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A model for surface effects in slow collisions of icy grains

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Abstract

A model is presented which describes the change of the relative velocity of slowly colliding solid grains in planar motion. It takes into account elasticity and viscosity of the material, effects of soft surface layers and adhesion, and friction between the surfaces. The equations of motion of the colliding bodies are solved numerically. Normal and tangential restitution coefficients are computed, and their dependence on the components of the relative velocity before impact is determined. The material constants in the examples have been chosen to reflect the behaviour of water ice as an important material in the outer solar system. Soft surface layers as well as surface energy lead to aggregation for low collision velocities, both normal and tangential relative velocity can be reduced to zero at the same time. A critical velocity exists below which rebound is inhibited. It can be related to the thickness and properties of the surface layer or the surface energy of the collision partners. The results are expected to be important for the evolution of the size distribution in granular media, in particular for the formation of larger bodies from centimetre-sized grains.

Keywords: Planet and satellite formation; Planetary rings; Granular media

1. Introduction

Non–elastic collisions are important for structure formation in granular systems like sand or artificial grainy materials, and even more in astrophysical systems like planetary rings or protoplanetary disks because they reduce the relative velocity of the grains and lead to a narrower velocity distribution. As the following examples will show, the collisional properties of particles in the planetary system can be expected to be crucial in particular for aggregation processes which, for instance, lead to the growth of planetesimals, as pointed out by Hartmann (1978).

A striking effect of dissipative collisions has been observed in numerical simulations of a situation corresponding to the Encke division of Saturn's A ring. Without dissipation, superposition of all subsequent gravitational wakes caused by an embedded satellite leads to the disappearance of the initially observed structures (Spahn et al., 1994). If the excitations are damped by non-elastic collisions, only the first generations of wakes can superpose (Hertzsch et al., 1997). Thus, collisional dissipation is responsible for the apparent persistence of wake structures. In this work, however, adhesion was still neglected.

The properties of chondrules which form some types of meteorites and which are considered primordial matter in the solar system may be determined by surface effects. On these small bodies, dust layers have been detected (Metzler et al., 1992) which will obviously affect the collisional behaviour of the grains they are covering. In the context of the formation of planets by collisional accretion of small bodies, this has been experimentally investigated by Hartmann (1978) and by Colwell and Taylor (1999) who found a significant reduction of the restitution coefficient by regolith layers covering the surface of the impacted body. Constraints on the formation of chondrites have recently been discussed on the grounds of the existence of such layers (Metzler and Bischoff, 1996; Morfill et al., 1998). Frost layers may play a similar role on ice particles in planetary rings or in the cometary belts (Bridges et al., 1996; Supulver et al., 1997).

In planetary rings, the so-called dynamic ephemeral bodies or DEBs (Davis et al., 1984) are believed to form temporarily. These are agglomerations of ring particles with similar orbital parameters which stay in close neighbourhood for some revolutions around the planet before separating again. One could imagine that they aggregate permanently to small moons like Pan in Saturn's A ring, whose gravitation causes the Encke gap and the associated wake structures. While it is generally understood that

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planetary rings are found in the Roche zone of the central body where self-gravity alone is not sufficient to counteract the tidal stresses, the size of this zone depends on the density of the potential satellite and is usually calculated for strengthless bodies. A sticking force provided by interactions between the grain surfaces may lead to permanent aggregation at least in the outer regions of a ring system. Depending on distance from the planet, size and surface energy of the particles, the adhesive force can be significantly larger than the tidal force exerted by the central body (see the appendix). However, moonlets which exist now in planetary ring systems may not originate directly from aggregation, but from the disruption of larger bodies by high-velocity impacts (Esposito and Colwell, 1989; Colwell and Esposito, 1992, 1993). Closely related to this problem is the low density of some asteroids like Mathilde (Veverka et al., 1999; Thomas et al., 1999) which suggests that these bodies are also loosely packed aggregates of smaller constituents. Whether aggregation or disruption has caused the existence of a particular body and whether particles will reaggregate after disruption, needs to be studied separately.

Non-elastic collisions of solid particles have been investigated in many experiments and for a long time (see for instance Haughton, 1864; Vincent, 1900; Raman, 1918; Ôkubo, 1922) and have yielded a wealth of data on the influence of the impact velocity on the degree of elasticity. The latter has generally been found to decrease for faster collisions. Surface effects, in particular adhesion, have been studied early, too, in investigations of plastic materials (Pochettino, 1914). An excellent overview of experiments on low-velocity impacts has been given by Goldsmith (1960). Recently, collisions of icy particles have been the focus of extensive experimental investigations (Bridges et al., 1984; Hatzes et al., 1988), and research concentrates increasingly on surface effects which may cause aggregation (Bridges et al., 1996; Supulver et al., 1997). Extensive laboratory and microgravity experiments have also been performed for collisions of microscopic dust grains with an emphasis on the formation and restructuring of fractal aggregates (Wurm and Blum, 1998; Blum and Wurm, 2000). Their results are of particular importance for the earliest stages of planet formation.

A model for low-velocity collisions of granular particles which includes aggregation due to surface phenomena and which can explain the behaviour observed in experiments will therefore be useful in future work. The present article is devoted to this topic and attempts to generalise earlier approaches (Maw et al. 1981; Hertzsch et al., 1995, Brilliantov et al., 1996). Planar motion is considered which is a good approximation for the situation in planetary rings as one of the most interesting practical applications. In contrast to the coagulation of small dust particles examined by Chokshi et al. (1993), or the collision of fractal dust aggregates (Wurm and Blum, 1998; Blum and Wurm, 2000), the contact of millimetre- to centimetre-sized solid grains will be examined. The relative velocity of the colliding bodies is assumed to be in the order of some mm/s which is expected for instance to be caused by Kepler shear for particles on neighbouring orbits in a planetary ring. The model is based on the analysis of elastic and viscous forces acting in the material and friction between the surfaces, and in addition to former work (Hertzsch et al., 1995; Brilliantov et al., 1996) the presence of dust-like surface layers and adhesion forces between the collision partners is taken into consideration. Parameters are the elastic and viscous constants of the material, the friction coefficient between the surfaces, thickness and viscosity of surface layers and surface energy. The influence of surface layers and adhesive surface forces on the normal contact of viscoelastic particles is analysed, and the results are included in a model of tangential restitution.

