Reinforcement Learning

5. Function approximation

Freek Stulp and Michèle Sebag

Université Paris-Saclay

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Credit: Richard Sutton's slides (NIPS 2015)

Damien Ernst slides (Busuniu et al., 2010)

Overview

Position of the problem

Notations

- State space $\mathcal{S}$
- Action space $\mathcal{A}$
- Transition model $p(s, a, s') \mapsto [0, 1]$
- Reward $r(s)$ bounded

Build

$$V^\pi(s) = r(s) + \gamma \sum_{s'} p(s, \pi(s), s') V^\pi(s')$$

$$V^*(s) = \max_\pi V^\pi(s')$$

$$\pi^*(s) = \underset{a \in \mathcal{A}}{argmax} \left\{ \sum_{s'} p(s, a, s') V^*(s') \right\}$$

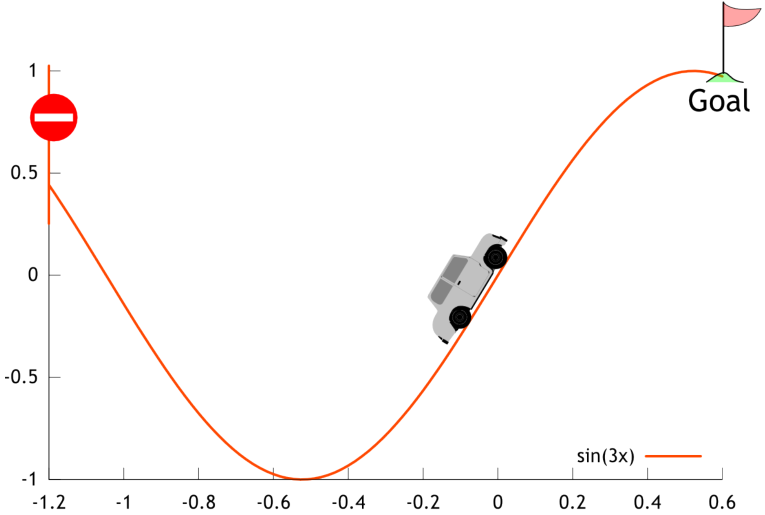
Context

- $\mathcal{S}$: finite (small or large) or infinite
- $\mathcal{A}$: finite (small or large) or infinite

Why function approximation ?

Exploration needed: in each state, try every action.
Impossible

- In continuous state space



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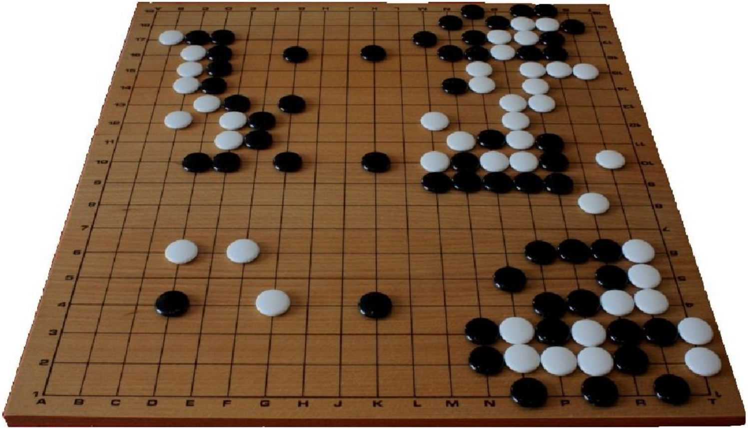

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More Playing Atari with Deep Reinforcement Learning, Mnih et al., 2015.
https://www.cs.toronto.edu/ vmnih/docs/dqn.pdf

Overview

A standard learning problem ? 1

Assume

$$\mathcal{S} \subset \mathbb{R}^d$$

Goal

$$\text{Build } V : \mathcal{S} \mapsto \mathbb{R}$$

Remind: Supervised Machine Learning

$$\mathcal{E} = \{(\mathbf{x}_i, y_i), \mathbf{x}_i \in \mathcal{X} \text{ (instance space) }, y_i \in \mathcal{Y} \text{ (label space) }, i = 1 \ldots n\}$$

- Classification: $\mathcal{Y} = \{-1, 1\}$ or $\{1, \ldots k\}$
- **Regression** $\mathcal{Y} = \mathbb{R}$

A standard learning problem ? 2

Assume we have the training set

$$\mathcal{E} = \{(s_i, V^*(s_i)), i = 1 \dots n\}$$

Then

- Find a hypothesis space $\mathcal{H}$
- Find an optimization criterion $\mathcal{L}$ (data fitting + **regularization**)
- Solve the optimization problem

$$\hat{V}^* = \underset{V \in \mathcal{H}}{arg\ opt}[\mathcal{L}(V)]$$

A standard learning problem ? NO, 1

Standard supervised ML criteria

$$\mathcal{L}(V) = \sum_{i=1}^{n} (V^*(s_i) - V(s_i))^2 + \mathcal{R}(V)$$

Minimize the average error.

But
In RL, one error is enough to lose the game... to fall down from the cliff... to kill the robot...

A standard learning problem ? NO, 2

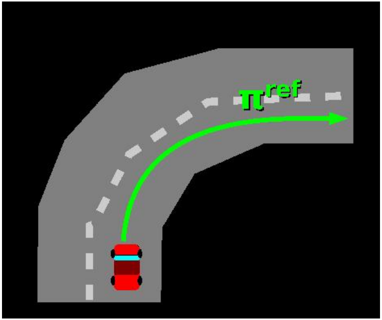
Standard supervised ML criteria

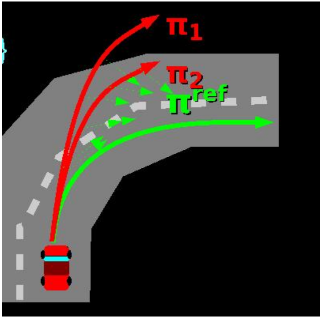
$$\mathcal{L}(V) = \sum_i (V^*(s_i) - V(s_i))^2 + \mathcal{R}(V)$$

Minimize the average error with respect to independent identically distributed s_i.

But

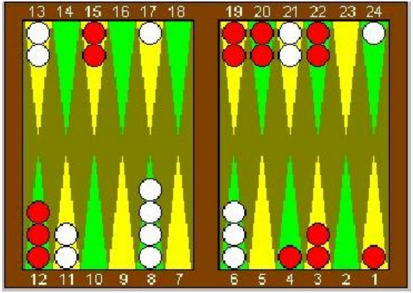
A wrong move, or the transition error can send you off the road... and then the error might be cumulative.





