Studies with Improved Renormalization Group Techniques

by

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Dedication

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Chapter 1

Strongly Coupled Physics Beyond the Standard Model

1.1 The Standard Model

In our current understanding of the universe, there are four fundamental forces in nature: gravity, electromagnetism, the strong nuclear force, and the weak nuclear force. The latter three forces are understood in terms of quantum field theories which form the Standard Model (SM). The Standard Model is a $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge theory.

The $SU(3)_C$ component of the theory explains the strong nuclear force, also called quantum chromodynamics (QCD). QCD is a $SU(3)$ Yang Mills theory with fermions in the fundamental representation. This force explains how spin 1/2 quarks possessing 'color' charge interact by exchanging gluons. It is important to note that this color charge has nothing to do with visible colors which are actually a narrow band of electromagnetic radiation that we can see. Rather the color charge is analogous to electric charge. Unlike electric charge which can be expressed by only one number, color charge is expressed by three numbers commonly labeled R, G, B, for red, green, and blue respectively. The Lagrangian for QCD is

$$\mathcal{L} = \sum_j \bar{\psi}_j (i\not{D})\psi_j - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a, \tag{1.1}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^b_\nu, \tag{1.2}$$

and

$$\not{D} = D_\mu \gamma^\mu = (\partial_\mu - igA^a_\mu \lambda^a)\gamma^\mu. \tag{1.3}$$

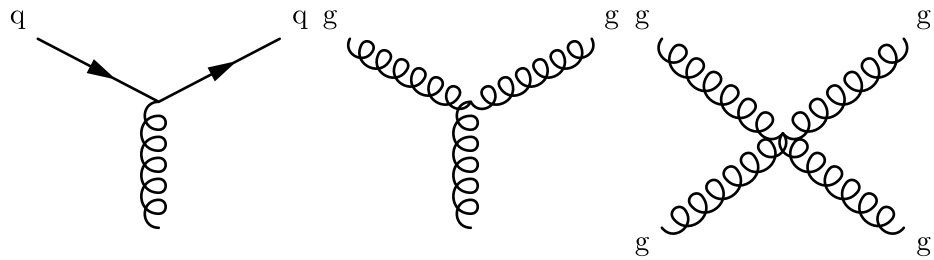

Figure 1.1: These are the Feynman Diagrams of the allowed QCD verticies. All interractions in the theory are built by combining these verticies. Quarks are represented by the solid lines and gluons are represented by curly lines. The three and four gluon vertecies are unique to non-Abelian gauge theories.

A^μ are the gluon fields, ψ_j is the jth quark flavor. Greek μ and ν are the usual spacetime indices while a, b, and c are color indices. The structure constant f^{abc} is given by the commutation relation between the generators of the group $[\lambda^a, \lambda^b] = \frac{i}{2} f^{abc} \lambda^c$ where λ^a are the Gell-Mann matricies.

The Lagrangian allows the three vertices shown in figure 1.1. The first vertex couples a fermion to a gluon and is analogous to diagrams in QED that couple an electron to the photon. The other two vertices have no analogue in QED, these are the three and four gluon vertices. The gluon verticies are a result of the non-abelian nature of the theory, consequently the gluons having a charge anti-charge moment. A repercussion of the gluons being charged is that the color field is anti-screened.

QCD exhibits two interesting properties: confinement and asymptotic freedom. Confinement means that the force between two quarks does not diminish as they are pulled apart. In fact if you try to pull two quarks apart eventually there will be enough energy in the field to produce a new quark anti quark pair. As a result we never observe free quarks in nature, they only exist in colorless bound states called hadrons. Hadrons come in two varieties $q\bar{q}$ pairs called mesons and qqq triplets called baryons. Despite the fact that confinement is easy to demonstrate on the lattice [120], there is an outstanding Millennium Prize for an analytic proof[1]. Asymptotic freedom reflects that at very large momentum transfers the quarks and gluons interract weakly. This is described by the β function and is elaborated on in section 1.4.3. The 2004 Nobel prize in physics was awarded for the discovery of asymptotic freedom by Frank Wilczek, David Gross, and David Politzer in 1973 [60, 101].

The strong nuclear force is responsible for the proton, composed of two up and one down quark, and neutron, composed of two down and one up quark. Additionally, the strong nuclear force binds these particles together to form atomic nuclei. Most of the understood mass in the universe is a result of the strong nuclear force. In QCD the up and down quarks have masses on the order of an MeV while the proton has a mass of 938GeV. Almost all of the proton mass results from its own binding energy.

The remaining $SU(2)_W \times U(1)_Y$ part of the standard model is the unified electroweak force [105, 116, 61, 73, 74, 44, 58]. This force accounts for the quantum theory of electrodynamics and how particles decay via the weak process. The electroweak force has three bosonic force carriers: the massless photon and the massive $W^{\pm}$ and Z. As a result of being carried by the massless photon, electrodynamics is an infinite range force. However, the weak force has an interraction range of $\mathcal{O} \approx 10^{16}$m due to the heavieness of the $W^{\pm}$ and Z. In addition to effecting the quark sector, the electroweak force also interracts with leptons. Leptons are spin 1/2 fermions that are not charged under the $SU(3)_C$ group. The electron, muon, tau, and their corresponding neutrinos are all leptons.

An essential feature of the electroweak theory is that $SU(2)_W \times U(1)_Y$ spontaneously breaks down to $U(1)_{EM}$, see section 1.2 for further details. This breaking gives the $W^{\pm}$ and Z bosons their mass and also accounts for the standard model fermion masses. $U(1)_{EM}$ is the theory of quantum electrodynamics in which spin 1/2 fermionic matter charged with electric charge interact by exchanging photos. Unlike in QCD, photons are not charged under U(1) and therefore don't interact at tree level. This yields a much simpler theory than QCD and can be understood perturbatively.

