ISyE8900 Project

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Abstract: This project proposes a nonparametric approach for the modeling and forecasting of weekly interest rate spread curves by using nonlinear dimension reduction, such as the locally linear embedding (LLE). We mainly focus on two objective spread curves: Swap Spread (LIBOR substract Treasury) and Basis Spread (LIBOR substract SOFR). Benchmarking on its linear dimension reduction counterparty – principle component analysis (PCA) – we show the LLE-based framework yields a higher out-of-sample forecast accuracy for specific underlying tenors as well as a better profit and loss (PnL) profile in backtesting various systematic term structure trading strategies.

1 Introduction

In the fixed-income trading industry, there is an increasing demand for accurately forecasting the short-term movement of spread term structure in a neat but efficient way as the low interest rate environment persists to squeeze the spread magnitude among multiple key interest rates curves. Besides, The Alternative Reference Rates Committee (ARRC) has identified the Secured Overnight Financing Rate (SOFR) as the successor rate of LIBOR, and continues supporting the launch of SOFR-based financial products in coming years. Thus, during the process of LIBOR's fallback, market participants are imperative to measure the LIBOR-SOFR spread when USD LIBOR-based activity gradually decreases until completely unusable.[1]

The recent development of statistical learning greatly inspired the interest in applying dimensionality reduction and predictive models with sparse and nonlinear features in the finance industry, where the curve spread modeling is a canonical example. The use of such methods are limited to principal component and latent factor analysis, because more complex models may not be suitable for structural analysis and parameter interpretation. However, as a forecasting problem instead of an identification problem, the main goal of modeling the time variation of interest rate spread curves should be pursuing a stronger out-of-sample forecast power.

In this paper we propose the following multistage approach for the short-term prediction of Swap/Basis spread changes following the work by Chen, Deng and Huo (2008): (a) dimension reduction (b) forecasting in the reduced dimension (c) mapping back to the original space. Forecasting time variation requires a

[1]https://www.newyorkfed.org/arrc/sofr-transition

careful approximation of an unknown encoder that maps information from high-dimensional (long spectrum of underlying maturities) to low-dimensional representations (latent drivers of the entire term structure). After obtaining such low-dimensional representation of spread curves, forecasts could be made by first predicting each new coordinate of the manifold using the ARMA model (or nonlinear time series models) and then map them back to the high-dimensional space utilizing the corresponding reconstruction method.
Benchmarking on the well-known PCA-based framework, the performance evaluation of different modeling objective curves (Swap and Basis spread) includes both statistical accuracy and the profitability of term structure relative-value trading strategies.

2 Background

Regarding the debate that how much predictability of curve movements one could capture using simply historical panel data, we confirm the superior out-of-sample forecasting performance of short-term interest rate dynamics if we model the spread curve as a whole.
Inspired by the idea of summarizing term structures by a small set of linear combinations of yields, Diebold and Li (2006) use the AR family models to obtain encouraging results for long-horizon ex-ante forecasts by reformulating the Nelson and Siegel (1987) model. Yet considering the substantial information about future curve dynamics (specifically the long-end tenors) embedded in the macroeconomic variables, researchers have also tried to extract macroeconomic information as a set of latent factors, then add these exogenous variables (e.g. real activity, inflation, and fed funds rate) into the term structure modeling framework (Ang and Piazzesi, 2003; Diebold, Rudebusch and Aruoba, 2006; Cooper and Priestley, 2008; Ludvigson and Ng, 2009). Though a large part of the term structure model specification has been deployed, a unified conclusion concerning the factor selection has not been achieved yet.
Specifically, focused on the context of dimension reduction techniques, PCA has been applied in the field of term structure modeling for decades, such as Steeley (1990) and Litterman and Scheinkman (1991). Usually, practitioners usually interpret the principal components as level, slope, and curvature effects. However, there is some unavoidable limit associated with the PCA approach, for example, encoding a great amount of information associated with curve shapes into a covariance matrix. This will miss a great amount of predictive information since the behavior of interest rates has been shown to depend on the absolute level of rates. Thus, to better capture the non-linear relationship among neighbor tenors, Kondratyev (2018) proposes a neural networks based term structure algorithm for brent oil forward price and USD swap curve.[2]
While the above-mentioned term structure modeling has been well developed in the Treasury markets, to the best of our current knowledge, advances in modeling the spread term structure using nonlinear dimension reduction methods are comparatively small. Inspired by the work of Kondratyev, we believe the heterogeneity of temporal evolution will be better captured by manifold learning combined with adaptive time series forecasting.

