EE 325: Probability and Random Processes

Assignment 1

Submission deadline on Moodle: Midnight, Tuesday, September 06.

1. Recall the 'capture-release-recapture' problem: Catch m fish, mark them and release them back into the lake. Allow the fish to mix well and then you catch m fish. Of these p are those that were marked before. Assume that the actual fish population in the lakes is n and has not changed between the catches. Let $P_{m,p}(n)$ be the probability of the event (for a fixed p recatches out of m) coming from n fish in the lake. Generate a plot for $P_{m,p}(n)$ as a function of n for the following values of m and $p : m = 100$ and $p = 10, 20, 50, 75$. For each of these p, use the plots to estimate (educated guess) the actual value of n, i.e., what is the best guess for n if $m = 100$ and you catch p of the marked fish after mixing them up. Call these four estimates $\hat{n}_1, \ldots, \hat{n}_4$. You define your notion of "best guess." Do not search, THINK!

2. Let us now simulate this process as follows. Let $n = 2000$. Initialise an array of 2000 integers to 0. Select 100 random locations and mark them as 1. This corresponds to catching $m = 100$ random fish. Now randomly select 100 locations in the array to check for the marked fish and note the number of 1s in these 100. This corresponds to the p. Use the best guess algorithm from the previous part to determine $\hat{n}$, the estimate for the number of fish. Let $e = \hat{n}_i - n = \hat{n}_i - 1000$ be error in the estimate. Repeat the experiment 500 times and collect 500 samples of the error and show a scatter plot of the errrors. Find the sample mean of the error and the sample variance of the error.

3. Consider the following discrete time system. Packets arrive randomly at at router to be transmitted on a link. The time between successive packets are independent geometric random variables with parameter λ. Packet transmission times are also random and have a geometric distribution with parameter μ. Only one packet can be transmitted at any time and packets that arrive during the transmission of a previously arrived packet wait in buffer memory. There is infinte memory and can accommodate any number of packets. This can be simulated as follows. At the beginning of each second, if there is a packet in the buffer, it leaves with probability μ. And a new packet is added to the queue with probability λ. Simulate this queue for 1,000,000 time steps. For $n = 0, \ldots 50$, plot $p(n)$, the fraction of time that there are are n packets in the queue. Also find the time average of the number of packets in the memory. Use $\lambda = 0.3$ and $\mu = 0.4$.

4. Now extend the program from the previous part to simulate 10,000 queues in simultaneously parallel. When you stop the simulation after 100,000 time steps you have 10,000 values for the number of packets in the system. Use this data to plot $p(n)$ the fraction of queues that have n packets in the system and calculate the sample average from this 10,000 samples.