Overview

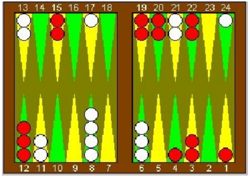
TD-Gammon, 1



The game of Backgammon Gerald Tesauro, 89-95

- State: vector of handcrafted features (e.g., number of White or Black checkers at each location) $\mathcal{S} \subset \mathbb{R}^D$
- Data: set of games
- A game: sequence of states $x_1, \ldots x_T$

TD-Gammon, 2. Where does the value come from ?



Assumptions

$$y_0 = \quad .5 \qquad \text{value of initial state}$$

$$y_T = \begin{cases} 1 & \text{if } x_T \text{ is a winning state} \\ 0 & \text{if } x_T \text{ is a losing state} \end{cases}$$

And for other states ?

Value is supposed to be continuous

TD-Gammon, 3. Learning the value

Search space $\mathcal{H}$ Neural Nets W, weight vector in $\mathbb{R}^d$

Learning criterion

$$\text{Minimize } \left(V(x_T) - y_T\right)^2 + \sum_\ell \left(V(x_\ell) - V(x_{\ell+1})\right)^2$$

Learning procedure: weight update

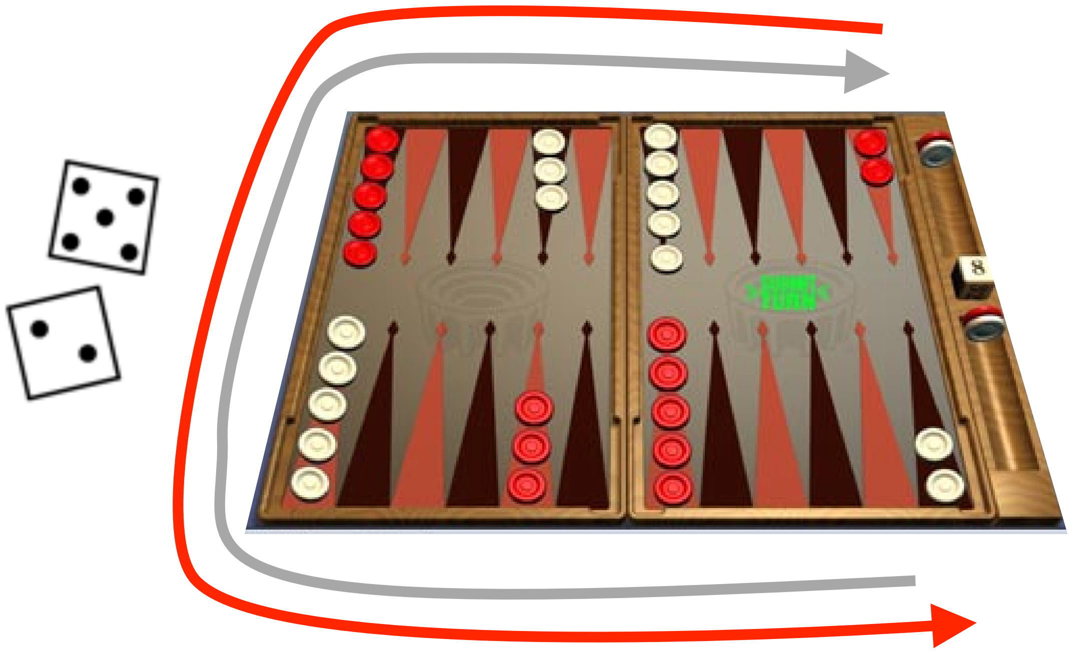
$$\Delta w = \alpha \left(V(x_{\ell+1}) - V(x_\ell)\right) \sum_{k=1}^{\ell} \lambda^{\ell-k} \nabla_w V(x_k)$$

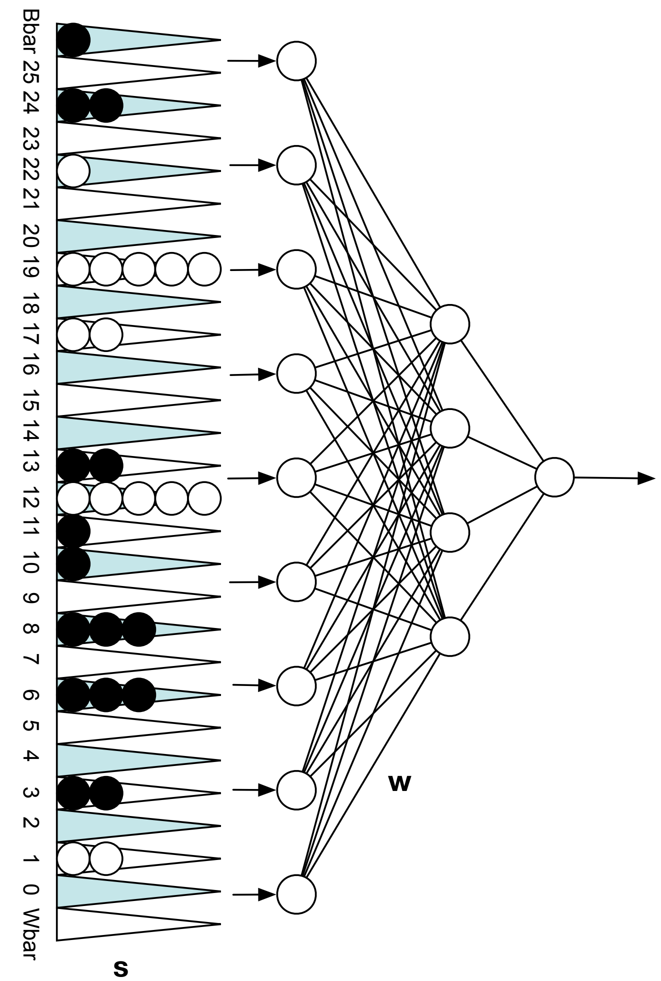
Learning by Self-play: Iteratively, 200,000 games

- Play using V_i as value function
- Use games to retrain weight vector W_i
- Increment i

Example: TD-Gammon

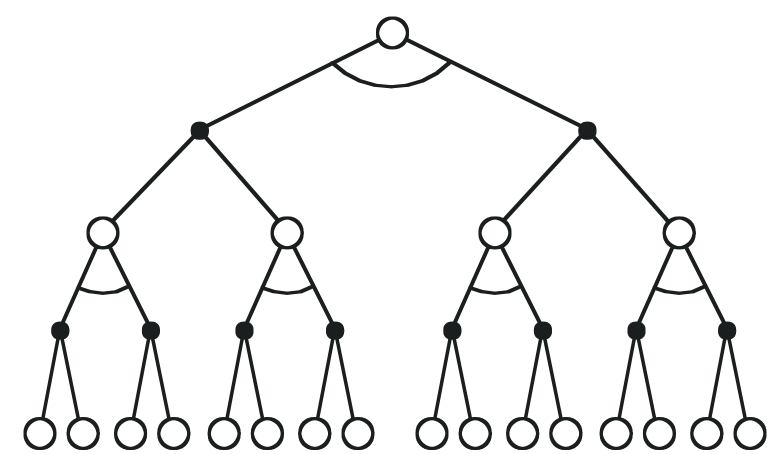
Tesauro, 1992-1995





estimated state value
(≈ prob of winning)

