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Dust to Planetesimals: Settling and Coagulation in the Solar Nebula

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The behavior of solid particles in a low-mass solar nebula during settling to the central plane and the formation of planetesimals is examined. Gravitational instability in a dust layer and collisional accretion are considered as possible mechanisms of planetesimal formation. Non-Keplerian rotation of the nebula results in shear between the gas and a dust layer. This shear produces turbulence within the layer which inhibits gravitational instability, unless the mean particle size exceeds a critical value, ~1 cm at 1 AU. The size requirement is less stringent at larger heliocentric distances, suggesting a possible difference in planetesimal formation mechanisms between the inner and outer nebula. Coagulation of grains during settling is expected in the solar nebula environment. Van der Waals forces appear adequate to produce centimeter-sized aggregates. Growth is primarily due to sweepup of small particles by larger ones due to size-dependent settling velocities. A numerical model for computing simultaneous coagulation and settling is described. Relative velocities are determined by gas drag and the non-Keplerian rotation of the nebula. The settling is very nonhomologous. Most of the solid matter reaches the central plane as centimeter-sized aggregates in a few times 10^3 revolutions, but some remains suspended in the form of fine dust. Drag-induced relative velocities result in collisions. The growth of bodies in the central plane is initially rapid. After sizes reach ~10^3 cm, relative velocities decrease and the growth rate declines. Gas drag rapidly damps the out-of-plane motions of these intermediate-sized bodies. They settle into a thin layer which is subject to gravitational instability. Kilometer-sized planetesimals are formed by this composite process.

I. INTRODUCTION

The formation of the planets required some degree of separation of heavy elements from cosmically abundant H and He. In those cosmogonical theories which involve a disk-shaped photoplanetary nebula, the method of separation depends on its assumed mass. In a massive nebula, of the order of a solar mass ($M_\odot$), gravitational instabilities produce giant spherical protoplanets of solar composition (Cameron, 1978). Dense cores form by settling of solid or liquid particles toward their centers, and the gaseous envelopes are partially or completely removed to produce the planets. The settling process within protoplanets has been modeled by McCrea and Williams (1965), Williams and Crampin (1971), Slattery (1978), and Slattery *et al.* (1980).

Another major class of theories assumes that the nebular mass was much less than that of the Sun (Safronov, 1969). The present paper deals with the mechanism of gas/solid separation in such a nebula. The low-mass model assumes that most of the nebula's content of heavy elements was incorporated into the planets (Weidenschilling, 1977a), and so implies a rather high efficiency for the separation process. The formation of gaseous protoplanets is not possible in a low-mass nebula. Instead, solid grains which condense from the gas settle toward the central plane of the disk, forming a dense layer there. By some means, larger bodies form from this matter. The most widely accepted mechanism is a gravitational instability in the dust layer (Safronov, 1969; Goldreich and Ward, 1973). This process produces planetesimals of a characteristic size (~1 km at r = 1 AU), which are held together by self-gravity. This result has been widely interpreted as showing that any nongravitational sticking of grains is unnecessary. However, there is a problem associated with forma-

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tion of planetesimals directly from dust in this manner. Small particles are easily disturbed by turbulence, which may prevent a dust layer from becoming sufficiently thin for the instability to occur. Consideration of this problem implies that particles must exceed some minimum size, and that some coagulation is necessary. Weidenschilling (1977b) has suggested that collisions among small bodies would be sufficiently frequent to produce growth on a time scale competitive with gravitational instability, if there is any sticking mechanism.

After large ($\gtrsim$ kilometer-sized) bodies have formed, their motions and interactions are dominated by gravitational forces. The accretion of these planetesimals into planets is studied within the formalism of celestial mechanics (Safronov, 1969; Weidenschilling, 1974; Greenberg *et al.*, 1978a, b; Wetherill, 1978). However, the earlier evolution of smaller bodies is controlled by nongravitational forces. The most important of these is aerodynamic drag on particles moving with respect to the nebular gas. There velocities depend on size, as well as the nebular structure. The size distribution depends on the nature of any attractive (sticking) forces between particles, and the relative velocities in collisions. The following sections will examine how these interrelated phenomena affect the settling of dust to the central plane of the solar nebula. The time scale and efficiency for formation of the dust layer will be determined. Also, the possibilities of forming planetesimals by collisional accretion and gravitational instability will be compared directly.

II. STRUCTURE OF A LOW-MASS SOLAR NEBULA: NON-KEPLERIAN ROTATION

In a "low-mass" solar-nebula model, the self-gravity of the disk can be neglected compared with the attraction of the Sun at its center. For most purposes, this is the case if the mass $<0.1 M_\odot$. The radial and vertical components of solar gravity are

$$g_r = GM_\odot/r^2,$$
$$g_z = g_r(z/r) = \Omega^2 z \qquad (1)$$

where r and z are distances from the nebular axis and central plane, G is the gravitational constant, and $\Omega = (GM_\odot/r^3)^{1/2}$ is the Keplerian orbital frequency. If the disk is in hydrostatic equilibrium, it can be shown (Safronov, 1969, Chap. 3) that the pressure

TABLE I

LIST OF SYMBOLS

d	Dust layer thickness	ΔV	Deviation of gas velocity from Kepler velocity
D	Boundary layer thickness	w	Out-of-plane velocity in dust layer (either gas eddies or particles)
G	Gravitational constant	z	Distance from central plane
g_r, g_z	Radial and vertical gravity components	δ	Space density of solids
H	Gas scale height	ϵ	Viscous energy-dissipation rate
l	Eddy length scale	η	Effective viscosity due to particle collisions
$M_\odot$	Solar mass	μ	Gas viscosity
P	Gas pressure	ν	Kinematic viscosity (μ/ρ)
r	Heliocentric distance	ρ	Gas density
Re	Reynolds number	ρ_s	Density of individual particles
Ri	Richardson number	σ	Surface density of gas
s	Particle radius	σ_s	Surface density of dust layer
S	Turbulent stress (ρu^2)	τ	Settling time scale without coagulation
T	Temperature	τ_g	Time scale for growth by coagulation
u	Characteristic eddy velocity	Ω	Kepler orbit frequency
v	Particle velocity (relative to gas)		
$\bar{v}$	Thermal velocity of gas molecules		
V^*	Critical velocity for sticking		

in the central plane is

$$P_c(r) = \Omega\sigma\bar{v}/4, \quad (2)$$

where σ is the surface density of the gas, and $\bar{v}$ the mean thermal velocity of the gas molecules. Equation (2) assumes that the temperature T is independent of z, but is nearly correct for an adiabatic temperature gradient as well. If $\partial T/\partial z = 0$, the pressure varies with z as

$$P(r, z) = P_c(r) \exp(-z^2/H^2), \quad (3)$$

where $H = \pi\bar{v}/2\Omega$ is the "scale height" of the gas.

It is reasonable to assume that σ and T decrease as r increases; such distributions imply $\partial P/\partial r < 0$. This radial pressure gradient partially supports the gas against the solar gravity. Therefore, the rotational velocity of the gas, V_g, must be slightly less than the Keplerian velocity, V_k. Typically, $\Delta V = V_k - V_g < 10^{-2}V_k$; this small deviation from Keplerian rotation does not significantly affect the structure of the gaseous component. However, Whipple (1972) pointed out that solid bodies in the nebula are not supported by the gas pressure. They move relative to the gas, influenced by both gravitational and drag forces. Their motions were analyzed in detail by Adachi *et al.* (1976) and Weidenschilling (1977b), whose results may be summarized as follows.

