

Studies with Improved Renormalization Group Techniques

by

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Dedication

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Chapter 1

Introduction

1.1 Motivation

The standard model (SM) was recently completed with the discovery of the Higgs particle at the Large Hadron Collider (LHC) in Geneva. While the standard model doesn't predict any new fundamental particles we know that it is not the complete description of nature. It was hoped by many that the LHC would discover new particles in addition to the Higgs particle. While this hasn't happened yet it is entirely possible that new particles could be discovered in the 2015 LHC run which will be at higher energies with new triggering strategies. Since any new particles are not be part of the standard model, new physics would be needed to explain their existence. Many beyond standard model (BSM) theories exist that predicted new particles and confirmation of one or more of these theories would usher in a new age of discovery at the energy frontier. This however has not come to pass, the Higgs arrived unaccompanied.

The Higgs particle in the standard model is an excitation of the Higgs field which itself is responsible for electroweak symmetry breaking (EWSB). EWSB explains how the W^\pm and Z bosons acquire a mass that would otherwise be forbidden by electroweak unification. Additionally in the EWSB process the Higgs develops a vacuum expectation value (VEV). The standard model fermions couple to the VEV and allow them to acquire mass without violating local gauge invariance.

The Higgs field in the standard model is a scalar field. This leads to the hierarchy problem (also equivalent to the naturalness or fine tuning problem) as the Higgs mass will be dependant on the UV cutoff of the standard model. If the standard model is the complete story and there is

no new physics just above the electroweak scale then the SM cutoff will be at a very high scale. Natively, we expect the Higgs mass to be at the same order as the cutoff. The fact the Higgs mass is 126 GeV requires fine tuning in the electroweak sector to cancel the UV divergences.

There have been many solutions proposed as an alternative to the Standard Model that don't require fine tuning. Supersymmetry (SUSY) is one such proposal that achieves cancellations by introducing a new symmetry between bosonic and fermionic fields. Since the bosons and fermions enter loop calculations with opposite signs there is no need for fine tuning. Currently SUSY is constrained by the Higgs mass and the fact we haven't seen any of the super partners of the standard model particles. Additionally most versions of SUSY that have not been ruled out replace the standard model hierarchy problem with a new 'little' hierarchy problem where fine tuning is still required but it is not as severe as it is in the standard model.

Another natural possibility is that EWSB is the result of new strong dynamics at the TeV scale. Under this prescription, EWSB happens much like spontaneous chiral symmetry breaking in quantum chromodynamics (QCD). QCD is the only strongly coupled sector of the standard model and is responsible for the strong nuclear force. Theories of EWSB that rely on strong dynamics are generally called Technicolor models. Technicolor is also highly constrained by the Higgs mass, the top quark mass, and precision electroweak measurements that show flavor changing neutral currents are highly suppressed.

It is natural to draw insights from the parallels of Technicolor and QCD or directly from perturbation theory, however such insights do not paint an accurate picture. The strongly coupled nature of these theories demands that they are studied through fully nonperturbative means. Therefore, lattice gauge theory is the only controlled way to study these theories. In lattice gauge theory we discretized euclidean space time into a regular grid of sites connected by links. The Lagrangian describing the theory is also discretized in such a way that in the continuum limit the original theory is recovered. The continuum limit is the limit that the lattice is taken to be infinitely large and the sites are infinitesimally close together. Lattice QCD, the study of QCD on the lattice, has been extremely successful in understanding QCD. Many nonperturbative effects

such as confinement, hadronization, chiral symmetry breaking, and instantons have been studied extensively. Many of these properties are completely inaccessible through analytic techniques while others could not be fully understood.

The ultimate goal of efforts in this field is to find a theory can explain electroweak symmetry breaking, as it is observed in nature, through new strong dynamics. Finding such a theory is highly nontrivial and a great deal of theoretical and computational effort has gone into looking for such a theory. The lattice community has set out to do this in a controlled and systematic way: map out the space of all possible theories and see what we find. To date a lot of the focus of the lattice community has been on locating the conformal window and developing techniques to distinguish conformal from chirally broken theories.

This thesis has two primary objectives. The first goal is a physics goal: map out the behavior of $SU(3)$ gauge theories with N_f fermions in the fundamental representation. For technical reasons we choose to study theories with 4, 8, 12, and 16 flavors. 4 and 16 flavor theories are already well understood so the focus of our work is on 8 and 12 flavor systems.