Action selection
by a shallow search



Start with a random Network

Play millions of games against itself

Learn a value function from this simulated experience

Six weeks later it's the best player of backgammon in the world

Originally used expert handcrafted features, later repeated with raw board positions

Overview

General priorities

1. Finding good data
2. Finding good representation
3. Finding good algorithm

Beware

- Big Data Motto (Data beat algorithms)...
- ... does not hold in RL

Finding a representation

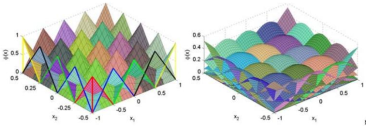
Using basis functions

$$\phi_1 \ldots \phi_K : \mathcal{S} \mapsto \mathbf{R}$$

- Usually ϕ are normalized,

$$\sum_{i=1}^{K} \phi(s) = 1$$

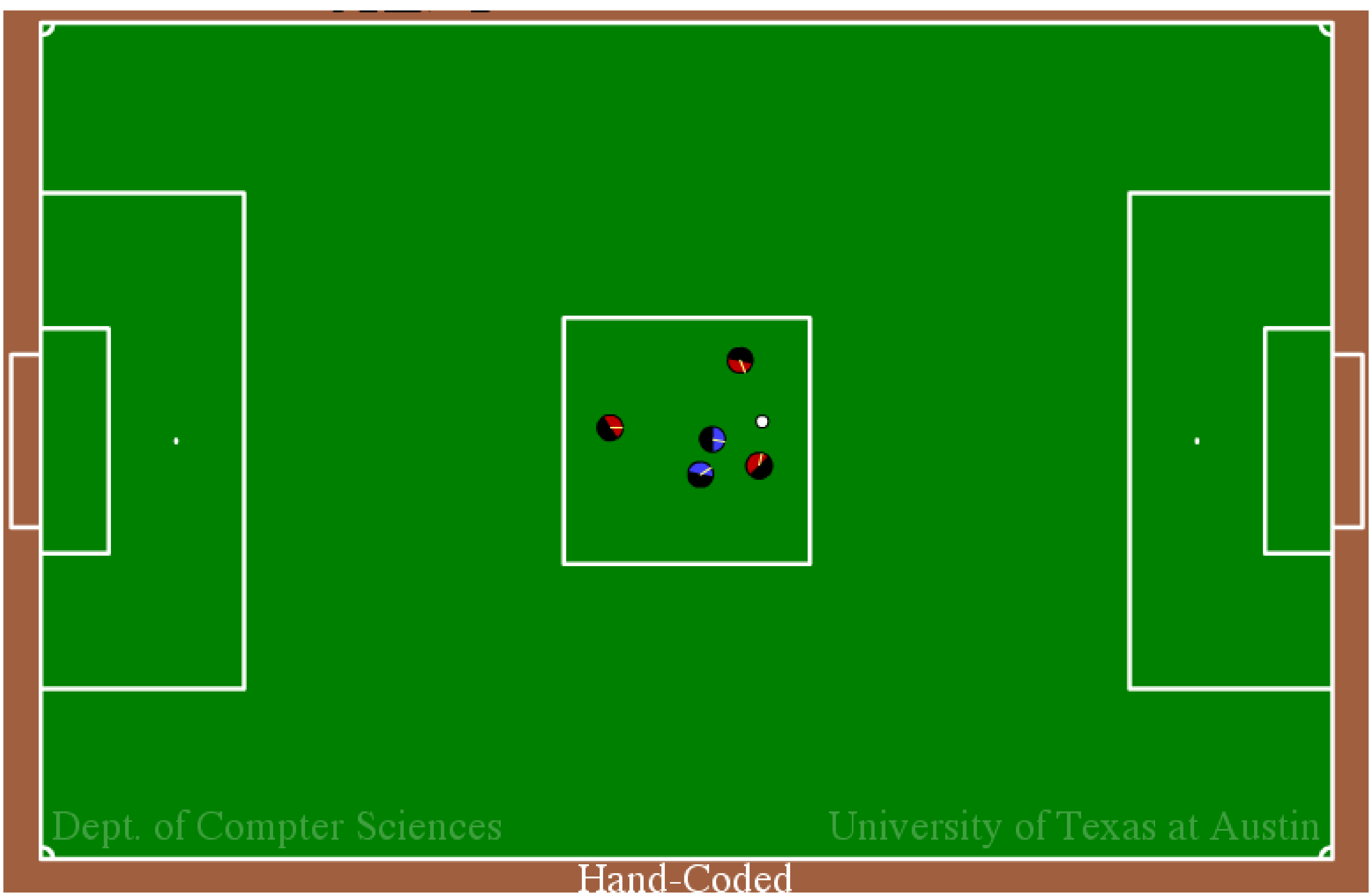
Fuzzy memberships Radius-basis functions

