Gravitational memory of matter shells

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We calculate the memory effect when a shell of matter crosses two freely falling points. The effect has a longitudinal component in addition to the usual transverse component carried by gravitational waves (Christodolou effect). Furthermore it is different from the Christodolou effect as the transverse component of this memory effect also has a term that grows with time. The calculations are performed with a physical setup in mind. We use a shell of neutrinos emitted by a supernova as our matter shell. Inspired by previous works [7, 11] we first calculate the effect this shell has on pulsar scintillation. Finally we calculate how the shell effects interferometers such as LIGO and whether such effects can be observed.

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I. INTRODUCTION AND SUMMARY

In recent papers [7, 11] we and Olum et al have calculated the effect a shell of matter can have on photon geodesics when it crosses them. In particular the effect is calculated for the physical scenario in which the photons are coming from a pulsar and the shell of matter is a shell of relativistic neutrinos coming from a nearby supernova. The physical change is in the path length of a single photon when there is a change in local geometry due to the shell crossing. Observationally this manifests itself as a shift in photon arrival times.

In this paper we extend this work to see what happens to multiple photon trajectories when a shell of matter crosses them. This situation arises naturally in pulsar scintillation where multiple photons from the pulsar arrive at an observer. The particular effect we calculate is the shift in two photon geodesics caused by the change in local geometry due to the shell crossing.

This effect is seen far away from the region of spacetime that is changing since the photons traverse a long distance before they arrive at the observer. It is natural to ask what happens if one was to make a local measurement of the change in geometry. A physical set-up that does precisely that is an interferometer. Thus we imagine having an interferometer in the place where the shell crosses the photons and calculate the effect of a shell crossing it.

We know from the recent detection of gravitational waves [1] that interferometers can provide great insight into the structure of local spacetime. Along with an oscillating change in length, gravitational waves also lead to a permanent displacement between two freely falling points. This is known as a memory effect[2, 3]. The memory effect caused by gravitational waves only has transverse traceless components. The memory effect we calculate in this paper will be different in two ways:

- It has a longitudinal component. The transverse-traceless limitation only applies to freely-propagating changes of the metric (i.e gravitational waves). While coupled to matter, which often have longitudinal (density) waves, it is natural to have an accompanying longitudinal change in the metric. The longitudinal component will be a permanent shift in the distance between geodesics.
- In addition to displacements of geodesics, the transverse component will also have a term that grows with time. This will be because there is relative velocity between the two freely falling points caused by shell crossing.

In terms of the dynamics, a change in velocity is higher order than a change in distance measured in gravitational waves. This however does not mean that our effect is harder to measure. The conventional gravitational memory effect needs a physical event that significantly breaks spherical symmetry to be detectable whereas our effect does not and therefore it can occur more generally. In addition, a change in velocity implies a distance change that grows in time even after the initial effect. That is an advantage for some detection methods.

The amplitude of the memory effect in ground based interferometers will be too small to measure in the interferometers being planned in the future such as LISA and BBO. However, the SKA telescope is expected to detect thousands of pulsars, [4], and one of them might be close enough to a supernova explosion to provide us with an observable value of the effect in pulsar scintillometry. This will also allow for a direct measurement of the energy emitted by a supernova in neutrinos. Currently there is no other way to obtain such information. Thus in addition to direct neutrino detection experiments such as in Super-Kamiokande [5], this memory effect can provide a new handle on constraining the explosion mechanism.

The rest of the paper is organized as follows. In section II we discuss how an astrophysical interferometer formed by pulsar scintillometry can measure the memory effect caused by a supernova. In section III we derive the value of this memory effect when the interferometer arm is both perpendicular and along the direction of the shell. In the final section IV we discuss potential observation of such an effect by experiments that are currently being planned such as LISA and BBO.

II. MEMORY EFFECT IN PULSAR SCINTILLATION

It is known that the images of many astronomical bodies scintillate [10]. A general reason for scintillation is that due to scattering or lensing, we receive multiple light rays from the same objects. These light rays are very close to each other, so they cannot be individually resolved and have to interfere. The scintillation pattern we see is the time dependence of their interference. If we consider two light rays from a faraway pulsar which happen to pass by a SN progenitor, as illustrated in Fig.2, they can probe the spacetime distortion when it explodes.

The scintillation/interference pattern is directly related to the path lengths of these light rays. The change in such path length during a SN explosion has been worked out in [11]

$$\Delta t = 2\delta M \left[\ln\left(1 + \frac{t^2}{b^2}\right) - \frac{t^2}{b^2 + t^2}\right]. \tag{1}$$

Here b is the impact parameter as shown in Fig.2, the shortest distance between the light ray and the SN. t is the proper time on earth, with $t = 0$ the time we directly observe the SN explosion. δM is the total energy of the neutrino shell, and Δt is the resulting time shift. A photon which should have reached the earth at time t, will arrive earlier at $(t - \Delta t)$ instead.

When the separation between two light rays has a component in the radial direction from the SN, Δb, there will be a nonzero relative change between their path lengths.

$$(\Delta t|_b - \Delta t|_{b+\Delta b}) \approx \frac{\partial \Delta t}{\partial b}\Delta b = -\frac{4\delta M t^4}{b(b^2+t^2)^2}\Delta b \ . \tag{2}$$

We can see that this effect grows from zero and approaches an asymptotic value,

$$(\Delta t|_b - \Delta t|_{b+\Delta b}) \longrightarrow \frac{4\delta M \Delta b}{b} \ , \tag{3}$$

at a characteristic time scale given by b.

