Supernova energy measurement in gravitational wave detectors

Darsh Kodwani,[1, 2, *] Ue-Li Pen,[1, 3, 4, 5, †] and I-Sheng Yang[1, 5, ‡]

[1] *Canadian Institute of Theoretical Astrophysics, 60 St George St, Toronto, ON M5S 3H8, Canada.*
[2] *University of Toronto, Department of Physics, 60 St George St, Toronto, ON M5S 3H8, Canada.*
[3] *Canadian Institute for Advanced Research, CIFAR program in Gravitation and Cosmology.*
[4] *Dunlap Institute for Astronomy & Astrophysics, University of Toronto, AB 120-50 St. George Street, Toronto, ON M5S 3H4, Canada.*
[5] *Perimeter Institute of Theoretical Physics, 31 Caroline Street North, Waterloo, ON N2L 2Y5, Canada.*

We calculate the gravitational memory effect when a spherically symmetric shell of energy passes through a spacetime region. In particular, this effect includes a longitudinal component, such that two radially separated geodesics pick up a relative velocity proportional to their separation. Such a measurement will allow us to obtain the total energy released by a supernova explosion in the form of neutrinos. We study the possibility to measure such an effect by space-based interferometers such as LISA and BBO, and also by astrophysical interferometers such as pulsar scintillometry.

I. INTRODUCTION AND SUMMARY

The recent detection of gravitational waves [1] has proved that gravitational waves leave an oscillating pattern in the amplitude of waveforms measured at detectors such as LIGO. It is also known that this is not the only effect that is potentially detectable. Strong gravitational waves imply a large flow of energy. Just like any other flow of energy, it leads to a gravitational memory effect [2, 3].

The memory effect discussed in [2, 3] causes permanent relative displacements between geodesics. It contains only transverse-traceless components and can leave an imprint in an interferometer. In this paper, we will introduce another memory effect that is different in two ways:

- It has a longitudinal component. The transverse-traceless limitation only applies to freely-propagating changes of the metric (i.e gravitational waves). While coupled to matter, which often have longitudinal (density) waves, it is natural to have an accompanying longitudinal change in the metric.

- Instead of displacements, it causes permanent changes in relative velocities between geodesics, with magnitudes proportional to their separations.

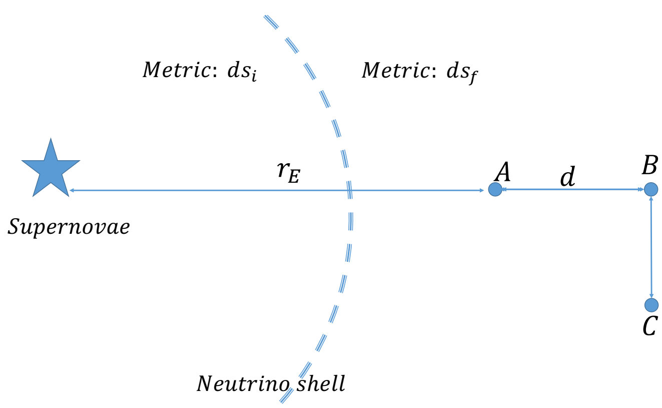

FIG. 1. Schematic of the effect being considered by a neutrino shell passing through the interferometer. The points A, B, C represent ends of the interferometer of arm length d. The three points A, B and C will pick up velocities v_A, v_B and v_C respectively after the shell crosses them. They are all different since they cross the shell at different locations.

In terms of the dynamics, a change in velocity is higher order than a change in distance. This however does not mean that our effect is harder to measure. The conventional gravitational memory effect needs a physical event that significantly breaks spherical symmetry to be detectable whereas our effect does not and therefore it can occur more generally. In addition, a change in velocity implies a distance change that grows in time even after the initial effect. That is an advantage for some detection methods.

