Mediation of Supersymmetry Breaking

I. THE MESSENGER PARADIGM

To make a viable model of SUSY breaking, we need either large loop corrections, or non-renormalizable terms in the Kähler potential. SUSY breaking from either of these sources is suppressed, by loop factors and/or by high mass scales. This means that the theory must contain a sector in which SUSY is broken at a scale much larger than the weak scale. This large primordial SUSY breaking will then be communicated to the MSSM fields through 'messenger' interactions.

This gives us a way of thinking about the SUSY flavor problem. Since the interactions of the messengers with the MSSM fields determine the pattern of SUSY breaking in the visible sector, a natural way to avoid additional flavor violation in the MSSM is if the messenger interactions do not violate the flavor symmetries, *i.e.* are 'flavor blind'. We will see that this paradigm gives rise to successful models of SUSY breaking.

II. HIDDEN SECTOR SUSY BREAKING

An obvious candidate for the messenger of SUSY breaking is gravity. From a particle physics perspective, the unique low-energy effective theory of gravity is general relativity.[1] Its consistency requires that gravity couples to matter through the stress-energy tensor, which is the origin of the equivalence principle. Because gravity couples to all forms of energy, it necessarily couples the SUSY breaking sector with the visible sector, even if there are no other interactions between the two sectors. In this case, we refer to the SUSY breaking sector as the 'hidden sector.'

The fact that gravity couples to the stress-energy tensor also means that general relativity is flavor-blind. (Different flavors have different masses, and in this sense couple differently to gravity. But what we want is that there be no *additional* flavor violation beyond the Yukawa couplings.) The difficulty with this in practice is the fact that general relativity is only an effective theory, and requires UV completion above the Planck scale (if not at

[1] This assumes that gravity is mediated by a spin-2 boson, and also assumes locality and Lorentz invariance.

lower scales). It is far from clear that the fundamental theory of gravity can have flavor symmetries that guarantee that the UV couplings of gravity are flavor-blind. In fact, there are strong hints from what little is known about the UV theory of gravity to suggest that the fundamental theory of gravity is unlikely to respect global symmetries such as flavor. One of these hints comes from studies of black holes, where Hawking radiation appears to be incapable of radiating away any global charge that was thrown into the black hole when it was formed. The final stages of black hole evaporation occur when the mass of the black hole becomes of order the Planck mass, and what happens there is not understood. However, requiring the conservation of global quantum numbers seems to require a large number of charged states at the Planck scale (corresponding to all the different possible charges of the initial black hole) which seems unlikely. Another hint comes from string theory, the only known candidate for a fundamental theory of gravity. String theory does not appear to allow exact global symmetries, although the full space of string theory vacua is still poorly understood.

From a low energy point of view, we can parameterize the most general effects of the unknown physics at the Planck scale by higher dimension operators suppressed by powers of the Planck scale M_P. Of particular interest to us are operators that connect the fields in the hidden sector with those of the observable sector. We assume that SUSY is broken in the hidden sector by the F component of a field X, and without loss of generality we shift X so that

$$\langle F_X \rangle \neq 0, \quad \langle X \rangle = 0. \tag{1}$$

We can write the most general interactions between X and the MSSM fields:

$$\begin{aligned} \Delta\mathcal{L} \;=\; & \int d^4\theta \left[\frac{(z_Q)^i_j}{M_P^2} X^\dagger X Q^\dagger_i Q^j + \cdots \right. \\ & \left. + \frac{b}{M_P} X H_u H_d + \frac{b'}{M_P^2} X^\dagger X H_u H_d + \text{h.c.} \right] \\ & + \int d^2\theta \left[\frac{s_1}{M_P} X W_1^\alpha W_{1\alpha} + \cdots \right] + \text{h.c.} \\ & + \int d^2\theta \left[\frac{a_{ij}}{M_P} X Q^i H_u (U^c)^j + \cdots \right]. \end{aligned} \tag{2}$$

When we substitute the SUSY breaking VEV $\langle F_X \rangle$, we find that this generates all the soft SUSY breaking terms of the MSSM: scalar masses-squared (first line), μ- and b-terms (second line), gaugino masses (third line), and a-terms (fourth line). They are characterized by a mass scale of order

$$m_{\rm susy} \sim \frac{\langle F_X \rangle}{M_P}. \tag{3}$$

Taking $m_{\rm susy} \sim TeV$ gives

$$\langle F_X \rangle \sim M_P m_{\rm susy} \sim (10^{11}\ GeV)^2. \tag{4}$$

The scale 10^{11} GeV is often called the 'intermediate scale.' Note that the μ and b terms are generated by the terms with coefficients b and b' in Eq. (2). This provides a very simple solution to the μ problem. Note that these terms are only allowed if the SUSY breaking field X is a singlet, as are the terms with couplings $s_1, \ldots$ that give rise to gaugino masses.

In this approach, it is easy to see how other SUSY breaking terms are further suppressed. For example, the so-called 'C-terms' are generated by operators of the form

$$\Delta\mathcal{L}_{\rm eff} \sim \int d^4\theta \frac{X^\dagger X}{M_P^3} Q H_d^\dagger U^c + \text{h.c.}, \tag{5}$$

which give rise to SUSY breaking trilinear couplings of order $m_{\rm susy}^2/M_P \ll m_{\rm susy}$. 'Hard' SUSY breaking is also small. For example, a fermion kinetic term arises from

$$\Delta\mathcal{L}_{\rm eff} \sim \int d^4\theta \frac{X^\dagger X}{M_P^4} D^\alpha Q \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \bar D^{\dot\alpha} Q^\dagger, \tag{6}$$

which gives $\Delta Z \sim m_{\rm susy}^2/M_P^2 \ll 1$. It is striking that simply writing all possible terms connecting the hidden sector to the visible sector suppressed by powers of a single large scale gives all required SUSY breaking terms (including μ and b terms) all of the same order.

