


For basis spread curve:

| Relative-value positions | level | fly | butterfly |
|---|---|---|---|
| PCA+ARMA | 56.8% | 50% | 61.4% |
| LLE+ARMA | 50% | 45.5% | 54.5% |




## V. Conclusion

## References

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