Identification of Frisbee aerodynamic coefficients using flight data

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***ABSTRACT*:** An alternate method to wind tunnel tests for the identification of aerodynamic coefficients of the Frisbee is presented. The method is based on matching actual flight data with predicted data from a simulation model. Three reflective markers or active LEDs are placed in a triangular array on the upper surface of the disc. Their positions during actual flights are tracked in three dimensions using high-speed video cameras. The Newton-Euler flight simulation model includes 10 aerodynamic coefficients relating linear approximations of aerodynamic forces and moments to the disc angle of attack and to its linear and angular velocities. Parameters (including coefficients and flight initial conditions) are estimated with an optimization algorithm that iteratively modifies the parameters to minimize the differences between predicted and estimated marker positions. Good estimator performance depends heavily on accurate measured data and on the set of flights spanning a wide range of angles of attack. Predicted aerodynamic coefficients compare reasonably with linearized approximations of wind tunnel results from the literature.

INTRODUCTION

More flying discs are sold each year than baseballs, basketballs, and footballs combined, but scientific research on its flight mechanics is still relatively scarce. Stilley (1972) investigated a self-suspended frisbee-like flare and used wind tunnel studies to measure the aerodynamic lift and drag forces on the Frisbee as a function of angle of attack, α, the angle between the velocity vector and the Frisbee plane. Spinning and non-spinning wind tunnel tests showed that the effect of spin on aerodynamic forces and moments is small. Stilley & Carstens (1972) analyzed flight stability and compared actual flights to free-fall tests. Mitchell (1999) also measured Frisbee lift and drag using wind tunnel tests.

Potts and Crowther (2000, 2001, 2002) have also measured pitching and rolling moments in an exhaustive series of wind tunnel tests. They verified the results of Stilley (1972) that spin effects, although observable, are not overly important in the generation of aerodynamic forces.

Hubbard and Hummel (2000) derived a dynamic model for Frisbee flight mechanics, proposed the use of aerodynamic coefficients to describe the forces and moments, and presented the results of several flight simulations. Lissaman (1993) investigated the flight stability of an oblate spheroid by considering the effect of each stability derivative (aerodynamic coefficient) on the characteristic equation. Recent work has studied the human biomechanics of the Frisbee throwing motion (Hummel and Hubbard, 2001).

Simulation of the motion of a complete Frisbee flight requires knowledge not only of initial conditions, but also of the dependence of all forces and moments experienced by the Frisbee on its dynamic motion through the air. As an alternative to expensive and time-consuming wind tunnel tests to measure these forces and moments, often analytic piecewise linear approximations are used for the functional dependence of the forces and moments on their arguments. The constants relating the linear approximations of these forces and moments to angles, velocities and angular velocities are called aerodynamic coefficients or stability derivatives. Such coefficients play a central role in stability investigations (Lissaman, 1993), but are also a simpler alternative to the use of look-up tables of measured force and moment data in the integration of the flight equations to predict Frisbee flight dynamics.

In this paper we describe the implementation of a scheme for the estimation of aerodynamic coefficients for the Frisbee. The algorithm is essentially an informed search in parameter space for the set of parameters that minimizes a performance index consisting of the mean squared difference between trajectories measured in actual flights and those predicted using the parameters in a simulation.

METHODS

SIMULATION MODEL

Since the Frisbee is a rigid body, the differential equations describing flight are similar to those for aircraft (Etkin & Reid, 1996). These are typically a Newton-Euler formulation relating the rates of change of linear and angular momenta to the forces and moments experienced from gravity and aerodynamics, and are usually expressed in body fixed coordinates. Because the Frisbee is axially symmetric, it is possible to coordinatize the equations in a frame associated with the velocity vector, rather than a body-fixed frame.

