Hidden sector SUSY breaking II: Gravity mediation

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08.07.2022

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Basis of GR

- Metric: $\eta_{\mu\nu} \to g_{\mu\nu}$
- Derivative: $\partial_\mu \to \nabla_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\nu{}_{\mu\lambda} A^\lambda$

$$\Gamma^\mu{}_{\rho\sigma} = \frac{g^{\mu\lambda}}{2}\left(\partial_\rho g_{\sigma\lambda} + \partial_\sigma g_{\rho\lambda} - \partial_\lambda g_{\rho\sigma}\right)$$

- Geodesic equation:

$$\frac{d^2x^\mu}{dt^2} = 0 \to \frac{d^2x^\mu}{dt^2} + \Gamma^\mu{}_{\rho\sigma}\frac{dx^\rho}{dt}\frac{d^2x^\sigma}{dt} = 0$$

- Canonical coordinates system:

$$\left\{T_p, T_p^*\right\} = \left\{\hat{e}_{(\mu)}, \hat{\theta}^{(\mu)}\right\} = \left\{\partial_\mu, dx^\mu\right\}$$

Vielbein and spin connection

- Vielbeins: ${e_a}^\mu$

$$\hat{e}_{(a)} = {e_a}^\mu \, \hat{e}_{(\mu)} = {e_a}^\mu \partial_\mu$$

- Spin connection : ${\omega^{ab}}_\mu$

$${\omega_\mu}^a{}_b = {e_\nu}^a {e_b}^\lambda \Gamma^\nu_{\mu\lambda} - {e_b}^\lambda \partial_\mu {e_\lambda}^a$$

- Torsion:

$$S^\lambda{}_{\mu\nu} = \frac{1}{2}\left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}\right) \neq 0$$

- Nonminimal covariant derivative :

$$\hat{\nabla}_\mu = \partial_\mu - \frac{i}{4}\omega^{ab}{}_\mu \sigma_{ab}$$

Curvature

- Riemann curvature tensor:

$$R^{\sigma}{}_{\mu\varrho\nu}(\Gamma) = \partial_\varrho \Gamma^{\sigma}{}_{\nu\mu} - \partial_\nu \Gamma^{\sigma}{}_{\varrho\mu} + \Gamma^{\lambda}{}_{\nu\mu}\Gamma^{\sigma}{}_{\varrho\lambda} - \Gamma^{\lambda}{}_{\varrho\mu}\Gamma^{\sigma}{}_{\nu\lambda}$$

$$R^{ab}{}_{\mu\nu}(\omega) = \partial_\mu \omega^{ab}{}_{\nu} - \partial_\nu \omega^{ab}{}_{\mu} + \omega^{ac}{}_{\mu}\omega_{c}{}^{b}{}_{\nu} - \omega^{ac}{}_{\nu}\omega_{c}{}^{b}{}_{\mu}$$

- Ricci scalar: $R = g^{\mu\nu}R_{\mu\nu} = g^{\mu\nu}R^{\varrho}{}_{\mu\varrho\nu}$

$$\begin{aligned} R &= \frac{1}{2}\left(e_a{}^{\mu}e_b{}^{\nu} - e_a{}^{\nu}e_b{}^{\mu}\right)R^{ab}{}_{\mu\nu}(\omega) \\ &= -\frac{1}{4e}e^{p}{}_{\lambda}e^{q}{}_{\varrho}\varepsilon_{pqab}\varepsilon^{\lambda\varrho\mu\nu}R^{ab}{}_{\mu\nu}(\omega) \end{aligned}$$

- Tetrad determinant: $e \equiv \det\left(e^{a}{}_{\mu}\right) = \frac{1}{\det(e_a{}^{\mu})} = \sqrt{-\det\left(g_{\mu\nu}\right)}$

Action and EoM

$$\mathcal{L}_{\mathcal{G}} = M_P^2 \sqrt{-g} R(g, \Gamma) \rightarrow -\frac{M_P^2}{2} e R(e, \omega)$$

- Variating w.r.t. the vielbein, one can obtain the Einstein equations:

$$e\left(R^a{}_\mu - \frac{1}{2} e^a{}_\mu R\right) = \frac{(T_M)^a{}_\mu}{M_P^2}$$

