Real Statistics Using Excel

Everything you need to do real statistical analysis using Excel

Power Regression

Another non-linear regression model is the **power regression** model, which is based on the following equation:

$$y = \alpha x^{\beta}$$

Taking the natural log (see Exponentials and Logs) of both sides of the equation, we have the following equivalent equation:

$$\ln y = \ln \alpha + \beta \ln x$$

This equation has the form of a linear regression model (where I have added an error term ε):

$$y' = \alpha' + \beta x' + \varepsilon$$

Observation: A model of the form ln y = β ln x + δ is referred to as a **log-log regression** model. Since if this equation holds, we have

$$y = e^{\ln y} = e^{\beta \ln x + \delta} = e^{\beta \ln x} e^{\delta} = \left(e^{\ln x}\right)^{\beta} e^{\delta} = e^{\delta} x^{\beta}$$

it follows that any such model can be expressed as a power regression model of form y = αx^{β} by setting $\alpha = e^{\delta}$.

Example 1: Determine whether the data on the left side of Figure 1 is a good fit for a power model.

	A	B	C	D	E
3	Original Data			Log Transformed Data	
4					
5	x	y		ln x	ln y
6	8.1	33		2.0918641	3.4965076
7	69.9	49		4.2470656	3.8918203
8	4.2	19		1.4350845	2.944439
9	14.1	27		2.6461748	3.2958369
10	5.6	23		1.7227666	3.1354942
11	52.1	51		3.9531649	3.9318256
12	44.6	34		3.7977339	3.5263605
13	19.6	32		2.9755296	3.4657359
14	33	28		3.4965076	3.3322045
15	6.7	36		1.9021075	3.5835189
16	30.1	43		3.4045252	3.7612001

Figure 1 – Data for Example 1 and log-log transformation

The table on the right side of Figure 1 shows y transformed into ln y and x transformed into ln x. We now use the Regression data analysis tool to model the relationship between ln y and ln x.

	G	H	I	J	K	L	M
3	SUMMARY OUTPUT						
4							
5	*Regression Statistics*						
6	Multiple R	0.75380689					
7	R Square	0.56822483					
8	Adjusted R Square	0.52024982					
9	Standard Error	0.21085977					
10	Observations	11					
11							
12	ANOVA						
13		*df*	*SS*	*MS*	*F*	*Significance F*	
14	Regression	1	0.526614162	0.526614162	11.84418164	0.007372583	
15	Residual	9	0.4001566	0.044461844			
16	Total	10	0.926770761				
17							
18		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
19	Intercept	2.8128629	0.20614121	13.64532061	2.55931E-07	2.346539088	3.279186717
20	ln x	0.23438143	0.068103695	3.441537686	0.007372583	0.080320171	0.388442694

Figure 2 – Log-log regression model for Example 1

Figure 2 shows that the model is a good fit and the relationship between ln *x* and ln y is given by

$$\ln y = 2.81 + .234 \ln x$$

Applying *e* to both sides of the equation yields

$$y = e^{2.81+.234\ln x} = e^{2.81}x^{.234} = \mathrm{EXP}(2.81) * x^{.234} = 16.7x^{.234}$$

We can also see the relationship between *x* and y by creating a scatter chart for the original data and choosing **Layout > Analysis|Trendline** in Excel and then selecting the Power Trendline option (after choosing More Trendline Options). We can also create a chart showing the relationship between ln *x* and ln y and use Linear Trendline to show the linear regression line (see Figure 3).

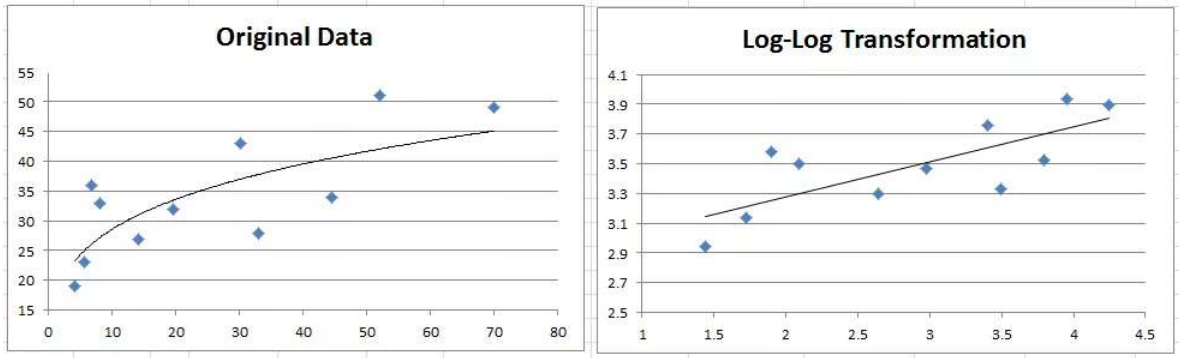

Figure 3 – Trend lines for Example 1

As usual we can use the formula described above for prediction. For example, if we want the y value corresponding to x = 26, using the above model we get

$$\hat{y} = 16.7(26)^{.234} = 35.748.$$

Excel doesn't provide functions like TREND/GROWTH (nor LINEST/LOGEST) for power/log-log regression, but we can use the TREND formula as follows:

=EXP(TREND(LN(B6:B16),LN(A6:A16),LN(26)))

to get the same result.

Observation: Thus the equivalent of the array formula GROWTH(R1, R2, R3) for log-log regression is =EXP(TREND(LN(R1), LN(R2), LN(R3))).

Observation: In the case where there is one independent variable x, there are four ways of making log transformations, namely level-level regression: $y = \beta x + \alpha$

log-level regression: $\ln y = \beta x + \alpha$

level-log regression: $y = \beta \ln x + \alpha$

log-log regression: $\ln y = \beta \ln x + \alpha$

We dealt with the first of these in ordinary linear regression (no log transformation). The second is described in Exponential Regression and the fourth is power regression as described on this webpage. We haven't studied the level-log regression, but it too can be analyzed using techniques similar to those described here.