We estimate b by assuming that the next SN is somewhere near the galactic centre. A sample of ~ 9000 pulsars from the SKA catalog in [4] shows that among those pulsars, the shortest b is about $10\ ly \sim 10^{14}\ km$. Δb is related to the scattering-broadening of images. We use the data from [12] that was observed on a scattering screen near the galactic centre. Scaling the frequency to $1\ GHz$ which is usually a good window to observe pulsar signals. We found that such a scattering screen can produce images separated by $\Delta b \sim 1000 A.U. \sim 10^{10}\ km$. We again use $\delta M \sim 1\ km$, and combining all these numbers we get $(\delta M \Delta b/b) \sim 1\ m$. This is comparable to the wavelength at $1\ GHz$, thus making the change in interference pattern easy to detect. Therefore, if we can monitor the pulsar scintillation pattern over ten years after a SN explosion we should see an order one change in the scintillation pattern predicted by Eq. (2).

III. MEMORY EFFECT IN INTERFEROMETERS

The pulsar scintillation pattern is usually observed far from where the shell is effecting the local spacetime. One way to measure the local spacetime is to use an interferometer. In this section we calculate what effect the shell of matter would have on an interferometer that is placed at the position where the photon geodesics are crossed by the shell.

A. Velocity change from junction conditions

Assuming the local spacetime is dominated by the SN proginator star, the metric governing the spacetime is the Schwarzschild metric. In the calculations we will need two Schwarzschild metrics. One with mass M and the other with mass $M - \delta M$, where δM will be the energy carried by the neutrinos after the SN explosion. The metric with mass M will have a bar over its coordinates and the one with $M - \delta M$ will not.

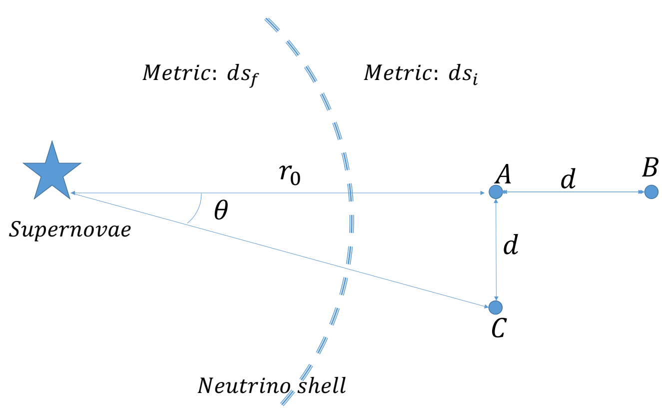

FIG. 1. Schematic of the effect being considered by a neutrino shell passing through the interferometer. The points A, B, C represent ends of the interferometer of arm length d. The three points A, B and C will pick up velocities v_A, v_B and v_C respectively after the shell crosses them. They are all different since they cross the shell at different locations.

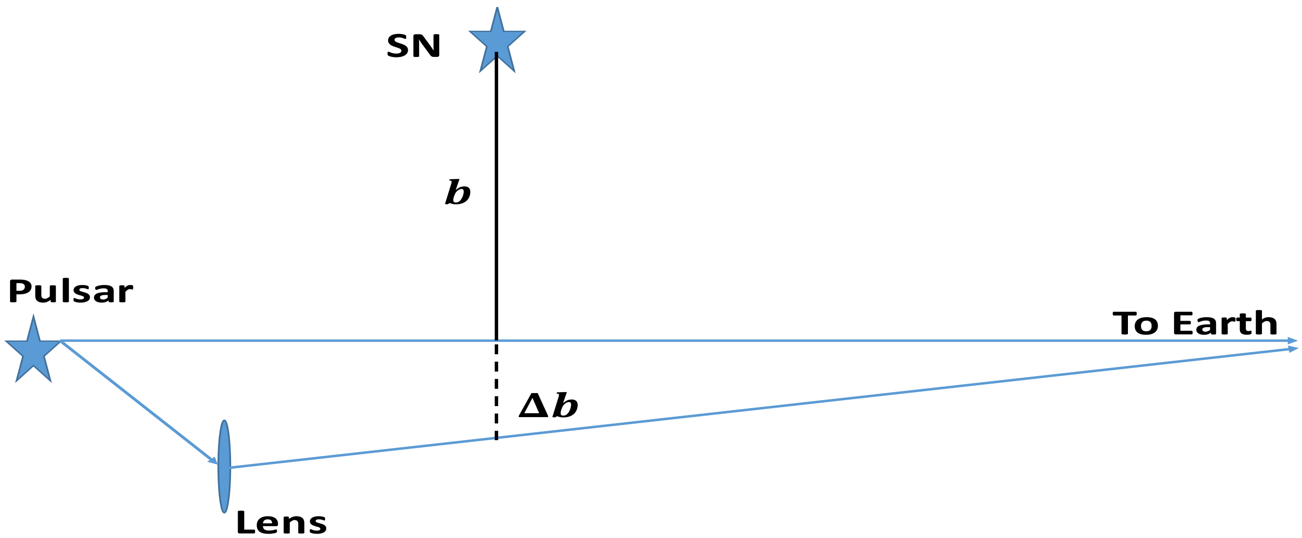

FIG. 2. Geometry of the astrophysical interferometer formed by pulsar scintillometry. Due to scattering or lensing, the image we see is an interference pattern of two light rays represented by the blue lines. If the separation of the two light rays has a component along the longitudinal (radial) direction from the SN, the spacetime distortion of the neutrino shell will change the interference pattern we see. We draw the lens to be behind the SN, but it could have been in front of it and the effect is the same.

