Additional Topics I: Member Releases

Lesson Objectives:

1) Derive the methodology to analyze structures for various member releases and secondary effects.
2) Derive the member local stiffness modifications to account for member releases.
3) Compute the structure fixed-joint force vector to account for support settlements.
4) Compute the member fixed-end force vector to account for temperature changes and fabrication errors.

Review of Previous Class Notes:

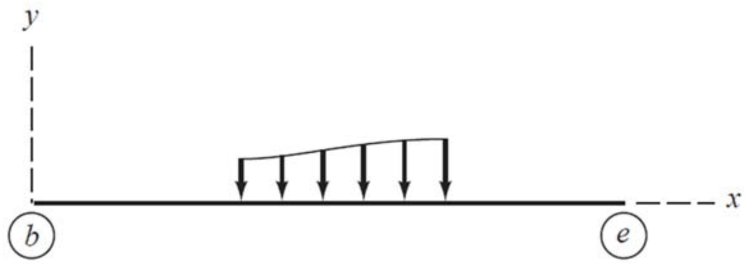
1) Previously the focus was on trusses, beams, and frames based on the assumptions that:
   a. The member ends are ________________________________ to nodes at both ends.
   b. Supports and restraints are sufficiently _________________ to prevent any _____ ___________________________________.
   c. Loads are ____.
      i. Examples of _____________________ loads include: __________________ and ____.
2) These additional considered topics build on the previous matrix based analysis already established (chapters refer to Kassimali textbook).
   a. Trusses: _Chapter 3_________________________.
   b. Beams: _Chapter 5___________________________.
   c. Frames: _Chapter 6__________________________.

Background Reading:

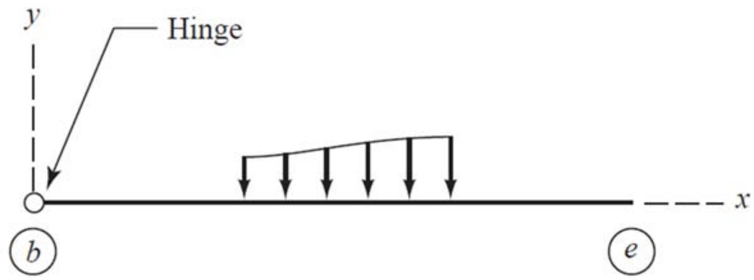
1) Read __Kassimali – Chapter 7 (focus on 7.1 for member releases)____________________

Member Releases:

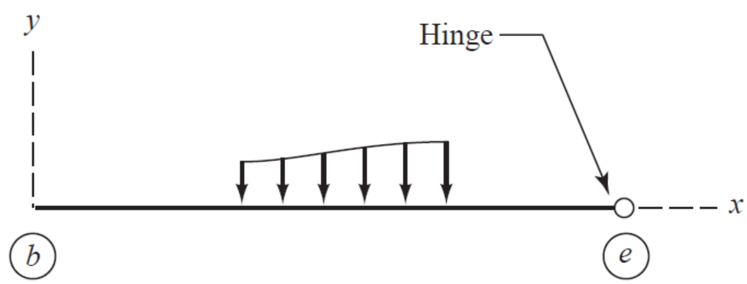
1) Member releases can be conveniently incorporated into the stiffness method by modifying the ____ to account for such releases.
2) Focus here is on __________________ releases, which may occur at one or both ends of a member.
    a. This release is the most commonly encountered type in practice.
    b. This release creates a ______________________________ at the location of release.
3) Figure 1 depicts the types of member releases that will be considered.
    a. Classified into _________ conditions.
    b. Nomenclature used in these notes aligns with the book definitions.
    c. MT denotes *member type*.
4) In previous notes and chapters, focus was placed on ______________.
    a. That is ____ releases are present.
    b. _______ and _________ matrices are valid.
5) How do one handle __________________________ releases?
    a. Modify the __________ and ___________ matrices as appropriate.
    b. Details are presented in the next section.