Where we set:

$$R^a{}_\mu = R^{ab}{}_{\nu\mu}(\omega) e_b{}^\nu$$

Supergravity multiplet

Multiplet of supergravity fields:

$$\left\{e_{\mu}{}^{a}, \Psi_{\mu}{}^{a}, b_a, M\right\}$$

- $e_{\mu}{}^{a}$ represents the graviton
- $\Psi_{\mu}{}^{a}$ is a Rarita Schwinger spin 3/2 field for the gravitino
- $\{M, b_a\}$ are the auxiliary terms

Supergravity Lagrangian

The Lagrangian density is:

$$\mathcal{L}_{SG} = -\frac{1}{2}eR - \frac{1}{3}eM^*M + \frac{1}{3}eb^a b_a + \frac{1}{2}e\varepsilon^{\mu\nu\varrho\sigma}\left(\bar{\Psi}_\mu\bar{\sigma}_\nu\nabla_\varrho\Psi_\sigma - \Psi_\mu\sigma_\nu\nabla_\varrho\bar{\Psi}_\sigma\right)$$

That is invariant under:

- general coordinate transformations
- local Lorentz transformations
- local $N = 1$ supersimmetry

Supergravity and d.o.f.

$$e_\mu{}^a : 4 \times 4 = 16$$
$$- 4 \text{ coordinates}$$
$$- 6 \text{ local Lorentz}$$
$$= 6 \text{ bosonic real components}$$

$$\Psi_\mu{}^a : 4 \times 4 = 16$$
$$- 4 \text{ local SUSY}$$
$$= 12 \text{ fermionic real components}$$

$$b_a : \text{real vector}$$
$$= 4 \text{ bosonic real components}$$

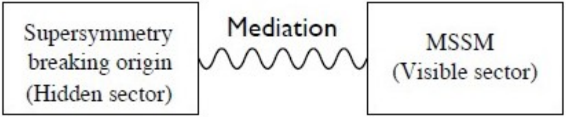
$$M : \text{complex scalar}$$
$$= 2 \text{ bosonic real components}$$

Supergravity as a local symmetry

- Taking into account gravity, SUSY must be promoted to a local symmetry.
- Any locally supersymmetric theory has to include gravity.

Hidden and visible sector

- Supersymmetry breaking occurs in a “hidden sector” of particles that have no direct couplings to the “visible sector” chiral supermultiplets of the MSSM.
- The two sectors share some interactions that are responsible for mediating supersymmetry breaking from the hidden sector to the visible sector, resulting in the MSSM soft terms.



Soft breaking

In the gravity mediated scenario, if SUSY is broken in the hidden sector by a VEV $\langle F \rangle \neq 0$, then the soft terms in the visible sector should be roughly, by dimensional analysis:

$$m_{\text{soft}} \sim \frac{\langle F \rangle}{M_P}$$

because m_{soft} must vanish:

- $\langle F \rangle \to 0$ where supersymmetry is unbroken.
- $M_P \to \infty$ ($G_{\text{Newton}} \to 0$) in which gravity becomes irrelevant.

Scales

$$M_P = \frac{1}{\sqrt{8\pi G_{Newton}}} = 2.4 \times 10^{18}\text{GeV}$$

$$\sqrt{\langle F \rangle} \sim 10^{10} - 10^{11}\text{GeV}$$

$$m_{\text{soft}} \sim 100\text{GeV} - 1\text{TeV}$$

Super-Higgs mechanism

- As long as supersymmetry is unbroken, the graviton and the gravitino are both massless, each with two spin helicity states.
- Once supersymmetry is spontaneously broken, the gravitino acquires a mass by absorbing the goldstino, which becomes its longitudinal (helicity $\pm 1/2$) components.
- The massive spin-3/2 gravitino now has four helicity states, of which two were originally assigned to the would-be goldstino.
- Its interactions will be of gravitational strength, so the gravitino will not play any role in collider physics, but it can be important in cosmology.

Gravity mediated SUSY Breaking

- Let X be the chiral superfield whose F term auxiliary field breaks supersymmetry, and consider first a globally supersymmetric effective Lagrangian, with the Planck scale suppressed effects that communicate between the two sectors included as non-renormalizable terms.
- Expand the superpotential W, the Kähler potential K and the gauge kinetic function f_{ab} for large M_P.