We will present a simple and natural occurrence of this effect. During a supernova explosion (SNe), most of the energy is released in a highly relativistic shell of neutrinos. As illustrated in Fig.1, when a neutrino shell passes through, the three free-falling points A, B and C, will pick up different velocities due to the change of geometry. If AB and BC are two arms of an interferometer, we will see a time-dependent change after the shell passes through. Note that this is purely a geometric change which happens even without the three actual objects. We will demonstrate this by showing that it is possibility to detect the same effect using pulsar interferometry, in which two parallel light rays get different time-delays after being hit by a neutrino shell.

The SKA telescope is expected to detect thousands of pulsars, [4], and one of them might be close enough to a SN explosion to provide us with a good estimation of the

* dkodwani@physics.utoronto.ca
† pen@cita.utoronto.ca
‡ isheng.yang@gmail.com

total energy in the neutrino shell of the explosion. Currently there is no other way to obtain such information. Thus in addition to direct neutrino detections such as in Super-Kamiokande [5], this memory effect can provide a new handle on constraining the explosion mechanism.

The rest of the paper is organized as follows. In section II we derive the change in velocities with the Israel Junction Conditions (IJC) [6], treating the neutrino shell as a co-dimension-one delta function. In section III we discuss potential observation of such an effect by experiments that are currently being planned such as LISA and BBO. The final section IV discusses how an astrophysical interferometer formed by pulsar scintillometry can measure the same effect.

II. CHANGE IN RELATIVE VELOCITY

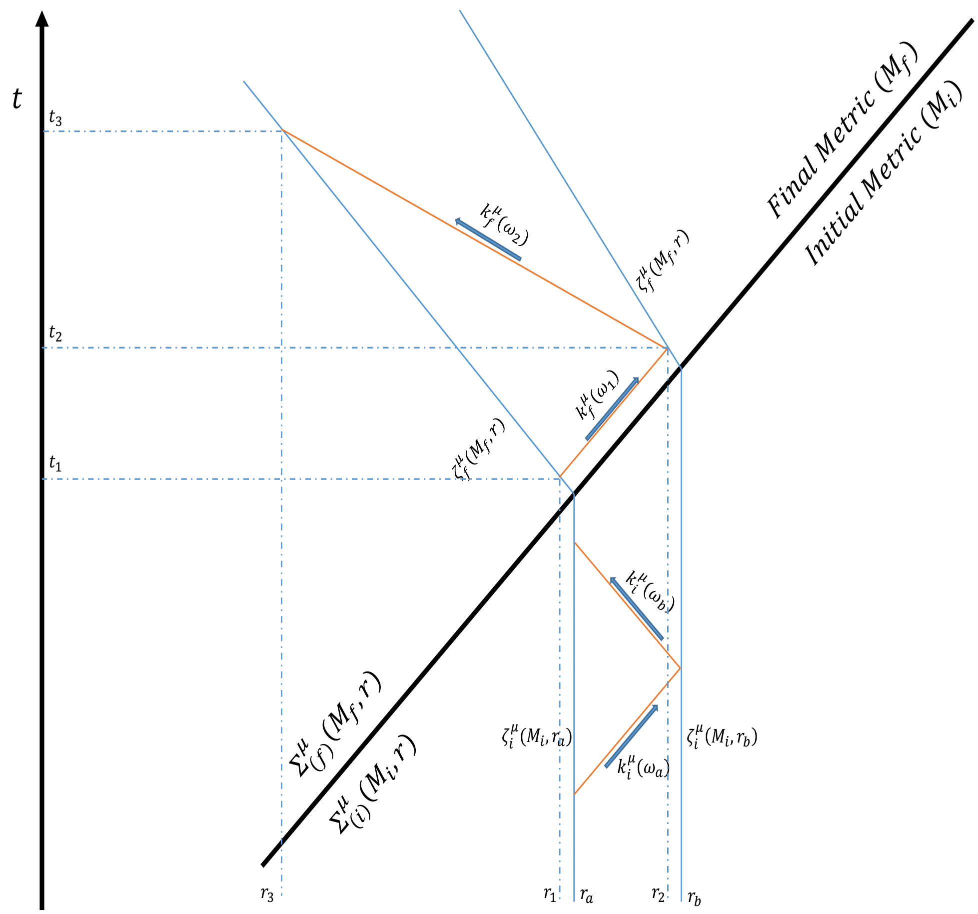

FIG. 2. Schematic of a spacetime diagram showing the paths of photons that are used in the interferometer to measure the change in the length of the interferometer arms. The orange lines represent the photon trajectories. The solid blue lines represent the trajectories of the two points A and B in figure 1. The dark black line is the null trajectory of the neutrino shell.

