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Mass dependence of energy loss in collisions of icy spheres: An experimental study

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Abstract. By varying the mass of icy spheres which collide with large ice blocks, we have determined the mass dependence of energy loss in such collisions. We find, at commercial deep freeze temperatures, that collisions at 1 cm/s become quite inelastic for small iceballs and that the rate at which elasticity decreases with decreasing mass roughly coincide with that predicted by a viscous dissipation model. If such a mass dependence persists at much lower temperatures, it could lead to very large effects in planetary ring systems (rings and dust, ice, surfaces, origin, and evolution).

Introduction

In ring systems, energy loss during collisions plays an important role in determining the mean free path between collisions, kinematic viscosity, ring thickness, and spreading rates [*Cook and Franklin*, 1964; *Goldreich and Tremaine*, 1978; *Stewart et al.*, 1984]. Also, in the Roche zone where most ring systems occur, substantial energy loss may be necessary for small grains to accrete into larger "particles" [*Wiedenschilling et al.*, 1984] or to reaccrete after disruptive collisions of satellites [*Esposito and Colwell*, 1989]. However, data on energy loss during collisions under typical ring conditions, impact speeds of a few centimeters per second or less and temperatures of 100–150°K, are difficult to obtain.

The coefficient of restitution, a key parameter for determining energy loss during collisions, was first measured for icy particles by *Bridges et al.* [1984]. Specifically, they observed low-velocity collisions of ice spheres with massive ice blocks (a proxy for spheres of infinite size and mass) at temperatures approximating those expected in ring systems. Later, *Hatzes et al.* [1988] simulated changes in "size" by modifying the radius of curvature of the spheres at the point of impact, leaving the mass unchanged. Thus, for the Bridges/Hatzes configuration, both velocity and size dependence were determined but mass dependence was not. What we would really like to know, of course, is the coefficient of restitution for collisions of any two spheres, for which both size and mass dependences are known.

Energy dissipation in collisions often involves a "critical velocity," below which the collisions are perfectly elastic [*Raman*, 1918; *Andrews*, 1931; *Barkan*, 1972]. Such behavior is thought to occur in both plastic deformation and brittle fractures [*Borderies et al.*, 1984]. In models based upon this type of energy dissipation, size and mass dependences tend to cancel, with particle radius (R) and mass (M) entering as a ratio M/R^3. If nature uses this mechanism for collisions of iceballs, then the size/mass of the balls may be of only secondary importance. However, *Dilley* [1993] found that existing models based on this mechanism failed to reproduce either the velocity or size dependence of *Hatzes et al.* [1988].

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As an alternative, Dilley introduced a viscous dissipation model, essentially a damped harmonic oscillator modified to accommodate collisions. Presumably, elastic forces are provided by hard ice and viscous dissipation mainly by frosts upon the surface. The importance of frosts had already been noted by *Hatzes et al.* [1988], who showed that smooth ice surfaces respond much more elastically than frosted ones. Also, *McDonald et al.* [1989] found that rough ice surfaces behave in much the same way as frosted ones during collisions, so ices roughened in space by collisions and radiation might also give viscous dissipation. In any event, the viscous dissipation model described the size and velocity dependence of the *Hatzes et al.* [1988] frosted ice data quite well but gave a mass dependence which adds to that of size, rather than canceling it. In fact, the predicted mass dependence was so large that *Dilley* [1993] modified the original model, introducing a mass dependence parameter whose value could be determined from further experiments.

This paper describes just such an experiment, carried out, however, at temperatures considerably higher than expected in ring systems. Rather surprisingly, the results turned out to be consistent with the original, unmodified model.

