Studies with Improved Renormalization Group Techniques

by

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Dedication

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1.1 Introduction

For the past several years many lattice groups have been involved in studying strongly-coupled near-conformal gauge–fermion systems. Some of these models may be candidates for new physics beyond the standard model, while others are simply interesting non-perturbative quantum field theories. Because the dynamics of these lattice systems are unfamiliar, it is important to study them with several complementary techniques. Not only does this allow consistency checks, it can also provide information about the most efficient and reliable methods to investigate near-conformal lattice theories.

Monte Carlo Renormalization Group (MCRG) two-lattice matching is one of several analysis tools that we are using to investigate SU(3) gauge theories with many massless fermion flavors. This technique predicts the step-scaling function s_b in the bare parameter space. In a previous work [99] we proposed an improved MCRG method that exploits the Wilson flow to obtain a bare step-scaling function that corresponds to a unique discrete β function. We briefly review our Wilson-flow-optimized MCRG (WMCRG) procedure in Sections 1.2. It is important to note that we are investigating a potential infrared fixed point (IRFP) where the coupling is irrelevant: its running slows and eventually stops. This is challenging to distinguish from a near-conformal system where the gauge coupling runs slowly but does not flow to an IRFP. The observation of a backward flow that survives extrapolation to the infinite-volume limit could provide a clean signal. In Section 1.3 we report WMCRG results for SU(3) gauge theory with $N_f = 12$ flavors of massless fermions in the fundamental representation.

This 12-flavor model has been studied by many groups, including Refs. [10, 39, 46, 7, 66, 33, 28, 80, 86, 6, 48, 49, 76, 27, 5, 69, 68, 26]. Using new ensembles of 12-flavor gauge configurations generated with exactly massless fermions, our improved WMCRG technique predicts a conformal IRFP where the step-scaling function vanishes. As with every method, it is essential to study the systematic effects. For WMCRG the most important systematic effects are due to the finite volume and limited number of blocking steps. While we are not able to carry out a rigorous infinite-volume extrapolation, the observed zero of the bare step-scaling function is present for all investigated lattice volumes and renormalization schemes, and agrees with the earlier MCRG results of Ref. [66]. The results of our complementary $N_f = 12$ investigations of finite-temperature phase transitions [107, 69], the Dirac eigenmode number [27, 26], and finite-size scaling [68] are also consistent with the existence of an infrared fixed point and IR conformality.

1.2 Wilson Flow Optimized MCRG

As an alternative to optimizing the RG blocking transformation, and thus changing the renormalization scheme at each coupling β_F, here we propose to use the Wilson flow to move the lattice system as close as possible to the renormalized trajectory of a fixed renormalization scheme.

The Wilson flow is a continuous smearing transformation [92] that can be related to the $\overline{\text{MS}}$ running coupling in perturbation theory [87]. Refs. [51, 50] recently used the Wilson flow to compute a renormalized step-scaling function in a way similar to Schrödinger functional methods. While this approach appears very promising, it is based on perturbative relations that are only fully reliable at weak coupling. Here we do not use this perturbative connection, instead applying the Wilson flow as a continuous smearing that removes UV fluctuations. The Wilson flow moves the system along a surface of constant lattice scale in the infinite-dimensional action-space; it is not a renormalization group transformation and does not change the IR properties of the system.

Our goal is to use a one-parameter Wilson flow transformation to move the lattice system as close as possible to the renormalized trajectory of our fixed RG blocking transformation. This is shown in figure 1.1. We proceed by carrying out two-lattice matching after applying the Wilson flow for a flow time t_f on all lattice volumes. (The Wilson flow is run only on the unblocked lattices, not in between RG blocking steps.) As above, since we can block our lattices only a few times, we must optimize t_f by requiring that consecutive RG blocking steps yield the same $\Delta\beta_F$, as shown in 1.2. As for traditional MCRG, increasing the number of blocking steps reduces the dependence on the optimization parameter; in the limit $n_b \to \infty$, our results would be independent of t_f.

With Wilson-flowed MCRG we can efficiently determine bare step-scaling functions that correspond to unique RG β functions. The uniquness of the β function is a result of using a fixed block transformation. In this work our block transformation consists of nHYP smearing the unblocked lattice and then multiplying adjacent links in the same direction.