Figure 1 shows a schematic of the Frisbee in flight. The coordinate axis $\mathbf{d_1}$ lies along the projection of the velocity vector on the Frisbee plane. The $\mathbf{d_3}$ axis of symmetry is perpendicular to the Frisbee plane and points generally downward, and $\mathbf{d_2}$ is the cross product of $\mathbf{d_3}$ and $\mathbf{d_1}$. A complete derivation of the Frisbee flight equations of motion (for a slightly different coordinate frame with $\mathbf{d_3}$ upward) has been presented by Hubbard and Hummel (2000). The equations of motion in schematic form are

$$m\,(d\mathbf{v}/dt + \boldsymbol{\omega}_\mathbf{D} \times \mathbf{v}) = \mathbf{F} - mg\mathbf{k} \tag{1}$$

$$I\, d\omega/dt + \boldsymbol{\omega}_\mathbf{D} \times I\,\omega = \mathbf{M} \tag{2}$$

where m is the mass, g is gravitational acceleration and I is the moment of inertia matrix, $\boldsymbol{\omega}_\mathbf{D}$ is the angular velocity of the coordinate frame, $\boldsymbol{\omega}$ is the angular velocity of the body, and the resultant vectors of aerodynamic force and moment **F** and **M** are due to aerodynamics alone (Hubbard and Hummel, 2000). The two contributors to **F**, the lift and drag forces **L** and **D,** act perpendicular and opposite to the velocity vector **v**, respectively, and are strong functions of the angle of attack, α, the angle (Fig. 1) between **v** and $\mathbf{d_1}$. A steadily spinning body experiences a Robins-Magnus side force perpendicular to the plane determined by the velocity and angular velocity vectors. Although this force has been measured by Potts and Crowther, it is neglected here. The only other force acting is gravity.

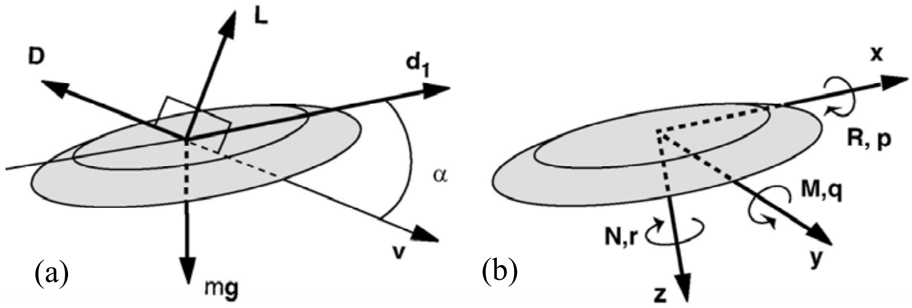

Fig 1 (a) Drag **D** and lift **L** forces act in a plane perpendicular to the disc plane and opposite and perpendicular to the velocity **v,** respectively**.** They are mostly functions of the angle of attack, α, the angle between **v** and $\mathbf{d_1}$ in the disc plane. (b) Roll, pitch and yaw axes.

While the previous model (Hubbard & Hummel, 2000) contained only eight coefficients, the present model has ten, denoted below by the symbol C_{Ij}. In each case the coefficient has two subscripts. The first I denotes the dependent variable (force or moment) being approximated, and the second j denotes the major independent variable on which the force or moment is assumed to depend. Thus each coefficient (except for C_{Lo} and C_{Do} which are constants and $C_{D\alpha}$, in which the dependence on α is quadratic) is a partial derivative of a force or moment with respect to an angle or angular velocity. To a first approximation the lift and drag are linear and quadratic functions, respectively, of the angle of attack (Anderson, 2001). The complete model for the aerodynamic forces and moments is

Lift force: $L = (C_{Lo} + C_{L\alpha}\alpha)\rho A v^2/2$

Drag force: $D = (C_{Do} + C_{D\alpha}\alpha^2)\rho A v^2/2$

Rolling moment: $R = (C_{Rr}r + C_{Rp}p)\rho d A v^2/2$

Pitching moment: $M = (C_{Mo} + C_{M\alpha}\alpha + C_{Mq}q)\rho d A v^2/2$

Spin-down moment: $N = C_{Nr}\, r\rho d A v^2/2$

where L and D are the lift and drag forces; R, M, and N and p, q, and r are the roll, pitch and yaw moments and angular velocities, respectively, in the $\mathbf{d_1},\mathbf{d_2},\mathbf{d_3}$ frame (Fig. 1); A and d are the Frisbee planform area and diameter, respectively; v is the speed and ρ the atmospheric density (Table 1).

The determination of these coefficients is the subject of the remainder of the paper. Given values for the ten coefficients, integration of the differential equations of motion yields the motion dynamics. Furthermore, using the time histories of the 6 rigid body configuration variables (including both center of mass positions and angular orientations), and the known positions of three reflective markers on the Frisbee surface (see description below), it is possible to calculate the predicted inertial xyz coordinates of the markers as functions of time. As the Frisbee translates and rotates the markers follow a sinusoidal path through space. The comparison of these predicted marker trajectories to those determined experimentally in actual flights is the basis of the estimation scheme for the determination of the aerodynamic coefficients.