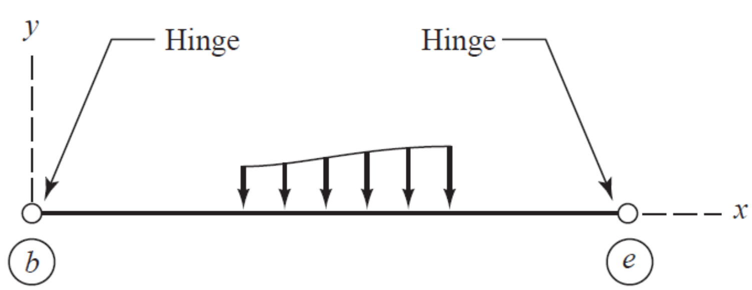
(a) Member with ____ hinges, denoted as _________.



(b) Member with ____ hinge, denoted as _________.



(c) Member with ____ hinge, denoted as _________.



(d) Member with ____ hinges, denoted as _________.

Figure 1. Types of member releases considered and definition of the nomenclature[1].

[1] All figures in Additional Topics I modified from: Kassimali, Aslam. (2012). *Matrix Analysis of Structures*. 2nd edition. Cengage Learning.

Local Stiffness Relations for Plane Frame Members with Hinge(s):

1) Let's begin to derive the modification of the local stiffness relationship matrices for plane frames.
2) Knowing the relationship of the local end forces, local stiffness matrix, and member fixed-end forces, one can write:

$$\{Q\} = [k]\{u\} + \{Q_f\}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} + \begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Q_{f3} \\ Q_{f4} \\ Q_{f5} \\ Q_{f6} \end{bmatrix}$$

3) Expanding this to a set of equations:

$$Q_1 = \frac{EA}{L}\,(u_1 - u_4) + FA_b$$

$$Q_2 = \frac{EI}{L^3}\,(12\,u_2 + 6\,L\,u_3 - 12\,u_5 + 6\,L\,u_6) + FS_b$$

$$Q_3 = \frac{EI}{L^3}\,(6\,L\,u_2 + 4\,L^2\,u_3 - 6\,L\,u_5 + 2\,L^2\,u_6) + FM_b$$

$$Q_4 = \frac{EA}{L}\,(-u_1 + u_4) + FA_e$$

$$Q_5 = \frac{EI}{L^3}\,(-12\,u_2 - 6\,L\,u_3 + 12\,u_5 - 6\,L\,u_6) + FS_e$$

$$Q_6 = \frac{EI}{L^3}\,(6\,L\,u_2 + 2\,L^2\,u_3 - 6\,L\,u_5 + 4\,L^2\,u_6) + FM_e$$

4) *Member Type 1*: End ____ of the member is connected to the adjacent joint by a hinged connection.

   a. Therefore ____.

   b. Solving for ____________, one can obtain an expression for the end rotation as:

   c. This indicates that _____ (end ____________) is no longer a __________________ ____.

   d. To eliminate ____ from the member stiffness relations, one can write the updated member stiffness equations as:

$$Q_1 = \frac{EA}{L}(u_1 - u_4) + FA_b$$

$$Q_2 = \frac{EI}{L^3}(3\,u_2 - 3\,u_5 + 3\,L\,u_6) + [FS_b - \frac{3}{2L}FM_b]$$

$$Q_3 = 0$$

$$Q_4 = \frac{EA}{L}(-u_1 + u_4) + FA_e$$

$$Q_5 = \frac{EI}{L^3}(-3\,u_2 + 3\,u_5 - 3\,L\,u_6) + [FS_e + \frac{3}{2L}FM_b]$$

$$Q_6 = \frac{EI}{L^3}(3\,L\,u_2 - 3\,L\,u_5 + 3\,L^2\,u_6) + [FM_e - \frac{1}{2}FM_b]$$

   e. Where this can be assembled into matrix form as:

$$\{Q\} = [k]\{u\} + \{Q_f\}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & 0 & 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & 0 & 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} \\ 0 & \frac{3EI}{L^2} & 0 & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} + \begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \\ Q_{f5} \\ Q_{f6} \end{bmatrix}$$

$$\begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \\ Q_{f5} \\ Q_{f6} \end{bmatrix} = \begin{bmatrix} FA_b \\ FS_b - \frac{3}{2L} FM_b \\ 0 \\ FA_e \\ FS_e + \frac{3}{2L} FM_b \\ FM_e - \frac{1}{2} FM_b \end{bmatrix}$$

f. The above equations are now valid for a plane frame member under the condition of ___________________.

5) *Member Type 2*: End ____ of the member is connected to the adjacent joint by a hinged connection.

a. Therefore ____.

b. Solving for _____________, one can obtain an expression for the end rotation as:

c. This indicates that _____ (end _____________) is no longer a ___________________ ____.

d. To eliminate ____ from the member stiffness relations, one can write the updated member stiffness equations as:

$$Q_1 = \frac{EA}{L}\,(u_1 - u_4) + FA_b$$

$$Q_2 = \frac{EI}{L^3}\,(3 + 3\,L\,u_3 - 3\,u_5) + [FS_b - \frac{3}{2L} FM_e\}$$

$$Q_3 = \frac{EI}{L^3}\,(3\,L\,u_2 + 3\,L^2\,u_3 - 3\,L\,u_5) + [FM_b - \frac{1}{2} FM_e]$$

$$Q_4 = \frac{EA}{L}\,(-u_1 + u_4) + FA_e$$

$$Q_5 = \frac{EI}{L^3}\,(-3\,u_2 - 3\,L\,u_3 + 3\,u_5) + [FS_e + \frac{3}{2L}FM_e]$$

$$Q_6 = 0$$

e. Where this can be assembled into matrix form as:

$$\{Q\} = [k]\{u\} + \{Q_f\}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 \\ 0 & \frac{3EI}{L^2} & \frac{3EI}{L} & 0 & -\frac{3EI}{L^2} & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} + \begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Q_{f3} \\ Q_{f4} \\ Q_{f5} \\ Q_{f6} \end{bmatrix}$$

$$\begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \\ Q_{f5} \\ Q_{f6} \end{bmatrix} = \begin{bmatrix} FA_b \\ FS_b - \frac{3}{2L}FM_e \\ FM_b - \frac{1}{2}FM_e \\ FA_e \\ FS_e + \frac{3}{2L}FM_e \\ 0 \end{bmatrix}$$

f. The above equations are now valid for a plane frame member under the condition of ____________________.

6) *Member Type 3*: Ends _______ of the member are connected to the adjacent joint by a hinged connection.

   a. Therefore ____.

   b. Solving for ____________________, one can obtain an expression for the end rotations as:

   c. This indicates that ______________ (ends __________________) are no longer a ____.

   d. To eliminate _____________ from the member stiffness relations, one can write the updated member stiffness equations as:

$$Q_1 = \frac{EA}{L}(u_1 - u_4) + FA_b$$

$$Q_2 = FS_b - \frac{1}{L}[FM_b + FM_e]$$

$$Q_3 = 0$$

$$Q_4 = \frac{EA}{L}(-u_1 + u_4) + FA_e$$

$$Q_5 = FS_e + \frac{1}{L}[FM_b + FM_e]$$

$$Q_6 = 0$$

e. Where this can be assembled into matrix form as:

$$\{Q\} = [k]\{u\} + \{Q_f\}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} + \begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \\ Q_{f5} \\ Qf_6 \end{bmatrix}$$

$$\begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \\ Q_{f5} \\ Q_{f6} \end{bmatrix} = \begin{bmatrix} FA_b \\ FS_b - \frac{1}{L}(FM_b + FM_e) \\ 0 \\ FA_e \\ FS_e + \frac{1}{L}(FM_b + FM_e) \\ 0 \end{bmatrix}$$

f. The above equations are now valid for a plane frame member under the condition of ____________________.