My second objective is to develop tools that make the first goal possible. Towards this end I introduce improvements to lattice step scaling calculation techniques. I explore how these techniques are different and how to implement them in a calculation. Finally these techniques are applicable to any lattice calculation and therefore will be of use to any lattice practitioner.

1.2 Organization

This thesis is organized into five chapters not including the introduction and conclusion. Briefly, I use chapter 2 to set the stage and chapter 3 to give a flavor for lattice calculations. Chapters 4-6 discuss my contributions, each chapter is its own calculation and reflects approximately a years worth of work.

I begin chapter 2 with a brief overview of the standard model. The Higgs mechanism is given its own section due to its relevance for the remainder of the thesis. I then summarized flaws in our current understanding of the standard model with emphasis given to the hierarchy problem. Finally

I discuss how a composite Higgs can resolve the hierarchy problem in a natural way. Because a composite Higgs arises from a strongly interacting quantum field theory we can not understand such a theory with the standard analytical tools of perturbation theory. Fortunately there is a well developed nonperturbative approach, lattice gauge theory. I conclude the chapter with an overview of current lattice studies.

Chapter 3 provides the reader with a brief overview of lattice gauge theory. Like any mature discipline, this topic is a subject of many books. My goal is not to cover this subject comprehensively; rather I want to provide the reader with a flavor of how a lattice calculation is performed and highlight areas that are useful in later sections. To that end I develop the pure gauge action and discuss the negative adjoint term that our group uses in our simulations. I also discuss the problems associated with the discretization of fermions on the lattice. I end the discussion of lattice fermions with a description of staggered fermions and the nHYP smearing we use in our calculations.

Chapter 4 - 6 document the body of work that is this thesis. Chapter 4 concerns my early work using Monte Carlo Renormalization Group (MCRG) on $SU(3)$ $N_f = 8$ and $SU(3)$ $N_f = 12$ theories. In chapter 5, I introduce the gradient flow and how it can be used to calculate the renormalized step scaling function. I also use this chapter to discuss an optimization improvement that our group developed. Results for $SU(3)$ $N_f = 4$ and $SU(3)$ $N_f = 12$ are given. Finally chapter 6 shows how the Wilson Flow can be used as an optimization step for MCRG. Results from $SU(3)$ $N_f = 12$ are given.

In my concluding chapter I make some final remarks about interpreting our results. I will also summarize results from other methods that our group used to study this system. All of the methods our group has used to study there systems have converged to the same answer making a strong case.

1.3 Notation and Conventions

Unless otherwise noted these are the standard conventions used in this thesis. I use standard particle theory units of $\hbar = c = 1$. The four vector is given by Greek indices

$$x^\mu = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad (1.1)$$

where x_0 is the temporal component t and the remaining three components are the spatial components of \vec{x} . For the metric I use the west coast convention for the metric

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (1.2)$$

On the lattice we use the Euclidian metric

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (1.3)$$

Standard Einstein summation notation applies to contract indices.

I work in the chiral representation for the Dirac spin matrices γ^μ

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (1.4)$$

where I is the 2×2 identity matrix and σ^i are the Pauli matrices:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (1.5)$$

I use the standard ‘slash’ notation to contract four vectors with γ^μ :

$$\not{p} \equiv p_\mu \gamma^\mu. \quad (1.6)$$

The momentum operator I use the notation

$$p^\mu = i\partial^\mu, \quad (1.7)$$

where

$$\partial^\mu = \left(\frac{\partial}{\partial x^0}, -\nabla \right) = \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right). \quad (1.8)$$

The Gell-Mann matrices λ_a are:

$$\begin{aligned} \lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \\ \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} & \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ \lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} & \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \end{aligned} \quad (1.9)$$