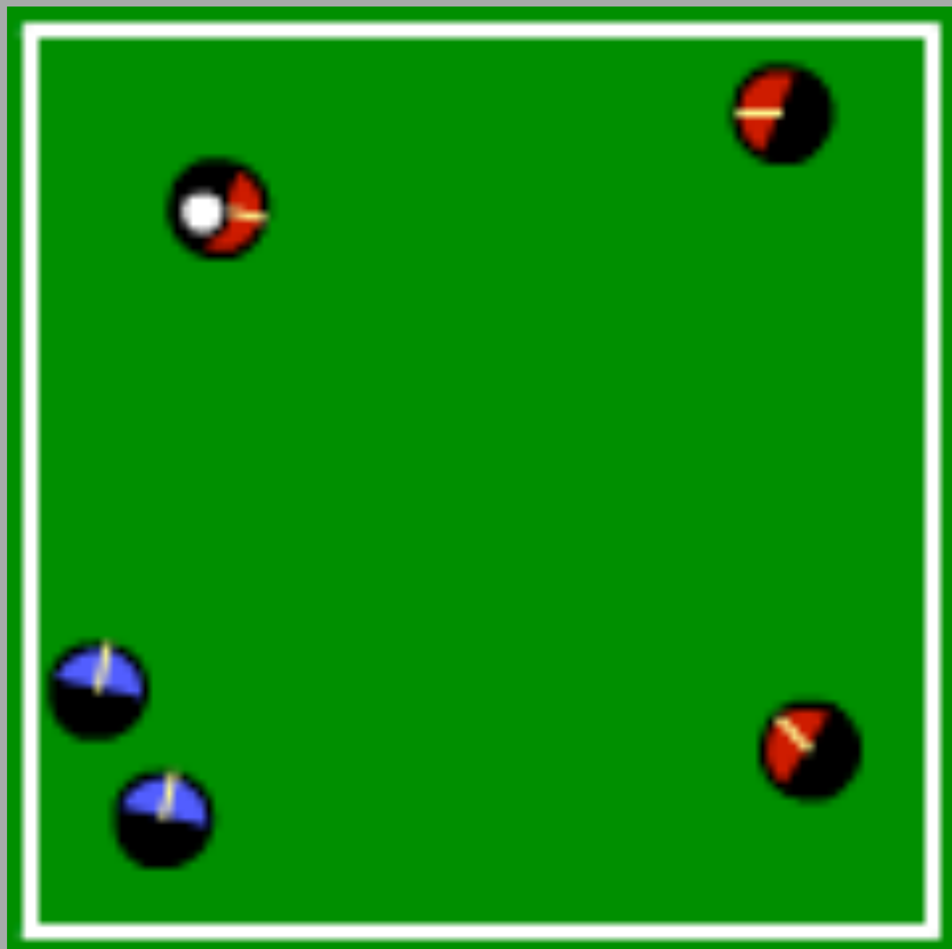


And then, back to Dynamic Programming.

