ENME808B: APPLIED NONLINEAR CONTROLS

Homework #10

(Due November 19, 2018)

1. [Lemma 8.1] To prove Lemma 8.1, express $e(t) = H(p)[k\phi^T(t)v(t)]$ into the following state-space model:

$$\dot{x} = Ax + b[k\phi^T(t)v(t)]$$
$$e = c^T x$$

Since $H(p)$ is SPR, the Kalman-Yakubovich lemma indicates that given a P.D. symmetric Q there exists a P.D. symmetric P that satisfies the Lyapunov equation:

$$A^T P + PA = -Q$$

Consider the adaptation law $\dot{\phi}(t) = -\mathrm{sgn}(k)\gamma e(t)v(t)$.

(1) Consider the following Lyapunov function candidate:

$$V(x, \phi) = x^T P x + \frac{|k|}{\gamma}\phi^T\phi$$

Show that the time derivative of $V(x, \phi)$ along the trajectory of the system with the given adaptation law is N.S.D., and $e(t)$ and $\phi(t)$ are globally bounded.

(2) Now assume that $v(t)$ is bounded. Show that $\dot{x}(t)$ is bounded. Then, show that $\dot{V}(x, \phi)$ is uniformly continuous. Then, show that $e(t)$ converges to zero.

2. [Example 8.3] Consider the 1st-order system $\dot{y} = -a_p + b_p = y + 3u$ and its reference model $\dot{y}_m = -4y_m + 4r$. Simulate the system with the MRAC law:

$$u = \hat{a}_r(t)r + \hat{a}_y(t)y, \qquad \begin{aligned} \dot{\hat{a}}_r &= -\mathrm{sgn}(b_p)\gamma er = -2er \\ \dot{\hat{a}}_y &= -\mathrm{sgn}(b_p)\gamma ey = -2ey \end{aligned}$$

Choose $\hat{a}_r(0) = \hat{a}_y(0) = 0$ as well as $y(0) = y_m(0) = 0$.

(1) $r(t) = 4$
(2) $r(t) = 4\sin 3t$

Plot the time responses of $y(t)$ and $y_m(t)$ as well as $\hat{a}_r(t)$ and $\hat{a}_y(t)$ in conjunction with a_r^* and a_y^* to assess the quality of on-line parameter estimation.