The Lagrangian for QED is similar to 1.1 however the field strength tensor $F^a_{\mu\nu}$ is replaced by $F_{\mu\nu}$, dropping the color indices. The simpler Abelian U(1) symmetry also means that the field strength tensor is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1.4}$$

Where A^μ is the four potential defined as $A^\mu \equiv (\phi, \vec{A})$. Only one vertex is allowed that couples a

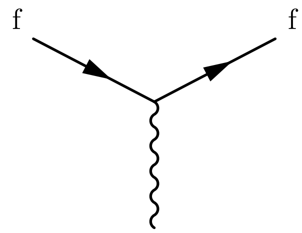

Figure 1.2: There is only one allowed vertex in QED which couples a fermion charged under U(1) with the photon.

fermion to a photon.

In both the QCD and Electroweak sector, matter is divided into three generations of matter. The first generation of matter is the most familiar. It consists of the up quark down quark, electron, and electron neutrino. All of the matter that we experience in our daily lives is made of this generation of matter. The other two generations of matter are essentially heavier replicas of the first generation. These heavier generations rapidly decay via the weak nuclear force to the first generation of matter and therefore are only detected in high energy events such as cosmic rays or particle physics experiments. The second generation consists of the strange quark, charm quark, muon, and muon neutrino. Finally the third consists of the top quark, bottom quark, tau, and tau neutrino. The properties of all the standard model particles is summarized in table **??**.

1.2 Higgs Mechanism

In the standard model the $W^{\pm}$ and Z bosons are massive but the photon is massless. Since gauge invariance dictates that a mass term is forbidden in the Lagrangian, this seems to pose a problem. Disaster is averted because $SU(2)_W \times U(1)_Y$ is spontaneously broken to $U(1)_{EM}$. In the standard model this is accomplished by adding a complex scalar doublet field to the theory. The field is named the Higgs field after one of its discoverers. The following discussion shows how electroweak symmetry breaking is facilitated by the Higgs [102, 103, 98].

Lets introduce a complex elementary scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$ that transforms in the $(1,2,\frac{1}{2})$ representation of $SU(3)_C \times SU(2)_L \times U(1)_Y$. We allow all terms in the Lagrangian that have mass

Particle Name	Symbol	Mass (MeV)	Generation	charge $(U(1)_{EM}$
electron	e	0.510999	1	-1
electron neutrino	ν_e	$< 2.2 \times 10^{-6}$	1	0
muon	μ	105.659	2	-1
muon neutrino	ν_μ	< 0.170	2	0
tau	τ	1.776×10^3	3	-1
tau neutrino	ν_τ	< 15.5	3	0

Table 1.1: The standard model fermions are spin 1/2 fundamental particles in the color singlet. The masses for the electron, muon, and tau are known to the percision given. The neutrinos are know to have masses but their masses are not percicly measured. Left handed leptons have weak hypercharge of -1 while right handed leptons have weak hyperchange of -2. Right handed neutrinos have not been observed.

Particle Name	Symbol	Mass	Generation	charge $U(1)_{\mathrm{EM}}$
up	u	$2.3^{+0.7}_{-0.5}$	1	$\frac{2}{3}$
down	d	$4.8^{0.5}_{-0.3}$	1	$-\frac{1}{3}$
strange	s	95 ± 5	2	$-\frac{1}{3}$
charm	c	$(1.275 \pm 0.025) \times 10^3$	2	$\frac{2}{3}$
top (truth)	t	$(173.07 \pm 1) \times 10^3$	3	$\frac{2}{3}$
bottom (beauty)	b	$(4.180 \pm 0.03) \times 10^3$	3	$-\frac{1}{3}$

Table 1.2: The standard model quarks are spin 1/2 fundamental particles that are charged under $SU(3)_C$. Left handed quarks have 1/3 hyperchage. Right handed quarks with 2/3 electric charge have weak hypercharge of 4/3 while the right handed quarks with -1/3 electric charge have weak hypercharge of -2/3.

Particle Name	Symbol	mass (MeV)	charge $U(1)_{\mathrm{EM}}$
gluon	g	0	0
photon	γ	0	0
W^+	W^+	$(80.385 \pm 0.015) \times 10^3$	1
W^-	W^-	$(80.385 \pm 0.015) \times 10^3$	-1
Z	Z	$(91.1876 \pm 0.0021) \times 10^3$	0
Higgs	H	$(125.9 \pm 0.4) \times 10^3$	0

Table 1.3: The standard model bosons. All partiles are spin 1 except for the Higgs particle which is spin 0.

dimension ≤ 4, thus ignoring irrelevant operators,

$$\mathcal{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - |\lambda| (\Phi^\dagger \Phi)^2, \tag{1.5}$$

with covariant derivative

$$D_\mu \Phi = (\partial_\mu - i g A^a_\mu \tau^a - i g' Y B_\mu) \Phi. \tag{1.6}$$

