Longitudinal gravitational memory

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We calculate the gravitational memory effect when a spherically symmetric shell of energy passes through a spacetime region. In particular, this effect includes a longitudinal component, such that two geodesics can pick up a relative velocity proportional to their separation. Such a measurement will allow us to obtain the total energy released by a supernova explosion in the form of neutrinos. We study the possibility to measure such an effect by space-based interferometers such as LISA and BBO, and also by astrophysical interferometers such as pulsar scintillometry.

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I. INTRODUCTION AND SUMMARY

The recent detection of gravitational waves [1] has proved that gravitational waves leave an oscillating pattern in the amplitude of waveforms measured at detectors such as LIGO. It is also known that this is not the only effect that is potentially detectable. Strong gravitational waves imply a large flow of energy. Just like any other flow of energy, it leads to a gravitational memory effect [2, 3].

The memory effect discussed in [2, 3] causes permanent relative displacements between geodesics. It contains only transverse-traceless components and can leave an imprint in an interferometer. In this paper, we will introduce another memory effect that is different in two ways:

- It has a longitudinal component. The transverse-traceless limitation only applies to freely-propagating changes of the metric (i.e gravitational waves). While coupled to matter, which often have longitudinal (density) waves, it is natural to have an accompanying longitudinal change in the metric.

- In addition to displacements of geodesics, it also causes permanent changes in relative velocities between geodesics, with magnitudes proportional to their separations.

In terms of the dynamics, a change in velocity is higher order than a change in distance measured in gravitational waves. This however does not mean that our effect is harder to measure. The conventional gravitational memory effect needs a physical event that significantly breaks spherical symmetry to be detectable whereas our effect does not and therefore it can occur more generally. In addition, a change in velocity implies a distance change that grows in time even after the initial effect. That is an advantage for some detection methods.

We start by analyzing an astrophysical scenario inspired by previous work in [7] . We calculate what happens to geodesics that are parallel when an energy shell crosses them. The particular set-up we consider is a neutrino shell emitted by supernova. The shell crosses the path of light rays coming from a pulsar that is scintillating due to lensing in the interstellar medium. The SKA telescope is expected to detect thousands of pulsars, [4], and one of them might be close enough to a supernova explosion to provide us with a good estimation of the total energy in the neutrino shell of the explosion. Currently there is no other way to obtain such information. Thus in addition to direct neutrino detections such as in Super-Kamiokande [5], this memory effect can provide a new handle on constraining the explosion mechanism.

Following from this we consider what happens if one considers an interferometer such as LISA instead of the two geodesics of the pulsar photons. Our calculation will show that the ends of the interferometer will see a change in their geodesic distance: this is a memory effect. When the interferometer arm is oriented perpendicular to the line of sight of the supernova we will find a more interesting effect that grows linearly with time.

The rest of the paper is organized as follows. In section II we discuss how an astrophysical interferometer formed by pulsar scintillometry can measure the memory effect caused by a supernova. In section III we derive the change in velocities caused by shell crossing the ends of an interferometer. This is done using the Israel Junction Conditions (IJC) [6] and by treating the neutrino shell as a co-dimension-one delta function. In the final section V we discuss potential observation of such an effect by experiments that are currently being planned such as LISA and BBO.

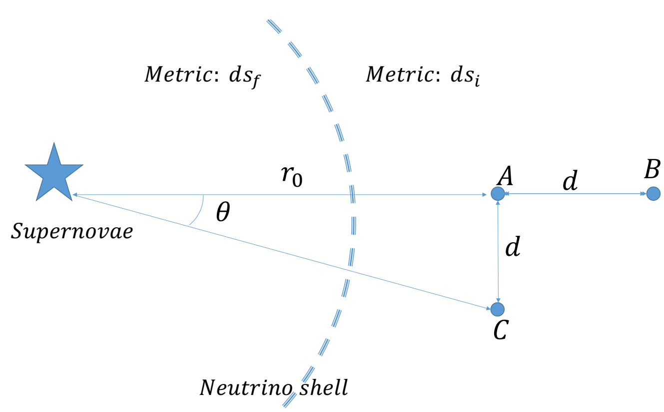

FIG. 1. Schematic of the effect being considered by a neutrino shell passing through the interferometer. The points A, B, C represent ends of the interferometer of arm length d. The three points A, B and C will pick up velocities v_A, v_B and v_C respectively after the shell crosses them. They are all different since they cross the shell at different locations.