Exercise: *Write the leading additional SUSY breaking allowed in the NMSSM coupled to a hidden sector. Does this automatically give rise to all allowed SUSY breaking of order $m_{\rm susy}$, as in the MSSM? Can we impose symmetries so that all required SUSY breaking is generated with size $m_{\rm susy}$?*

The difficulty with this approach is that the soft masses and a terms can violate flavor. The a terms arise from the terms with coefficients a_{ij}, and we can imagine forbidding these by symmetries acting on the field X. However, the soft scalar masses-squared are generated by the operators with coefficients z_i^j, which are invariant under all the symmetries. Unless there are flavor symmetries at the Planck scale, there appears no reason for these coefficients to be flavor-diagonal or universal. This is the flavor problem of hidden sector models of SUSY breaking.

One way to avoid the flavor problem is to assume that there is a gauged flavor symmetry at the Planck scale. The existence of gauge symmetries (as opposed to global symmetries) is compatible with what is known about string theory and black hole physics. A gauge symmetry must be free of anomalies, but extra fermions can always be added to cancel the anomalies, and once the flavor symmetries are broken, all these extra fermions can in principle become massive. The flavor gauge symmetry must be broken at a high scale to avoid dangerous flavor-changing neutral currents.

A. The 'Minimal SUGRA' ansatz

An ansatz that has been extensively analyzed in the literature assumes that the couplings that give rise to scalar masses are equal to a universal value at $\mu = M_P$:

$$(z_Q)^i_j = (z_L)^i_j = \cdots = z_0 \delta^i_j, \qquad z_{H_u} = z_{H_d} = z_0. \tag{7}$$

This is called 'minimal SUGRA' for historical reasons. One feature of this ansatz is that the up-type Higgs mass runs negative because of the large top Yukawa coupling. This is called 'radiative electroweak symmetry breaking.' We have argued above that, if there is no flavor symmetry at the Planck scale, this ansatz is not natural. Nonetheless, there is an extensive literature on this, so you should at least know what it is.

III. THE GOLDSTINO AND THE GRAVITINO

The spontaneous breaking of global supersymmetry implies the existence of a massless Weyl fermion, the goldstino. The goldstino is the fermionic component of the supermultiplet whose auxiliary field obtains a VEV.

To prove that, consider a general supersymmetric model with both gauge and chiral supermultiplets. The fermionic degrees of freedom consist of gauginos (λ^a) and chiral fermions (ψ_i). After some of the scalar fields in the theory obtain VEVs, the fermion mass matrix has the form

$$m_F = \begin{pmatrix} 0 & \sqrt{2} g_b (\langle \phi^* \rangle T^b)^i \\ \sqrt{2} g_a (\langle \phi^* \rangle T^a)^j & \langle W^{ji} \rangle \end{pmatrix} \tag{8}$$

in the (λ^a, ψ_i) basis. Now observe that m_F annihilates the vector

$$\tilde{G} = \begin{pmatrix} \langle D^a \rangle / \sqrt{2} \\ \langle F_i \rangle \end{pmatrix}. \tag{9}$$

The first row of m_F annihilates $\tilde{G}$ by virtue of the requirement that the superpotential is gauge invariant,

$$\delta_{\text{gauge}} W = W^i (T^a \phi)_i = 0. \tag{10}$$

The second row vanishes because of the condition $\langle \partial V/\partial \phi_i \rangle = 0$, which must be satisfied at a minimum of a scalar potential. Equation (9) is therefore proportional to the goldstino wavefunction; it is non-trivial if and only if at least one of the auxiliary fields has a VEV, breaking supersymmetry.

By using Noether's procedure, one find the conserved supercurrent:

$$\begin{aligned} J^\mu_\alpha &= (\sigma^\nu \bar{\sigma}^\mu \psi_i)_\alpha D_\nu \phi^{*i} - i(\sigma^\mu \psi^{\dagger i})_\alpha W_i^* \\ &+ \frac{1}{2\sqrt{2}} (\sigma^\nu \bar{\sigma}^\rho \sigma^\mu \lambda^{\dagger a})_\alpha F^a_{\nu\rho} - \frac{i}{\sqrt{2}} g_a \phi^* T^a \phi (\sigma^\mu \lambda^{\dagger a})_\alpha. \end{aligned} \tag{11}$$

We can derive an important property of the goldstino by considering (11). Suppose for simplicity that the only non-vanishing auxiliary field VEV is $\langle F \rangle$ with goldstino superpartner $\tilde{G}$. Then the supercurrent conservation equation tells us that

$$0 = \partial_\mu J^\mu_\alpha = i\langle F \rangle (\sigma^\mu \partial_\mu \tilde{G}^\dagger)_\alpha + \partial_\mu j^\mu_\alpha + \cdots, \tag{12}$$

where j^μ_α is the part of the supercurrent that involves all the other supermultiplets, and the ellipses represent other contributions of the goldstino supermultiplet to $\partial_\mu J^\mu_\alpha$, which we can ignore. The first term in Eq. (12) comes from the second term in Eq. (11) , using the equation of motion $F_i = -W_i^*$ for the goldstino's auxiliary field. This equation of motion for the goldstino field allows us to write an effective Lagrangian,

$$\mathcal{L}_{\tilde{G}} = -i\tilde{G}^\dagger \bar{\sigma}^\mu \partial_\mu \tilde{G} - \frac{1}{\langle F \rangle} (\tilde{G} \partial_\mu j^\mu + \text{h.c.}), \tag{13}$$

which describes the interactions of the goldstino with all the other fermion-boson pairs. In particular, since

$$j^\mu_\alpha = (\sigma^\nu \bar{\sigma}^\mu \psi_i)_\alpha \partial_\nu \phi^{*i} + \sigma^\nu \bar{\sigma}^\rho \sigma^\mu \lambda^{\dagger a} F^a_{\nu\rho}/2\sqrt{2} + \cdots, \tag{14}$$

