Copulas, models of dependence

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Introduction

Questions :

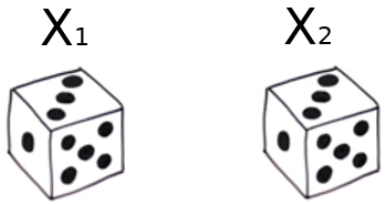
- What is the type of interrelation between variables ?
- How much information about variables can be gained from the observation of one ?

Copulas are tools for modeling dependence between random variables.

Dependence

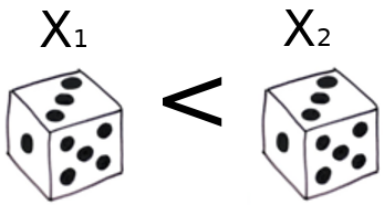
Dependence : how the occurrence of one variable affects the probability of occurrence of the others.

- **Independent** : X_1 gives no information about X_2



- **Totally dependent** : X_1 have full information of X_2

- **Dependent** : X_1 gives information about X_2 (but the relation can be difficult to describe)



Marginal distribution

- Cumulative distribution function (CDF) : marginal distribution
  $F_X(x) = P(X \leq x)$

- The CDFs give us no information about the **joint behaviour**

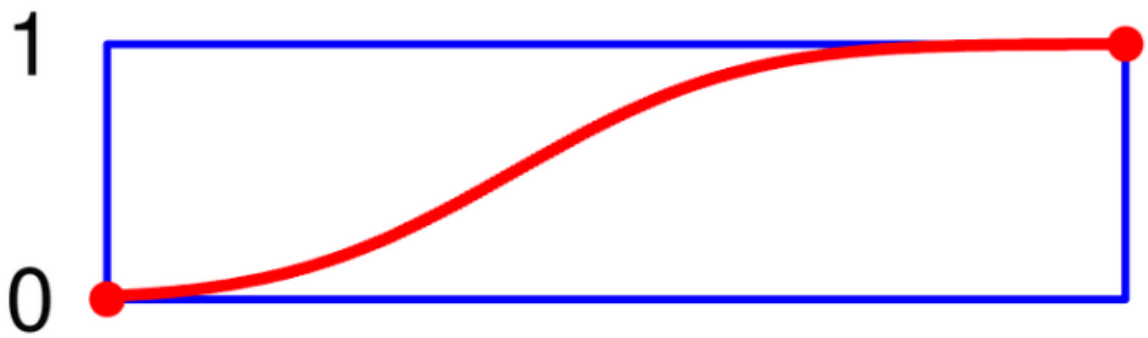


Figure – Example of CDF of continuous variable

Copula

A d-dimensional **copula** is a function :

$$C : [0,1]^d :\rightarrow [0,1]$$

With those properties :

$$C(1, ..., 1, u_i, 1, ..., 1) = u_i$$

$$C(u_1, ..., u_{i-1}, 0, u_{i+1}, ..., u_d) = 0$$

Sklar theorem says that every distribution function on $\mathbb{R}^d$ inherently embodies a copula function. On the other side, if we choose a copula and some marginal distributions and entangle them in the right way, we will end up with a proper multivariate distribution function.

Training copulas with data

1. Estimate the marginal CDFs $P(x_1)$ and $P(x_2)$ as the **marginal ECDFs** (empirical) $P_n(x_1)$ and $P_n(x_2)$.
2. Obtain the **copula pseudo-sample** using empirical copula transformation : transform each X to follow an uniform distribution.
3. Choose a parametric copula function C_θ (e.g. gaussian copula) and **estimate its parameters** θ (optimization)

References

1. Little introduction, “Copulas made easy” : https://dahtah.wordpress.com/2011/10/28/hello-world/
2. A good course, “Coping with copulas” : http://archiv.stochastik.uni-freiburg.de/homepages/schmidt/publications/TSchmidt_Copulas.pdf
3. Thesis, “From dependance to causation”, chapter 4.4 : https://arxiv.org/pdf/1607.03300.pdf