II. MEMORY EFFECT IN PULSAR SCINTILLATION

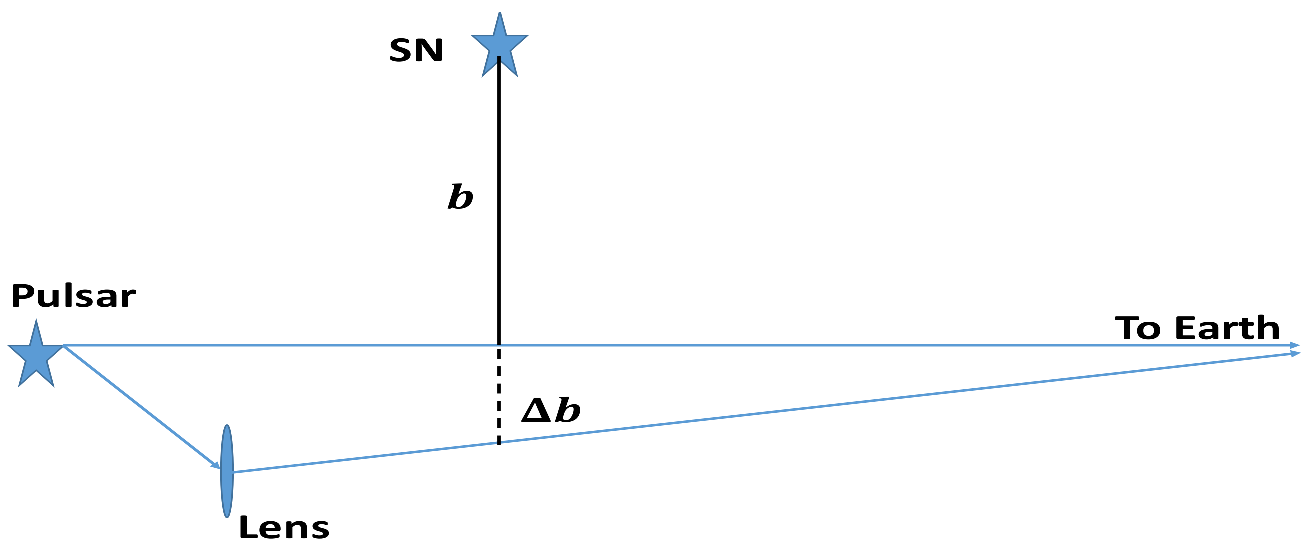

FIG. 2. Geometry of the astrophysical interferometer formed by pulsar scintillometry. Due to scattering or lensing, the image we see is an interference pattern of two light rays represented by the blue lines. If the separation of the two light rays has a component along the longitudinal (radial) direction from the SN, the spacetime distortion of the neutrino shell will change the interference pattern we see. We draw the lens to be behind the SN, but it could have been in front of it and the effect is the same.

It is known that the images of many astronomical bodies scintillate [10]. A general reason for scintillation is that due to scattering or lensing, we receive multiple light rays from the same objects. These light rays are very close to each other, so they cannot be individually resolved and have to interfere. The scintillation pattern we see is the time dependence of their interference. If we consider two light rays from a faraway pulsar which happen to pass by a SN progenitor, as illustrated in Fig.2, they can probe the spacetime distortion when it explodes.

The scintillation/interference pattern is directly related to the path lengths of these light rays. The change in such path length during a SN explosion has been worked out in [11]

$$\Delta t = 2\delta M \left[\ln\left(1+\frac{t^2}{b^2}\right) - \frac{t^2}{b^2+t^2} \right] . \tag{1}$$

Here b is the impact parameter as shown in Fig.2, the shortest distance between the light ray and the SN. t is the proper time on earth, with $t=0$ the time we directly observe the SN explosion. δM is the total energy of the neutrino shell, and Δt is the resulting time shift. A photon which should have reached the earth at time t, will arrive earlier at $(t-\Delta t)$ instead.

When the separation between two light rays has a component in the radial direction from the SN, Δb, there will be a nonzero relative change between their path lengths.

$$(\Delta t|_b - \Delta t|_{b+\Delta b}) \approx \frac{\partial \Delta t}{\partial b}\Delta b = -\frac{4\delta M t^4}{b(b^2+t^2)^2}\Delta b \ . \tag{2}$$

We can see that this effect grows from zero and approaches an asymptotic value,

$$(\Delta t|_b - \Delta t|_{b+\Delta b}) \longrightarrow \frac{4\delta M \Delta b}{b} \ , \tag{3}$$

at a characteristic time scale given by b.

We estimate b by assuming that the next SN is somewhere near the galactic centre. A sample of $\sim$ 9000 pulsars from the SKA catalog in [4] shows that among those pulsars, the shortest b is about 10 $ly \sim 10^{14}$ km. Δb is related to the scattering-broadening of images. We use the data from [12] that was observed on a scattering screen near the galactic centre. Scaling the frequency to 1 GHz which is usually a good window to observe pulsar signals. We found that such a scattering screen can produce images separated by $\Delta b \sim 1000 A.U. \sim 10^{10}$ km. We again use $\delta M \sim 1$ km, and combining all these numbers we get $(\delta M \Delta b/b) \sim 1$ m. This is comparable to the wavelength at 1 GHz, thus making the change in interference pattern easy to detect. Therefore, if we can monitor the pulsar scintillation pattern over ten years after a SN explosion we should see an order one change in the scintillation pattern predicted by Eq. (2).