there are goldstino-scalar-chiral fermion and goldstino-gaugino-gauge boson couplings. Since our derivation depends only on supercurrent conservation, Eq. (13) holds independently of the details of how supersymmetry breaking is communicated from $\langle F \rangle$ to the MSSM fields (ϕ_i, ψ_i) and (λ^a, A^a). It may appear strange at first that the interaction couplings in Eq. (13) get larger in the limit $\langle F \rangle \to 0$. However, the interaction term $\tilde{G} \partial_\mu j^\mu$ contains two derivatives, which turn out to always give a kinematic facto proportional to the mass-squared difference of the superpartners when they are on-shell, *i.e.* $m_\phi^2 - m_\psi^2$ and $m_\lambda^2 - m_A^2$. These can be non-zero only by virtue of supersymmetry breaking, so they must also vanish as $\langle F \rangle$, and the interction is well-defined in that limit. Nevertheless, for fixed $m_\phi^2 - m_\psi^2$ and $m_\lambda^2 - m_A^2$, the interaction term in Eq. (13) can be phenomenologically important if $\langle F \rangle$ is not too large.

The above remarks apply to the breaking of global supersymmetry. However, taking into account gravity, supersymmetry must be promoted to a local symmetry. This means that the spinor parameter ϵ^α is no longer a constant, but can vary from point to point in spacetime. The resulting locally supersymmetric theory is called *supergravity*. It necessarily unifies the spacetime symmetries of ordinary general relativity with local supersymmetry transformations. In supergravity, the spin-2 graviton has a spin-3/2 fermion superpartner called the gravitino, which we denote by $\tilde{\Psi}_\mu^\alpha$. The gravitino is R-parity odd ($P_R = -1$). It carries both a vector index μ and a spinor index α, and transforms inhomogeneously under local supersymmetry transformations:

$$\delta \tilde{\Psi}_\mu^\alpha = \partial_\mu \epsilon^\alpha + \cdots. \tag{15}$$

Thus the gravitino should be thought of as the "gauge" field of local supersymmetry transformations. As long as supersymmetry is not broken, the graviton and the gravitino are both massless, each with two spin helicity states. Once supersymmetry is spontaneously broken, the gravitino acquires a mass by absorbing ("eating") the goldstino, which becomes its longitudinal (helicity $\pm 1/2$) components. This is called the *super Higgs mechanism*, and it is analogous to the ordinary Higgs mechanism for gauge theories. The massive spin-3/2 gravitino now has four helicity states, of which two were originally assigned to the would-be goldstino. The gravitino mass is called $m_{3/2}$, and in the case of F-term breaking it can be estimated as

$$m_{3/2} \sim \langle F \rangle / M_P. \tag{16}$$

This follows simply from dimensional analysis, since $m_{3/2}$ must vanish in the limits that supersymmetry is restored ($\langle F \rangle \to 0$) and that gravity is turned off ($M_P \to \infty$). Eq. (16) implies very different expectations for the mass of the gravitino in gravity-mediated and gauge-mediated models, because they usually make very different predictions for $\langle F \rangle$.

In the Planck scale mediated supersymmetry breaking case, the gravitino mass is comparable to the masses of the MSSM sparticles. Therefore $m_{3/2}$ is expected to be at least of order 100 GeV or so. Its interactions will be of gravitational strength, so the gravitino will not play any role in collider physics, but it can be important in cosmology. If it is the LSP, then it is stable and its primordial density could easily exceed the critical density, causing the universe to become matter-dominated too early. Even if it is not the LSP, the gravitino can cause problems unless its density is diluted by inflation at late times, or it decays sufficiently rapidly.

In contrast, GMSB models predict that the gravitino is mush lighter than the MSSM sparticles as long as $M_{\text{mess}} \ll M_P$. The gravitino is (almost certainly) the LSP, and all the MSSM sparticles will eventually decay into final states that include it. Naively, one might expect that these decays are extremely slow. This, however, is not necessarily true, because the gravitino inherits the non-gravitational interactions of the goldstino it has absorbed. This means that the gravitino or, more precisely, its longitudinal (goldstino) components can play an important role in collider experiments. The mass of the gravitino can be ignored for kinematical purposes, as can its transverse (helicity $\pm 3/2$) components, which really do have only gravitational interactions. Therefore, in collider phenomenology discussions, one may interchangeably use the same symbol $\tilde{G}$ for the goldstino and for the gravitino of which it is the longitudinal (helicity $\pm 1/2$) part. By using the effective Lagrangian of Eq. (13), one can compute that the decay rate of any sparticle $\tilde{X}$ into its standard model partner X plus a goldstino/gravitino $\tilde{G}$ is

$$\Gamma(\tilde{X} \to X\tilde{G}) = \frac{m_{\tilde{X}}^5}{16\pi \langle F \rangle^2} \left(1 - \frac{m_X^2}{m_{\tilde{X}}^2}\right)^4 . \tag{17}$$

One factor of $(1 - m_X^2/m_{\tilde{X}}^2)^2$ comes from the derivatives in the interaction term in Eq. (13) evaluated for on-shell final states, and another such factor from the kinematic phase space integral with $m_{3/2} \ll m_{\tilde{X}}, m_X$.

If the supermultiplet containing the goldstino and $\langle F \rangle$ has canonically normalized kinetic terms, and the tree level vacuum energy is required to vanish, then the estimate of Eq. (16) is sharpened to

$$m_{3/2} = \langle F \rangle / (\sqrt{3} M_P). \tag{18}$$

In that case, one can write Eq. (17) as

$$\Gamma(\tilde{X} \to X\tilde{G}) = \frac{m_{\tilde{X}}^5}{48\pi M_P^2 m_{3/2}^2}\left(1 - \frac{m_X^2}{m_{\tilde{X}}^2}\right)^4, \tag{19}$$

and this is how the formula is often presented, although it is less general since it assumes Eq. (18). The decay is faster for smaller $\langle F \rangle$ or, equivalently, for smaller $m_{3/2}$, if the other masses are fixed.