Non-destructive collisions are conveniently described using the restitution coefficients ε_N and ε_T for the normal and tangential component of the relative motion:

$$\varepsilon_\mathrm{N} = -\frac{g'_\mathrm{N}}{g_\mathrm{N}} \quad (0 \leqslant \varepsilon_\mathrm{N} \leqslant 1) \quad \text{and}$$

$$\varepsilon_\mathrm{T} = \frac{g'_\mathrm{T}}{g_\mathrm{T}} \quad (-1 \leqslant \varepsilon_\mathrm{T} \leqslant 1), \tag{1}$$

where $\vec{g}$ is the relative velocity of the surfaces with its normal and tangential components g_N and g_T, respectively. The primes in Eq. (1) indicate the values of the velocities after the collision, non-primed quantities are measured immediately before contact. $\vec{g}$ is calculated from the translation velocities $\vec{v}_{i,j}$ and angular velocities $\vec{\omega}_{i,j}$ of the particles i, j and their radii $a_{i,j}$ in the point of contact:

$$(g_\mathrm{N}, g_\mathrm{T}) = \vec{g} = (\vec{v}_i - \vec{\omega}_i \times \vec{n}a_i) - (\vec{v}_j + \vec{\omega}_j \times \vec{n}a_j). \tag{2}$$

This concept has the advantage that quick and simple simulations with event-driven algorithms become possible (instead of having to solve equations of motion or to evaluate contact potentials for each single collision) once the dependence of ε_N and ε_T on the impact velocity is known. In this article, the influence of surface phenomena on these dependences is examined, and predictions of $\varepsilon_\mathrm{N}(g_\mathrm{N})$ and $\varepsilon_\mathrm{T}(g_\mathrm{N}, g_\mathrm{T})$ are made which are expected to be useful in numerical simulations.

Exemplary velocity dependences of restitution coefficients in the present paper are calculated using material constants of water ice as one of the most common, and therefore most relevant materials of small bodies in the outer solar system. Non-withstanding this choice, the results of the model are applicable for studies of granular gases with aggregating particles in general.

2. Normal restitution

2.1. Stresses in a viscoelastic body

Experiments which show that the normal restitution coefficient decreases with increasing impact velocity

(Goldsmith, 1960; Bridges et al., 1984) can be explained by the assumption that the colliding bodies consist of a viscoelastic material (Kuwabara and Kono, 1987; Hertzsch et al., 1995). In the current section, the main results of this theory will be introduced because they will be used in the later extensions of the model.

A viscoelastic body can be represented by a Kelvin–Voigt model with an elastic and a viscous element in parallel. The total stress σ exerted on such a body is the sum of elastic and viscous stresses σ^{el} and σ^{vi}, and its tensor components take the form

$$\begin{aligned}\sigma_{ik} &= \sigma_{ik}^{\mathrm{el}} + \sigma_{ik}^{\mathrm{vi}}\\ &= 2\lambda_{\mathrm{I}}(u_{ik} - \tfrac{1}{3}\,\delta_{ik}u_{ll}) + \lambda_{\mathrm{II}}\delta_{ik}u_{ll}\\ &\quad + 2\eta_{\mathrm{I}}(\dot{u}_{ik} - \tfrac{1}{3}\,\delta_{ik}\dot{u}_{ll}) + \eta_{\mathrm{II}}\delta_{ik}\dot{u}_{ll},\end{aligned} \tag{3}$$

where $\lambda_{\mathrm{I,II}}$ are the Lamé constants, $\eta_{\mathrm{I,II}}$ denote the shear and bulk viscosity, respectively, and $\{u_{ik}\}$ is the deformation tensor. Provided the collision occurs at a velocity much slower than the speed of sound in the material (which is true for the velocity range of well below 1 m/s considered here), a dynamical equation for the compression can be derived for curved surfaces (Brilliantov et al., 1996) in a similar way as in the theory of elastic contact (Hertz, 1881). Let Y denote the Young modulus of elasticity and v the Poisson ratio, $R_{\mathrm{eff}} = R_1R_2/(R_1 + R_2)$ the effective radius in the point of contact and $m_{\mathrm{eff}} = m_1m_2/(m_1 + m_2)$ the effective mass of the colliding bodies numbered 1 and 2. Then, for spherical particles consisting of the same material the dynamical equation for the compression $\xi = R_1 + R_2 - R$ (where R is the distance of the centres of the colliding bodies) reads

$$\ddot{\xi} + \frac{2Y\sqrt{R_{\mathrm{eff}}}}{3m_{\mathrm{eff}}(1-v^2)}\left(\xi^{3/2} + \frac{3}{2}A\sqrt{\xi}\dot{\xi}\right) = 0 \tag{4}$$

with $A = \frac{1}{3}\,[(3\eta_2 - \eta_1)^2/(3\eta_2 + 2\eta_1)](1-v^2)(1-2v)/Yv^2$. ξ is to be taken positive if the bodies approach each other. This differential equation is solved numerically. If the collision is assumed to start at time $t = 0$, and the duration of the collision (until separation of the grains) is t_{c}, the restitution coefficient will be

$$\varepsilon_{\mathrm{N}} = \frac{\dot{\xi}(t_{\mathrm{c}})}{\dot{\xi}(0)}. \tag{5}$$

It can be shown (Hertzsch et al., 1995) by evaluation of the coefficients in the derivation of Eq. (4), that the equation of motion for particles with any finite macroscopical curvature of their surfaces takes the same form as the one for spherical surfaces except for an additional factor in the second term which depends only on the eccentricity of the contact ellipse. This means that the normal contact problem for arbitrarily curved surfaces can always be reduced to the simpler one of spherical grains by rescaling of the coordinates (as long as the size of asperities is much smaller than the macroscopical curvature of the surfaces).

This model can explain several experimental results, in particular for collisions of ice surfaces (Bridges et al., 1984; Hatzes et al., 1988), but already early measurements (Pochettino, 1914) indicate that the normal restitution coefficient is not always a monotonously falling function of the impact velocity. Instead, it may assume low values and even reach zero for slow collisions. In particular, much interest has recently been devoted to surface effects in the contact of ice particles (Hatzes et al., 1988; Bridges et al., 1996) where $\varepsilon_{\mathrm{N}} = 0$ (sticking) has been observed below a certain impact velocity. Therefore, a realistic model of collisions must incorporate surface effects, among which the presence of soft surface layers (dust, frost, liquids) and surface energy are most important.

In the following sections these effects will be examined, and the results of the simple viscoelastic model will serve as a common base of comparison. Throughout this paper, the grains will be assumed to be spherical with an effective radius of $R_{\mathrm{eff}} = 1$ cm, and they shall consist of water ice with a density of $\rho = 0.9$ g/cm^3. Throughout the article, the other necessary material constants for this material are assigned the following (rounded) values for ice at low temperatures (Landolt–Börnstein, 1952, 1982): Youngs modulus $Y = 10$ GPa, Poissons ratio $v = 0.3$, coefficient of internal friction $\eta_1 = 10^6$ Pa s, bulk viscosity $\eta_2 = 10^{12}$ Pa s. Any other parameters will be introduced and their values be set where appropriate.

2.2. Effects of soft surface layers

A powdery layer such as dust or frost may be considered as a material with a very low elasticity, caused by the loose packing, but a notable apparent viscosity due to friction between the grains. Its behaviour will therefore be approximated by a simple viscous element.