A small particle has negligible velocity with respect to the gas in the direction of orbital motion. In a reference frame rotating with the gas, it moves radially inward at all values of z; if $z \neq 0$, it also settles toward the central plane. A large body moves in a nearly Keplerian orbit, with its eccentricity and inclination gradually damped by drag. It has a mean transverse velocity of magnitude ΔV with respect to the gas, and orbital decay gives it a radial velocity inversely proportional to its size. In a transitional size range (typically $\sim 10^2$ cm), the radial velocity reaches a maximum value of ΔV.

Weidenschilling (1977b) pointed out that this effect causes relative motion between particles of different sizes. As will be shown below, the drag-induced relative velocities are dominant in a size range of roughly micrometers to kilometers. Mutual gravitational perturbations are significant for larger bodies, and thermal motions for smaller ones. Any analysis of collision-dependent processes, including coagulation or breakup, must take the appropriate velocity distribution into account.

In order to produce quantitative results, I define a "standard" model nebula with properties given in Table II. This model is based on Weidenschilling's (1977a) reconstruction of a minimal-mass solar nebula, based on the heavy element content of the planets. Figure 1 shows the radial and transverse velocity components induced at $r = 1$ AU. The assumed density of an individual particle is 1 g cm^{-3}. For simplicity, this calculation ignores a transitional drag regime (Weidenschilling 1977b) which would change the shape of the curves slightly. I emphasize that the qualitative results derived below are not sensitive to the choice of such parameters as the mass or temperature of the nebula. The effects of varying these assumptions are discussed by Weidenschilling (1977b). One important assumption in the calculations of settling is that the nebula as a whole is not turbulent. There is no energy source capable of sustaining large-scale turbulence in the nebula (Safronov, 1969). After a dust layer has formed, there is a possible source of localized turbulence near the central plane. Its effects will be considered in the next section.

III. TURBULENCE AND THE POSSIBILITY OF GRAVITATIONAL INSTABILITY IN A DUST LAYER

In a dust layer which is near gravitational instability, the space density of solids is much greater than that of the gas, by two or three orders of magnitude. If the particles are sufficiently small, gas molecules do not freely pass through the dust layer without colliding with the grains. For a surface

TABLE II

PROPERTIES OF THE MODEL SOLAR NEBULA

	Mass:	0.05 $M_\odot$	
	Surface density:	$\sigma \propto r^{-3/2}$	
	Temperature:	$T \propto r^{-1}$	
	Pressure:	$P \propto r^{-7/2}$	
		r = 1 AU	r = 10 AU
σ	Surface density of gas, g cm^{-2}	6000	190
σ_s	Surface density of solids, g cm^{-2}	10.2	0.94
ρ_c	Gas density at $z = 0$, g cm^{-3}	3.2×10^{-9}	1.0×10^{-10}
ρ_s	Assumed particle density, g cm^{-3}	1.0	0.5
T	Temperature, °K	600	60
H	Gas scale height, AU	0.0706	0.706
Ω	Kepler orbit frequency, sec^{-1}	2.0×10^{-7}	6.3×10^{-9}
ΔV	Kepler velocity—gas velocity, cm sec^{-1}	1.3×10^4	4.2×10^3
	Mean free path, cm	0.56	178
μ	Gas viscosity, P	2.1×10^{-4}	6.8×10^{-5}
	Solids/gas mass ratio	0.0034	0.01

density of solids σ_s, of 10 g cm^{-2} (roughly that required in the region of terrestrial planets), this condition is met if the particle size is less than about 1 cm. In this case, the particles do not have individual velocities as indicated in Fig. 1. Instead, the dust layer maintains Keplerian rotation. There is then a velocity difference of magnitude ΔV between the dust layer and the gas on either side of the central plane. This effect was recognized by Goldreich and Ward (1973), who estimated the drag on the dust disk and its lifetime against orbital decay. However, they did not consider the possibility that this stress could affect the structure of the dust layer itself. We shall now examine this question in detail.

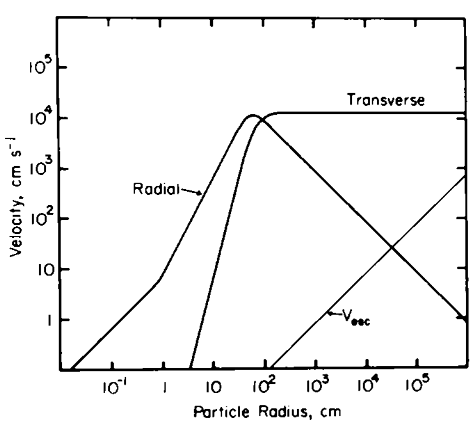

FIG. 1. Radial and transverse velocity components, relative to the gas, of a particle at r = 1 AU, z = 0 in the model solar nebula. A particle density of 1 g cm^{-3} is assumed. The escape velocity from a particle's surface is also shown.

A. *Boundary Layer Structure*

I assume, as did Goldreich and Ward (1973), that the boundary layer is an Ekman layer, similar in structure to one which would be produced by a rotating solid disk. The properties of turbulence are estimated only as to order of magnitude by dimensional arguments, as described by Tennekes and Lumley (1972; hereafter TL). Quantitative results, when given, refer to conditions at r = 1 AU in the standard nebula model. The derived formulas include the fact that there are actually two boundary layers, one on each side of the dust layer, though this level of precision is seldom justified. I assume σ_s = 10 g cm^{-2}, and a particle density ρ_s = 1 g cm^{-3} for the calculations. The critical density for gravitational instability corresponds to a thickness of the layer, d, of the order of 10^7 cm. This close to the central plane, the vertical component of solar gravity can be neglected, compared to the self-gravity of the dust layer, so

$$g_z \simeq 2\pi G\sigma_s. \tag{4}$$

The thickness of the dust layer is related to the random out-of-plane particle velocity, w, by

$$d \sim w^2/g_z \sim w^2/2\pi G\sigma_s. \tag{5}$$

The velocity w may refer to either random motions of individual particles or turbulent eddies, depending on whether or not the particles are large enough to move independently of the gas. The critical thickness $d \sim 10^7$ cm corresponds to $w \sim 10$ cm sec^{-1}.

The Reynolds number is a nondimensional parameter which indicates the importance of viscosity effects. It is defined by

$$Re = VD\rho/\mu, \tag{6}$$

where V and D are a characteristic velocity and length, respectively, and μ is the gas viscosity. Values of Re greater than $\sim 10^3$ are usually associated with turbulence. The boundary layer has $Re \gtrsim 10^4$ (Goldreich and Ward, 1973). The Richardson number, a measure of stability due to density stratification (Chandrasekhar, 1961), is given by

$$Ri \sim g_z(d\rho/dz)/\rho(dV/dz)^2. \tag{7}$$

Its value is $\sim 10^{-12}$, much less than the critical value of $\frac{1}{4}$ needed to inhibit turbulence. If the boundary layer is turbulent, there must also be an energy source adequate to maintain the turbulence (it is the lack of an energy source which suggests that the nebula as a whole is not turbulent (Safronov, 1969)). It will be shown that the required energy is supplied from the potential energy of the dust layer in the Sun's gravitational field, as it moves radially inward.

