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Discrete Adjoint Approach to the Spalart-Allmaras Turbulence Model for Incompressible Flow in OpenFOAM

Fluid dynamic optimal control design problems are ubiquitous in the aerospace and automotive industry. In case of gradient based optimization, the so called adjoint method has long been identified as the method of choice for the computation of shape sensitivities [1]. A discrete adjoint solver is implemented in OpenFOAM. The focus lies on incompressible and steady flow for high Reynolds numbers, governed by the Reynolds-Averaged Navier-Stokes equations completed with the Spalart-Allmaras turbulence model. The discrete adjoint equation is derived using the method of Lagrange multiplier. The NACA 2412 airfoil is presented in order to verify the discrete adjoint solver against a total finite difference approach.

Governing Flow Equations

Reynolds-Averaged Navier-Stokes equations (RANS) closed with the Spalart-Allmaras turbulence model equation for incompressible, steady state and turbulent

$$\frac{\partial U_i}{\partial x_i} = 0$$

$$U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(2(\nu + \nu_t) S_{ij}\right)$$

$$U_j \frac{\partial \tilde{\nu}}{\partial x_j} = c_{b1}\tilde{S}\tilde{\nu} + (1 + c_{b2})\frac{\partial}{\partial x_j}\left(\tilde{\nu}_{\mathrm{eff}}\frac{\partial \tilde{\nu}}{\partial x_j}\right) - c_{b1}\tilde{\nu}_{\mathrm{eff}}\frac{\partial^2 \tilde{\nu}}{\partial x_j^2} - c_{w1} f_w \left(\frac{\tilde{\nu}}{d}\right)^2$$

flow for $i = \{1,2,3\}$ and $\nu_t = f_{v1}\tilde{\nu}$.

Equality Contrained Shape Optimization

$$\min_{\boldsymbol{x}} \quad f(\boldsymbol{x}, \tilde{\boldsymbol{\nu}}(\boldsymbol{x})), \quad \boldsymbol{x} \in \mathbb{R}^n, \ n \in \mathbb{N}_+$$
$$\text{s.t.} \quad \boldsymbol{R}_{\tilde{\nu}}(\boldsymbol{x}, \tilde{\boldsymbol{\nu}}(\boldsymbol{x})) = \boldsymbol{0}$$

Lagrange Multiplier Method for Shape Sensitivities

$$L = f(\boldsymbol{x}, \tilde{\boldsymbol{\nu}}(\boldsymbol{x})) + \boldsymbol{\psi}_{\tilde{\nu}}^{\mathrm{T}} \boldsymbol{R}_{\tilde{\nu}}(\boldsymbol{x}, \tilde{\boldsymbol{\nu}}(\boldsymbol{x}))$$

$$\frac{\partial L}{\partial \boldsymbol{x}} = \frac{\partial f}{\partial \boldsymbol{x}} + \boldsymbol{\psi}_{\tilde{\nu}}^{\mathrm{T}}\frac{\partial \boldsymbol{R}_{\tilde{\nu}}}{\partial \boldsymbol{x}} + \underbrace{\left(\frac{\partial f}{\partial \tilde{\boldsymbol{\nu}}} + \boldsymbol{\psi}_{\tilde{\nu}}^{\mathrm{T}}\frac{\partial \boldsymbol{R}_{\tilde{\nu}}}{\partial \tilde{\boldsymbol{\nu}}}\right)}_{:=\boldsymbol{0}^{\mathrm{T}}}\frac{\mathrm{d}\tilde{\boldsymbol{\nu}}}{\mathrm{d}\boldsymbol{x}}$$

Discrete Adjoint Equation

$$\left[\frac{\partial \boldsymbol{R}_{\tilde{\nu}}}{\partial \tilde{\boldsymbol{\nu}}}\right]^{\mathrm{T}} \boldsymbol{\psi}_{\tilde{\nu}} = -\left[\frac{\partial f}{\partial \tilde{\boldsymbol{\nu}}}\right]^{\mathrm{T}}$$

Discrete Adjoint Solver Verification

Once the adjoint variables $\boldsymbol{\psi}_{\tilde{\nu}}$ are known, the shape sensitivities can be computed with a semi-analytic finite difference approach

$$\frac{\partial L}{\partial \boldsymbol{x}} \approx \frac{\partial f}{\partial \boldsymbol{x}} + \boldsymbol{\psi}_{\tilde{\nu}}^{\mathrm{T}}\frac{\boldsymbol{R}_{\tilde{\nu}}(\boldsymbol{x}+\Delta \boldsymbol{x}) - \boldsymbol{R}_{\tilde{\nu}}(\boldsymbol{x}+\Delta \boldsymbol{x})}{2\Delta \boldsymbol{x}}$$