The viscous stress is not proportional to the deformation, but the deformation velocity (see Eq. (3), viscous term). Therefore, in the analysis of the compression of such material in analogy to the elastic contact theory (Hertz, 1881, see also Landau and Lifshitz, 1986) the integral over the force must be used instead of the force itself. This leads directly to

$$\int F\,\mathrm{d}t = \frac{\sqrt{R_{\mathrm{eff}}}}{K}\,\xi^{3/2} \tag{6}$$

which consequently yields

$$F = -m_{\mathrm{eff}}\ddot{\xi} = \frac{3}{2}\,\frac{\sqrt{R_{\mathrm{eff}}}}{K}\,\sqrt{\xi}\,\dot{\xi} \tag{7}$$

with the constant

$$K = \frac{3}{4}\left(\frac{4\eta_1' + 3\eta_2'}{4\eta_1'(\eta_1' + \eta_2')} + \frac{4\eta_1'' + 3\eta_2''}{4\eta_1''(\eta_1'' + \eta_2'')}\right).$$

Here, primes and double primes serve to distinguish between the materials of the two bodies in case they are

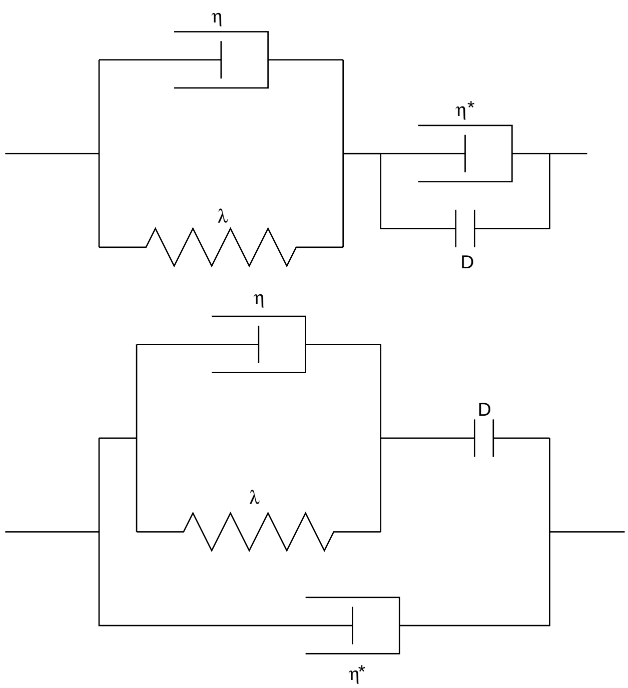


Fig. 1. Top: modified Letherisch model, consisting of a Kelvin–Voigt element with an elastic and a viscous part, symbolised by effective Lamé constant λ and viscosity η, in series with another viscous element with effective viscosity η^*. In contrast to the original model, the latter part is only a layer with a given thickness D, represented by the distance of the buffers. Bottom: simplified model, where the very soft outer layer with thickness D is almost decoupled from the bulk of the material. Its viscosity is so low that this layer is first compressed without affecting the bulk, and the later stage of the compression is completely governed by the bulk properties.

different. The right side of Eq. (7) is strikingly similar to the non-elastic term in Eq. (4), which describes dissipation due to the viscous part of the stress.

Particles covered by soft layers may be represented by a Kelvin–Voigt element (for the bulk) and a viscous element (for the surface layer) in series, similar to the Letherisch model of viscoelasticity which had been constructed for the purpose of describing "elastic sols" like tar or pitch (see e.g. Reiner, 1958) whose deformational behaviour depends on the speed of deformation. If the stress is applied slowly, they resemble a viscous fluid, while they behave like an imperfectly elastic solid for faster deformations. This model, however, needs to be adjusted for the purpose of the present investigation. First of all, one has to take into account the finite thickness D of the surface layer. This is represented by a pair of "stoppers" or "buffers" with the distance D in parallel to the viscous element (Fig. 1, top).

A further simplification is desirable because coupling between the elements in the original Letherisch model complicates the analysis and makes it impossible to formulate an ordinary differential equation for the time dependence of the compression. It can be achieved because dust–like surface layers are usually much more easily deformed than the viscoelastic bulk of the material. One can therefore assume that the latter remains unaffected by the collision until the former is completely compressed. Thus, one is lead to a model with two independent stages of the collision: first, the viscous outer layer is compressed while the bulk material is not affected, then viscoelastic deformation of the bulk occurs during which the influence of the soft surface layer is negligible, and the two parts may be considered separately (Fig. 1, bottom).

The time dependence of the compression ξ in the two-stage model can now be formulated as follows (D_1 and D_2 are the thicknesses of the layers which cover the surfaces of the colliding bodies):

$$\ddot{\xi} + \frac{3}{2}\frac{\sqrt{R_{\text{eff}}}}{Km_{\text{eff}}}\sqrt{\xi}\,\dot{\xi} = 0 \quad \text{if } \xi \leqslant D_1 + D_2,$$

$$\ddot{\xi} + \frac{2Y\sqrt{R_{\text{eff}}}}{3m_{\text{eff}}(1-\nu^2)}\left(\xi^{3/2} + \frac{3}{2}A\sqrt{\xi}\dot{\xi}\right) = 0 \quad \text{if } \xi > D_1 + D_2 \tag{8}$$

with the constants defined above. The first of these equations can be solved analytically using $(\mathrm{d}/\mathrm{d}t)(\frac{2}{3}\xi^{3/2}) = \sqrt{\xi}\,\dot{\xi}$ and separation of the variables, but the resulting expression for $t(x)$ is rather complicated and cannot be resolved in x. However, integrating it only once results in the relation

$$\dot{\xi} - \dot{\xi}(0) = -\frac{\sqrt{R_{\text{eff}}}}{Km_{\text{eff}}}\,\xi^{3/2} \tag{9}$$

which can be used to determine the thickness of a surface layer necessary to stop the impact of a body with a given velocity, or the critical velocity g_{s} below which rebound is inhibited by a layer of thickness $D_1 + D_2$:

$$g_{\text{s}} = \frac{\sqrt{R_{\text{eff}}}}{Km_{\text{eff}}}\,(D_1 + D_2)^{3/2}. \tag{10}$$

Although Schwager and Pöschel (1998) have found an expression of the dependence $\varepsilon_{\text{N}}(g_{\text{N}})$ in the viscoelastic case (Eq. (4)) in form of a power series, a numerical solution of Eqs. (8) seems to be more convenient and appropriate for practical purposes. It has been performed with the same self-adjusting Runge–Kutta integrator which has been used for the solution of the purely viscoelastic problem.