Experiments

In the studies previously cited, most impact velocities were 1 cm/s or less, values not easy to obtain experimentally. To circumvent this difficulty, *Bridges et al.* [1984] and *Hatzes et al.* [1988] employed a compound pendulum, with the weight of an iceball being mostly balanced by a counterweight. For such a setup the inertial mass of the system is not equal to the mass of the iceball, greatly complicating any interpretation of mass effects in collisions. The simplest way to get around this problem is to suspend the colliding iceball by long, light lines, so as to obtain a quasi-free collision. (The mass of our heaviest line was 0.3 g, less than a hundredth that of our heaviest iceball.) Accordingly, we decided to use this technique to find out whether changes in mass make a noticeable effect in the coefficient of restitution and, if so, to see if the mass dependence coincides with that predicted by the viscous dissipation model. For the second "iceball" we used, as had others before us, a massive (4 kg) ice block. This is not, of course, the situation expected in real ring collisions, but if the viscous dissipation model correctly describes our results, it can be used to predict results for iceballs of arbitrary mass.

Table 1. Average Coefficient of Restitution for Individual Runs and Overall Averages for Iceballs of Different Masses

	Run			
	1	2	3	4
Mass, g	34	58	112	565
Radius, cm	2.1	2.5	3.1	5.3
Average coefficient of restitution	0.46 ± 0.12	0.59 ± 0.06	0.67 ± 0.03	0.66 ± 0.02
	0.24 ± 0.04	0.37 ± 0.04	0.67 ± 0.03	0.73 ± 0.02
	0.29 ± 0.08	0.28 ± 0.03	0.62 ± 0.06	0.76 ± 0.02
	0.32 ± 0.07	0.30 ± 0.06		0.63 ± 0.11
	0.16 ± 0.05	0.63 ± 0.04		
	0.33 ± 0.09			
Final average	0.27	0.43	0.65	0.70

In general, variations within individual runs are small compared to those between runs. For this reason, final standard deviations are not very meaningful, and we use only the overall averages in our figures.

A window was installed between the coils of a commercial deep freezer, which was placed on the basement floor of our physics building at the base of a stairwell. In addition, the top of the freezer was replaced by moveable Styrofoam sheets thereby allowing access to the interior. In the interior an ice block was placed at a convenient point behind the window, and an iceball, attached via small screws to two lightweight lines about 10 m long, was hung vertically at a point just touching the ice block. With this length line, releasing the iceball at a distance of 1 cm from the block leads to an expected impact velocity of about 1 cm/s. An electromagnet was placed behind the iceball, which had a tiny metal pin inserted into the iceball's rear surface. Thus the iceball could be pulled back to the magnet using the lines above the freezer and released rather cleanly by cutting the power to the magnet. Using a video camera with a zoom lens that looks through the window, collisions were viewed on a video monitor and recorded on tape.

The procedure was as follows: We inserted a ruler just above the collision area and measured, on the monitor, a distance corresponding to 1 cm on the ruler. Then, using the monitor as a guide, we moved the end of the electromagnet to a distance of about 1 cm behind the vertically hanging iceball. After a preliminary series of collisions to compact frosts on the ice, a set of collisions was made and recorded. Later, the collisions were viewed on the monitor in slow motion, and the minimum distance of the iceball from the magnet (corresponding to the maximum distance from the ice block) following the collision was measured. Given the length of the lines and gravitational acceleration, the initial distance from the ice block determines the velocity of impact v_i, the maximum distance following the collision determines the velocity after impact v_f, and the resulting coefficient of restitution ε is $\varepsilon = v_f/v_i$.

Iceballs of four different masses were used, 34 g, 58 g, 112 g, and 538 g. The three smallest iceballs were made by freezing water inside a sphere machined from plastic. The largest mold was a miniature rubber basketball used to throw out into crowds at local basketball games. Unfortunately, significantly larger masses were precluded by space considerations.