For a free stream velocity ΔV, the magnitude of a typical velocity fluctuation in the turbulent boundary layer is $u \sim \Delta V/30$ (TL, p. 12). The velocity and time scales determine its thickness, $D \sim u/\Omega \sim 10^9$ cm. The rate of viscous energy dissipation per unit mass of gas is $\epsilon \sim u^3/l$, where l is the length scale of the largest eddies (TL, p. 20). For $l \sim D$, $\epsilon \sim 1$ erg g^{-1} sec^{-1}. The smallest eddies have associated scales:

$$\begin{aligned} &\text{length:} && (\nu^3/\epsilon)^{1/4} \sim 10^4 \text{ cm}, \\ &\text{time:} && (\nu/\epsilon)^{1/2} \sim 10^3 \text{ sec}^{-1}, \\ &\text{velocity:} && (\nu\epsilon)^{1/4} \sim 10 \text{ cm sec}^{-1} \end{aligned} \tag{8}$$

(TL, p. 19), where $\nu = \mu/\rho$ is the kinematic viscosity. For particles smaller than the mean free path of the gas (~ 1 cm), the response time to a change in gas velocity is (Weidenschilling, 1977b)

$$t_e \simeq s\rho_s/\rho\bar{v} \sim (10^3 s) \text{ sec}, \tag{9}$$

so particles smaller than ~ 1 cm can respond to the smallest eddies. The turbulent stress on the surface of the dust layer is $S \sim \rho u^2 \sim 10^{-3}$ dyne cm^{-2}. The change in angular momentum due to the torque on the disk leads to a radial velocity

$$dr/dt \sim 4S/\Omega\sigma_s \sim 10^3 \text{ cm sec}^{-1}. \tag{10}$$

The rate of change of potential energy (per unit area) is

$$g_r\sigma_s \frac{dr}{dt} \sim 4g_rS/\Omega \sim 10^4 \text{ ergs cm}^{-2} \text{ sec}^{-1}, \tag{11}$$

independent of σ_s. Since the disk's rotation remains Keplerian, half of this change goes to increase its kinetic energy; the rest can drive turbulence. This figure can be compared with possible energy sinks to show that this energy source is adequate.

Viscous dissipation in the boundary layer. The dissipation rate per unit mass is $\sim u^3/l$. Per unit area, this is

$$2\rho Du^3/l \sim \rho u^3 \sim 1 \text{ erg cm}^{-2} \text{ sec}^{-1}, \tag{12}$$

independent of D.

Particle collisions. Assuming random relative velocities of magnitude v, and completely inelastic collisions, the dissipation rate is

$$3\pi G\sigma_s^3 v/32(2)^{1/2}s\rho_s \sim (10^{-5}(v/s)) \text{ erg cm}^{-2} \text{ sec}^{-1} \tag{13}$$

when v and s are in centimeter-gram-seconds. If the particles are large enough to move independently of the gas within the dust layer, then $v \sim u \sim 10$ cm sec^{-1}. If the particle motions are controlled by the gas ($s \ll 1$ cm), their relative velocity is

$$v \sim s(\epsilon/\nu)^{1/2} \sim s(u^3/\nu d)^{1/2} \sim (10^{-4}s) \text{ cm sec}^{-1} \quad (14)$$

(Saffman and Turner, 1956), where I take $u \sim 10$ cm sec^{-1} to be the eddy velocity within the dust layer. In either regime, the dissipation rate is <1 erg cm^{-2} sec^{-1}.

Particle/gas relative motion. The Epstein drag law is applicable when the particle size is less than the mean free path of gas molecules (Weidenschilling, 1977b). This gives a drag force $F = (4\pi/3)\rho\bar{v}s^2v$, where v is the particle's velocity relative to the gas. Assuming a dissipation rate per particle of Fv, the rate per unit area is

$$\rho\bar{v}\sigma_s v^2/s\rho_s \sim (10^{-3}\, v^2/s) \text{ erg cm}^{-2} \text{ sec}^{-1}. \quad (15)$$

For v given by Eq. (14), or for a maximum value of 10 cm sec^{-1} within the dust layer, this dissipation rate is small. Some particles will be entrained in the boundary layer, and exposed to high-velocity eddies. If $s \gtrsim 1$ cm, $v \sim \Delta V/30$, the dissipation rate is $\sim(10^3\, f/s)$ ergs cm^{-2} sec^{-1}, where f is the fraction of particles entrained in the boundary layer. For smaller s, the gas/particle relative velocity is lower. It seems unlikely that f could be large enough to make this a significant energy sink.

Gas outflow. Angular momentum is transferred from the dust layer to the gas in the boundary layer, which flows outward. The mean outflow velocity in a turbulent Ekman layer is $\sim 0.2\Delta V$ (Davies, 1959), giving a rate of gain of potential energy

$$0.2g_r\, \rho D\Delta V \sim 10^4 \text{ ergs cm}^{-2} \text{ sec}^{-1} \quad (16)$$

(the kinetic energy decreases at half this rate for nearly Keplerian rotation).

We conclude that a description of the boundary layer as a turbulent Ekman layer is self-consistent. The energy released by the inward motion of the dust layer is roughly balanced by the mean outflow of the gas in the boundary layer. Other energy sinks are too small to inhibit turbulence.

B. Effect of Stress on the Dust Layer

The turbulent stress $\sim\rho u^2$ exerted on the surface of the dust layer is transmitted within the layer, by either gas viscosity or particle collisions. The gas viscosity μ is $\sim 10^{-4}$ P. By analogy with the hard-sphere model for atomic viscosity of a gas, the effective viscosity due to particle collisions is

$$\eta = s\rho_s v/6(2)^{1/2}, \quad (17)$$

where v is the mean random velocity. For small particles with v given by Eq. (14), $\eta \ll \mu$. For larger particles which are not dominated by the gas, so that $v \sim 10$ cm sec^{-1}, η (in P) is approximately equal to s (in cm). Particle collisions can dominate the viscosity only for $s \gtrsim 1$ cm.

We first assume that the dust layer is laminar, and show that this leads to a contradiction. The surface stress condition $S \sim \rho u^2$) implies a parabolic velocity profile within the layer; this is the classical solution for laminar viscous flow between parallel surfaces. The velocity difference between the center of the dust layer and its surface is

$$\Delta V_s \sim Sd/4\mu \quad \text{or} \quad \sim Sd/4\eta, \quad (18)$$

whichever is smaller. For the nominal parameters at $r = 1$ AU, with $d \sim 10^7$ cm, use of μ implies $\Delta V_s \gg \Delta V$, which is physically impossible. Using η, $\Delta V_s \sim (10^4/sv)$ cm sec^{-1}; the assumption of laminar flow breaks down for $sv \lesssim 1$ cm^2 sec^{-1}. Using η in the case when $s \gtrsim 1$ cm, the effective "Reynolds number" of the dust layer is

$$Re_s \sim (\sigma_s/d)(d\, \Delta V_s/\eta) \sim 10^{-2}(\rho\Delta V^2/\pi G s^2\rho_s^2). \quad (19)$$

For the nominal parameters, $Re_s \sim (10^4/s^2)$ when s is in centimeters. Use of μ instead of η yields $Re_s \sim 10^{12}$.