In Fig. 2, results of this simplified Letherisch model are shown in comparison to the situation without surface layers (clean surfaces), described by Eq. (4). The restitution coefficient has been calculated according to Eq. (5) and plotted against the normal impact velocity g_{N}. As a parameter for the surface layer, the quotient $1/K$, which can be regarded as an effective viscosity, denoted by η^* in Fig. 1, was assumed to yield several values ranging from 500 to 5000 Pa s. The thickness of the layer was chosen to be 1 μm which is less than usually that of the mineral dust layers found in solar system chondrites (Metzler et al., 1992) but sufficient to show significant effects which are well possible for frost

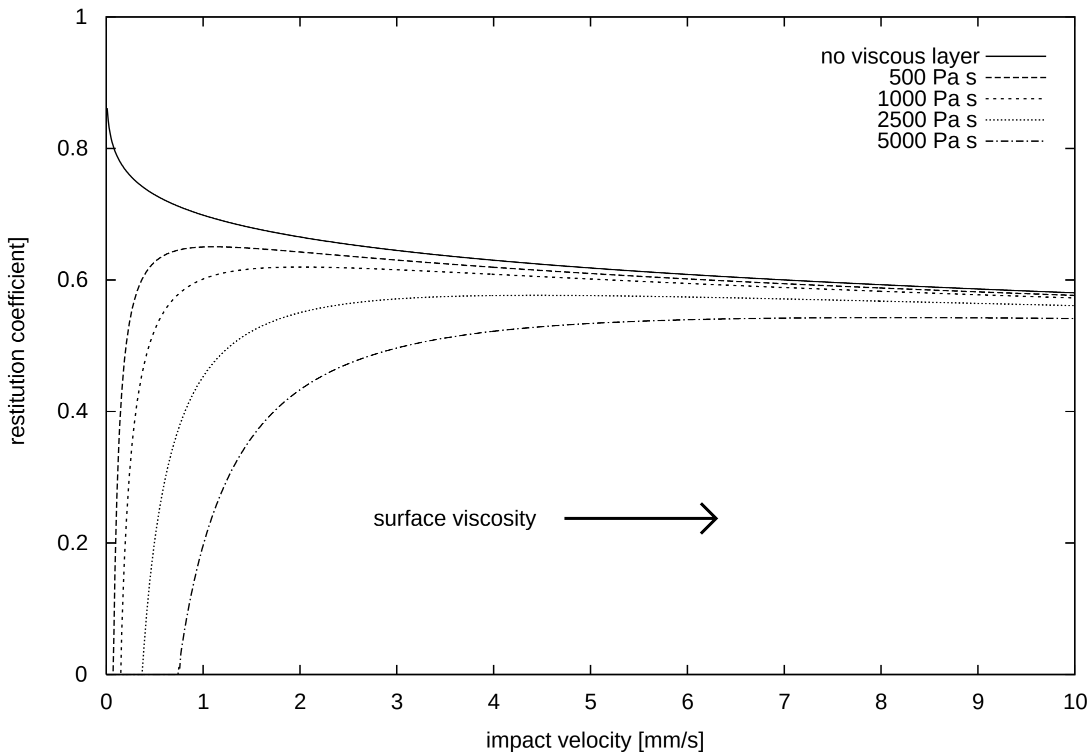


Fig. 2. Normal restitution coefficient ε_N vs. normal impact velocity g_N for different viscosities of the surface layer. The relative motion of the particles stops ($\varepsilon_N = 0$) if the velocity is not high enough for "penetration" or complete compression of the soft outer layer. When this layer is penetrated, it has still the effect that the restitution coefficient is reduced compared to a "clean" surface.

layers. From the figure it is obvious that a certain minimum velocity or kinetic energy which is a monotonous function of viscosity and thickness of the outer layer is needed to penetrate, i.e. completely compress the latter. The relative motion of the particles stops ($\varepsilon_N = 0$) if the velocity is not high enough to reach this point, and in this case the kinetic energy is completely dissipated in the surface layer. Even in the case of penetration, this dissipation layer reduces the restitution coefficient in comparison to a "clean" surface, the more effectively, the more viscous the layer is.

2.3. *Effects of attractive surface forces*

It is well known that molecular adhesive forces act between two surfaces in contact. Their effects are generally expressed in terms of the Dupré energy of adhesion γ which is equal to the sum of the surface energies of the two materials in contact, diminished by the interfacial energy. For simplicity, γ will be referred to as "surface energy" throughout the remainder of this article. The analysis of the (static) adhesive elastic contact has been performed independently by Sperling (1964) and Johnson et al. (1971). Using these results, the influence of the energy of adhesion on the impact of elastic spheres has been discussed by Johnson and Pollock (1994). Recently, these findings have been used in the analysis of the contact of plastic grains (Thornton and Ning, 1998). The surface energy W_s can be obtained by an integration of the contact force F resulting from compression and adhesion over the relative approach ξ of the grains. According to Johnson (1976), these quantities are related by

$$\frac{\xi}{\xi_f} = \frac{3(F/F_c) + 2 \pm 2(1 + (F/F_c))}{3^{2/3}((F/F_c) + 2 \pm 2(1 + (F/F_c))^{1/2})^{1/3}} \qquad (11)$$

with $F_c = \frac{3}{2}\pi\gamma R_{\text{eff}}$ being the force needed to separate elastic spheres in adhesive contact, and $\xi_f = \frac{3}{4}(\pi^2\gamma^2 R_{\text{eff}}/(Y/(1-\nu^2))^2)^{1/3}$ is the value of the relative approach at which this separation occurs. Integration over the area in the F–ξ plane where $-\xi_f < \xi < 0$ (Thornton and Ning, 1998) yields the surface energy

$$W_s = \int_0^{-\xi_f} F \, d\xi \approx 7.09 \left(\frac{\gamma^5 R_{\text{eff}}^4}{(Y/(1-\nu^2))^2} \right)^{1/3}. \qquad (12)$$

If separation shall be possible, this energy must be supplied by the kinetic energy of the particles corresponding to the elastic energy stored in them. If the grains consist of a viscoelastic material, energy is not only dissipated in breaking the surface contact which requires the energy W_s just mentioned, but also by the viscous properties of the material as described in Section 2.1. The energy loss due to the latter effects shall be denoted with W_v. The total energy balance can therefore be expressed using the kinetic energies before