We assume the geometry of the spacetime is governed by the SN progenitor star. We assume the star is not rotating very rapidly and thus the surrounding spacetime is parametrized by the Schwarzschild metric,

$$ds_i^2 = -\left(1 - \frac{2M_i}{r}\right) dt_i^2 + \left(1 - \frac{2M_i}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2, \tag{1}$$

where are working in units with $G = c = 1$. After the star explodes, it emits a shell of neutrinos carrying energy δM. The metric surrounding the original star then changes since the mass has changed. In particular the i index used above to denote the "initial" metric is replaced by f to denote the "final" metric with a smaller mass $M_f = M_i - \delta M$. Notice that the time component of the metric is different in both geometries whereas the radial component is the same as it corresponds to the radius of a two sphere separating the two geometries. Since we are describing the shell as a delta function travelling at roughly the speed of light it will follow a null geodesic. The null vector of the shell can be written in both metrics as follows[1]

$$\Sigma^\mu_{(i,f)} = \left(\left(1 - \frac{2M_{(i,f)}}{r}\right)^{-1}, 1, 0, 0 \right). \quad (2)$$

We choose a coordinate chart in which the interferometer points are initially at rest (denoted by the points A, B, C in figure 1) and are described by the following geodesic

$$\zeta^\mu_i = \left(\left(1 - \frac{2M_i}{r}\right)^{-\frac{1}{2}}, 0, 0, 0 \right). \quad (3)$$

The r will be different for points A, B and C as shown in figure 1. After the shell has crossed the points we expect ζ to have a velocity component. This can be found using the IJC,

$$g_{\mu\nu} \Sigma^\mu_{(i)} \zeta^\nu_{(i)} = \hat{g}_{\mu\nu} \Sigma^\mu_{(f)} \zeta^\nu_{(f)}, \quad (4)$$

where $\hat{g}_{\mu\nu}$ represents the metric of the "final" spacetime and $g_{\mu\nu}$ is the metric of the initial spacetime. This expression comes from demanding the continuity of geodesics from one geometry to another and was used in [7] to calculate a similar effect. Substituting Eqs (2, 3) into Eq (4) gives the final vector for the interferometer, to leading order,

$$\zeta^\mu_f = \left(\left(1 - \frac{2M_f}{r_{crossing}}\right)^{-\frac{1}{2}}, -\frac{\delta M}{r_{crossing}}, 0, 0 \right). \quad (5)$$

Note that $\frac{\delta M}{r_{crossing}}$ is a coordinate velocity and to convert to proper velocity it will need to be multiplied by a factor of $\left(1 - \frac{2M}{r_{crossing}}\right)^{\frac{1}{2}}$. Since this is a higher order correction it is not important here. $r_{crossing}$ is a fixed distance at which the shell crosses a point. For A it is r_E, for B it is $r_E + d$ and for C it is $\approx r_E + d$ as well. [2].

A. The effect of change in velocity on photons

An interferometer works by sending photons from a source to another point that reflects them back to the source. Usually this is done using two arms of equal length so that if the length of one of the arms increases the photons that leave the source will not arrive back in phase with the photons that went along the other arm (as there will be a path difference in their travel distance) and it is this phase change that is observed in an interferometer. There is an equivalent way of calculating this effect which is to count the number of wavelengths of the photon along its total trajectory. In general, a photon trajectory, k^μ, is of the form

$$k^\mu = (E, p_r, p_\theta, p_\phi), \quad (6)$$

for a photon moving in a radial direction $p_r = \frac{1}{\lambda}$ where λ is the wavelength of the photon. A photon moving in a Schwarzschild geometry will have the following four vector

$$k^\mu(\omega_\infty, M, r) = \omega_\infty \left(\frac{1}{1 - \frac{2M}{r}}, \sqrt{1 - \frac{b^2}{r^2}\left(1 - \frac{2M}{r}\right)}, \frac{b}{r^2}, 0 \right), \quad (7)$$