IV. GAUGE-MEDIATED SUPERSYMMETRY BREAKING

In gauge mediated supersymmetry breaking (GMSB) models, the ordinary gauge interactions, rather than gravity, are responsible for the appearance of soft supersymmetry breaking in the MSSM. The basic idea is to introduce some new chiral supermultiplets, called messengers, that couple to the ultimate source of supersymmetry breaking, and also couple to the (s)quarks, (s)leptons and Higgs(inos) of the MSSM through the ordinary $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge boson and gaugino interactions. There is still gravitational comunication between the MSSM and the source of supersymmetry breaking, of course, but that effect is now relatively unimportant compared to the gauge interaction effects,

In contrast to Planck scale mediation, GMSB can be understood entirely in terms of loop effects in a renormalizable framework. In the simplest such model, the messenger fields are a set of left-handed chiral supermultiplets $q, \bar{q}, \ell, \bar{\ell}$ transforming under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$q(3,1)_{-1/3}, \quad \bar{q}(\bar{3},1)_{+1/3}, \quad \ell(1,2)_{+1/2}, \quad \bar{\ell}(1,2)_{-1/2}. \tag{20}$$

These supermultiplets contain messenger quarks $\psi_q, \psi_{\bar{q}}$ and scalar quarks $q, \bar{q}$, and messenger leptons $\psi_\ell, \psi_{\bar{\ell}}$ and scalar leptons $\ell, \bar{\ell}$. All of these paticles must get very large masses so as not to have been discovered already. Assume that they do so by coupling to a gauge-singlet chiral supermultiplet S through a superpotential:

$$W_{\text{mess}} = y_2 S\ell\bar{\ell} + y_3 Sq\bar{q}. \tag{21}$$

The scalar component of S and its auxiliary (F-term) component are each supposed to acquire VEVs, denoted by $\langle S \rangle$ and $\langle F_S \rangle$ respectively. They do so by participating in another part of the superpotential that we call W_{break} but do not specify.

Let us consider the mass spectrum of the messenger fermions and bosons. The fermionic messenger fields pair up to get their masses:

$$\mathcal{L} = -y_2\langle S\rangle\psi_\ell\psi_{\bar{\ell}} - y_3\langle S\rangle\psi_q\psi_{\bar{q}} + \text{h.c.}. \tag{22}$$

The scalar messenger partners have a scalar potential given by (neglecting D-term contributions that do not affect the following discussion):

$$V = \left|\frac{\delta W_{\text{mess}}}{\delta\ell}\right|^2 + \left|\frac{\delta W_{\text{mess}}}{\delta\bar{\ell}}\right|^2 + \left|\frac{\delta W_{\text{mess}}}{\delta q}\right|^2 + \left|\frac{\delta W_{\text{mess}}}{\delta\bar{q}}\right|^2 + \left|\frac{\delta(W_{\text{mess}} + W_{\text{break}})}{\delta S}\right|^2. \tag{23}$$

Suppose that at the minimum of the potential

$$\langle S\rangle \neq 0, \tag{24}$$

$$\langle\delta W_{\text{break}}/\delta S\rangle = -\langle F_S^*\rangle \neq 0, \tag{25}$$

$$\langle\delta W_{\text{mess}}/\delta S\rangle = 0. \tag{26}$$

Replacing S and F_S by their VEVs, one finds quadratic mass terms in the potential for the messenger scalars:

$$\begin{aligned} V &= |y_2\langle S\rangle|^2(|\ell|^2 + |\bar{\ell}|^2) + |y_3\langle S\rangle|^2(|q|^2 + |\bar{q}|^2) - \left(y_2\langle F_S\rangle\ell\bar{\ell} + y_3\langle F_S\rangle q\bar{q} + \text{h.c.}\right) \\ &+ \text{quartic terms.} \end{aligned} \tag{27}$$

The complex scalar messengers $\ell, \bar{\ell}$ thus obtain a mass-squared matrix equal to

$$\begin{pmatrix} |y_2\langle S\rangle|^2 & -y_2^*\langle F_S^*\rangle \\ -y_2\langle F_S\rangle & |y_2\langle S\rangle|^2 \end{pmatrix}, \tag{28}$$

with eigenvalues $|y_2\langle S\rangle|^2 \pm |y_2\langle F_S\rangle|$. In just the same way, the scalar quarks $q, \bar{q}$ get masses-squared $|y_3\langle S\rangle|^2 \pm |y_3\langle F_S\rangle|$. Thus, the effect of supersymmetry breaking is to split each messenger supermultiplet pair apart:

$$\begin{aligned} \ell, \bar{\ell}: \quad m_f^2 = |y_2\langle S\rangle|^2, \quad m_s^2 = |y_2\langle S\rangle|^2 \pm |y_2\langle F_S\rangle|; \\ q, \bar{q}: \quad m_f^2 = |y_3\langle S\rangle|^2, \quad m_s^2 = |y_3\langle S\rangle|^2 \pm |y_3\langle F_S\rangle|. \end{aligned} \tag{29}$$

The supersymmetry violation apparent in this messenger spectrum for $\langle F_S\rangle \neq 0$ is communicated to the MSSM fields through radiative corections. The MSSM gauginos obtain masses from one-loop Feynman diagrams. The scalar and fermion lines in the loop are messenger fields. Gauge-mediation provides that $q, \bar{q}$ messenger loops give masses to the gluino and the bino, and $\ell, \bar{\ell}$ messenger loops give masses to the wino and the bino fields. Computing the one-loop diagrams, one finds that the resulting MSSM gaugino masses are given by

$$M_a = \frac{\alpha_a}{4\pi}\Lambda, \qquad (a = 1, 2, 3), \tag{30}$$

where

$$\Lambda \equiv \langle F_S \rangle / \langle S \rangle. \tag{31}$$

If $\langle F_S \rangle$ were 0, the messenger scalars would be degenerate with their fermionic superpartners and there would be no contribution to the MSSM gaugino masses ($\Lambda = 0$). In contrast, the corresponding MSSM gauge bosons cannot get masses, since their masslessness is protected by gauge invariance. So supersymmetry breaking has been successfully mediated to the MSSM. To a good approximation, Eq. (30) holds for the running gaugino masses at an RG scale Q_0 corresponding to the typical scale of the heavy messenger masses, $M_{\rm mess} \sim y_I \langle S \rangle$ for $I = 2, 3$. The running mass parameters can then be RG-evolved down to the electroweak scale to predict the physical masses.

