# 《工程硕士数学》第二次作业

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## 第二题 （1）

编程求解：

```
format long;
A = [10, -1, 0; -1, 10, -2; 0, -2, 10];
b = [9; 7; 6];

D = diag(diag(A));
L = -tril(A, -1);
U = -triu(A, 1);
BJ = D \ (L + U);
fJ = D \ b;
BG = (D - L) \ U
fG = (D - L) \ b

x = [0; 0; 0];
for i = 1:1:5
    disp(i)
    x = BG * x + fG
    % x = BJ * x + fJ
end
```

计算结果为：

- J法：

```
BJ =

                   0   0.100000000000000                   0
   0.100000000000000                   0   0.200000000000000
                   0   0.200000000000000                   0


fJ =

   0.900000000000000
   0.700000000000000
   0.600000000000000

     1


x =

   0.900000000000000
   0.700000000000000
   0.600000000000000

     2


x =

   0.970000000000000
   0.910000000000000
   0.740000000000000

     3


x =

   0.991000000000000
   0.945000000000000
   0.782000000000000

     4


x =

   0.994500000000000
   0.955500000000000
   0.789000000000000

     5


x =

   0.995550000000000
   0.957250000000000
   0.791100000000000
```

则解得$x=\begin{bmatrix}0.99555\\0.95725\\0.79110\end{bmatrix}$。

- GS法：

```
BG =

                   0   0.100000000000000                   0
                   0   0.010000000000000   0.200000000000000
                   0   0.002000000000000   0.040000000000000


fG =

   0.900000000000000
   0.790000000000000
   0.758000000000000

     1


x =

   0.900000000000000
   0.790000000000000
   0.758000000000000

     2


x =

   0.979000000000000
   0.949500000000000
   0.789900000000000

     3


x =

   0.994950000000000
   0.957475000000000
   0.791495000000000

     4


x =

   0.995747500000000
   0.957873750000000
   0.791574750000000

     5


x =

   0.995787375000000
   0.957893687500000
   0.791578737500000
```

则解得$x=\begin{bmatrix}0.99579\\0.95789\\0.79159\end{bmatrix}$。

## 第三题 （1）

$B_J=\begin{bmatrix}0&-2&2\\-1&0&-1\\-2&-2&0\end{bmatrix}$，特征值均为0，因此$\rho(B_J)=0<1$，收敛

$B_G=\begin{bmatrix}0&-2&2\\0&2&-3\\0&0&2\end{bmatrix}$，特征值为0、2、2，因此$\rho(B_G)=2>1$，不收敛

> 如将第三行第二列换为1，则$B_J=\begin{bmatrix}0&-2&2\\-1&0&-1\\-2&-1&0\end{bmatrix}$，$B_G=\begin{bmatrix}0&-2&2\\0&2&-3\\0&2&-1\end{bmatrix}$，两矩阵在复数域上的所有特征值均不小于1，因此均不收敛。

## 第六题

$A=\begin{bmatrix}1&a&0\\a&1&a\\0&a&1\end{bmatrix}$，$2D-A=\begin{bmatrix}1&-a&0\\-a&1&-a\\0&-a&1\end{bmatrix}$。

令$A$正定，则$\begin{cases}1>0\\1-a^2>0\\-2a^2+1>0\end{cases}$，解得$-\frac{\sqrt{2}}{2}<a<\frac{\sqrt{2}}{2}$。此时GS法收敛

由于$2D-A$即是把$A$中的$a$全部取相反数，因此其正定条件仍然是$\begin{cases}1>0\\1-a^2>0\\-2a^2+1>0\end{cases}$成立（该不等式组在$a$变为$-a$时不变），故仍然解得$-\frac{\sqrt{2}}{2}<a<\frac{\sqrt{2}}{2}$。故$-\frac{\sqrt{2}}{2}<a<\frac{\sqrt{2}}{2}$时J法收敛。

## 第八题

1. $x^{(k+1)}=L_\omega x^{(k)}+\omega(D-\omega L)^{-1}b$，式中$L_\omega=(D-\omega L)^{-1}[(1-\omega)D+\omega U]$
2. 由于$A$为对称正定的三对角矩阵，因此可按$\omega_b=\frac{2}{1+\sqrt{1-\rho(B)^2}}$计算，解得$\omega_b=1.01282$，$R=-\ln(\omega_b-1)=4.35654$
3. 编程求解：

```
format long;
A = [10, -1, 0; -1, 10, -2; 0, -2, 10]
b = [9; 7; 6];

D = diag(diag(A));
L = -tril(A, -1);
U = -triu(A, 1);
BJ = D \ (L + U);
fJ = D \ b;
```

```
BG = (D - L) \ U;
fG = (D - L) \ b;
wb = 2 / (1 + sqrt(1 - max(eig(BJ) * max(eig(BJ)))));
Lw = (D - wb * L) \ ((1 - wb) * D + wb * U);

x = [0; 0; 0];
for i = 1:1:3
    disp(i)
    x = Lw * x + wb * (D - wb * L) \ b
    % x = BG * x + fG
    % x = BJ * x + fJ
end
```

解得:

```
     1


x =

   0.888605745516403
   0.781137802068314
   0.750634637518024

     2


x =

   0.956326894619684
   0.930032467600919
   0.771170311301206

     3


x =

   0.970538920543274
   0.933722472893268
   0.771654454310314
```

故$x^{(1)} = \begin{bmatrix} 0.88861 \\ 0.78114 \\ 0.75063 \end{bmatrix}$, $x^{(2)} = \begin{bmatrix} 0.95633 \\ 0.93003 \\ 0.77117 \end{bmatrix}$, $x^{(3)} = \begin{bmatrix} 0.97054 \\ 0.93372 \\ 0.77165 \end{bmatrix}$。