Gravity mediated SUSY Breaking in mSUGRA

In general, the superpotential and Kähler potential have terms coupling X to the MSSM fields as:

$$W = W_{\mathrm{MSSM}} - \frac{1}{M_P}\left(\frac{\alpha}{6}y^{ijk}X\Phi_i\Phi_j\Phi_k + \frac{\beta}{2}\mu^{ij}X\Phi_i\Phi_j\right) + \dots,$$

$$K = |\Phi^2| + \frac{n}{M_P}\left(X + X^*\right)|\Phi^2| - \frac{k}{M_P^2}XX^*|\Phi^2| + \dots,$$

$$f_{ab} = \frac{\delta_{ab}}{g_a^2}\left(1 - \frac{2}{M_P}fX + \dots\right).$$

Where Φ^i represent the chiral superfields of the MSSM, $\alpha, \beta, n, k \in \mathbb{R}$ and y^{ijk}, f are dimensionless coupling, while μ^{ij} has mass dimensions, W_{MSSM} is:

$$W_{MSSM} = \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j$$

Gravity mediated SUSY Breaking in mSUGRA

Effective field theory non-renormalizable Lagrangian (part of) that couples F to the MSSM scalar fields ϕ_i and gauginos λ^a:

$$\begin{aligned}\mathcal{L}_{\text{GMSB}} &= -\left(\frac{F}{2M_P} f \lambda^a \lambda^a + \text{ c.c. }\right) - \frac{|F^2|}{M_P^2}(k + n^2)|\phi^2| \\ &\quad - \left(\frac{F}{6M_P} \alpha y^{ijk} \phi_i \phi_j \phi_k + \frac{F}{2M_P} \beta \mu^{ij} \phi_i \phi_j + \text{ c.c. }\right)\end{aligned}$$

When we replace F by its VEV $\langle F \rangle$, we get exactly the MSSM soft SUSY-breaking Lagrangian.

Soft terms

- Gaugino masses:

$$m_{1/2} = \langle M_a \rangle = \frac{\langle F \rangle}{M_P} f$$

- Scalar couplings:

$$A_0 = \langle a^{ijk} \rangle = \frac{\langle F \rangle}{M_P} (\alpha + 3n)$$

- Scalar squared mass:

$$B_0 = \langle b^{ij} \rangle = \frac{\langle F \rangle}{M_P} (\beta + 2n)$$

$$m_0^2 = \left\langle \left(m^2 \right)^i_j \right\rangle = \frac{|\langle F \rangle|^2}{M_P} (k + n^2)$$

Soft SUSY breaking

Let's introduce the Kähler function:

$$G = M_P^2 \left[K\left(\frac{\phi_i}{M_P}, \frac{\bar{\phi}^{*i}}{M_P} \right) - \ln \left(\frac{W(\phi_i) W(\phi_i)^*}{M_P^6} \right) \right]$$

with:

- K : Kähler potential
- W : Superpotential

$$G^i = \frac{\delta G}{\delta \phi_i};\ G_i = \frac{\delta G}{\delta \phi^{*i}};\ G^i{}_j = \frac{\delta^2 G}{\delta \phi_i \delta \phi^{*j}} = K^j{}_i M_P^2$$

Soft SUSY breaking

$$V = V_F + V_D$$

We also make the following ansatz:

- minimal Kähler potential:

$$K = \varphi^{*i}\varphi_i + X^* X$$

- divide the fields φ_i into a visible sector including the MSSM fields φ_i, and a hidden sector containing a field X that breaks supersymmetry:

$$W = W_{\text{vis}}(\varphi_i) + W_{\text{hid}}(X)$$

Soft SUSY breaking

We can write the generalization of the F-term contribution to the scalar potential in ordinary renormalizable global supersymmetry:

$$V_F = -M_P^2 e^{-G/M_P^2}\left[G^i (G^{-1})^j{}_i G_j + 3M_P^2\right] = G^j{}_i F_j F^{*i} - 3e^{-G/M_P^2}\frac{WW^*}{M_P^2}$$

where F_i are order parameters for SUSY breaking in supergravity:

$$F_i = \frac{\partial W}{\partial \phi_i} = M_P e^{-G/(2M_P^2)}(G^{-1})^j{}_i G_j = e^{-G/(2M_P^2)}\left(W_j^* + W^* K_j\right)$$

Gravitino mass

Local Supersimmetry will be broken if one or more of the F_i obtain a VEV. Te gravitino then absorbs the would-be goldstino and obtains a mass:

$$m_{3/2}^2 = \frac{\langle G^i_{\ j} F_i F^{*j} \rangle}{3M_P^2}$$

Thus, if one impose the constraint: $\langle V \rangle = 0$, one obtain:

$$\langle G^i_{\ j} F_i F^{*j} \rangle = 3e^{-\langle G \rangle / M_P^2} \frac{|\langle W \rangle^2|}{M_P^2} = 3M_P^4 e^{-\langle G \rangle / M_P^2}$$

Giving the gravitino mass:

$$m_{3/2} = M_P e^{-\langle G \rangle / (2M_P^2)}$$

Results

- We treated the spontaneously broken N=1 supergravity theory in which we have highlighted the transmission of supersymmetry breaking from the hidden to the observable sector via gravitational interactions.