Table 1 Values of non-aerodynamic simulation parameters.

Parameter units	m kg	I_a kg-m^2	I_d kg-m^2	d m	A m^2	ρ kg/m^3	g m/s^2
	0.175	0.00235	0.00122	0.269	0.057	1.23	9.794

EXPERIMENTAL FLIGHT DATA

High speed (120 Hz) video kinematic data acquisition cameras and standard DLT techniques were used to record the xyz locations of the three markers on the Frisbee during multiple flights between 2 and 20 meters in length. In initial short flights, three reflective tape markers were placed on the top Frisbee surface. Reflectivity of the markers is a strong function of the angle of incidence from the surface normal direction. Thus tracking of the markers proved difficult in later experiments with the longer flights and reflective markers were replaced by active light emitting diodes (LEDs) in a similar triangular configuration on the Frisbee upper surface. This position data, with the dynamic flight model (Hubbard and Hummel, 2000), allows the estimation of the aerodynamic force and moment coefficients using an iterative estimation algorithm.

ESTIMATION ALGORITHM

A MATLAB computer program was written implementing the following steps:

1. Guess 12 initial conditions (for 6 rigid body degrees of freedom and 6 associated velocities) for each of n experimental flights and values for the 10 coefficients. This results in a total of 10+12n unknown parameters to be estimated. The vector of parameters is denoted by **p**.
2. Simulate (integrate the ODE's for) each flight given the parameter vector **p**.
3. Use the simulated state variables to predict the xyz motion of the markers.
4. Calculate the residual, defined as the sum of squared differences between the predicted and measured xyz marker motions for all times over all flights.
5. Using 10+12n other simulations calculate both the gradient and Hessian of the residual with respect to **p**.
6. Determine a correction to the parameters **p** (both initial conditions and coefficients) that reduces the residual using the method described by Hubbard and Alaways (1989).
7. Return to step 2 until the residual is below a minimum value.

We found it necessary to use data from several (four) flights containing trajectory information over a wide range of α to yield accurate coefficient estimates.

RESULTS

The results of the estimation algorithm are shown in Table 2. The first six coefficients are compared to linear approximations of the lift, drag and pitching moment measured by Potts and Crowther (2000). Our results using short flights agree well with these slopes verifying that the method is indeed valid. Generally agreement is good, with errors ranging from about 5% in the case of C_{Lo} to about a factor of two in several of the other coefficients.

Table 2 Comparison of estimated coefficients to measured values from Potts & Crowther (2000).

	C_{Lo}	$C_{L\alpha}$	C_{Do}	$C_{D\alpha}$	C_{Mo}	$C_{M\alpha}$	C_{Mq}	C_{Lr}	C_{Lp}	C_{Nr}
Four Flights	0.188	2.37	0.15	1.24	-0.06	0.38	0.0008	0.0004	-0.013	-2.8E-5
P & C	0.2	2.96	0.08	2.60	-0.02	0.13	---	---	---	---

The final four coefficients in Table 2 correspond to the dependence of pitching rolling and yawing moments on the three components of angular velocity. Because wind tunnel tests to determine these coefficients are, generally, extremely difficult if not impossible to perform, parameter estimation provides an attractive alternative. Although a wind tunnel test including single axis rotation could be designed to measure the spin-down coefficient, C_{Nr}, this technique in this paper using parameter estimation is the only way we know to obtain estimates of the other three coefficients C_{Mq}, C_{Rr}, and C_{Rp}.

DISCUSSION

Residual prediction errors in the x, y, and z components of marker positions are shown in Fig. 2. In the figure, each component of position error for three markers and for all four flights is shown on the same axis. The rms position error ranges from about 1 mm to roughly 4 mm. This indicates that the measurement accuracy probably varied between flights. Nevertheless, there is very little correlation observable in the apparently nearly white random noise of the position error traces.

