

# 《工程硕士数学》第一次作业

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## 第一章 13(2)

$$\|A\|_1 = \max(2+1, 2+1+1, 2+1) = 4$$

$$\|A\|_2 = \sqrt{\rho(A^T A)} = \sqrt{\max(4, 6+4\sqrt{2}, 6-4\sqrt{2})} = 2 + \sqrt{2}$$

$$\|A\|_\infty = \max(2+1, 2+1+1, 2+1) = 4$$

$$\rho(A) = \max(2, 2+\sqrt{2}, 2-\sqrt{2}) = 2 + \sqrt{2}$$

## 第一章 19

使用顺序主子式法。

- $i = 1$ , 顺序主子矩阵为  $A_{11} = [2]$ , 顺序主子式显然为正
- $i = 2$ , 顺序主子矩阵为  $A_{22} = \begin{bmatrix} 2 & a \\ a & 2 \end{bmatrix}$ , 行列式  $|A_{22}| = 4 - a^2$ , 其为正的条件为  $-2 < a < 2$
- $i = 3$ , 顺序主子矩阵为  $A_{33} = A$ , 行列式  $|A_{33}| = -4a^2 - 2a + 12$ , 其为正的条件为  $-2 < a < \frac{3}{2}$

综上,  $a$  的取值范围为  $-2 < a < \frac{3}{2}$ 。

## 第二章 2

见下图



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解: ① 取 2.68 为第一列主元, 有  $\left[ \begin{array}{cccc|c} 2.68 & 3.04 & -1.48 & -0.53 & \\ 1.48 & 0.93 & -1.30 & 1.03 & \\ 2.51 & 1.48 & 4.53 & 0.25 & \end{array} \right]$

消元:  $l_{21} = 1.48 / 2.68 = 0.55224$

$l_{31} = 2.51 / 2.68 = 0.93657$

作变换  $(-l_{21}E_1 + E_2) \rightarrow (E_2)$

$(-l_{31}E_1 + E_3) \rightarrow (E_3)$

有  $\left[ \begin{array}{cccc|c} 2.68 & 3.04 & -1.48 & -0.53 & \\ 0 & -0.74881 & -0.48268 & 1.3227 & \\ 0 & -1.3672 & 5.9161 & 0.54638 & \end{array} \right]$

② 消元时取 1.3672 为第二列主元.

有  $\left[ \begin{array}{cccc|c} 2.68 & 3.04 & -1.48 & -0.53 & \\ -1.3672 & 5.9161 & & 0.54638 & \\ -0.74881 & -0.48268 & & 1.3227 & \end{array} \right]$

$l_{32} = -0.74881 / -1.3672 = 0.54772$

作变换  $(-l_{32}E_2 + E_3) \rightarrow (E_3)$

有  $\left[ \begin{array}{cccc|c} 2.68 & 3.04 & -1.48 & -0.53 & \\ -1.3672 & 5.9161 & & 0.54638 & \\ 0 & -3.7229 & & 1.0234 & \end{array} \right]$

③ 回代:  $x = \begin{bmatrix} 1.4530 \\ -1.5891 \\ -0.27489 \end{bmatrix}$

## 第二章 3

LU分解:  $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & 1 & 0 & 0 \\ \frac{1}{6} & \frac{1}{5} & 1 & 0 \\ -\frac{1}{6} & \frac{1}{10} & -\frac{9}{37} & 1 \end{pmatrix} \begin{pmatrix} 6 & 2 & 1 & -1 \\ 0 & \frac{10}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & \frac{37}{10} & -\frac{9}{10} \\ 0 & 0 & 0 & \frac{191}{74} \end{pmatrix}$

$$\begin{aligned} \text{解} \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{3} & 1 & 0 & 0 \\ \frac{1}{6} & \frac{1}{5} & 1 & 0 \\ -\frac{1}{6} & \frac{1}{10} & -\frac{9}{37} & 1 \end{pmatrix} y &= \begin{pmatrix} 6 \\ 1 \\ 5 \\ -5 \end{pmatrix}, \text{得} y = \begin{pmatrix} 6 \\ -1 \\ \frac{21}{5} \\ -\frac{213}{74} \end{pmatrix} \\ \text{解} \begin{pmatrix} 6 & 2 & 1 & -1 \\ 0 & \frac{10}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & \frac{37}{10} & -\frac{9}{10} \\ 0 & 0 & 0 & \frac{191}{74} \end{pmatrix} x &= \begin{pmatrix} 6 \\ -1 \\ \frac{21}{5} \\ -\frac{213}{74} \end{pmatrix}, \text{得} x = \begin{pmatrix} \frac{151}{191} \\ -\frac{69}{191} \\ \frac{165}{191} \\ -\frac{213}{191} \end{pmatrix} \end{aligned}$$