7) It should be noted that while the ____________ has been reduced due to member releases, the ______________________________ of the matrices remain consistent in size.

a. One can use the concept of __________________________ to reduce the size of the matrices due to the eliminated degrees of freedom.

Local Stiffness Relations for Beam Members with Hinge(s):

1) Recall from the beam notes, that beams:
   a. ____.
   b. ____.
2) Therefore a beam has ______ degrees of freedom.
3) To derive the updated stiffness relations for a beam, it is simplest to delete _____________ ________________ from the plane frame members corresponding to each end condition.

4) *Member Type 1*: End ____ of the member is connected to the adjacent joint by a hinged connection.
   a. Therefore ________________________________.
   b. To obtain the updated stiffness for a beam, delete rows and columns ___________ from the plane frame member.

$$\{Q\} = [k]\{u\} + \{Q_f\}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} \frac{3EI}{L^3} & 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} \\ 0 & 0 & 0 & 0 \\ -\frac{3EI}{L^3} & 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} \\ \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \end{bmatrix}$$

$$\begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \end{bmatrix} = \begin{bmatrix} FS_b - \frac{3}{2L}FM_b \\ 0 \\ FS_e + \frac{3}{2L}FM_b \\ FM_e - \frac{1}{2}FM_b \end{bmatrix}$$

c. The rotation of ______ (hinged end of member) can be expressed as:

5) *Member Type 2*: End ____ of the member is connected to the adjacent joint by a hinged connection.

a. Therefore _______________________________.

b. To obtain the updated stiffness for a beam, delete rows and columns ___________ from the plane frame member.

$$\{Q\} = [k]\{u\} + \{Q_f\}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} \frac{3EI}{L^3} & \frac{3EI}{L^2} & -\frac{3EI}{L^3} & 0 \\ \frac{3EI}{L^2} & \frac{3EI}{L} & -\frac{3EI}{L^2} & 0 \\ -\frac{3EI}{L^3} & -\frac{3EI}{L^2} & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \end{bmatrix}$$

$$\begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \end{bmatrix} = \begin{bmatrix} FS_b - \frac{3}{2L} FM_e \\ FM_b - \frac{1}{2} FM_e \\ FS_e + \frac{3}{2L} FM_e \\ 0 \end{bmatrix}$$

c. The rotation of ______ (hinged end of member) can be expressed as:

6) *Member Type 3*: Ends _________ of the member are connected to the adjacent joints by a hinged connection.

   a. Therefore ______________________________.

   b. To obtain the updated stiffness for a beam, delete rows and columns ___________ from the plane frame member.

   c. Through inspection of these row and column deletions, it is noted that: _________.

      i. This indicates that no _________________________ against small end displacements in the direction perpendicular to its centroidal axis.

         1. This behaves like a ______________________________.

   d. The member end force vector can be expressed as:

$$\begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Q_{f3} \\ Q_{f4} \end{bmatrix} = \begin{bmatrix} FS_b - \frac{1}{L}(FM_b + FM_e) \\ 0 \\ FS_e + \frac{1}{L}(FM_b + FM_e) \\ 0 \end{bmatrix}$$

   e. The rotation of _________ (hinged ends of member) can be expressed as:

Procedure for Analysis:

1) The general analysis procedure for beam and frames as previously outlined can be applied.
    a. Account for modified local stiffness matrices, as appropriate.
    b. The previous relationships are still valid:

2) Examine Figure 2 for examples of hinges introduced into a plane frame structure.
3) How to account for hinged joints in beams and frames?
    a. Note a hinged joint is defined as: all framing members of the joint are connected by hinged connections.
    b. In Figure 2, __________ is a hinged joint.
    c. Why is joint _________ not considered a hinged joint? *Only the first end of the beam is hinged. The column ends that frame the joint can transfer a moment.*