The MSSM scalars do not get any radiative corrections to their masses at one-loop. The leading contributions come from two-loop diagrams, with messenger scalars and fermions and gauge bosons and gauginos in the loops. Computing these diagrams, one finds that each MSSM scalar ϕ_i gets a mass-squared given by

$$m^2_{\phi_i} = 2\Lambda^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3(i) + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2(i) + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1(i) \right], \tag{32}$$

with the quadratic Casimir invariants $C_a(i)$ given by $(T^a T^a)_i^j = C_a(i)\delta_i^j$, and so

$$C_3(i) = \begin{cases} 4/3 & \text{for } \phi_i = Q, \bar{u}, \bar{d}, \\ 0 & \text{for } \phi_i = L, \bar{e}, H_u, H_d, \end{cases} \tag{33}$$

$$C_2(i) = \begin{cases} 3/4 & \text{for } \phi_i = Q, L, H_u, H_d, \\ 0 & \text{for } \phi_i = \bar{u}, \bar{d}, \bar{e}, \end{cases} \tag{34}$$

$$C_1(i) = (3/5) Y_i^2. \tag{35}$$

The masses-squared in Eq. (32) are positive (fortunately!).

The trilinear scalar couplings a_u, a_d, a_e arise at two-loop order and are suppressed by an extra factor of $\alpha_a/4\pi$ compared to the gaugino masses. So, to a very good approximation, one has, at Q_0,

$$a_u = a_d = a_e = 0. \tag{36}$$

Eqs. (32) and (36) apply at the messenger scale. Evolving the RG equations down to the electroweak scale generates non-zero a_u, a_d, a_e proportional to the corresponding Yukawa matrices and the non-zero gaugino masses. These will only be large for the third generation squarks and sleptons.

The bilinear scalar coupling (the b term) may also vanish at the messenger scale, but this is quite model dependent. In any case, b will become non-zero when it is RG-evolved to the electroweak scale.

Because the gaugino masses arise at one-loop while scalar masses-squared appear at two-loop order, both Eqs. (30) and (32) correspond to the estimate

$$m_{\text{soft}} \sim \frac{\alpha_a}{4\pi} \frac{\langle F_S \rangle}{M_{\text{mess}}}, \tag{37}$$

with $M_{\text{mess}} \sim y_I \langle S \rangle$. Eqs. (30) and (32) hold in the limit of small $\langle F_S \rangle / y_I \langle S \rangle^2$, corresponding to mass splittings within each messenger supermultiplet that are small compared to the overall messenger mass scale. The sub-leading corrections in an expansion in $\langle F_S \rangle / y_I \langle S \rangle^2$ turn out to be quite small unless there are very large messenger mass splittings.

The model we described is often called the minimal model of GMSB. Let us now generalize it to a more complicated messenger sector. Suppose that $q, \bar{q}$ and $\ell, \bar{\ell}$ are replaced by a collection of messengers $\Phi_I, \bar{\Phi}_I$ with a superpotential

$$W_{\text{mess}} = \sum_I y_I S \Phi_I \bar{\Phi}_I. \tag{38}$$

The bar is used to indicate that the left-handed chiral superfields $\bar{\Phi}_I$ transform as the complex comjugate representations of the left-handed chiral superfields Φ_I (that is, $\Phi_I + \bar{\Phi}_I$ consitute a vector-like representation of the gauge group). As before, the fermionic components of each pair $\Phi_I + \bar{\Phi}_I$ pair up to get a mass-squared of $|y_I \langle S \rangle|^2$ and their scalar partners mix to get masses-squared of $|y_I \langle S \rangle|^2 \pm |y_I \langle F_S \rangle|$. The induced MSSM gaugino masses are now

$$M_a = \frac{\alpha_a}{4\pi} \Lambda \sum_I n_a(I), \qquad (a = 1, 2, 3), \tag{39}$$

where $n_a(I)$ is the dynkin index for each $\Phi_I + \bar{\Phi}_I$, in a normalization where $n_3 = 1$ for a $3 + \bar{3}$ of $SU(3)_C$ and $n_2 = 1$ for a pair of $SU(2)_L$ doublets. For $U(1)_Y$, one has $n_1 = 6Y^2/5$ for each messenger pair with hypercharge $\pm Y$. For example, $(n_1, n_2, n_3) = (2/5, 0, 1)$ for $q + \bar{q}$ and $(3/5, 1, 0)$ for $\ell + \bar{\ell}$. Thus, for the minimal model, $\sum_I (n_1, n_2, n_3) = (1, 1, 1)$ and Eq.

(39) is in agreement with Eq. (30). On general group-theoretic grounds, n_2 and n_3 must be integers, and n_1 is always an integer multiple of 1/5.

