《工程硕士数学》第七次计算实习

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第二、三题

理论依据

Gauss-Legendre求积公式（五个节点）

Romberg算法

算法推导

- Gauss-Legendre求积公式：$\int_{-1}^{1} f(x)dx = \sum_{k=0}^{n} A_k f(x_k)$，五个节点对应$n=4$，其中$x_k$与$A_k$的值如下所示：

表 8.4

n	x_k	A_k
0	0	2
1	±0.577 350 269 2	1
2	±0.774 596 669 2	0.555 555 555 6
	0	0.888 888 888 9
3	±0.861 136 311 6	0.347 854 845 1
	±0.339 981 043 6	0.652 145 154 9
4	±0.906 179 845 9	0.236 926 885 1
	±0.538 469 310 1	0.478 628 670 5
	0	0.568 888 888 9
5	±0.932 469 514 2	0.171 324 492 4
	±0.661 209 386 5	0.360 761 573 0
	±0.238 619 186 1	0.467 913 934 6
6	±0.949 107 912 3	0.129 484 966 2
	±0.741 531 185 6	0.279 705 391 5
	±0.405 845 151 4	0.381 830 050 5
	0	0.417 959 183 7
7	±0.960 289 856 5	0.101 228 536 3
	±0.796 666 477 4	0.222 381 034 5
	±0.525 532 409 9	0.313 706 645 9
	±0.183 434 642 5	0.362 683 783 4

- Romberg算法：
  1. $h=b-a$，$T(0,0)=\frac{h}{2}(f(a)+f(b))$
  2. 将区间$[a,b]$分半，$T(1,0)=T(\frac{b-a}{2})$，$T(1,1)=\frac{4T(1,0)-T(0,0)}{4^1-1}$，$1\to j$，转4
  3. 对区间作2^j等分，$T(j,0)=T(\frac{b-a}{2^j})$，$T(j,k)=\frac{4^kT(j,k-1)-T(j-1,k-1)}{4^k-1}$，$k=1,2,\cdots,j$，求出$T(j,j)$，转4
  4. $|T(j,j)-T(j-1,j-1)|<\varepsilon$，则$T(j,j)$即为所求；否则$j+1\to j$，转3

计算代码

```
% Gauss-Legendre
xk = [0, 0.9062, -0.9062, 0.5385, -0.5385];
Ak = [0.5689, 0.2369, 0.2369, 0.4786, 0.4786];
s = 0;
for k = 1.1:0.2:2.9
    ss = 0;
    for n = 1:5
        ff = @(t) 1 / 10 * ((10 / (t / 10 + k)) ^ 2) * sin(10 / (t / 10 + k));
        ss = ss + Ak(n) * ff(xk(n));
```

```
    end
    s = s + ss;
end
s

% Romberg
a = 1;
b = 3;
f = @(x) (10 / x) ^ 2 * sin(10 / x);
k = 0;
n = 1;
h = b - a;
T = h / 2 * (f(a)+f(b));
err = 1;
while err >= 1e-4
    k = k + 1;
    h = h/2;
    tmp = 0;
    for i = 1:n
        tmp = tmp + f(a + (2 * i - 1) * h);
    end
    T(k+1, 1)=T(k) / 2 + h * tmp;
    for j = 1:k
        T(k+1, j+1)=T(k+1, j) + (T(k+1, j) - T(k, j)) / (4 ^ j - 1);
    end
    n = n * 2;
    err = abs(T(k+1, k+1) - T(k, k));
end
T
R = T(k+1, k+1)
```

计算结果

两种方法最终计算结果：

- Gauss-Legendre： `s=-1.4260`
- Romberg： `R=-1.4260` ，$T-$表如下：

```
T =

  -56.5195         0         0         0         0         0         0         0
  -52.2329  -50.8040         0         0         0         0         0         0
  -23.8564  -14.3976  -11.9705         0         0         0         0         0
   -6.8278   -1.1516   -0.2685   -0.0828         0         0         0         0
   -2.6815   -1.2994   -1.3093   -1.3258   -1.3307         0         0         0
   -1.7327   -1.4164   -1.4242   -1.4260   -1.4264   -1.4265         0         0
   -1.5022   -1.4254   -1.4260   -1.4260   -1.4260   -1.4260   -1.4260         0
   -1.4450   -1.4260   -1.4260   -1.4260   -1.4260   -1.4260   -1.4260   -1.4260
```