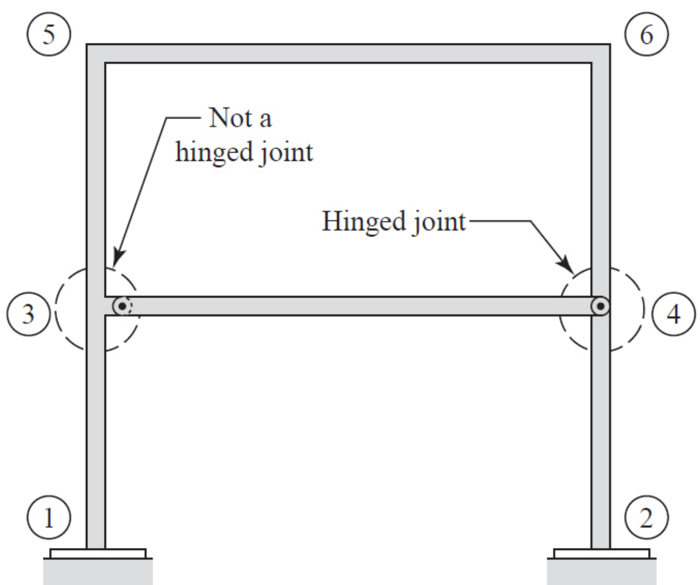


Figure 2. Example frame structure with hinges.

4) Inclusion of the rotation degree of freedom in the analysis for a hinged joint will result in a ____________________________.

a. That is the ____.

5) What are two possible discretizations of the example frame structure in Figure 2?

6) Discretization Type One:

a. Figure 3 introduces the concept of a _______________________________ __________________________________.

i. Herein the ______________at joint ____ is eliminated as an imaginary clamp is applied at this joint.

ii. Therefore this joint has ____ DOFs, namely: _______ and _________.

iii. This method is straightforward and efficient.