## 第二章 6

$$\text{Cholesky分解: } A = \begin{pmatrix} 4 & & \\ 1 & 2 & \\ 2 & -3 & 3 \end{pmatrix} \begin{pmatrix} 4 & 1 & 2 \\ & 2 & -3 \\ & & 3 \end{pmatrix}.$$

$$\text{解} \begin{pmatrix} 4 & & \\ 1 & 2 & \\ 2 & -3 & 3 \end{pmatrix} y = \begin{pmatrix} -4 \\ 3 \\ 10 \end{pmatrix}, \text{得} y = \begin{pmatrix} -1 \\ 2 \\ 6 \end{pmatrix}$$

$$\text{解} \begin{pmatrix} 4 & 1 & 2 \\ & 2 & -3 \\ & & 3 \end{pmatrix} x = \begin{pmatrix} -1 \\ 2 \\ 6 \end{pmatrix}, \text{得} x = \begin{pmatrix} -\frac{4}{9} \\ 4 \\ 2 \end{pmatrix}$$

## 第二章 10

LU分解:

$$\begin{aligned} A &= \begin{pmatrix} 1 & & & & & & \\ -\frac{1}{2} & 1 & & & & & \\ & -\frac{2}{3} & 1 & & & & \\ & & -\frac{3}{4} & 1 & & & \\ & & & -\frac{4}{5} & 1 & & \\ & & & & -\frac{5}{6} & 1 & \\ & & & & & -\frac{6}{7} & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & & & & & \\ & \frac{3}{2} & -1 & & & & \\ & & \frac{4}{3} & -1 & & & \\ & & & \frac{5}{4} & -1 & & \\ & & & & \frac{6}{5} & -1 & \\ & & & & & \frac{7}{6} & -1 \\ & & & & & & \frac{8}{7} \end{pmatrix} \\ \text{解} \begin{pmatrix} 1 & & & & & & \\ -\frac{1}{2} & 1 & & & & & \\ & -\frac{2}{3} & 1 & & & & \\ & & -\frac{3}{4} & 1 & & & \\ & & & -\frac{4}{5} & 1 & & \\ & & & & -\frac{5}{6} & 1 & \\ & & & & & -\frac{6}{7} & 1 \end{pmatrix} y &= \begin{pmatrix} 1 \\ \\ \\ \\ \\ \\ \end{pmatrix}, \text{得} y = \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{5} \\ \frac{1}{6} \\ \frac{1}{7} \end{pmatrix} \end{aligned}$$

$$\text{解} \begin{pmatrix} 2 & -1 & & & & & \\ & \frac{3}{2} & -1 & & & & \\ & & \frac{4}{3} & -1 & & & \\ & & & \frac{5}{4} & -1 & & \\ & & & & \frac{6}{5} & -1 & \\ & & & & & \frac{7}{6} & -1 \\ & & & & & & \frac{8}{7} \end{pmatrix} x = \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{5} \\ \frac{1}{6} \\ \frac{1}{7} \end{pmatrix}, \text{得} x = \begin{pmatrix} \frac{7}{8} \\ \frac{3}{4} \\ \frac{5}{8} \\ \frac{1}{2} \\ \frac{3}{8} \\ \frac{1}{4} \\ \frac{1}{8} \end{pmatrix}$$

## 第二章 11

$$cond(A)_\infty = ||A||_\infty ||A^{-1}||_\infty = 6$$

$$cond(B)_2 = \rho(B)\rho(B^{-1}) = (2 + \sqrt{2})(1 + \frac{\sqrt{2}}{2}) = 3 + 2\sqrt{2} \text{ (矩阵B对称)}$$