$$d\bar{s}^2 = \bar{g}_{\mu\nu} d\bar{x}^\mu d\bar{x}^\nu = -\left(1 - \frac{2M}{r}\right) d\bar{t}^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2. \tag{4}$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\left(1 - \frac{(M - \delta M)}{r}\right) dt^2 + \left(1 - \frac{2(M - \delta M)}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2. \tag{5}$$

We are working in units with $G = c = 1$. Notice that the time component of the metric is different in both geometries whereas the radial component is the same as it corresponds to the radius of a two sphere separating the two geometries. We describe the shell as a delta function travelling at roughly the speed of light and thus following a null geodesic. The null vector of the shell can be written in both metrics as follows[1]

$$k^\mu = \left(g_{tt}^{-1}, 1, 0, 0\right), \qquad \bar{k}^\mu = \left(\bar{g}_{tt}^{-1}, 1, 0, 0\right) \tag{6}$$

Since we expect the interferometer points (denoted by the points A, B, C in figure 1) to be very far from the massive object we assume they have a negligible initial velocity.

$$\bar{u}^\mu = \left((-1/\bar{g}_{tt})^{\frac{1}{2}}, 0, 0, 0\right). \tag{7}$$

After the shell has crossed the points will pick up a velocity v. We can calculate v using the IJC,

$$g_{\mu\nu} u^\mu k^\nu = \bar{g}_{\mu\nu} \bar{u}^\mu \bar{k}^\nu. \tag{8}$$

[1] One could write this vector in different ways and still have it satisfy the null normalization condition. However the vectors should have the same radial component as we do not expect any jumps in the radial coordinate.

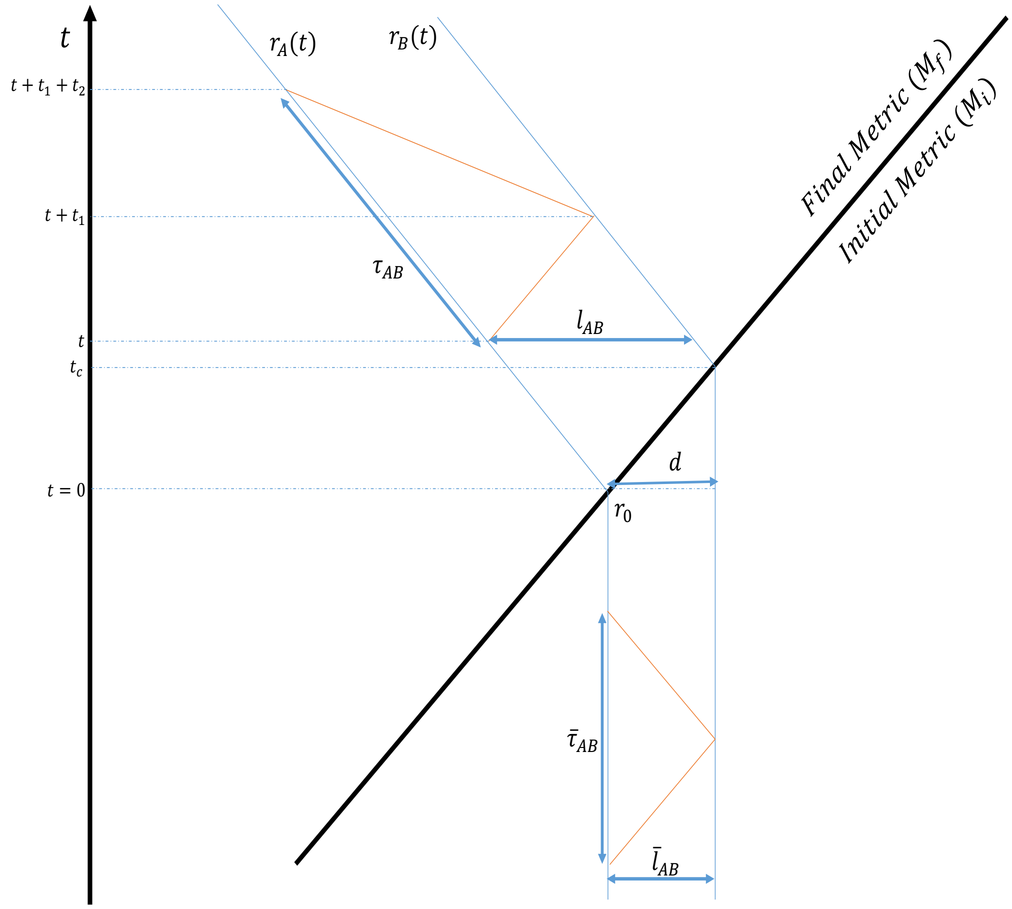

FIG. 3. Spacetime diagram showing the paths of photons that are used in the interferometer to measure the change in the length of the interferometer arms. The orange lines represent the photon trajectories. The solid blue lines represent the trajectories of the two points A and B in figure 1. The dark black line is the null trajectory of the neutrino shell. the proper lengths before(after) shell crossing are $\bar{l}_{AB}(l_{AB})$. Proper times before(after) shell crossing are $\bar{\tau}_{AB}(\tau_{AB})$. The coordinate time at which the shell crosses point B is different in both metrics. t_c in metric of mass $M - \delta M$ and $\bar{t}_c$ in the metric of mass M.