The ability to study a unique β function opens up interesting directions for future studies. By comparing different β functions around the perturbative gaussian FP, we can study scaling violations in the lattice system. In IR-conformal systems, we can investigate the scheme-dependence of the β function near the IRFP, an issue explored in perturbation theory by Ref. [104].

1.3 12 Flavor Results

Our WMCRG results for the 12-flavor system are obtained on gauge configurations generated with exactly massless fermions. Our lattice action uses nHYP-smeared staggered fermions as described in Ref. [28], and to run with $m = 0$ we employ anti-periodic boundary conditions in all four directions. All of our analyses are carried out at couplings weak enough to avoid the unusual strong-coupling "$\cancel{S^4}$" phase discussed by Refs. [28, 69].

We perform three-lattice matching with volumes 6^4–12^4–24^4 and 8^4–16^4–32^4. Three-lattice matching is based on two sequential two-lattice matching steps, to minimize finite-volume effects [66]. Both two-lattice matching steps are carried out on the same final volume V_f. We denote the number of blocking steps on the largest volume by n_b, and tune the length of the initial Wilson flow by requiring that the last two blocking steps predict the same step-scaling function. Using the 8^4–16^4–32^4 data we determine the bare step-scaling function for $n_b = 3$ and $V_f = 4^4$ as well as $n_b = 4$ and $V_f = 2^4$, while the 6^4–12^4–24^4 data set is blocked to a final volume $V_f = 3^4$ ($n_b = 3$). This allows us to explore the effects of both the final volume and the number of blocking steps. We investigate three renormalization schemes by changing the HYP smearing parameters in our blocking transformation [99]: scheme 1 uses smearing parameters (0.6, 0.2, 0.2), scheme 2 uses (0.6, 0.3, 0.2) and scheme 3 uses (0.65, 0.3, 0.2).

Figs. 1.3, 1.4 and 1.5 present representative results for 12 flavors. All of the bare step-scaling functions clearly show $s_b = 0$, signalling an infrared fixed point, for every n_b, V_f and renormalization scheme. Appropriately for an IR-conformal system, the location of the fixed point is scheme dependent. We observe that the fixed point moves to stronger coupling as the HYP smearing parameters in the RG blocking transformation increase.

When we block our 8^4, 16^4 and 32^4 lattices down to a final volume $V_f = 2^4$ (corresponding to $n_b = 4$), the observables become very noisy, making matching more difficult. The problem grows worse as the HYP smearing parameters increase, and our current statistics do not allow reliable three-lattice matching for $V_f = 2^4$ in schemes 2 and 3. To resolve this issue, we are accumulating more statistics in existing 32^4 runs, and generating additional 32^4 ensembles at more values of the gauge coupling β_F. These additional data will also improve our results for scheme 1, which we show in Fig. 1.5. Different volumes and n_b do not produce identical results in scheme 1, suggesting that the corresponding systematic effects are still non-negligible. We can estimate finite-volume effects by comparing $n_b = 3$ with $V_f = 3^4$ and $V_f = 4^4$. Systematic effects due to n_b can be estimated from $n_b = 4$ and $V_f = 2^4$, but this is difficult due to the noise in the 2^4 data. Even treating the spread in the results shown in Fig. 1.5 as a systematic uncertainty, we still obtain a clear zero in the bare step-scaling function, indicating an IR fixed point.

In this chapter we have shown how the Wilson-flow-optimized MCRG two-lattice matching procedure proposed in Ref. [99] improves upon traditional lattice renormalization group techniques. By optimizing the flow time for a fixed RG blocking transformation, WMCRG predicts a bare step-scaling function s_b that corresponds to a unique discrete β function. Applying WMCRG to new 12-flavor ensembles generated with exactly massless fermions, we observe an infrared fixed point in s_b. The fixed point is present for all investigated lattice volumes, number of blocking steps and renormalization schemes, even after accounting for systematic effects indicated by Fig. 1.5. This result reinforces the IR-conformal interpretation of our complementary $N_f = 12$ studies of phase transitions [107, 69], the Dirac eigenmode number [27, 26], and finite-size scaling [68].