The Richardson number of the layer is

$$Ri \sim \pi G \sigma_s \, d/\Delta V_s^2. \tag{20}$$

Using μ or η, $Ri \sim 10^{-13}$ or $\sim(10^{-6}s^2)$, respectively. By these criteria, Ri is too small for density stratification to stabilize the dust layer, which should therefore be turbulent unless $s \gtrsim 1$ cm. The energy-dissipation mechanisms which might damp the turbulence are the same as those considered for the boundary layer. A similar analysis shows that the dissipation rate is small unless the particles are large enough to cross eddies ($s \gtrsim 1$ cm). This condition is equivalent to Saffman's (1962) criterion for a flow to be stabilized by the presence of dust. If $s \gtrsim 1$ cm, the damping becomes appreciable when the turbulent velocity in the dust layer is $\gtrsim 10^2$ cm $\sec^{-1}$ [cf. Eq. (15)]; this implies a thickness $\sim 10^9$ cm, much too large for gravitational instability. If $s \ll 1$ cm, all damping mechanisms can account for only a small fraction of the total energy released by radial motion. The probable result is that the dust layer thickens until the density becomes so low that the dust layer no longer behaves as a unit, or until turbulent entrainment in the boundary layer reduces the radial velocity of the dust.

We conclude that a layer of fine dust ($s \lesssim 1$ cm) in a gaseous nebula is subject to "self-induced turbulence" which can prevent it from becoming dense enough for gravitational instability. It is of interest that Re_s should decrease with increasing r, since ρ and ΔV will both decrease. A laminar dust layer may be possible in the outer nebula; this might be a factor in different formation mechanisms for terrestrial and Jovian planets. The formation of planetesimals in the terrestrial planet region would be possible after the gas had dispersed. However, a simpler explanation is that dust grains were able to coagulate into centimeter-sized, or larger, aggregates as they settled to the central plane; this would invalidate the original assumption of a coherent dust layer in Keplerian rotation. Such coagulation is reasonable, as will be shown below.

IV. COAGULATION OF GRAINS

The type of process by which solid matter settles to the central plane of the nebula depends on whether the initial population of grains can coagulate to form larger bodies during their descent. Bodies smaller than ~ 1 cm will not accrete by gravitational attraction alone, since their thermal Brownian velocity exceeds their escape velocity (Greenberg *et al.*, 1978b). Nongravitational sticking mechanisms depend on the composition and physical state of the grains, which are poorly known. Nonetheless, there is empirical and theoretical evidence for effective sticking of micron-sized particles under a wide variety of conditions (Hartmann, 1979). It is more realistic to allow coagulation, at least in a limited size range, than to neglect it completely. Laboratory simulations of condensation in a solar nebula environment generally yield branching-chain aggregates of grains (Stephens and Kothari, 1978). Interplanetary dust particles collected in the Earth's upper atmosphere typically consist of highly heterogeneous aggregates of submicron grains (Fraundorf and Shirck, 1979). Their origin cannot be proved, but these may be nearly unaltered products of the solar nebula. Also, the evolution of terrestrial aerosols is generally dominated by coagulation (Hidy and Brock, 1970).

Sticking of small particles is usually due to van der Waals attraction. This is a short-range force between surfaces in contact, due to induced electrical dipoles in the adjacent layers of atoms (Corn, 1966). The strength of the attraction depends on the surface roughness and geometry of the contact area, but is relatively insensitive to composition. Other attractive forces, such as ferromagnetism or electrostatic charge, may also promote coagulation. However, they are effective only for certain materials or under more restrictive conditions. It will be shown that the van der Waals force alone is sufficient to produce macroscopic aggregates under solar nebula conditions.

Coagulation requires more than the presence of an attractive force. Energy must also be dissipated in a collision, or the particles will bounce apart. A sticking criterion has been developed by Dahneke (1971, 1972). His model includes the van der Waals force, elastic deformation of particles near their contact area, and a coefficient of restitution in the particle material. The theory yields a critical velocity V^*; impacts at less than this value result in sticking. V^* is a function of particle size, as well as mechanical properties. The theory agrees with experiment for micron-sized particles (Dahneke, 1975). However, it predicts that V^* approaches an unreasonably large value, independent of size, for large bodies. This result is due to the assumption that the contact surface is always microscopically smooth. I have modified the theory by assuming that there is a characteristic size scale of surface roughness. For bodies much larger than this scale, the number of contacts is assumed proportional to s^2, where s is the particle radius. The attractive force per contact is a constant, given by Dahneke's original theory for a particle the size of the roughness scale. This modified theory give $V^* \propto s^{-1/2}$ for large spherical particles.

Figure 2 shows V^* for two hypothetical materials, compared with typical velocity regimes in the model solar nebula at $r = 1$ AU. The "hard" material has an elastic modulus, Y, of 5×10^{11} dynes cm^{-2}, similar to that of fused quartz, and a coefficient of restitution, e (defined as fraction of kinetic energy remaining after rebound), of 0.98. The "soft" material has $Y = 5 \times 10^{10}$ dynes cm^{-2} and $e = 0.5$. Both have a Hamaker constant (a measure of van der Waals bond strength; see Dahneke (1972) for definition) of 10^{-12} erg. The horizontal dashed lines give V^* for a particle of $s = 10^{-4}$ cm colliding with a much larger body of a flat surface. The diagonal lines are for equal-sized spheres, using the modified theory with a surface roughness scale of 10^{-4} cm. Also shown are the thermal Brownian velocity for $T = 600°$K, gravitational escape velocity from a particle's surface, vertical settling velocities at $z = H$ and $Z = 2H$, and radial drift velocity at $Z = 0$, for a particle density of 1 g cm^{-3}.

The thermal velocity is less than V^*, even for the "hard" material, if $s \gtrsim 10^{-5}$ cm. Therefore, it appears that Brownian motion could not inhibit accretion in this size range. For reasonable mechanical properties, van der Waals bonding alone appears to allow production of centimeter-sized bodies if they grow by accreting much smaller grains. The radial and vertical settling velocities may inhibit growth of larger bodies. However, the calculated values of V^* are for competent, homogeneous bodies, while the centimeter-sized bodies

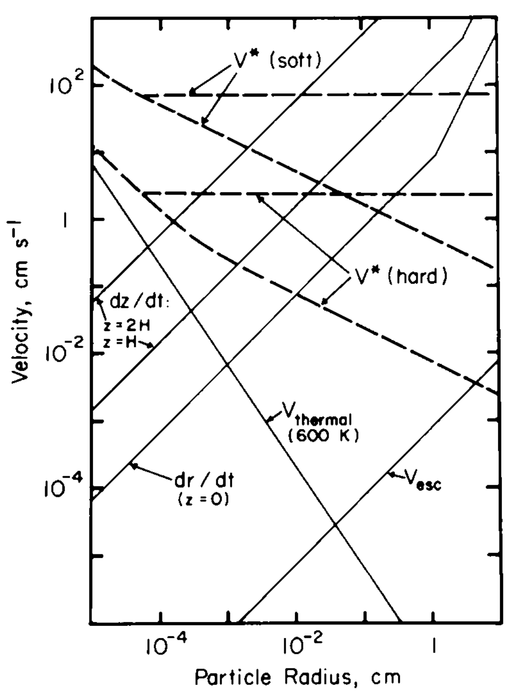

FIG. 2. Velocities vs particle size at $r = 1$ AU. Shown by solid lines are gravitational escape velocity, thermal velocity, vertical settling velocities at $z = H$ and $z = 2H$, and radial velocity at $z = 0$. Heavy dashed lines give critical velocities (V^*) for sticking by van der Waals force, according to modified Dahneke theory (see text for explanation). Horizontal lines are V^* for a 10^{-4}-cm particle to stick to a much larger body; slanted lines are for collisions of equal-sized bodies.

would be aggregates of smaller grains. Their effective coefficient of restitution would probably be $\sim 10^{-2}$, as found by Hartmann (1978) for regolith powder layers which are deep relative to the size of the impacting body. Impacting particles could be embedded within aggregates, if the impact energy were less than that required to break the preexisting bonds between grains. Larger bodies might accrete in this manner.