Bibliography

- [1]
- [2] David Adams. Fourth root prescription for dynamical staggered fermions. Phys. Rev. D, 72:114512, Dec 2005.
- [3] Yasumichi Aoki, Tatsumi Aoyama, Masafumi Kurachi, Toshihide Maskawa, Kohtaroh Miura, Kei-ichi Nagai, Hiroshi Ohki, Enrico Rinaldi, Akihiro Shibata, Koichi Yamawaki, and Takeshi Yamazaki. Light composite scalar in eight-flavor QCD on the lattice. 2014.
- [4] Yasumichi Aoki, Tatsumi Aoyama, Masafumi Kurachi, Toshihide Maskawa, Kei-ichi Nagai, Hiroshi Ohki, Enrico Rinaldi, Akihiro Shibata, Koichi Yamawaki, and Takeshi Yamazaki. Light composite scalar in twelve-flavor QCD on the lattice. Phys. Rev. Lett., 111:162001, 2013.
- [5] Yasumichi Aoki, Tatsumi Aoyama, Masafumi Kurachi, Toshihide Maskawa, Kei-ichi Nagai, Hiroshi Ohki, Enrico Rinaldi, Akihiro Shibata, Koichi Yamawaki, and Takeshi Yamazaki. The scalar spectrum of many-flavour QCD. 2013.
- [6] Yasumichi Aoki, Tatsumi Aoyama, Masafumi Kurachi, Toshihide Maskawa, Kei-ichi Nagai, Hiroshi Ohki, Akihiro Shibata, Koichi Yamawaki, and Takeshi Yamazaki. Lattice study of conformality in twelve-flavor QCD. Phys. Rev., D86:054506, 2012.
- [7] T. Appelquist, G. T. Fleming, M. F. Lin, E. T. Neil, and D. Schaich. Lattice Simulations and Infrared Conformality. Phys. Rev., D84:054501, 2011.
- [8] Thomas Appelquist, Richard Brower, Simon Catterall, George Fleming, Joel Giedt, Anna Hasenfratz, Julius Kuti, Ethan Neil, and David Schaich. Lattice Gauge Theories at the Energy Frontier. 2013.
- [9] Thomas Appelquist, George T. Fleming, and Ethan T. Neil. Lattice study of the conformal window in QCD-like theories. Phys. Rev. Lett., 100:171607, 2008.
- [10] Thomas Appelquist, George T. Fleming, and Ethan T. Neil. Lattice Study of Conformal Behavior in SU(3) Yang-Mills Theories. Phys. Rev., D79:076010, 2009.
- [11] Thomas Appelquist and Ethan T. Neil. Lattice gauge theory beyond the standard model. pages 699–729, 2009.
- [12] Janos Balog, Ferenc Niedermayer, and Peter Weisz. Logarithmic corrections to $O(a^{*2})$ lattice artifacts. Phys. Lett., B676:188–192, 2009.

- [13] Janos Balog, Ferenc Niedermayer, and Peter Weisz. The Puzzle of apparent linear lattice artifacts in the 2d non-linear sigma-model and Symanzik's solution. Nucl. Phys., B824:563–615, 2010.
- [14] Tom Banks and A. Zaks. On the Phase Structure of Vector-Like Gauge Theories with Massless Fermions. Nucl. Phys., B196:189, 1982.
- [15] Claude Bernard, Maarten Golterman, Yigal Shamir, and Stephen R. Sharpe. Comment on: chiral anomalies and rooted staggered fermions [phys. lett. b 649 (2007) 230]. Physics Letters B, 649(23):235 – 240, 2007.
- [16] Gyan Bhanot. $Su(3)$ lattice gauge theory in 4 dimensions with a modified wilson action. Physics Letters B, 108(45):337 – 340, 1982.
- [17] Gyan Bhanot and Michael Creutz. Variant actions and phase structure in lattice gauge theory. Phys. Rev. D, 24:3212–3217, Dec 1981.
- [18] J. Binney, N.J. Dowrick, A.J. Fisher, and M.E.J. Newman. The Theory of Critical Phenomena: An Introduction to the Renormalization Group. Oxford University Press, Oxford, 1992.
- [19] T. Blum, C. DeTar, Urs M. Heller, Leo Krkkinen, K. Rummukainen, and D. Toussaint. Thermal phase transition in mixed action $\{SU\}$ (3) lattice gauge theory and wilson fermion thermodynamics. Nuclear Physics B, 442(12):301 – 316, 1995.
- [20] A. Bode. Two loop expansion of the schrödinger functional coupling sf in $\{SU\}$ (3) lattice gauge theory. Nuclear Physics B - Proceedings Supplements, 63(13):796 – 798, 1998. Proceedings of the $\{XVth\}$ International Symposium on Lattice Field Theory.
- [21] Szabolcs Borsanyi, Stephan Durr, Zoltan Fodor, Christian Hoelbling, Sandor D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, C. McNeile, and K. K. Szabo. High-precision scale setting in lattice QCD. JHEP, 1209:010, 2012.
- [22] William E. Caswell. Asymptotic Behavior of Nonabelian Gauge Theories to Two Loop Order. Phys. Rev. Lett., 33:244, 1974.
- [23] Simon Catterall and Francesco Sannino. Minimal walking on the lattice. Phys.Rev., D76:034504, 2007.
- [24] Anqi Cheng, Anna Hasenfratz, Yuzhi Liu, Gregory Petropoulos, and David Schaich. Finite size scaling of conformal theories in the presence of a near-marginal operator. 2013.
- [25] Anqi Cheng, Anna Hasenfratz, Yuzhi Liu, Gregory Petropoulos, and David Schaich. Step scaling studies using the gradient flow running coupling. 2014, in preparation.
- [26] Anqi Cheng, Anna Hasenfratz, Gregory Petropoulos, and David Schaich. Determining the mass anomalous dimension through the eigenmodes of Dirac operator. PoS, LATTICE 2013:088, 2013.
- [27] Anqi Cheng, Anna Hasenfratz, Gregory Petropoulos, and David Schaich. Scale-dependent mass anomalous dimension from Dirac eigenmodes. JHEP, 1307:061, 2013.