This expression comes from demanding the continuity of geodesics from one geometry to another. Throughout this calculation we assume that the interferometer is very far from the supernova and the interferometers' arm length is much smaller then the distance to the supernova. Thus we assume there are three small quantities, $\frac{M}{r_0}, \frac{\delta M}{r_0}, \frac{d}{r_0} << 1$. To simplify notation we will use the symbol $\mathcal{O}(r^{-1})$ to represent suppression by any one of the three small quantities. Substituting Eqs (6, 7) into Eq (8) gives the final vector for the interferometer to order $\mathcal{O}(r_{crossing}^{-2})$,

$$u^\mu = \left((-1/g_{tt})^{\frac{1}{2}}\sqrt{1 - v^2 g_{rr}^{-1}}, v, 0, 0\right). \tag{9}$$

The change in velocity due a shell crossing is $v = -\frac{\delta M}{r_{crossing}}\bar{g}_{tt}^{-\frac{1}{2}}$. $r_{crossing}$ is a fixed distance at which the shell crosses a point. For A it is r_0, for B it is $r_0 + d$ and for C it is $r_0 + \mathcal{O}(r_0^{-1})$. Note that this is the proper velocity $\frac{dr}{d\tau}$. We will need the coordinate velocity for our calculations which is given by

$$\begin{aligned} \frac{dr}{dt} &= \frac{dr}{d\tau}\frac{d\tau}{dt} = v g_{tt}^{\frac{1}{2}} = \frac{\delta M}{r_{crossing}}\bar{g}_{tt}^{-\frac{1}{2}} g_{tt}^{\frac{1}{2}} \\ &= -\frac{\delta M}{r_{crossing}}\left(1 + \frac{\delta M}{r_{crossing}}\right). \end{aligned} \tag{10}$$

B. Distance between A and B

Points A and B are the end points of an interferometer arm that is radially oriented towards the SN (as shown in figure 1). Here we calculate the distance between the points before and after shell crossing.

1. Defining trajectories

We parametrize the trajectories of the points in a piecewise form. The trajectory for point A is

$$\begin{aligned} r_A(\bar{t}) &= r_0 - \underbrace{\frac{1}{2}\bar{a}^{(A)}_{(2)}\bar{t}^2}_{T1}, \qquad (\bar{t} \leq 0) \\ r_A(t) &= r_0 - \underbrace{\frac{1}{2}a^{(A)}_{(2)}t^2}_{T2} - \underbrace{\left(v^{(A)}_{(1)} + v^{(A)}_{(2)}\right)t}_{T3}, \qquad (t > 0). \end{aligned} \tag{11}$$

We have introduced new notation to represent the velocity and acceleration of the particles. The subscript represents the number of factors of r_0 the term is suppressed by, i.e $a^{(A)}_{(2)}$ is term of $\mathcal{O}(r_0^{-2})$.[2] The superscript is a label for the particle. Here $\bar{a}^{(A)}_{(2)}$ is the acceleration of particle A in the initial metric. $a^{(A)}_{(2)}$ is the acceleration of particle A in the final metric. $v^{(A)}_{(1)}(v^{(A)}_{(2)})$ represent the first(second) order velocities of particle A in the final metric. The equations for these accelerations and velocities are

$$\bar{a}^{(A)}_{(2)} = \frac{M}{r_0^2}, a^{(A)}_{(2)} = \frac{M - \delta M}{r_0^2}, v^{(A)}_{(1)} = \frac{\delta M}{r_0}, v^{(A)}_{(2)} = \frac{\delta M^2}{r_0^2}. \tag{12}$$

- $t = 0$ is defined to be the time when the shell crosses point A at $r_A(0) = r_0$. For $t < 0$ we assume there is no velocity. The only term that contributes to the trajectory is the acceleration of point A, $\bar{a}^{(A)}_{(2)}$ towards mass M. This is given by $T1$.

- For $t > 0$ there is still the acceleration term, $a^{(A)}_{(2)}$, but the mass is different since the shell has carried δM away. This is given by $T2$.

- $T3$ comes from the velocity that is picked up by shell crossing.

The trajectory of point B is given by

$$\begin{aligned} r_B(\bar{t}) &= r_0 + d - \underbrace{\frac{1}{2}\bar{a}^{(B)}_{(2)}\bar{t}^2}_{T4}, \qquad (\bar{t} \leq \bar{t}_c) \\ r_B(t) &= r_0 + d - \underbrace{\frac{1}{2}a^{(B)}_{(2)}(t - t_c)^2}_{T5} - \underbrace{\frac{1}{2}\bar{a}^{(B)}_{(2)}t_c^2}_{T6} - \underbrace{\left(\bar{a}^{(B)}_{(2)}t_c + v^{(B)}_{(1)} + v^{(B)}_{(2)}\right)(t - t_c)}_{T7}, \qquad (t > t_c). \end{aligned} \tag{13}$$

where the accelerations and velocities are defined analogously to point A. In fact some of them are the same as shown below

[2] We will only keep terms up to order $\mathcal{O}(r_0^{-2})$.

$$
\begin{aligned}
\bar{a}^{(A)}_{(2)} &= \bar{a}^{(B)}_{(2)} = \frac{M}{r_0^2} \\
a^{(A)}_{(2)} &= a^{(B)}_{(2)} = \frac{M - \delta M}{r_0^2} \\
v^{(A)}_{(1)} &= v^{(B)}_{(1)} = \frac{\delta M}{r_0} \\
v^{(A)}_{(2)} &= \frac{\delta M^2}{r_0^2} \\
v^{(B)}_{(2)} &= \frac{\delta M^2}{r_0^2} - \frac{\delta M d}{r_0^2}.
\end{aligned}
\tag{14}
$$

- For $\bar{t} < \bar{t}_c$ there is just the acceleration, $\bar{a}^{(B)}_{(2)}$, of the point towards mass M. This is given by $T4$.
- After $t > t_c$ the shell has crossed. The particle now accelerates in the final metric.[3] Term $T5$ takes into account the distance moved due to this.
- $T6$ corresponds to the distance travelled while accelerating in the initial metric for time t_c.
- $T7$ is a term that contains the velocity of the particle. The $\bar{a}^{(B)}_{(2)} t_c$ term is the velocity gained while accelerating in the metric of mass M. $v^{(B)}_{(1)}, v^{(B)}_{(2)}$ are the velocities picked up due to shell crossing.