V. SETTLING OF PARTICLES

The time scale for settling to the central plane of the nebula is strongly influenced by coagulation. Consider a particle settling under the influence of gravitational and drag forces. Assuming the Epstein drag law applies, the equation of motion is

$$\frac{d^2z}{dt^2} + \frac{\rho\bar{v}}{s\rho_s}\frac{dz}{dt} + \Omega^2 z = 0. \tag{20}$$

Within about one scale height of the central plane, we can take ρ as constant. Equation (20) then has the solution

$$z = z_0 \exp(-t/\tau), \tag{21}$$

where

$$1/\tau = (\rho\bar{v}/2s\rho_s)\left[1 - (1 - 4s^2\rho_s^2\Omega^2/\rho^2\bar{v}^2)^{1/2}\right]. \tag{22}$$

If the root is imaginary, the particle overshoots the central plane and oscillates about it. Critical damping occurs for $s = s_c = \rho\bar{v}/2\rho_s\Omega$. For the standard nebula at 1 AU, $s_c \sim 10^3$ cm (actually, the Stokes drag law is applicable for $s \gtrsim 1$ cm, but in either case, particles $\lesssim 10^2$ cm do not overshoot the central plane).

For particles much smaller than s_c, d^2z/dt^2 can be neglected. Then $\tau = \rho\bar{v}/\rho_s s\Omega^2$ is the e-folding time for settling. Since τ is independent of z, the settling is homologous, i.e., the vertical distribution does not change, except for a scale factor, provided s is constant (no coagulation). If grains originally have the same scale height H as the gas, the elapsed time to reach the critical thickness for gravitational instability ($z/H \sim 10^{-4}$) is $\sim 10\tau$. In the standard nebula at 1 AU, $\tau \sim (10^2/s)$ year. The presence of submicron grains in undifferentiated meteorites would imply a settling time $\sim 10^7$ years in the absence of coagulation. This time scale exceeds the probable lifetime of the nebula; it would also rule out any effects of short-lived radionuclides or an active pre-main-sequence sun on the thermal evolution of planetesimals.

A detailed calculation of settling without coagulation has been published by Coradini *et al.* (1980). Homologous settling models which included coagulation were published by Kusaka *et al.* (1970) and Wesson and Lermann (1978). Both of the latter models assumed that particle relative velocities were due solely to thermal motion. The homologous assumption implied that while particles could grow, they all had the same size at any time. In each case, $\sim 10^6$ years was required for particles to reach the central plane. However, neither of their key assumptions appears correct. As seen in Fig. 2, settling velocities generally exceed thermal velocities. If settling velocities influence coagulation, they will also cause spreading of the size distribution. Suppose that one particle is initially larger than its neighbors. It settles faster, and the collisions due to the relative motion result in growth. The larger particle falls still faster, allwing it to collide with more of the small grains and grow further. This effect results in runaway growth. Such a process is well known in terrestrial cloud physics as a means of producing raindrops (it is called "gravitational coagulation" by atmospheric scientists, but should not be confused with accretion by gravitational binding).

Safronov (1969) considered settling with coagulation in the solar nebula. He showed that if a particle's motion is only vertical, the maximum size it can attain by the time it reaches the central plane is

$$s_{max} = \sigma_s/8\rho_s \tag{23}$$

if it accretes all other particles with which it collides. Note that s_{max} is independent of the initial size s_0 (provided $s_0 \ll s_{max}$). The initial growth of s is exponential, with a time constant

$$\tau_g \sim 4\rho\bar{v}/\delta\Omega^2 z, \qquad (24)$$

where δ is the space density of solid grains (Weidenschilling, 1977c). Taking $\delta = 0.0034\rho$, the expected dust/gas ratio after condensation of metal and silicates, and $z = H$, $\tau_g \simeq 200$ years at 1 AU. Since $\tau_g \propto z^{-1}$, particles initially at larger z grow faster. As a result, settling is nonhomologous; rather, matter at large z "rains out" first, reaching the central plane after $\sim 10\tau_g$. Note also that τ_g is independent of ρ_s; the higher settling velocity of a denser particle is offset by its smaller collisional cross section. If the dust/gas ratio is constant, τ_g is also independent of the assumed mass of the nebula, since $\delta \propto \rho$.

A detailed integration by Handbury and Williams (1977) of the equations of motion for a particle which accretes other grains while settling showed good agreement with analytic approximations of this type. Unfortunately, they assumed a highly artificial nebular structure (a uniform slab in Keplerian rotation) which limits the usefulness of their results. I have recalculated the simultaneous settling and growth of such a particle in model nebulae which include a finite scale height of the gas and radial motion due to non-Keplerian rotation. Settling was assumed to be at the terminal velocity ($d^2z/dt^2 = d^2r/dt^2 = 0$), a good approximation since $s \ll s_c$. The results confirm the nonhomologous nature of the settling, the time scale $\sim 10\ \tau_g$ for the first aggregates to reach the central plane, and the lack of dependence of ρ_s or the nebular mass. Radial motion allows more growth than does the case of purely vertical settling. A particle initially at $z = H$ settling to $\sim 10^{-4}$ H undergoes about a 10% decrease in r, and reaches a size about twice the limit of Eq. (23). Since radial motion does not cease, the particle can continue to grow beyond this point.

Such single-particle settling models refer only to the first bodies to reach the central plane. Their further growth rate depends on the number of other particles present, their size distribution, and the rate at which continued settling adds to the mass density in the central plane. It is also of interest to know whether most of the dust eventually reaches the plane and becomes available for planetesimal formation, or whether some significant fraction remains suspended. These questions require the calculation of the simultaneous evolution of the particle size distribution, and changes in the local mass density due to settling, at all levels in the nebula. Such a calculation is described in the following sections.

VI. NUMERICAL SIMULATION OF COMPETITIVE GROWTH AND SETTLING: THE MODEL

For a system of particles interacting by coagulation and/or fragmentation, the evolution of the mass distribution is usually described by an integro-differential "coagulation equation" (Pechernikova, 1975). Such an equation can, in principle, be solved numerically (Pechernikova *et al.*, 1976), though the solution is generally very time consuming. However, its formulation involves the implicit assumption that the calculated distribution is uniform over the volume considered, and that all particles can interact. Thus, the coagulation equation does not accurately describe a system in which the mass distribution varies strongly with location. It is just this variation which is of interest for the settling of particles in the solar nebula.

