A Dynamic Model of the U.S. Beef Cattle Industry

by

Dinesh Reddy Poddaturi

A thesis submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:
Lee L. Schulz, Co-major Professor
Chad E. Hart, Co-major Professor
John M. Crespi
Dermot J. Hayes
Daniel D. Loy

Iowa State University

Ames, Iowa

2022

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

The primary purpose of this thesis is the development of an economic framework that incorporates the essential dynamic process of the U.S. beef cattle industry. In particular, a dynamic economic model of the U.S. beef cattle industry is conceptualized and developed. The economic model is conceived upon a bottom-up framework, where the behavior of a representative farmer, the biology, age, and gender composition of cattle, evolving changes in the U.S. cattle structure, and micro-foundations are incorporated. The model exclusively depends on the data (measured consistently), and the data are collected and compiled from various USDA organizations. The data-driven economic model is calibrated to fit the observed data appropriately. The calibrated model is further utilized to project the beef cattle prices and quantities several years into the future. To validate the model projections, the projected prices and quantities are compared with the USDA and FAPRI long-term projections. The model fitness is tested by computing an error in unit-free form and by replicating U.S. cattle inventories from the fitted model.

In this proposal packet, the conceptual model framework, the analytical solution, numerical solution algorithm, and projections framework are developed and presented. In addition, the data used to solve the model, the numerical solution of the model which includes estimated parameters, fitted results, the model fitted errors, and the replication of the cattle inventories are presented. Also included are the model projections and the comparisons of the model projections to the USDA and FAPRI projections. The numerical solution of the model is written in R programming language, the code and related materials are intended to be open-source and will be made public in the near future.

CHAPTER 1. INTRODUCTION

1.1 Topic overview

American agriculture, food, and related industries contributed $1.05 trillion to the U.S. gross domestic product (GDP) in 2020, which is a 5% share of overall GDP [USDA-ERS (2022)]. Cattle production in the U.S. is crucial to the economy and is a major contributor to the agriculture industry. With its diverse agriculture resources and unique production practices, the U.S. has developed a beef industry that is separate from the dairy sector. The unique and high-quality beef production makes the U.S. competitive in the international beef markets. The U.S. has the world's largest fed cattle industry and produced up to 27 billion pounds of meat in 2020 and exported 3 billion pounds which is valued at $6.80 billion [USDA-ERS (2021)].[1] The domestic cash receipts from beef production alone in 2020 are $63.10 billion [USDA-ERS (2021)]. Figures 1.1 and 1.2 illustrate the export value and domestic cash receipts of American beef from 2001-2020 respectively. In addition to being the largest producer of beef, the United States is also the world's largest consumer of beef, mainly consuming high-quality, grain-fed beef.

[1]The United States was the third-largest exporter of beef in 2020.

Figure 1.1 Value of the U.S. beef exports

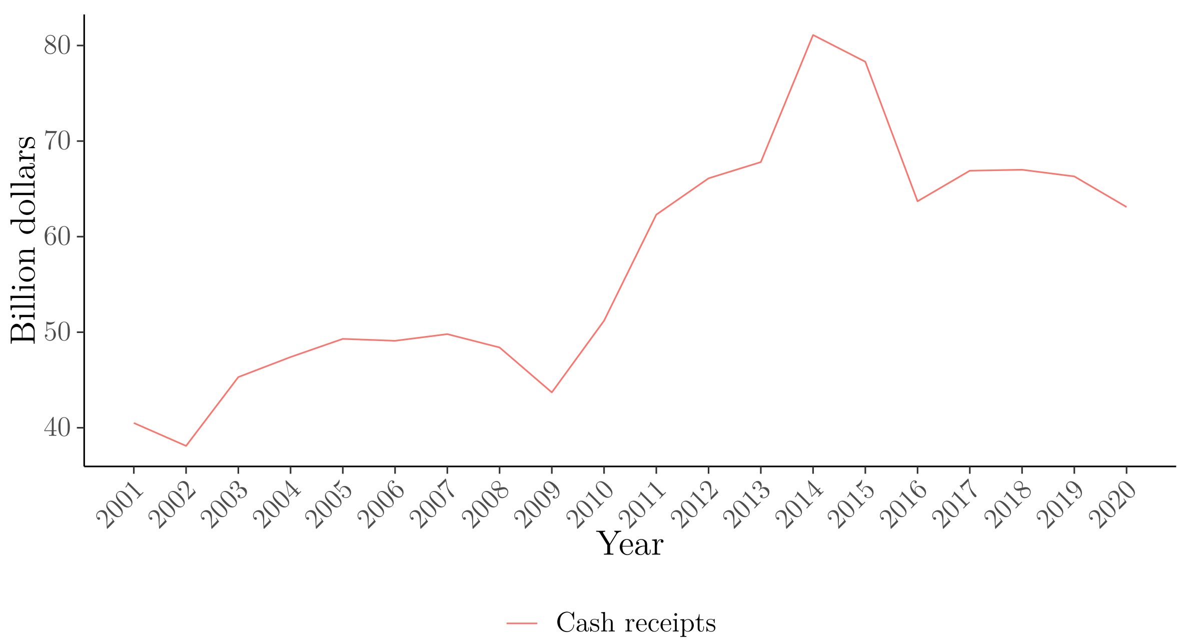

Figure 1.2 Domestic cash receipts from beef production in the United States

Beef production in the United States is divided into several stages - primarily cow-calf, backgrounding, feedlot, and harvest facilities. Farmers move cattle through these production stages to increase profits and minimize costs. The majority of U.S. calves are born in the spring [Schulz et al. (2016), USDA-NASS (2016)]. The newborn calves spend the first few months of their lives with the mother in a cow-calf operation. At around six months of age, a majority of young calves are weaned and sent to backgrounding where they are fed dry forage, silage, and grains for about four to six months. A share of the calves, mostly female, are not sent into backgrounding. They are kept on the farm for replacement breeding stock. The calves in backgrounding are then sent to feedlots where they are fed high-energy grain feed until they reach harvesting age which is typically between 12 and 24 months. Calf-feds typically are 12 to 16 months old at the harvest, depending upon the length of the finishing period. Most cattle fed as yearlings or long-yearlings are harvested between 16 and 24 months of age [Stuttgen (2019)]. Ultimately, production timelines for different regions depend on the availability and costs of feed and other resources. For example, in regions with more access to pastureland, cattle often spend more time and gain more weight on pastureland than in feedlots.

For farmers, cattle are both capital and consumption goods [Rosen et al. (1994)]. A calf destined for slaughter as a fed animal is a production good, since it will be sold at the market price. In contrast, a calf that is kept on the farm and added to the breeding stock is a capital good because it will contribute to future production for up to 10 years. In the literature, this process is recognized as a *dynamic* process [Rosen et al. (1994); Rosen (1987); Jarvis (1974)]. The dynamic process involves the biology of the female breeding cow, a one-year birth delay and a two-year maturation lag, and the decisions made by the farmer about adult cows (whether to consume or breed the cow) and young calves (whether to consume or add the female calves to the breeding stock). The population mechanics in the below-simplified flow chart 1.3 depict the dynamic process of the beef cattle.[2] Once born, every surviving calf enters the production line and stays in the line for two years before joining the mature stock. Then, a representative farmer makes a decision about which animals are sent for consumption (includes young steers and heifers and older, mature cows) and production (replacement heifers and mature cows).[3]

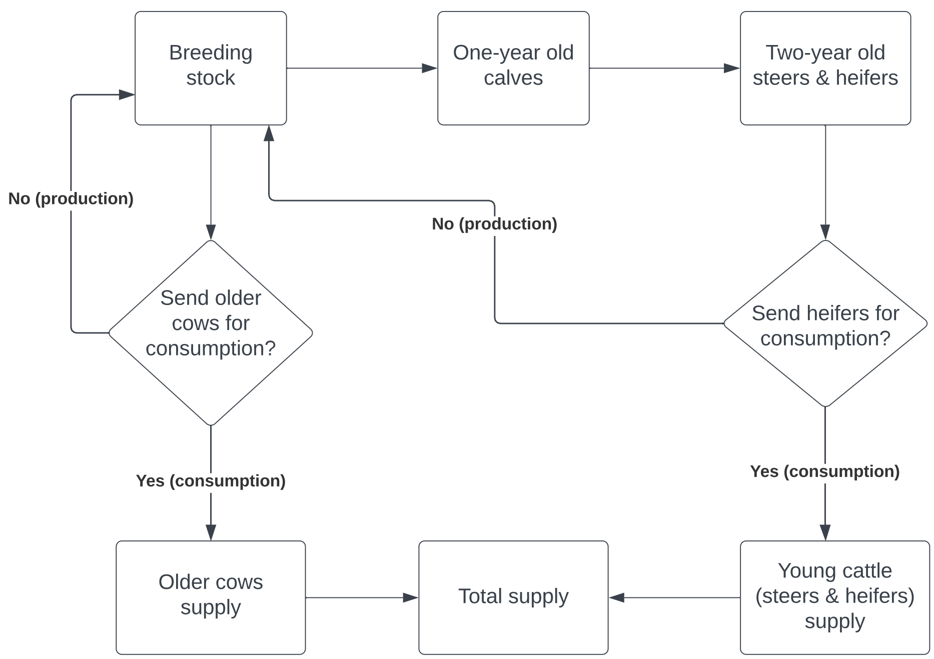

Figure 1.3 Population Mechanics

The biology of a mature cow, specifically the gestation and birth delays encapsulates a natural *time-to-build* [Kydland and Prescott (1982)] feature in the age structure of the beef cattle. This natural time-to-build feature creates cyclical feedback between current consumption and future production. Any exogenous shock can have persistent effects and

[2]The flowchart assumes a closed system. For simplicity, the imports and exports of live animals are not included. The flowchart is inspired from Rosen et al. (1994).

[3]A representative farmer makes these decisions annually.

can naturally alter the production and investment decisions about beef cattle made by the farmers. Ultimately, these decisions change the age distribution of the breeding inventories and cause a cyclical response as the cattle approach equilibrium [Rosen et al. (1994)]. This process leads to an expansion and contraction of the cattle inventories. The breeding stock inventory decisions, and the result of the decisions, have been theorized as *cattle cycles*. The following Figure 1.4 illustrates the occurrences of cattle cycles in the United States from 1970 to 2020. A typical cattle cycle averages between eight and twelve years with the latest cattle cycle being ten years (2004-2014). Along with the biology of the cattle and the decisions of a farmer, dry pasture lands and feed supplies may also alter a cattle cycle's length.

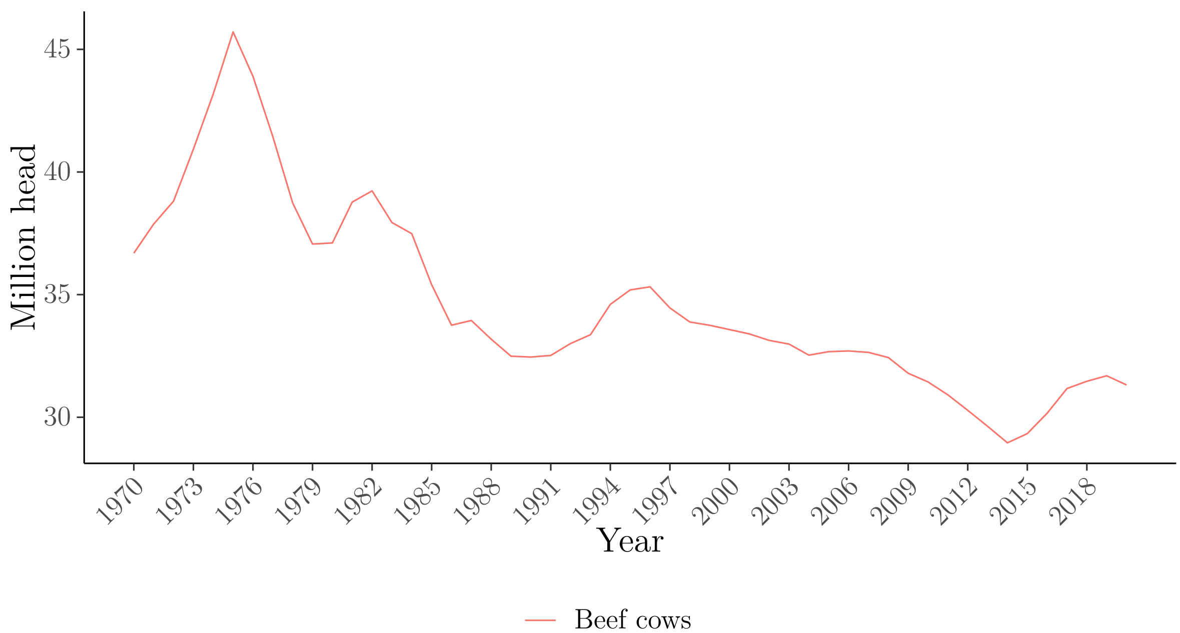

Figure 1.4 Total U.S. beef cow inventories

Even with the existence of natural cattle cycle phenomena, a representative farmer makes decisions such that the profits are maximized at every stage of the cycle. This is due to expectations made by a farmer. In particular, expectations about future prices play an important role in determining the movement of cattle through production stages. Especially, expectation about the price of a young animal and an adult cow is crucial in a farmer's decision-making process. A representative farmer observes the present price, supply, and demand for the meat, makes expectations about the price of the cattle in the future, leading to a dynamic decision process (on whether to keep the animal for production or use it for consumption). A high expected price leads to building the stocks by adding more replacement heifers to breeding inventory, and a low expected price leads to shrinking the stocks by sending the older cows to harvest facilities and adding fewer replacement heifers to the breeding stock eventually looping back to the cattle cycle phenomena.