The adjoint approach is verified against a total finite difference approach

$$\frac{\mathrm{d}f}{\mathrm{d}\boldsymbol{x}} \approx \frac{f(\boldsymbol{x}+\Delta \boldsymbol{x}) - f(\boldsymbol{x}-\Delta \boldsymbol{x})}{2\Delta \boldsymbol{x}}$$

Note that for each shape perturbation a complete CFD solution is needed, however, the adjoint approach needs only two solver calls. The objective functions is chosen to be

$$f = \sum_{i \in I} \tilde{\nu}_i$$

where I is a set of numbers corresponding to respective cells.

NACA 2412 Airfoil

The NACA 2412 airfoil is presented as an example. The source term is visualized in

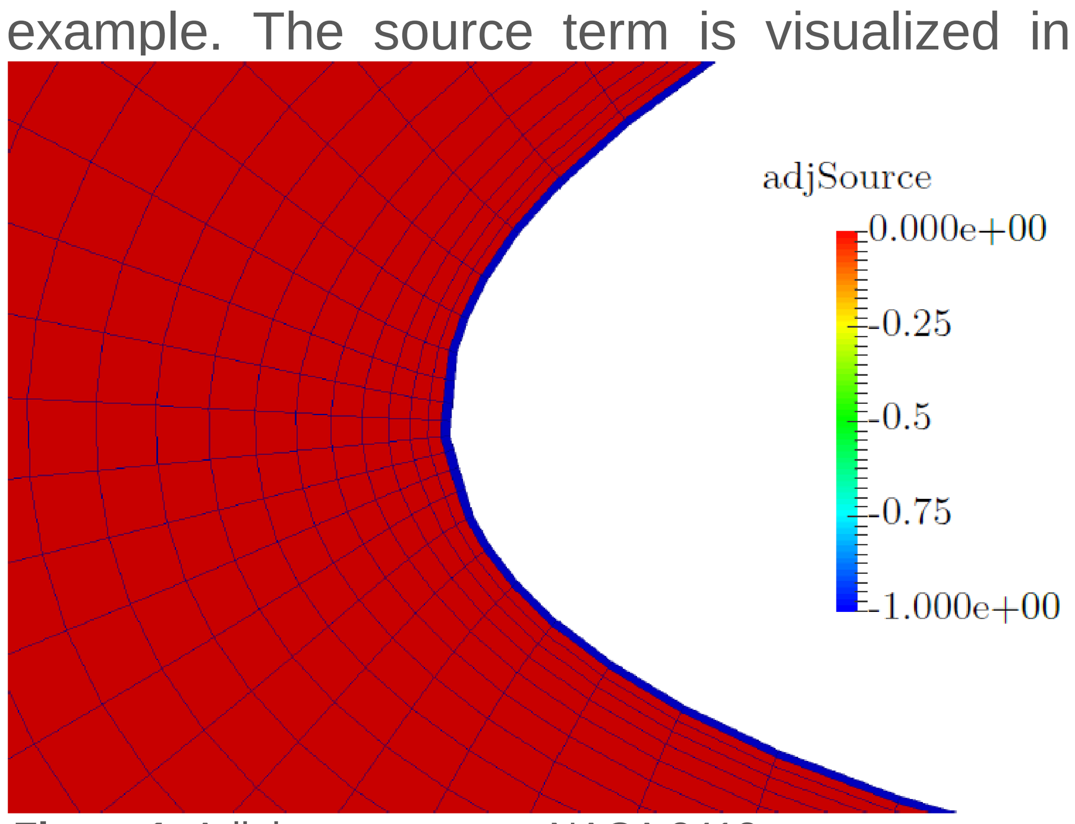


Figure 1: Adjoint source term NACA 2412

Fig. 1. It is chosen such that all cells adjacent to the airfoil take the value negative one. The scaled adjoint solution is shown in Fig. 2. Note that the solution is distributed