An exact solution is not practical. I have developed a computer program which models simultaneous growth and settling with certain approximations. The nebula is divided into a series of discrete levels at a given r. The size distribution, with changes due to coagulation and fragmentation, is computed for each level. Simultaneously, particles are transferred to the next lower level at rates proportional to their settling velocities. The coagulation equation is not used. A spatial resolution fine enough to justify its use would require too many levels and prohibitive computing times. Instead, a discrete approximation procedure is used, similar in concept to that described by Greenberg *et al.* (1978a).

At a given level, the mass distribution is modeled as a series of bins, each containing particles in a specified mass range. Each bin has a width of a factor of 10 in mass; some 30 bins are needed to span the size range of interest, from microns to kilometers. The velocity components (radial, transverse, and vertical) relative to the gas are computed for the mean mass in each bin; random thermal velocities are also included. The relative velocity between members of different bins is used to calculate their collision rate, using particle-in-a-box statistics. Collisions also occur between members of the same bin, using the thermal velocity, or half the relative velocity spread between the upper and lower size limits, as appropriate.

Collisions may result in coagulation or destruction, according to criteria described below. When small particles are accreted by larger ones, their mass is removed from the former bin and added to the latter. A gain in mass by particles near the upper end of a bin may shift them into the next larger one. This process is modeled by assuming that at the beginning of each time step there is an even distribution in log mass across the bin,

$$dn\,(m)/d\log m = C, \tag{24}$$

where C depends on the mass in the bin. For a total mass M in the bin,

$$C = M/(m_2 - m_1), \tag{25}$$

where m_1 and m_2 are the lower and upper bounds. The total number of particles in the bin is

$$N = C\log(m_2/m_1). \tag{26}$$

Suppose that particles from a smaller bin, with mean mass m_0, are accreted. Particles in the larger bin with a mass greater than $m_2 - m_0$ are moved into the next larger bin by the mass gain. The fraction of such particles in the bin is

$$F = \log[m_2/(m_2 - m_0)]/\log[m_2/m_1], \tag{27}$$

and their mean mass is

$$\bar{m} = m_0/\log[m_2/(m_2 - m_0)]. \tag{28}$$

The mass transferred into the next-larger bin is the total number of collisions between the first two bins, times $F(\bar{m} + m_0)$. The program keeps track of such mass shifts, as well as direct transfers of mass due to accretion and disruption, for all possible pairs of interacting bins. M and C are recomputed, and the process repeated. Note that mass is conserved in this algorithm, but the number of particles is adjusted according to the total mass in each bin.

The method used to determine the size of the largest particles (i.e., the largest bin occupied) differs from that used by Greenberg *et al.* (1978a). Their algorithm kept track of the mean mass gain of the largest particles. When this gain equaled the mean mass of the next-larger bin, one new particle was created in that bin. This algorithm appears valid when the size distribution does not vary too steeply near the large end of the size distribution (Greenberg, unpublished calculations). However, when the distribution is steep, the number of large particles may be overestimated; this was found to occur in the case of purely thermal coagulation, for which a numerical solution (Lai *et al.*, 1972) is known. Since thermal velocities are significant in the earliest stage of dust coagulation in the solar nebula, it was feared that the extent of runaway growth during settling could be exaggerated. The following algorithm was developed to avoid this problem.

A new bin is first occupied by particles which have been shifted across the upper bound of the adjacent bin by a small gain in mass. They are shifted into the bottom of the new bin, and yet another bin cannot be started until some particles can gain mass equivalent to a full bin width. This is modeled by allowing the largest occupied bin to have a variable width. The lower mass bound of this bin is fixed, remaining tied to the rest of the distribution. The mean mass gain of the particles in a time step is computed. The upper mass bound is shifted (bin width increased) by an amount consistent with the change in mean mass. When the width of the largest bin exceeds that of a normal bin, it is split into two bins with the same instantaneous value of $dn/d \log m$. The lower part becomes a normal bin, the upper is the "new" largest bin. This algorithm produces a size distribution in good agreement with the known solution for thermal coagulation. However, the large size end of the distribution is truncated, presumably due to neglect of large shifts of the upper mass bound due to coagulation among the few largest bodies. We may be fairly confident that this algorithm does not overestimate the size of the largest body in thermal or nonthermal coagulation.

The appropriate collisional cross section depends on the sizes of the particles. For sufficiently small particles, glancing collisions may allow coagulation; for radii s_1 and s_2, the cross section should be proportional to $(s_1 + s_2)^2$. For macroscopic particles, the cross section should be proportional to the area of the larger body. I assume the former behavior whenever both bodies are smaller than the mean free path of the gas molecules, or the Reynolds number of the larger is less than unity. There is also an aerodynamic effect on collision rates; gas flowing past a large body can sweep aside small particles. The geometric cross section is, therefore, multiplied by a collection efficiency which depends on particle size, gas density and viscosity, as given by Whipple (1972).

If impact velocities are high enough, collisions may cause disruption. Following Hartmann's (1980) experimental data, half of the impact kinetic energy in the center-of-mass frame is assumed to be partitioned into each body. If the energy density exceeds some specified "impact strength," a body is disrupted. Disruption of only the smaller body can still result in accretion. Since both objects are aggregates, fragments of the smaller one will tend to become embedded in the larger. Depending on the assumed strength, some collisions may disrupt both bodies. Experimental data on disruption of aggregates are scarce. A single experiment by Hartmann (1980) resulted in a mass distribution with a peak at the size of the constituent particles, as well as a power law distribution of larger fragments. Since data are so limited, I adopt the assumption that disruption places all of the mass into the smallest size bin. This is conservative in the sense that it probably causes an underestimate of the number of larger bodies (a variation which distributes fragments over a range of bins according to a power law has also been tested; the results presented below are not altered significantly). Note that the maximum dynamic pressure on the particles is $\sim\rho\Delta V^2 \sim 1$ dyne cm^{-2}; this is inadequate to disrupt them, or even to dislodge grains from their surfaces.

At each time step, some of the particles at each level are transferred to corresponding size bins at the next-lower level. The number of particles shifted downward is proportional to the settling velocity and the length of the timestep. For small particles, the settling velocity is the terminal velocity due to the vertical component of solar gravity. For bodies large enough to maintain quasi-Keplerian orbits, the "settling rate" used is $0.5\ r(di/dt)$, where the orbital inclination i is taken as z/r at that level. The inclination damping rate is given by Adachi *et al.* (1976).

The numerical model includes radial velocities in computing collision rates and outcomes, but ignores any changes in r during settling. Single-particle settling models indicate $\Delta r/r \sim 10\%$ before reaching the central plane. The accompanying changes in ρ and Ω can be neglected.

Larger changes in r are possible in principle after a particle reaches the central plane. However, it will be seen below that accretion there is generally so rapid that further radial displacement is unimportant.

Drag due to particle collisions has also been neglected. If the space density of small particles is high, they are, in effect, part of the resisting medium for any large bodies. Weidenschilling (1977c) showed that this drag force changes the shape of the velocity vs size curve (Fig. 1), and shifts the velocity peak toward larger sizes, but does not change the maximum velocity. In practice, particle drag was found to be insignificant, except in the central plane in the late stages of the simulations. Its effects on the quantitative results were judged to be minor.