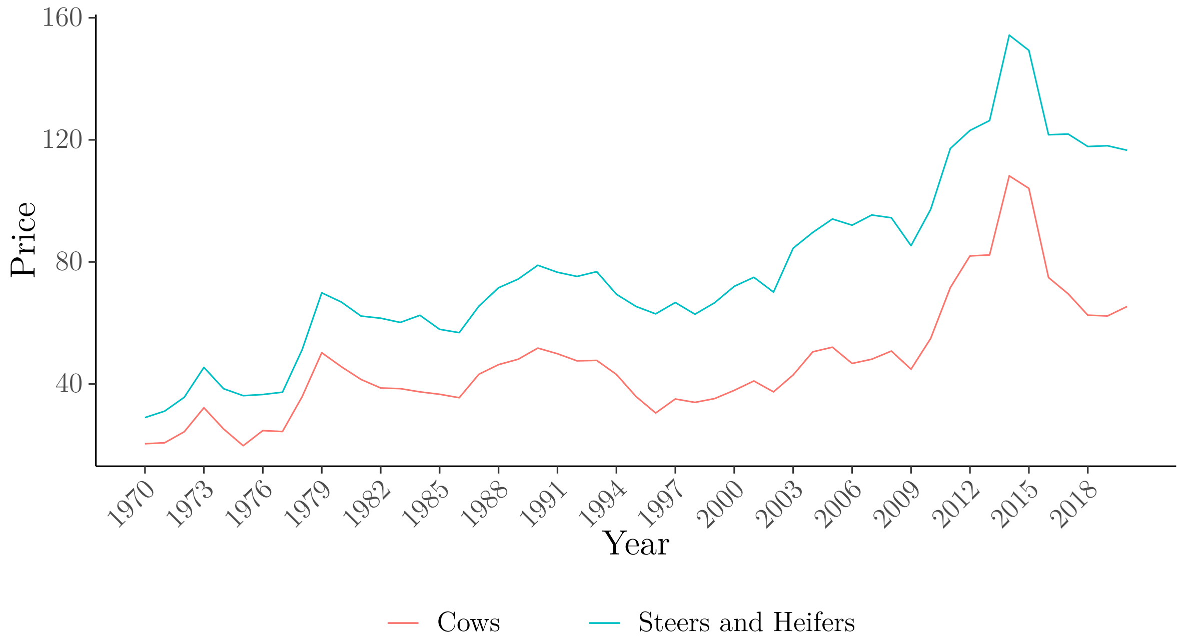

Figure 1.5 Price received by U.S. farmers ($/CWT)

The prices received by the farmer reflect the cattle cycle phenomena. From the above Figure 1.5, it is evident the prices received by farmers for both fed cattle and adult cows are trending upwards with occasional peaks and troughs. These peaks and troughs in the price are associated with the cattle cycles. For example, from the detrended beef cow inventories and prices in Figures 1.6 and 1.7 respectively, the cattle inventory reached a trough in 2014 (end of the latest cattle cycle) with a peak observed in the prices in the same year.

Figure 1.6 Detrended total U.S. beef cow inventories

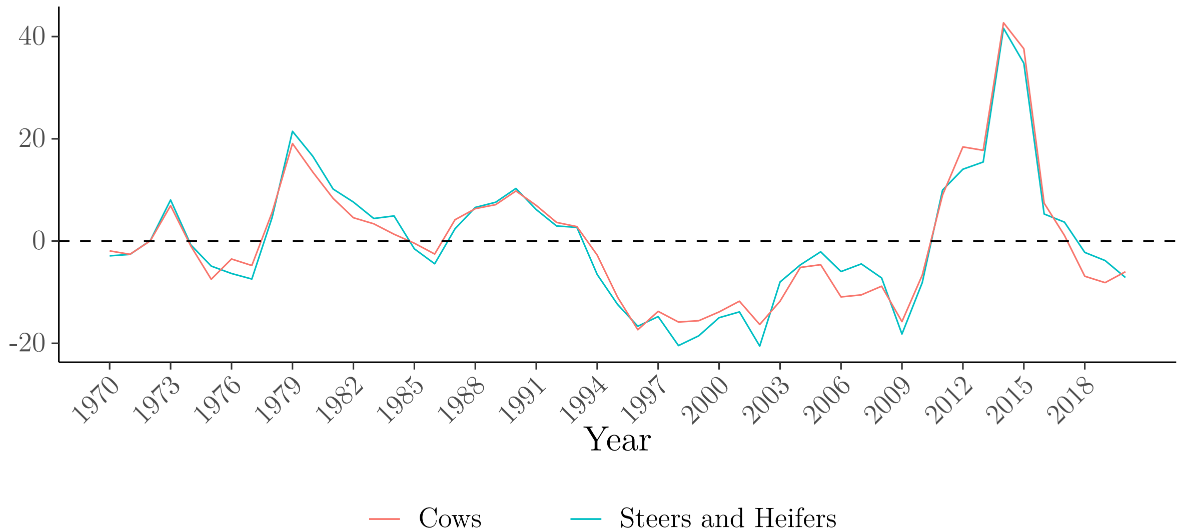

Figure 1.7 Detrended prices received by U.S. farmers

The persistence of periodical cattle cycles, price cycles (not as pronounced as cattle cycles), and the dynamic process itself in the beef cattle industry, poses some unique and intriguing questions from a theoretical standpoint. In order to fully understand these phenomena, to make important policy recommendations, and to analyze any exogenous impacts on the beef cattle industry, an eloquent economic model that considers the biology of the animal, the dynamic decisions of a farmer, and the evolving changes in the beef cattle industry is required. This thesis aims to provide such an economic model. In particular, a structural dynamic economic model of the U.S. beef cattle industry is developed. The dynamic model is built upon a bottom-up framework, where the fundamentals of farmers' behavior, biological constraints of the beef cattle, age distribution and gender composition of cattle, and micro-foundations are incorporated. The data-driven structural model is calibrated to adequately capture the dynamics observed in the U.S. beef cattle industry. The calibrated model is then used to project the beef cattle market variables (prices and quantities) into the future. Besides studying the beef industry structure and the dynamics of the beef cattle, the dynamic model can also be used to run counterfactual simulations to analyze policy impacts on the beef cattle industry and make important recommendations, making the model an interest to policymakers, industry stakeholders, and beef cattle farmers.

1.2 Literature review

The literature on beef cattle is primarily divided into two categories. The first category aims to explain the expansion and contraction of the cattle stocks, the expectation formation among the farmers, and to find the dynamic processes in the beef cattle sector. The second category aims to study policies affecting the U.S. livestock, in particular, quantifying the economic impacts of a policy introduction or modification, trade bans, and disease outbreaks using static economic models.

1.2.1 Price expectations

Expectation formation in economics is widely studied in general and is applied in various sub-disciplines of economics. Muth (1961) originally suggested the rational expectation hypothesis and explained the outcome of an economic phenomenon depends on the decisions made by the economic agents, and to a certain extent, on what the economic agents expect to happen. The rational expectation hypothesis postulates that the economy does not ignore information and the expectations depend on the systems involved in the economy.

Rational expectation formulation in agricultural economics has been used and applied in different settings. In the context of American agriculture, Cooley and DeCanio (1977) showed that the American farmers' price expectations (in aggregate) were consistent with the rational expectation theory. Using a simultaneous equation model, Goodwin and Sheffrin (1982) tested the existence of the rational expectation hypothesis in the agricultural markets. When applied to the chicken broiler industry, the concept of rational expectation (according to Muth (1961)) was present in the supplier behavior. Furthermore, Eckstein (1984) developed a dynamic linear rational expectation model to study the impact of price changes on the production decisions and land allocation in agriculture settings under exogenous price assumptions. These studies provide a piece of evidence that American farmers indeed make expectations about the price. In particular, their price expectations are consistent with the rational expectation hypothesis postulated by Muth (1961).

In the context of beef cattle, the decision-making process of the farmers when deciding whether to send an animal to a harvest facility (slaughterhouse) or keep it in the breeding stock has been studied in various settings. The debate about the farmers' expectation formation (naïve, rational, and quasi-rational) of beef cattle prices still exists among researchers. Several studies used different expectation formations to explain the evolution of U.S. cattle stocks and the decision-making process of a farmer. The economic models developed using different expectation formations are further used to examine the behavior of the farmer and their decision to slaughter the cattle with respect to their price expectations.

1.2.2 Economic models with price expectations in cattle industry

The seminal work by Jarvis (1974) was the first to coin "cattle are capital goods and producers are portfolio managers" (p.489). In order to explain producer behavior, using Argentian beef cattle data, Jarvis (1974) analyzed the farmer's decision-making process in a micro-economic setting. Rosen (1987) developed a dynamic rational expectation equilibrium model for the cattle to explain the supplier behavior. However, the model does not consider the age and gender composition of the cattle. By mathematically modeling the U.S. cattle inventory stocks, Rosen et al. (1994) was the first to explain the *cattle cycle* phenomena in the United States. The simple and straightforward model by Rosen et al. (1994) fits the US cattle inventory data and explains for the first time the expansion and contraction of the U.S. adult cattle stocks. Mundlak and Huang (1996) showed that even under different economic settings and different technologies, cattle cycles were observed in Argentina and Uruguay, indicating regardless of the economic conditions, technologies, or location, the decision process of a farmer depends on the biology of the cattle, and price expectations.

Nerlove and Fornari (1998) however, developed a quasi-rational expectation model as an alternative to rational expectations to explain the farmers' behavior. Using both published and constructed quarterly and monthly data, Nerlove and Fornari (1998) explained the dynamic optimizing behavior of U.S. cattle farmers. In contrast, Chavas (2000) showed rational expectations indeed exist in the farmers' behavior. The empirical results in Chavas (2000) indicate the presence of heterogeneous price expectations, where a large proportion of the farmers simply use the most recent data ("naïve") to make their production decisions. Although the number of farmers with forward-looking price expectations is less compared to naïve expectations, Chavas (2000) provided evidence of the existence of Muth's rational expectation hypothesis [Muth (1961)] among farmers. Additionally, Chavas (2000) concluded it is costly to access and analyze the market information and the cost varies among different market participants. In a related study, Baak (1999) developed a bounded rationality model to test for the existence of the different expectation formation of the U.S. beef cattle farmers and found the existence of bounded rationality among one-third of the farmers.

Recognizing the existence of heterogeneous price expectation formation and the criticism of Nerlove and Fornari (1998) about rational expectations, Aadland and Bailey (2001) examined the response of the beef cattle producers to changes in beef prices using an economic model with rational expectations. By separating the fed and un-fed cattle and the corresponding decisions of the producer, the model predicted a positive response by the producer to higher prices on one margin and build the stocks on the other margin. In related work, Foster and Burt (1992) developed a dynamic model of investments in U.S. beef cattle. The model suggests that incorporating the age distribution of the herd is critical in explaining farmers' choices or investment decisions. Schmitz (1997) studied the dynamics of the beef cow herd size using an inventory approach. The study included the age distribution of the cattle and by using a simulation strategy, the study estimated the short-run and long-run impacts of shocks on the retention and slaughtering of the cattle. However, the model abstracted away from price expectations.

In order to explain the recurring 10-year cattle cycles, Aadland (2004) developed a forward-looking economic model that takes into account heterogeneous expectations and the age distribution of animals. By allowing two different types of farmers (i) fully rational, where the farmers treat the future variables as endogenous and forecast them, (ii) boundedly rational, where the farmers use naïve expectations, Aadland (2004) replicated the 10-year U.S. cattle cycles.

Almost all previous work on the economics of beef cattle has focused on finding evidence for the existence of different expectation formation, explaining the cyclical nature of cattle inventories, determining what the dynamic processes are, and understanding the response of a cattle farmer to price changes through rational price expectations. The economic models mentioned above do not facilitate the inclusion of age distribution, gender composition, and price expectations of the beef cattle, simultaneously. Although the economic models in these studies are constructed to explain farmers' behavior in the event of temporary or permanent changes in the demand or price, they abstracted away from policy recommendations. These studies, by their construction, often do not have direct applications in investigating the impacts of policies affecting the beef cattle industry. An economic model of the beef cattle that not only explains the producer behavior, but also assists policymakers to develop and assess beef cattle policies, can be more beneficial at large.

1.2.3 Policy studies of the U.S. beef cattle industry

There is a host of studies that analyzed the policy impacts on the beef cattle industry. Traditionally, studies that analyzed policy or disease or trade impacts on the beef cattle industry have deviated from dynamics and have heavily relied on static Equilibrium Displacement Models (EDMs), Input-Output (I-O) models, and partial equilibrium models.