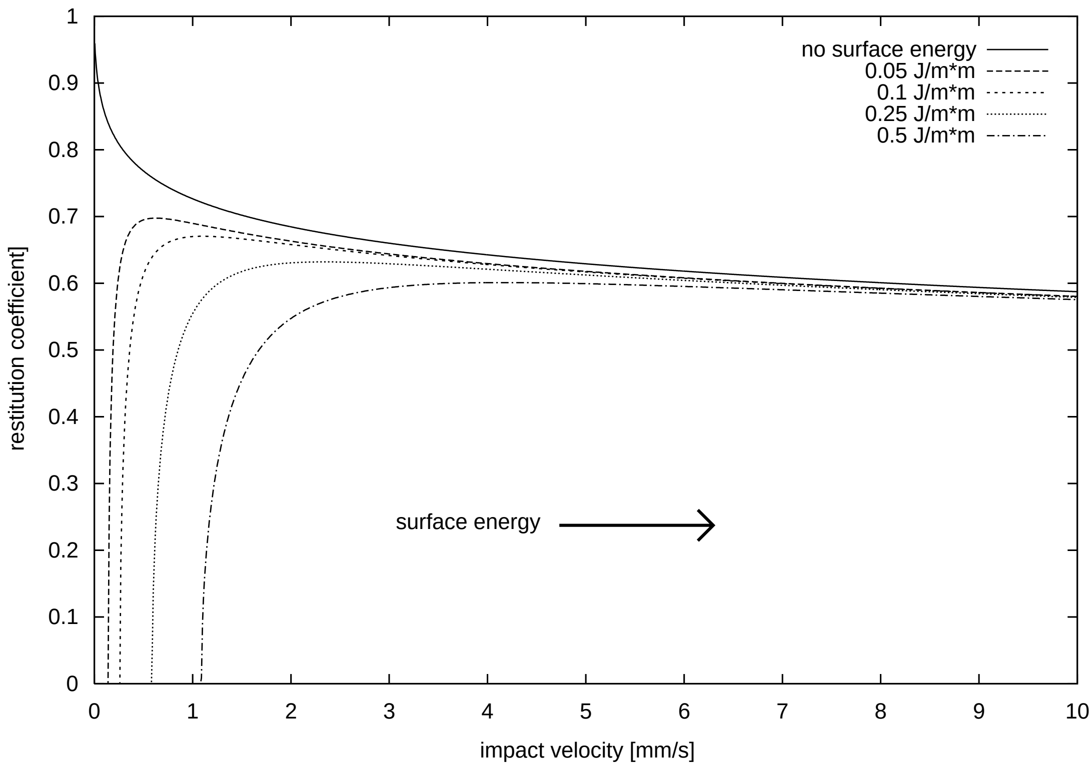


Fig. 3. Normal restitution coefficient ε_N vs. normal impact velocity g_N for different values of the surface energy. If the kinetic energy of the rebounding particles is not sufficient to overcome the attraction of their surfaces, the relative motion of the particles stops ($\varepsilon_N = 0$). For higher velocities, attractive surface forces still reduce the restitution coefficient compared to grains without surface effects.

and after the collision by

$$\frac{m_{\text{eff}}}{2}\, g_N'^2 = \frac{m_{\text{eff}}}{2}\, g_N^2 - W_v - W_s, \tag{13}$$

where one can substitute according to Section 2.1:

$$\frac{m_{\text{eff}}}{2}\, g_N^2 - W_v = \frac{m_{\text{eff}}}{2}\, g_{N,v}'^2. \tag{14}$$

Here $g'_{N,v}$ shall denote the rebound velocity for viscoelastic particles without surface attraction which is calculated from Eq. (4) or (8). Following Thornton and Ning (1998), one can calculate the critical value g_s for $g'_{N,v}$ where the rebound velocity g'_N becomes zero and below which the surfaces still stick together if W_s is non-zero. This critical velocity for separation is obtained from Eqs. (12)–(14) by setting $g'_N = 0$ and takes the value

$$g_s \approx \left(\frac{14.18}{m_{\text{eff}}}\right)^{1/2} \left(\frac{\gamma^5 R_{\text{eff}}^4}{(Y/(1-\nu^2))^2}\right)^{1/6}. \tag{15}$$

If the grains separate after the collision, $g'_{N,v} > g_s$, and one can derive from Eqs. (13) and (14) the relation

$$1 - \left(\frac{g_s}{g'_{N,v}}\right)^2 = \left(\frac{g'_N}{g'_{N,v}}\right)^2. \tag{16}$$

Using the normal restitution coefficient resulting from the viscoelastic model without or with soft surface layers obtained from Eq. (4) or (8) which here shall be denoted with $\varepsilon_{N,v} = g'_{N,v}/g_N$, one obtains finally for the restitution coefficient of grains with adhesive surface forces:

$$\varepsilon_N = \begin{cases} 0 & \text{if } g_N < g_s, \\ \sqrt{1 - \left(\dfrac{g_s}{g'_{N,v}}\right)^2}\, \varepsilon_{N,v} & \text{if } g_N > g_s. \end{cases} \tag{17}$$

In the case of missing surface layers, one may alternatively use for $\varepsilon_{N,v}(g_N)$ the analytical expression obtained by Schwager and Pöschel (1998).

In order to demonstrate the effects of surface energy alone, ε_N has been calculated for viscoelastic spheres with $R_{\text{eff}} = 1$ cm and the same other material constants as used previously in this article. The surface energy ranges between values of 0.05 and 0.5 J/m^2 which can be assumed to be typical for many materials. The resulting curves are shown in Fig. 3. If the kinetic energy of the rebounding particles is not sufficient to overcome the attraction of their surfaces, the relative motion of the particles stops ($\varepsilon_N = 0$). The higher the surface energy, the greater the value of the critical velocity, as is already clear from Eq. (15). For higher velocities, attractive surface forces still reduce the restitution coefficient compared to the case without surface effects because breaking the contact always dissipates part of the kinetic energy.

3. Phenomenological model of tangential restitution

Energy loss in the tangential motion is commonly attributed to the contact of surface asperities. This contact can only be broken if the tangential force component is larger than the frictional resistance determined by the normal force between the surfaces. Otherwise, a reversal of the direction of the relative motion can be expected. In other words, the ratio between tangential and normal component of the force—or the relative velocity—between the surfaces determines the kinematic behaviour. The threshold value at which the sign of the post-collisional velocity changes will be determined below.

At low values of this ratio a gear-like reversal of spin occurs, accompanied by energy dissipation due to non-elastic rebound of the deformed asperities. This situation is similar to the normal restitution described above. The tangential restitution coefficient is determined by the properties of the material, and $g'_{\mathrm{T}} = -\varepsilon_{\mathrm{N}}(g_{\mathrm{T}})g_{\mathrm{T}}$ or $\varepsilon_{\mathrm{T}} = -\varepsilon_{\mathrm{N}}(g_{\mathrm{T}})$. Of course, it will depend on the impact velocity as shown in the previous section.