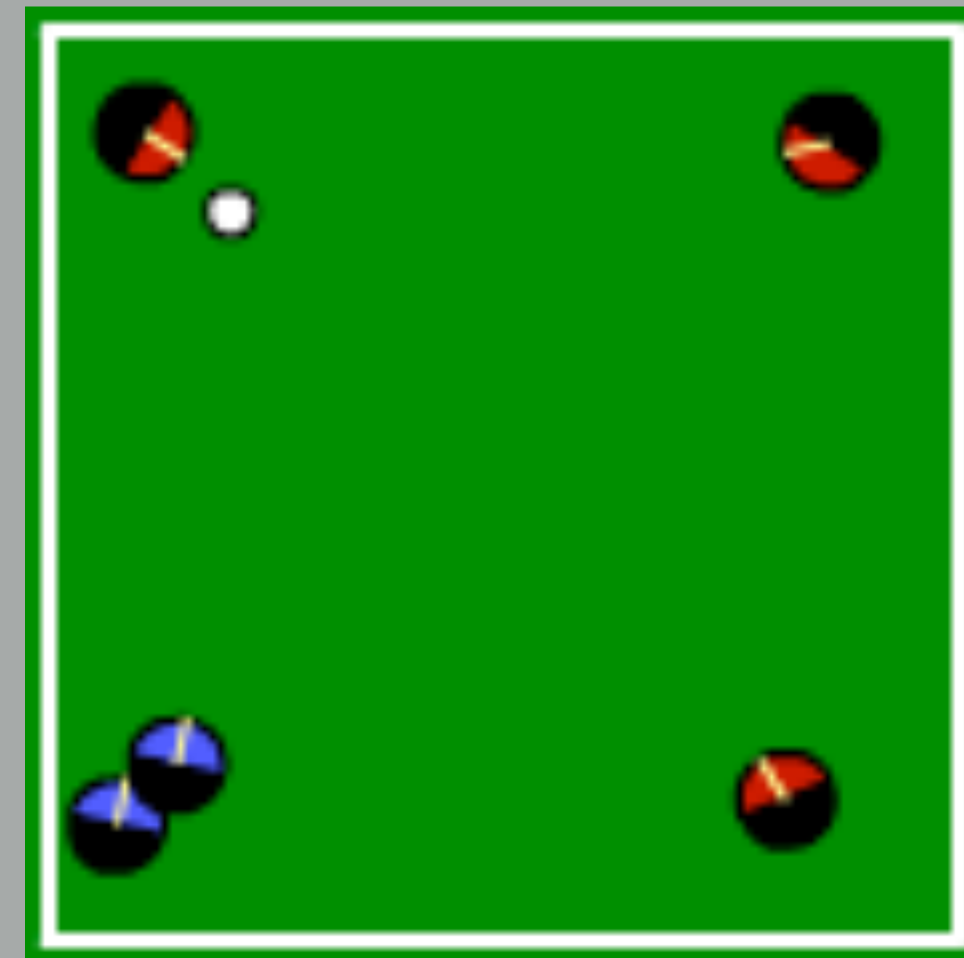
RoboCup soccer keepaway

Stone, Sutton & Kuhlmann, 2005



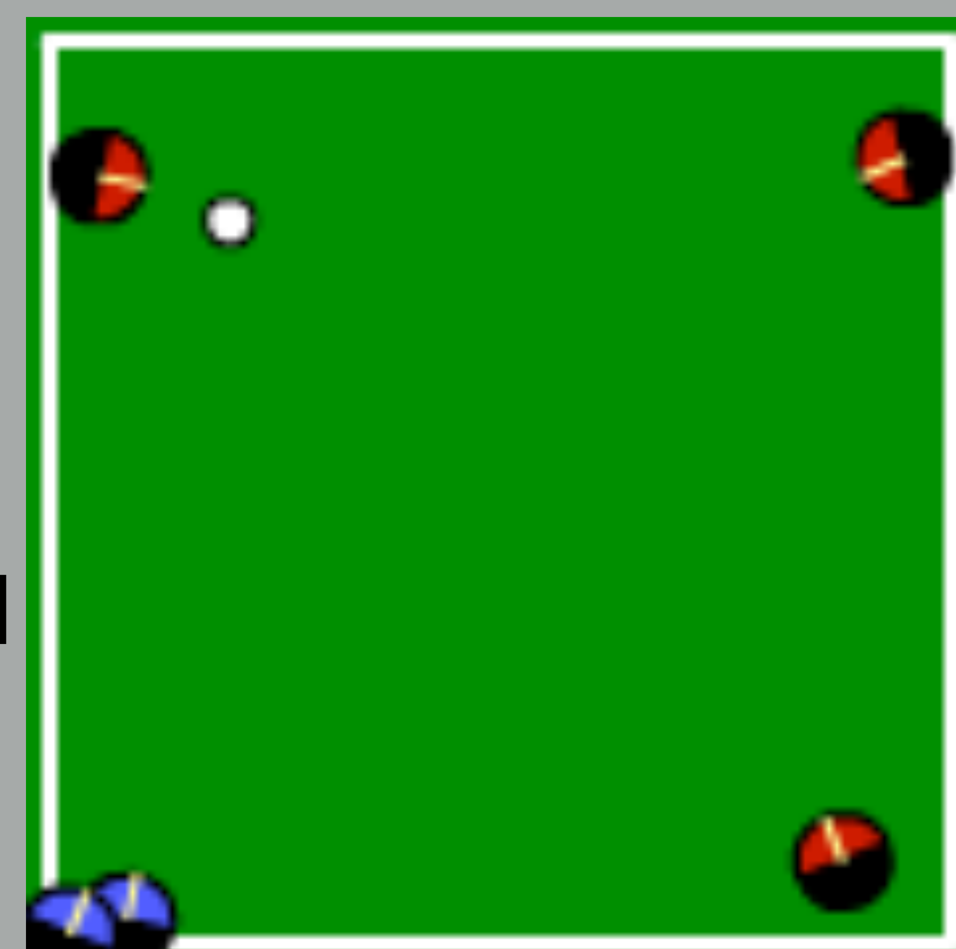


Random



Learned

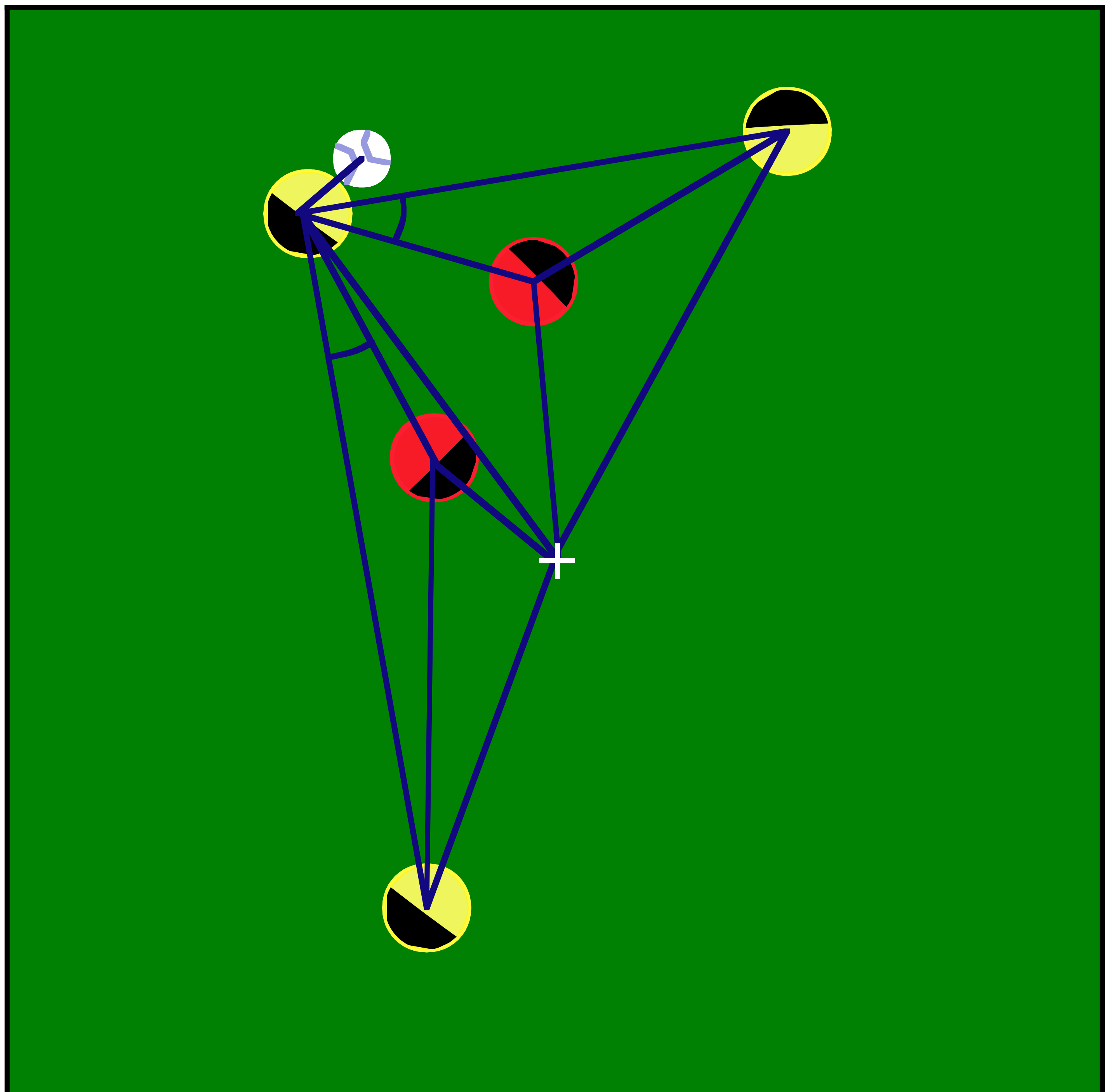
Hand-coded



Hold

Stone, Sutton & Kuhlmann, 2005

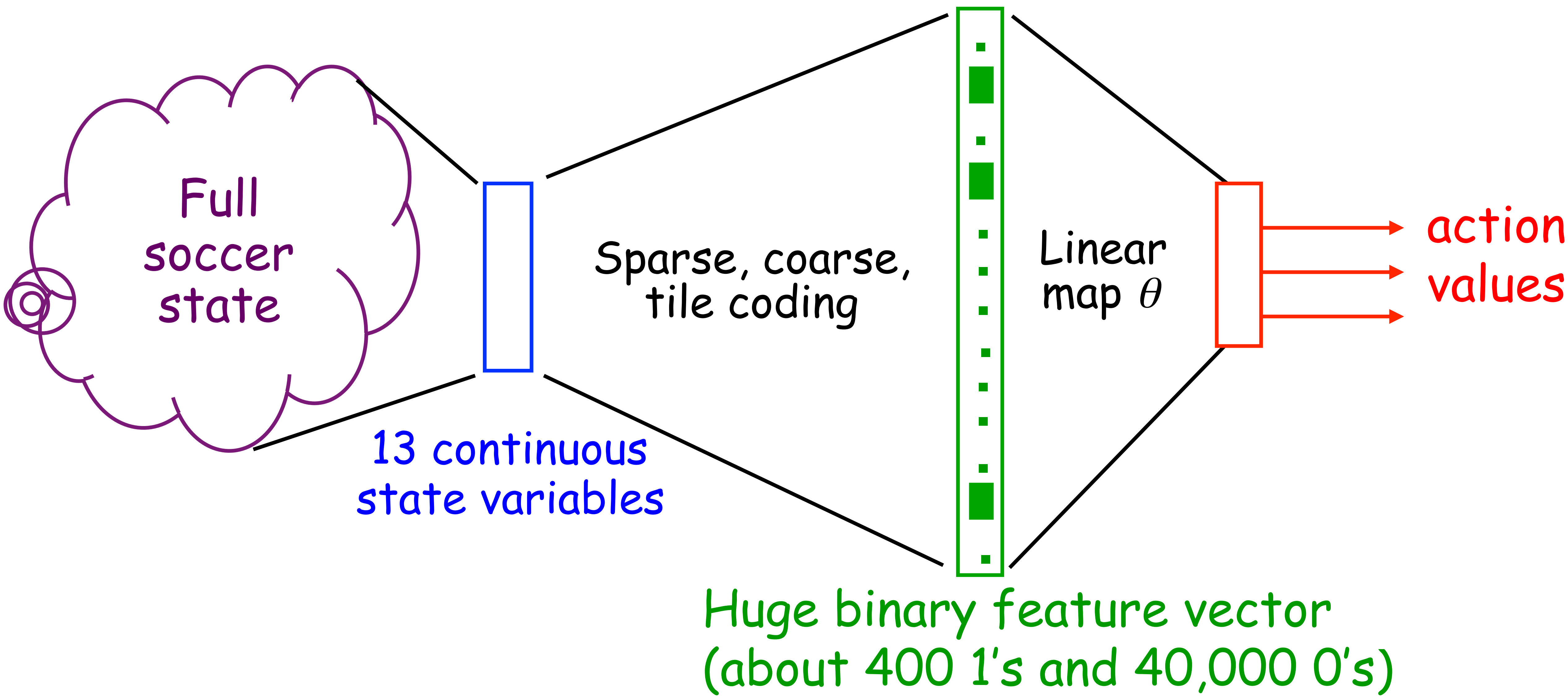
How is the state encoded?
In 13 continuous state variables