VII. RESULTS AND DISCUSSION

Due to a limited computing budget, only two cases have been investigated in detail, at $r = 1$ AU and 10 AU in the standard nebula model. Each simulation used eight levels extending from $z = 0$ to $z = 2H$. The thickness of the levels varied, with a finer resolution toward the central plane. Symmetry about the central plane was assumed. The particle properties are given in Table II. The initial solids/gas mass ratio, as-

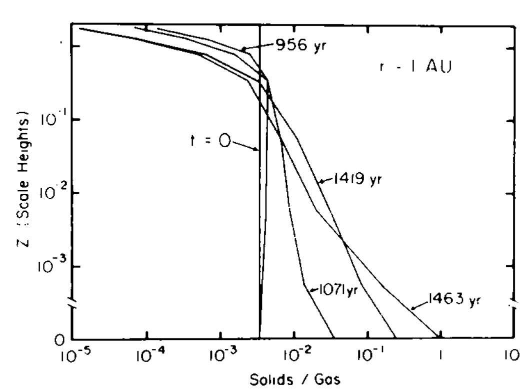

FIG. 3. Solids/gas ratio (by mass) vs z during coagulation and settling at $r = 1$ AU. Initial ratio is 0.0034 at all values of z, corresponding to condensation of metal + silicates.

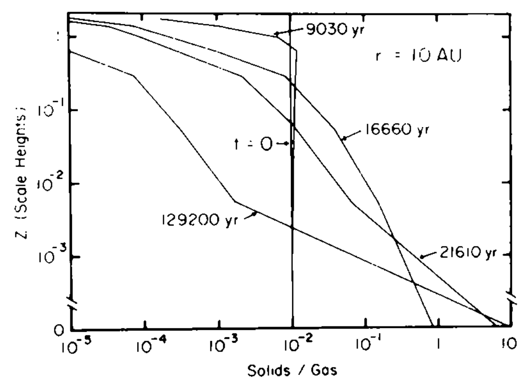

FIG. 4. Solids/gas mass ratio vs z at $r = 10$ AU. Initial ratio is 0.01, corresponding to condensation of metal + silicates + ices.

sumed constant with z, is consistent with a (silicate + metal) dust composition at $r = 1$ AU, and the addition of ices at $r = 10$ AU. The densities used assume significant porosity of particle aggregates.

Figures 3 and 4 show the evolution of the solids/gas ratio as a function of z. The settling is strongly nonhomologous in each case. The region above $z \sim 0.5H$ clears rapidly by settling. As could be expected, the solids/gas ratio in the central plane increases monotonically. Between these levels, it first increases, then decreases. The settling time scale is about an order of magnitude greater at $r = 10$ AU than at $r = 1$ AU. The solids/gas ratio in the central plane is actually a lower limit, since it is assumed to be uniform throughout a level of finite thickness. In the late stages of settling, solids may form a thinner layer than the spatial resolution used here (note that the z/H values of the levels are different for the two runs).

The evolution of the size distribution in the central plane is shown in Figs. 5 and 6. The display format is such that a line of slope -1 would represent an equal mass in each bin. Consider first the evolution at $r = 1$ AU. At $t = 599$ years, the solids/gas ratio is still near the original value. The size distribution for the smallest particles is essentially thermal, but radial drift has produced an excess of particles with $s > 10^{-3}$

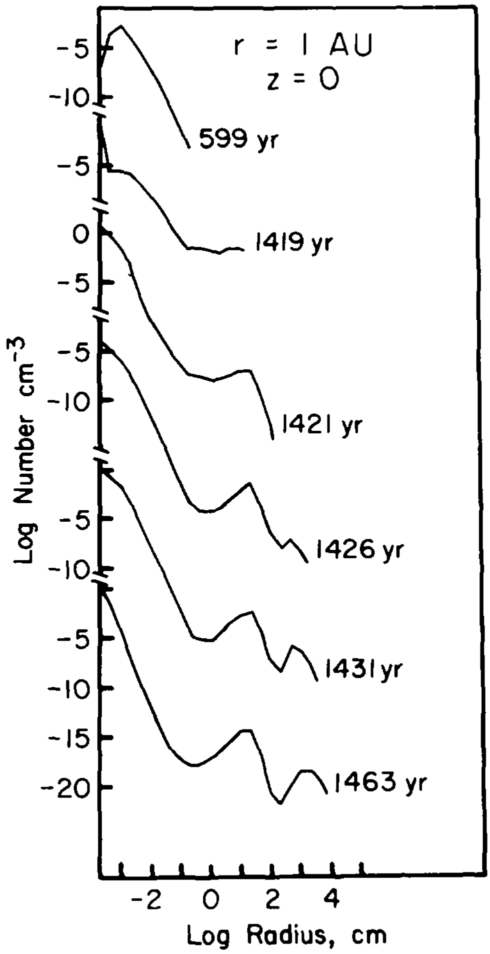

FIG. 5. Evolution of size distribution in the central plane at r h 1 AU. See text for details.

cm. At 1419 years, settling has increased the local solids/gas ratio by about two orders of magnitude. Most of this additional mass has arrived in the form of particles with $1 \lesssim s \lesssim 10$ cm. Particles with $s \gtrsim 10$ cm have relative velocities large enough to shatter each other in collisions; this returns mass to the smallest-sized bin. Further evolution is rapid, as "runaway" growth occurs. At 1421 years, the size distribution is bimodal. The two peaks consist of collisional fragments ($s \lesssim 10^{-3}$ cm), and the largest bodies which can avoid fragmentation ($s \simeq 10$ cm). Nonetheless, some particles manage to grow to larger sizes by accreting the small fragments. At 1426 years, a third peak appears in the distribution. This consists of bodies large enough to escape collisional destruction ($s \gtrsim 10^3$ cm). As seen in Fig. 1, in this size regime radial velocities begin to decrease, while transverse velocities have leveled off at a constant value. The relative velocity between two such large bodies is too low for a collision to break them up. A small body will still have a high velocity relative to one in this size range, but its mass is too small for a collision to be catastrophic. By 1463 years, most of the mass is in bodies with $10^3 \lesssim s \lesssim 10^4$ cm. At this time, the central plane level contains only about one-third of the total mass. The peak in the distribution at s

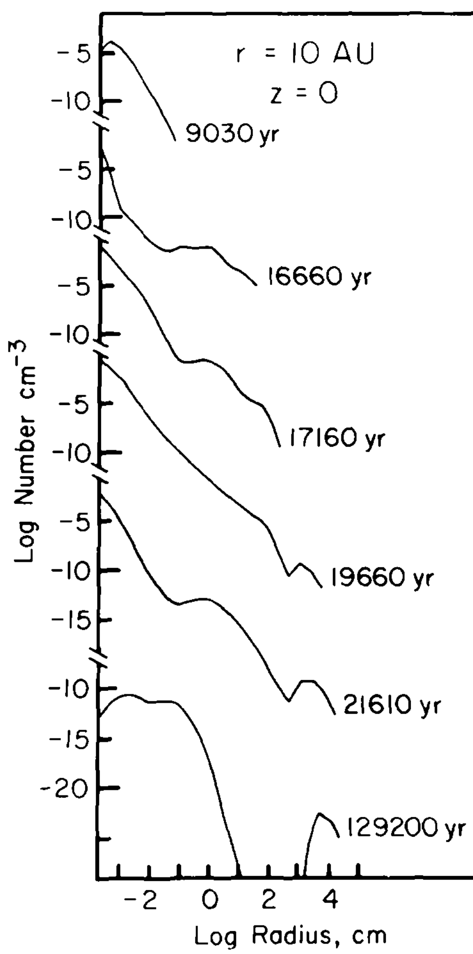

FIG. 6. Evolution of size distribution in the central plane at r = 10 AU.

