

8 Supersymmetry breaking and the Standard Model

In this lecture we would like to elaborate a little bit on how the machinery we have been constructing can be used to describe physics beyond the Standard Model under the hypothesis that this is described by a supersymmetry theory.

The basic idea is that the SM should be viewed as an effective theory, valid only up to, say, the TeV scale or slightly higher, and that Nature, at energy above such scale, is described by some suitable $\mathcal{N} = 1$ supersymmetric extension of the SM itself. The most economic option we can think of would be a $\mathcal{N} = 1$ Lagrangian which just includes known particles (gauge bosons, Higgs fields, leptons and quarks) and their superpartners (up to the Higgs sector, which should be doubled, for reasons I mentioned in my first lecture). Then, we might ask whether it is possible to break supersymmetry spontaneously in this theory and be consistent with phenomenological constraints and expectations. Addressing this question will be our main concern, in this lecture.

So far we have discussed possible supersymmetry breaking scenarios at *tree level*: the Lagrangian is supersymmetric but the classical potential is such that the vacuum state breaks supersymmetry (or at least there exist metastable but sufficiently long-lived supersymmetry breaking vacua, besides supersymmetric ones). Can these scenarios, i.e. either F or D-term supersymmetry breaking at tree level, occur in such a minimal supersymmetric extension of the Standard Model (MSSM)? In what follows, we will claim the answer is no: things cannot be as simple as that.

8.1 Towards dynamical supersymmetry breaking

As far as tree level supersymmetry breaking is concerned, from a purely theoretical view point there is at least one point of concern. For several reasons, most notably the hierarchy problem, we would like to have the sparticle masses around the TeV scale (which is not much different from the EW scale, in fact roughly of the same order). This scale is much smaller than any natural UV cut-off one can think of, like the Planck mass. If supersymmetry is broken at tree level, the mass setting the scale of supersymmetry breaking, M_s , would be some mass parameter entering the bare Lagrangian, which would in turn set the scale of all other masses of the SM. For instance, in the O’Raifeartaigh model this scale is μ , the coefficient of the linear term in the superpotential. This way, we would have a scenario where an unnaturally small mass scale has been introduced in a theory in order to solve the

unnatural hierarchy between the EW scale and, say, the Planck scale, and avoid a fine-tuning problem for the Higgs mass. What we gain introducing supersymmetry, would then just be that this small parameter, put by hand into the Lagrangian, would be protected against quantum corrections. Is this a satisfactory solution of the hierarchy problem?

It would be much more natural for this small mass parameter to be explained in some dynamical way. This is possible, in fact. In order to understand how it comes, we should first recall two pieces of knowledge.

First recall that, due to non-renormalization theorems, in a supersymmetric theory of chiral and vector superfields the superpotential is tree-level exact in perturbation theory, meaning that its structure looks like

$$W_{\text{eff}} = W_{\text{tree}} + W_{\text{non-pert}} . \quad (8.1)$$

As we already observed, this implies that if supersymmetry is unbroken at tree level, then it cannot be broken at any order in perturbation theory, but only non-perturbatively.

The second piece of knowledge we need comes from a well-known property that many gauge theories share, i.e. dimensional transmutation. Due to the running of the gauge coupling, which becomes bigger and bigger towards the IR, any UV-free gauge theory possesses an intrinsic (dynamical) scale, Λ , which governs the strong-coupling IR dynamics of the theory. This scale is naturally very small with respect to the scale M_X at which the theory is weakly coupled, according to

$$\Lambda \sim M_X e^{-\frac{\#}{g^2(M_X)}} \ll M_X , \quad (8.2)$$

where $\#$ is a number which depends on the details of the specific theory, but is roughly of order 1.

Suppose now we have some complicated supersymmetric gauge theory which does not break supersymmetry at tree level (so all F-terms coming from W_{tree} are zero), but whose strong coupling dynamics generates a contribution to the superpotential $W_{\text{non-pert}}$ which does provide a non-vanishing F-term. This F-term will be order the dynamical scale Λ , and so will be the scale of supersymmetry breaking. This would imply

$$M_s \sim \Lambda \ll M_X , \quad (8.3)$$

hence giving a natural hierarchy between M_s and the UV scale M_X (which can be the GUT scale, $M_{\text{GUT}} \sim 10^{15}$ GeV, or any other scale of the UV-free theory

under consideration). This idea is known as *Dynamical Supersymmetry Breaking* (DSB). For the reasons I outlined above, it can be considered as the most natural way we can think of supersymmetry breaking in a fully satisfactory way. We will discuss several DSB models in later lectures. For now, let me simply notice that even if supersymmetry were broken dynamically in Nature, what we have learned in the previous lecture has not been by any means a waste of time. As we will see, in DSB models the effective superpotential W_{eff} has typically a O’Raifeartaigh-like structure: at low enough energy gauge degrees of freedom typically disappear from the low energy spectrum (because of confinement, higgsing and alike) and the effective theory ends-up being a theory of chiral superfields, only. The analysis will then follow the one of the previous lecture, but with the great advantage that the mass parameter setting the scale of supersymmetry breaking and sparticle masses has been dynamically generated (and with the complication that in general the Kähler potential will be non-canonical, of course).

8.2 The Supertrace mass formula

There is yet another reason against tree-level supersymmetry breaking in the MSSM, which is more phenomenological in nature, and related to the so-called supertrace mass formula.