Unfortunately, our procedures continually exposed the iceballs to the atmosphere under a variety of different temperatures and humidities. In particular, iceballs were frozen and stored in one freezer, transported under ambient conditions to the collision freezer where they were stored overnight, then exposed again to warmer, more humid air while being attached to the lines which would suspend them. Moreover, conditions undoubtedly changed to some extent while collisions were being monitored, owing to the entry of outside air from above the freezer. All that can really be said therefore is that our results are averages over a number of iceballs having a variety of frost conditions. On the other hand, surface conditions of real ring particles are likely to vary at least this widely. Table 1 shows the average coefficient of restitution for individual runs, together with overall averages for iceballs of different masses: error bars are statistical only, indicating one standard deviation. The corresponding nominal radius is also shown, assuming a density of 0.92 g/cm^3. The final averages are also shown in Figure 1. Interpretations aside, the data show that elasticity increases rapidly with mass for small masses but levels out at much higher masses.

To check that these results were not some artifact resulting from the way the experiment was done, we made control runs using steel balls and a steel plate. Since elasticity for metals is known to be relatively independent of size/mass [*Tabor*, 1948; *Barkan*, 1972], these collisions provide a convenient control for our experimental methods. The collisions were done exactly as before, that is, steel balls were suspended in the same manner as the iceballs, and collisions within the freezer were monitored and measured in the same way. Our findings, shown in

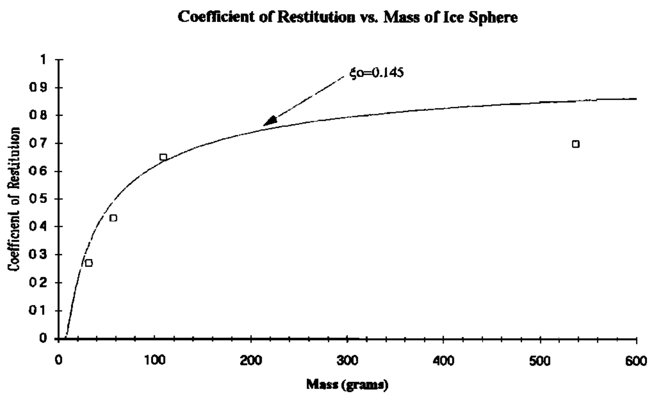

Figure 1. Average coefficient of restitution for ice spheres of different masses with a large ice block. The fit is from the model of *Dilley* [1993] (original version) with $\xi_0 = 0.145$ and $K = 0.6$.

Figure 2, show no systematic effect of size/mass for steel spheres, the coefficient of restitution being nearly constant over the entire mass range. For this reason, the observed change in elasticity of the iceballs is unlikely to be spurious.

Discussion

In evaluating the significance of these experimental results, it is useful to compare them to results predicted by the viscous dissipation model. In this model, size and mass are separate, size referring to a radius of curvature and mass to inertial properties. However, $M \propto R^3$ for a physical iceball, so that size and mass dependences can both be expressed in terms of either the iceball's radius or mass. For collisions of two ice spheres having radii R_1 and $R_2 \geq R_1$ (with corresponding masses M_1 and M_2), the coefficient of restitution is *Dilley* [1993]

$$\varepsilon = \exp\left(-\pi\xi/\sqrt{1-\xi^2}\right) \tag{1}$$

with inverse

$$\xi = \sqrt{\frac{(\ln\varepsilon)^2}{\pi^2 + (\ln\varepsilon)^2}}, \tag{2}$$

$$\xi = \xi_0(1+\delta)^{0.2}(1+\delta^3)^K r^{-3K-0.2} v^p, \tag{3}$$

where v is the relative velocity of the colliding iceballs, and

$$\delta = R_1/R_2 \leq 1. \tag{4}$$

(The factor v^p was inadvertently omitted by *Dilley* [1993].) In practice, the constants ξ_0, p, and K are determined from some reference experiment using collisions of an iceball with an ice block. Assuming that,

$$r = R_1/R_0 = (M_1/M_0)^{1/3}, \tag{5}$$

$R_0(M_0)$ being the radius (mass) of the colliding iceball in the reference experiment. Using $M_0 = 58$ g ($R_0 = 2.5$ cm), $\delta = 0$, and $\xi_0 = 0.145$, together with $K = 0.6$, the original model prediction, produces the fit to the data shown in Figure 1.