III. VELOCITY CHANGE FROM JUNCTION CONDITIONS

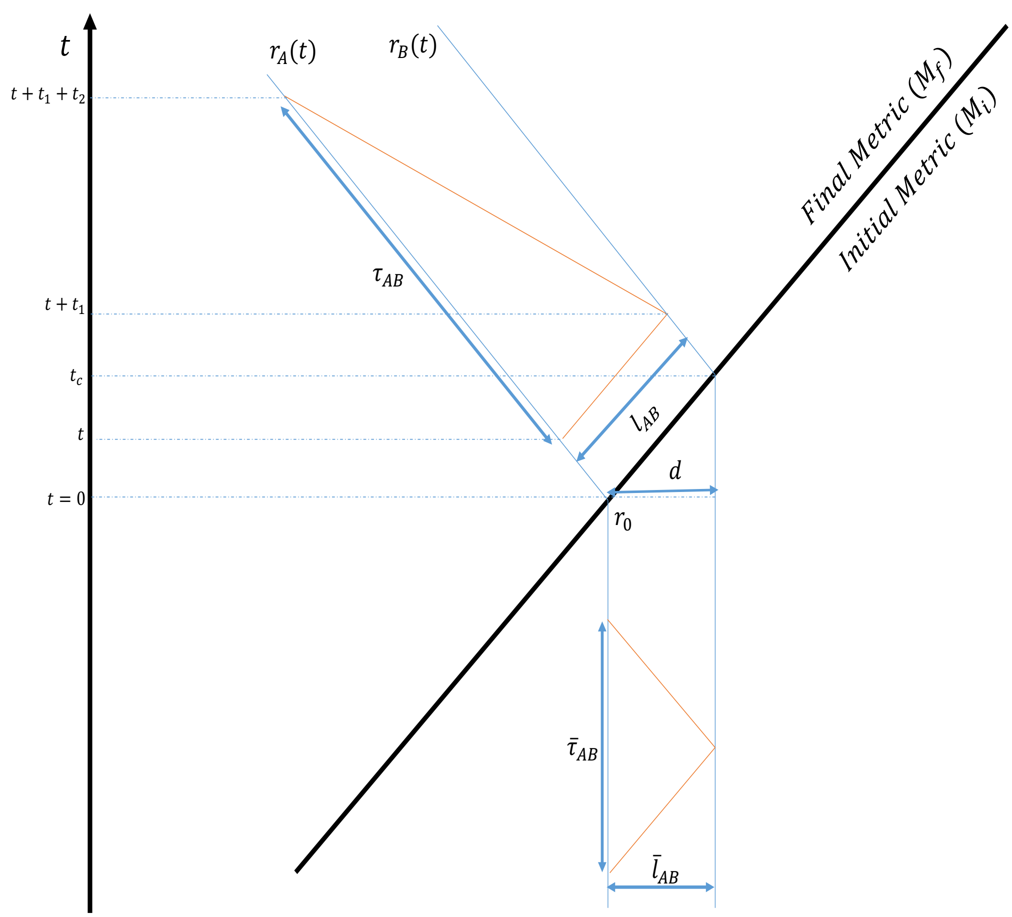

FIG. 3. Spacetime diagram showing the paths of photons that are used in the interferometer to measure the change in the length of the interferometer arms. The orange lines represent the photon trajectories. The solid blue lines represent the trajectories of the two points A and B in figure 1. The dark black line is the null trajectory of the neutrino shell. the proper lengths before(after) shell crossing are $\bar{l}_{AB}(l_{AB})$. Proper times before(after) shell crossing are $\bar{\tau}_{AB}(\tau_{AB})$. The coordinate time at which the shell crosses point B is different in both metrics. t_c in metric of mass $M - \delta M$ and $\bar{t}_c$ in the metric of mass M.

In this section we discuss how a similar effect can be also seen in traditional interferometers such as LIGO. We assume the geometry of the spacetime is governed by the SN progenitor star. Assuming the star is not rotating very rapidly and thus the surrounding spacetime is parametrized by the Schwarzschild metric. In the calculations we will need two Schwarzschild metrics. One with mass M and the other with mass $M - \delta M$, where δM will be the energy carried by the neutrinos after the SN explosion. The metric with mass M will have a bar over its coordinates and the one with $M - \delta M$ will not.

$$d\bar{s}^2 = \bar{g}_{\mu\nu} d\bar{x}^\mu d\bar{x}^\nu = -\left(1 - \frac{2M}{r}\right) d\bar{t}^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2. \tag{4}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\left(1 - \frac{(M - \delta M)}{r}\right) dt^2 + \left(1 - \frac{2(M - \delta M)}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2. \tag{5}$$

We are working in units with $G = c = 1$. Notice that the time component of the metric is different in both geometries whereas the radial component is the same as it corresponds to the radius of a two sphere separating the two geometries. Since we are describing the shell as a delta function travelling at roughly the speed of light it will follow a null geodesic. The null vector of the shell can be written in both metrics as follows[1]

$$k^\mu = \left(g_{tt}^{-1}, 1, 0, 0\right) \tag{6}$$

and similarly for $\bar{k}^\mu$. Since we expect the interferometer points (denoted by the points A, B, C in figure 1) to be very far for the massive object we assume they have a negligible initial velocity.

$$\bar{u}^\mu = \left((-1/\bar{g}_{tt})^{\frac{1}{2}}, 0, 0, 0\right). \tag{7}$$

The r will be different for points A, B and C as shown in figure 1. After the shell has crossed the points will pick up a velocity v. We can calculate v using the IJC,

$$g_{\mu\nu} u^\mu k^\nu = \bar{g}_{\mu\nu} \bar{u}^\mu \bar{k}^\nu, \tag{8}$$

This expression comes from demanding the continuity of geodesics from one geometry to another and was used in [7] to calculate a similar effect. Throughout this calculation we assume that the interferometer is very far from the SNe. Thus we assume there are three small quantities, $\frac{M}{r_0}, \frac{\delta M}{r_0}, \frac{d}{r_0} \ll 1$. To simplify notation we will use the symbol $\mathcal{O}(r^{-1})$ to represent suppression by any one of the three small quantities. Substituting Eqs (6, 7) into Eq (8) gives the final vector for the interferometer, to order $\mathcal{O}(r_{crossing}^{-2})$,

$$u^\mu = \left((-1/g_{tt})^{\frac{1}{2}} \sqrt{1 - v^2 g_{tt}^{-1}}, v, 0, 0\right). \tag{9}$$

The change in velocity due a shell crossing is $v = -\frac{\delta M}{r_{crossing}} \bar{g}_{tt}^{-\frac{1}{2}}$. $r_{crossing}$ is a fixed distance at which the shell crosses a point. For A it is r_0, for B it is $r_0 + d$ and for C it is $r_0 + \mathcal{O}(d/r_0)$. Note that this is the proper velocity $\frac{dr}{d\tau}$. We will need the coordinate velocity for the our calculations which is given by

$$\begin{aligned} \frac{dr}{dt} &= \frac{dr}{d\tau}\frac{d\tau}{dt} = v g_{tt}^{\frac{1}{2}} = \frac{\delta M}{r_{crossing}} \bar{g}_{tt}^{-\frac{1}{2}} g_{tt}^{\frac{1}{2}} \\ &= -\frac{\delta M}{r_{crossing}} \left(1 + \frac{\delta M}{r_{crossing}}\right) \end{aligned} \tag{10}$$

IV. MEMORY EFFECT IN INTERFEROMETER

A. Distance between A and B

1. *Defining trajectories*

An interferometer can measure the displacement between geodesics. There are two ways to calculate this displacement. The first is to directly calculate the photon geodesics from the metric and find the difference between them.