ω_∞ is a normalization factor that corresponds to the frequency of the photon as observed by an observer at rest sitting infinitely far away and b represents the impact parameter of the photon. Since we are in a situation where the photon is very far away from the supernova this photon vector is well approximated by

$$k^\mu(\omega_\infty, M, r) = \omega_\infty \left(\frac{1}{1 - \frac{2M}{r}}, 1, 0, 0 \right). \quad (8)$$

This photon has wavelength $\lambda_\infty = \frac{1}{\omega_\infty}$. As an illustration, we consider the effect a neutrino shell has on just one arm of an interferometer, i.e two free falling points separated by distance $r_a - r_b = d$,[3] as shown in the bottom part of figure 2. A photon leaves from r_a with four vector $k^\mu_i(\omega_a)$ and goes to point r_b. The frequency measured, ω_{O1}, by an observer at r_b of this photon is

$$\omega_{O1} = -g_{\mu\nu}(M_i, r_b) k^\mu(M_i, \omega_a, r_b) \zeta^\nu_i(M_i, r_b). \quad (9)$$

This must be the same as the frequency observed for a photon coming from the opposite direction which, in figure 2, is denoted by $k^\mu_i(\omega_b)$,

$$\omega_{O1} = -g_{\mu\nu}(M_i, r_b) k^\mu(M_i, \omega_b, r_b) \zeta^\nu_i(M_i, r_b). \quad (10)$$

Since the photon is travelling in the opposite direction in this case, the velocity component of the $k^\mu_i(\omega_b)$ will have the opposite sign

$$k^\mu(\omega_b) = \omega_b \left(\frac{1}{1 - \frac{2M}{r}}, -1, 0, 0 \right). \quad (11)$$

[1] One could write this vector in different ways and still have it satisfy the null normalization condition, however, the fact the shell has some surface area with a *fixed* radius, the two vectors must have the same radial component and this enforces the given form of the time component.

[2] The exact expression is $r_E \left(1 + \frac{2d}{r_E} + \frac{2d^2}{r_E^2}\right)^{\frac{1}{2}}$ but we are working to leading order so we can make the given approximation.

[3] This is representative of the two points A and B in figure 1 as the two points are assumed to be in line with the SN.

By equating Eq (9) and (10) we see $\omega_a = \omega_b$ and therefore the wavelengths of the photons must be the same. Since the distance between the two points, d, does not change, the total number of wavelengths traversed by the photons along there paths is a constant number, N_i, given by

$$N_i = \frac{d}{\lambda_a} = d\omega_a = d\omega_b \tag{12}$$

When a neutrino shell crosses the two points, they pick up a velocity as is shown in Eq (5). This means both, the frequency of the photons and the distance between the two points will change. Let's begin by calculating the change in the frequency. Consider the photon $k^\mu_f(\omega_1)$ emitted from a distance r_1, shown in the upper part of figure 2. An observer $O2$ at r_2 will measure the frequency of photon $k^\mu_f(\omega_1)$ to be

$$\begin{aligned}\omega_{O2} &= -g_{\mu\nu}(M_f, r_2)k^\mu(M_f, \omega_1, r_2)\zeta^\nu(M_f, r_2) \\ &= \left(1 + \frac{M_f + \delta M}{r_2} + \frac{2M_f\delta M}{r_2^2}\right)\omega_1. \end{aligned}\tag{13}$$

Similarly the photon $k^\mu_f(\omega_2)$ has the measured frequency

$$\begin{aligned}\omega_{O2} &= -g_{\mu\nu}(M_f, r_2)k^\mu(M_f, \omega_2, r_2)\zeta^\nu(M_f, r_2) \\ &= \left(1 + \frac{M_f - \delta M}{r_2} - \frac{2M_f\delta M}{r_2^2}\right)\omega_2. \end{aligned}\tag{14}$$