If the ratio of the force components is sufficiently high, i.e. the tangential force F_{T} is high enough to overcome the frictional resistance μF_{N}, the asperities will be deformed in a way that sliding is possible with frictional dissipation of energy. The energy loss in tangential motion can be written as

$$\Delta Q = \mu \int_0^{t_c} F_{\mathrm{N}} g_{\mathrm{T}}\,\mathrm{d}t, \tag{18}$$

where t_c is the duration of the collision, and the friction coefficient μ is assumed to be independent of the tangential relative velocity of the surfaces. The normal force F_{N} can be approximated using the definition of the force as time derivative of the momentum p as

$$F_{\mathrm{N}} = \frac{\mathrm{d}p_{\mathrm{N}}}{\mathrm{d}t} \approx \frac{\Delta p_{\mathrm{N}}}{t_c} \approx \frac{m_{\mathrm{eff}}\Delta g_{\mathrm{N}}}{t_c}. \tag{19}$$

This, inserted in Eq. (18), yields:

$$\Delta Q = \mu \int_0^{t_c} m_{\mathrm{eff}} \frac{\Delta g_{\mathrm{N}}}{t_c} g_{\mathrm{T}}\,\mathrm{d}t \approx \mu m_{\mathrm{eff}} \Delta g_{\mathrm{N}} g_{\mathrm{T}} \tag{20}$$

and using definition (1) of the restitution coefficients one obtains

$$\Delta Q \approx -\mu m_{\mathrm{eff}}(1+\varepsilon_{\mathrm{N}}) g_{\mathrm{N}} g_{\mathrm{T}}. \tag{21}$$

The loss in kinetic energy of the tangential motion can also be written using the restitution coefficient:

$$\Delta Q = \frac{m_{\mathrm{eff}}}{2} \kappa_{ij} g_{\mathrm{T}}^2 (\varepsilon_{\mathrm{T}}^2 - 1), \tag{22}$$

where κ_{ij} denotes the reduced moment of inertia of the two bodies in contact. If the two expressions for ΔQ are set equal, one obtains the following expression for the tangential restitution in the case of sliding motion:

$$\varepsilon_{\mathrm{T}} = \sqrt{1 - \frac{2\mu g_{\mathrm{N}}(1+\varepsilon_{\mathrm{N}})}{\kappa_{ij} g_{\mathrm{T}}}}. \tag{23}$$

The condition for sliding, simply given as $F_{\mathrm{T}} > \mu F_{\mathrm{N}}$, can be formulated using $F_{\mathrm{N,T}} = \mathrm{d}p_{\mathrm{N,T}}/\mathrm{d}t \approx \Delta p_{\mathrm{N,T}}/t_c$ as

$$m_{\mathrm{red}} \kappa_{ij} g_{\mathrm{T}} > \mu m_{\mathrm{eff}} g_{\mathrm{N}} \tag{24}$$

and can therefore be expressed in the components of the relative velocity of the surfaces

$$\tan\psi = \frac{g_{\mathrm{T}}}{g_{\mathrm{N}}} > \frac{\mu}{\kappa_{ij}} = \tan\psi_0, \tag{25}$$

where ψ is the so-called “sliding angle”.

Summarizing the above results, the tangential restitution coefficient equals

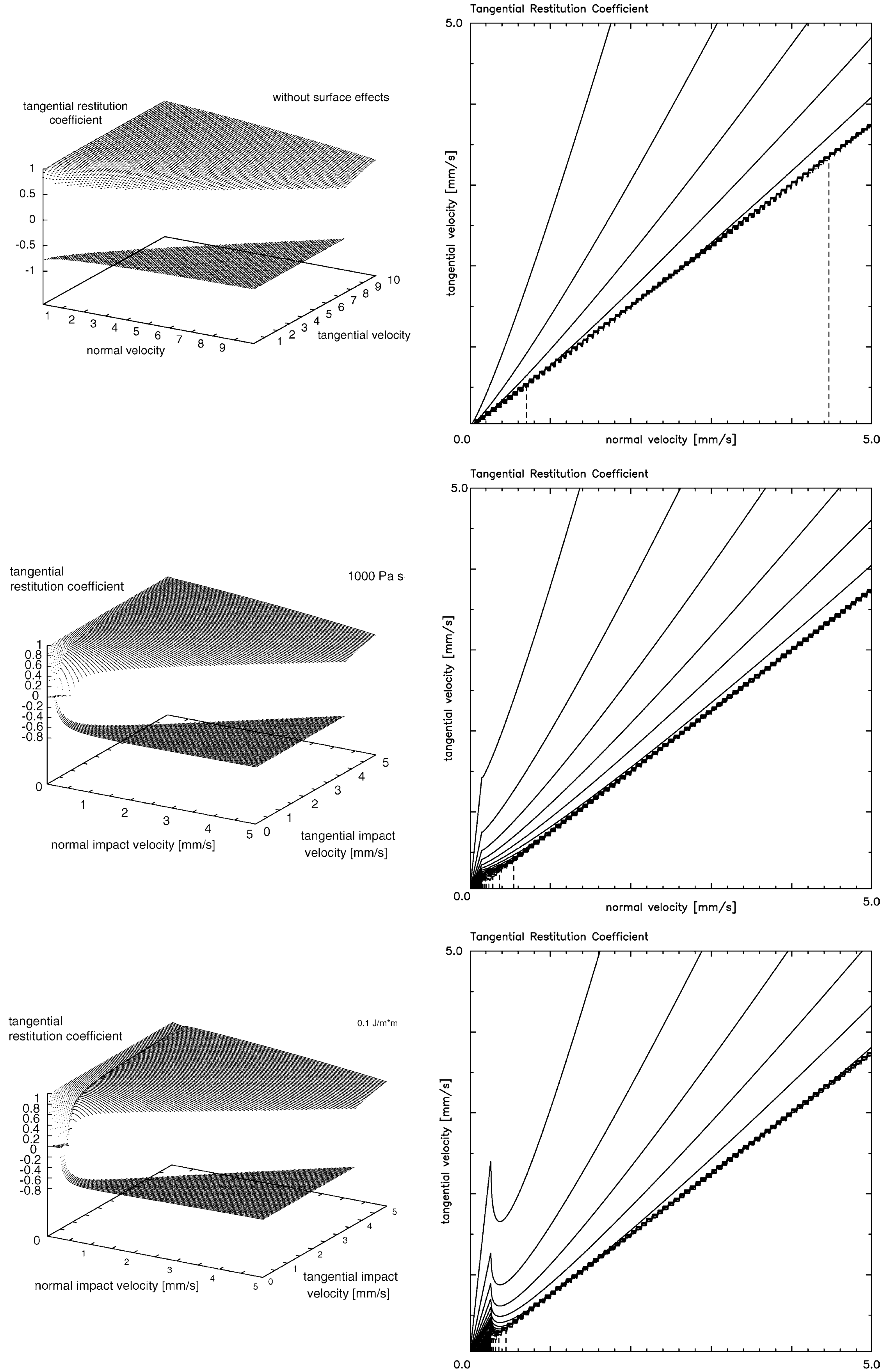
$$\varepsilon_{\mathrm{T}}(g_{\mathrm{N}}, g_{\mathrm{T}}) = \begin{cases} -\varepsilon_{\mathrm{N}}(g_{\mathrm{T}}) & \text{for } \psi \leqslant \psi_0, \\ \sqrt{1 - \dfrac{2\mu g_{\mathrm{N}}(1+\varepsilon_{\mathrm{N}}(g_{\mathrm{N}}))}{\kappa_{ij} g_{\mathrm{T}}}} & \text{for } \psi > \psi_0, \end{cases} \tag{26}$$

where ε_{N} is calculated using the appropriate model of normal contact.