1. Easy to implement in computer structural analysis codes.

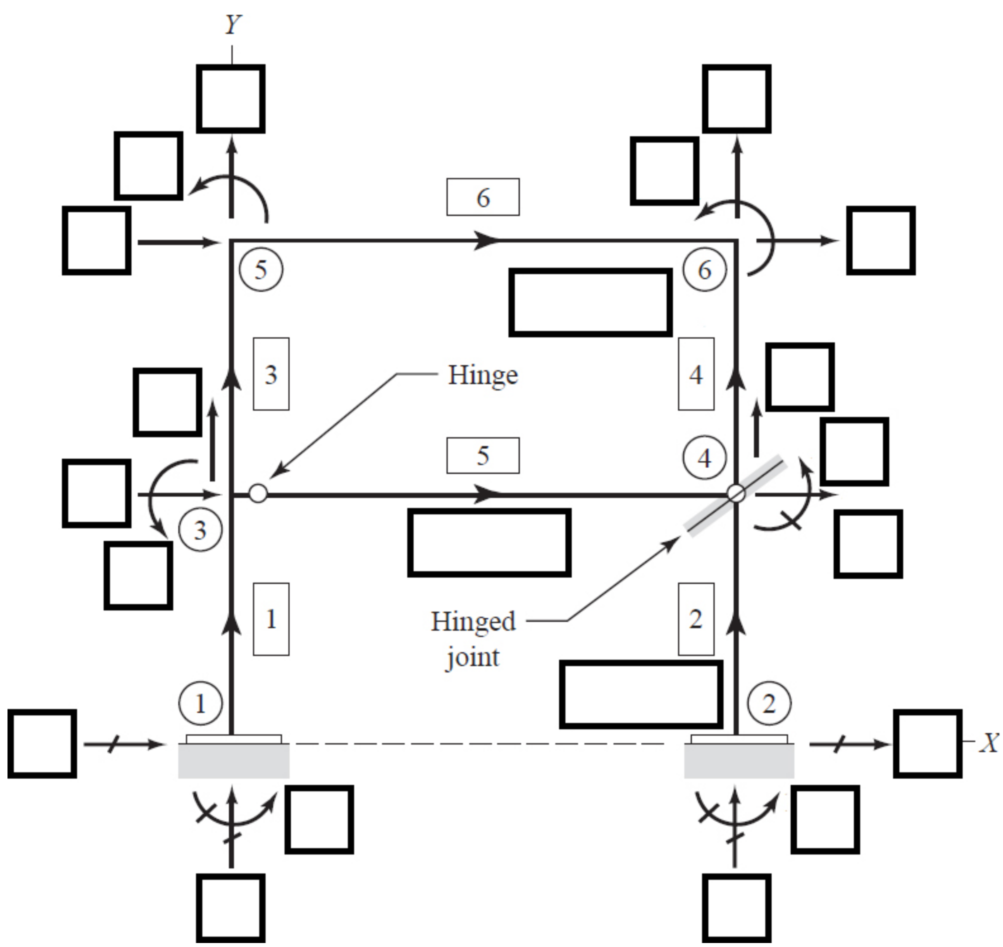


Figure 3. Example discretization of the frame structure with hinge. Note ____ DOFs are present.

7) Discretization Type Two:
   a. Figure 4 introduces the concept of releasing all the hinges except one.
      i. Herein the beam frames into the joint where the rotation is free.
         1. One and only one framing member can have a restrained rotation.
      ii. This approach is based on the following concepts:
         1. No external moment is applied to the hinged node.
         2. The moment of the restrained member end is 0 (along with all other end moments framing into the member).
            a. Ensures moment equilibrium is satisfied.
      iii. This method is not as efficient as demonstrated in Figure 3a, why?

