91.542 - Natural Language Processing
Problem Set 1

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1 Part II. Combinatorics, Probability, Information Theory

1.1 Problem 5. [30 pts]

(a) **Solution:** Assume that $S_k = \sum_{k=1}^{K} k * r^k$, then $S_{k+1} = \sum_{k=1}^{K} (k+1) * r^{k+1}$, then we need to split the equation of S_{k+1}, and get the following equation:

$$\begin{aligned} S_{k+1} &= r * \sum_{k=1}^{K} k * r^k + r * \sum_{k=1}^{K} r^n \\ &= r * S_k + r^2/(1-r) \end{aligned}$$

Now I will deduction the further equation from the above one:

$$\begin{aligned} S_{k+1} &= r * S_k + r^2/(1-r) \\ &= r * (S_k - \frac{r^2}{(1-r)^2}) + r * \frac{r^2}{(1-r)^2} + \frac{r^2}{(1-r)} \\ &= r * (S_k - \frac{r^2}{(1-r)^2}) + \frac{r^2}{(1-r)^2} \end{aligned}$$

Now that from the above equation, if I move the right r-relate value to the left, I can then get the following equation

$$S_{k+1} - \frac{r^2}{(1-r)^2} = r * (S_k - \frac{r^2}{(1-r)^2})$$

And then we can conclude that $S_k - r^2/(1-r)^2$ is a geometrical sequence, which meets the basic form of

$$\frac{S_{k+1} - r^2/(1-r)^2}{S_k - r^2/(1-r)^2} = r$$

Each item of the $S_k - \frac{r^2}{(1-r)^2}$ is a multiple of r with the previous one.

According to the questions conclusion, $\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$, the sum of the geometrical sequence with r^n, the sum should be $\frac{r}{1-r}$.

Now we have another geometric sequence of $S_k - \frac{r^2}{(1-r)^2} = [S_1 - \frac{r^2}{(1-r)^2}]^{k-1}$. It has similar form of the r^n, so we can just replace the variable with similar items, using the equation to calculate the items for a geometrical sequence, we can get the general form for S_k. By combining the limitation of the $\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$, we can get the general item:

$$\begin{aligned} S_n - r^2/(1-r)^2 &= r * \frac{1}{1-r} \\ S_n &= r^2/(1-r)^2 + r * \frac{1}{1-r} \\ &= \frac{r}{(1-r)^2} \end{aligned}$$

So we can get the value of S_n, as follows:

$$S_n = \frac{r}{(1-r)^2}$$