Combining these two equations gives, to second order in small quantities,

$$\omega_1 = \left(1 - \frac{2\delta M}{r_2} - \frac{4M_f\delta M - M_f^2 + \delta M^2}{r_2^2}\right)\omega_2. \tag{15}$$

The distance, d, will also change as both the points will have different velocities. Therefore the distance, d_1, traversed by photon $k^\mu_f(\omega_1)$ is

$$d_1 = r_2 - r_1 = d - \frac{\delta M}{r_b}t_2 + \frac{\delta M}{r_a}t_1. \tag{16}$$

We want to get everything in terms of one time t_1, so lets find t_2 in terms of t_1

$$t_2 = t_1 + d - \frac{\delta M}{r_a}t_1 + \frac{\delta M}{r_b}t_2 \tag{17}$$

$$t_2\left(1 - \frac{\delta M}{r_b}\right) = t_1 + d - \frac{\delta M t_1}{r_a} \tag{18}$$

which to leading order is

$$t_2 = t_1 + d + \mathcal{O}(r^{-1}). \tag{19}$$

and so d_1 becomes,

$$d_1 = d\left(1 - \frac{\delta M}{r_a} - \frac{\delta M t_1}{r_a^2} + \mathcal{O}(r_a^{-3})\right). \tag{20}$$

Therefore the number of wavelengths, N_{f1}, traversed by photon $k^\mu_f(\omega_1)$ is

$$N_{f1} = \frac{d_1}{\lambda_1} = \frac{d\left(1 - \frac{\delta M}{r_a} - \frac{\delta M t_1}{r_a^2}\right)}{\lambda_1} \tag{21}$$

So we see that there is a term that increases linearly with time and so the number of wavelengths increases linearly with time. Similarly we can calculate the number of wavelengths, N_{f2}, traversed by the photon $k^\mu_f(\omega_2)$ on its way back to r_3. The distance d_3 traveresed by the photon is

$$d_3 = r_2 - r_3 = d - \frac{\delta M}{r_a}t_3 + \frac{\delta M}{r_b}t_2 \tag{22}$$

Again lets get t_3 in terms of t_2 (which we know how to write in terms of t_1),

$$t_3 = t_2 + d + \frac{\delta M}{r_a}t_3 - \frac{\delta M}{r_b}t_2 \tag{23}$$

$$t_3\left(1 - \frac{\delta M}{r_a}\right) = t_2 + d - \frac{\delta M}{r_b}t_2 \tag{24}$$

and so to leading order

$$t_3 = t_2 + d = t_1 + 2d + \mathcal{O}(r^{-1}) \tag{25}$$

which is again expected. So d_3 becomes

$$\begin{aligned} d_3 &= d - \frac{\delta M d}{r_a} - \frac{\delta M t_1}{r_a} + \frac{\delta M}{r_b}t_1 \\ &= d\left(1 - \frac{\delta M}{r_a}\right) + \frac{\delta M d}{r_a^2}t_1 \end{aligned}\tag{26}$$

And the number of wavelengths N_{f2} is

$$N_{f2} = \frac{d\left(1 - \frac{\delta M}{r_a}\right) + \frac{\delta M d}{r_a^2}t_1}{\lambda_2} \tag{27}$$

where $\lambda_2 = \frac{1}{\omega_2}$.

B. Distance change in interferometer

We have seen in the previous section how the effect of the nuetrino shell crossing the points manifests itself to an interferometer. Here we expand on this calculation and apply it to a general interferometer like the one shown in figure 1.

The relative velocity between the ends of a horizontal interferometer, A and B, that is in radial alignment with the SN as shown in figure 1 is given by

$$\Delta L_{AB} = (v_A - v_B)t = d\frac{\delta M}{r_E^2}t + \mathcal{O}(r_E^{-3}) \ . \tag{28}$$

where v_A and v_B are the velocities picked up by points A and B after shell crossing and t is the time passed after shell crossing. Note that since the point A picks up a larger velocity toward the supernova, ΔL_{AB} is *increasing*.