A^a_μ and B_μ are the $SU(2)$ and $U(1)$ gauge bosons respectively. The coupling constant g belongs to $SU(2)$ and the coupling constant g' belongs to $U(1)$. Finally Y is the generator of hypercharge, τ^a are the generators of $SU(2)$ and are related to the Pauli matrices

$$\tau^a = \frac{1}{2} \sigma^a. \tag{1.7}$$

We can now identify the potential $V(\Phi)$ as

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + |\lambda| (\Phi^\dagger \Phi)^2, \tag{1.8}$$

this potential is shown in figure 1.3. As long as μ^2 is positive the potential has a spontaneously broken symmetry. Gauge invariance allows us to choose the vacuum state to correspond to the vacuum expectation value (VEV)

$$\langle \Phi \rangle_0 = \frac{\nu}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix}, \tag{1.9}$$

where $\nu = \sqrt{\frac{\mu^2}{|\lambda|}}$. Moreover, ν is related to the Fermi constant, G_F, by

$$\nu = \frac{1}{\sqrt{\sqrt{2} G_F}} \approx 246 \text{ GeV}. \tag{1.10}$$

If we assign our theory to have hypercharge $Y = +\frac{1}{2}$, a complete gauge transformation in this theory is

$$\Phi \rightarrow e^{i \alpha^a(x) \tau^a} e^{i \beta(x)/2} \Phi. \tag{1.11}$$

Through a clever choice of $\alpha^1 = \alpha^2 = 0$ and $\alpha^3 = \beta$ we see that $\langle \Phi \rangle$ is invariant. Therefore the theory still contains an unbroken U(1) symmetry which we identify with electromagnetism. This unbroken U(1) symmetry contains one massless gauge boson which is the photon. The other three

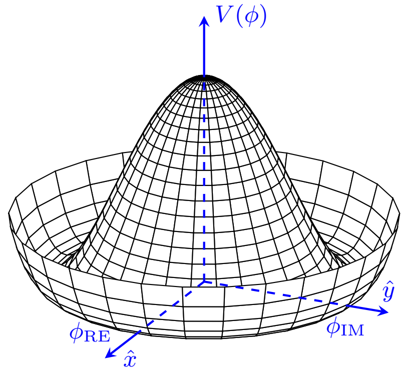

Figure 1.3: The Higgs potential with $\mu^2 > 0$ has a spontaneously broken global symmetry. We can think of this classically. The origin is a local maximum of potential energy and therefore an unstable equilibrium. If we place a particle at the origin any perturbation will push the particle 'down hill' to a minimum of the potential. Since there are a number of degenerate minima, choosing one spontaneously breaks the symmetry.

gauge bosons corresponding to the broken generators of the symmetry group become massive. The massive gauge bosons get their mass from the square of the kinetic term evaluated at the VEV

$$(D_\mu\Phi)^\dagger(D^\mu\Phi) = \frac{1}{2}(0 \quad \nu)\left(gA^a_\mu\tau^a + \frac{1}{2}g'B_\mu\right)\left(gA^{b\mu}\tau^\mu + \frac{1}{2}g'B^\mu\right)\begin{pmatrix}0\\ \nu\end{pmatrix}. \tag{1.12}$$

Substituting τ^a and taking the product we get

$$(D_\mu\Phi)^\dagger(D^\mu\Phi) = \frac{\nu^2}{8}\left[g^2(A^1_\mu)^2 + g^2(A^2_\mu)^2 + (-gA^3_\mu + g'B_\mu)^2\right]. \tag{1.13}$$

We can now perform a field redefinition and recover

$$W^\pm_\mu = \frac{1}{sqrt2}\left(A^1_\mu \mp A^2_\mu\right) \qquad \text{with mass} \quad m_W = g\frac{\nu}{2} \tag{1.14}$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(gA^3_\mu - g'B_\mu\right) \qquad \text{with mass} \quad m_Z = \sqrt{g^2+g'^2}\frac{\nu}{2} \tag{1.15}$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g'A^3_\mu + gB_\mu\right) \qquad \text{with mass} \quad m_A = 0, \tag{1.16}$$

which are the standard model $W^\pm, Z$ bosons and photon.

The change in basis from the original massless bosons to massive bosons and photon is characterized by the weak mixing angle θ_w:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix}, \tag{1.17}$$

with

$$\cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}}, \quad \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}. \tag{1.18}$$

When the $W^\pm$ and Z bosons become massive they each gain an additional longitudinal degree of freedom. These new degrees of freedom aren't free; they come from the three components of the Higgs doublet Φ associated with the original broken symmetry. The Goldstone mechanism facilitates this transfer of degrees of freedom. The vernacular is that the massless Goldstone bosons of the original theory are "eaten" by the $W^\pm$ and Z.

Additionally, the Higgs mechanism provides a mass for the standard model fermions. Fermion mass terms are not explicitly allowed in the standard model because they would violate local gauge invariance since left and right fermions transform differently. Now that we have a scalar field Φ we can couple it to our fermion with a Yukawa coupling, $-c\bar{f}_L\Phi f_R$. In such a term f_L and f_R are the right and left components of the fermion, c is an arbitrary coupling characterizing the strength of the interaction with the Higgs VEV, and Φ is the Higgs VEV. Heavy particles like the top have a large value of c while light particles like the up quark have a small value of c.

Finally, the Higgs Boson, a particle excitations of the Higgs field, was recently discovered at the Large Hadron Collider (LHC).The discovery was announced to the pulbic July 4, 2012. Current experimental results pin the Higgs mass at 126 GeV. Peter Higgs and Franois Englert shared the Nobel prize in 2013 for their work in developing the theory of EWSB thorugh the Higgs mechanism.

1.3 Beyond Standard Model Physics

While the Standard Model is the pinnacle of our current understanding of the universe it is not the last word on the subject. There are many phenomena that the standard model does not explain. Furthermore there are theoretically troubling aspects to the standard model that leaves more to be desired.