[1] One could write this vector in different ways and still have it satisfy the null normalization condition, however, the fact the shell has some surface area with a *fixed* radius, the two vectors must have the same radial component and this enforces the given form of the time component.

The second is to integrate the geodesics deviation equation. In this paper we will use the first approach and calculate the trajectories of the interferometer points. We consider points A, B and C in figure 1 as the ends of the arms of the interferometer. We parametrize the trajectories of the points in a piecewise form. We will use the following terms to represent the acceleration and velocity of the points:

$$
\begin{aligned}
\bar{a}^{(A)}_{(2)} = \bar{a}^{(B)}_{(2)} = \bar{a}^{(C)}_{(2)} &= \frac{M}{r_0^2} \\
a^{(A)}_{(2)} = a^{(B)}_{(2)} = a^{(C)}_{(2)} &= \frac{M-\delta M}{r_0^2} \\
v^{(A)}_{(1)} = v^{(B)}_{(1)} = v^{(C)}_{(1)} &= \frac{\delta M}{r_0} \\
v^{(A)}_{(2)} = v^{(C)}_{(2)} &= \frac{\delta M^2}{r_0^2} \\
v^{(B)}_{(2)} &= \frac{\delta M^2}{r_0^2} - \frac{\delta M d}{r_0^2}.
\end{aligned} \tag{11}
$$

The notation is to take the superscript to be the label of the particle. The subscript represents the number of factors of r_0 the term is suppressed by, i.e $a^{(A)}_{(2)}$ is term of $\mathcal{O}(r_0^{-2})$. We will only keep terms up to order $\mathcal{O}(r_0^{-2})$. The trajectory for point A is

$$
\begin{aligned}
r_A(\bar{t}) &= r_0 - \frac{1}{2}\bar{a}^{(A)}_{(2)}\bar{t}^2, \qquad (\bar{t} \leq 0) \\
r_A(t) &= r_0 - \frac{1}{2}a^{(A)}_{(2)}t^2 - \left(v^{(A)}_{(1)} + v^{(A)}_{(2)}\right)t, \qquad (t > 0).
\end{aligned} \tag{12}
$$

- $t = 0$ is defined to be the time when the shell crosses point A and the distance $r_A(0) = r_0$. For $t < 0$ we assume there is no velocity and the only term that contributes to the trajectory is the acceleration of point A towards mass M.
- For $t > 0$ there is still the acceleration term but the mass is different since the shell has carried δM away.
- Along with that there is a term that corresponds to the velocity change (as we calculated from the IJC) coming from shell crossing.

The trajectory of point B is given by

$$
\begin{aligned}
r_B(\bar{t}) &= r_0 + d - \frac{1}{2}\bar{a}^{(B)}_{(2)}\bar{t}^2, \qquad (\bar{t} \leq \bar{t_c}) \\
r_B(t) &= r_0 + d - \frac{1}{2}\bar{a}^{(B)}_{(2)}t_c^2 - \left(\bar{a}^{(B)}_{(2)}t_c + v^{(B)}_{(1)} + v^{(B)}_{(2)}\right)(t - t_c) - \frac{1}{2}a^{(B)}_{(2)}(t-t_c)^2, \qquad (t > t_c).
\end{aligned} \tag{13}
$$

- For $\bar{t} < \bar{t}_c$ there is just the acceleration, $\bar{a}^{(B)}_{(2)}$, of the point towards mass M.
- After $t > t_c$ the shell has crossed and now there is a change in distance in the time t_c that the point is accelerating with $a^{(B)}_{(2)}$ towards $M - \delta M$.[2] This is the second term in the expression.
- The $\bar{a}^{(B)}_{(2)}$ term corresponds to the acceleration of the point towards mass $M - \delta M$.
- The terms with $v^{(B)}_{(1)} + v^{(B)}_{(2)}$ are the velocities picked up from the IJC. The $\bar{a}^{(B)}_{(2)}t_c$ term is the velocity gained while accelerating in the metric of mass M.

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[2] Note that $\bar{t}_c$ and t_c are different since they are coordinates in two different metrics.

2. Relation between proper length and proper time

The interferometer infers the distance between two points by measuring the proper time it takes for a photon to go from one end of the arm to the other and come back. We will show that *this proper time τ is the same as twice the proper length l up to $\mathcal{O}(r_0^{-2})$, as long as their is no relative velocity between the two points* (which in our case are the ends of the arms of the interferometer). The most general expression for the proper distance for a stationary observer is

$$l_{AC} = \int \sqrt{g_{ab} dx^a dx^b}. \tag{14}$$

Before shell crossing, the only difference between two points is their acceleration. Which means we can choose a constant (coordinate) time slice where the relative velocity of the two points is the same up to $\mathcal{O}(r_0^{-2})$. In this section we calculate the proper distance before shell crossing $\bar{l}_{AB}$ between the two points A and B at time $\bar{t}$. Since we are only considering radial motion Eq (14) simplifies to

$$\bar{l}_{AB} = \int_{r_A(\bar{t})}^{r_B(\bar{t})} dr\ \bar{g}_{rr}^{\frac{1}{2}} = \int_{r_A(\bar{t})}^{r_B(\bar{t})} dr \left(1 + \frac{M}{r} + \frac{3}{2}\frac{M^2}{r^2} + \mathcal{O}((M/r)^3)\right). \tag{15}$$

In all the calculations we will only need terms of $\mathcal{O}(r_0^{-2})$. Thus we can use $r_B(\bar{t}) = r_0 + d$ and $r_A(\bar{t}) = r_0$.

$$\bar{l}_{AB} = \int_{r_A(\bar{t})}^{r_B(\bar{t})} dr\ \bar{g}_{rr}^{\frac{1}{2}} = d + M \ln\left(\frac{r_0 + d}{r_0}\right) - \frac{3}{2} M^2 \left(\frac{1}{r_0 + d} - \frac{1}{r_0}\right) \tag{16}$$

This equation for $\bar{l}_{AB}$ is a general expression up to $\mathcal{O}(r_0^{-2})$. It simplifies to

$$\bar{l}_{AB} = d\left(1 + \frac{M}{r_0} - \frac{1}{2}\frac{Md}{r_0^2} + \frac{3}{2}\frac{M^2}{r_0^2}\right). \tag{17}$$

We calculate the proper time $\bar{\tau}_{AB}$ in the appendix and notice that indeed $2\bar{l}_{AB} = \bar{\tau}_{AB}$.