[2] https://github.com/DarseZ/CurveFrcst-Using-ManifoldLrn/blob/main/papers/ANNLearnCurveDynamics.pdf

The rest of the paper is organized as follows. Section 3 starts from the economic meaning of four interest rate curves used in this project (LIBOR, SOFR, Treasury, and OIS), then provides a structured formulation of how to build the required curves and implement nonlinear dimension reduction, finally outlines the times series forecasting architectures and trading strategies design. Section 4 describe the results of the empirical analysis. Section 5 concludes and discusses future work.

3 Problem formulation

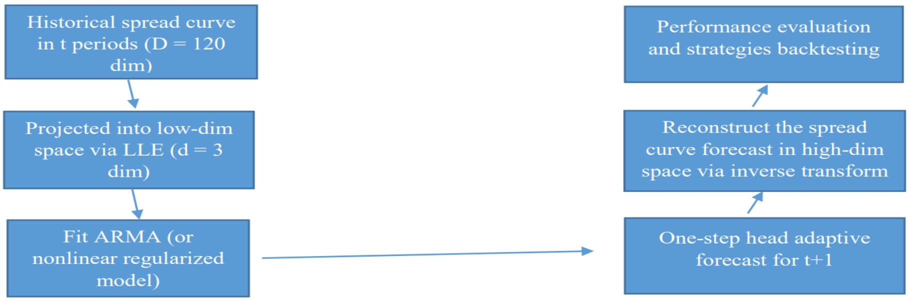

Figure 1: Conceptual flowchart of the framework

Given the panel dataset of LIBOR rate $L_1, L_2, \dots, L_n$ where n denotes the number of time periods and each $L_i \in R^D$ and similar panel datasets of Treasury rate $T_1, T_2, \dots, T_n$ and SOFR rate $S_1, S_2, \dots, S_n$, we first calculate the two required spread datasets: swap spread $SS_i = L_1 - T_1, L_2 - T_2, \dots, L_n - T_n$ and basis spread $BS_i = L_1 - S_1, L_2 - S_2, \dots, L_n - S_n$. Then, by using a pre-specified dimension reduction technique, we obtain their low-dimensional representations $SSlow_i$ and $BSlow_i$ respectively, where each sample is in the R^d space for i from 1 to n. For each objective spread, we do a one-step head prediction using univariate time series modeling to get $SSlow_{n+1}$ and $BSlow_{n+1}$ respectively. Finally, map them back to the original R^D space using the corresponding inverse transform to get $\hat{SS}_{n+1}$ and $\hat{BS}_{n+1}$, then evaluate the forecasting accuracy and design trading strategies according to the comparison with realized value SS_{n+1} and BS_{n+1}.

3.1 Curves Building and Interest Rate Processes Simulation

Since there are a great amount of frameworks to construct interest rate curves using data from different providers, in order to eliminate some ambiguity, we first go through our data preparation process.
Considering the purpose of understanding the treasury curve's fundamental determinants, we will employ a parametric yield curve specification described in Svensson (1994). As shown below, this specification could largely rule out variation resulting from a few specific securities at a given maturity.
From the website of Fed Reserve Bank of New York, we get the functional parameters of the Nelson-Siegel-Svensson structural models, assuming that instan-

taneous forward rates n years ahead are characterized by a continuous function with only four parameters: $\beta_0 + \beta_1$ measures the initial level at horizon zero, while the asymptote level will be β_0. β_2 and β_3 determine the convexity of two humps located in between the entire maturity spectrum.
Empirically, using the calibrated parameters as shown in equation (1) and (2), we could construct the zero (forward) rate curve by plugging the maturity index into the zero rate expression.[3]
Equation (1):

$$y_t(n) = \beta_0 + \beta_1 \frac{1 - exp(\frac{-n}{\tau_1})}{\frac{n}{\tau_1}} + \beta_2 (\frac{1 - exp(\frac{-n}{\tau_1})}{\frac{n}{\tau_1}} - exp(\frac{-n}{\tau_1})) + \beta_3 (\frac{1 - exp(\frac{-n}{\tau_2})}{\frac{n}{\tau_2}} - exp(\frac{-n}{\tau_2}))$$

Equation (2):

$$f_t(n, 0) = \beta_0 + \beta_1 exp(\frac{-n}{\tau_1}) + \beta_2 \frac{n}{\tau_1} exp(\frac{-n}{\tau_1}) + \beta_3 \frac{n}{\tau_2} exp(\frac{-n}{\tau_2})$$

The data format should be daily data with a continuous maturity spectrum from overnight to 30y (360 dimensions). To make it consistent with the other two swap curves, we re-sample it to get a weekly dataset with a discrete maturity spectrum (120 dimensions). For the LIBOR and SOFR curves, we directly download them from the Bloomberg terminal by manually changing the "As of Date" variable. If time permits, we will replicate the whole curve bootstrapping pipeline to get these curves using market instruments prices (deposits, futures/forwards, swaps).[4]
To generate more datasets for the performance evaluation, we will add parametric simulations of underlying interest rates to serve as a more solid numerical experiment. More details will be addressed in Section 5.