Let us consider the most general $\mathcal{N} = 1$ renormalizable Lagrangian (5.78) and suppose that supersymmetry is spontaneously broken at tree level. We want to compute the trace over all bosonic and fermionic fields of the mass matrix squared in an arbitrary supersymmetry breaking vacuum. To this aim, let us suppose that, generically, all F and D auxiliary fields have some non-vanishing VEV.

- *Vectors*

If F and D -fields are non vanishing, this means that some scalar fields ϕ^i have acquired a non-vanishing VEV. If such fields are charged under the gauge group, due to gauge covariant derivatives in the scalar kinetic terms, a mass for some vector fields will be induced since $\overline{D_\mu \phi^i} D_\mu \phi^i \supset g^2 \bar{\phi} T^a T^b \phi A_{a,\mu} A_b^\mu$. Hence, we have for the mass matrix squared of vector bosons

$$[(\mathcal{M}_1)^2]^{ab} = 2g^2 \langle \bar{\phi} T^a \rangle \langle T^b \phi \rangle = 2 \langle D_i^a \rangle \langle D^{bi} \rangle, \quad (8.4)$$

where the lower index refers to the spin, which is one for vectors and we have defined $D_i^a = \partial D^a / \partial \phi^i$, $D^{ai} = \partial D^a / \partial \bar{\phi}_i$.

- *Fermions*

The fermion mass matrix can be easily read from the Lagrangian (5.78) to be

$$\mathcal{M}_{1/2} = \begin{pmatrix} \langle F_{ij} \rangle & \sqrt{2}i\langle D_i^b \rangle \\ \sqrt{2}i\langle D_j^a \rangle & 0 \end{pmatrix}, \quad (8.5)$$

where $F_{ij} = \partial^2 W / \partial \phi^j \partial \phi^i$. The matrix squared reads

$$\mathcal{M}_{1/2} \mathcal{M}_{1/2}^\dagger = \begin{pmatrix} \langle F_{il} \rangle \langle F^{lj} \rangle + 2\langle D_i^b \rangle \langle D_b^j \rangle & -\sqrt{2}i\langle F_{il} \rangle \langle D_b^l \rangle \\ \sqrt{2}i\langle D_l^a \rangle \langle F^{lj} \rangle & 2\langle D_l^a \rangle \langle D_b^l \rangle \end{pmatrix} \quad (8.6)$$

where, with obvious notation, $F^{ij} = \partial^2 \bar{W} / \partial \bar{\phi}_j \partial \bar{\phi}_i$.

- *Scalars*

The scalar mass matrix squared is instead

$$(\mathcal{M}_0)^2 = \begin{pmatrix} \langle \frac{\partial^2 V}{\partial \phi^i \partial \phi_k} \rangle & \langle \frac{\partial^2 V}{\partial \phi^i \partial \phi^l} \rangle \\ \langle \frac{\partial^2 V}{\partial \phi_j \partial \phi_k} \rangle & \langle \frac{\partial^2 V}{\partial \phi_j \partial \phi^l} \rangle \end{pmatrix}. \quad (8.7)$$

Recalling that $V = F^i F_i + \frac{1}{2} D^a D_a$, one can write it as

$$(\mathcal{M}_0)^2 = \begin{pmatrix} \langle F_{ip} \rangle \langle F^{kp} \rangle + \langle D^{ak} \rangle \langle D_i^a \rangle + \langle D^a \rangle D_i^{ak} & \langle F^p \rangle \langle F_{ilp} \rangle + \langle D_i^a \rangle \langle D_l^a \rangle \\ \langle F_p \rangle \langle F^{jkp} \rangle + \langle D^{aj} \rangle \langle D^{ak} \rangle & \langle F_{lp} \rangle \langle F^{jp} \rangle + \langle D^{aj} \rangle \langle D_l^a \rangle + \langle D^a \rangle D_l^{aj} \end{pmatrix}, \quad (8.8)$$

where $D_j^{ai} = -gT_j^{ai}$, $F_{ijk} = \partial^3 W / \partial \phi^i \partial \phi^j \partial \phi^k$, $F^{ijk} = \partial^3 \bar{W} / \partial \bar{\phi}_i \partial \bar{\phi}_j \partial \bar{\phi}_k$.

Taking the trace over gauge and flavor indexes of the three matrices (8.4), (8.6) and (8.8)

$$\begin{aligned} \text{Tr } (\mathcal{M}_1)^2 &= 2\langle D_i^a \rangle \langle D^{ai} \rangle \\ \text{Tr } \mathcal{M}_{1/2} \mathcal{M}_{1/2}^\dagger &= \langle F_{il} \rangle \langle F^{li} \rangle + 4\langle D_i^a \rangle \langle D_a^i \rangle \\ \text{Tr } (\mathcal{M}_0)^2 &= 2\langle F_{il} \rangle \langle F^{il} \rangle + 2\langle D^{ai} \rangle \langle D_i^a \rangle - 2g\langle D^a \rangle \text{Tr } T^a \end{aligned}$$

and finally for the supertrace

$$\text{STr } \mathcal{M}^2 = 3\text{Tr } (\mathcal{M}_1)^2 + \text{Tr } (\mathcal{M}_0)^2 - 2\text{Tr } (\mathcal{M}_{1/2})^2 = -2g\langle D^a \rangle \text{Tr } T^a. \quad (8.9)$$

This formula puts severe phenomenological constraints. First notice that, because of the trace on gauge generators, the r.h.s. is non vanishing only in presence of