The tangential restitution coefficient ε_{T} has been calculated for values of the impact velocity components ranging between 0 and 5 cm/s, assuming a friction coefficient of $\mu = 0.3$. Results are shown in Fig. 4 for three different situations: particles without surface effects (“hard spheres”), particles with a surface layer with $1/K = 1000$ Pa s and particles with a surface energy of $\gamma = 0.1$ J/m^2. Lacking any definite information on the properties of the asperities in comparison to the bulk, the restitution coefficient ε_{N} was assumed to be the same as calculated in the previous sections in the case of rebound.

The resulting dependence $\varepsilon_{\mathrm{T}}(g_{\mathrm{N}}, g_{\mathrm{T}})$ shows clearly a transition between rolling and sliding behaviour with changing ratio $g_{\mathrm{N}}/g_{\mathrm{T}}$. For hard surfaces, i.e. grains without any soft cover or surface energy, no combination of the velocity components $(g_{\mathrm{N}}, g_{\mathrm{T}})$ exists where the kinetic energy of the tangential motion is completely dissipated. If particles are covered with a viscous layer with $1/K = 1000$ Pa s, the latter causes both components of the relative motion to stop in a certain area in the plane $(g_{\mathrm{N}}, g_{\mathrm{T}})$. Very similarly, a surface energy of $\gamma = 0.1$ J/m^2 causes the particles to stick together in a part of the $(g_{\mathrm{N}}, g_{\mathrm{T}})$ plane.

As one can infer from Eq. (26), points $(g_{\mathrm{N}}, g_{\mathrm{T}})$ may exist in the latter two cases where $1 - 2\mu g_{\mathrm{N}}(1+\varepsilon_{\mathrm{N}})/\kappa_{ij} g_{\mathrm{T}} < 0$. These, however, have been found to lie within the region where $\varepsilon_{\mathrm{T}} = 0$, and may consequently be interpreted as belonging to the area where the relative motion comes to rest.


tangential restitution coefficient
without surface effects
1
0.5
0
-0.5
-1
1 2 3 4 5 6 7 8 9
normal velocity
1 2 3 4 5 6 7 8 9 10
tangential velocity
Tangential Restitution Coefficient
5.0
tangential velocity [mm/s]
0.0
normal velocity [mm/s]
5.0
tangential restitution coefficient
1000 Pa s
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
1 2 3 4 5
normal impact velocity [mm/s]
0 1 2 3 4 5
tangential impact velocity [mm/s]
Tangential Restitution Coefficient
5.0
tangential velocity [mm/s]
0.0
normal velocity [mm/s]
5.0
tangential restitution coefficient
0.1 J/m*m
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
1 2 3 4 5
normal impact velocity [mm/s]
0 1 2 3 4 5
tangential impact velocity [mm/s]
Tangential Restitution Coefficient
5.0
tangential velocity [mm/s]
0.0
normal velocity [mm/s]
5.0

4. Results and conclusion

In this article, the principal influences of both viscous surface layers and attractive surface forces on the restitution of viscoelastic grains have been addressed in a theoretical model. For the normal component of the relative motion, the effects are essentially very similar for both kinds of surface effects:

- A critical velocity g_s exists which depends on the viscosity and thickness of the surface layer in the former case according to Eq. (10) or the magnitude of the surface energy in the latter case where it is determined by Eq. (15). Collisions with impact velocities below the critical value are stopped.
- For impact velocities above g_s, the restitution coefficient is reduced compared to "hard particles", which effect is less pronounced for higher impact velocities.

This agrees with the results of experiments in which a frost layer has been found to lower the value of the restitution coefficient of ice bodies (Hatzes et al., 1988; Bridges et al., 1996), or where the collisional contact of adhesive surfaces has been examined (Wall et al., 1991), and it confirms the result of Hartmann (1978) that "regolith strongly favors accretion by inhibiting rebound", not only for head-on collisions, but also for glancing encounters.

However, a direct quantitative comparison with results of experiments, in which projectiles consisting of rock (Hartmann, 1978) or teflon (Colwell and Taylor 1999) impacted regolith layers of a depth comparable to the projectile size, is not possible due to the lack of data on the mechanical properties of the target material. Also, the high impact velocities in particular in Hartmann's experiments which exceed those used in the present simulations by a factor of 100 and more, may be cause of additional mechanical effects like fracturing or even cratering which are not covered by this model. The depth of the regolith layer and the critical velocity below which rebound does not occur can directly be related to each other using Eq. (10) provided the mechanical properties of the material remain unchanged over the range of velocities considered. Therefore, this equation can be used for an estimation of the constant K which is related to the apparent viscosity of the surface layer. If for instance the results of Hartmann (1978) (Fig. 2, l.c.) are used, where basalt spheres of masses ranging from 0.13 to 5 g impacted pulverized rock at velocities between 1.4 and 6.2 m/s, and a ratio of about 0.65 between regolith depth and impactor diameter was found to be sufficient to stop the motion, one would obtain values for K of about 0.01 Pa s or $1/K \approx 100$ Pa s if the highest mass is paired with the lowest velocity and vice versa, or $K \approx 0.04$ Pa s if the result was assumed to have been achieved for the low mass–low speed impact. However, these experiments were performed with priorities other than systematic measurements of the relation between regolith depth, impactor size, and collision velocity, which latter appear therefore highly desirable.

The velocity dependence of the normal restitution coefficient for particles with soft surface layers can be approximately fitted by a function

$$\varepsilon_N(g_N) \approx a g_N^b (1 - \exp(-c g_N^d) \qquad (27)$$

while the modification of $\varepsilon_N(g_N)$ by surface energy can be expressed analytically as shown above in Eq. (17).

The influence of surface effects on tangential restitution has been discussed in a simple phenomenological model. For grains without surface layers or adhesion (hard surfaces), the model predicts only a transition from sliding to rolling motion with changing (g_N, g_T). In contrast to this result, surface effects may also cause the tangential motion to stop in a certain range of impact velocity components. Again, the behaviour is very similar for soft outer layers and for adhesive forces.

When both components of the relative velocity of two colliding particles become zero at the same time, aggregation of the grains in contact has occurred. The simulations show that already a thin viscous layer is sufficient to achieve this. However, for a permanent aggregate which can withstand external forces, a certain tensile strength is necessary. This is provided by adhesive surface forces. From an approximate estimation of their value (see the appendix) in a typical case one can see that they can be strong enough to counteract the tidal forces resulting from different distances of grains in neighbouring orbits from their central body, so that, e.g. grains in planetary rings could grow into larger bodies. In practical applications, it can be necessary to consider both effects together because surface forces are always present. This is easily achieved insofar as the restitution coefficient resulting from Eq. (8) is further reduced by adhesion according to Eq. (17), and one would only need to insert the results for surface layers without adhesion in the latter formula.