The total velocities for points B and C have the same magnitudes but are pointing in slightly different directions. It is easy to work out the geometry to see that

$$\begin{aligned}\Delta L_{BC} &= -\left(\frac{\delta M}{r_E}\sin\theta\right)t \\ &= -d\frac{\delta M}{r_E^2}t + \mathcal{O}(r_E^{-3}) \ .\end{aligned} \tag{29}$$

Here $\theta \approx (d/r_E)$ is the angle between point B and C to the SN. Since they both fall toward the SN, they are getting closer to each other thus ΔL_{BC} is *decreasing*[4].

In summary, we have an interferometer whose one arm decreases in length while the other increases, resulting in a detectable change in the interference pattern.

ΔL_{AB}	ΔL_{BC}
$\frac{\delta M d}{r_E^2}t$	$-\frac{\delta M d}{r_E^2}t$

Note that for the conventional memory effect, the signal is maximized when the interferometer is face-on to the source. In our case, a face-on interferometer would get zero signal, since both arms will be decreasing in length. Our signal is maximized by having a longitudinal arm, in this case AB.

III. OBSERVATION WITH SPACE-BASED INTERFEROMETERS

Taking the generic form of the change in distance as $\Delta L \sim \frac{\delta M d}{r_E^2}t$ we can estimate the distance a SN would have to be from the interferometer to have an observable change in strain, which is a unitless number quantifying the amount of space-time distortion.

$$h \sim \frac{\Delta L}{d} \sim \frac{\delta M t}{r_E^2} \ . \tag{30}$$

Since our strain grows linearly with time, we do not expect detections from ground based experiments as for those setups the three points A, B, C cannot remain in free fall for a long enough amount of time such that the signal builds up to an observable value.

If we plot our effect on the strain-frequency diagram [8] that is usually used to compare different interferometers, it will be a 45-degree line. Thus the first point at which the sensitivity curve of a device crosses with a 45-degree line will give the best chance for our effect being detected. In all these estimations, we take δM to be a faction of a solar mass, and take the corresponding Schwarzschild radius to be 1 km for simplicity.

For LISA, the best observing frequency is $\sim 0.5 \times 10^{-2} Hz$ with a sensitivity in strain $\sim 10^{-21}$. Using Eq. (30), we can solve for the distance to the SN, r_E, for our effect to be detectable.

$$r_E = \left(\frac{\delta M}{h}t\right)^{\frac{1}{2}} \tag{31}$$

$$= \left(\frac{1\ km}{10^{-21}} \times 10^7\ km\right)^{\frac{1}{2}} \approx 10^{14}\ km = 10\ ly. \tag{32}$$

This is clearly too close. It has been estimated that only once in 10^8 years will a SN go off within a distance of 30 ly [9].[5] By a naive volume scaling, an explosion within 10 ly only occurs once every billion years.

If we look at the Big Bang Observer (BBO) instead, the best observing frequency is $\sim 0.5 Hz$ with a sensitivity in strain $\sim 10^{-24}$. First of all, this frequency range does not have as many background signals from compact binaries, making it a much better device to measure our effect. The improved sensitivity gives a value for r_E of $\sim 100\ ly$. This is a factor of 10^3 increase in the volume for detectable events, thus improves the expectation of one SN that is within 10 ly to occur in less than a million years . That is unfortunately still a long shot.

In this type of simple estimation, we cannot go lower in the frequency. The exact duration of the neutrino-shell passage is not known, but we do no expect it to be much less than a second. Thus for higher frequencies, the co-dimension-one delta function approximation breaks down, and the effect will be weaker than Eq. (30).

Finally, we expect 2 to 3 SN per century in our galaxy and we can assume that the next SN would be at a distance comparable to the galactic diameter of $\sim 10^5\ ly$. If we are going to measure such effect at 1 Hz, Again using

[4] This is important; if the distance ΔL_{BC} was also increasing, since the magnitude of the increase is the same as ΔL_{AB}, we would not see an effect since the photons in the interferometer arms would still have the same phase when they come back to point B.