In order to study the effects of non-governmental costs of mandating the animal identification system on livestock prices, quantities, and producer and consumer surplus, Blasi et al. (2009) developed an EDM. Using different simulation scenarios on animal identification adoption rates, the study quantified the annual costs for beef, pork, lamb, and poultry sectors. In a similar study, using a multi-market EDM, Pendell et al. (2010) examined the impact of animal identification and tracing on the meat and livestock in the United States. The multi-market EDM facilitates all the stages in the supply chain (farm, wholesale, and retail), and quantifies the short-run impacts on the prices and quantities. Pendell et al. (2013) used EDM to analyze and quantify the economic impacts of changes in the source and age verification requirements and related adjustment in international trade of U.S. beef. To quantify the economic impacts, Pendell et al. (2013) simulated the EDM by assuming the loss of exports to South Korea. The study quantified the producer surplus and consumer surplus in the short-run and in a 10-year period in the U.S. livestock industry.

EDMs are widely used to study the implementation of country of origin labeling (COOL) in the United States. Brester et al. (2004) employed EDM to estimate short-run and long-run changes in equilibrium quantities and prices in the beef, pork, and poultry industries resulting from the implementation of COOL. The study simulated two different scenarios (no demand increase, 4.05% beef and 4.45% pork demand increase) and found that producer surplus declined (absence of demand increase) by \$647.80 million and \$220.40 million in beef and pork industries respectively. The study also reported the increased poultry demand further increased the producer surplus up to \$198.30 million. In a similar study, Tonsor et al. (2015) estimated the economic impacts of the 2009 COOL rule and 2013 amendments to the 2009 COOL rule on the U.S. livestock industry. The study reported that the implementation of the 2009 COOL rule would result in welfare losses of \$405 million in the beef industry in the first year, and short-run gains of \$105 million and \$405 million in pork and poultry industries respectively. The impacts of the 2013 amendments to the COOL rule would increase the economic welfare losses additionally by $494 million for beef and $403 million for the pork industries over the first 10-years. Lusk and Anderson (2004) developed an EDM for retail, wholesale, and farm markets for beef, pork, and poultry to study the impact of COOL on producers and consumers. The empirical results from the study indicate that, after implementing COOL, the costs are shifted from producer to the processor and then to the retailer. Lusk and Anderson (2004) also concluded, with the adoption of COOL, producers are made better off and the consumers are made worse off.

EDMs were also used to study the welfare impacts of trade bans and meat recalls. Mutondo et al. (2009) used EDM to evaluate the welfare impacts of trade bans by Japan and South Korea in 2003. The analysis included domestic meat, imports, and exports. Using a simulation strategy, the study found that with the Japanese ban on U.S. Beef, the welfare of the U.S. producers and retailers decreases, and the welfare of the Australian and Japanese beef market increases. The same phenomenon is observed with South Korea banning U.S. beef. Using EDM, Shiptsova et al. (2002) evaluated the effects of recall costs on the beef, pork, and poultry industries. The study reported the producer welfare losses in the presence and absence of substitution effects on the demand side.

Static partial equilibrium models, I-O models, and EDMs were heavily employed to study the impacts of animal vaccination and foreign animal diseases on the U.S. economy. Pendell et al. (2007) employed a partial equilibrium and I-O approach to estimate the economic impacts of a Foot-And-Mouth Disease (FMD) outbreak in southwest Kansas. Pendell et al. (2007) simulated the models under different scenarios such as introducing the FMD outbreak at a cow-calf operation, a medium-sized feedlot, and simultaneously at five large feedlots. For each scenario, the study reported the producer surplus losses for the beef industry. Tonsor and Schroeder (2015) studied the market impacts of E. Coli vaccination in U.S. feedlot cattle using an EDM and estimated the economic impacts of adopting the vaccination program by the feedlot operators. In particular, the study illustrated the costs associated with the use of a vaccine in fed cattle and reported the economic welfare loss of producers with adopting the program.

The economic models used in all the aforementioned policy studies did not consider dynamics and were simplified significantly (a common theme among all the studies is employing static EDMs to quantify economic impacts). Some of the simplifications include linear supply and demand curves and parallel shifts in supply and demand. Simplified static models without dynamics can facilitate other species such as swine, poultry, dairy, and the inclusion of several stages of the supply chain from farm to consumer. In addition, static models are useful for informing policies and can provide fairly accurate predictions in the short run. However, dynamics are an essential feature in beef cattle production and markets. This implies the static models can be biased when trying to predict several years into the future. The inclusion of dynamics can yield more accurate estimates not only in the short run, but also in the long run and show how the estimates vary over time.

1.3 Purpose and contribution to literature

Although dynamics is an important driver of beef cattle markets, current policy analysis of the beef cattle markets discounts its importance and relies heavily on simplified static models. The calibration of these static models uses relatively old and fragile estimates; hence, there is a potential to significantly improve beef industry policy analysis by using a dynamic model that calibrates directly from the data rather than relying on static counterfactuals and parameter estimates from the literature. Policies such as restricting the movements of cattle or requiring traceability, identification, vaccination, mitigation of greenhouse gas emissions, and foreign animal disease management strategies routinely reappear in policy discussions of the beef cattle sector. The recurrence of discussions about these policy issues reflects technological improvements that lower costs and open new approaches to old problems, new knowledge, pressure from trading partners, and environmental concerns.

Considering all these potential changes in the U.S. beef cattle industry, the primary purpose of this thesis is to develop an economic framework to more appropriately quantify the economic impacts of numerous policy proposals in the U.S. beef cattle sector. We develop a dynamic model that will yield more accurate estimates of the economic impacts of policies affecting the beef cattle sector, and how these impacts vary over time. Using extensive beef cattle industry data, we calibrate the model to capture the dynamics of U.S. beef cattle and the calibrated model is used to project prices and quantities several years into the future.

Our structural dynamic model is modeled with naïve and rational price expectations. We acknowledge there are mixed results and beliefs in the literature on the formation of the price expectations and fully rational expectations are highly unlikely among all farmers. Some past studies disputed the fully rational expectation formation. In particular, Nerlove and Fornari (1998) used quasi-rational expectations as an alternative to fully rational expectations and used time-series model forecasts to replace the future exogenous and endogenous variables.

However, Rosen et al. (1994) used fully rational expectations and their model is widely recognized as the leading study to formally explain cattle cycles and made a significant contribution to research on cattle cycles in general. Also, the Rosen et al. (1994) model is simple and easy to navigate. Our dynamic model is inspired by previous studies, and model terminology is borrowed from Aadland and Bailey (2001), Chavas (2000), and Rosen et al. (1994).

In contrast to the existing literature, our dynamic model incorporates both the age distribution and the gender composition of the cattle. The inclusion of age distribution and gender composition can complicate the model, but previous research [Foster and Burt (1992)] suggested that they are crucial. In addition to decision-making for young heifers and steers (hereafter referred to as fed cattle), we also include decision-making for older animals, adding another layer to a farmers' decision-making process. Using previous research [Aadland (2004); Trapp (1986)] and observed data, an assumption of "younger cows (six years and less) in the breeding stock are never slaughtered as they are productive" is maintained in the model. The possibility of cows exiting the breeding stock due to natural causes or premature death is also incorporated in the model. The exogenous assumption of the prices and quantities (widely made in the literature) is relaxed in the model. The model distinguishes the production of fed cattle and adult cows, uses the total derived demand for the meat to determine the proportion of consumer demand for fed cattle and older cow meat, and utilizes equilibrium conditions to solve for the prices and quantities. Our equilibrium system of equations is nonlinear, and it is very unlikely to find a closed-form analytical solution. Therefore, numerical methods are employed to find solutions (the model, methods, and solution algorithm are presented in the next chapter).

Our model, based solely on data and numerical methods, can be used for policy analysis and to study the dynamics of beef cattle. The solution algorithm of our model can be adapted to different species with biological lags. For example, inspired by the biological constraints and the production lags in the beef industry, Asche et al. (2017) developed a partial equilibrium model for the production of fish when subjected to environmental shocks. This shows our model can be adapted to other industries where biological lags similar to those in beef cattle are present.

Our work complements the existing literature in several ways. First, the model extends the existing literature explaining beef cattle stocks by modeling farmer behavior and price expectations. We distinguish between the type of cattle and include various age groups of cattle (2-year-old heifers and steers, 3 to 10-year-old cows) to determine the production of fed cattle and adult cows in a given year. By applying micro-foundations on consumer preferences for meat, we determine the demand share for fed cattle meat and cull cow meat separately. We present solutions for the model under naïve price expectations and rational price expectations.

Second, and more importantly, our model can be applied to study a variety of policy impacts, animal disease impacts, and any other exogenous impacts affecting the U.S. beef cattle industry. Knowing not only the short-run impacts, but also knowing the long-run impacts and how the impacts change over time is crucial when policymakers are designing, debating, and deploying any beef cattle policy. The model can be used to run counterfactual simulations to better understand the short-run and long-run impacts of a policy change or implementation and make necessary adjustments to the policy in question.

Third, we use numerical methods to find a solution to the model. Numerical methods are rarely used in cattle models. Our model is non-linear, so a full closed-form analytical solution cannot be achieved, hence we use numerical methods to find a solution. We borrow some of the solution methods widely appear in competitive storage literature and apply them to our cattle model. To our best knowledge, this is the first cattle model that utilizes a variety of data (multiple sources are used to collect and compile data), carefully tracks and accounts for observed changes in the beef cattle industry (e.g., dressed weights of cattle), and use machine-based algorithms (developed and programmed by us) to find a solution for the model.

The novelty of our model comes in different pieces. In contrast to the extensive literature on beef cattle models, we include age distribution, gender composition, and price expectations at the same time. The presence of both fed cattle and adult cows is perhaps one of the important novel features in the model. The model also explains the substantial dynamics of breeding inventories and prices (for both fed cattle and adult cows) in the United States. The model's use for policy analysis and recommendations to policymakers is also another novelty.

CHAPTER 2. CONCEPTUAL MODEL FRAMEWORK AND SOLUTION TO THE MODEL

2.1 The model

The temporal arbitrage conditions are specific to the age of cattle, in the following all the conditions are mentioned. But first, we introduce the notation and the assumptions that are made in the model.

2.1.1 Notation

- $V_{j,t+1}$ is the value of a cow of age j.
- $p_{s,t}$ is the price of fed cattle for slaughter.
- $p_{c,t}$ is the value of culling [1] a cow at time t. All cows have the same culling value.
- h_t is the unit holding cost of an animal. We assume that it is exogenous, stochastic and depends on the price of corn.
- γ_0 and γ_1 are proportion factors for the cost of holding respectively new-born and one-year-old calves.
- β is the discount factor.
- g is the breeding rate.

[1]Culling is defined as the departure of cows from the herd because of sale, slaughter, salvage, or death [Fetrow et al. (2006)].

- $k_{j,t}$ is the stock of cows of age j at time t.

- $K_t = \sum_{j=3}^{9} k_{j,t}$ is the total breeding stock.

2.1.2 Assumptions

- Cows can have a calf every year.

- Use the word slaughter for fed cattle and the word cull for cows.

- Once a cow is ten-year old it is culled and has a value $p_{c,t}$.

- A cow is first bred when it is two years old.

- We assume that a cow must survive to the next period for her calf to survive as well. Thus, the size of the calf crop is proportional to the stock of cows in the next period.

- A cow of age j survives to the next period with a probability δ_j.

- Half of the calves are steers and they are all slaughtered when they are 2 years old. The number of steers at time t is $0.5gK_{t-1}$.

- We ignore the bull population because it is small.

- Heifers are either slaughtered or added to the breeding stock. The number of heifers at time t is $0.5gK_{t-1}$.

- The number of fed cattle slaughtered equals consumption and is written as $q_t = gK_{t-1} - k_{3,t+1}$, where $k_{3,t+1} \leq 0.5gK_{t-1}$.

2.1.3 Holding costs

The discounted cost of holding cows for one more year is given by:

$$z_t = h_t + \beta g \gamma_0 h_{t+1} + \beta^2 g \gamma_1 h_{t+2}. \tag{2.1}$$

The equation 2.1 states that if a farmer keeps a cow for another year, the farmer commits to the cost of feeding that cow for the next year and its progeny for the next two years. Rosen et al. (1994) specify holding costs in the same way.

The unit holding cost, h_t, depends in large part on the price of feed, mainly the price of corn and on the price of forage, in varying proportions depending on the region. Here, we will take holding costs based on the price of corn, assuming that the price of forage and other feeds are correlated with the price of corn and assuming that other costs are fixed. We calibrate the parameters β, g, γ_0, and γ_1 with information from the literature and from existing cattle models.

2.1.4 Arbitrage conditions by cohort

Here we describe the arbitrage conditions by cohort. The equations take a similar form but it is important to understand timing, in particular for older and younger cows.

Farmers with nine-year old cows can either breed them for another year or cull them in the current period. If a farmer breeds a nine-year old cow for one more year, the farmer will naturally cull the cow within our model in the next period. In practice, the number of breeding cows over 10-years old is small and it is a common practice to cull cows when they reach about that age.

2.1.4.1 Nine-year old cows

The value of a nine-year old cow is

$$V_{9,t} = \max\Big\{p_{c,t}, \mathrm{E}_t[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t]\Big\}, \tag{2.2}$$

where $p_{c,t}$ is the value of culling the cow this year and $\mathrm{E}_t[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t]$ is the net value of breeding the cow this year, culling the cow next year, and capturing the slaughter value from the calf.