$U(1)$ factors. If this is the case, then one needs non trivial FI terms to let the r.h.s. being non-vanishing, since we know that if $\xi = 0$ then also $\langle D^a \rangle = 0$. Now, suppose supersymmetry is broken spontaneously, at tree level, in the MSSM. We have only two $U(1)$ factors we can play with, the hypercharge generator $U(1)_Y$ and, eventually, $U(1)_{\text{em}}$. The latter cannot be of any use since if the corresponding FI parameter ξ were non-vanishing, some squarks or sleptons would get a VEV and hence would break EM interactions (comparing with the FI model, being all MSSM scalars massless at tree level, we will be in the mixed F and D-term phase, and hence the potential would have a minimum at non-vanishing value of some scalar field VEV). As for the hypercharge, this again cannot work, since the trace of $U(1)_Y$ taken over all chiral superfields vanishes in the SM (this is just telling us that the SM is free of gravitational anomalies). The upshot is that within the MSSM, formula (8.9) reduces to

$$\text{STr } \mathcal{M}^2 = 0 . \quad (8.10)$$

It is easy to see that this formula is hardly compatible with observations. Since supersymmetry commutes with internal quantum numbers the vanishing of the supertrace would imply that for any given SM set of fields with equal charge, we should observe at least a real component of a sparticle with a mass smaller than all particles with the same charge. Take a charged $SU(3)$ sector. Gluons are massless, since $SU(3)$ is unbroken. From (8.4) it then follows that $\langle D_i^a \rangle = \langle D^{bi} \rangle = 0$, which, by (8.5), implies that the corresponding gluinos are also massless. Then, in such charged sector, only quarks and squarks can contribute non-trivially to (8.10). Since they contribute with opposite sign, the squarks cannot all be heavier than the heaviest quark, and some must be substantially lighter. For instance, in the color-triplet sector with electric charge $e = -1/3$, to which down, strange and bottom quarks belong, a charged scalar with mass smaller than 7 GeV should exist! This is clearly excluded experimentally.

The upshot of this discussion is that we should give up with the idea that the whole story is as simple as (just) tree level supersymmetry breaking in the MSSM.

8.3 Beyond Minimal Supersymmetric Standard Model

The supertrace condition derived above holds at tree-level, and masses get modified by loop effects. However, within the MSSM such modifications are small since the Standard Model is a weakly interacting theory at the electro-weak scale. So this

does not help much. A way to avoid the supertrace mass formula severe constraints, while still keeping the MSSM, would be to allow for supersymmetry breaking beyond tree-level, that is dynamical supersymmetry breaking. If supersymmetry breaking were transmitted to the MSSM by quantum corrections, there would be effective corrections to kinetic terms from wave-function renormalization which could violate the supertrace mass formula by in principle large amounts, hence allowing for phenomenologically meaningful sparticle spectra. We do have a dynamical scale we can play with in the Standard Model, the $SU(3)$ strong coupling scale Λ_{QCD} . However, DSB driven by QCD strong coupling dynamics could not work either. Looking at eq. (8.3) we would expect in this case a supersymmetry breaking scale of order 300 MeV, which is by far too low for accommodating any sensible phenomenology.

The punchline is that we need something more than just the MSSM, to describe beyond the Standard Model physics. We might need new particles and fields and/or new strong interactions. The options we can play with are many, and understanding the correct path of supersymmetry breaking beyond the SM has been, and still is, a matter of concern and great challenge for theoretical physicists. There are, however, at least two basic properties a competitive model should have. Supersymmetry should be broken dynamically, so to generate the low scale we need (much lower than, say, the Planck scale) in a natural way. Second, in order to avoid the unpleasant constraints coming from the supertrace mass formula, we should better rely on non-renormalizable couplings, or loop effects, to transmit this breaking to the MSSM. As we will see shortly, besides invalidating formula (8.10), such an option would also have the free-bonus of providing an extra suppression between the natural scale of the underlying UV theory and the scale of MSSM sparticle masses. Essentially, supersymmetry breaking from either these sources would be suppressed by either loop factors and/or high masses setting the scale of non-renormalizable terms in the Kähler potential. Hence, the primordial supersymmetry breaking scale would not need to be comparable with electro-weak scale; it could be sensibly higher.

8.4 Spurions, soft terms and the messenger paradigm

Let us deviate, for a while, from what we have been saying so far, and come back to what we said at the very beginning of the previous lecture about possible mechanisms for supersymmetry breaking. We have a second option we have not yet considered: explicit supersymmetry breaking by soft terms. Let us suppose we add explicit supersymmetry breaking terms to the MSSM Lagrangian. In order to save

the nice UV properties of supersymmetry, these terms should be UV irrelevant. For instance, if we were to add non-supersymmetric dimensionless couplings, like Yukawa couplings and scalar quartic couplings, we would certainly destroy the pattern of UV cancellations which makes supersymmetry solving, e.g. the hierarchy problem. We can instead add mass terms, and more generally, positive dimension couplings, like cubic scalar couplings. These would simply tell us below which scale UV cancellations will stop working. Such soft supersymmetry breaking Lagrangian will schematically be of the form

$$\mathcal{L}_{\text{soft}} = m_\lambda \lambda \lambda - m^2 \bar{\phi} \phi + b \phi \phi + a \phi^3 + h.c. , \quad (8.11)$$

where λ represents gauginos and ϕ any possible scalar of the MSSM. The first two terms provide masses for gauginos (wino, zino, photino, gluino) and scalar particles (squarks and sleptons, Higgs particles), respectively. The third term, known as B-term, may arise in the Higgs sector and couples the up and down scalar Higgs H_u and H_d . Finally the fourth, known as A-term, corresponds to cubic gauge and flavor singlet combinations of MSSM scalars, e.g. Higgs and left and right squark components. A-terms are in one-to-one correspondence with Yukawa couplings (which belong to the supersymmetric part of the MSSM Lagrangian): each quark and lepton is just substituted by its scalar superpartner.