The MSSM scalar masses iin generalized GMSB framework are

$$m_{\phi_i}^2 = 2\Lambda^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3(i) \sum_I n_3(I) + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2(i) \sum_I n_2(I) + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1(i) \sum_I n_1(I) \right]. \quad (40)$$

In writing Eqs. (39) and (40) as simple sums, we implicitly assume that the messengers are all approximately equal in mass, with

$$M_{\rm mess} \approx y_I \langle S \rangle. \quad (41)$$

Eq. (40) is still not a bad approximation if the y_I are not very different from each other, because the dependence of the MSSM mass spectrum on the y_I is only logarithmic (due to RGE) for fixed Λ. However, if large hierarchies in the messenger sector are present, then the additive contributions to the gaugino and scalar masses from each individual messenger multiplet I should instead be incorporated at the mass scale of that messenger multiplet. Then RGE is used to run these contributions down to the electroweak scale; the individual contributions can be thought of as threshold corrections to this RG running.

Messengers with masses far below the GUT scale affect the running of the gauge couplings and might therefore be expected to ruin the apparent coupling unification. However, if the messengers come in complete $SU(5)$ multiplets, and are not very different in mass, then approximate gauge coupling unification still occurs, but at a higher unification scale M_U. For this reason, a popular class of models is obtained by taking the messengers to consist of N_5 copies of the $5+\bar{5}$ of $SU(5)$, resulting in

$$\sum_I n_1(I) = \sum_I n_2(I) = \sum_I n_3(I) = N_5. \quad (42)$$

Equations (39) and (40) reduce to

$$\begin{aligned} M_a &= \frac{\alpha_a}{4\pi} \Lambda N_5, \\ m_{\phi_i}^2 &= 2\Lambda^2 N_5 \sum_{a=1}^{3} C_a(i) \left(\frac{\alpha_i}{4\pi}\right)^2. \end{aligned} \quad (43)$$

For example, the minimal model corresponds to $N_5 = 1$. A single copy of $10 + \overline{10}$ has dynkin indices $\sum_I n_a(I) = 3$, and so can be substituted for 3 copies of $5+\bar{5}$. Note that the gaugino masses scale like N_5, while the scalar masses like $\sqrt{N_5}$. This means that sleptons and squarks tend to be lighter relative to gauginos for larger values of N_5 in non-minimal models. If, however, N_5 is too large, then the running gauge couplings diverge before they unify at M_U. For messenger masses of order 10^6 GeV or less, one needs $N_5 \leq 4$.

There are many other possible generalizations of the basic gauge-mediation scenario as described above. An important general expectation in these models is that the strongly interacting particles (squarks and the gluino) are heavier than the weakly coupled ones (slepton, bino, winos). The common feature that makes these models highly attractive is that the soft masses-squared of squarks and sleptons depend only on their gauge quantum numbers, leading automatically to degeneracy and suppression of flavor changing effects. The most distinctive phenomenological prediction of GMSB is the fact that the gravitino is the LSP. This can have crucial consequences for both cosmology and collider physics.

APPENDIX A: THE GAUGE MEDIATED SPECTRUM

We compute the induced SUSY breaking masses using an elegant method due to Giudice and Rattazzi [1] that makes essential use of superfield couplings. We treat the messenger scale as a chiral superfield,

$$\mathcal{M} = M + \theta^2 F. \tag{A1}$$

This reduces the problem to how the superfield couplings in the effective theory below the scale M depend on $\mathcal{M}$. The leading dependence for large M is given by RG, making the calculation of the loop diagrams very simple. For example, the value of the standard model gauge coupling at $\mu < M$ can be obtained from the one loop RG equation:

$$\frac{1}{g^2(\mu)} = \frac{1}{g'^2(\Lambda)} + \frac{b'}{8\pi^2} \ln \frac{M}{\Lambda} + \frac{b}{8\pi^2} \ln \frac{\mu}{M}. \tag{A2}$$

Here g' is the gauge coupling in the theory above the scale M, and b' is the beta function coefficient in this theory, while g and b are the corresponding quantities in the effective theory below the scale M. We started the running at an arbitrary scale $\Lambda > M$. For a non-Abelian group,

$$b - b' = N_m, \tag{A3}$$

where N_m is the number of messengers (in the fundamental representation) that get a mass at the scale M. Note that N_m is always positive.

The idea of [1] is to extend the formula Eq. (A2) to superfield couplings. We therefore have

$$\tau(\mu) = \tau'(\Lambda) + \frac{b'}{16\pi^2} \ln \frac{\mathcal{M}}{\Lambda} + \frac{b}{16\pi^2} \ln \frac{\mu}{\mathcal{M}}. \tag{A4}$$

Here τ is the chiral superfield containing the (holomorphic) gauge coupling. Both sides of this equation are now well-defined chiral superfields. We can then compute the gaugino mass just by taking the higher θ component of both sides:

$$\frac{m_\lambda(\mu)}{g^2(\mu)} = -[\tau(\mu)]_{\theta^2} = \frac{b - b'}{16\pi^2} [\ln \mathcal{M}]_{\theta^2} = \frac{N_m}{16\pi^2} \frac{F}{M}. \tag{A5}$$

We have assumed that the only SUSY breaking is contained in $\mathcal{M}$. In particular, the couplings at the cutoff scale Λ have no higher θ components, which means that SUSY is unbroken in the fundamental theory above the scale M. In components, this would have been a finite one-loop computation, but this method reduces it ti a simple RG calculation.