If $\mathrm{E}_t\left[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t}$, then a farmer keeps the cow for one more year such that $k_{10,t+1} = \delta_9 k_{9,t}$ where δ_9 is the survival rate of nine-year-old cows. That is, if that inequality holds, the farmer maximizes profit by breeding all of the nine-year-old cows. Conversely, if $\mathrm{E}_t\left[\beta p_{c,t+1} + g\beta^3 p_{t+3} - z_t\right] < p_{c,t}$, a farmer culls all of the nine-year old cows such that $k_{10,t+1} = 0$. Finally, if $\mathrm{E}_t\left[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t\right] = p_{c,t}$, then the farmer will cull only a fraction of the cows such that $k_{10,t+1} \in (0, \delta_9 k_{9,t})$.

2.1.4.2 Eight-year old cows

The arbitrage condition for an eight-year old cow is similar. Farmers with eight-year old cows can either breed them for one more year or cull them in the current period. Of course, a farmer would only cull an 8-year cow if the farmer had already culled all of the 9-year cows.

If a farmer breeds a cow for one more year, they expect to earn $\mathrm{E}_t V_{9,t+1}$ in the next period. Therefore, we can write the value of an eight-year old cow as

$$V_{8,t} = \max\left\{p_{c,t}, \mathrm{E}_t[\beta V_{9,t+1} + g\beta^3 p_{s,t+3} - z_t]\right\}. \tag{2.3}$$

At equilibrium, $V_{8,t} \geq V_{9,t}$ because the lowest value it can take is $p_{c,t}$ and $\mathrm{E}_t\left[\beta V_{9,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > \mathrm{E}_t\left[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t\right]$. The value for the 8-year old cow contains the expected value for a 9-year old cow which we can write as

$$\mathrm{E}_t V_{9,t+1} = \max\left\{\mathrm{E}_t p_{c,t+1}, \mathrm{E}_t[\beta p_{c,t+2} + g\beta^3 p_{s,t+4} - z_{t+1}]\right\}. \tag{2.4}$$

Substituting 2.4 in 2.3 yields

$$V_{8,t} = \max\left\{p_{c,t}, \mathrm{E}_t\left[\beta \max\{p_{c,t+1}, \beta p_{c,t+2} + g\beta^3 p_{s,t+4} - z_{t+1}\} + g\beta^3 p_{s,t+3} - z_t\right]\right\}. \tag{2.5}$$

2.1.4.3 Cows between 3 and 7 years old

For cows between 3 and 7 years old, the arbitrage conditions are analogous to the arbitrage condition for 8-year old cows. We will not fully expand upon all these conditions as they are repetitive and grow in complexity rapidly.

The value of a seven-year old cow is

$$V_{7,t} = \max\left\{p_{c,t}, \mathrm{E}_t[\beta V_{8,t+1} + g\beta^3 p_{s,t+3} - z_t]\right\}. \tag{2.6}$$

We can iteratively substitute for $V_{8,t+1}$ and then $V_{9,t+2}$ to find an expression containing the observed and expected prices.

Farmers rarely cull younger cows as they will cull older and lower performing cows first, so culling cows that are six years old or younger is an unlikely event. The main reason is that the conditions that would lead farmers to cull younger cows are quite extreme and in practice, we do not expect such conditions to occur regularly. Younger cows have many years of useful life ahead and a farmer would need to have dire expectations about the future to cull a young cow. Another reason is that risk-loving farmers may prefer to maintain a sufficiently large breeding herd even though market conditions indicate otherwise. Farmers may also have positive outlooks on the future despite the market signaling the opposite.

Based on this observation, we assume that farmers never cull cows that are six years old or younger.[2] The assumption has to do with the comparison of the expected future value and their value for culling. For $k \in \{3, 4, 5, 6\}$, we can write

$$V_{k,t} = \mathrm{E}_t[\beta V_{k+1,t+1} + g\beta^3 p_{s,t+3} - z_t] \geq p_{c,t}. \tag{2.7}$$

The implication is that for younger cattle the annual transition is determined by the survival rate of each cohort such that we can write $k_{4,t+1} = \delta_3 k_{3,t}$, $k_{5,t+1} = \delta_4 k_{4,t}$, $k_{6,t+1} = \delta_5 k_{5,t}$, and $k_{7,t+1} = \delta_6 k_{6,t}$.

[2] In practice, farmers will cull sick or injured cows. We consider these cases as natural mortality using the parameter for the survival rate, $\delta_{k,t}$.

2.1.4.4 Two-year old heifers

Farmers with two-year old heifers have the choice to either send them to slaughter house or add them to the breeding stock. The value of a heifer is

$$V_{2,t} = \max\Big\{p_{s,t}, \mathrm{E}_t[\beta V_{3,t+1} + g\beta^3 p_{s,t+3} - z_{t+1}]\Big\}, \tag{2.8}$$

where $p_{s,t}$ is the slaughter price. Only heifers can be kept in the breeding herd, therefore putting an upper limit to the number of heifers bred given by $k_{3,t+1} \leq 0.5gK_{t-1}$. In practice, it is never the case that $k_{3,t+1} = 0$, i.e., no heifers are added to breeding herd. Thus, we only consider the interior solution where $k_{3,t+1} \in (0, 0.5gK_{t-1})$ such that

$$p_{s,t} = \mathrm{E}_t\left[\beta V_{3,t+1} + g\beta^3 p_{s,t+3} - z_{t+1}\right]. \tag{2.9}$$

2.1.5 Demand for beef and cattle

Most meat from fed cattle makes higher value cuts (e.g., steaks), while meat from the cull cows makes lower value beef (e.g., ground beef). Beef products from fed cattle and from cows are imperfect substitutes, with fed cattle meat being of higher quality. Prices for fed cattle and cull cows reflect the difference in the quality of meat products from cattle of different ages.

Our model recognizes the quality difference between beef products from fed cattle and cull cows. The demand for cattle is derived from the demand for beef. Accordingly, we proceed in two steps. We begin by modeling consumer demand for beef products, assuming that distinct products are made out of fed cattle and cull cows. We then turn to beef packing production technology, which allows us to derive an expression for the demand for fed cattle and cull cows.

2.1.5.1 Consumer demand for beef products

Beef from fed cattle is considered a higher quality product than beef from cows. In practice, this means that if prices for beef from fed cattle and beef from cows are the same, consumers will choose beef from fed cattle. This is a simplification because an animal yields many cuts with a wide range of values.

The intensity preference for beef will vary across consumers depending on their intrinsic characteristics. We model these preferences using a standard choice model where the diversity of preferences is captured using a distribution function. The utility a consumer derives from one unit of beef is

$$\theta_j - w_j, \tag{2.10}$$

where w_j is the retail price of beef for $j \in \{s, c\}$. The parameter θ_j is the utility to a consumer of beef of type j, excluding the purchase cost w_j. A consumer purchases beef from a fed cattle if

$$\theta \equiv \theta_s - \theta_c > w_s - w_c \equiv w. \tag{2.11}$$

Equation 2.11 says that a consumer purchases beef that yields the most utility. We can interpret θ as consumer willingness to pay for beef from fed cattle over beef from cows, and we can interpret w as the premium for fed cattle beef over beef from cows.

The parameter θ summarizes consumer preferences. Consumers are not identical, and we expect some to have strong preferences for a steak from fed cattle while others are content with ground beef coming mostly from cows. We write that $h(\theta)$ is the marginal distribution of willingness to pay, defined between a lower bound of $\underline{\theta}$ and an upper bound of $\overline{\theta}$. Consumers who purchase beef from fed cattle over beef from cows are those with a willingness to pay a greater price premium as equation 2.11 shows. We write the share of consumers who purchase beef from fed cattle as

$$\int_{w}^{\overline{\theta}} h(\theta)d\theta = 1 - H(w). \tag{2.12}$$

and the share of consumers who purchase beef from cows as

$$\int_{\underline{\theta}}^{w} h(\theta)d\theta = H(w). \tag{2.13}$$

Because the cumulative distribution function $H(w)$ is increasing, the share of beef derived from fed cattle purchased by beef packers decreases as the price premium w increases. Conversely, the share of beef derived from cull cows purchased by packers increases as the price premium w increases.

Equations 2.12 and 2.13 give the consumption shares for a given premium w. We multiply these shares by the total consumption of beef. We write the total consumption of beef as

$$Q_b(w_s, w_c) = Q_s(w_s, w_c) + Q_c(w_s, w_c), \tag{2.14}$$

where we can write $Q_s(w_s, w_c) = \Big(1 - H(w)\Big)Q_b(w_s, w_c)$ and $Q_c(w_s, w_c) = H(w)Q_b(w_s, w_c)$. The total demand for beef, $Q_b(w_s, w_c)$, depends on prices for the beef categories and depends on other variables that we do not identify to avoid notational clutter. The specification of $H(w)$ is discussed in the next section.

We simplify by assuming that the aggregate demand for beef is perfectly inelastic [Aadland (2004); Aadland and Bailey (2001); Capps et al. (1994); Marsh (1991)] in the short run such that $A = Q_b(w_s, w_c)$. The reason for this assumption is that it will make the calibration of the model much simpler. Although we assume a perfectly inelastic demand in the short run, we can allow for the value of A to change over time to adjust for trends and to vary according to changes in the price of beef relative to other meat products. We can also introduce stochasticity in the demand through the demand parameter A or by using an additive stochastic parameter to the demand equation.

In what follows, it is useful to work with inverse demand equations. We write the inverse demand functions as $w_s(Q_s, Q_c)$ and $w_c(Q_s, Q_c)$.

2.1.5.2 Derived demand by packing houses

The beef supply chain is as follows: beef packers purchase cattle from farmers, the cattle are then processed in harvest facilities, the meat is then sold to wholesalers, then to retailers, finally reaching consumers. We simplify the supply chain by assuming the beef packers purchase cattle from farmers and sell meat to consumers.[3] Therefore, the beef packers demand cattle from farmers and supply meat to consumers. We derive these supply and demand curves based on the packer's production technology.

We will focus on packers who process both fed cattle and cull cows. In practice, most packers specialize in the processing of fed cattle or the processing of cull cows. We do not expect these plants to switch to cattle of another age category unless there is a large change in their relative prices. Some plants do accept both fed cattle and cull cows. These plants will typically accept fed cattle on certain days of the week and cull cows on other days of the week. The number of days dedicated to each age category depends on their relative profitability. Plants that can arbitrage between cattle of the two age categories are the ones that matter at the margin.[4] This is why we focus on the profit of a plant that can process animals of the two age categories.

[3] This simplification is done as we do not model the wholesalers, retailers and also to decrease the complexity. The final product is purchased by the consumer and in aggregate consumers and farmers matter the most in the model.

[4] This assumes no corner solution where the capacity to process cattle of a certain age category is binding.

The production technology of transforming live cattle into meat is simple. A live cattle carcass yield is on average about $62-64\%$.[5] The yield varies by cattle weight, cattle breed as well as cattle age.[6]

We assume that packers have Leontief production technology where the total quantity of beef produced by a plant n is given by

$$q_n = q_{ns} + q_{nc} = \min(\phi_s X_{ns} + \phi_c X_{nc}, z), \tag{2.15}$$

where ϕ_i is meat yield, X_{ni} is the quantity of cattle of category $i \in \{s, c\}$ and z is the quantity of other inputs. The literature provides evidence of increasing returns to scale in meatpacking [Xia and Steven (2002); Azzam and Anderson (1996); Ball and Chambers (1982)]. However, assuming constant returns to scale simplifies the model and is consistent with fixed processing capacity in the short run.

We write the profit of a packing plant as

$$\Pi_n = w_s\phi_s X_{ns} + w_c\phi_c X_{nc} - p_s X_{ns} - p_c X_{nc} - (\phi_s X_{ns} + \phi_c X_{nc})p_z, \tag{2.16}$$

where p_z is the price of other inputs. Taking the first order conditions and assuming perfect competition yields

$$\phi_s w_s - p_s - \phi_s p_z \leq 0, \tag{2.17}$$

$$\phi_c w_c - p_c - \phi_c p_z \leq 0. \tag{2.18}$$

For a competitive packer, the first-order conditions permit three solutions: two corner solutions where the plant processes only either fed cattle or cows and an interior solution where

[5] Penn State Extension; Understanding beef carcass yields and losses during processing - August 4, 2016.

[6] The meat yield is lower, as not all bones and fat are sold to consumers at retail. For example, fat rendering is used as an input in the production of biodiesel. Out of 750 pound fed steer carcass, about 450 pounds is boneless trimmed beef, 150 pounds fat trim, and 110 pounds bone. See https://extension.sdstate.edu/how-much-meat-can-you-expect-fed-steer for more information.

it processes both. Our interest here is in a marginal plant that will process both (cattle) categories, such that the two first-order conditions hold with equality. These equations give us that $p_s = \phi_s(w_s - p_z)$ and $p_c = \phi_c(w_c - p_z)$, such that we can write the inverse demand for fed cattle as

$$p_s(X_s, X_c) = \phi_s(w_s(\phi_s X_s, \phi_c X_c) - p_z), \tag{2.19}$$

and for cows as

$$p_c(X_s, X_c) = \phi_c(w_c(\phi_s X_s, \phi_c X_c) - p_z). \tag{2.20}$$

Thus, the demands for fed cattle and cows are proportional to the beef products for the two categories and shifted down reflecting processing costs.