[5] And if that happens, it might kill us.

Eq. (30), we find

$$h=\frac{1\ km\times\ (3\times10^{8}\ m)}{(10^{5}\ ly)^{2}}\sim10^{-30}\ . \tag{33}$$

This requires a measurement of the strain that is six orders of magnitude better than BBO and is not yet achievable by interferometers that are currently being planned.

IV. OBSERVATION WITH PULSAR SCINTILLATION

We learned from the previous section how difficult it is to measure the spacetime distortion. It is strongly suppressed by a factor of r_E^{-2}, which is proportional to the energy density of the shell when it reaches us. If we can have an interferometer much closer to the SN, the signal will be larger.

In this final section we discuss a possibility to do just that. It is known that the images of many astronomical bodies scintillate [10]. A general reason for scintillation is that due to scattering or lensing, we receive multiple light rays from the same objects. These light rays are very close to each other, so they cannot be individually resolved and have to interfere. The scintillation pattern we see is the time dependence of their interference. If we consider two light rays from a faraway pulsar which happen to pass by a SN progenitor, as illustrated in Fig.3, they can probe the spacetime distortion when it explodes.

The scintillation/interference pattern is directly related to the path lengths of these light rays. The change in such path length during a SN explosion has been worked out in [11]

$$\Delta t=2\delta M\left[\ln\left(1+\frac{t^{2}}{b^{2}}\right)-\frac{t^{2}}{b^{2}+t^{2}}\right]. \tag{34}$$

Here b is the impact parameter as shown in Fig.3, the shortest distance between the light ray and the SN. t is the proper time on earth, with $t=0$ the time we directly observe the SN explosion. δM is the total energy of the neutrino shell, and Δt is the resulting time shift. A photon which should have reached the earth at time t, will arrive earlier at $(t-\Delta t)$ instead.

When the separation between two light rays has a component in the radial direction from the SN, Δb, there will be a nonzero relative change between their path lengths.

$$(\Delta t|_b-\Delta t|_{b+\Delta b})\approx\frac{\partial\Delta t}{\partial b}\Delta b=-\frac{4\delta Mt^{4}}{b(b^{2}+t^{2})^{2}}\Delta b\ . \tag{35}$$

We can see that this effect grows from zero and approaches an asymptotic value,

$$(\Delta t|_b-\Delta t|_{b+\Delta b})\longrightarrow\frac{4\delta M\Delta b}{b}\ , \tag{36}$$

at a characteristic time scale given by b.

We estimate b by assuming that the next SN is somewhere near the galactic center. A sample of ~ 9000 pulsars from the SKA catalog in [4] shows that among those pulsars, the shortest b is about $10\ ly\sim10^{14}\ km$. Δb is related to the scattering-broadening of images. We use the data from [12] that was observed on a scattering screen near the galactic center. Scaling the frequency to $1\ GHz$ which is usually a good window to observe pulsar signals. We found that such a scattering screen can produce images separated by $\Delta b\sim 1000 A.U.\sim10^{10}\ km$. We again use $\delta M\sim1\ km$, and combining all these numbers we get $(\delta M\Delta b/b)\sim1\ m$. This is comparable to the wavelength at $1\ GHz$, thus making the change in interference pattern easy to detect. Therefore, if we can monitor the pulsar scintillation pattern over ten years after a SN explosion we should see an order one change in the scintillation pattern predicted by Eq. (35).

ACKNOWLEDGMENTS

This work is supported by the Canadian Government through the Canadian Institute for Advance Research and Industry Canada, and by Province of Ontario through the Ministry of Research and Innovation.