      ____.

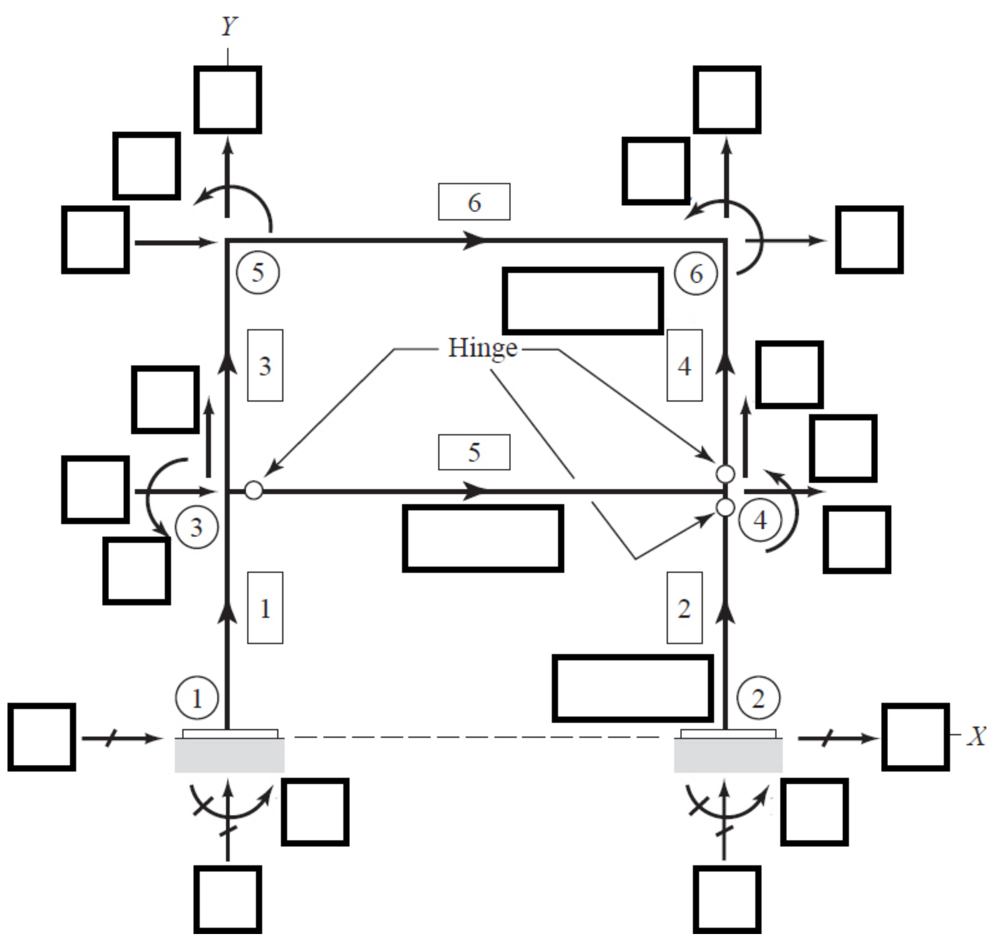


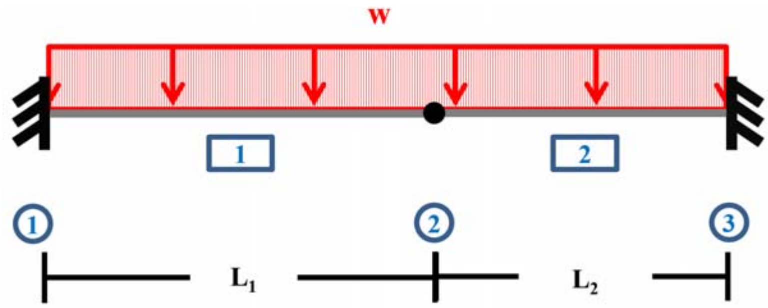
Figure 4. Example discretization of the frame structure with hinge. Note ____ DOFs are present.

8) In summary, two different discretization approaches are possible for frame where hinges are present.
    a. Preference may be given to the first approach, since:
        i. ____.
        ii. ____

Member Releases: Example

Example #1

For the steel beam illustrated below, determine the joint displacements, member end forces, and the support reactions. Use the matrix stiffness method.



Additional Information:
Nodes 1 and 3 are fixed
Node 2 is hinged
$w = 1.5$ kip/ft
$L_1 = 20$ feet
$L_2 = 15$ feet
$E = 29{,}000$ ksi
$I = 350$ in^4

Solution:

Identify the number of degrees of freedom. NDOF = ________.

Identify the number of reactions. NR = ________.

E = ______________.

I = ______________.

Examine member 1.

MT = ____________

L = 240 in

Code numbers: ___ ___ ___ ___

$$FS_b = FS_e =$$

$$FM_b = -FM_e =$$

Local stiffness matrix and end force vector:

$$[k_1] = \begin{bmatrix} \frac{3EI}{L^3} & \frac{3EI}{L^2} & -\frac{3EI}{L^3} & 0 \\ \frac{3EI}{L^2} & \frac{3EI}{L} & -\frac{3EI}{L^2} & 0 \\ -\frac{3EI}{L^3} & -\frac{3EI}{L^2} & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \end{bmatrix} =$$

Examine member 2.

MT = ____________

L = 180 in

Code numbers: ___ ___ ___ ___

$FS_b = FS_e =$

$FM_b = -FM_e =$

Local stiffness matrix and end force vector:

$$[k_2] = \begin{bmatrix} \frac{3EI}{L^3} & 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} \\ 0 & 0 & 0 & 0 \\ -\frac{3EI}{L^3} & 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} \\ \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix} =$$

$$\begin{bmatrix} Q_{f1} \\ Q_{f2} \\ Qf_3 \\ Q_{f4} \end{bmatrix} =$$

Assembling the Structure Stiffness Matrix.

[S] =

Assembling the Structure Fixed-Joint Force and Joint Load Vectors.

$\{P_f\} =$

$\{P\} =$

Find the Joint Displacements.

Compute Member End Displacements, Forces, and Reactions.