After shell crossing the two points will have no relative velocity between them (schematic diagram of the velocities is shown in figure 4) up to order $\mathcal{O}(r_0^{-2})$. Thus for the after shell crossing scenario we can also calculate the proper length l_{AB} and know that it will be half the proper time τ_{AB}. First lets show that there is no relative velocity between A and B explicitly.

3. Relative velocity between A and B

We have assumed point A has velocity zero at the time of shell crossing. After the shell crosses point A, it will pick up a velocity $\frac{\delta M}{r_0}$. In the time the shell takes to go from A to B, A will also gain a velocity from its acceleration towards the mass $M - \delta M$. The magnitude of the velocity gain from acceleration is $\frac{M-\delta M}{r_0^2} d$. The final velocity of A at time t_c is, therefore, given by

$$v_A(t = t_c) = \frac{\delta M}{r_0}\left(1 + \frac{\delta M}{r_0}\right) + \frac{M - \delta M}{r_0^2} t_c. \tag{18}$$

Similarly, point B will gain a velocity from the acceleration towards mass M with magnitude $\frac{M}{r_0^2} d$. The velocity change from shell crossing is $\frac{\delta M}{r_0 + d}$. The final velocity of point B at time d is

$$v_B(t = t_c) = \frac{\delta M}{r_0 + d}\left(1 + \frac{\delta M}{r_0}\right) + \frac{M}{(r_0 + d)^2} t_c. \tag{19}$$

We see that v_A is the same as v_B to $\mathcal{O}(r_0^{-2})$. This also implies that there shouldn't be any change in distance that grows with time at order $\mathcal{O}(r_0^{-2})$.

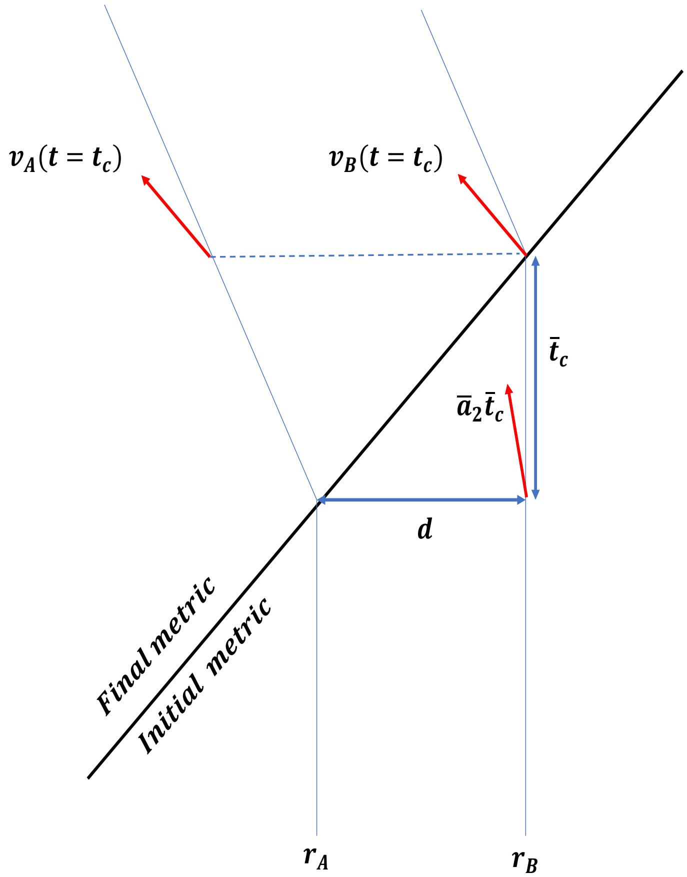

FIG. 4. Schematic of a spacetime diagram showing what happens to the velocities around shell crossing. The colour of the lines is defined in the same way as is done in figure 3. The new red lines represent the velocity of points A and B. We show in Eqs (18, 19) that the velocities of both points A and B are the same after shell crossing at time t_c. This comes from the fact that point B is accelerating for some time in the metric of mass M.

The memory effect, Δl_{AB} is the difference between l_{AB} and $\bar{l}_{AB}$. In the appendix we have the full calculation of the proper length l_{AB}, here we quote the resulting memory effect.

$$\Delta l_{AB} \equiv l_{AB} - \bar{l}_{AB} = -\frac{\delta M^2 d}{r_0^2}. \tag{20}$$

We find that indeed there is no time dependence in the change in proper length at $\mathcal{O}(r_0^{-2})$. Next we calculate an effect which is much more interesting since it grows with time.