[1] B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. **116**, 061102 (2016).
[2] D. Christodoulou, Phys. Rev. Lett. **67**, 1486 (1991).
[3] K. S. Thorne, Phys. Rev. D **45**, 520 (1992).
[4] D. R. Lorimer, *Neutron Stars and Pulsars: Challenges and Opportunities after 80 years*, IAU Symposium, **291**, 237 (2013), arXiv:1210.2746.
[5] M. Malek *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **90**, 061101 (2003).
[6] W. Israel, Nuovo Cim. **B44S10**, 1 (1966).
[7] D. Kodwani, U.-L. Pen, and I.-S. Yang, Phys. Rev. D **93**, 103006 (2016).
[8] C. J. Moore, R. H. Cole, and C. P. L. Berry, Class. Quant. Grav. **32**, 015014 (2015), arXiv:1408.0740 [gr-qc].
[9] J. R. Ellis and D. N. Schramm, (1993), arXiv:hep-ph/9303206 [hep-ph].
[10] R. Narayan, Philosophical Transactions: Physical Sciences and Engineering **341**, 151 (1992).
[11] K. D. Olum, E. Pierce, and X. Siemens, Phys. Rev. **D88**, 043005 (2013), arXiv:1305.3881 [gr-qc].
[12] G. C. Bower, A. Deller, P. Demorest, A. Brunthaler, R. Eatough, H. Falcke, M. Kramer, K. J. Lee, and L. Spitler, Astrophys. J. **780**, L2 (2014), arXiv:1309.4672 [astro-ph.GA].

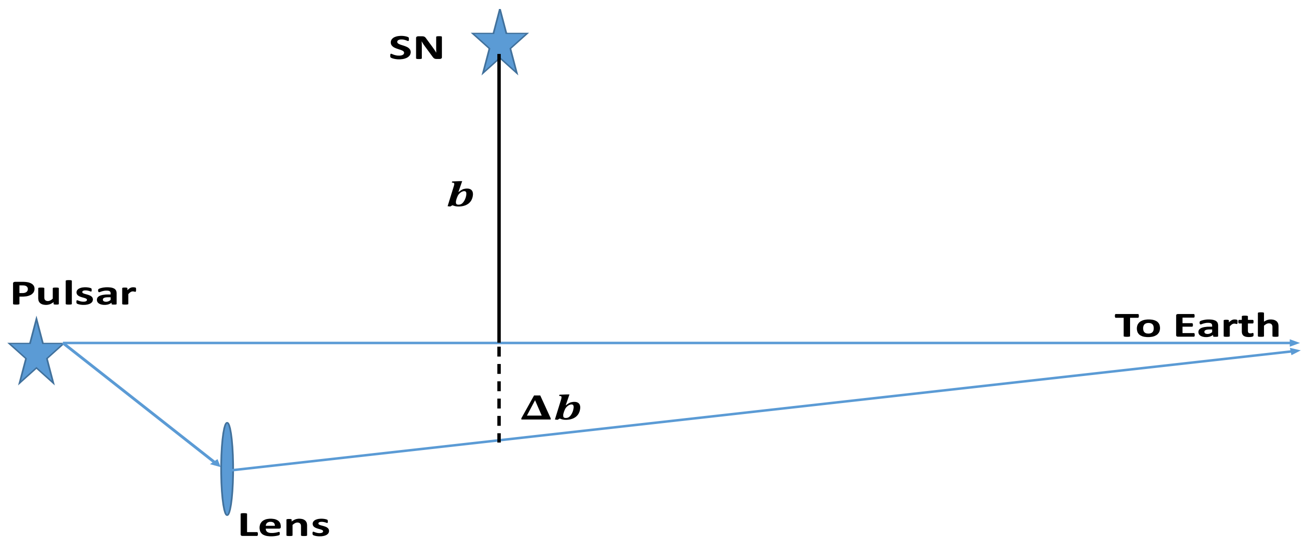

FIG. 3. Geometry of the astrophysical interferometer formed by pulsar scintillometry. Due to scattering or lensing, the image we see is an interference pattern of two light rays represented by the blue lines. If the separation of the two light rays has a component along the longitudinal (radial) direction from the SN, the spacetime distortion of the neutrino shell will change the interference pattern we see. We draw the lens to be behind the SN, but it could have been in front of it and the effect is the same.