2. Relation between proper length and proper time

The interferometer infers the distance between two points by measuring the proper time it takes for a photon to go from one end of the arm to the other and come back. In this section we will relate the proper time seen by one end of the arm to the proper length of the arm (in figure 3). In particular we show that *the proper time is twice the proper length at a fixed coordinate time up to* $\mathcal{O}(r_0^{-2})$. This will be very useful as we can compute the proper length (which is comparatively easier) and still infer the effect seen in the interferometer.

The most general expression for the proper distance is

$$
l = \int \sqrt{g_{ab} dx^a dx^b}. \tag{15}
$$

Using this we calculate the proper distance before shell crossing $\bar{l}_{AB}$ between the two points A and B at time $\bar{t}$. Since we are only considering radial motion Eq (15) simplifies to

$$
\bar{l}_{AB} = \int_{r_A(\bar{t})}^{r_B(\bar{t})} dr\ \bar{g}_{rr}^{\frac{1}{2}} = \int_{r_A(\bar{t})}^{r_B(\bar{t})} dr \left(1 + \frac{M}{r} + \frac{3}{2}\frac{M^2}{r^2} + \mathcal{O}((M/r)^3)\right). \tag{16}
$$

In all the calculations we will only need terms of $\mathcal{O}(r_0^{-2})$. Thus we can use $r_B(\bar{t}) = r_0 + d$ and $r_A(\bar{t}) = r_0$.

$$
\bar{l}_{AB} = \int_{r_A(\bar{t})}^{r_B(\bar{t})} dr\ \bar{g}_{rr}^{\frac{1}{2}} = d + M \ln\left(\frac{r_0 + d}{r_0}\right) - \frac{3}{2} M^2 \left(\frac{1}{r_0 + d} - \frac{1}{r_0}\right) \tag{17}
$$

This can be fully simplified to $\mathcal{O}(r_0^{-2})$,

$$
\bar{l}_{AB} = d\left(1 + \frac{M}{r_0} - \frac{1}{2}\frac{Md}{r_0^2} + \frac{3}{2}\frac{M^2}{r_0^2}\right). \tag{18}
$$

The calculation for $\bar{\tau}_{AB}$ is much longer and so we do that in the appendix. We refer the reader to Eq (A12) which shows that $\bar{\tau}_{AB} = 2\bar{l}_{AB}$, thus proving our claim.

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[3] Note that $\bar{t}_c$ and t_c are different since they are coordinates in two different metrics.

3. Longitudinal memory effect

The memory effect in the longitudinal direction, Δl_{AB} is the difference between l_{AB} and $\bar{l}_{AB}$. In the appendix we have the full calculation of the proper length l_{AB} (see Eq (B9)), here we simply use the result

$$\Delta l_{AB} \equiv l_{AB} - \bar{l}_{AB} = -\frac{\delta M^2 d}{r_0^2}. \tag{19}$$

We note that there is no time dependent term in this expression. This follows from the fact that the velocity of both points is the same up to $\mathcal{O}(r_0^{-2})$.[4] It will turn out that when we calculate the distance between A and C there will be a relative velocity and therefore a time dependent turn.

C. Distance between A and C

In this subsection we calculate the proper distance change between A and C. First lets write down the trajectory equation for point C

$$\begin{aligned} r_C(\bar{t}) &= d - \frac{1}{2}\bar{a}^{(C)}_{(2)}\bar{t}^2, \qquad (\bar{t} \leq 0) \\ r_C(t) &= d - \frac{1}{2}a^{(C)}_{(2)}t^2 - \left(v^{(C)}_{(1)} + v^{(C)}_{(2)}\right)t, \qquad (t > 0). \end{aligned} \tag{22}$$

The acceleration and velocities are defined below

$$\begin{aligned} \bar{a}^{(A)}_{(2)} = \bar{a}^{(B)}_{(2)} = \bar{a}^{(C)}_{(2)} &= \frac{M}{r_0^2} \\ a^{(A)}_{(2)} = a^{(B)}_{(2)} = a^{(C)}_{(2)} &= \frac{M - \delta M}{r_0^2} \\ v^{(A)}_{(1)} = v^{(B)}_{(1)} = v^{(C)}_{(1)} &= \frac{\delta M}{r_0} \\ v^{(A)}_{(2)} = v^{(C)}_{(2)} &= \frac{\delta M^2}{r_0^2} \end{aligned} \tag{23}$$