In interpreting the adequacy of the fit, it is useful to discuss the general nature of mass dependency of viscous energy loss in a damped harmonic oscillator. First of all, since the coefficient of restitution is defined as $\varepsilon = v_f/v_i$, what is being measured is the fraction of the original energy retained after a collision. As the mass of an impacting particle increases, its initial energy increases linearly with mass. On the other hand, the dissipative (velocity-dependent) force is independent of

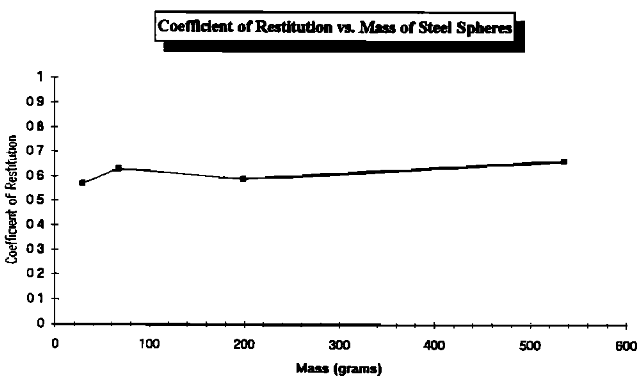

Figure 2. Average coefficient of restitution for steel spheres of different masses with a large steel plate.

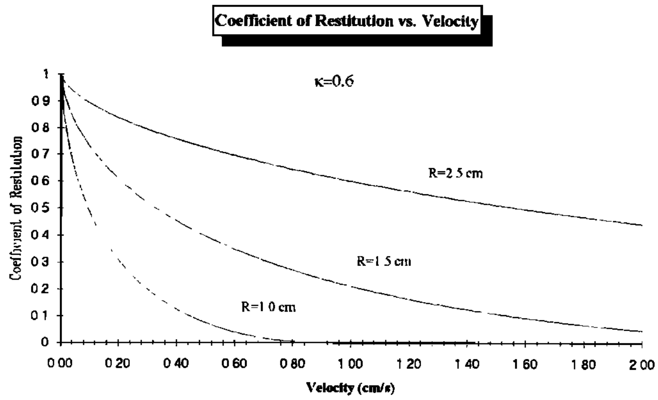

Figure 3. The coefficient of restitution for spheres of different radii (and mass) as predicted by the viscous dissipation model (original form, $K = 0.6$) for spheres and a large ice block.

mass, and the added distance traveled, hence energy lost, increases (roughly) only as the square root of the mass. (Our exact mass dependence is taken from the Hertz model for a sphere rather than a simple harmonic oscillator [*Dilley*, 1993], but this does not change the general argument.) As masses, hence initial energies, get small, the velocity-dependent energy losses get increasingly important, and the fraction of energy retained drops rapidly. On the other hand, as mass and initial energy increase, the velocity-dependent energy loss cannot keep up, and the fraction retained should rise slowly toward unity. In our own collisional experiments the increase of elasticity with mass, for the largest iceball, drops below that predicted by the model. It could be, of course, that the model, for unknown reasons, simply breaks down at large masses. On the other hand, the finite size of the ice block, whose smallest dimension was about 14 cm, might be responsible. That is, as iceball mass increases, an increasing amount of kinetic energy is transferred to the block, some portion of which might be lost owing to edge effects. In any event, the measured coefficient of restitution has a mass dependence whose general nature is very much like that predicted by the model.

The major question, then, is whether the energy loss mechanism observed in our collisions at about −20°C (253°K) also acts at the much lower ring temperatures. At this point the approximate agreement of our present measurements with predictions of the viscous dissipation model becomes crucial, because the same model also describes the data of *Hatzes et al.* [1988], taken at ring-like temperatures. It is quite plausible therefore that the observed mass effects also apply at ring temperatures. If so, then mass effects are almost surely important in ring systems. One can see this quite simply by extending the results of *Hatzes et al.* [1988] from 2.5-cm-radius (60 g) iceballs to smaller sizes using our model and a mass dependence corresponding to $K = 0.6$, as shown in Figure 3. Here we see that an iceball having a radius of 2.5 cm retains nearly 70% of its initial speed following a collision at 0.8 cm/s, while a $R = 1$ cm ball does not rebound at all. If this type of mass dependence is present within ring systems, large effects would presumably appear. For example, the inelasticity of small particle collisions might well promote accretion and growth of "particle" size within the Roche zone. What can be done to see if the mass effects discussed here apply to icy ring particles?