Perhaps the biggest omission in the standard model is a quantum field theory of Gravity. Our current understanding of gravity is Einstein's theory of General Relativity (GR). GR is a thoroughly verified theory and famously explained the perihelion of mercury and the deflection of light rays massive objects. Unfortunately, general relativity is incompatible with quantum mechanics, and to date a force carrying boson, the graviton, has eluded detection. A related problem is that gravity is so much weaker than the other three fundamental forces. Another unexplained phenomena is neutrino oscillations. In nature we have observed the flavor of neutrinos to change. These oscillations prove that the neutrino's are not massless particles as the standard model previously had us believe. Finally we currently understand from astrophysical observations that the standard model only accounts for 4% of the energy budget of the universe. Dark matter, which has eluded detection, composes of another 21% of the energy budget. Dark Energy, which is responsible for the acceleration of the expansion of the universe, composes 75% of our energy budget. Taking this view the standard model actually explains a relatively small part of the universe.

In addition to not explaining the entire range of physical phenomena that we observe (or fail to observe) there are theoretical shortcomings that indicate a lack of complete understanding. The first indication that something is awry is the number of parameters. The standard model in its most general form has nineteen free parameters. All of these parameters are fixed by experiment and cover a large range of scales. We expect that a more nuanced understanding of the universe will require less parameters.

Another theoretical quirk of the standard model is the strong CP problem [42]. Unlike the electroweak interactions, QCD does not violate CP symmetry. CP symmetry is the symmetry of charge conjugation (C) and parity (P). The QCD Lagrangian allows terms that violate CP symmetry but these terms appear to be zero which is a type of fine tuning.

Finally there is the hierarchy problem. In the standard model the coupling constants of the theory change with energy scale. If you run the theory into the UV the coupling constants nearly converge at the GUT scale of 10^{16} GeV. The standard model can be run further to the plank scale and it remains a consistent theory. The fact remains that the standard model is an effective theory and at some point it must be cut off. The cutoff of the theory is removed from several orders of magnitude from the weak scale. This is a problem because the Higgs is an elementary scalar field and therefore has a relevant quartic self coupling. The renormalized Higgs mass is

$$m_H^2 = m_0^2 + \frac{3}{4\pi}\lambda\Lambda^2, \tag{1.19}$$

where m_0 is the bare Higgs mass, λ is the quartic coupling, and Λ is the cutoff scale. The renormalized mass has a quadratically divergent additive renormalization to its mass that is proportional to the cutoff. To keep the renormalized Higgs mass at its physical value an unnatural degree of fine tuning to its bare mass, m_0, and quartic term, λ, is required. A more natural solution to this problem is for Λ to be at a scale similar to the Higgs mass.

These issues have pushed researchers to look for extensions or alternatives to the standard model that answer one or more of these unresolved questions. One area of active research has been the hierarchy problem. The state of affairs for almost 40 years was that we knew there had to be a Higgs mechanism to complete the standard model. We also had a very good guess where to look for the Higgs boson because of the mass of the $W^{\pm}$ and the Z. Solving the hierarchy problem and predicting the Higgs mass became a popular game, guess right and win a Nobel Prize. Most methods of solving the hierarchy problem introduce new physics at the electroweak scale. This moves the SM cutoff much closer to the electroweak scale, removing the need for fine tuning. Because the Tevatron and more recently the LHC could reach energies high enough to probe the electroweak scale it was possible to build and test falsifiable theories.

Super Symmetry is one such approach that postulates a symmetry between fermions and bosons. This symmetry results in a cancellation of the quadratic divergence that the scalar Higgs theory has. Another approach which I will describe further in the next section is for the Higgs particle to be a composite particle. A consequence of all of these theories is new particles. Now we know the value of the Higgs mass. Additionally, no other particles have yet been discovered by the

LHC. This has proven to be a deadly combination, ruling out the simplest incarnations of these models.

1.4 Composite Higgs

One solution to the hierarchy problem is for the Higgs to be a composite composed of particles from a new strongly interacting sector [81, 55, 117, 112, 118, 43, 41]. This new sector is responsible for electroweak symmetry breaking. There are many types of theories that use such a modus operandi. While these theories have the benefits of solving the heirarchy and fine tuning problems with the standard model Higgs they typically favor a heavier Higgs mass than what has been observed and pose other theoretical challenges that I will elaborate on below. In the next three subsections I will briefly discuss technicolor, extended techniclolor, introduce the conformal window, and discuss the state of Lattice endeavors in this area.

1.4.1 Technicolor

Technicolor seeks to replace the Higgs sector of the electroweak theory with a new strongly interacting sector. In this section I will show how this is accomplished. Some good reviews on the subject can be found in references [75, 85, 108, 90, 106] For simplicity lets consider a $SU(3)_{TC}$ gauge group that couples to a pair of massless fermions $\Psi = (\psi_1, \psi_2)$ in the fundamental representation. It is worth noting that so far this theory is identical to QCD with massless up and down quarks. The Lagrangian for our theory is

$$\mathcal{L}_{TC} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \sum_i \bar{\Psi}(i\not{D})\Psi_i. \tag{1.20}$$

Recall that fermions have a right handed and left handed component $\phi = (\phi_L, \phi_R)$. We know from QCD that such a theory possesses a global $SU(2)_L \times SU(2)_R$ symmetry and that this symmetry is spontaneously broken to the vector subgroup $SU(2)_V$. This symmetry manifests itself in the nonzero VEV of the chiral condensate

$$< \bar{\Psi}\Psi > = < \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L > \neq 0. \tag{1.21}$$

To see how spontaneous chiral symmetry breaking translates to spontaneous electroweak symmetry breaking we have to use chiral perturbation theory [54, 59]. Chiraly perturbation theory is an effective field theory whose fundamental degrees of freedom are the Goldstone bosons associated with the symmetry breaking. In our case the broken $SU(2)$ symmetry has 3 degrees of freedom and therefore our effective theory will have three massless Goldstone bosons. Continuing our analogy to QCD, these bosons are the pions.