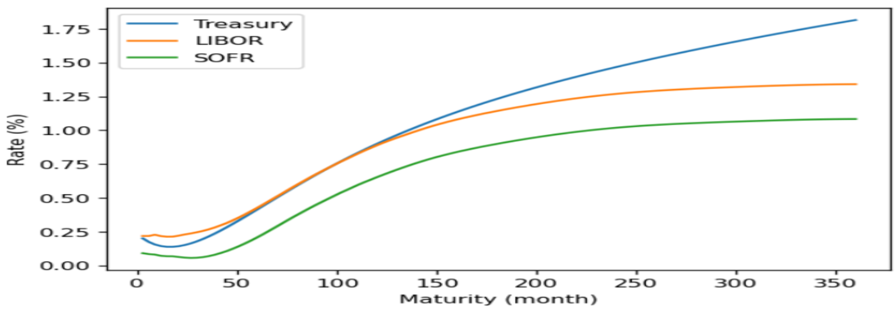

Figure 2: Original interest rate curves snapshot on 2020-10-30

As shown in Fig. 2, on 2020-10-30, all of the three curves are upward sloping. The LIBOR and SOFR curve are quite similar in this cross-sectional view, while the Treasury curve starts below the LIBOR curve at short-end and then crosses over the LIBOR curve around mid to long tenors.

As shown in Fig. 3, on 2020-10-30, the basis spread curve is quite flat and constantly positive, while the swap spread shows a hump shape at short end

[3]https://www.federalreserve.gov/data/nominal-yield-curve.htm
[4]https://github.com/DarseZ/CurveFrcst-Using-ManifoldLrn/blob/main/CurveBuild.ipynb

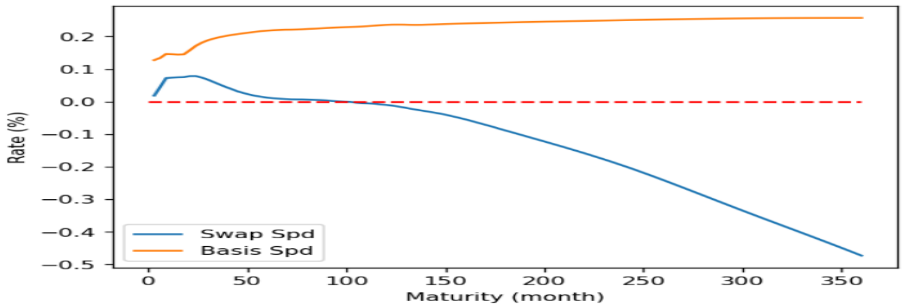

Figure 3: Interest rate spread curves snapshot on 2020-10-30

and sharply goes down into the negative territory as tenor increases.

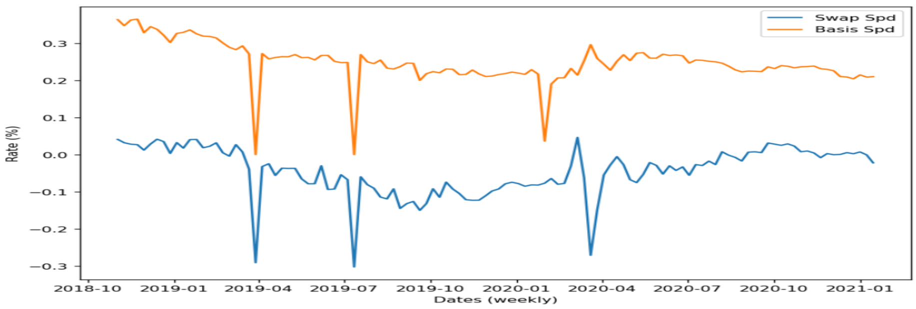

Figure 4: Interest rate spread curves time series (10year tenor)

As shown in Fig. 4, for the 10year tenor, the two spread time series demonstrate a similar pattern in general. Within the observation window, there are three downward jumps caused by the LIBOR around Apr 2019, Jul 2019, and Feb 2020.

3.2 Nonlinear Dimension Reduction

3.3 Time Series Forecasting and Inverse Transform

3.4 Performance Evaluation and Systematic Trading Strategies

4 Results

5 Conclusion and Future Work