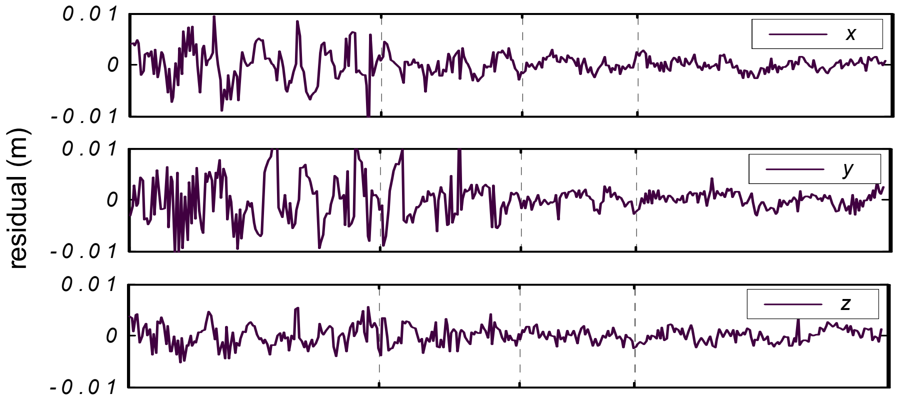

Fig. 2 Prediction errors in x, y, and z for all three markers for four flights: bsfl5, fffl10, fffl14, and ffsl5, shown sequentially.

The first six coefficients relate not only to their mean values of lift, drag and pitching moment, but also their sensitivities to angle of attack. In order for their dependence on angle of attack to be discernible, it is necessary that data be available over a wide range of angles of attack.

Shown in Fig. 3 are the calculated angle of attack time histories for the four flights. Although the angles of attack of the second and third flights (fffl10 and fffl14) varied over very small ranges (only about 6 and 5 deg, respectively), the entire ensemble of four flights experienced angles of attack spanning roughly $0 < \alpha < 30$ degrees, and this provided enough information for a robust determination of the six coefficients associated with angle of attack. This illustrates an important feature of the estimation algorithm. Although it is difficult in practice to obtain data for a single flight with large variations in angle of attack, this failure can be compensated by data from other flights.

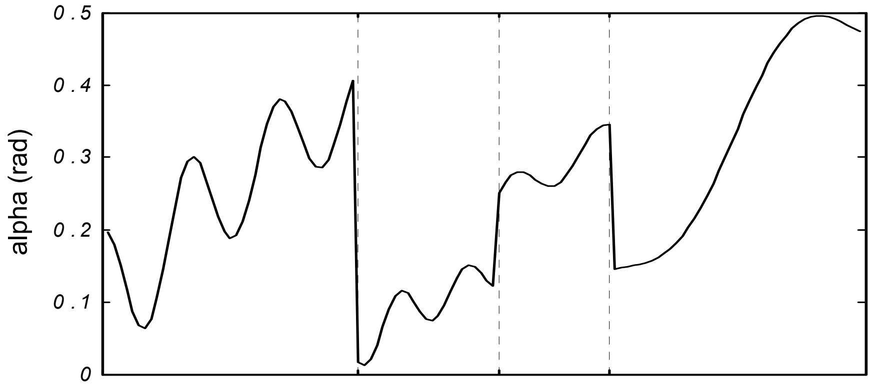

Fig. 3 Angle of attack versus time for four flights: bsfl5, fffl10, fffl14, and ffsl5 shown sequentially. Wide range of angle of attack are essential for accurate coefficient estimates.

Some of the lack of agreement in Table 2 between estimated coefficients and those determined from wind tunnel tests can be attributed to approximations in the model. For example the Robins-Magnus side force, neglected here, may cause errors in other coefficients in order to account for motion actually produced by the neglected force.

CONCLUSIONS

A method has been presented for the determination of aerodynamics coefficients for the Frisbee, that is a practical alternative to expensive and time-consuming wind tunnel tests. The method uses an iterative algorithm to find the set of parameters (aerodynamic coefficients and rigid body initial conditions for each flight) that, when used with a numerical simulation model, result in minimum rms position errors between predicted and measured positions for three markers on the Frisbee surface.

Not only does this technique give results comparable to those achieved in the direct measurement of forces and moments, but it is the only practical way of obtaining estimates for certain of the coefficients relating moments about one axis to angular velocities about a second, perpendicular, axis. The accuracy of the estimation algorithm is highly dependent on the accuracy of the measured position data, in the sense that the variances of the coefficient estimates are directly proportional to the mean squared positions measurement noise (Hubbard and Alaways, 1989). Future work will focus on using the method with a more complex aerodynamic model including lateral Magnus-Robins forces arising from spin and incorporating data from the longer flights.

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