B. Distance between A and C

In this subsection we calculate the proper distance change between A and C. First lets write down the trajectory equation for point C

$$\begin{aligned} r_C(\bar{t}) &= d - \frac{1}{2}\bar{a}^{(C)}_{(2)}\bar{t}^2, \qquad (\bar{t} \leq 0) \\ r_C(t) &= d - \frac{1}{2}a^{(C)}_{(2)}t^2 - \left(v^{(C)}_{(1)} + v^{(C)}_{(2)}\right)t, \qquad (t > 0). \end{aligned} \tag{21}$$

From the geometry in figure 1 we see that only θ and r will change in the motion of A and C. Thus we can drop the ϕ dependence from the start and write the above integral as

$$l_{AC} = \int_{\theta_i}^{\theta_f} d\theta\ g_{\theta\theta}^{\frac{1}{2}} \left(1 + \underbrace{(g_{rr}/g_{\theta\theta})(dr/d\theta)^2}_{T3} \right)^{\frac{1}{2}} . \tag{22}$$

where $\theta_f = d/r_0$ and $\theta_i = 0$. To evaluate $T3$ we would need the geodesic equation for r as function of θ. Instead of evaluating it explicitly we note that $\frac{dr}{d\theta}$ will be of order d. Since $g_{\theta\theta} = \frac{1}{r_0^2} + \mathcal{O}(r_0^{-3})$, to leading order we can ignore the effect of $T3$. Integrating the first term gives

$$l_{AC} = r_C(t)(\theta_f - \theta_i) = r_C(t)\frac{d}{r_0}. \tag{23}$$

Before shell crossing the proper length $\bar{l}_{AC} = d + \mathcal{O}(r_0^{-3})$. Whereas after shell crossing, there will be an additional velocity kick term in the proper length, l_{AC}, which will give a time dependent term at $\mathcal{O}(r_0^{-2})$

$$l_{AC} = d - \frac{\delta M d}{r_0^2} t. \tag{24}$$

The difference in proper lengths will be

$$\Delta l_{AC} \equiv l_{AC} - \bar{l}_{AC} = -\frac{\delta M d}{r_0^2} t. \tag{25}$$

V. OBSERVATION WITH SPACE-BASED INTERFEROMETERS

Taking the generic form of the change in distance as $\Delta L \sim \frac{\delta M d}{r_0^2} t$ we can estimate the distance a SN would have to be from the interferometer to have an observable change in strain, which is a unitless number quantifying the amount of space-time distortion.

$$h \sim \frac{\Delta L}{d} \sim \frac{\delta M t}{r_0^2} . \tag{26}$$

Since our strain grows linearly with time, we do not expect detections from ground based experiments as for those setups the three points A, B, C cannot remain in free fall for a long enough amount of time such that the signal builds up to an observable value.

If we plot our effect on the strain-frequency diagram [8] that is usually used to compare different interferometers, it will be a 45-degree line. Thus the first point at which the sensitivity curve of a device crosses with a 45-degree line will give the best chance for our effect being detected. In all these estimations, we take δM to be a faction of a solar mass, and take the corresponding Schwarzschild radius to be 1 km for simplicity.

For LISA, the best observing frequency is $\sim 0.5 \times 10^{-2} Hz$ with a sensitivity in strain $\sim 10^{-21}$. Using Eq. (26), we can solve for the distance to the SN, r_0, for our effect to be detectable.

$$r_0 = \left(\frac{\delta M}{h} t \right)^{\frac{1}{2}} \tag{27}$$

$$= \left(\frac{1\ km}{10^{-21}} \times 10^7\ km \right)^{\frac{1}{2}} \approx 10^{14}\ km = 10\ ly. \tag{28}$$

This is clearly too close. It has been estimated that only once in 10^8 years will a SN go off within a distance of 30 ly [9].[3] By a naive volume scaling, an explosion within 10 ly only occurs once every billion years.

[3] And if that happens, it might kill us.

If we look at the Big Bang Observer (BBO) instead, the best observing frequency is $\sim 0.5Hz$ with a sensitivity in strain $\sim 10^{-24}$. First of all, this frequency range does not have as many background signals from compact binaries, making it a much better device to measure our effect. The improved sensitivity gives a value for r_0 of $\sim 100\ ly$. This is a factor of 10^3 increase in the volume for detectable events, thus improves the expectation of one SN that is within $10\ ly$ to occur in less than a million years . That is unfortunately still a long shot.

In this type of simple estimation, we cannot go lower in the frequency. The exact duration of the neutrino-shell passage is not known, but we do no expect it to be much less than a second. Thus for higher frequencies, the co-dimension-one delta function approximation breaks down, and the effect will be weaker than Eq. (26).

Finally, we expect 2 to 3 SN per century in our galaxy and we can assume that the next SN would be at a distance comparable to the galactic diameter of $\sim 10^5\ ly$. If we are going to measure such effect at $1\ Hz$, Again using Eq. (26), we find

$$h = \frac{1\ km \times\ (3 \times 10^8\ m)}{(10^5\ ly)^2} \sim 10^{-30}\ . \tag{29}$$

This requires a measurement of the strain that is six orders of magnitude better than BBO and is not yet achievable by interferometers that are currently being planned.

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Appendix A: Relating proper time and proper distance before shell crossing

1. Proper time for photon to go between A and B before shell crossing

From the metric, we can set $ds = 0$, for the photon to get

$$\int_t^{t+t_1} dt = \int_{r_A(t)}^{r_B(t+t_1)} \frac{dr}{(1-\frac{2M}{r})} \tag{A1}$$

Integrating this gives

$$t_1 = r_B(t+t_1) - r_A(t) + 2M \ln\left(\frac{r_B(t+t_1)-2M}{r_A(t)-2M}\right) \tag{A2}$$

Now we can expand the log using $\ln(x) \approx (x-1) - \frac{1}{2}(x-1)^2 + ...$ For us $x = \frac{r_B(t+t_1)-2M}{r_A(t)-2M}$ Thus $x - 1 = \frac{r_B(t+t_1)-r_A(t)}{r_A(t)-2M} = \frac{r_B(t+t_1)-r_A(t)}{r_A(t)}\left(1-\frac{2M}{r_A(t)}\right)^{-1}$ and we get

$$2M \ln\left(\frac{r_B(t+t_1)-2M}{r_A(t)-2M}\right) = 2M\left(\frac{r_B(t+t_1)-r_A(t)}{r_A(t)}\left(1+\frac{2M}{r_A(t)}\right) - \frac{1}{2}\left(\frac{r_B(t+t_1)-r_A(t)}{r_A(t)}\right)^2\left(1-\frac{2M}{r_A(t)}\right)^{-2}\right) \tag{A3}$$