All terms appearing in eq. (8.11) are UV irrelevant and renormalizable, and it was indeed shown time ago that the full Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{MSSM}} + \mathcal{L}_{\text{soft}} \quad (8.12)$$

is free of quadratic divergences to all orders in perturbation theory. Notice, in passing, that such a Lagrangian would automatically solve the supertrace mass formula problem. Indeed, a Lagrangian like the one above would violate eq.(8.10) precisely by terms of order the sparticle masses, see the expression (8.11), which is, by construction, compatible with observations.

There is a number of very important issues one should discuss regarding the Lagrangian (8.12), including a number of potential problems some of the soft terms could pose, like the so-called supersymmetry flavor, CP and fine-tuning problems, to name a few. This is however beyond the scope of this course, for which we refer instead to the complementary course on Beyond the Standard Model (BSM) physics.

Here we would like instead to emphasize a crucial point. We want to reconnect to our previous discussion and show how such a rather ad hoc soft Lagrangian,

where supersymmetry is broken explicitly, can actually be generated by *spontaneous* supersymmetry breaking in a bigger theory, including fields and interactions beyond the MSSM ones.

First, let us introduce the idea of *spurion* fields. The basic observation is that in a supersymmetric theory any constant, non-zero value for the lowest component of a superfield (a VEV) does not break supersymmetry. Hence, in a supersymmetric Lagrangian each coupling constant can be promoted to a background superfield, a spurion in fact, with a non-vanishing such VEV. Let us consider, for concreteness, the WZ model

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} Z \bar{\Phi} \Phi + \int d^2\theta \left(\frac{1}{2} M \Phi^2 + \frac{1}{6} \lambda \Phi^3 \right) + h.c. , \quad (8.13)$$

and think of Z, M and λ as real and chiral background superfields, respectively. If only their lowest components have a non-vanishing VEV, this is just the WZ model itself. We can include supersymmetry breaking terms in the above Lagrangian by allowing these superfields having higher (scalar!) component VEVs. We can have in general

$$\begin{aligned} \langle Z \rangle &= 1 + \theta^2 B + h.c. + c \theta^2 \bar{\theta}^2 \\ \langle M \rangle &= \mu - \theta^2 F_M \\ \langle \lambda \rangle &= \lambda - \theta^2 F_\lambda . \end{aligned}$$

Plugging these expressions into the Lagrangian (8.13), after integrating out the auxiliary fields of Φ we find for the potential

$$V = V_{\text{susy}} - (c - |B|^2) \bar{\phi} \phi + \left[(F_M + B\mu) \phi^2 + \left(\frac{1}{3} F_\lambda + \frac{1}{2} B\lambda \right) \phi^3 + h.c. \right] , \quad (8.14)$$

where $V_{\text{susy}} = \left| \mu \phi + \frac{1}{2} \lambda \phi^2 \right|^2$. We see that the non supersymmetric contribution to the potential exactly reproduces the second, third and fourth soft terms of the Lagrangian (8.11), upon the trivial identifications

$$\begin{aligned} m^2 &= c - |B|^2 \\ b &= F_M + B\mu \\ a &= \frac{1}{3} F_\lambda + \frac{1}{2} B\lambda . \end{aligned}$$

Following the same logic for the SYM action

$$\mathcal{L} = \int d^2\theta \, \tau W_a^\alpha W_\alpha^a , \quad (8.15)$$

one can seemingly reproduce gaugino masses by promoting the complexified gauge coupling τ to a chiral superfield and provide a non-vanishing constant VEV for its F-term

$$\langle \tau \rangle = \tau + \theta^2 m_\lambda . \quad (8.16)$$

Applying this logic to the Lagrangian (8.12), one can actually write all soft terms by means of spurion couplings, and re-write (8.12) using a pure supersymmetric formalism. This turns out to be a very convenient thing to do when it comes to compute the divergence structure of the theory and prove, e.g., the absence of quadratic divergences.

The whole picture we get, although phenomenologically viable and logically consistent, is still not completely satisfactory. The softly broken MSSM Lagrangian (8.12) has more than 100 free parameters (masses, phases, mixing angles, etc...), meaning there are few unambiguous predictions one can really make. One might want to find some organizing principle, where these many parameters may be naturally explained in terms of some simpler underlying theory.

Here is where we can finally close the gap between soft term breaking and spontaneous supersymmetry breaking. It is enough to promote spurions to fully fledged superfields with their own Lagrangian and kinetic terms. By some suitable and for the time being unspecified mechanism, they acquire non-vanishing F and D-terms spontaneously, and then generate soft terms by their interactions with the MSSM fields via couplings of the kind (8.13). This is the basic idea of the so-called *messenger paradigm*: one imagines a fully renormalizable theory where supersymmetry is broken spontaneously in some hidden sector at some high scale and then communicated to the MSSM fields by non-renormalizable interactions and/or loop effects. After integrating out heavy fields, this will generate effective couplings precisely as those in the Lagrangian (8.13), with non-vanishing F and D-components for some fields. These F and D-terms will then give rise to soft terms through a procedure like the one above. This way, all specific properties that MSSM supersymmetric breaking soft terms should have, will be ultimately generated (and explained) by a larger theory in which supersymmetry breaking occurs spontaneously.