The chiral Lagrangian is nonlinear and possesses an infinite number of terms. The standard approach is to perform an expansion about momentum, p, that are small with respect to the cutoff Λ. We are then free to choose what order in $\frac{p}{\Lambda}$ we work with. The lowest order in the expansion is

$$\mathcal{L}_\chi = \frac{F^2}{4} Tr\Big[(D^\mu U)^\dagger (D_\mu U)\Big]. \tag{1.22}$$

U is a non-linear function of the Goldstone fields ϕ_a

$$U \equiv e^{\sigma^a \phi_a \frac{2i}{f}}. \tag{1.23}$$

The derivative D_μ is coupled to the electroweak and gauge field because our technifermions are charged under $SU(2) \times U(1)$

$$D_\mu = \partial_\mu - ig\frac{\sigma^a}{2} A^a_\mu + ig'\frac{\sigma^3}{2} B_\mu. \tag{1.24}$$

We can substitute our derivative into our lowest order effective Lagrangian and find that

$$\mathcal{L}_\chi = \frac{F^2}{4} Tr\left[\frac{g^2}{4}\left[\left(A^1_\mu - \frac{4}{Fg}\partial_\mu\phi_1\right)^2 + \left(A^2_\mu - \frac{4}{fg}\partial_\mu\phi_2\right)^2 + \left(A^3_\mu - \frac{g'}{g}B_\mu - \frac{4}{fg}\partial_\mu\phi_3\right)^2\right]\right]. \tag{1.25}$$

We can now make the field redefinitions

$$\begin{aligned} W^{1,2}_\mu &\equiv A^{1,2}_\mu - \frac{4}{fg}\partial_\mu\phi_{1,2} \\ Z_\mu &\equiv \frac{g}{\sqrt{g^2+g'^2}}\left(A^3_\mu - \frac{g'}{g}B_\mu - \frac{4}{fg}\partial_\mu\phi_3\right) \\ A_\mu &\equiv \frac{g}{\sqrt{g^2+g'^2}}\left(\frac{g'}{g}A^3_\mu + B_\mu\right). \end{aligned} \tag{1.26}$$

The field redefinition completely removes ϕ_a from our theory to first order. The field A_μ is the massless photon field. Our Lagrangian now reads

$$\mathcal{L}_\chi = \mathcal{L}_{\text{kinetic}} + \frac{1}{2}m^2_W\left[(W^1_\mu)^2 + (W^2_\mu)^2\right] + \frac{1}{2}m^2_Z Z^2_\mu, \tag{1.27}$$

where the masses for the heavy bosons are

$$\begin{aligned} m_W &= \frac{1}{2}Fg \\ m_Z &= \frac{1}{2}F\sqrt{g^2+g'^2}. \end{aligned} \tag{1.28}$$

With the substitution $F = \nu$ we reproduce m_W and m_Z that we found in section 1.2 exactly! This is quite a shock, as I mentioned earlier so far everything we are doing is simply QCD with 2 flavors of massless quarks, why did we introduce the Higgs field in the first place? Simply put the effect in QCD is much too small. In QCD, $F = F_\pi \approx 93MeV$, this is more than a factor of a thousand too small to account for the observed $W^\pm$ and Z masses.

Despite the fact that spontaneous chiral symmetry breaking in QCD is too small to account for the observed electroweak sector, there is no reason that we can't introduce a new strongly interacting theory where the equivalent $F = \nu \approx 246$. Furthermore we can choose a different gauge group, number of fermions, and representation if we wish. The only constraint is that chiral symmetry is broken and the results are phenomenologically consistent with experimental observation. It is worth noting that more flavors of fermions will produce additional pions that will acquire masses. In general $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$ will produce $N_f^2 - 1$ Goldstone bosons, three can be eaten to form the $W^\pm$ and Z and the remaining $N_f^2 - 4$ become massive pseudo-Goldstone bosons. The masses of the pseudo-Goldstone bosons in a viable theory need to be large enough to not have been discovered yet.

In summary we have shown how strong dynamics can explain electroweak symmetry breaking. Because of asymptotic freedom, such a theory does not have UV divergences, solving the fine tuning problem. One failure of our model is that we have lost an explanation for the fermion masses. It is possible to construct an effective theory of fermion masses using a four fermion operator and adjusting the couplings for each quark and lepton flavor by hand. Unfortunately a four-Fermi interaction is non-renormalizable, for a UV complete technicolor that gives fermion masses we need extended Technicolor.

1.4.2 Extended Technicolor

In order for a theory of electroweak symmetry breaking to replace the Higgs mechanism it must also generate masses for the quarks and leptons of the standard model. This is accomplished through a framework called extended technicolor (ETC)[1] . Under this framework an extended technicolor group is introduced $SU(3 + N_{TC})_{ETC}$. This extended group is also asymptotically free and therefore UV complete. Both the usual standard model content as well as $SU(N_{TC})$ are charged under ETC. At some scale $\Lambda_{ETC} >> \nu$ the ETC group spontaneously breaks to $SU(N)_{TC} \times SU(3)_c \times SU(2)_L \times U(1)_Y$. The gauge bosons of ETC theories have masses $M_{ETC} \approx g_{ETC}\Lambda_{ETC}$. Because of the wide range of masses for standard model fermions, most models ETC models spontaneously break at multiple scales. Most models have three breaking scales, corresponding to the three generations of quarks and leptons. ETC is an ambitions theory, a successful model of ETC will solve almost all the outstanding theoretical issues with the SM. Both the flavor hierarchy problem and the number of free parameters in the standard model would be solved dynamically. It is not surprising then that no 'reasonable' ETC model has been produced yet.