$\simeq$ 10 cm is maintained by continued infall from the higher levels. At the next level ($z/H \simeq 5 \times 10^{-3}$), the largest particles only reach a few tens of centimeters in size.

The evolution at r = 10 AU (Fig. 6) is generally similar, but takes longer, due to the lower velocities and smaller surface density of solids. This case was run to a relatively later stage. At 21,610 years, about two-thirds of the solids have reached the level of the central plane; most of this mass is in bodies $\sim 10^4$ cm in size. At 129,200 years, 99% of the mass is in the central plane. Growth has virtually ceased, due to the low relative velocities of the large bodies (actually, other processes such as gravitational stirring would have become effective before this stage).

The impact strengths assumed (10^5 ergs cm^{-3} at 1 AU, 10^4 ergs cm^{-3} at 10 AU) are arbitrary, but may not be unreasonable. Van der Waals bonding alone should produce aggregate strengths of the order of 10^3 ergs cm^{-3}. Limited test runs were performed for strengths of 10^6 and 10^4 ergs cm^{-3} at r = 1 AU. As might be expected, the stronger bodies accreted more rapidly. The weaker ones showed similar evolution on a slightly longer time scale up to bodies $\sim 10^2$ cm in size, but the run was terminated due to excessive use of computer time. Obviously, some finite strength is necessary for collisional accretion, but the general result does not appear to depend strongly on the value assumed. To some extent, the accretion process is self-regulating: if objects of a certain size can destroy each other, they are depleted, and their collisions become rarer. Those which escape destruction can grow by accreting the more abundant small fragments from those collisions. It should also be noted that the maximum velocities developed are proportional to the nebular temperature (Weidenschilling, 1977b); accretion would be favored in a cooler nebula than assumed here.

When most of the mass is in bodies larger than the size corresponding to the maximum radial velocity, the growth rate decreases sharply. This is due to the lower relative velocities and collision rates. In Fig. 6, the size of the largest bodies increases only slightly in 10^5 years. This result contradicts Cameron's (1973) suggestion that drag-induced relative velocities could produce objects of planetary size. Such a process could occur only if a single body in a large zone were allowed to grow beyond $\sim 10^2$ cm in size, so that all of the remaining mass would be available for its growth.

For the largest formed in the simulations, the inclination damping rate exceeds the rate of further growth. Therefore, they will tend to concentrate in a thin layer near the central plane (these bodies are too large and widely spaced to form a coherent layer of the sort described in Section III). In the absence of large-scale turbulence in the nebula, the only mechanisms for stirring this layer appear to be the mutual gravitational perturbations and turbulent wakes of the bodies. For $s \sim 10^3$ cm, the escape velocity is ~ 1 cm sec^{-1}. Taking this as a typical out-of-plane perturbation velocity, the thickness of the layer would be $\sim 10^5$ cm. Dimensional analysis suggests the maximum width of a turbulent wake is of the order $\Delta V s^2/\nu$ (TL, p. 118); this value is also $\sim 10^5$ cm at 1 AU.

Such a thin layer is subject to gravitational instability. At r = 1 AU, bodies $\sim 10^3$ cm in size will have a mean horizontal spacing $\sim 10^4$–10^5 cm. The critical wavelength for perturbations in the layer is $4\pi^2 G\sigma_s/\Omega^2 \sim 10^9$ cm (Goldreich and Ward, 1973). Since this is much greater than the mean spacing, the layer is unstable. It should fragment into gravitationally bound clusters, each containing $\sim 10^8$–10^9 bodies. These clusters eventually collapse into kilometer-sized planetesimals, as described by Goldreich and Ward. Their further evolution will be dominated not by aerodynamic forces, but by their mutual gravitational perturbations.

VIII. CONCLUSIONS

It is widely believed that the formation of planetesimals by gravitational instability in a dust layer eliminates the need for "ad hoc" particle sticking mechanisms. However, non-Keplerian rotation of the gaseous component is a virtually inescapable feature of any model of the solar nebula. This results in shear between a dust layer and the gas. Unless the "dust" particles are larger than a critical size, ~ 1 cm at 1 AU, turbulence is induced within the layer, and gravitational instability is inhibited. Conditions are less conducive to such turbulence at larger heliocentric distances. If particles cannot coagulate to sizes $\gtrsim 1$ cm in the inner nebula, planetesimal formation may be delayed until after the gas dissipates.

Some coagulation of grains is plausible under nebular conditions. Van der Waals attraction alone may suffice to allow formation of centimeter-sized aggregates during settling to the central plane. Since the van der Waals force is rather insensitive to composition, it is unlikely that any significant chemical fractionation occurs by differences in sticking efficiency during settling. The settling velocities exceed thermal velocities for particles larger than $\sim 10^{-4}$ cm; this specifically rules out fractionation mechanisms which depend on the assumption of thermal velocities (e.g., Wesson and Lermann, 1978).

Coagulation strongly affects settling behavior and time scales. The dominant coagulation process involves the sweeping up of small particles by larger ones. This behavior produces nonhomologous settling; even after most of the mass of solids reaches the central plane, some remains suspended in the form of fine particles which may maintain significant opacity in the nebula. The settling time scales are $\sim 10^3$ years in the inner nebula, and $\sim 10^4$ years in its outer region. These are minimum values which assume all solids are condensed initially. If grains are condensing from a cooling gas, coagulation and settling will proceed more slowly.

Particles can form large aggregates by collisional accretion if a reasonable impact strength is assumed. For bodies less than $\sim 10^2$ cm in size, drag-induced velocities increase with size. This behavior produces runaway growth in the central plane; the largest bodies can grow from ~ 10 cm to 10^3–10^4 cm in a few tens of orbital periods. Relative velocities decrease for still larger sizes, inhibiting further growth. Even if mechanical properties are such that coagulation is efficient, bodies as large as 1 km are not produced directly by drag-induced collisions. Instead, efficient damping of inclinations by gas drag concentrates the smaller bodies into a thin layer which is subject to gravitational instability.

The formation of planetesimals appears to be a composite process, involving a stage of collisional accretion, followed by gravitational instability. The latter produces objects of about the same characteristic size as those calculated for a dust layer without coagulation. However, the material in these planetesimals has undergone extensive collisional breakup and mixing of fragments before this stage. This collisional evolution may be a factor in accounting for the small-scale texture of undifferentiated meteorites. The further accretion of planetesimals into planets is dominated by gravitational, rather than aerodynamic, forces.

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Note added in proof. After this paper was written, I received a preprint by Y. Nakagawa, K. Nakazawa, and C. Hayashi, entitled "Growth and Sedimentation of Dust Grains in the Primordial Solar Nebula." This describes a similar calculation of competitive coagulation and settling, using a completely different numerical method. Their results are in substantial agreement with those presented here. Minor differences appear to be due to their assumption of coagulation without fragmentation, and use of a slightly different nebular model.

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