11 distances among the players, ball, and the center of the field

2 angles to takers along passing lanes

The Feature-Construction Pipeline



Overview

1. Position of the problem
2. Principle
   - Learning the value
   - A first example: TD-gammon
3. Representation of the state space
4. Approximating the Q-value
   - Gradient-based approaches
   - Decision tree-based approaches
5. Summary

Parametric action-value function

Find

$$v(s, \theta) \approx V^*(s)$$

$$q(s, a, \theta) \approx Q^*(s, a)$$

Search spaces

- Linear approximation: (many) handcrafted features, and then find linear weights
- NN approximation **Deep Reinforcement Learning**

What matters

- **Linear** Learning complexity required to scale up to large problems
- **Self-play** to acquire examples in critical regions
- Online learning; dealing with **non-stationary** target value function

Mean-square error, 1

Optimization problem

$$\mathcal{L}(\theta) = \sum_{s \in \mathcal{S}} \left(v(s,\theta) - V^*(s)\right)^2$$

Any difficulties with this formulation ?

Mean-square error, 1

Optimization problem

$$\mathcal{L}(\theta) = \sum_{s \in \mathcal{S}} \mathbf{P(s)} \left(v(s,\theta) - V^*(s)\right)^2$$

Why using distribution P ?

- $v(s,\theta)$ is an approximation: it has to make errors
- Not all errors are equally harmful: harmful errors must weight more.
- P might reflect a uniform distribution;
  or the distribution associated to the current policy π (on-policy learning);
  or to another policy used to acquire data (off-policy learning)
- Most generally, a new point $(s_t, V_t(s_t))$ is drawn and θ_t is updated using stochastic gradient.

Mean-square error, 2

$$\theta_{t+1} = \theta_t - \frac{1}{2}\alpha\nabla_{\theta_t}\left(V_t(s_t) - v(s,\theta_t)\right)^2$$

$$= \theta_t + \alpha\left(V_t(s_t) - v(s,\theta_t)\right).\nabla_{\theta_t} v(s,\theta_t)$$

Requirements

- $v(s,\theta_t)$ must be an unbiased estimate of the desired $V_t(s_t)$.
- not the case in general (except for Monte-Carlo); but practical.
- The approximation of the value function must allow for optimization, to define the policy by greedification:

$$\hat{\pi}(s) = \underset{a\in\mathcal{A}}{argmax}\left(\hat{q}(s,a,\theta^*)\right)$$

Learning Criteria

Notations

- For state s, push value toward backed-up value v $\qquad s \mapsto v$

Backed-up value

Dynamic programming

$$s \mapsto \mathbf{E}\left[r(s) + \gamma V(s')\right]$$

Monte-Carlo

$$s \mapsto r(s) + \sum_{t=1}^{T} \gamma^t r_t$$

TD(0)

$$s_t \mapsto r(s_t) + \gamma V(s_{t+1})$$

Semi-gradient SARSA

Sutton 89, Rummery 94

Loss function

Bellman expectation equation

$$\mathcal{L}(\theta) = \mathbf{E}\left[\left(\underbrace{R_{t+1} + \gamma q(S_{t+1}, A_{t+1}, \theta)}_{target\ \ value} - q(S_t, A_t, \theta)\right)^2\right]$$

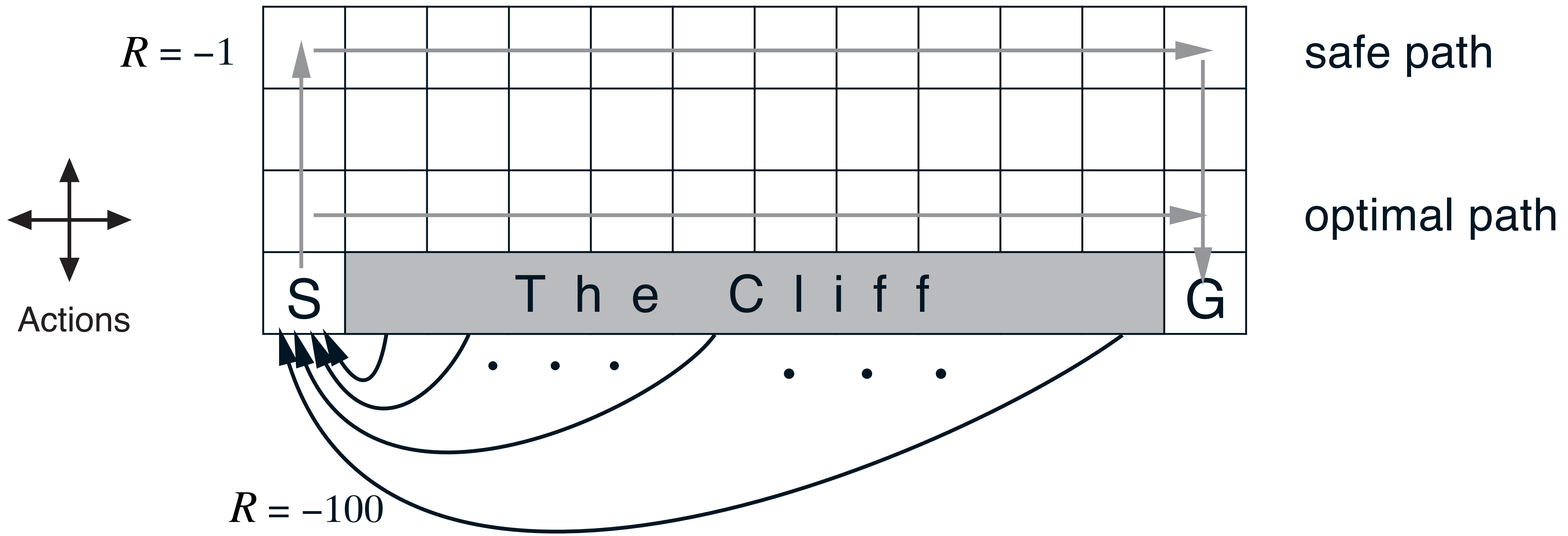
- again target depends on θ and we ignore this,
- taking the derivative wrt $q(S_t, A_t, \theta)$:

$$\Delta\theta_t = (R_{t+1} + \gamma q(S_{t+1}, A_{t+1}, \theta_t) - q(S_t, A_t, \theta_t)) \cdot \frac{\partial q(S_t, A_t, \theta_t)}{\partial \theta_t}$$

Remark

- This is an on-policy algorithm: it approximates q^π not Q^*.
- Therefore π should incorporate some exploration (be ϵ-greedy)

Cliff-walking example (on-policy vs off-policy)



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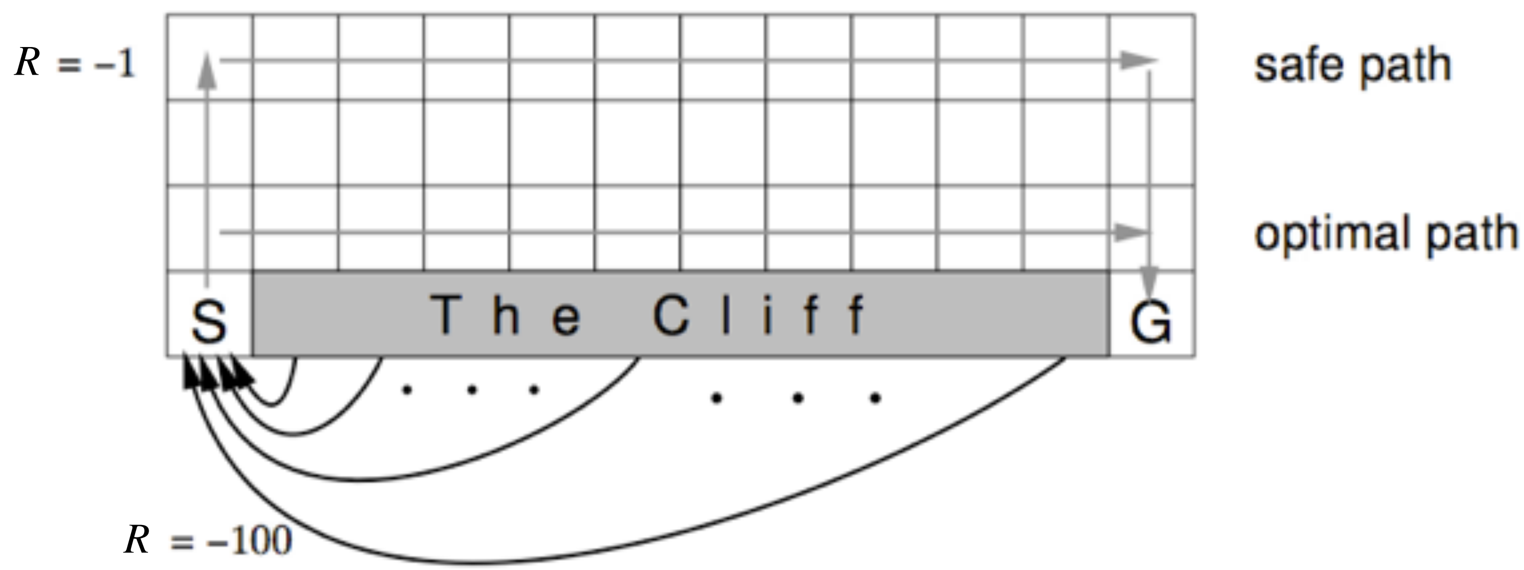
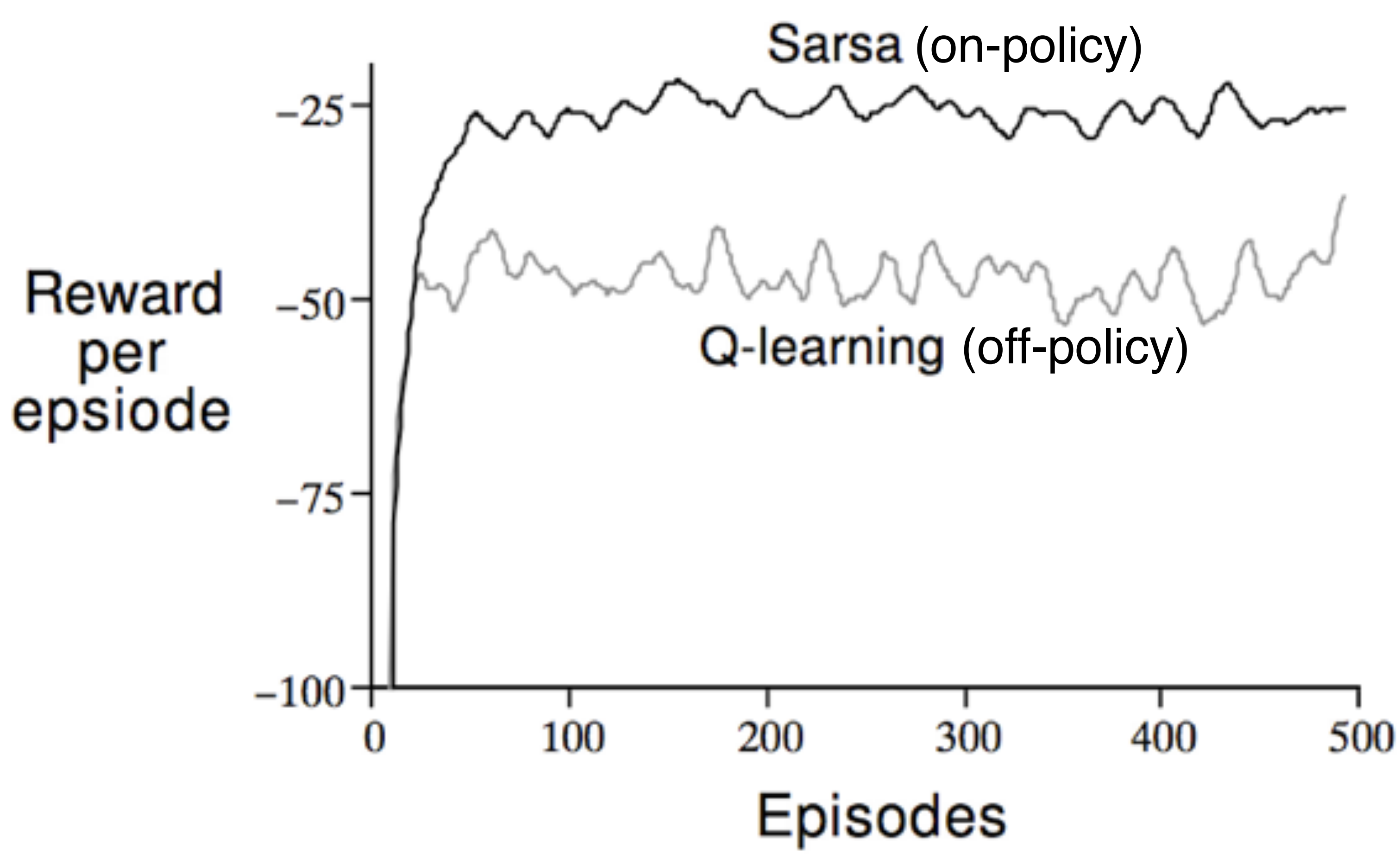