- [28] Anqi Cheng, Anna Hasenfratz, and David Schaich. Novel phase in $SU(3)$ lattice gauge theory with 12 light fermions. Phys. Rev., D85:094509, 2012.
- [29] Christian B. Lang Christof Gattringer. Quantum Chromodynamics on the Lattice. Springer, 2010.
- [30] Michael Creutz. Chiral anomalies and rooted staggered fermions. Physics Letters B, 649(23):230 – 234, 2007.
- [31] Michael Creutz. Reply to: comment on: chiral anomalies and rooted staggered fermions [phys. lett. b 649 (2007) 230] [phys. lett. b 649 (2007) 235]. Physics Letters B, 649(23):241 – 242, 2007.
- [32] Thomas DeGrand. Lattice studies of QCD-like theories with many fermionic degrees of freedom. 2010.
- [33] Thomas DeGrand. Finite-size scaling tests for spectra in $SU(3)$ lattice gauge theory coupled to 12 fundamental flavor fermions. Phys. Rev., D84:116901, 2011.
- [34] Thomas DeGrand and Anna Hasenfratz. Remarks on lattice gauge theories with infrared-attractive fixed points. Phys.Rev., D80:034506, 2009.
- [35] Thomas DeGrand, Yigal Shamir, and Benjamin Svetitsky. Gauge theories with fermions in the two-index symmetric representation. PoS, LATTICE2011:060, 2011.
- [36] Thomas DeGrand, Yigal Shamir, and Benjamin Svetitsky. Infrared fixed point in $SU(2)$ gauge theory with adjoint fermions. Phys.Rev., D83:074507, 2011.
- [37] Luigi Del Debbio, Biagio Lucini, Agostino Patella, Claudio Pica, and Antonio Rago. Mesonic spectroscopy of Minimal Walking Technicolor. Phys.Rev., D82:014509, 2010.
- [38] T. DeGrand & C DeTar. Lattice Methods for Quantum Chromodynamics. World Scientific, 2006.
- [39] A. Deuzeman, M. P. Lombardo, and E. Pallante. Evidence for a conformal phase in $SU(N)$ gauge theories. Phys. Rev., D82:074503, 2010.
- [40] Albert Deuzeman, Maria Paola Lombardo, Tiago Nunes da Silva, and Elisabetta Pallante. The bulk transition of QCD with twelve flavors and the role of improvement. 2012.
- [41] Savas Dimopoulos and Leonard Susskind. Mass Without Scalars. Nucl.Phys., B155:237–252, 1979.
- [42] Michael Dine. Tasi lectures on the strong cp problem.
- [43] Estia Eichten and Kenneth D. Lane. Dynamical Breaking of Weak Interaction Symmetries. Phys.Lett., B90:125–130, 1980.
- [44] F. Englert and R. Brout. Broken Symmetry and the Mass of Gauge Vector Mesons. Phys.Rev.Lett., 13:321–323, 1964.
- [45] Zoltan Fodor, Kieran Holland, Julius Kuti, Daniel Nogradi, and Chris Schroeder. Nearly conformal gauge theories on the lattice. Int.J.Mod.Phys., A25:5162–5174, 2010.