Since we only want to keep terms that are $\mathcal{O}(r_0^{-2})$ we can replace $\frac{M}{r_A(t)}$ with $\frac{M}{r_0}$. Thus Eq (A2) becomes

$$\begin{aligned} t_1 &= (r_B(t+t_1) - r_A(t))\left(1+\frac{2M}{r_0}\left(1+\frac{2M}{r_0}-\frac{1}{2}\frac{r_B(t+t_1)-r_A(t)}{r_0}\right)\right) \\ &= (r_B(t+t_1) - r_A(t))\left(1+\frac{2M}{r_0}+\frac{4M^2-M(r_B(t+t_1)-r_A(t))}{r_0^2}\right) \end{aligned} \tag{A4}$$

to leading order $r_B(t+t_1) - r_A(t) = d$

$$t_1 = (r_B(t+t_1) - r_A(t))\left(1+\frac{2M}{r_0}+\frac{4M^2-Md}{r_0^2}\right) \tag{A5}$$

$$r_B(t+t_1) - r_A(t) = d - \frac{1}{2}\frac{M}{r_0^2}(t+d)^2 + \frac{1}{2}\frac{Mt^2}{r_0^2} = d\left(1-\frac{1}{2}\frac{Md}{r_0^2}-\frac{Mt}{r_0^2}\right) \tag{A6}$$

Therefore t_1 becomes

$$t_1 = d\left(1+\frac{2M}{r_0}+\frac{4M^2}{r_0^2}-\frac{3}{2}\frac{Md}{r_0^2}-\frac{Mt}{r_0^2}\right) \tag{A7}$$

Now we can compute t_2.

$$t_2 = (r_B(t+t_1) - r_A(t+t_1+t_2))\left(1+\frac{2M}{r_0}+\frac{4M^2-M(r_B(t+t_1)-r_A(t+t_1+t_2))}{r_0^2}\right) \tag{A8}$$

$$\begin{aligned} r_B(t+t_1) - r_A(t+t_1+t_2) &= d - \frac{1}{2}\frac{M}{r_0^2}(t+d)^2 + \frac{1}{2}\frac{M}{r_0^2}(t+2d)^2 \\ &= d\left(1+\frac{Mt}{r_0^2}+\frac{3}{2}\frac{Md}{r_0^2}\right) \end{aligned} \tag{A9}$$

Thus t_2 becomes

$$\begin{aligned} t_2 &= d\left(1+\frac{Mt}{r_0^2}+\frac{3}{2}\frac{Md}{r_0^2}\right)\left(1+\frac{2M}{r_0}+\frac{4M^2-Md}{r_0^2}\right) \\ &= d\left(1+\frac{2M}{r_0}+\frac{4M^2}{r_0^2}+\frac{1}{2}\frac{Md}{r_0^2}+\frac{Mt}{r_0^2}\right) \end{aligned} \tag{A10}$$

Summing t_1 and t_2

$$t_1+t_2 = 2d\left(1+\frac{2M}{r_0}+\frac{4M^2}{r_0^2}-\frac{1}{2}\frac{Md}{r_0^2}\right) \tag{A11}$$

The proper time measured is

$$\begin{aligned} \tau &= \left(1-\frac{2M}{r}\right)^{\frac{1}{2}}(t_1+t_2) \\ &= 2d\left(1-\frac{M}{r}-\frac{1}{2}\frac{M^2}{r_0^2}\right)\left(1+\frac{2M}{r_0}+\frac{4M^2}{r_0^2}-\frac{1}{2}\frac{Md}{r_0^2}\right) \\ &= 2d\left(1+\frac{M}{r_0}+\frac{3}{2}\frac{M^2}{r_0^2}-\frac{1}{2}\frac{Md}{r_0^2}\right) \end{aligned} \tag{A12}$$

Appendix B: Proper length after shell crossing

In this section we calculate the proper length between A and B after shell crossing. Integrating Eq (14) with the trajectories for $r_B(t)$ and $r_A(t)$ gives

$$l_{AB} = r_B(t) - r_A(t) + (M - \delta M)\underbrace{\ln\left(\frac{r_B(t)}{r_A(t)}\right)}_{T1} \underbrace{- \frac{3}{2}(M-\delta M)^2\left(\frac{1}{r_B(t)} - \frac{1}{r_A(t)}\right)}_{T2} \tag{B1}$$

Thus we see the only new thing to compute is $r_B(t) - r_A(t)$. Lets expand these terms individually, starting with $T1$

$$\begin{aligned}
\ln\left(\frac{r_B(t)}{r_A(t)}\right) &= \ln\left(\frac{(r_B - r_0) + r_0}{(r_A - r_0) + r_0}\right) \\
&= \ln\left(\left(1 + \frac{r_B - r_0}{r_0}\right)\left(1 + \frac{r_A - r_0}{r_0}\right)^{-1}\right) \\
&= \ln\left(1 + \frac{r_B - r_A}{r_0} - \frac{(r_B - r_0)(r_A - r_0)}{r_0^2} - \left(\frac{r_A - r_0}{r_0}\right)^2\right).
\end{aligned} \tag{B2}$$

Note that $r_A - r_0 = \mathcal{O}(r_0^{-1})$ and we are only interested in terms up to $\mathcal{O}(r_0^{-2})$ thus we simplify the above equation to

$$\ln\left(\frac{r_B(t)}{r_A(t)}\right) = \ln\left(1 + \frac{r_B - r_A}{r_0}\right). \tag{B3}$$