From the geometry in figure 1 we see that only θ and r will change in the motion of A and C. Thus we can drop the ϕ dependence from the start and write the integral for proper length in Eq (15) as

$$l_{AC} = \int_{\theta_i}^{\theta_f} d\theta \; g_{\theta\theta}^{\frac{1}{2}} \left(1 + \underbrace{(g_{rr}/g_{\theta\theta})(dr/d\theta)^2}_{T8}\right)^{\frac{1}{2}}. \tag{24}$$

where $\theta_f = d/r_0$ and $\theta_i = 0$. To evaluate $T8$ we would need the geodesic equation for r as function of θ. Instead of evaluating it explicitly we note that $\frac{dr}{d\theta}$ will be of order d. Since $g_{\theta\theta}^{-1} = \frac{1}{r_0^2} + \mathcal{O}(r_0^{-3})$, to leading order we can ignore the effect of $T8$. Integrating the first term gives

$$l_{AC} = r_C(t)(\theta_f - \theta_i) = r_C(t)\frac{d}{r_0}. \tag{25}$$

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[4] After the shell crosses point A, it will pick up a velocity $\frac{\delta M}{r_0}\left(1 + \frac{\delta M}{r_0}\right)$. In the time the shell takes to go from A to B, A will also gain a velocity from its acceleration towards the mass $M - \delta M$. The magnitude of the velocity gain from acceleration is $\frac{M-\delta M}{r_0^2}t_c$. The final velocity of A at time t_c is, therefore, given by

$$v_A(t = t_c) = \frac{\delta M}{r_0}\left(1 + \frac{\delta M}{r_0}\right) + \frac{M - \delta M}{r_0^2}t_c. \tag{20}$$

Similarly, point B will gain a velocity from the acceleration towards mass M with magnitude $\frac{M}{r_0^2}t_c$. The velocity change from shell crossing is $\frac{\delta M}{r_0+d}\left(1 + \frac{\delta M}{r_0}\right)$. The final velocity of point B at time t_c is

$$v_B(t = t_c) = \frac{\delta M}{r_0 + d}\left(1 + \frac{\delta M}{r_0}\right) + \frac{M}{(r_0 + d)^2}t_c. \tag{21}$$

We see that v_A is the same as v_B to $\mathcal{O}(r_0^{-2})$.

Before shell crossing the proper length $\bar{l}_{AC} = d + \mathcal{O}(r_0^{-3})$. Whereas after shell crossing, there will be an additional velocity term in the proper length, l_{AC}, which will give a time dependent term at $\mathcal{O}(r_0^{-2})$

$$l_{AC} = d - \frac{\delta M d}{r_0^2} t. \tag{26}$$

The difference in proper lengths will be

$$\Delta l_{AC} \equiv l_{AC} - \bar{l}_{AC} = -\frac{\delta M d}{r_0^2} t. \tag{27}$$

IV. OBSERVATION WITH SPACE-BASED INTERFEROMETERS

Taking the generic form of the change in distance as $\Delta L \sim \frac{\delta M d}{r_0^2} t$ we can estimate the distance a SN would have to be from the interferometer to have an observable change in strain, which is a unitless number quantifying the amount of space-time distortion.

$$h \sim \frac{\Delta L}{d} \sim \frac{\delta M t}{r_0^2} \ . \tag{28}$$

Since our strain grows linearly with time, we do not expect detections from ground based experiments as for those setups the three points A, B, C cannot remain in free fall for a long enough amount of time such that the signal builds up to an observable value.

If we plot our effect on the strain-frequency diagram [8] that is usually used to compare different interferometers, it will be a 45-degree line. Thus the first point at which the sensitivity curve of a device crosses with a 45-degree line will give the best chance for our effect being detected. In all these estimations, we take δM to be a faction of a solar mass, and take the corresponding Schwarzschild radius to be 1 km for simplicity.

For LISA, the best observing frequency is $\sim 0.5 \times 10^{-2} Hz$ with a sensitivity in strain $\sim 10^{-21}$. Using Eq. (28), we can solve for the distance to the SN, r_0, for our effect to be detectable.

$$r_0 = \left(\frac{\delta M}{h} t\right)^{\frac{1}{2}} \tag{29}$$

$$= \left(\frac{1\ km}{10^{-21}} \times 10^7\ km\right)^{\frac{1}{2}} \approx 10^{14}\ km = 10\ ly. \tag{30}$$

This is clearly too close. It has been estimated that only once in 10^8 years will a SN go off within a distance of 30 ly [9].[5] By a naive volume scaling, an explosion within 10 ly only occurs once every billion years.

If we look at the Big Bang Observer (BBO) instead, the best observing frequency is $\sim 0.5 Hz$ with a sensitivity in strain $\sim 10^{-24}$. First of all, this frequency range does not have as many background signals from compact binaries, making it a much better device to measure our effect. The improved sensitivity gives a value for r_0 of $\sim 100\ ly$. This is a factor of 10^3 increase in the volume for detectable events, thus improves the expectation of one SN that is within 10 ly to occur in less than a million years . That is unfortunately still a long shot.

In this type of simple estimation, we cannot go lower in the frequency. The exact duration of the neutrino-shell passage is not known, but we do no expect it to be much less than a second. Thus for higher frequencies, the co-dimension-one delta function approximation breaks down, and the effect will be weaker than Eq. (28).

Finally, we expect 2 to 3 SN per century in our galaxy and we can assume that the next SN would be at a distance comparable to the galactic diameter of $\sim 10^5\ ly$. If we are going to measure such effect at 1 Hz, Again using Eq. (28), we find

$$h = \frac{1\ km \times\ (3 \times 10^8\ m)}{(10^5\ ly)^2} \sim 10^{-30}\ . \tag{31}$$

This requires a measurement of the strain that is six orders of magnitude better than BBO and is not yet achievable by interferometers that are currently being planned.

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[5] And if that happens, it might kill us.

ACKNOWLEDGMENTS

This work is supported by the Canadian Government through the Canadian Institute for Advance Research and Industry Canada, and by Province of Ontario through the Ministry of Research and Innovation.