Extended technicolor faces many practical problems. One major hurdle is flavor changing neutral currents (FCNC). Flavor changing neutral currents such as $\mu \rightarrow e\bar{e}e$ and mixing between neutral mesons are highly constrained in the standard model. Most ETC theories typically introduce FCNC that are proportional to M_{ETC}^2.

Another issue with ETC theories is size of the top quark mass. The top quark is so massive, $m_t = 172GeV$, that its associated ETC scale is $M_{ETC} \approx 3TeV$. This is comparable to the electroweak scale! Additionally incorporating both the top and the bottom with the same ETC breaking would require an accompanying large isospin breaking. This has conundrum has forced many models to give the top special treatment in so called top assisted ETC.

Skirting these and other issues without producing particles light enough to have already been discovered is a daunting task. There are many models that have been able to trade one defect

[1] The first rule of ETC is no scalar fields, the second rule of ETC is no scalar fields.

for another, but to date no complete model exists. There is a general consensus that any viable ETC model probably will have a slowly running (walking) coupling. In a walking theory $\gamma(\alpha(\mu))$ is large between Λ_{TC} and M_{ETC} It has been shown that walking behavior addresses all of the problems I have discussed to a point. However any model that breaks at multiple scales and has the peculiarities of walking between those scales is likely to be very baroque. I discuss how walking a walking theory can be generated in more detail below.

1.4.3 Conformal Window

A general class of strongly interacting theories that are of interest for Technicolor and extended Technicolor dynamics are Yang-Mills gauge theories [32, 34]. Pure gauge Yang-Mills theories have $SU(N_C)$ interactions while more general theories include N_f flavors of fermions in some representation. At low energies QCD is effectively a $SU(3)$ gauge theory with N_f=2. This description includes only the up and down quarks which are nearly massless and respect an approximate $SU(2)$ isospin symmetry. The most general Yang-Mills Lagrangian with a $SU(N_c)$ local gauge symmetry and N_f flavors of massless fermions in a representation R is:

$$\mathcal{L}_{YM} = -\frac{1}{4g^2}\sum_{a=1}^{N_c} F^a_{\mu\nu}F^{\mu\nu,a} + \sum_{i=1}^{N_f} \bar{\Psi}_i \left(i\not{D}\right) \Psi_i. \tag{1.29}$$

$F^a_{\mu\nu}$ is the field strength tensor, shown here for an arbitrary group

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{ijk} A^i_\mu A^j_\nu, \tag{1.30}$$

where A^a_μ is the field, a is a group index and the structure coefficient f^{ijk} is determined by the commutation relationship

$$[T^i, T^j] = i f^{ijk} T^k, \tag{1.31}$$

and T^a are the generators of the group. The Lagrangians may look similar however their behavior in both UV and IR can be dramatically different from theory to theory. While the Lagrangian is classically scale invariant; the quantum theory is not. This is understood through the beta function,

$$\beta(g^2) = -\mu\frac{\partial g^2}{\partial\mu}. \tag{1.32}$$

Representation	$\dim(R)$	$T(R)$	$C_2(R)$
F	N	$\frac{1}{2}$	$\frac{N^2-1}{2N}$
S_2	$\frac{N(N+1)}{2}$	$\frac{N+2}{2}$	$\frac{(N+2)(N-1)}{N}$
A_2	$\frac{N(N-1)}{2}$	$\frac{N-2}{2}$	$\frac{(N-2)(N+1)}{N}$
G	N^2-1	N	N

Table 1.4: This table summarizes some commonly used groups, their dimension, first Casimir invariant, and second Casimir invariant. F, S_2, A_2 and G are the is the fundamental, 2-index symmetric, 2-index antisymmetric, and adjoint representations respectively.

The beta function describes how the gauge coupling evolves as the renormalization scale μ is changed. This function can be expanded in perturbation theory and is universal to two loops:

$$\beta\left(g^2\right) = -\frac{b_1}{16\pi^2}g^4 - \frac{b_2}{(16\pi^2)^2}g^6 \tag{1.33}$$

Any terms beyond two loops are renormalization scheme dependant and are not relevant for the following discussion. The values for b_1 and b_2 are:

$$\begin{aligned} b_1 &= \frac{11}{3}N_c - \frac{4}{3}N_f T(R) \\ b_2 &= \frac{34}{3}N_c^2 - \frac{4}{3}T(R)N_f\Big[5N_c + 3C_2(R)\Big]. \end{aligned} \tag{1.34}$$

$T(R)$ and $C_2(R)$ are the first and second Casimir invariants and depend on the representation R of the group. Table 1.4.3 give these invariants for a few common representations.

Clearly, tuning N_c, N_f and R offers a great deal of freedom in specifying the gauge theory. We can see from equation 1.34 that if the coefficients b_1 and b_2 are both positive than the beta function is negative. In this scenario the theory asymptotically free, confining, and spontaneously breaks chiral symmetry. The dynamics in the IR will be strongly coupled and non-perturbative. Figure 1.4 shows what this scenario looks like. This occurs in QCD ($SU(3)$ $N_f = 2$ or 3 depending on your treatment of the light quarks), which is an example of a theory where the beta function is negative.