- [46] Zoltan Fodor, Kieran Holland, Julius Kuti, Daniel Negradi, and Chris Schroeder. Twelve massless flavors and three colors below the conformal window. Phys. Lett., B703:348–358, 2011.
- [47] Zoltan Fodor, Kieran Holland, Julius Kuti, Daniel Negradi, Chris Schroeder, and Chik Him Wong. Can the nearly conformal sextet gauge model hide the Higgs impostor? Phys. Lett., B718:657–666, 2012.
- [48] Zoltan Fodor, Kieran Holland, Julius Kuti, Daniel Negradi, Chris Schroeder, and Chik Him Wong. Confining force and running coupling with twelve fundamental and two sextet fermions. PoS, Lattice 2012:025, 2012.
- [49] Zoltan Fodor, Kieran Holland, Julius Kuti, Daniel Negradi, Chris Schroeder, and Chik Him Wong. Conformal finite size scaling of twelve fermion flavors. PoS, Lattice 2012:279, 2012.
- [50] Zoltan Fodor, Kieran Holland, Julius Kuti, Daniel Negradi, and Chik Him Wong. The gradient flow running coupling scheme. PoS, Lattice 2012:050, 2012.
- [51] Zoltan Fodor, Kieran Holland, Julius Kuti, Daniel Negradi, and Chik Him Wong. The Yang-Mills gradient flow in finite volume. JHEP, 1211:007, 2012.
- [52] Zoltan Fodor, Kieran Holland, Julius Kuti, Daniel Negradi, and Chik Him Wong. Can a light Higgs impostor hide in composite gauge models? PoS, LATTICE 2013:062, 2014.
- [53] Patrick Fritzsche and Alberto Ramos. The gradient flow coupling in the Schrödinger Functional. JHEP, 1310:008, 2013.
- [54] J. Gasser and H. Leutwyler. Chiral perturbation theory to one loop. Ann. Phys., 158:142, 1984.
- [55] Howard Georgi and David B. Kaplan. Composite Higgs and Custodial SU(2). Phys.Lett., B145:216, 1984.
- [56] Joel Giedt. Confining force and running coupling with twelve fundamental and two sextet fermions. PoS, Lattice 2012:006, 2012.
- [57] Joel Giedt. Lattice gauge theory and physics beyond the standard model. PoS, Lattice 2012:006, 2012.
- [58] Sheldon L. Glashow. Partial-symmetries of weak interactions. Nuclear Physics, 22(4):579 – 588, 1961.
- [59] Maarten Golterman. Applications of chiral perturbation theory to lattice QCD. pages 423–515, 2009.
- [60] David Gross and Frank Wilczek. Ultraviolet behavior of non-abelian gauge theories. Phys. Rev. Lett., 30:1343–1346, Jun 1973.
- [61] G.S. Guralnik, C.R. Hagen, and T.W.B. Kibble. Global Conservation Laws and Massless Particles. Phys.Rev.Lett., 13:585–587, 1964.
- [62] A. Hasenfratz, R. Hoffmann, and F. Knechtli. The Static potential with hypercubic blocking. Nucl.Phys.Proc.Suppl., 106:418–420, 2002.