First of all, quasi-free collisions can be made at much lower

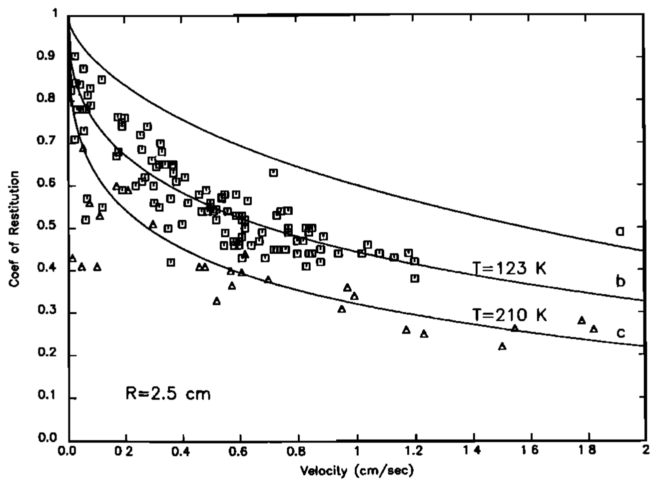

Figure 4. Fits to the coefficient of restitution for 2.5-cm-radius spheres having different frost coverings. Curve a is a model fit for thin frosts [*Dilley*, 1993], where the corresponding data, taken at 133°K, also appear (omitted here to avoid crowding). Cases b and c, with corresponding model fits, refer to thicker frosts (see text). Data are from *Hatzes et al.* [1988].

temperatures. This would require a more sophisticated cooling system and making provisions for adjusting ice block orientation and magnet distance remotely. Even with iceballs exposed to the atmosphere, an experiment of this type could settle the reality of large mass effects for collisions of ring particles.

Second, it might be possible to untangle the effects of counterweights in Hatzes/Bridges type apparatus by looking at different combinations of physical mass and counterweight mass. A combination of the two approaches might even provide a good model for free collisions.

Again, one might use our model in an exploratory fashion, that is, varying parameters to see what effects in ring systems result and then seeing if such effects produce features observed in real rings. To this end, we note that the number of velocity-dependent parameters may be reducible. *Hatzes et al.* [1988] made measurements using iceballs having the same radius, 2.5 cm, but which had different frost thickness and different amounts of elasticity. Figure 4 shows the *Dilley* [1993] fit (curve a) to their main data (where the iceballs have relatively thin frosts), as compared to data from cases with higher temperatures and thicker frosts [*Hatzes et al.*, 1988, Figure 14]. The general trend of the data suggests that the constants ξ_0, which determines the elasticity at 1 cm/s, and p, which determines the velocity dependence, are correlated. Accordingly, we tried simple linear relations between the two parameters and found that

$$p = 1.07 - 2.6\xi_0 \tag{6}$$

works quite well (curves b and c). The data of *Hatzes et al.* [1988] at $T = 210°$K (curve c), by the way, agrees well with the earlier data of *Bridges et al.* [1984], so (6) produces results agreeing with the coefficient of restitution for both sets of data. Note, however, that none of these results, even with $T = 210°$K, can be compared to ours because the Bridges/Hatzes collisions were not free collisions.

Finally, in passing, we speculate that thick frosts might approximate conditions in which large ring particles are coated by a layer of loose, smaller particles [*Weidenschilling et al.*, 1984], the so-called "dynamic ephemeral bodies," or DEBs. If so, then our model could be used to probe the effect of having DEBs within ring systems.

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