Appendix A: Proper time for photon to go between A and B before shell crossing

From the metric, we can set $ds = 0$, for the photon to get

$$\int_t^{t+t_1} dt = \int_{r_A(t)}^{r_B(t+t_1)} \frac{dr}{\left(1-\frac{2M}{r}\right)} \tag{A1}$$

Integrating this gives

$$t_1 = r_B(t+t_1) - r_A(t) + 2M \ln\left(\frac{r_B(t+t_1)-2M}{r_A(t)-2M}\right) \tag{A2}$$

Now we can expand the log using $\ln(x) \approx (x-1) - \frac{1}{2}(x-1)^2 + ...$ For us $x = \frac{r_B(t+t_1)-2M}{r_A(t)-2M}$ Thus $x - 1 = \frac{r_B(t+t_1)-r_A(t)}{r_A(t)-2M} = \frac{r_B(t+t_1)-r_A(t)}{r_A(t)}\left(1-\frac{2M}{r_A(t)}\right)^{-1}$ and we get

$$2M \ln\left(\frac{r_B(t+t_1)-2M}{r_A(t)-2M}\right) = 2M\left(\frac{r_B(t+t_1)-r_A(t)}{r_A(t)}\left(1+\frac{2M}{r_A(t)}\right) - \frac{1}{2}\left(\frac{r_B(t+t_1)-r_A(t)}{r_A(t)}\right)^2\left(1-\frac{2M}{r_A(t)}\right)^{-2}\right) \tag{A3}$$

Since we only want to keep terms that are $\mathcal{O}(r_0^{-2})$ we can replace $\frac{M}{r_A(t)}$ with $\frac{M}{r_0}$. Thus Eq (A2) becomes

$$\begin{aligned} t_1 &= (r_B(t+t_1) - r_A(t))\left(1 + \frac{2M}{r_0}\left(1 + \frac{2M}{r_0} - \frac{1}{2}\frac{r_B(t+t_1)-r_A(t)}{r_0}\right)\right) \\ &= (r_B(t+t_1) - r_A(t))\left(1 + \frac{2M}{r_0} + \frac{4M^2 - M(r_B(t+t_1)-r_A(t))}{r_0^2}\right) \end{aligned} \tag{A4}$$

to leading order $r_B(t+t_1) - r_A(t) = d$

$$t_1 = (r_B(t+t_1) - r_A(t))\left(1 + \frac{2M}{r_0} + \frac{4M^2 - Md}{r_0^2}\right) \tag{A5}$$

$$r_B(t+t_1) - r_A(t) = d - \frac{1}{2}\frac{M}{r_0^2}(t+d)^2 + \frac{1}{2}\frac{Mt^2}{r_0^2} = d\left(1 - \frac{1}{2}\frac{Md}{r_0^2} - \frac{Mt}{r_0^2}\right) \tag{A6}$$

Therefore t_1 becomes

$$t_1 = d\left(1 + \frac{2M}{r_0} + \frac{4M^2}{r_0^2} - \frac{3}{2}\frac{Md}{r_0^2} - \frac{Mt}{r_0^2}\right) \tag{A7}$$

Now we can compute t_2.

$$t_2 = (r_B(t+t_1) - r_A(t+t_1+t_2))\left(1 + \frac{2M}{r_0} + \frac{4M^2 - M(r_B(t+t_1)-r_A(t+t_1+t_2))}{r_0^2}\right) \tag{A8}$$

$$\begin{aligned} r_B(t+t_1) - r_A(t+t_1+t_2) &= d - \frac{1}{2}\frac{M}{r_0^2}(t+d)^2 + \frac{1}{2}\frac{M}{r_0^2}(t+2d)^2 \\ &= d\left(1 + \frac{Mt}{r_0^2} + \frac{3}{2}\frac{Md}{r_0^2}\right) \end{aligned} \tag{A9}$$

Thus t_2 becomes

$$
\begin{aligned}
t_2 &= d\left(1+\frac{Mt}{r_0^2}+\frac{3}{2}\frac{Md}{r_0^2}\right)\left(1+\frac{2M}{r_0}+\frac{4M^2-Md}{r_0^2}\right) \\
&= d\left(1+\frac{2M}{r_0}+\frac{4M^2}{r_0^2}+\frac{1}{2}\frac{Md}{r_0^2}+\frac{Mt}{r_0^2}\right)
\end{aligned} \tag{A10}
$$

Summing t_1 and t_2

$$
t_1+t_2 = 2d\left(1+\frac{2M}{r_0}+\frac{4M^2}{r_0^2}-\frac{1}{2}\frac{Md}{r_0^2}\right) \tag{A11}
$$

The proper time measured is

$$
\begin{aligned}
\tau &= \left(1-\frac{2M}{r}\right)^{\frac{1}{2}}(t_1+t_2) \\
&= 2d\left(1-\frac{M}{r}-\frac{1}{2}\frac{M^2}{r_0^2}\right)\left(1+\frac{2M}{r_0}+\frac{4M^2}{r_0^2}-\frac{1}{2}\frac{Md}{r_0^2}\right) \\
&= 2d\left(1+\frac{M}{r_0}+\frac{3}{2}\frac{M^2}{r_0^2}-\frac{1}{2}\frac{Md}{r_0^2}\right)
\end{aligned} \tag{A12}
$$

Appendix B: Proper length after shell crossing

In this section we calculate the proper length between A and B after shell crossing. Integrating Eq (15) with the trajectories for $r_B(t)$ and $r_A(t)$ gives

$$l_{AB} = r_B(t) - r_A(t) + \underbrace{(M - \delta M) \ln\left(\frac{r_B(t)}{r_A(t)}\right)}_{T9} \underbrace{- \frac{3}{2}(M - \delta M)^2 \left(\frac{1}{r_B(t)} - \frac{1}{r_A(t)}\right)}_{T10} \tag{B1}$$