8.5 Mediating the breaking

What are the possible ways in which a scenario as the one outlined above can actually be realized?

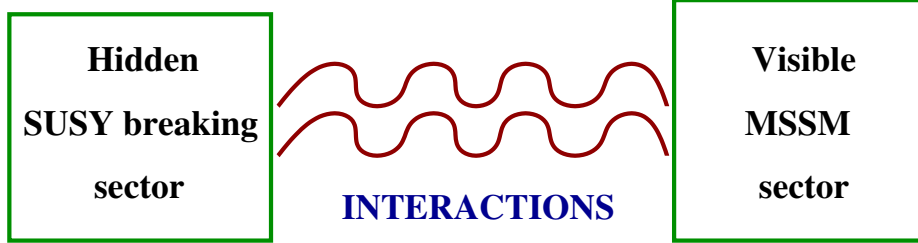


Figure 8.1: The messenger paradigm: supersymmetry is broken in a hidden sector and then communicated to the visible MSSM sector (or any viable supersymmetric extension of the Standard Model) via interactions felt by the MSSM particles.

An obvious candidate as messenger of supersymmetry breaking is gravity, since any sort of particle couples universally to it. Gravity is inherently non-renormalizable, at least as it manifests itself at energies lower than the Planck scale. Hence, couplings like those appearing in eqs. (8.13) and (8.15) are precisely what one expects, in this scenario.

Another possibility is that supersymmetry breaking is mediated by gauge interactions. We can imagine that supersymmetry is broken in the hidden sector, and that some fields, known as *messenger fields*, feeling (or directly participating in, this is a model-dependent property) supersymmetry breaking are also charged under SM gauge interactions. Gauginos will get a mass at one-loop, by direct coupling with messenger fields. Scalar MSSM sparticles, instead, would get mass at two loops, interacting with messenger fields via intermediate MSSM vector superfields, to which gauginos belong to. In this scenario, soft terms will be generated after integrating out heavy fields, ending-up again with effective couplings of the kind (8.13) and (8.15). Obviously, the main source of mediation can be gauge interactions only in a regime where the always present gravity mediation is suppressed. Below we will give an estimate of the regime where such a situation can occur.

In what follows, we are not going to discuss these two mediation mechanisms in detail, nor any of their diverse phenomenological benchmarks, neither the many variants of the basic models which have appeared in the literature, with their pros and cons. For this, we refer again to the BSM course. Here, we just want to show how such scenarios may naturally generate, at low energy, spurion-like couplings with MSSM fields and, eventually, give rise to soft terms.

8.5.1 Gravity mediation

From a low energy point of view, one can parameterize the effect of unknown physics at the Planck scale M_P by higher order operators, suppressed by M_P . Suppose that some hidden sector field X gets a non-vanishing F -term, that is

$$\langle X \rangle = 0 \quad , \quad \langle F_X \rangle \neq 0 . \quad (8.17)$$

The most general form of the Lagrangian describing the gravitational interaction between X and the visible sector fields will be something like

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \int d^2\theta d^2\bar{\theta} \left(\frac{c}{M_P^2} X^\dagger X Q_i^\dagger Q^i + \frac{b'}{M_P^2} X^\dagger X H_u H_d + \frac{b}{M_P} X^\dagger H_u H_d + h.c. \right) \\ & + \int d^2\theta \left(\frac{s}{M_P} X W_a^\alpha W_\alpha^a + \frac{a}{M_P} X Q^i H_u \tilde{Q}_i + h.c. \right) \end{aligned} \quad (8.18)$$

plus, possibly, higher order operators. The Q_i 's represent the up and down Higgs chiral superfields plus all matter superfields, while H_u and H_d obviously refer to the up and down Higgs only. For the sake of simplicity, we have taken all order one dimensionless coefficients in each term to be the same, that is i -independent.

Plugging the values (8.17) into the above Lagrangian one gets all possible MSSM soft terms! The first term on the r.h.s. of eq. (8.18) gives rise to non-supersymmetric masses for all sfermions (squarks, sleptons and scalar Higgs particles), while the second and third terms provide mass terms for the scalar Higgs only (more below). The first term of the second line provides gaugino masses; finally, the last term generates all A-terms. We see that we get a rather simple pattern of soft terms. Up to order one coefficients, they share one and the same mass scale, which we dub m_{SOFT}

$$m_{\text{SOFT}} \sim \frac{\langle F_X \rangle}{M_P} . \quad (8.19)$$

Imposing m_{SOFT} to be order the TeV scale we see that in a gravity mediated scenario the primordial supersymmetry breaking scale, the so-called intermediate scale, is order

$$M_s = \sqrt{\langle F_X \rangle} \sim \sqrt{m_{\text{SOFT}} M_P} \sim 10^{11} \text{ GeV} , \quad (8.20)$$

somewhat in between the EW scale and the Planck scale.

Let us spend a few more words on Higgs mass terms. From the Lagrangian (8.18) we see three contributions to scalar Higgs mass. The first gives rise to mass terms for the up and down Higgs, respectively (they are proportional to $H_u^\dagger H_u$ and $H_d^\dagger H_d$).

The second term is the B-term, which gives rise to a quadratic term mixing H_u and H_d . Finally, as for the third term, notice that it can be re-written as

$$\int d^2\theta d^2\bar{\theta} \frac{b}{M_P} X^\dagger H_u H_d = b \frac{\langle F_X^\dagger \rangle}{M_P} \int d^2\theta H_u H_d . \quad (8.21)$$

This contribution is a so-called μ -term contribution and upon integration in chiral superspace it gives a quadratic contribution similar in structure to the first term.