both algorithms are ε−greedy
ε = 0.1

Overview

Fitted Q iteration

Ernst et al. 2005

Principle iterating over the time horizon

- Given a set of four-tuples (s, a, r, s')
- First iteration:

$$\hat{q}_1(s, a) \approx r(s, a)$$

- iteration N:

$$\hat{q}_n(s_t, a_t) \approx r(s_t, a_t) + \gamma \max_{a \in \mathcal{A}} \hat{q}_{n-1}(s_{t+1}, a)$$

- Successive calls to the supervised learning algorithm are independent: possible to adapt the resolution/complexity depending on the iteration and the available sample.

Search space: Decision trees

- Non parametric; flexible
- Scalability wrt high-dimensional spaces
- Robustness wrt irrelevant features, noise, outliers.

Trees in Fitted Q iteration

Decision tree

Quinlan 89; Breiman 86

- Select cutting feature and cutting threshold to maximize the average variance reduction of the output variable
- Select hyper-parameter (min number of examples in a leaf) by cross-validation

Bagged trees

Breiman 96

- *M* times — *M* hyper-parameter
- Bootstrap the training set
- Grow a decision tree from the bootstrapped data

KD-tree

- In each node at depth d: cutting feature is i-th feature if $d < \#$ features
- cutting threshold: median of the f_i value in the training set
- (does it change among iterations ?)

Random Forests

Breiman 01; Geurts 04

- Like Bagged trees, except
- Sample a number K of (cutting feature, cutting threshold), return the best one

Trees in Fitted Q iteration, 2

Note l a leaf in a tree

$$q(s,a) = \sum_{trees} \sum_{l} k(s,a,l) v(l)$$

with

$$k(s,a,l) = \frac{1_{(s,a) \in l}}{\sum_i 1_{(s_i,a_i) \in l}}$$

Property

$$||\hat{q}_n(s,a)||_\infty \leq B + \gamma ||\hat{q}_{n-1}(s,a)||_\infty$$

with $\hat{q}_0(s,a) = 0$.
Therefore

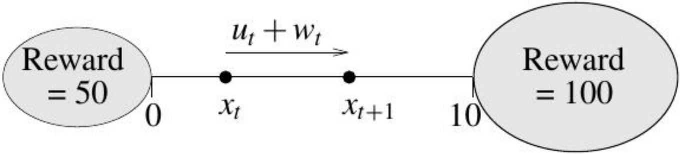
$$||\hat{q}_n(s,a)||_\infty \leq \frac{B}{1-\gamma}$$

with B a bound on the reward.

The RiverSwim

Ernst et al, 05

The problem



11 states $(0, 1, \ldots 10)$
2 actions, right or left
rewards on terminal states 0 or 10.

The results

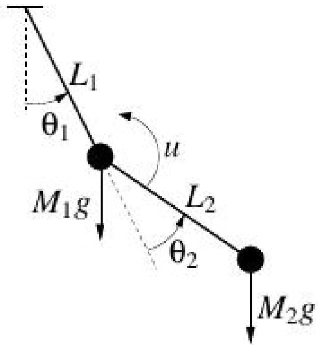
1. Bellman residuals wrt number $\#\mathcal{F}$ of 4-tuples.

Tree-based method	$\#\mathcal{F}$		
	720	2010	6251
Pruned CART Tree	2.62	1.96	1.29
Pruned Kd-Tree	1.94	1.31	0.76
Pruned Tree Bagging	1.61	0.79	0.67
Pruned Extra-Trees	1.29	0.60	0.49
Pruned Tot. Rand. Trees	1.55	0.72	0.59

The Acrobot

Ernst et al, 05

The problem



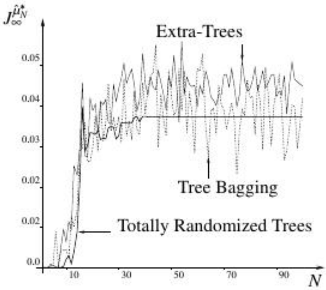
state in $\mathbb{R}^4$: $(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$
action: torque u = -5 or 5
reward: distance to up-equilibrium position, if < 1 (then terminates)

The Acrobot

The results

$\#\mathcal{F} \approx 150{,}000$ tuples

1. The return



2. Comparative performances

Tree-based method	Policy which generates $\mathcal{F}$	
	ε-greedy	Random
Pruned CART Tree	0.0006	0.
Kd-Tree (Best n_{min})	0.0004	0.
Tree Bagging	0.0417	0.0047
Extra-Trees	0.0447	0.0107
Totally Rand. Trees	0.0371	0.0071

Overview

Function approximation for RL: Summary

Goal

Learn an approximation $\hat{V}$ of the value function; define $\hat{\pi}$ from $\hat{V}$

Ingredients

- Data **off-line; online**
- Learning criterion **data fitting; Bellman residual**
- Learning procedure **knn; decision trees; gradient (linear or NN)**

Function approximation for RL: Summary, 2

Comments

1. Required to scale up
2. Pitfalls:
   - Sufficient representation needed (if large representation, robust learning required, e.g. decision trees)
   - Self-play / replay mandatory
   - A further stage of optimization is required to define $\hat{\pi}$
   - Pathologies: gradient can blow up (see Fig. 8.13, Sutton Barto)

After all

- **Value is a means for building a policy**
- **Can we build the policy directly ? next course**