Cosmology

- LSP, then it is stable and its primordial density could easily exceed the critical density, causing the universe to become matter-dominated too early.
  Upper bound: $T_{\text{max}} \sim 10^{10}\text{GeV}$, the temperature at which the ordinary radiation-dominated Universe starts or the reheating temperature after an inflationary epoch. This bound is rather uncomfortable for many inflation scenarios.
- Not LSP, lifetime: $\tau \sim 10^{6}\text{sec}(\text{TeV}/m_{3/2})^{3}$.This late decay leads to an enormous entropy production after nucleosynthesis, unless its has not been diluted after the original thermalization.

Flavour Problem

Supergravity does not guarantee flavor-blindness of the soft terms. The soft terms are generated at the Planck scale. There is then no obvious reason why the supersymmetry-breaking masses for squarks and sleptons should be flavour-invariant.

Thank you!

Literature

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Backup 1 - Effective lagrangian M_P expansion

- Let X be the chiral superfield whose F term auxiliary field breaks supersymmetry, and consider first a globally supersymmetric effective Lagrangian, with the Planck scale suppressed effects that communicate between the two sectors included as non-renormalizable terms.

$$W = W_{\mathrm{MSSM}} - \frac{1}{M_P}\left(\frac{1}{6}y^{Xijk}X\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{Xij}X\Phi_i\Phi_j\right) + \ldots,$$

$$K = \Phi^{*i}\Phi_i + \frac{1}{M_P}\left(n_i^j X + \bar{n}_i^j X^*\right)\Phi^{*i}\Phi_j - \frac{1}{M_P^2}k_i^j XX^*\Phi^{*i}\Phi_j + \ldots,$$

$$f_{ab} = \frac{\delta_{ab}}{g_a^2}\left(1 - \frac{2}{M_P}f_a X + \ldots\right).$$

Backup 2 - Effective lagrangian M_P expansion

$$\mathcal{L}_{\text{soft}} = -\frac{F}{2M_{\text{P}}} f_a \lambda^a \lambda^a - \frac{|F|^2}{M_{\text{P}}^2}\left(k^i_j + n^i_p \bar{n}^p_j\right)\phi^{*j}\phi_i - \frac{F}{M_{\text{P}}} n^j_i \phi_j W^i_{\text{MSSM}} + \text{ c.c.}$$
$$- \frac{F}{6M_{\text{P}}} y^{Xijk}\phi_i\phi_j\phi_k - \frac{F}{2M_{\text{P}}}\mu^{Xij}\phi_i\phi_j$$

And the visible sector superpotential:

$$W_{\text{MSSM}} = \frac{1}{6} y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j$$

Backup 3 - Effective lagrangian M_P expansion

- Gaugino masses: $M_a = \frac{\langle F \rangle}{M_P} f_a$
- Scalar couplings: $a^{ijk} = \frac{\langle F \rangle}{M_P} \left(y^{Xijk} + n_p^i y^{pjk} + n_p^j y^{pik} + n_p^k y^{pij} \right)$
- Scalar squared mass: $\left(m^2 \right)_j^i = \frac{|\langle F \rangle|^2}{M_P^2} \left(k_j^i + n_p^i \bar{n}_j^p \right)$
- Scalar squared mass: $b^{ij} = \frac{\langle F \rangle}{M_P} \left(\mu^{Xij} + n_p^i \mu^{pj} + n_p^j \mu^{pi} \right)$

With dimensionless coupling constants. These SUSY breaking masses are generated at the messenger scale (M_P), we then must use the renormalization group to evolve these parameters from the Planck scale to the weak scale

Backup 4

The largest possible sources for them are non-renormalizable Kähler potential terms, which lead to:

$$\mathcal{L} = -\frac{|F|^2}{M_{\mathrm{P}}^3} x_i^{jk} \phi^{*i} \phi_j \phi_k + c.c.$$

where $x^j k_i$ is dimensionless. In principle, the parameters f_a, k_j^i, n_j^i, y^{Xijk} and μ^{Xij} ought to be determined by the fundamental underlying theory. The familiar flavor blindness of gravity expressed in Einstein's equivalence principle does not, by itself, tell us anything about their form. Therefore, the requirement of approximate flavor blindness in $\mathcal{L}_{\mathrm{soft}}$ is a new assumption in this framework, and is not guaranteed without further structure.