Notice that *all* these three couplings are needed in order to trigger EW breaking. The first such terms gives masses to scalar Higgs particle, and it can actually give a negative mass square to some of them, something we certainly need to trigger spontaneous symmetry breaking. The second one is also necessary. One can show the B-term to be proportional to $\sin 2\beta$ where $\tan \beta$ is the ratio between the VEVs of the up and down Higgses, $\tan \beta = v_u/v_d$. Clearly, if $B = 0$, either the up or the down Higgs do not get a VEV, and therefore one cannot provide masses to all SM particles. Finally, the μ -term is the only possible contribution which can provide higgsino a mass, and therefore should certainly be there.

The way we have re-written the μ -term makes it clear that it can also be (and generically is) generated from a perfectly supersymmetric superpotential coupling in the MSSM Lagrangian

$$W = \mu_{\text{SUSY}} H_u H_d . \quad (8.22)$$

There is no a priori reason why the above term, which comes from a supersymmetric contribution and is then not related to the dynamics driving the breaking of supersymmetry, should come to be the same scale of the soft terms, as it should. In principle, it could be any scale between m_{SOFT} and M_P . This is the famous μ (or $\mu/B\mu$) problem: how to avoid large μ -terms *and* have them the same order of magnitude of B-terms. Gravity mediation provides an elegant and simple way to solve this problem. First, one can impose some (discrete) symmetry on the MSSM Lagrangian which forbids a tree level μ -term in the superpotential, $\mu_{\text{SUSY}} = 0$. This way, both the μ and the B-terms are generated radiatively. The non trivial thing is to make them be the same order of magnitude. However, as we have seen above, this is exactly what happens in a gravity mediation scenario: up to coefficients of order unity, all soft terms, including the B and μ -terms, are the same order, eq. (8.19)!

A typical problem of gravity mediation scenarios, instead, is the so-called supersymmetry flavor problem. In order not to spoil the excellent agreement between FCNC effects predicted by the SM and known experimental bounds, any sort of new physics should not induce any sensible extra FCNC. In order for this to be the case

the interactions mediating supersymmetry breaking better be flavor-blind. This is not the case for gravity whose high energy UV completion is not actually guaranteed to couple universally to flavor. Therefore, in general, in gravity mediation scenarios one has to confront with the flavor problem. We will not discuss this further here. Let us just remark that there exist different proposals on how to overcome this problem, the most compelling and natural one being possibly the so-called anomaly mediation scenario.

8.5.2 Gauge mediation

Any gauge mediation model is characterized by the assumption that there exist messenger fields. The latter, by definition, are those hidden sector fields which are charged under the SM gauge group. The basic idea of gauge mediation is as follows.

Messengers couple (in a model-dependent way) to hidden sector supersymmetry breaking dynamics and this affects their mass matrix which, besides a supersymmetric contribution (which is supposed to be large enough not to make messengers appear at energies of order the EW scale), receives a non-supersymmetric contribution. By coupling radiatively with MSSM fields, supersymmetry breaking is communicated to MSSM fields and provides soft terms for MSSM sparticles. As we are going to show in the following, gaugini get a mass at one-loop while squarks, sleptons and Higgs fields feel supersymmetry breaking at two loops through ordinary $SU(3) \times SU(2) \times U(1)_Y$ gauge boson and gauginos interactions. One of the beauties of gauge mediation as opposed to gravity mediation, is that gauge mediation supersymmetry breaking can be understood entirely in terms of loop effects in a renormalizable framework. Hence, it has a high level of reliability and calculability.

There are different schemes for gauge mediation, e.g. minimal, direct, semi-direct gauge mediation, which differ, ultimately, by the way the messenger mass matrix is affected by the hidden sector supersymmetry breaking dynamics. This provides different patterns for the MSSM soft terms texture. As an exemplification, in what follows we will briefly discuss minimal gauge mediation (MGM) which is a simple, still rich enough scenario to let us get a feeling on how things work. In MGM all complicated hidden sector dynamics is parameterized in terms of a single chiral superfield X which couples to the messenger sector. The latter is made of two set of chiral superfields Φ and $\tilde{\Phi}$ transforming in complex conjugate representation of the SM gauge group, so not to generate gauge anomalies. The interaction term

is as simple as

$$W = X \tilde{\Phi} \Phi . \quad (8.23)$$

A rough scheme of MGM is depicted in Figure 8.2.

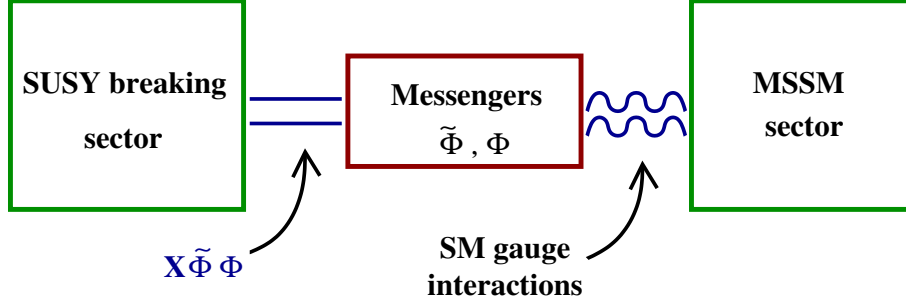


Figure 8.2: Minimal gauge mediation. Messengers feel supersymmetry breaking via a cubic coupling with a spurion-like chiral superfield X which has a non-vanishing F-term VEV inherited from the hidden sector non supersymmetric dynamics.