For the purpose of calibrating the model to the data, we must specify a functional form for the demand, which requires specifying an expression for $H(w)$. We want a distribution function defined over the positive and negative intervals, capable of capturing a wide range of preferences for beef products and possessing few parameters, so we can more easily calibrate it to the data. One such function is the logistic distribution:

$$H(w) = \frac{1}{1 + \exp\left(\frac{\mu - w}{\sigma}\right)}, \tag{2.21}$$

where μ is the mean and median willingness to pay for beef from fed cattle over beef from cows and σ is a scale parameter. The variance of the logistic distribution is $(\sigma^2\pi^2)/3$.

We can express the distribution of willingness to pay as it applies to the derived demand for cattle by a packer. From the first order conditions for a packer's profit maximization, we can write $w_s = \frac{p_s}{\phi_s} + p_z$ and $w_c = \frac{p_c}{\phi_c} + p_z$ such that $w \equiv w_s - w_c = \frac{p_s}{\phi_s} - \frac{p_c}{\phi_c} = \widetilde{p}_s - \widetilde{p}_c \equiv \widetilde{p}$. We must also adjust the units for μ and σ. In the distribution of willingness to pay for beef, μ and σ are measured in dollars per pound of beef. To modify these parameters into dollars per pound of live cattle, we multiply them by ϕ which is measured in pounds of beef per pound of live cattle such that $\widetilde{\mu} = \phi\mu$ and $\widetilde{s} = \phi s$. The distribution function becomes

$$H(\widetilde{p}) = \frac{1}{1 + \exp\left(\frac{\tilde{\mu} - \tilde{p}}{\tilde{s}}\right)}. \tag{2.22}$$

We must also adjust the parameter for the demand intensity by writing that $\widetilde{A} = \frac{A}{\phi}$ such that the demand for fed cattle is given by $X_s = \widetilde{A}(1 - H(\widetilde{p}))$ and the demand for cull cows is given by $X_c = \widetilde{A}H(\widetilde{p})$.

Calibrating the demand equations will require finding values for the parameters A, μ, s, ϕ_s and ϕ_c. We can find values for μ and s from the observed value for p and from the data. In equilibrium, as equation 2.11 shows, $\theta = p$ so that the empirical distribution parameters for the premium will follow those for θ, meaning that we can calculate $\mu = p$ and $s = \sigma_p\sqrt{3}/\pi$, where σ_p is the standard deviation for p. The only remaining parameter is A, which captures the intensity of the demand for cattle. We will find the value for A using the observed data.

2.2 Solving the model

The model comprises several equations and variables. In what follows, we will focus on the equations that describe how farmers optimize their profits by choosing to breed, slaughter, or cull cattle. After solving these equations, it will be straightforward to solve for prices and quantities.

The equations of the model are as follows:

$$gK_{t-1} - k_{3,t+1} = A\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}; \tag{2.23}$$

$$k_{10,t} + \sum_{j=7}^{9}(k_{j,t} - k_{j+1,t+1}) = A\frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}; \tag{2.24}$$

$$p_{s,t} = E_t\left[\beta V_{3,t+1} + g\beta^3 p_{s,t+3} - z_t\right]; \tag{2.25}$$

$$V_{3,t} = E_t\left[\beta V_{4,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t} \implies k_{4,t+1} = \delta_3 k_{3,t}; \tag{2.26}$$

$$V_{4,t} = E_t\left[\beta V_{5,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t} \implies k_{5,t+1} = \delta_4 k_{4,t}; \tag{2.27}$$

$$V_{5,t} = E_t\left[\beta V_{6,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t} \implies k_{6,t+1} = \delta_5 k_{5,t}; \tag{2.28}$$

$$V_{6,t} = E_t\left[\beta V_{7,t+1} + g\beta^3 p_{s,t+3} - z_t\right] > p_{c,t} \implies k_{7,t+1} = \delta_6 k_{6,t}; \tag{2.29}$$

$$V_{7,t} = \max\left\{p_{c,t}, E_t\left[\beta V_{8,t+1} + g\beta^3 p_{s,t+3} - z_t\right]\right\}; \tag{2.30}$$

$$V_{8,t} = \max\left\{p_{c,t}, E_t\left[\beta V_{9,t+1} + g\beta^3 p_{s,t+3} - z_t\right]\right\}; \tag{2.31}$$

$$V_{9,t} = \max\left\{p_{c,t}, E_t\left[\beta p_{c,t+1} + g\beta^3 p_{s,t+3} - z_t\right]\right\}; \tag{2.32}$$

$$V_{10,t} = p_{c,t} \implies k_{11,t} = 0. \tag{2.33}$$

Equation 2.23 says that the supply of fed cattle equals the demand for fed cattle. Similarly, equation 2.24 says that the supply of cull cows equals the demand for cull cows. Equation 2.25 is the arbitrage condition for fed cattle. Equations 2.26-2.29 are the arbitrage conditions for cows between 3 and 6 years of age. Note that we assume farmers never cull cows six years or younger which implies that the number of cows for these younger cohorts carries to the next year, adjusting for natural death. Equations 2.30-2.32 determine the choice between keeping cows for one more year or culling cows between 7 and 9 years of age. Finally, equation 2.33 says that all 10-year cows are culled such that there are no 11-year old cows.

The next step is to specify how farmers form their expectations about future prices. In fact, the equations 2.23-2.32 imply that farmers form price expectations for prices several years into the future. In the following subsections, we present analytical solution and numerical solution algorithm under naïve and rational price expectations. To simplify the model, we assume $\delta_j \forall j \in [3, 9] = \delta$.[7]

2.2.1 Naïve price expectations

We refer to *naïve* expectations as a situation where producers use prices in the current period as the expected prices in all future periods.[8] For example, under naïve expectations, we can write that

$$\mathrm{E}_t p_{s,t+3} = \mathrm{E}_t p_{s,t+2} = \mathrm{E}_t p_{s,t+1} = p_{s,t}, \tag{2.34}$$

and that

$$\mathrm{E}_t p_{c,t+3} = \mathrm{E}_t p_{c,t+2} = \mathrm{E}_t p_{c,t+1} = p_{c,t}. \tag{2.35}$$

Assuming naïve expectations simplifies the model quite significantly. Equations 2.23 and 2.24 remain unchanged as they only contain contemporaneous variables. For cohort from

[7]This assumption is consistent with Aadland (2004), Aadland and Bailey (2001), and Chavas (2000). Relaxing this assumption will not change the model solution.

[8]We can alternatively use the price in the previous period. This would actually make it simpler to solve the model, but it would be less realistic, as it would assume that farmers ignore the information provided by the current price.

$j = 2$ to $j = 9$, we have the following equalities

$$p_{s,t} = \beta \mathrm{E}_t V_{3,t+1} + g\beta^3 p_{s,t} - \mathrm{E}_t h_t \left(1 + \beta g(\gamma_0 + \beta\gamma_1)\right); \tag{2.36}$$

$$k_{j+1,t+1} = \delta k_{j,t} \quad \forall j \in [3, 4, 5, 6] \tag{2.37}$$

$$V_{j,t} = max\Big\{p_{c,t}, \beta \mathrm{E}_t \left[V_{j+1,t+1} + g\beta^3 p_{s,t} - E_t h_t(1 + \beta g(\gamma_0 + \beta\gamma_1))\right]\Big\} \quad \forall j \in [7, 8, 9] \tag{2.38}$$

$$V_{10,t} = p_{c,t} \implies k_{11,t} = 0. \tag{2.39}$$

Where we assume that h_t is stationary such that $\mathrm{E}_t h_t = \mathrm{E}_t h_{t+1} = \mathrm{E}_t h_{t+2}$. Under naïve expectations, our model reduces to four cases. We describe the equations for cows aged between 7 and 10 years of age below for these four cases.

2.2.1.1 Case I: Only 10-year old cows are culled

The first case is where farmers only cull 10-year old cows. In that situation, the equations for cows aged between 7 and 10 years of age below are:

$$\begin{gathered} k_{j+1,t+1} = \delta k_{j,t} \quad \forall j \in [7, 8] \\ \beta p_{c,t} + g\beta^3 p_{s,t} - \mathrm{E}_t h_t \left(1 + \beta g(\gamma_0 + \beta\gamma_1)\right) > p_{c,t} \implies k_{10,t+1} = \delta k_{9,t}; \\ V_{10,t} = p_{c,t} \implies k_{11,t} = 0. \end{gathered} \tag{2.40}$$

Observe that we do not need to solve for $\mathrm{E}_t V_{10,t+1}$, $\mathrm{E}_t V_{9,t+1}$ or $\mathrm{E}_t V_{8,t+1}$ because we assume that farmers cull older cows first. Thus, because farmers do not cull 9-year old cows, they do not cull 8 and 7-year-old cows either.

2.2.1.2 Case II: Some nine-year old cows are culled

In the second case farmers cull some nine-year old cows. The equations for cows aged between 7 and 10 years of age below are:

$$\begin{aligned} k_{j+1,t+1} &= \delta k_{j,t} \quad \forall j \in [7,8] \\ \beta p_{c,t} + g\beta^3 p_{s,t} - \mathrm{E}_t h_t \left(1 + \beta g(\gamma_0 + \beta\gamma_1)\right) = p_{c,t} &\implies k_{10,t+1} \leq \delta k_{9,t}; \\ k_{11,t} &= 0. \end{aligned} \tag{2.41}$$

Again, we do not need to solve for $\mathrm{E}_t V_{9,t+1}$ or $\mathrm{E}_t V_{8,t+1}$ because we assume that farmers cull older cows first. We solve for $k_{10,t+1}$ using the arbitrage equality for 10-year old cows.

2.2.1.3 Case III: Some eight-year old cows are culled

The third case is when farmers cull some eight-year old cows. The equations for cows aged between 7 and 10 years of age for that case are:

$$\begin{aligned} k_{8,t+1} &= \delta k_{7,t}; \\ \beta p_{c,t} + g\beta^3 p_{s,t} - \mathrm{E}_t h_t \left(1 + \beta g(\gamma_0 + \beta\gamma_1)\right) = p_{c,t} &\implies k_{9,t+1} \leq \delta k_{8,t}; \\ k_{j+1,t+1} &= 0 \quad \forall j \in [9,10] \end{aligned} \tag{2.42}$$

2.2.1.4 Case IV: Some seven-year old cows are culled

Finally, the last case is when farmers cull some or all of their seven-year old cows. The equations for cows aged between 7 and 10 years of age below are:

$$\begin{aligned} \beta p_{c,t} + g\beta^3 p_{s,t} - \mathrm{E}_t h_t \left(1 + \beta g(\gamma_0 + \beta\gamma_1)\right) \leq p_{c,t} &\implies k_{8,t+1} \leq \delta k_{7,t}; \\ k_{j+1,t+1} &= 0 \quad \forall j \in [8,9,10] \end{aligned} \tag{2.43}$$

2.2.1.5 Expected return for a two-year old heifer

Equation 2.36 contains $V_{3,t+1}$, the expected value of a three-year-old cow in the next period. This value contains expectations for the return to a cow for its entire life. We must compute the expression $V_{3,t+1}$ and assuming naïve expectations significantly simplifies the process. After a few substitutions, we can write the expression for the discounted expected return of a three-year-old cow as

$$\beta V_{3,t+1} = \beta^5 \mathrm{E}_t V_{7,t+5} + g\beta^4(1+\beta+\beta^2+\beta^3)p_{s,t} - (1+\beta+\beta^2+\beta^3)(1+\beta g(\gamma_0+\beta\gamma_1))\mathrm{E}_t h_t \quad (2.44)$$

In this expression, we stopped computing expectations for cows that are seven years old or older because we have not described yet how farmers form expectations about how long they will keep a cow.

We assume that farmers will only cull cows that are seven years or older. This means that a foreseeing farmer would assign probabilities that they will cull a cow at ages between 7 and 10 years old. In practice, we could assign these probabilities and use them to calculate $\mathrm{E}_t V_{7,t+5}$. However, we will adopt a simpler approach because in practice most cows are culled either at 9 or 10 years old. Therefore, when calculating $\mathrm{E}_t V_{7,t+5}$, we assume that a cow is culled only at the end of its useful life, i.e., 10 years old, and that market conditions do not play a role in early culling. This simplifying assumption does not play an important role in our model because the expected value of older cows is quite heavily discounted.[9]

2.2.1.6 Solution for Case I: Only 10-year old cows are culled

With the assumption that farmers expect to keep their cows until they are 10 years old, we can write after a few manipulations that

$$\beta V_{3,t+1} = \beta^8 p_{c,t} + g\beta^4 \frac{1-\beta^7}{1-\beta} p_{s,t} - \beta(1+\beta g(\gamma_0+\beta\gamma_1))\frac{1-\beta^7}{1-\beta}\mathrm{E}_t h_t. \quad (2.45)$$

We can insert 2.45 in equation 2.36 to complete and solve the model under naïve expectations.

[9]The assumption is also consistent with the often optimistic view that farmers have regarding their livestock. It might indeed be the case that farmers always expect to keep their cows for 10 years [Powell and Ward (2009)].