Now we look ahead to $r_B(t) - r_A(t)$ in Eq (B7). We will need to keep terms up to $\mathcal{O}(r_0^{-1})$ in $r_B(t) - r_A(t)$. Thus we expand the log to get

$$\ln\left(\frac{r_B(t)}{r_A(t)}\right) = \frac{r_B - r_A}{r_0} - \frac{1}{2}\left(\frac{r_B - r_A}{r_0}\right)^2 = \frac{d}{r_0} + \frac{\delta M d}{r_0^2} - \frac{1}{2}\frac{d^2}{r_0^2} \tag{B4}$$

Evaluating $T2$ to $\mathcal{O}(r_0^{-2})$ is straightforward

$$-\frac{3}{2}(M-\delta M)^2\left(\frac{1}{r_B(t)} - \frac{1}{r_A(t)}\right) = \frac{3}{2}\frac{(M-\delta M)^2 d}{r_0^2} \tag{B5}$$

Putting in the expansion of $T1$ and $T2$ into l_{AB}

$$l_{AB} = r_B(t) - r_A(t) + \frac{(M-\delta M)d}{r_0} + \frac{\delta M(M-\delta M)d}{r_0^2} - \frac{1}{2}\frac{d^2(M-\delta M)}{r_0^2} + \frac{3}{2}\frac{(M-\delta M)^2 d}{r_0^2} \tag{B6}$$

Now we can compute $r_B(t) - r_A(t)$.

$$\begin{aligned}
r_B(t) - r_A(t) &= d - \frac{1}{2}\frac{M}{r_0^2}t_c^2 - \left(\frac{M}{r_0^2}t_c + \frac{\delta M}{r_0} + \frac{\delta M^2}{r_0^2} - \frac{\delta M d}{r_0^2}\right)(t - t_c) - \frac{1}{2}\frac{(M-\delta M)}{r_0^2}(t-t_c)^2 \\
&\quad + \frac{1}{2}\frac{(M-\delta M)t^2}{r_0^2} + \left(\frac{\delta M}{r_0} + \frac{\delta M^2}{r_0^2}\right)t \\
&= d + \frac{\delta M t_c}{r_0} + \frac{\delta M^2 d}{r_0^2} - \frac{1}{2}\frac{\delta M d^2}{r_0^2} \\
&= d + \frac{\delta M t_c}{r_0} + \frac{\delta M^2 d}{r_0^2} - \frac{1}{2}\frac{\delta M d^2}{r_0^2}
\end{aligned} \tag{B7}$$

Finally we get the expression for l_{AB}

$$l_{AB} = d + \frac{\delta M t_c}{r_0} + \frac{\delta M^2 d}{r_0^2} - \frac{1}{2}\frac{\delta M d^2}{r_0^2} + \frac{(M-\delta M)d}{r_0} + \frac{\delta M(M-\delta M)d}{r_0^2} - \frac{1}{2}\frac{d^2(M-\delta M)}{r_0^2} + \frac{3}{2}\frac{(M-\delta M)^2 d}{r_0^2} \tag{B8}$$

Substituting in for $t_c = d\left(1 + \frac{2(M-\delta M)}{r_0}\right)$ gives

$$\begin{aligned}
l_{AB} &= d + \frac{\delta M d}{r_0} + \frac{2\delta M(M-\delta M)d}{r_0^2} + \frac{\delta M^2}{r_0^2}d - \frac{1}{2}\frac{\delta M d^2}{r_0^2} + \frac{(M-\delta M)d}{r_0} + \frac{\delta M(M-\delta M)d}{r_0^2} - \frac{1}{2}\frac{d^2(M-\delta M)}{r_0^2} \\
&\quad + \frac{3}{2}\frac{M^2 d}{r_0^2} - \frac{3M\delta M d}{r_0^2} + \frac{3}{2}\frac{\delta M^2 d}{r_0^2} \\
&= d + \frac{Md}{r_0} - \frac{1}{2}\frac{\delta M^2 d}{r_0^2} + \frac{3}{2}\frac{M^2 d}{r_0^2} - \frac{1}{2}\frac{Md^2}{r_0^2}
\end{aligned} \tag{B9}$$

[1] B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. **116**, 061102 (2016).
[2] D. Christodoulou, Phys. Rev. Lett. **67**, 1486 (1991).
[3] K. S. Thorne, Phys. Rev. D **45**, 520 (1992).
[4] D. R. Lorimer, *Neutron Stars and Pulsars: Challenges and Opportunities after 80 years*, IAU Symposium, **291**, 237 (2013), arXiv:1210.2746.
[5] M. Malek *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **90**, 061101 (2003).
[6] W. Israel, Nuovo Cim. **B44S10**, 1 (1966).
[7] D. Kodwani, U.-L. Pen, and I.-S. Yang, Phys. Rev. D **93**, 103006 (2016).
[8] C. J. Moore, R. H. Cole, and C. P. L. Berry, Class. Quant. Grav. **32**, 015014 (2015), arXiv:1408.0740 [gr-qc].
[9] J. R. Ellis and D. N. Schramm, (1993), arXiv:hep-ph/9303206 [hep-ph].
[10] R. Narayan, Philosophical Transactions: Physical Sciences and Engineering **341**, 151 (1992).
[11] K. D. Olum, E. Pierce, and X. Siemens, Phys. Rev. **D88**, 043005 (2013), arXiv:1305.3881 [gr-qc].
[12] G. C. Bower, A. Deller, P. Demorest, A. Brunthaler, R. Eatough, H. Falcke, M. Kramer, K. J. Lee, and L. Spitler, Astrophys. J. **780**, L2 (2014), arXiv:1309.4672 [astro-ph.GA].