Appendix A
Collision Risk Index (CRI) Formulae

This appendix contains all necessary formulae in order to calculate the encountering vessels' CRI, given their kinematic characteristics.

The distance and time to CPA (DCPA and TCPA, respectively) are calculated as follows.:

$$DCPA = D \times \sin(\phi_O^T - \alpha_O^T - \pi) \tag{1}$$

$$TCPA = \frac{D \times \cos(\phi_O^T - \alpha_O^T - \pi)}{V_O^T} \tag{2}$$

where D_O^T, V_O^T ϕ_O^T, α_O^T, correspond to the vessels' distance, as well as their relative speed, movement, and azimuth angle of V_O with respect to V_T, respectively. In particular, given the two vessels' speed and direction, V_O^T is calculated using the following formula:

$$\begin{aligned} V_O^T &= \sqrt{\left[V_O^T\right]_x^2 + \left[V_O^T\right]_y^2} \\ &= \sqrt{(V_{Tx} - V_{Ox})^2 + (V_{Ty} - V_{Oy})^2} \end{aligned} \tag{3}$$

where $V_{\{T,O\}x}$, $V_{\{T,O\}y}$, are calculated as follows:

$$\begin{aligned} V_{\{T,O\}x} &= V_{\{T,O\}} \times \sin\left(\phi_{\{T,O\}}\right) \\ V_{\{T,O\}y} &= V_{\{T,O\}} \times \cos\left(\phi_{\{T,O\}}\right) \end{aligned} \tag{4}$$

Additionally, the vessels' relative direction, bearing, and azimuth angle are calculated using the following formulas:

$$\begin{aligned} \theta_O^T &= \phi_T - \phi_O \\ \phi_O^T &= \arctan2\left(\frac{\left[V_O^T\right]_x}{\left[V_O^T\right]_y}\right) \\ \alpha_O^T &= \arctan2\left(\frac{x_T - x_O}{y_T - y_O}\right) \end{aligned} \tag{5}$$

In turn, the vessels' speed ratio K is calculated using the following formula:

$$K = \frac{V_T}{V_O} \tag{6}$$

The membership function of DCPA is calculated using the following formula [2]:

$$U_{DCPA} = \begin{cases} 1 & , |DCPA| \le d_1 \\ \left(\dfrac{d_2 - |DCPA|}{d_2 - d_1}\right)^2 & , d_1 < |DCPA| \le d_2 \\ 0 & , |DCPA| > d_2 \end{cases}$$

where $d_2 = 2 \times d_1$ and ,

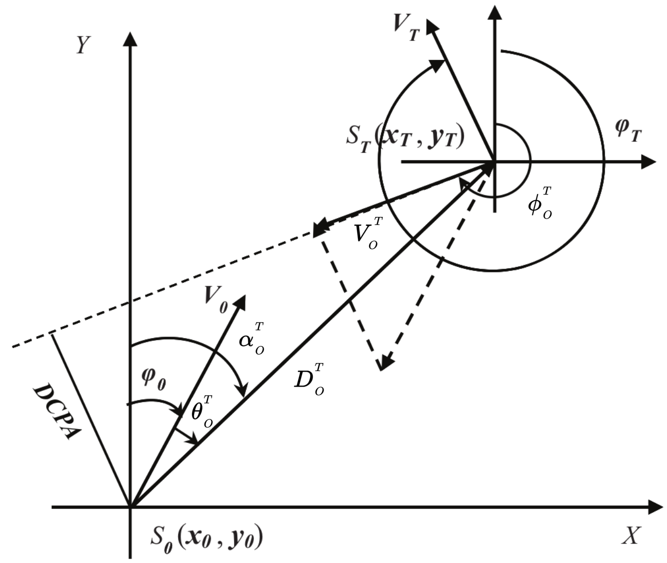

Fig. 1: The diagram of vessel collision geometry, adapted from [1].

$$d_1 = \begin{cases} 1.1 - 0.2\dfrac{\alpha_O^T}{\pi_T} & , 0 \le \alpha_O^T < \dfrac{5\pi}{8} \\ 1.0 - 0.4\dfrac{\alpha_O^T}{\pi} & , \dfrac{5\pi}{8} \le \alpha_O^T < \pi \\ 1.0 - 0.4\dfrac{2\pi - \alpha_O^T}{\pi} & , \pi \le \alpha_O^T < \dfrac{11\pi}{8} \\ 1.1 - 0.4\dfrac{2\pi - \alpha_O^T}{\pi} & , \dfrac{11\pi}{8} \le \alpha_O^T < 2\pi \end{cases}$$

Additionally, the membership function of TCPA is calculated using the following formula:

$$U_{TCPA} = \begin{cases} 1 & , |TCPA| \le t_1 \\ \left(\dfrac{t_2 - |TCPA|}{t_2 - t_1}\right)^2 & , t_1 < |DCPA| \le t_2 \\ 0 & , |TCPA| > d_2 \end{cases}$$

where $t_{\{1,2\}}$ is calculated as follows [2], [3]:

$$t_1 = \begin{cases} \dfrac{\sqrt{d_1^2 - DCPA^2}}{V_O^T} & , DCPA \le d_1 \\ \dfrac{d_1 - DCPA}{V_O^T} & , else \end{cases}$$

$$t_2 = \frac{\sqrt{d_2^2 - DCPA^2}}{V_O^T}$$

The membership function of D is as follows [3]:

$$U_D = \begin{cases} 1 & , D \le D_1 \\ \left(\dfrac{D_2 - D}{D_2 - D_1}\right)^2 & , D_1 < D \le D_2 \\ 0 & , D > D_2 \end{cases}$$

where

$$D_1 = 12 \times length_O$$

and

$$D_2 = 1.7 \cos(\theta_O^T - 19^o) + \sqrt{4.4 + 2.89 \cos^2(\theta_O^T - 19^o)}$$

(According to the domain experts) D_1 denotes critical safety distance and D_2 denotes the distance at which the final action of collision avoidance can be taken. The membership functions of θ_T and K are calculated as follows:

$$U_{\theta_O^T} = \frac{1}{2}\left[\cos(\theta_O^T - 19^o) + \sqrt{\frac{440}{289} + \cos^2(\theta_O^T - 19^o)}\right] - \frac{5}{17}$$

$$U_K = \frac{1}{1 + \frac{2}{K * \sqrt{K^2 + 1 + 2K * \sin(\phi_O^T)}}}$$

Finally, regarding the weights of the membership values, in [3], the authors derived the following set of weights for each term with advice by domain experts:

$$W = [W_{DCPA}, W_{TCPA}, W_D, W_{\theta_T}, W_K] = [0.4457, 0.2258, 0.1408, 0.1321, 0.0556] \tag{7}$$

References

[1] L. Gang, Y. Wang, Y. Sun, L. Zhou, and M. Zhang, "Estimation of vessel collision risk index based on support vector machine," *Advances in Mechanical Engineering*, vol. 8, no. 11, p. 1687814016671250, 2016.

[2] S. Wang, Y. Zhang, and Y. Zheng, "Multi-ship encounter situation adaptive understanding by individual navigation intention inference," *Ocean Engineering*, vol. 237, p. 109612, 2021.

[3] J. Park and J.-S. Jeong, "An estimation of ship collision risk based on relevance vector machine," *Journal of Marine Science and Engineering*, vol. 9, no. 5, 2021.