By plugging in 2.45 in equation 2.36 and after some manipulations, the system for Case I becomes

$$k_{11,t} = 0 \tag{2.46}$$

$$k_{j+1,t+1} = \delta k_{j,t} \quad \forall j \in [3,4,5,6,7,8,9] \tag{2.47}$$

$$p_{s,t} = \beta^8 p_{c,t} + g\beta^3 p_{s,t}\frac{1-\beta^8}{1-\beta} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\frac{1-\beta^8}{1-\beta}\mathrm{E}_t h_t \tag{2.48}$$

$$k_{10,t} = A\frac{1}{1+\exp(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}})} \tag{2.49}$$

$$gK_{t-1} - k_{3,t+1} = A\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.50}$$

2.2.1.7 Solution for Case II: Some nine-year old cows are culled

In this case we assume that farmers keep their cows until they are nine years old and then cull them. After a few manipulations $\beta V_{3.t+1}$ becomes

$$\beta V_{3,t+1} = \beta^7 p_{c,t} + g\beta^4\frac{1-\beta^6}{1-\beta}p_{s,t} - \beta(1 + \beta g(\gamma_0 + \beta\gamma_1))\frac{1-\beta^6}{1-\beta}\mathrm{E}_t h_t \tag{2.51}$$

Similar to Case I we insert 2.51 in equation 2.36 and solve the model under naïve expectations. After some manipulations, the system for Case II becomes

$$k_{11,t} = 0 \tag{2.52}$$

$$\beta p_{c,t} + g\beta^3 p_{s,t} - \mathrm{E}_t h_t(1 + \beta g(\gamma_0 + \beta\gamma_1)) = p_{c,t} \implies k_{10,t+1} \leq \delta k_{9,t} \tag{2.53}$$

$$k_{j+1,t+1} = \delta k_{j,t} \quad \forall j \in [3, 4, 5, 6, 7, 8] \tag{2.54}$$

$$p_{s,t} = \beta^7 p_{c,t} + g\beta^3 p_{s,t}\frac{1-\beta^7}{1-\beta} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\frac{1-\beta^7}{1-\beta}\mathrm{E}_t h_t \tag{2.55}$$

$$k_{9,t} + \sum_{j=7}^{8}(k_{j,t} - k_{j+1,t+1}) = A\frac{1}{1 + \exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.56}$$

$$gK_{t-1} - k_{3,t+1} = A\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1 + \exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.57}$$

For all other cases (cases III and IV) we can solve the model analogously to the previous cases. For brevity, we do not present them here. Once we have analytic expressions, all we have to do is solve for the parameters using the system of equations. We use the observed data and numerical methods to solve for the parameter values. Note that here we always have a boundary condition that is $0.5gK_{t-1} > k_{3,t+1}$. This boundary condition ensures that we calculate realistic quantities and prices.

2.2.2 Rational price expectations

We refer to *rational* expectations as a situation where the farmers use all the information available in the economy to make price expectations. The information may include the present and past prices, production, and disappearance of the fed cattle and cull cows. The system of equations under rational price expectations is as follows:

$$gK_{t-1} - k_{3,t+1} = A\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.58}$$

$$k_{10,t} + \sum_{j=7}^{9}(k_{j,t} - k_{j+1,t+1}) = A\frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.59}$$

$$p_{s,t} = \beta \mathrm{E_t} V_{3,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t \tag{2.60}$$

$$k_{4,t+1} = \delta k_{3,t} \tag{2.61}$$

$$k_{5,t+1} = \delta k_{4,t} \tag{2.62}$$

$$k_{6,t+1} = \delta k_{5,t} \tag{2.63}$$

$$k_{7,t+1} = \delta k_{6,t} \tag{2.64}$$

$$V_{7,t} = max\Big\{p_{c,t}, \beta \mathrm{E_t} V_{8,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t\Big\} \tag{2.65}$$

$$V_{8,t} = max\Big\{p_{c,t}, \beta \mathrm{E_t} V_{9,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t\Big\} \tag{2.66}$$

$$V_{9,t} = max\Big\{p_{c,t}, \beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t\Big\} \tag{2.67}$$

$$V_{10,t} = p_{c,t} \tag{2.68}$$

Assuming a farmer never culls a cow younger than six years of age, we have different conditions under different decisions of the farmers about culling the older cows. These decisions reduce to four different cases.

2.2.2.1 Case I: Only 10-year old cows are culled

In this case, farmers only cull the 10-year old cows. Hence, the equations for cows aged between 7 and 10 are as below:

$$k_{j+1,t+1} = \delta k_{j,t} \forall j \in [7, 8] \tag{2.69}$$

$$\beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t > p_{c,t} \implies k_{10,t+1} = \delta k_{9,t} \tag{2.70}$$

$$V_{10,t} = p_{c,t} \implies k_{11,t} = 0 \tag{2.71}$$

2.2.2.2 Case II: Some nine-year old cows are culled

In this case, farmers cull some 9-year old cows in addition to all 10-year old cows. The set of expressions for cows aged between 7 and 10, in this case, are as below:

$$k_{j+1,t+1} = \delta k_{j,t} \forall j \in [7, 8] \tag{2.72}$$

$$\beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t = p_{c,t} \implies k_{10,t+1} \leq \delta k_{9,t} \tag{2.73}$$

$$k_{11,t} = 0 \tag{2.74}$$

In Case I and Case II, we don't need to solve for $\mathrm{E_t} V_{9,t+1}$ and $\mathrm{E_t} V_{8,t+1}$. This is because we assume that farmers cull all the older cows first. Similar to Case I and Case II, we can also express the equation system for the cases where some 8-year old cows are culled and some 7-year old cows are culled. For brevity, we do not present them here.

2.2.2.3 Expected return for a two-year old heifer

The equation 2.60 contains the expected value of the three-year-old cow $\mathrm{E_t} V_{3,t+1}$. Contrary to naïve expectations where we simply replace the expected price with the price at t, we write the expected price as is and by iterative substitution, the equation 2.60 can be rewritten as

$$\mathrm{E_t} V_{3,t+1} = \beta^4 \mathrm{E_t} V_{7,t+5} + g \sum_{i=3}^{6} \beta^i p_{s,t+1+i} - \sum_{i=0}^{3} \beta^i z_{t+1+i} \tag{2.75}$$

2.2.2.4 Solution for Case I: Only 10-year old cows are culled

Assuming farmers expect to keep their cows until they are 10 years old, after some few manipulations 2.75 can be written as

$$\mathrm{E_t}V_{3,t+1} = \beta^7 p_{c,t+8} + g\sum_{i=3}^{9}\beta^i p_{s,t+1+i} - \sum_{i=0}^{5}\beta^i z_{t+1+i} \tag{2.76}$$

where z is the discounted holding hosts. By replacing $\mathrm{E_t}V_{3,t+1}$ in 2.60 with 2.76, we get

$$p_{s,t} = \beta^8 p_{c,t+7} + g\sum_{i=4}^{10}\beta^i p_{s,t+i} + g\beta^3\mathrm{E_t}p_{s,t+3} - \sum_{i=1}^{7}\beta^i z_{t+i} \tag{2.77}$$

Finally, by replacing 2.60 with 2.77 the final solution system for Case I is as follows

$$gK_{t-1} - k_{3,t+1} = A\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.78}$$

$$k_{10,t} + \sum_{j=7}^{9}(k_{j,t} - k_{j+1,t+1}) = A\frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.79}$$

$$p_{s,t} = \beta^8 p_{c,t+7} + g\sum_{i=4}^{10}\beta^i p_{s,t+i} + g\beta^3\mathrm{E_t}p_{s,t+3} - \sum_{i=1}^{7}\beta^i z_{t+i} \tag{2.80}$$

$$k_{j+1,t+1} = \delta k_{j,t} \quad \forall j \in [3,4,5,6,7,8] \tag{2.81}$$

$$p_{c,t} = \beta\mathrm{E_t}p_{c,t+1} + g\beta^3\mathrm{E_t}p_{s,t+3} - (1+g\beta(\gamma_0+\beta\gamma_1))\mathrm{E_t}h_t \tag{2.82}$$

2.2.2.5 Solution for Case II: Some nine-year old cows are culled

In this case, we assume farmers keep their cows until they are nine years old and then cull them. With some substitutions 2.75 can be written as

$$\mathrm{E_t}V_{3,t+1} = \beta^6 p_{c,t+7} + g\sum_{i=3}^{8}\beta^i p_{s,t+1+i} - \sum_{i=0}^{5}\beta^i z_{t+1+i} \tag{2.83}$$

where z is the discounted holding hosts. By replacing $\mathrm{E_t}V_{3,t+1}$ in 2.60 with 2.76, we get

$$p_{s,t} = \beta^7 p_{c,t+7} + g\sum_{i=4}^{9}\beta^i p_{s,t+i} + g\beta^3\mathrm{E_t}p_{s,t+3} - \sum_{i=1}^{6}\beta^i z_{t+i} \tag{2.84}$$

Finally, by replacing 2.60 with 2.84 the final solution system for Case II becomes

$$gK_{t-1} - k_{3,t+1} = A\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.85}$$

$$k_{10,t} + \sum_{j=7}^{9}(k_{j,t} - k_{j+1,t+1}) = A\frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.86}$$

$$p_{s,t} = \beta^7 p_{c,t+7} + g\sum_{i=4}^{9}\beta^i p_{s,t+i} + g\beta^3 \mathrm{E_t} p_{s,t+3} - \sum_{i=1}^{6}\beta^i z_{t+i} \tag{2.87}$$

$$k_{j+1,t+1} = \delta k_{j,t} \quad \forall j \in [3,4,5,6,7,8] \tag{2.88}$$

$$p_{c,t} = \beta \mathrm{E_t} p_{c,t+1} + g\beta^3 \mathrm{E_t} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E_t} h_t \tag{2.89}$$

The above system is the analytical solution of the rational expectations model (Case I and Case II). The solution system is solved to obtain equilibrium prices and quantities. Note that, the system of equations contains the expected price of the fed cattle three periods ahead, and the expected price of the cull cow one period ahead. We must know these expected prices to find the equilibrium solution.[10]

Under rational expectations, the producers have the ability to make expectations about the price by using all the information available to them. Forming expectations requires knowing the production (in the number of head) of the cattle in future periods. Although it is not possible to know future production with absolute certainty, an approximation of future production is sufficient to make expectations about the price. At time t, a producer knows the total breeding stock $K_t = k_{3,t} + \ldots + k_{10,t}$, the production of fed cattle sl_t, and the production of cull cows cl_t. However, in order to make expectations about the price, the production (or an approximation of the future production) must be used. We rely on the competitive storage model to construct the production of the fed cattle and cull cows into the future. A simple storage type model is specified as $Q_{t+1} = Q_t\epsilon_t + \mathrm{Storage}_t$, where Q_{t+1}

[10] We rely on numerical methods to compute the expected price.

is the production at $t+1$, Q_t, ϵ_t, Storage$_t$ are the production, production shock, and storage respectively at t. Note that the solution system includes the expected price in the future, and computing the expected price requires knowledge of random variables, hence the use of a competitive storage model to construct the production.[11]

Fed Cattle Production:

$$\begin{aligned} sl_{t+1} &= sl_{t-1}\epsilon^s_{t-1} + \text{storage}_{t-1} && (2.90)\\ &= \Big(gK_{t-2} - k_{3,t}\Big)\epsilon^s_{t-1} + \text{storage}_{t-1} && (2.91)\\ &= \Big(g - gr\Big)K_{t-2}\epsilon^s_{t-1} + \text{storage}_{t-1} && (2.92)\\ &= g\Big(1 - r\Big)K_{t-2}\epsilon^s_{t-1} + \text{storage}_{t-1} && (2.93)\\ & && (2.94) \end{aligned}$$

where g is the breeding rate, r is the rate of the new born progeny entering the breeding stock. storage$_{t-1}$ can be further decomposed as:

$$\begin{aligned} \text{storage}_{t-1} &= K_{t-1} - k_{3,t+1} && (2.95)\\ &= (1 - gr)K_{t-1} && (2.96)\\ &= (1 - gr)\delta g\Big[K_{t-2} - sl_{t-2} - cl_{t-2}\Big] && (2.97)\\ &= (1 - gr)\delta g\Big[K_{t-2} - g(1-r)K_{t-3} - (k_{9,t-2} + (1-\delta)k_{8,t-2} + (1-\delta)k_{7,t-2})\Big] && (2.98) \end{aligned}$$

[11] Numerical methods to compute expected price require integration over random variables. The production shocks in the storage model are used as random variables to compute the expected price.

Cull Cow Production:

$$cl_{t+1} = cl_t \epsilon_t^c + \text{storage}_t \tag{2.99}$$

$$= \Big[k_{9,t} + (1-\delta)k_{8,t} + (1-\delta)k_{7,t}\Big]\epsilon_t^c + \Big[(k_{9,t+1} - k_{9,t}) + (k_{8,t+1} - k_{8,t} + (k_{7,t+1} - k_{7,t}))\Big] \tag{2.100}$$

$$= \Big[k_{9,t} + (1-\delta)k_{8,t} + (1-\delta)k_{7,t}\Big]\epsilon_t^c + \Big[\delta(k_{8,t} + k_{7,t} + k_{6,t}) - (k_{7,t} + k_{8,t} + k_{9,t})\Big] \tag{2.101}$$

where the production shock of fed cattle ϵ_t^s and the production shock of cull cows ϵ_t^c are a random variable following a Gaussian distribution. The production shocks are constructed by taking the ratio of observed historical production to the constructed production from the model specification. Then, the standard deviation of the constructed production shocks is used to define the Gaussian distribution.