- [63] Anna Hasenfratz. Investigating the critical properties of beyond-qcd theories using monte carlo renormalization group matching. Phys. Rev. D, 80:034505, Aug 2009.
- [64] Anna Hasenfratz. Conformal or Walking? Monte Carlo renormalization group studies of SU(3) gauge models with fundamental fermions. Phys. Rev., D82:014506, 2010.
- [65] Anna Hasenfratz. MCRG study of 12 fundamental flavors with mixed fundamental-adjoint gauge action. PoS, Lattice 2011:065, 2011.
- [66] Anna Hasenfratz. Infrared fixed point of the 12-fermion SU(3) gauge model based on 2-lattice MCRG matching. Phys. Rev. Lett., 108:061601, 2012.
- [67] Anna Hasenfratz, Anqi Cheng, Gregory Petropoulos, and David Schaich. Mass anomalous dimension from Dirac eigenmode scaling in conformal and confining systems. PoS, Lattice 2012:034, 2012.
- [68] Anna Hasenfratz, Anqi Cheng, Gregory Petropoulos, and David Schaich. Finite size scaling and the effect of the gauge coupling in 12 flavor systems. PoS, LATTICE 2013:075, 2013.
- [69] Anna Hasenfratz, Anqi Cheng, Gregory Petropoulos, and David Schaich. Reaching the chiral limit in many flavor systems. 2013.
- [70] Anna Hasenfratz, Roland Hoffmann, and Stefan Schaefer. Hypercubic smeared links for dynamical fermions. JHEP, 0705:029, 2007.
- [71] Anna Hasenfratz and Francesco Knechtli. Flavor symmetry and the static potential with hypercubic blocking. Phys. Rev., D64:034504, 2001.
- [72] Anna Hasenfratz, David Schaich, and Aarti Veernala. Nonperturbative beta function of eight-flavor SU(3) gauge theory. 2014.
- [73] Peter W. Higgs. Broken symmetries and the masses of gauge bosons. Phys. Rev. Lett., 13(16):508–509, October 1964.
- [74] P.W. Higgs. Broken symmetries, massless particles and gauge fields. Physics Letters, 12(2):132 – 133, 1964.
- [75] Christopher T. Hill and Elizabeth H. Simmons. Strong dynamics and electroweak symmetry breaking. Physics Reports, 381(46):235 – 402, 2003.
- [76] Etsuko Itou. Properties of the twisted Polyakov loop coupling and the infrared fixed point in the SU(3) gauge theories. PTEP, 2013:083B01, 2013.
- [77] Y. Iwasaki, K. Kanaya, S. Kaya, S. Sakai, and T. Yoshie. Phase structure of lattice QCD for general number of flavors. Phys.Rev., D69:014507, 2004.
- [78] Xiao-Yong Jin and Robert D. Mawhinney. Lattice QCD with Eight Degenerate Quark Flavors. PoS, LATTICE2008:059, 2008.
- [79] Xiao-Yong Jin and Robert D. Mawhinney. Lattice QCD with 8 and 12 degenerate quark flavors. PoS, LAT2009:049, 2009.

- [80] Xiao-Yong Jin and Robert D. Mawhinney. Lattice QCD with 12 Degenerate Quark Flavours. PoS, Lattice 2011:066, 2012.
- [81] David B. Kaplan, Howard Georgi, and Savas Dimopoulos. Composite Higgs Scalars. Phys.Lett., B136:187, 1984.
- [82] D.B. Kaplan. Chiral symmetry and lattice fermions.
- [83] John Kogut and Leonard Susskind. Hamiltonian formulation of wilson’s lattice gauge theories. Phys. Rev. D, 11:395–408, Jan 1975.
- [84] Andreas S. Kronfeld. Lattice gauge theory with staggered fermions: How, where, and why (not). PoS, LAT2007:016, 2007.
- [85] Kenneth Lane. Two lectures on technicolor.
- [86] C.-J. David Lin, Kenji Ogawa, Hiroshi Ohki, and Eigo Shintani. Lattice study of infrared behaviour in $SU(3)$ gauge theory with twelve massless flavours. JHEP, 1208:096, 2012.
- [87] Martin Luscher. Properties and uses of the Wilson flow in lattice QCD. JHEP, 1008:071, 2010.
- [88] Martin Luscher. Trivializing maps, the Wilson flow and the HMC algorithm. Commun. Math. Phys., 293:899–919, 2010.
- [89] M. Lüscher and P. Weisz. Computation of the action for on-shell improved lattice gauge theories at weak coupling. Physics Letters B, 158(3):250 – 254, 1985.
- [90] Adam Martin. Technicolor signals at the lhc.
- [91] Shinya Matsuzaki and Koichi Yamawaki. Holographic techni-dilaton at 125 GeV. Phys. Rev., D86:115004, 2012.
- [92] R. Narayanan and H. Neuberger. Infinite N phase transitions in continuum Wilson loop operators. JHEP, 0603:064, 2006.
- [93] Ethan T. Neil. Exploring Models for New Physics on the Lattice. PoS, Lattice 2011:009, 2011.
- [94] H.B. Nielsen and M. Ninomiya. Absence of neutrinos on a lattice: (i). proof by homotopy theory. Nuclear Physics B, 185(1):20 – 40, 1981.
- [95] H.B. Nielsen and M. Ninomiya. Absence of neutrinos on a lattice: (ii). intuitive topological proof. Nuclear Physics B, 193(1):173 – 194, 1981.
- [96] H.B. Nielsen and M. Ninomiya. A no-go theorem for regularizing chiral fermions. Physics Letters B, 105(23):219 – 223, 1981.
- [97] Paula Perez-Rubio and Stefan Sint. Non-perturbative running of the coupling from four flavour lattice QCD with staggered quarks. PoS, Lattice 2010:236, 2010.
- [98] Michael E. Peskin and Dan V. Schroeder. An Introduction To Quantum Field Theory (Frontiers in Physics). Westview Press, 1995.