If we keep b_1 positive and allow b_2 to become negative we can force the two terms to compete. In perturbation theory this occurs at at a critical value of fermions $N_f^{crit} < \frac{17N_c^2}{2T(R)[5N_c+C(R)]}$. Such a beta function would start out negative, then at some coupling it would pass through a local minimum after which it would grow. Eventually, at some g_*^2, the beta function would have a zero where $\beta(g_*^2) = 0$. Perturbatively this zero occurs at $g_*^2 = -b_1/b_2$ and is a Banks-Zaks infrared fixed point. This is illustrated in figure 1.5, such a theory is governed by the conformal dynamics at the infrared fixed point and is scale invariant. If the fixed point is at very weak coupling it is possible to gain insights from perturbation theory, however many theories are known to have strongly coupled IRFP's where insights from perturbation theory are unreliable. Conformal theories, like the scenario described above, do not support bound states of particles. As such they are often referred to as 'unparticle theories'.

Allowing both b_1 and b_2 to be negative results in a trivial theory. Such a theory, shown in figure 1.6 is not asymptotically free. The beta function is very similar to that of QED. Perturbatively this occurs when $N_f < \frac{11N_c}{4T(R)}$.

The region in theory space between where the IRFP appears and where asymptotic freedom is lost is referred to as the conformal window. Limiting our consideration to theories using the fundamental representation fermions and $N_c = 3$ we see that perturbation theory predicts the conformal window is between $N_f \approx 9.4$ and $N_f = 16.5$. It is important to note that all of the discussion in this section are perturbative results. On the upper end of the conformal window before asymptotic freedom is lost, the IRFP of many theories is weak enough that perturbation theory is reasonable. However, as we pass through the conformal window and consider theories with less fermionic degrees of freedom, the IRFP becomes strongly coupled. Here perturbation theory is not to be trusted. The location of the lower bound of the conformal window is an area of active research.

The conformal window itself is not a very interesting system to study but understanding the lower bound of the conformal window is of great interest for potential Technicolor theories. Right below the conformal window it is believed that a walking theory can exist. In such a theory, shown in figure 1.7 the beta function would start out like the conformal scenario. However, as the beta function approaches the IRFP, chiral symmetry is spontaneously broken and the beta function would turn around away from the 'would be' IRFP. If this occurs, the coupling g will run very slowly (walk) over a wide range of scales where the β function approaches zero. Walking is necessary to achieve a wide separation of scales in Technicolor theories without generating flavor changing neutral currents.

1.4.4 Lattice Studies

Today there is a rich ecosystem of lattice studies of BSM studies, a survey of the current state of the field can be found in [**?**]. It is extremely important that different ways of probing for IR conformality on different lattice discretization schemes converge to common conclusions. In general this is accomplished but there are still systems that cause debate because different groups draw different conclusions from their findings. Below I summarize the sate of the field both in terms of calculation types and methods of theories studied.

Many studies that seek to locate the conformal window have proceeded by calculating the RG flow of the theory under consideration. The first example of this type of study is [10]. If an IRFP is found then the theory is conformal while a lack of an IRFP at strong coupling indicates that chiral symmetry is broken and the theory confines. One benefit of running coupling calculations is that they are not as computationally intensive as many other alternatives. There are multiple types of calculations that fall under this umbrella. Schrödinger functional, MCRG (section **??**), and now the Wilson flow step scaling (section **??**) are examples.

Another common method uses lattice spectroscopy calculations. In these methods the masses of light bound states are measured at a variety of input quark masses. The fit of the bound state as a function of the bare quark mass will determine if the theory is best described by a chiraly broken theory or conformal one. References [78, 79, 45] are examples of studying the conformal window using spectroscopy. In practice these fits are extremely difficult, in particular fits for volume squeezed conformal theories has proved troublesome.

Several groups have studied the finite temperature phase diagram [77, 107]. These studies identify an IRFP with a deconfining bulk transition. Additionally techniques using Dirac eigenvalue spectrum [27] has proven useful in calculating γ_m over a wide range of scales.

The majority of calculations have been for theories with $N_c = 3$ with fermions in the fundamental representation. This is in part because the wide availability of optimized code bases for three colors. Within these studies staggered fermions are commonly employed because they are cheap to simulate and preserve a U(1) remnant of chrial symmetry **??**. As a consequence theories with $N_f = 4, 8, 12, 16$ are most widely studied although others have been studied as well. The consensus is that $N_f \leq 8$ are confining, $N_f = 12$ is probably conformal, and $N_f = 16$ is conformal.

There is also a grown number of calculations with $SU(2)$. The two flavor model in the adjoint representation is widly studied and is clearly in the conformal window[23, 37, 36, 35]. Studies with fundamental fermions exist for $N_f = 2, 4, 6, 8, 10$ with the conformal window being identified as existing between $4 < N_f < 8$. Studies of $SU(3)$ and $SU(4)$ in the 2-index symmetric representation also exist.

Clearly, beyond standard model physics is inherently a difficult subject to study on the lattice. Unlike studies of QCD we don't know the answer before hand. Taking a holistic approach and finding consensus is vital to our understanding. Additionally, as more of these theories have been studied, vital improvements have been made. This is true of the lattice code base, which prior to BSM studies was highly optimized and sometimes only available for QCD simulations. This is also true for improvements in how we analyze lattice data. I will discuss two improvements we have developed in chapters **??** and **??**. Chapter **??** introduces an improvement to the gradient flow method for calculating the renormalized step scaling function. Chapter **??** introduces an improvement to the traditional MCRG method of calculating the bare step scaling function. Finally work in BSM physics on the lattice has pushed forward our knowledge of formulating lattice actions and understanding the nature of lattice artifacts. Many of the theories that have been studied are sufficiently un-QCD like that lattice QCD intuitions are being upended with a new tribal knowledge.