Thus we see the only new thing to compute is $r_B(t) - r_A(t)$. Lets expand these terms individually, starting with $T9$

$$\begin{aligned} \ln\left(\frac{r_B(t)}{r_A(t)}\right) &= \ln\left(\frac{(r_B - r_0) + r_0}{(r_A - r_0) + r_0}\right) \\ &= \ln\left(\left(1 + \frac{r_B - r_0}{r_0}\right)\left(1 + \frac{r_A - r_0}{r_0}\right)^{-1}\right) \\ &= \ln\left(1 + \frac{r_B - r_A}{r_0} - \frac{(r_B - r_0)(r_A - r_0)}{r_0^2} - \left(\frac{r_A - r_0}{r_0}\right)^2\right). \end{aligned} \tag{B2}$$

Note that $r_A - r_0 = \mathcal{O}(r_0^{-1})$ and we are only interested in terms up to $\mathcal{O}(r_0^{-2})$ thus we simplify the above equation to

$$\ln\left(\frac{r_B(t)}{r_A(t)}\right) = \ln\left(1 + \frac{r_B - r_A}{r_0}\right). \tag{B3}$$

Now we look ahead to $r_B(t) - r_A(t)$ in Eq (B7). We will need to keep terms up to $\mathcal{O}(r_0^{-1})$ in $r_B(t) - r_A(t)$. Thus we expand the log to get

$$\ln\left(\frac{r_B(t)}{r_A(t)}\right) = \frac{r_B - r_A}{r_0} - \frac{1}{2}\left(\frac{r_B - r_A}{r_0}\right)^2 = \frac{d}{r_0} + \frac{\delta M d}{r_0^2} - \frac{1}{2}\frac{d^2}{r_0^2} \tag{B4}$$

Evaluating $T10$ to $\mathcal{O}(r_0^{-2})$ is straightforward

$$-\frac{3}{2}(M - \delta M)^2\left(\frac{1}{r_B(t)} - \frac{1}{r_A(t)}\right) = \frac{3}{2}\frac{(M - \delta M)^2 d}{r_0^2} \tag{B5}$$

Putting in the expansion of $T9$ and $T10$ into l_{AB}

$$l_{AB} = r_B(t) - r_A(t) + \frac{(M - \delta M)d}{r_0} + \frac{\delta M(M - \delta M)d}{r_0^2} - \frac{1}{2}\frac{d^2(M - \delta M)}{r_0^2} + \frac{3}{2}\frac{(M - \delta M)^2 d}{r_0^2} \tag{B6}$$

Now we can compute $r_B(t) - r_A(t)$.

$$\begin{aligned} r_B(t) - r_A(t) &= d - \frac{1}{2}\frac{M}{r_0^2}t_c^2 - \left(\frac{M}{r_0^2}t_c + \frac{\delta M}{r_0} + \frac{\delta M^2}{r_0^2} - \frac{\delta M d}{r_0^2}\right)(t - t_c) - \frac{1}{2}\frac{(M - \delta M)}{r_0^2}(t - t_c)^2 \\ &\quad + \frac{1}{2}\frac{(M - \delta M)t^2}{r_0^2} + \left(\frac{\delta M}{r_0} + \frac{\delta M^2}{r_0^2}\right)t \\ &= d + \frac{\delta M t_c}{r_0} + \frac{\delta M^2 d}{r_0^2} - \frac{1}{2}\frac{\delta M d^2}{r_0^2} \\ &= d + \frac{\delta M t_c}{r_0} + \frac{\delta M^2 d}{r_0^2} - \frac{1}{2}\frac{\delta M d^2}{r_0^2} \end{aligned} \tag{B7}$$

Finally we get the expression for l_{AB}

$$l_{AB} = d + \frac{\delta M t_c}{r_0} + \frac{\delta M^2 d}{r_0^2} - \frac{1}{2}\frac{\delta M d^2}{r_0^2} + \frac{(M - \delta M)d}{r_0} + \frac{\delta M(M - \delta M)d}{r_0^2} - \frac{1}{2}\frac{d^2(M - \delta M)}{r_0^2} + \frac{3}{2}\frac{(M - \delta M)^2 d}{r_0^2} \tag{B8}$$

Substituting in for $t_c = d\left(1 + \frac{2(M-\delta M)}{r_0}\right)$ gives

$$
\begin{aligned}
l_{AB} &= d + \frac{\delta M d}{r_0} + \frac{2\delta M(M-\delta M)d}{r_0^2} + \frac{\delta M^2}{r_0^2}d - \frac{1}{2}\frac{\delta M d^2}{r_0^2} + \frac{(M-\delta M)d}{r_0} + \frac{\delta M(M-\delta M)d}{r_0^2} - \frac{1}{2}\frac{d^2(M-\delta M)}{r_0^2} \\
&\quad + \frac{3}{2}\frac{M^2 d}{r_0^2} - \frac{3M\delta M d}{r_0^2} + \frac{3}{2}\frac{\delta M^2 d}{r_0^2} \\
&= d + \frac{Md}{r_0} - \frac{1}{2}\frac{\delta M^2 d}{r_0^2} + \frac{3}{2}\frac{M^2 d}{r_0^2} - \frac{1}{2}\frac{Md^2}{r_0^2}
\end{aligned} \tag{B9}
$$

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