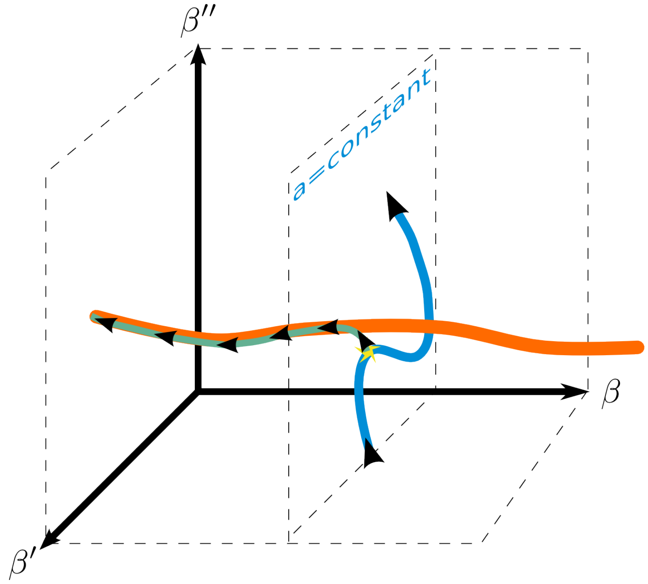

Figure 1.1: In Wilson Flow MCRG we use the Wilson Flow (blue) to approach the renormalized trajectory. An optimization step similar to that used in MCRG allows us to locate the flow time that gets us closest to the renormalized trajectory (orange). We then block our lattice using a fixed block transformation (green).

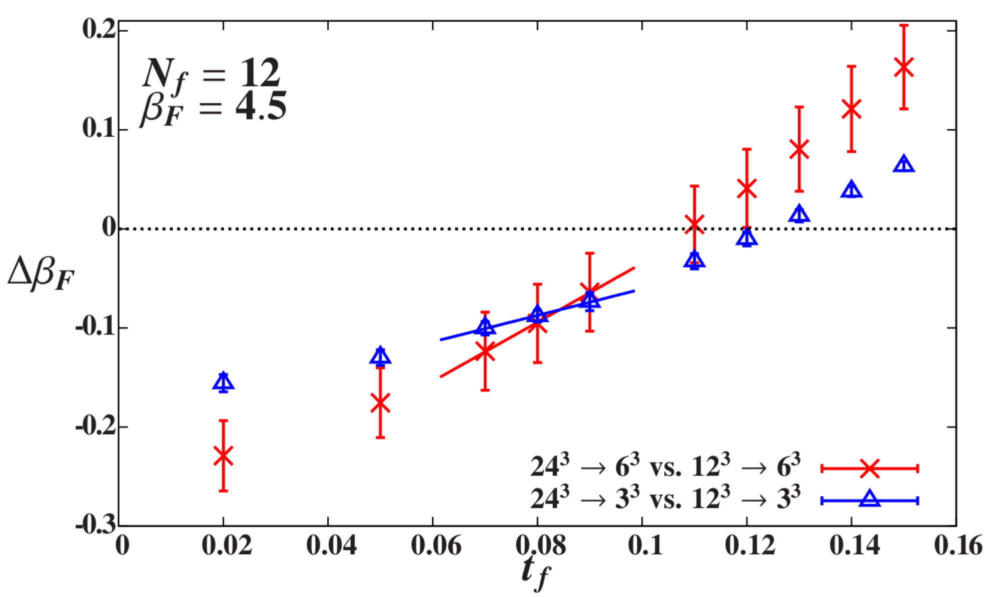

Figure 1.2: Optimization of the Wilson flow time t_f with fixed $\alpha = 0.5$, for $\beta_F = 4.5$. The uncertainties on the data points are dominated by averaging over the different observables.