The equilibrium system of equations can be solved when the price expectations of fed cattle and cull cows are known. The constructed production from the competitive storage type model is utilized to determine the price and the expected price of the fed cattle and cull cows. The rational expectations model is a functional equation problem. The solution takes the form of a function rather than a finite-sized vector of prices and quantities. Therefore, the model cannot be solved analytically and requires numerical methods to solve it. The competitive storage models with rational expectations is also a functional equation problem and the solution takes functional form instead of a finite-sized vector. Therefore, numerical methods from the competitive storage literature are borrowed to find a solution to the rational expectations model.

Collocation methods are widely used in the competitive storage literature [Miranda and Fackler (2002)]. Although there are multiple methods for functional approximation, Miranda (1997) showed that solutions under collocation methods are accurate and efficient. Gouel (2013) provided various fast and precise numerical methods to solve competitive storage models which include projection methods such as the collocation method. Miranda (1997) and Gouel (2013) compared different numerical methods for solving competitive storage models and presented solution methods that are suitable to use (collocation method is one of the solution methods presented). Using the collocation method, Miranda and Schnitkey (1995) proposed a solution method to a dairy production model (collocation method is applied for a function that possessed no closed-form analytical solution). Ritten et al. (2010) used the collocation method in their stochastic dynamic model to solve for optimal rangeland stocking decisions under uncertain weather and climate conditions. Miranda and Glauber (2021) used the Chebyshev collocation method to find equilibrium market price for a production model of global agricultural commodity markets. Hence, to find a solution to the rational price expectations model, we apply the projection method. Specifically, a collocation method is applied to find an equilibrium solution.

Using the collocation method, the following system of equations is solved to determine the equilibrium prices and quantities.

$$gK_{t-1} - k_{3,t+1} = A\frac{\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.102}$$

$$k_{10,t} + \sum_{j=7}^{9}(k_{j,t} - k_{j+1,t+1}) = A\frac{1}{1+\exp\left(\frac{\tilde{\mu}-\tilde{p}}{\tilde{s}}\right)} \tag{2.103}$$

$$p_{s,t} = \beta^7 p_{c,t+7} + g\sum_{i=4}^{9}\beta^i p_{s,t+i} + g\beta^3 \mathrm{E}_{\mathrm{t}} p_{s,t+3} - \sum_{i=0}^{6}\beta^i z_{t+i} \tag{2.104}$$

$$p_{c,t} = \beta \mathrm{E}_{\mathrm{t}} p_{c,t+1} + g\beta^3 \mathrm{E}_{\mathrm{t}} p_{s,t+3} - (1 + g\beta(\gamma_0 + \beta\gamma_1))\mathrm{E}_{\mathrm{t}} h_t \tag{2.105}$$

The above system of equations is non-linear, so to solve the system of equations we need to provide the initial values for the price and the expected price. First, we determine the cattle price and use it to compute the expected price. We posit that the price of the fed cattle (cull cows) depends on the total supply of fed cattle (cull cows), demand shock, and corn price. Without assuming a functional form of the relationship, we approximate the price function by a linear combination of independent basis functions $\phi_1, \phi_2, \ldots, \phi_m$ of the supply of fed cattle (sl_t) for the fed cattle price, supply of cull cows (cl_t) for cull cow price, demand shock (ϵ_t^D), and corn price (c_t^p).

Specifically, the price of fed cattle is expressed as

$$p_{s,t} = p_{s,t}\Big(sl_t, \epsilon_t^D, c_t^p\Big) \tag{2.106}$$

$$\begin{aligned} p_{s,t} &= p_{s,t}\Big(sl_t, \epsilon_t^D, c_t^p\Big) \approx \tilde{p}_{s,t}\Big(sl_t, \epsilon_t^D, c_t^p\Big) & (2.107)\\ &= \sum_{l_1=1}^{m_1}\sum_{l_2=1}^{m_2}\sum_{l_3=1}^{m_3} c_{l_1 l_2 l_3}\phi_{s1}^{(l_1)}\Big(sl_t\Big)\phi_2^{(l_2)}\Big(\epsilon_t^D\Big)\phi_3^{(l_3)}\Big(c_t^p\Big) & (2.108)\end{aligned}$$

and the price of cull cows is expressed as

$$p_{c,t} = p_{c,t}\Big(cl_t, \epsilon_t^D, c_t^p\Big) \tag{2.109}$$

$$\begin{aligned} p_{c,t} &= p_{c,t}\Big(cl_t, \epsilon_t^D, c_t^p\Big) \approx \tilde{p}_{c,t}\Big(cl_t, \epsilon_t^D, c_t^p\Big) & (2.110)\\ &= \sum_{l_1=1}^{m_1}\sum_{l_2=1}^{m_2}\sum_{l_3=1}^{m_3} c_{l_1 l_2 l_3}\phi_{c1}^{(l_1)}\Big(cl_t\Big)\phi_2^{(l_2)}\Big(\epsilon_t^D\Big)\phi_3^{(l_3)}\Big(c_t^p\Big) & (2.111)\end{aligned}$$

where $\Big\{\phi_{s1}^{(1)}, \phi_{s1}^{(2)}, \phi_{s1}^{(3)}, \ldots, \phi_{s1}^{(m_1)}\Big\}$ and $\Big\{\phi_{c1}^{(1)}, \phi_{c1}^{(2)}, \phi_{c1}^{(3)}, \ldots, \phi_{c1}^{(m_1)}\Big\}$ are univariate basis functions of sl_t and cl_t, $\Big\{\phi_2^{(1)}, \phi_2^{(2)}, \phi_2^{(3)}, \ldots, \phi_2^{(m_2)}\Big\}$ are basis functions of the demand shock ϵ_t^D, and

$\Big\{\phi_3^{(1)}, \phi_3^{(2)}, \phi_3^{(3)}, \ldots, \phi_3^{(m_3)}\Big\}$ are basis functions of corn price c_t^p. And m_1, m_2, m_3 are the number of univariate basis functions for sl_t, cl_t, ϵ_t^D, and c_t^p respectively. The coefficient vector $c_{l_1 l_2 l_3}$ contains $M = m_1 \times m_2 \times m_3$ elements. These elements must be solved to determine the price.

With the above specification, the equations 2.85 - 2.89 are required to hold at a selected number of collocation nodes $x_0, x_1, \ldots, x_m$. The nodes must cover the range of possible values of the variables included in the approximation and don't necessarily need to be equidistant. The polynomial specification is determined by studying the functional properties of the price. The price must be non-negative and must be greater than zero indicating that there will be no corner solution and the range will be positive. Therefore a Chebyshev polynomial interpolation is used [Miranda and Fackler (2002); Judd (1998); Miranda (1997); Miranda and Glauber (1995)]. The nodes that are used are Chebyshev nodes (Chebyshev nodes are not evenly spaced and are more concentrated on the boundaries of the interval), over a bounded interval $[a, b]$ and takes the following form

$$x_i = \frac{a+b}{2} + \frac{b-a}{2}\cos\left(\frac{m-i+0.5}{m}\pi\right), \forall i = 1, 2, \ldots, m \tag{2.112}$$

Additionally, the chebyshev polynomial basis are defined recursively as

$$T_0(z) = 1 \tag{2.113}$$

$$T_1(z) = z \tag{2.114}$$

$$T_2(z) = 2z^2 - 1 \tag{2.115}$$

$$T_{m+1}(z) = 2zT_{m-1}(z) - T_{m-2}(z) \tag{2.116}$$

where $z_i = \frac{2(x_i-a)}{b-a} - 1, \forall i = 1, 2, \ldots, m$ is the normalized node such that the polynomials are defined on the domain $[-1, 1]$. Alternatively, the chebyshev polynomials can also be formulated using a trigonometric definition $T_m(z) = \cos(\text{arccos(z)m})$.

The system of equations that needs to be solved contain the expected price of the fed cattle and cull cows. Determining the expected price requires numerical integration. A Gaussian Quadrature for integration is used to compute the expected price [Miranda and Fackler (2002); Judd (1998)]. In the Gaussian quadrature for the integration method, the continuous distribution is approximated by a finite number of discrete points. The expected price is then computed by assigning weights to the Gaussian nodes and then by taking a weighted average of all the nodes. The expected price for fed cattle and cull cows are specified as below:

$$\mathrm{E}_t\Big[p_{s,t+3}\Big] = \frac{1}{n}\sum_{i=1}^{n}\int\int \tilde{p}_{s,t+3}\Big(sl\left(sl_{t+1}, \mathrm{E}_t(p_{s,t+3})\right)\epsilon^{s}_{t+1,j} + \text{storage}_{t+1}, \epsilon^{D}_{t+3,l}, c^{p}_{t}\Big)\mathrm{d}\epsilon^{s}\mathrm{d}\epsilon^{D} \tag{2.117}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\sum_{j,l=1}^{M} w_{j,l}\tilde{p}_{s,t+3}\Big(sl\left(sl_{t+1}, \mathrm{E}_t(p_{s,t+3})\right)\epsilon^{s}_{t+1,j} + \text{storage}_{t+1}, \epsilon^{D}_{t+1,l}, c^{p}_{t}\Big) \tag{2.118}$$

$$\mathrm{E}_t\Big[p_{c,t+1}\Big] = \frac{1}{n}\sum_{i=1}^{n}\int\int \tilde{p}_{c,t+1}\Big(cl\left(cl_{t}, \mathrm{E}_t(p_{c,t+1})\right)\epsilon^{c}_{t,j} + \text{storage}_{t}, \epsilon^{D}_{t+1,l}, c^{p}_{t}\Big)\mathrm{d}\epsilon^{c}\mathrm{d}\epsilon^{D} \tag{2.119}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\sum_{j,l=1}^{M} w_{j,l}\tilde{p}_{c,t+1}\Big(cl\left(cl_{t}, \mathrm{E}_t(p_{c,t+1})\right)\epsilon^{s}_{t,j} + \text{storage}_{t}, \epsilon^{D}_{t+1,l}, c^{p}_{t}\Big) \tag{2.120}$$

The price of the fed cattle and cull cows must satisfy the equilibrium conditions. In equilibrium, the system of equations is non-linear. Therefore, we provide the estimated price and expected price from the above price approximation using Gaussian quadrature method as starting points to solve the system of equations. Additionally, we provide bounds for the price so the price satisfies the equilibrium conditions.

Equations 2.93 and 2.101 are used to compute the fed cattle and cull cow production and corresponding chebyshev nodes. In order to attain the equilibrium price, a multi-year iterative method is performed using the chebyshev nodes of fed cattle production, cull cow production, production shocks, and demand shocks. Using the iterative algorithm, the prices are approximated, expected prices are computed, and the results are used as initial values to solve the system of equations 2.85 - 2.89. In particular, we solve 2.85, 2.86, 2.87, and 2.89. Where equations 2.85 and 2.86 are the equilibrium conditions (supply equal demand) for the fed cattle and cull cows respectively. Equations 2.87 and 2.89 contain both price and expected price conditions.

An initial guess of the price is made and the price coefficients are determined (from the linear price function relationship) as a beginning step before the following multi-year iteration begins. The initial guess of the price (both fed cattle and cull cows) along with the computed expected price are used to solve the system of equations simultaneously. A non-linear least squares estimation method is used to solve the system. The prices that solve the system are then used to update the coefficients and are compared with the previous iteration coefficients. A simple *euclidean* distance is measured between the updated and previous iteration coefficients. The euclidean distance is then compared with a predetermined tolerance level. If the euclidean distance is below the predetermined tolerance level, the iteration stops and the updated coefficient vector is the part of the solution. If the euclidean distance is above the predetermined tolerance level, the guessed coefficient vector is replaced with the updated coefficient vector, and the iteration continues until the tolerance level is met.