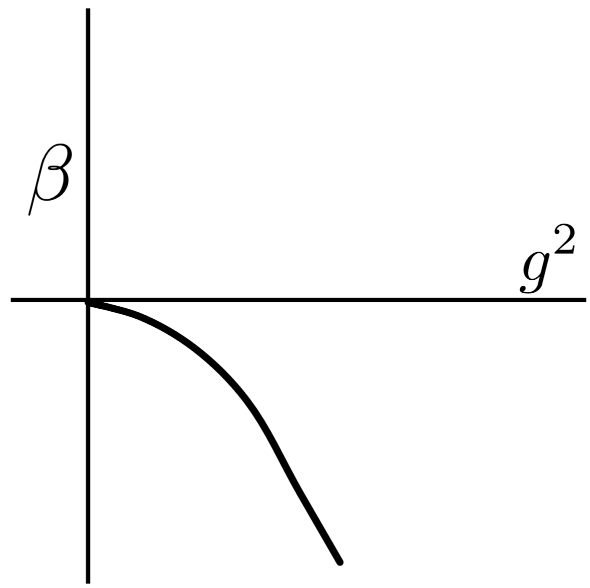

Figure 1.4: In a confining theory, such as QCD, the running of the coupling starts out negative. As g^2 grows the running becomes increasingly negative. Chiral symmetry is broken and the theory is described by confined bound states.

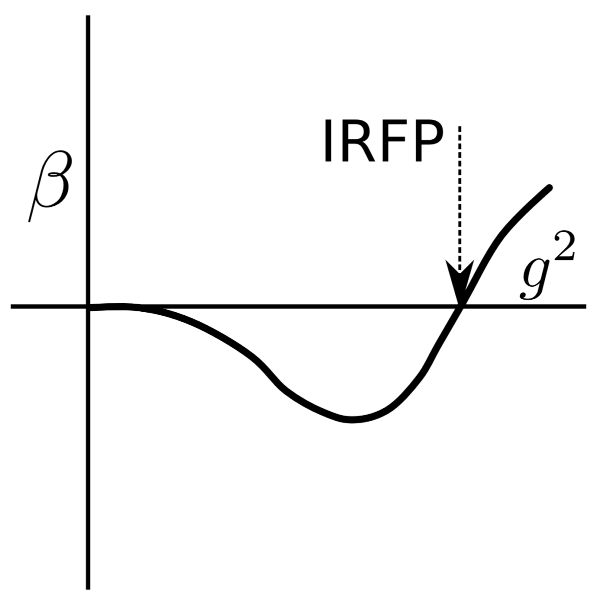

Figure 1.5: In a conformal field theory the β function starts out negative but turns around and crosses the $\beta = 0$ axis. At this crossing the theory has an infrared fixed point. The renormalized flow for the theory will run to the IR fixed point and the theory is conformal.

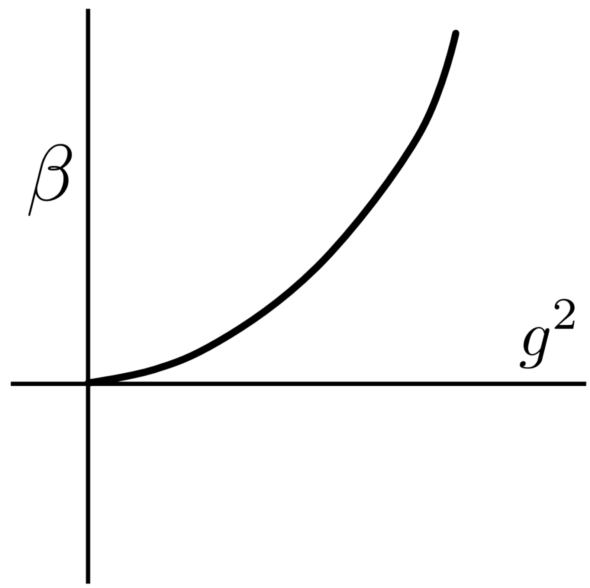

Figure 1.6: If the fermionic degrees of freedom overwhelm those of the bosons it the beta function will start positive. This is the loss of asymptotic freedom. The beta function for such a theory resembles that of QED.

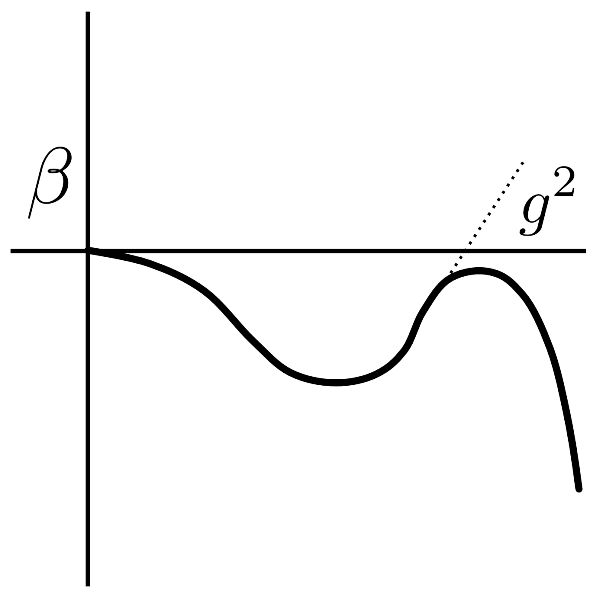

Figure 1.7: In a walking theory the beta function looks similar to the conformal picture of figure 1.5. The crucial difference is that before the β function develops a zero, shown by the dashed line, chiral symmetry is spontaneously broken driving the beta function back down. Near the would be IRFP where the dashed line has a zero, the coupling runs very slowly and is said to be walking.

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