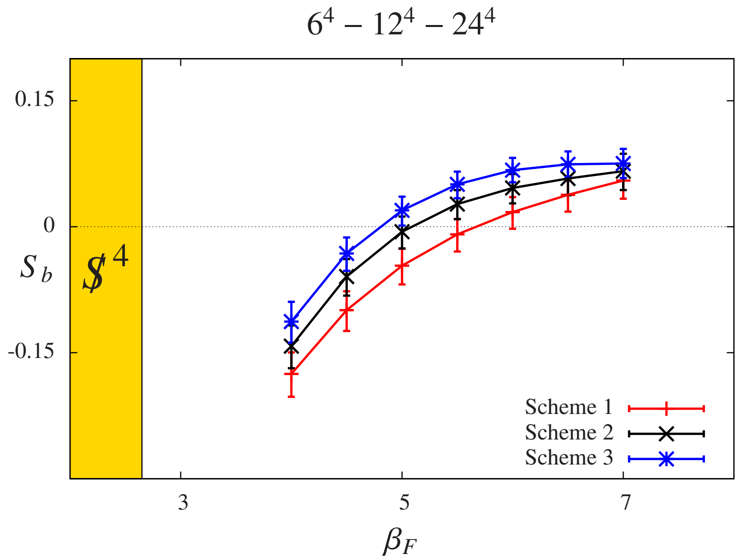

Figure 1.3: The bare step-scaling function s_b predicted by three-lattice matching with 6^4, 12^4 and 24^4 lattices blocked down to 3^4, comparing three different renormalization schemes. The error bars come from the standard deviation of predictions using the different observables.

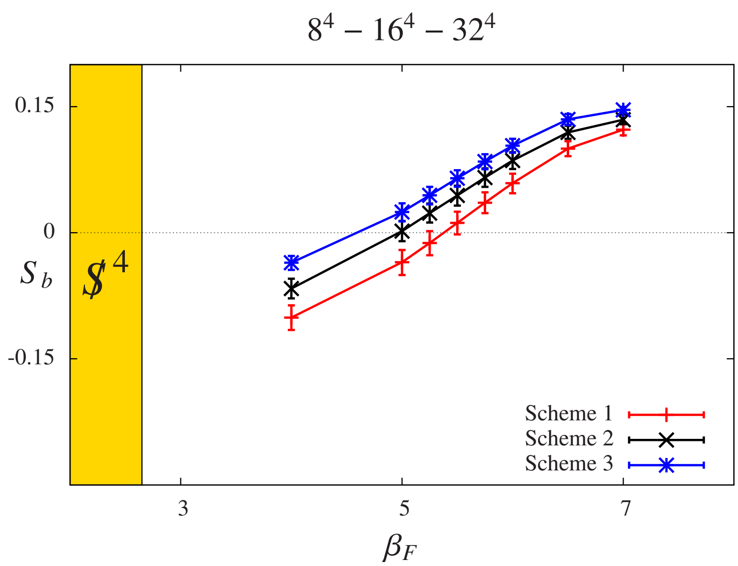

Figure 1.4: As in Fig. 1.3, the bare step-scaling function s_b for three different renormalization schemes from three-lattice matching, now using 8^4, 16^4 and 32^4 lattices blocked down to 4^4.

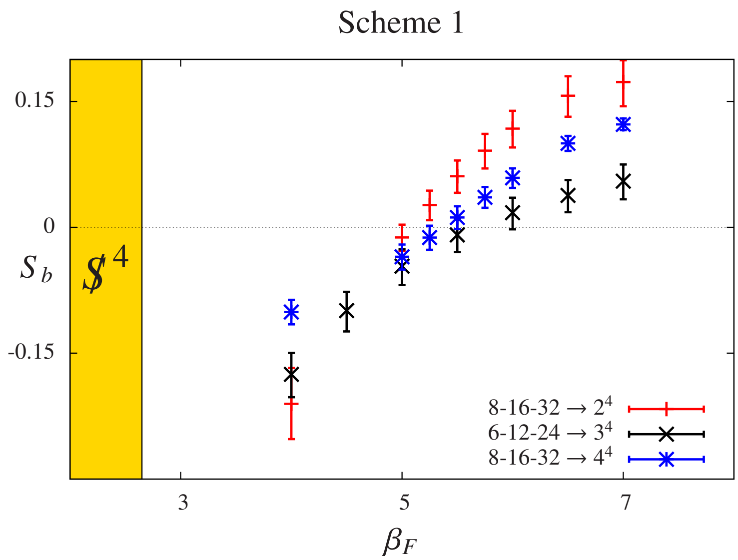

Figure 1.5: The bare step-scaling function s_b for scheme 1, comparing three-lattice matching using different volumes: 6^4, 12^4 and 24^4 lattices blocked down to 3^4 (black $\times$s) as well as 8^4, 16^4 and 32^4 lattices blocked down to 4^4 (blue bursts) and 2^4 (red crosses).

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