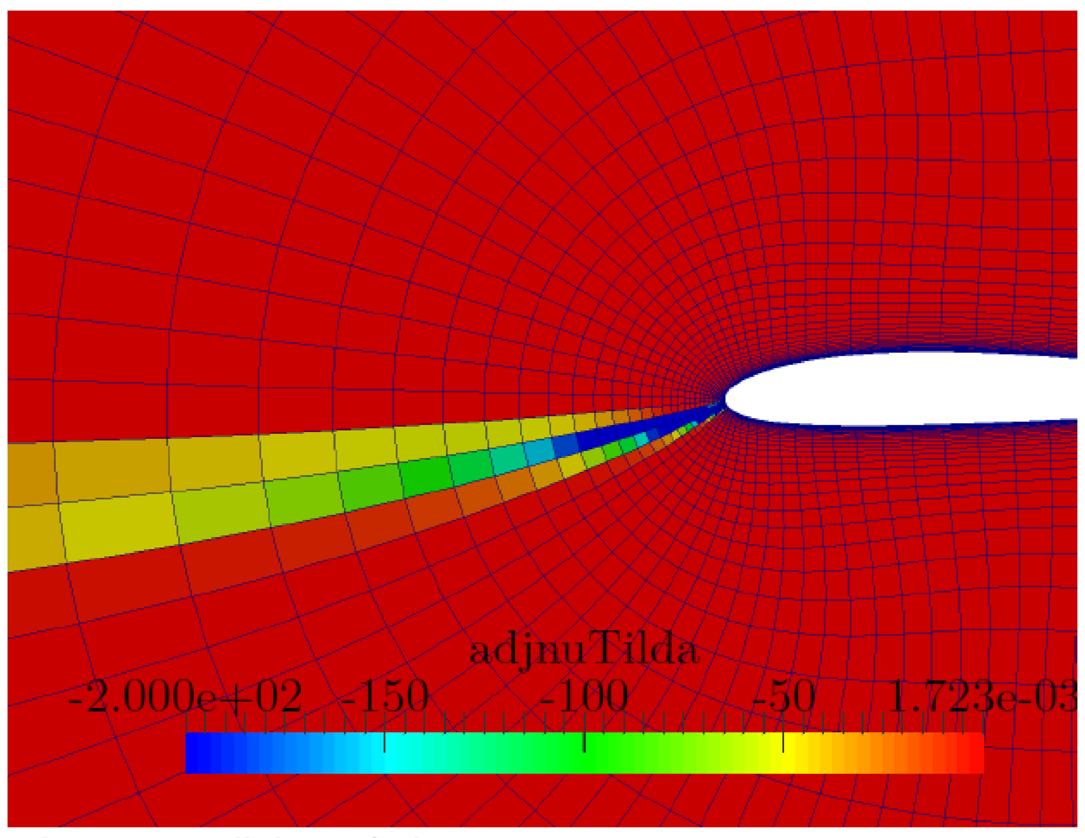


Figure 2: Adjoint solution NACA 2412

in the reversed flow direction. The shape sensitivities are presented in Fig. 3(a) for the adjoint and in Fig. 3(b) for the total finite difference approach respectively. As expected the results match very well. However, for the trailing and leading edge minor differences are denoted. A step size study is carried out for two points chosen to be in the hot spot and medium region as depicted in Fig. 4. It is observed that the sensitivities are not