The model can be refined using a microscopic theory of tangential restitution (Brilliantov et al., 1996). It may also be required to consider a possible dependence of the friction coefficient on the impact velocity. In non-planar systems it will be necessary to take into account the third Cartesian component of the relative motion, i.e. effects of drilling friction will have to be considered in such a case. The (approximative) functional formulation of the dependences $\varepsilon_N(g_N)$ and $\varepsilon_T(g_N, g_T)$ obtained from the models will be useful in

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Fig. 4. Dependence of the tangential restitution coefficient ε_T on the components of the relative velocity of the surfaces. Left column: surface plots. Right column: projections on the (g_N, g_T) plane. With changing ratio g_N/g_T, a transition between rolling and sliding behaviour occurs. For "hard" surfaces (top), i.e. without soft cover or surface energy, no combination (g_N, g_T) exists where the tangential motion comes to rest. If particles are covered with a viscous layer with $1/K = 1000$ Pa s (centre), the latter causes both components of the relative motion to stop in a certain area in the plane (g_N, g_T), as does a surface energy of $\gamma = 0.1$ J/m^2 (bottom). These areas are marked with black in the diagrams in the right column.

many-particle simulations of granular gases. Experimental estimations of the values of the material constants (which can at present only be regarded as approximate ones) as well as a more thorough comparison of the results of the theoretical model and of collision experiments will enhance the knowledge of the material properties and lead to improved predictions of the behaviour of grains in contact. A systematic study of the mechanical properties of dust, in particular powders of rock and ice, and frost of various volatiles as well as estimations of the surface energy of relevant materials under solar system conditions will enable a direct comparison of the present model with experimental data and its practical application.

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Appendix. Comparison of tidal forces and adhesion

A detailed examination of the behaviour of orbiting adhesive grains is outside the scope of this article. This would require solving the equations of motion under consideration of gravitational and surface forces and warrant a separate investigation. The following example is intended to demonstrate the relative magnitude of the principal forces acting on two particles in a typical situation.

Two identical spherical grains of density $\rho = 0.9$ g/cm^3 with a radius of $r = 2$ cm (effective radius $R_{\mathrm{eff}} = 1$ cm), which have a mass of $m \approx 30$ g each, are assumed to revolve around a planet with the mass $M = 5.688 \times 10^{26}$ kg of Saturn on circular orbits. They are assumed to be in contact, and the distance of the contact point from the centre of the planet shall be $d = 1.336 \times 10^8$ m corresponding to the mean radius of the Encke division in Saturn's rings. It is assumed that the straight line connecting the centres of the two spheres is parallel to the radius vector of the centre of the contact figure. Other forces due to, e.g. electrostatic charging or the gravitation of other celestial bodies, in particular other satellites of the same planet, will be neglected in this estimation, and static contact will be assumed so that Kepler shear due to differences in orbital velocity will not be taken into account here.

The difference of the gravitational forces exerted by the central body on these two grains is expressed using Newton's law

$$\Delta F = GMm\left(\frac{1}{(d-r)^2} - \frac{1}{(d+r)^2}\right) \approx 4GMm\,\frac{r}{d^3} \quad \text{for } r \ll d \tag{A.1}$$

with $G = 6.672$ N m^2/kg^2 being the gravitational constant. The surface force F_{s} of the two particles in static contact can be approximately calculated according to Johnson et al. (1971):

$$F_{\mathrm{s}} \approx \pi\gamma R_{\mathrm{eff}}. \tag{A.2}$$

In the current example, the difference of the gravitative forces equals $\Delta F \approx 3.84 \times 10^{-11}$ N while, assuming an adhesion energy of $\gamma = 0.1$ J/m^2, the attractive force between the grains takes the value $F_{\mathrm{s}} \approx 6.28 \times 10^{-3}$ N which is larger than the tidal force ΔF by several orders of magnitude. This is a striking example that surface forces may significantly affect structure formation in a planetary system by causing adhesion of protoplanetary grains or by holding small bodies together against tidal forces.

From Eqs. (A.1) and (A.2) and the fact that for identical spheres $R_{\mathrm{eff}} = r/2$, taking into account their mutual gravitational attraction over a distance of $2r$

$$F_{\mathrm{g}} = \frac{Gm^2}{4r^2}, \tag{A.3}$$

using the relation $m = \frac{4}{3}\pi\rho r^3$ and setting $F_{\mathrm{g}} + F_{\mathrm{s}} - \Delta F = 0$, one obtains the condition

$$\frac{4}{9}\pi^2 G\rho^2 r^4 + \pi\gamma\frac{r}{2} - \frac{16}{3}\pi GM\rho\,\frac{r^4}{d^3} = 0 \tag{A.4}$$

from which one can derive for particles with surface attraction, orbiting at a given distance from the centre of the planet, the approximate maximum size $r_{\max}$ below which they will not be pulled apart by the gravitation of the central body

$$r_{\max} = \left(\frac{3\gamma}{8G\rho(4(M/d^3) - (\pi/3)\rho)}\right)^{1/3} \tag{A.5}$$

and the minimum distance $d_{\min}$ for particles with radius r to remain in contact

$$d_{\min} = \left(\frac{\frac{32}{3}GM\rho r^3}{\gamma + \frac{8}{9}\pi G\rho^2 r^3}\right)^{1/3} = \left(\frac{\pi}{12}\,\frac{\rho}{M} + \frac{3}{32}\,\frac{\gamma}{GM\rho r^3}\right)^{-1/3}. \tag{A.6}$$

The classical radius of the Roche zone without any additional force between the particles is determined by the condition $F_{\mathrm{g}} - \Delta F = 0$ and takes the value

$$d_{\mathrm{Roche}} = \left(\frac{12}{\pi}\,\frac{M}{\rho}\right)^{1/3} \tag{A.7}$$

and is independent on the particle radius. Obviously, it is larger than $d_{\min}$ for any positive value of γ. In the current example, $d_{\mathrm{Roche}} \approx 1.341 \times 10^8$ m, and $d_{\min} \approx 5.43 \times 10^4$ m which would place the "tidal breakup" of neighbouring spheres with 2 cm radius inside the planet if it were of Saturn's diameter. On the other hand, $r_{\max} \approx 37.6$ m which is not quite in the order of magnitude of the known small satellites of the giant planets, but the present estimate is only based on the assumption of surface adhesion, not on the formation of a coherent solid which may require thermal modification and to which adhesive aggregation can only be a first step. The Roche limit of solid bodies with higher tensile strength has been investigated by Jeffreys (1947), who discussed approaches of asteroids and icy bodies to the giant planets, Öpik (1966), who revised the results of the former and examined the fate of comets in the inner solar system, and Aggarwal and Oberbeck (1974), who considered different fracture modes in detail. A refined analysis of the forces in and on particles in orbit, including the results of the work just mentioned, effects of shear, and also using better values for the material constants, can be useful for an approximate prediction of possible sizes of satellites formed by aggregation of grains consisting of a given material.

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