The result Eq. (A5) includes the running from the matching scale M down to scales $\mu < M$. We can find the value of the gaugino mass at the matching scale M by expanding about $\mu = \mathcal{M}$. We illustrate this method here because it is very useful to the scalar masses to be discussed below. For the gaugino mass, we write

$$\tau(\mu) = \tau(\mathcal{M} + \left.\frac{d\tau}{d \ln \mu}\right|_{\mu = M} \ln \frac{\mu}{\mathcal{M}} + \mathcal{O}\left(\ln^2 \frac{\mu}{\mathcal{M}}\right). \tag{A6}$$

When we take the θ^2 component of both sides, the terms of order $\ln^2(\mu/\mathcal{M})$ do not contribute in the limit $\mu \to M$. We then have

$$\lim_{\mu \to M} [\tau(\mu)]_{\theta^2} = [\tau(\mathcal{M}]_{\theta^2} + \left.\frac{d\tau}{d \ln \mu}\right|_{\mu = M} \left[\ln \frac{\mu}{\mathcal{M}}\right]_{\theta^2}. \tag{A7}$$

We then compute

$$[\tau(\mathcal{M}]_{\theta^2} = \frac{F}{M} \frac{\partial \tau(M)}{\partial \ln M} = \frac{F}{M} \frac{\partial \tau'(M)}{\partial \ln M} = \frac{F}{M} \frac{b'}{8\pi^2}. \tag{A8}$$

Note that in our expansion, the UV couplings are held fixed, which is why the result is proportional to the beta function in the theory above the scale M. Putting this together, we obtain

$$\lim_{\mu \to M} [\tau(\mu)]_{\theta^2} = \frac{b' - b}{8\pi^2} \frac{F}{M}, \tag{A9}$$

in agreement with Eq. (A5).

This method is even more powerful when used to compute scalar masses. These can be extracted from the wavefunction coefficient Z via

$$m^2 = -[\ln Z]_{\theta^2 \bar{\theta}^2}. \tag{A10}$$

Here Z is a real superfield, so it depends on $\mathcal{M}$ via the real superfield

$$\ln M \to \ln|\mathcal{M}| = \ln|M| + \frac{1}{2}\left(\theta^2 \frac{F}{M} + \text{h.c.}\right). \tag{A11}$$

Expanding about $\mu = \mathcal{M}$, we have

$$\lim_{\mu\to M}[\ln Z(\mu)]_{\theta^2\bar\theta^2} = [\ln Z(\mathcal{M})]_{\theta^2\bar\theta^2} + \left([\gamma(\mathcal{M})]_{\theta^2}\left[\ln\frac{\mu}{\mathcal{M}}\right]_{\bar\theta^2} + \text{h.c.}\right) + \frac{1}{2}\frac{d\gamma}{d\ln\mu}(M)\left[\ln^2\frac{\mu}{\mathcal{M}}\right]_{\theta^2\bar\theta^2}, \tag{A12}$$

where

$$\gamma(\mu) = \frac{d\ln Z}{d\ln\mu} \tag{A13}$$

is the anomalous dimension in the effective theory below the scale M. As in the calculation of the gaugino mass, we must perform the expansion keeping the UV cutoff fixed, which means that we expand in M in the fundamental theory. We therefore have

$$[\ln Z(\mathcal{M})]_{\theta^2\bar\theta^2} = \frac{1}{4}\left|\frac{F}{M}\right|^2\left(\frac{\partial}{\partial\ln M}\right)^2 \ln Z'(M) = \frac{1}{4}\left|\frac{F}{M}\right|^2\frac{\partial\gamma'}{\partial\ln\mu}(M), \tag{A14}$$

where

$$\gamma'(\mu) = \frac{d\ln Z'}{d\ln\mu} \tag{A15}$$

is the anomalous dimension in the theory above the scale M. Similarly,

$$[\gamma(\mathcal{M})]_{\theta^2} = \frac{1}{2}\frac{F}{M}\frac{\partial}{\partial\ln M}\gamma(g'(M)) = \frac{1}{2}\frac{F}{M}\frac{\partial\gamma}{\partial g_i}(M)\beta'(M), \tag{A16}$$

where $g_i(g_i')$ denotes the dimensionless coupling of the theory below (above) the scale M, and $\beta_i(\beta_i')$ is the corresponding beta function,

$$\beta_i = \frac{dg_i}{d\ln\mu}. \tag{A17}$$

Putting it all together, we obtain

$$\begin{aligned} m^2(M) &= -\lim_{\mu\to M}[\ln Z(\mu)]_{\theta^2\bar\theta^2} \\ &= \frac{1}{4}\left|\frac{F}{M}\right|^2\left[-\frac{\partial\gamma'}{\partial g_i'}\beta_i' + 2\frac{\partial\gamma}{\partial g_i}\beta_i' - \frac{\partial\gamma}{\partial g_i}\beta_i\right]. \end{aligned} \tag{A18}$$

Here all anomalous dimensions are evaluated at $\mu = M$. This shows that the gauge mediated scalar mass at the threshold is a simple function of the anomalous dimensions of the theory. From this equation, we see that the scalar masses arise at two loops, since both γ and β start at one loop.

Note that we have performed a two-loop finite matching calculation using only the RG equations. The threshold corrections are determined compleyely by the anomalous dimensions and beta functions of the theory. This is another illustration of the power of superfield couplings.

Squarks and sleptons do not couple directly to the messengers, so they have $\gamma' = \gamma$ at one loop. (This means that γ' is the same function of the coupling g' as γ is of the coupling γ.) In this case, the expression for the scalar mass simplifies further:

$$m^2(M) = \frac{1}{4}\left|\frac{F}{M}\right|^2 \frac{\partial\gamma}{\partial g_i}(\beta_i' - \beta_i). \tag{A19}$$

The one-loop RGE for a kinetic coefficient (of, say, a quark field) from a gauge loop is

$$\mu\frac{d\ln Z}{d\mu} = \frac{c}{4\pi^2}g^2, \tag{A20}$$

where c is the quadratic Casimir of the field. For a fundamental representation of an $SU(N)$ gauge group, $c = (N^2-1)/(2N)$. Putting this in, we obtain

$$m^2(\mu = M) = \frac{g^4}{(16\pi^2)^2}2cN\left|\frac{F}{M}\right|^2. \tag{A21}$$

Note that the scalar masses are positive at the matching scale $\mu = |M|$, which is certainly a good starting point for a realistic model. RG evolution down to the weak scale can make the up-type Higgs mass turn negative (due to the large top Yukawa coupling), triggering electroweak symmetry breaking.