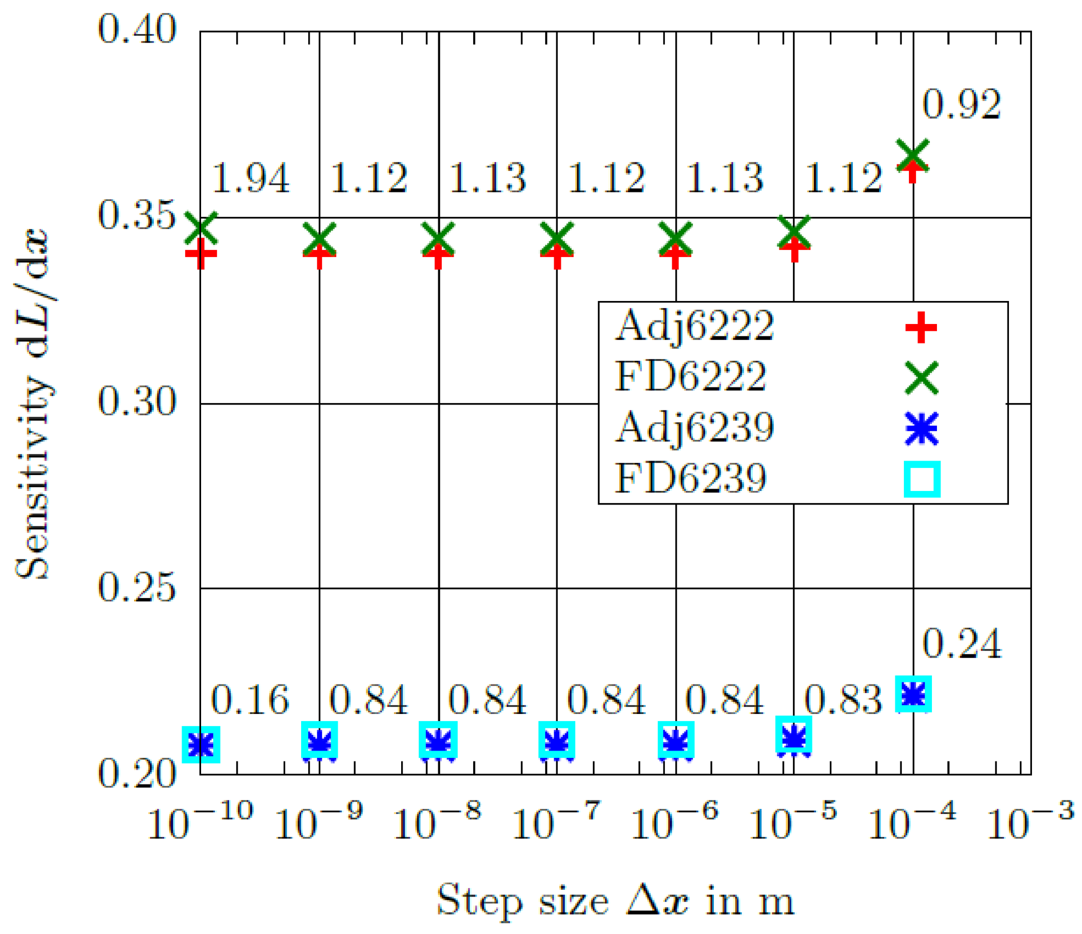
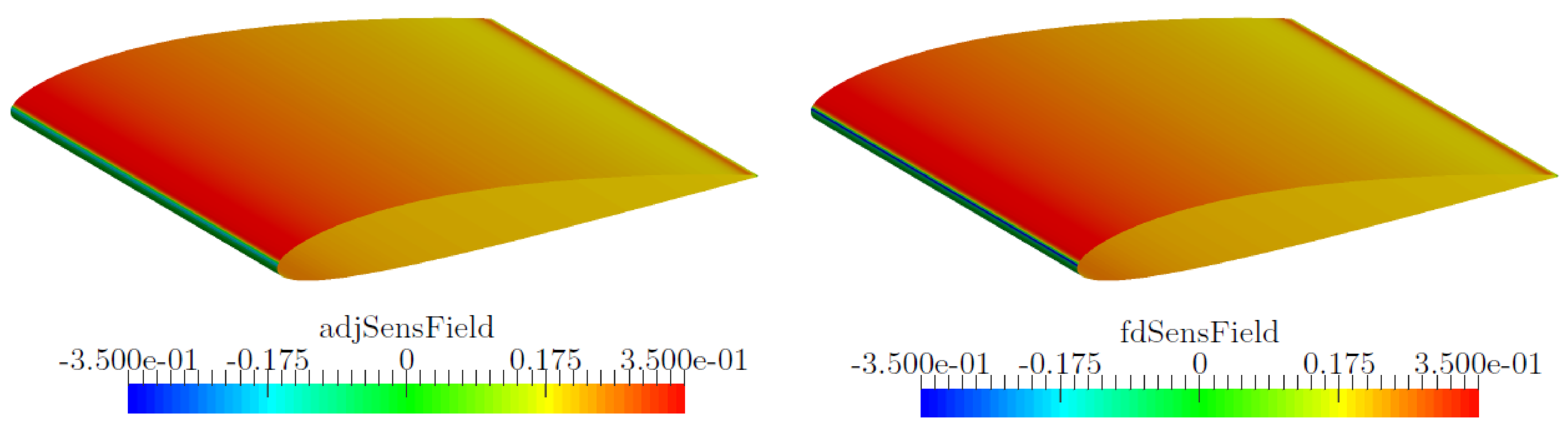


Figure 4: Step size study NACA 2412

sensitive w.r.t. the step size. Concluding that the adjoint solver is verified. More examples can be found in the thesis.

[1] C. Othmer. A continous adjoint formulation for the computation of topological and surface sensitivities of ducted flows. International Journal for Numerical Methods in Fluids, 58(8):861–877, 2008.



(a) Adjoint method

(b) Total finite difference method

Figure 3: Sensitivity scaled NACA 2412