The spurion-like field X inherits non-vanishing F and lower component term VEVs from the hidden sector,

$$\langle X \rangle = M + \theta^2 \langle F_X \rangle . \quad (8.24)$$

Once plugged into the messenger Lagrangian, this gives a splitted messenger mass spectrum

$$m_{\phi, \tilde{\phi}}^2 = M^2 \pm \langle F_X \rangle \quad , \quad m_{\psi, \tilde{\psi}} = M . \quad (8.25)$$

While fermions receive only the supersymmetric contribution, scalars receive both supersymmetric and non supersymmetric contributions. Recalling that messenger fields are charged under the SM gauge group we see there is a stability bound which forces us to take $M^2 > \langle F_X \rangle$ (if not, some messenger scalars would get a non-vanishing VEV and would break part of the SM gauge group). If M is large enough we can then integrate the messengers out and the effective low energy theory at scale lower than M breaks supersymmetry. The net low energy effect boils down to radiative corrections to gaugino propagators, which get a mass at one loop, while gauge bosons remain massless since they are protected by gauge invariance. Via intermediate SM gauge coupling interactions, also MSSM scalar fields will eventually get a non supersymmetric mass contribution at two loop order (such contributions come from inserting messenger loop corrections in the one-loop sfermions mass diagrams, which in the MSSM, that is without contribution from

the messenger sector, consistently sum-up to zero). Feynman diagrams contributing to gaugino and scalar masses are reported in Figures 8.3 and 8.4, respectively.



Figure 8.3: The one-loop diagram providing gaugino mass. Black lines are MSSM fields, green lines are messenger fields. Dashed lines correspond to scalar fields and continuous lines to fermion fields.

The gaugino mass computation is rather easy, since only one type of diagram contributes. Summing-up all two-loop contributions renormalizing scalar masses is instead not an easy task. However, the end result is surprisingly simple and reads (quite interestingly, and in agreement with the general philosophy advocated in section 8.4, one can equivalently get these results upon integrating out messenger fields and using the RG)

$$m_\lambda \sim \frac{g^2}{16\pi^2} \frac{\langle F_X \rangle}{M} \left[1 + \mathcal{O}\left(\left|\frac{\langle F_X \rangle}{M^2}\right|^2\right) \right] \quad (8.26)$$

$$m_{sf}^2 \sim \left(\frac{g^2}{16\pi^2} \right)^2 \left| \frac{\langle F_X \rangle}{M} \right|^2 \left[1 + \mathcal{O}\left(\left|\frac{\langle F_X \rangle}{M^2}\right|^2\right) \right] . \quad (8.27)$$

We see that in MGM all soft terms come naturally of the same order of magnitude

$$m_{\text{SOFT}} \sim \frac{g^2}{16\pi^2} \frac{\langle F_X \rangle}{M} . \quad (8.28)$$

Imposing again that soft masses are order the TeV scale one then gets

$$\frac{\langle F_X \rangle}{M} \sim 10^5 \text{GeV} , \quad (8.29)$$

which implies that in MGM the primordial supersymmetry breaking scale M_s can be as low as

$$M_s = \sqrt{\langle F_X \rangle} \sim 10 \sqrt{m_{\text{SOFT}} M} \geq 10^5 \text{ GeV} , \quad (8.30)$$

where the lower bound is reached for $M^2 \sim \langle F_X \rangle$.

As we have already observed, gravity mediation is an always present contribution to supersymmetry breaking mediation mechanisms (the field X would also