Using the same technique, we can see that

$$\lim_{\mu\to M}[\ln Z(\mu)]_{\theta^2} = \frac{1}{2}\frac{F}{M}(\gamma' - \gamma). \tag{A22}$$

Again, for particles that do not couple directly to the messengers, $\gamma' = \gamma$ at one loop, and sowe do not get A terms at one loop. (This is also obvious from the fact that there are no one-loop diagrams that could give an A term.) Direct couplings of the quarks and leptons to the messengers violate flavor symmetries, but the Higgs can have nontrivial couplings to the messengers.

It is important to remember that the results above are only the leading resulty in an expansion in powers of F/M^2. In the effective theory below the messenger scale M, these additional terms are parameterized by terms with additional SUSY covariant derivatives, such as

$$\Delta\mathcal{L}_{\text{eff}} \sim \int d^4\theta \left|\frac{D^2 M}{M^2}\right|^2 Q^\dagger Q \sim \left|\frac{F^2}{M^3}\right|^2 \tilde{Q}^\dagger\tilde{Q} + \cdots. \tag{A23}$$

Unlike the leading terms computed above, these terms are not related to the dimensionless couplings of the low-energy theory, and therefore require an independent calculation. It turns out that the scalar mass is very insensitive to corrections unless F is very near $|M|^2$.

[1] G. F. Giudice and R. Rattazzi, Nucl. Phys. B **511**, 25 (1998) [arXiv:hep-ph/9706540].

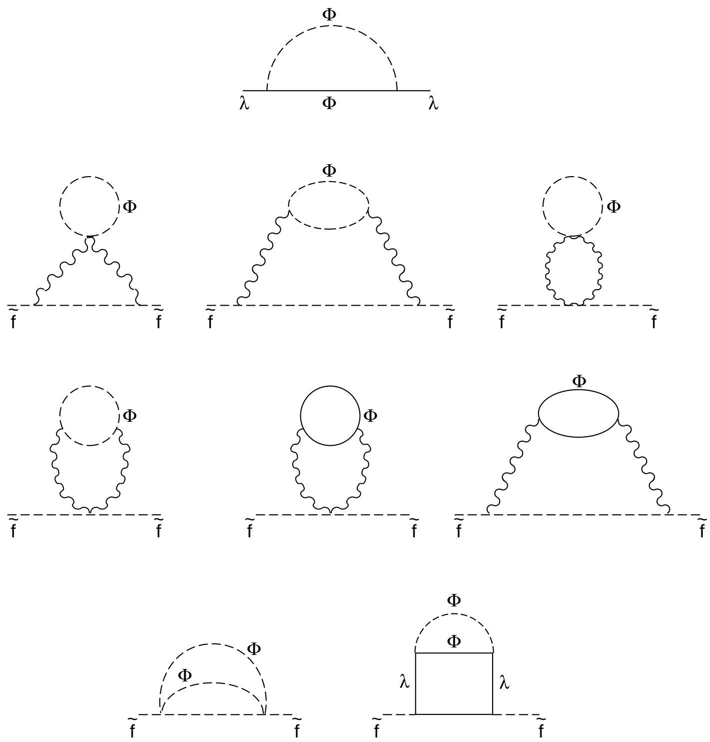

Figure 1: *Feynman diagrams contributing to supersymmetry-breaking gaugino* (λ) *and sfermion* ($\tilde{f}$) *masses. The scalar and fermionic components of the messenger fields* Φ *are denoted by dashed and solid lines, respectively; ordinary gauge bosons are denoted by wavy lines.*

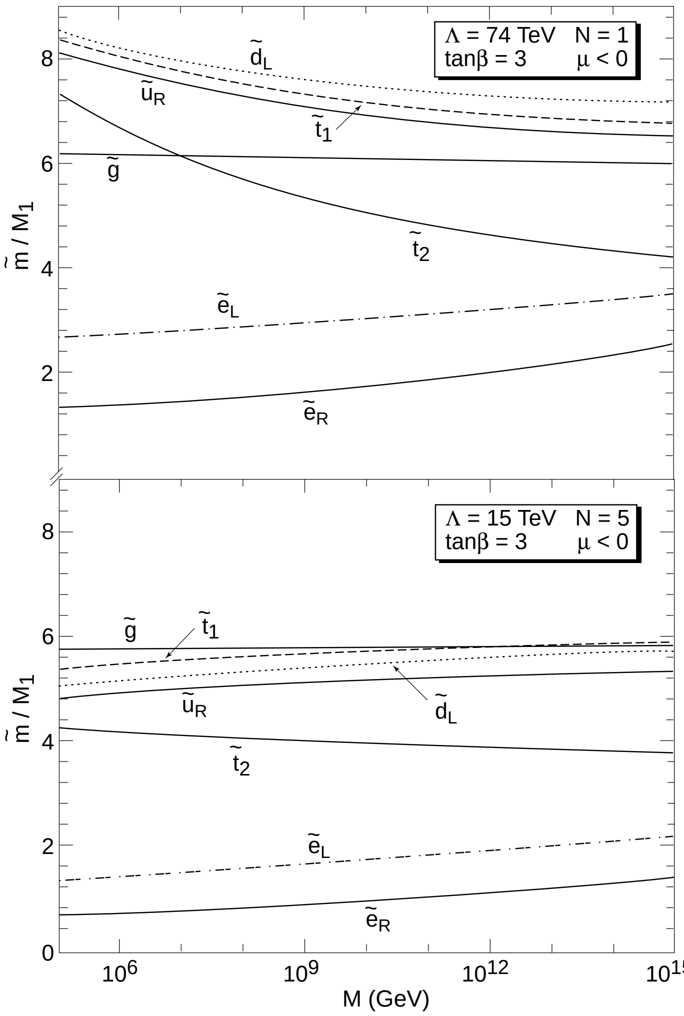

Figure 3: *Different supersymmetric particle masses in units of the B-ino mass M_1, as a function of the messenger mass M. The choice of parameters is indicated, and in both cases it corresponds to a B-ino mass of 100 GeV.*