- [99] Gregory Petropoulos, Anqi Cheng, Anna Hasenfratz, and David Schaich. PoS, Lattice 2012:051, 2012.
- [100] Gregory Petropoulos, Anqi Cheng, Anna Hasenfratz, and David Schaich. Improved Lattice Renormalization Group Techniques. PoS, LATTICE 2013:079, 2013.
- [101] H. David Politzer. Reliable Perturbative Results for Strong Interactions? Phys.Rev.Lett., 30:1346–1349, 1973.
- [102] C. Quigg. Spontaneous symmetry breaking as a basis of particle mass. Rept. Prog. Physics, pages 1019–1054, 2007.
- [103] C. Quigg. Unanswered questions in the electroweak theory. Annual Review of Nuclear and Particle Science, pages 505–555, 2009.
- [104] Thomas A. Ryttov and Robert Shrock. An Analysis of Scheme Transformations in the Vicinity of an Infrared Fixed Point. Phys.Rev., D86:085005, 2012.
- [105] Abdus Salam and John Clive Ward. Electromagnetic and weak interactions. Phys. Lett., 13:168–171, 1964.
- [106] Francesco Sannino. Conformal Dynamics for TeV Physics and Cosmology. Acta Phys.Polon., B40:3533–3743, 2009.
- [107] David Schaich, Anqi Cheng, Anna Hasenfratz, and Gregory Petropoulos. Bulk and finite-temperature transitions in SU(3) gauge theories with many light fermions. PoS, Lattice 2012:028, 2012.
- [108] Robert Shrock. Some recent results on models of dynamical electroweak symmetry breaking. pages 227–241, 2007.
- [109] Stefan Sint. On the schrödinger functional in {QCD}. Nuclear Physics B, 421(1):135 – 156, 1994.
- [110] Jan Smit. Introduction to Quantum Fields on a Lattice. Cambridge University Press, 2002.
- [111] Rainer Sommer. Scale setting in lattice QCD. PoS, LATTICE 2013:015, 2014.
- [112] Leonard Susskind. Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory. Phys.Rev., D20:2619–2625, 1979.
- [113] R.H. Swendsen. Phys. Rev. Lett., 42:859, 1979.
- [114] K. Symanzik. Continuum limit and improved action in lattice theories : (ii). $o(n)$ non-linear sigma model in perturbation theory. Nuclear Physics B, 226(1):205 – 227, 1983.
- [115] Fatih Tekin, Rainer Sommer, and Ulli Wolff. The Running coupling of QCD with four flavors. Nucl. Phys., B840:114–128, 2010.
- [116] Steven Weinberg. A Model of Leptons. Phys.Rev.Lett., 19:1264–1266, 1967.
- [117] Steven Weinberg. Implications of Dynamical Symmetry Breaking. Phys.Rev., D13:974–996, 1976.

- [118] Steven Weinberg. Implications of Dynamical Symmetry Breaking: An Addendum. Phys.Rev., D19:1277–1280, 1979.
- [119] P. Weisz. Continuum limit improved lattice action for pure yang-mills theory (i). Nuclear Physics B, 212(1):1 – 17, 1983.
- [120] Kenneth Wilson. Confinement of quarks. Phys. Rev. D, 10:2445–2459, Oct 1974.