Backup 5 - mSUGRA

Nevertheless, it has historically been popular to make a dramatic simplification by assuming a "minimal" form for the normalization of kinetic terms and gauge interactions in the non-renormalizable Lagrangian. Specifically, it is often assumed that there is a common $f_a = f$ for the three gauginos, that, $k_i^j = k\delta_i^j$, and $n_i^j = n\delta_i^j$ are the same for all scalars, with $k, n \in \mathbb{R}$, and that the other couplings are proportional to the corresponding superpotential parameters, so that $y^{Xijk} = \alpha y^{ijk}$ and $\mu^{Xij} = \mu^{ij}$ with universal real dimensionless constants α, β.

Backup 6 - mSUGRA

A popular approximation is to start this RG running from the unification scale $M_U \simeq 1.5 \times 10^{16}$GeV instead of M_P. The reason for this is more practical than principled; the apparent unification of gauge couplings gives us a strong hint that we know something about how the RG equations behave up to M_U, but unfortunately gives us little guidance about what to expect at scales between M_U and M_P. The errors made in neglecting these effects are proportional to a loop suppression factor times $\ln(M_P/M_U)$. These corrections hopefully can be partly absorbed into a redefinition of $m_0^2, m_{1/2}, A_0$, B_0, at M_U, but in many cases will lead to other important effects that are difficult to anticipate.

Majorana gaugino

$$\mathcal{L}^{M_\lambda}_{\mathrm{SUGRA}} = \frac{1}{4} M_P e^{-\langle G\rangle/(2M_P^2)} \langle G^l (G^{-1})^k{}_l f^*_{ab,k}\rangle \lambda^a \lambda^b + h.c.$$

$$M_\lambda = \frac{1}{2} m_{3/2}\,\mathrm{Re}\left\{\langle G^l (G^{-1})^k{}_l f^*_{ab,k}\rangle\right\}$$

The point, as already mentioned once, is that supergravity is a nonrenormahzable effective field theory and, in consequence, canonical gaugino kinetic energy terms can in no way be singled out. With the requirement that nonrenormahzable terms be suppressed by inverse powers of the Planck mass, the theory will become renormalizable in the 'flat limit' $M_P \to \infty$ for a fixed $m_{3/2}$.

Majorana gaugino

The statement that the mass $m_{3/2}$ of the gravitino is expected in the gravity mediated supersymmetry breaking scheme to be of the same order as the gaugino massM_λ, can be put on a stronger foundation. One can, in fact, derive an upper bound on the ratio $M_\lambda/m_{3/2}$ from the requirement of the validity of perturbative unitarity till near- Planckian energies. The latter is a reasonable demand since generally one does not expect any nonperturbative behavior in the observable sector till those energies.

Majorana gaugino

Consider the scattering of two massless gauge bosons with helicities λ_1 and λ_2 into two longitudinally polarized gravitinos of helicities σ_1 and σ_2 . The relevant diagrams at the tree level involve exchanges of the gravitino and the graviton as well as of light scalar and pseudoscalar fields which occur in the N=1 supergravity theory, broken spontaneously by the super-Higgs mechanism. The helicity amplitudes for this process have been computed from the vertex Feynman rules of that theory. Let us specifically consider the Jth partial wave projected helicity amplitude:

$$T^J_{\lambda_1,\lambda_2;\sigma_1,\sigma_2} = T^2_{1,-1;1/2,-1/2} = \frac{E^2_{CM}}{288\pi M_P^2}\left(\frac{M_\lambda^2}{m_{3/2}^2} - \frac{6}{5}\right)$$

Majorana gaugino

Partial wave unitary requirement:

$$\mathrm{Re}\left\{T^2_{1,-1;1/2,-1/2}\right\} < \frac{1}{2}$$

That gives the upper bound

$$\frac{M_\lambda}{m_{3/2}} < \sqrt{144\pi + 6/5} \simeq 21$$