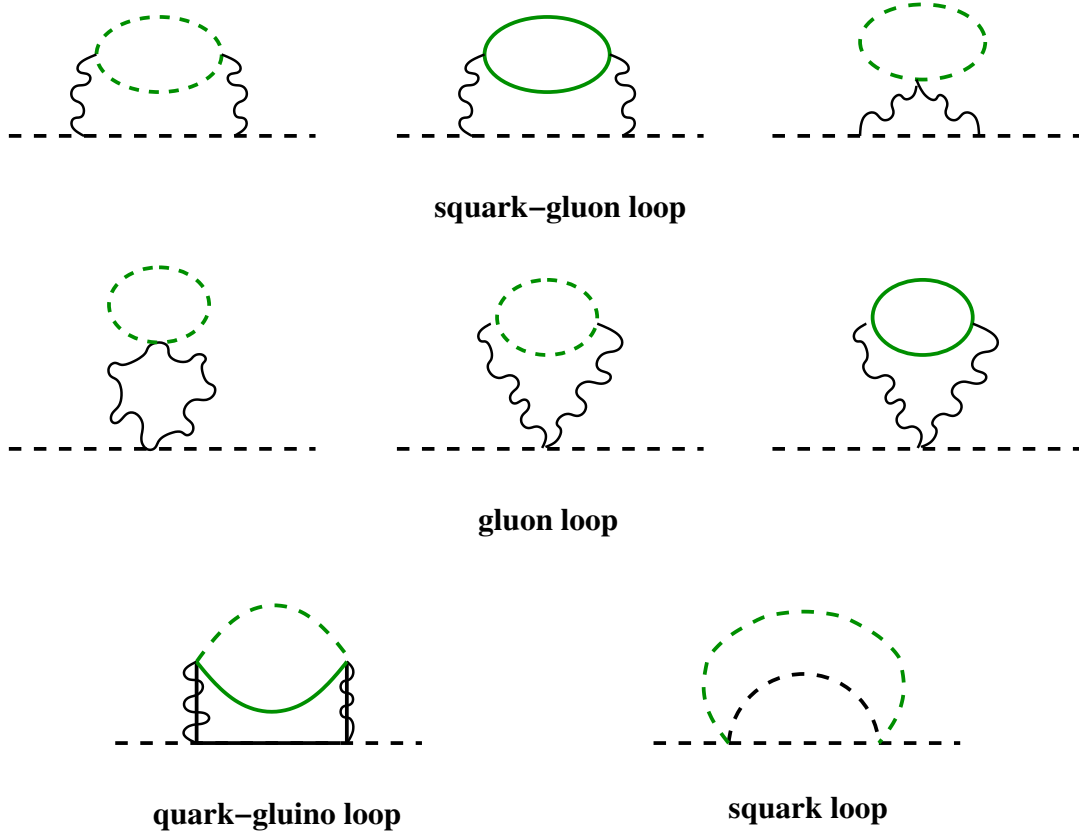


Figure 8.4: Two-loops diagrams providing scalar sparticles mass. There are four different class of diagrams. Conventions are as in Figure 8.3.

interact gravitationally with the visible sector via a Lagrangian like (8.18), in general). Hence, it is only when its contribution is suppressed with respect to that of gauge mediation that the latter can play a role. In order for gravity effects to be negligible, say to contribute no more than $1/1000$ to soft mass squared, one gets an upper bound for the scale M (and hence for $M_s \sim \sqrt{\langle F_X \rangle}$)

$$\frac{g^2}{16\pi^2} \frac{\langle F_X \rangle}{M} \geq 10^{3/2} \frac{\langle F_X \rangle}{M_P} \longrightarrow M \leq \frac{g^2}{16\pi^2} 10^{-3/2} M_P \sim 10^{15} \text{ GeV} . \quad (8.31)$$

Using the relation $M_s \sim 10\sqrt{m_{\text{SOFT}} M}$ this gives an upper bound for M_s of order 10^{10} GeV. Together with the lower bound (8.30) this implies that the supersymmetry breaking scale M_s can range from 10^5 to up to 10^{10} GeV, in gauge mediation scenarios.

Let us notice, in passing, that the simple mass pattern (8.26)-(8.27) is not a generic feature of gauge mediation. Indeed, in another popular scheme, direct gauge

mediation, as well as in semi-direct gauge mediation, the soft spectrum tends to be split, that is gauginos are typically suppressed with respect to scalar particles.

Let us close this brief overview on gauge mediation saying a few words about flavor and μ problems. We are in a sort of reversed situation with respect to gravity mediation. Gauge interactions are intrinsically flavor-blind. Hence, gauge mediation does not provide any further FCNC contribution to the SM, and the flavor problem is automatically solved in this framework. On the contrary, the μ problem is much harder. One can again avoid a supersymmetric μ -term by means of some discrete symmetry to be imposed on the Higgs sector supersymmetric Lagrangian. What is problematic, though, is to generate radiatively μ and B-terms of the same order of magnitude (note that the two-loops diagrams we have discussed above do not provide B and μ -terms). The simplest possible way one can think of, does not work. Indeed, allowing a cubic coupling between H_u , H_d and the field X

$$W_H = \lambda_H X H_u H_d , \quad (8.32)$$

one could in principle generate both a μ and a B-term from supersymmetry breaking dynamics. In order for the μ -term being of the order of other soft masses, as it should be, we need

$$\mu = \lambda_H M \sim 1 \text{ TeV} . \quad (8.33)$$

This implies that λ_H is order 10^{-2} or smaller. This enhances the B-term. Indeed, recalling that $\langle F_X \rangle \leq M^2$, the non-supersymmetric to supersymmetric mass ratio contribution coming from the superpotential coupling (8.32) is

$$\frac{B}{\mu^2} \sim \frac{\lambda_H \langle F_X \rangle}{\lambda_H^2 M^2} \sim \frac{\langle F_X \rangle}{\mu M} \sim 10^2 , \quad (8.34)$$

giving an unacceptably large B-term. This problem is not specific to MGM nor to the actual way we have generated μ and B-terms here. It is a problem which generically plagues any gauge mediation scenario. Even though several proposals has been put forward to solve the μ -problem in gauge mediation, it is fair to say that a fully satisfactory and natural framework to solve this problem is not yet available.

Let me conclude stressing again what is the main point of this all business. What all these mediation models are about, is to provide a theory of the soft terms, a predictive pattern for these extra terms that one can (and has to) add to the MSSM Lagrangian or any desired supersymmetric extension of the Standard Model. We

have been trying to give an idea on how things might work, and reviewed few aspects of the most basic mediation mechanisms. A throughout analysis of the phenomenology of these schemes and their variants is not our goal here. In the remainder of these lectures, we will instead focus on the hidden sector dynamics, trying to deepen our understanding of supersymmetric dynamics at strong coupling. Equipped with new tools to attack non-perturbative regimes of supersymmetric theories, we will eventually be able to study concrete models of dynamical supersymmetry breaking.

8.6 Exercises

1. Derive the gaugino mass formula (8.26) from the Feynman diagram of Figure 8.3.
2. Compute the contribution of two diagrams arbitrarily chosen out of those depicted in Figure 8.4 to the sfermion mass formula (8.27).

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