2.2.2.6 The iterative algorithm

1. Specification of the Chebyshev nodes and polynomials

   Using 2.112 and 2.116 chebyshev nodes and chebyshev polynomials are defined. The selection criteria of the number of polynomials depend on the execution time and precision. A higher number of polynomials means more precision, but the execution time increases as well. In this work, for both fed cattle and cull cows, $m_1 = m_2 = m_3 = m$ number of independent chebyshev polynomials for each variable is used to approximate the price function. The variables for fed cattle and cull cows are $(sl_t, \epsilon_t^D, c_t^p)$ and $(cl_t, \epsilon_t^D, c_t^p)$ respectively.[12] Therefore, a total number of $M = m_1 \times m_2 \times m_3 = m^3$ independent chebyshev polynomials are constructed separately for fed cattle and cull cow price approximation. The chebyshev nodes are selected over the domains $[sl_{min}, sl_{max}]$, $[cl_{min}, cl_{max}]$, $[\epsilon_{min}^D, \epsilon_{max}^D]$, and $[c_{min}^p, c_{max}^p]$. Historical data are used to define the do-

[12]Note that the state variables for both fed cattle and cull cows price approximation are the same except for the production and the corresponding production shock.

mains of each variable that go into the price approximation. Following the chebyshev polynomial structure, m chebyshev nodes for each variable are determined. For both fed cattle and cull cow price approximation, a grid of $M = m^3$ interpolation nodes is constructed by using a cartesian product of each univariate interpolation node. The number of nodes is chosen to optimize the accuracy and execution time:

$$\left\{ sl_{l_1}, \epsilon^D_{l_2}, c^p_{l_3} \mid l_1, l_2, l_3 = 1, 2, \ldots, m \right\} \tag{2.121}$$

$$\left\{ cl_{l_1}, \epsilon^D_{l_2}, c^p_{l_3} \mid l_1, l_2, l_3 = 1, 2, \ldots, m \right\} \tag{2.122}$$

2. Start with initial guess for the coefficients

   Using the guessed price, the initial guess for the coefficients are determined. The initial guess of the coefficients are then applied to the price approximation functions

$$\tilde{p}_{s,t} = \sum_{l_1=1}^{m_1} \sum_{l_2=1}^{m_2} \sum_{l_3=1}^{m_3} c_{l_1 l_2 l_3,t} \phi^{(l_1)}_{s1} \phi^{(l_2)}_{2} \phi^{(l_3)}_{3} \tag{2.123}$$

$$\tilde{p}_{c,t} = \sum_{l_1=1}^{m_1} \sum_{l_2=1}^{m_2} \sum_{l_3=1}^{m_3} c_{l_1 l_2 l_3,t} \phi^{(l_1)}_{c1} \phi^{(l_2)}_{2} \phi^{(l_3)}_{3} \tag{2.124}$$

   Once the price is approximated, using 2.118 and 2.120 the expected price of fed cattle and cull cow is computed, the chebyshev node $(sl^1_t, \epsilon^{D1}_t, c^{p1}_t)$, $(cl^1_t, \epsilon^{D1}_t, c^{p1}_t)$ is applied to the system of equations and solved with approximated price and the expected price as initial values. This is repeated for all the m^3 chebyshev nodes to get a complete set of fed cattle price, expected fed cattle price, cull cow price, and expected cull cow price:

$$\mathbf{p_{s,t}} = \begin{pmatrix} p^1_{s,t} \\ p^2_{s,t} \\ \vdots \\ p^M_{s,t} \end{pmatrix}; \mathbf{p_{c,t}} = \begin{pmatrix} p^1_{c,t} \\ p^2_{c,t} \\ \vdots \\ p^M_{c,t} \end{pmatrix}; \mathrm{E}_{\mathbf{t}}[\mathbf{p_{s,t+3}}] = \begin{pmatrix} \mathrm{E}_t[p_{s,t+3}]^1 \\ \mathrm{E}_t[p_{s,t+3}]^2 \\ \vdots \\ \mathrm{E}_t[p_{s,t+3}]^M \end{pmatrix}; \mathrm{E}_{\mathbf{t}}[\mathbf{p_{c,t+1}}] = \begin{pmatrix} \mathrm{E}_t[p_{c,t+1}]^1 \\ \mathrm{E}_t[p_{c,t+1}]^2 \\ \vdots \\ \mathrm{E}_t[p_{c,t+1}]^M \end{pmatrix} \tag{2.125}$$

3. Update the coefficients

   A $M \times M$ interpolation matrix Φ (for both fed cattle and cull cow) is determined. Each element in the interpolation matrix is defined by evaluating each chebyshev polynomial at each interpolation node and the matrix is specified as below

$$\Phi_{M\times M} = \begin{bmatrix} \phi_1^{(1)}\phi_2^{(1)}\phi_3^{(1)}\mid_1 & \cdots & \phi_1^{(m_1)}\phi_2^{(m_2)}\phi_3^{(m_3)}\mid_1 \\ \phi_1^{(1)}\phi_2^{(1)}\phi_3^{(1)}\mid_2 & \cdots & \phi_1^{(m_1)}\phi_2^{(m_2)}\phi_3^{(m_3)}\mid_2 \\ \vdots & \ddots & \vdots \\ \phi_1^{(1)}\phi_2^{(1)}\phi_3^{(1)}\mid_M & \cdots & \phi_1^{(m_1)}\phi_2^{(m_2)}\phi_3^{(m_3)}\mid_M \end{bmatrix} \tag{2.126}$$

   In a simpler matrix notation, the above $M \times M$ interpolation matrix can be specified by a tensor product of univariate $(m \times m)$ interpolation matrices as $\Phi_{M\times M} = \Phi_1 \otimes \Phi_2 \otimes \Phi_3$. Using the relationship of the price, the approximation function is :

$$\mathbf{p}_{h,t} = \begin{pmatrix} p_{h,t}^1 \\ p_{h,t}^2 \\ \vdots \\ p_{h,t}^M \end{pmatrix} = \Phi \times c_1 \quad \text{for} \quad h = \{s, c\} \tag{2.127}$$

   The coefficients are computed and updated by simply solving the linear interpolation equation 2.127, as $c_1 = (\Phi_{M\times M})^{-1}\mathbf{p}_{h,t}$ for $h = \{s, c\}$. Note that, if used higher number of nodes, we don't have to invert the $M \times M$ interpolation matrix as this will increase the complexity with each additional node. Instead, we can invert the individual univariate interpolation matrices and multiply them together.

4. Equilibrium conditions

   After the first iteration, using the updated price, for each node, the differences between the estimated supply and demand are calculated for both fed cattle and cull cows. The computed differences are then used to give prices a specific direction to hold in equilibrium (to avoid local optimum solutions and to increase the precision). At a given node, if the difference between the supply and demand is positive, then the price is reduced and vice versa. The updated prices, along with the expected prices, are used again to solve the model. This is repeated until the price reaches equilibrium and the difference between supply and demand reaches a predetermined tolerance level. The fed cattle production of fed cattle and the production of cull cows at the equilibrium price are also determined for each node.

Finally, the equilibrium prices are used to update the coefficients and the iteration is continued with the equilibrium prices and quantities until the coefficients converge.

2.3 Projections framework

The parameter estimates from the fitted model are used to project the prices and quantities into the future. In particular, we use $\tilde{\mu}$ and $\tilde{s}$. In addition, we also use the fitted fed cattle price $\hat{p}_s$, cull cow price $\hat{p}_c$, fed cattle expected price $\mathrm{E}[\hat{p}_{s,t+3}]$, cull cow expected price $\mathrm{E}[\hat{p}_{c,t+1}]$, fed cattle production $\hat{sl}$, cull cow production $\hat{cl}$, and the total demand for beef $\hat{A}$. All of these estimates are from the year t.

Projecting future prices and quantities require the knowledge of replacement heifers. In particular, the number of heifers that will be added to the breeding stock each year in the future. The following system of equations is solved to determine future replacement heifers:

$$sl_{t+1} = gK_{t+1} - k_{3,t+2} \tag{2.128}$$

$$\begin{aligned} cl_{t+1} &= k_{9,t+1} + (k_{8,t+1} - k_{9,t+2}) + (k_{7,t+1} - k_{8,t+2}) \\ &= \frac{\delta^4}{\gamma^7}\Big[\delta^2 + (1-\delta)\gamma(\delta+\gamma)\Big] \\ &\Big[k_{3,t+2} - \eta\gamma^4 k_{0,t-6} + \gamma^3 k_{0,t-5} + \gamma^2 k_{0,t-4} + \gamma k_{0,t-3} + k_{0,t-2}\Big] \\ &- \frac{\delta^5}{\gamma^2}\eta\Big[\delta\gamma k_{0,t-8} + (\delta + (1-\delta)\gamma)k_{0,t-7}\Big] \end{aligned} \tag{2.129}$$

where sl_{t+1}, cl_{t+1}, K_{t+1} are the fed cattle production, cull cow production, and total breeding stock respectively at $t+1$. $k_{3,t+2}$ is the replacement heifers that need to be determined. To elaborate further, $k_{0,t-i} \forall j \in [2,8]$ is the calf crop at $t-i$. Additionally, γ and η are the parameters determined from the linear relationship $k_{3,t+1} = \gamma k_{3,t} + \eta k_{0,t-3}$, where $k_{3,t+1}, k_{3,t}$ are replacement heifers and a lag of replacement heifers, and $k_{0,t-3}$ is the calf-crop at $t-3$. The proof for 2.129 is provided in the appendix.

In order to solve 2.128 and 2.129, we must know the total breeding stock in those years. Using the historical data of the total breeding inventory in the U.S., we fit a linear model. An Autoregressive Integrated Moving Average (ARIMA) model is selected to fit the breeding inventory. Briefly, a linear regression model is built with specified lags to breeding stock observations with a moving average window. The breeding stock data from 1924 to 2020 are used to fit the following linear model.[13]

$$K_t = \sum_{i=1}^{2} \psi_i K_{t-1} + \sum_{j=1}^{3} \theta_j Z_{t-j} + Z_t \tag{2.130}$$

The 2.130 model estimates are used to determine projections (with a 95% confidence interval) of the total breeding stock into the future.

A multi-year algorithm is performed to project the production and prices into the future. This is a cyclical algorithm, meaning the result of each iteration is used to project prices and quantities of the next subsequent year. The breeding stock projections are used as input in the cyclical algorithm.

2.3.1 Multi-year cyclical algorithm for projection of prices and quantities

1. For the first iteration, using the fitted price $\hat{p}_{s,t}$, $\hat{p}_{c,t}$, demand $\hat{A}_t$, and the model parameter estimates $\tilde{\mu}_t$ and $\tilde{s}_t$, the fed cattle production sl_{t+1} and cull cow production cl_{t+1} are computed (The estimated prices and model parameters are used to obtain the share metric containing the proportion of consumer demand for fed cattle meat and cull cow meat separately. This metric is then used along with the estimated demand to determine the corresponding production).

2. sl_{t+1}, cl_{t+1} from the step 1, along with the projected breeding stock K_{t+1} for $t+1$ are used in equations 2.128 and 2.129 to solve for replacement heifers $k_{3,t+2}$.

3. Using the estimated $k_{3,t+2}$ (in step 2) along with K_{t+1}, the fed cattle production sl_{t+1}, cull cow production cl_{t+1}, and total demand for meat A_{t+1} is updated.

[13] Our linear model is consistent with Rosen et al. (1994).

4. The updated production and demand from step 3, the prices and the expected prices from step 1 (used as starting values) are used to solve the system of equations 2.102 - 2.105. The result of this step contain projected prices ($\hat{p}_{s,t+1}$ and $\hat{p}_{c,t+1}$), and quantities ($\hat{sl}_{t+1}$, $\hat{cl}_{t+1}$, and $\hat{A}_{t+1}$). This is the end of the first iteration.

5. For the second iteration, the results from step 4 are used together with the model parameter estimates ($\tilde{\mu}_t$ and $\tilde{s}_t$) and the projected total breeding stocks, to repeat steps 2 through 4.

6. Step 5 is repeated until the data for the projected total breeding stock is available. This is the end of the cyclical iteration to project the prices and quantities into the future.

To project a 95% confidence interval, the above cyclical algorithm is repeated by simply replacing the projected breeding stock with its 95% projected upper and lower bound counterparts.

2.4 Data

The dynamic model depends exclusively on the data which are measured consistently over time and are publicly available. We use livestock industry data compiled and distributed by various U.S. Department of Agriculture agencies. Our data includes total beef cattle inventory, calf crop, inventory of replacement heifers, prices received by the farmers for fed cattle and cows, numbers of animals slaughtered, and dressed weight of slaughtered animals.

The National Agricultural Statistics Service (NASS) of the USDA [USDA-NASS (2022c)] compiles beef cow inventory data every year and provides a measure in January and July. January measures of annual total beef cows inventory and replacement heifers are used in the model. With these measures, we construct the cattle of all age groups in any year.[14] Knowing the age groups in a given year is important as we use these numbers to determine the annual domestic fed cattle production and cull cow production. Table 2.1 provides the constructed age distribution of cattle in the U.S.[15]

The constructed domestic supply of fed cattle and cull cows is converted from the units of the number of head to pounds of meat. We use dressed weights of the slaughtered animals to calculate the meat supply in pounds. Using the monthly dressed weights provided by USDA NASS [USDA-NASS (2022d)], we compute the annual average dressed weights of fed cattle (includes steers & heifers) and cull cows.[16] The annual average dressed weights are used to determine the domestic production of fed cattle and cull cow meat in pounds. With changes in the weight of the cattle over time, it is important to use dressed weights (using a constant weight could deviate the model from the trends we observe in the real world).

[14]The survival rate of cows δ in our model makes it easy to compile the age distribution of cows.

[15]For brevity, we provide data for the latest decade.

[16]USDA NASS compiles these data from the reports of Food Safety and Inspection Service (FSIS) and the USDA along with data from state-administered non-federally inspected (NFI) slaughter plants.

Table 2.1 Age distribution of cattle in the U.S.(in number of head)

Year	K	k_3	k_4	k_5	k_6	k_7	k_8	k_9
2010	31439900	5550200	5364270	5266448	5027218	4592268	4262218	1377278
2011	30912600	5443000	5272690	5096056	5003126	4775857	4362654	959216
2012	30281900	5134600	5170850	5009056	4841254	4752970	4537065	836107
2013	29631300	5280600	4877870	4912308	4758603	4599191	4515321	687408
2014	28956400	5429200	5016570	4633976	4666692	4520673	4369231	320057
2015	29332100	5556300	5157740	4765742	4402278	4433358	4294639	722044
2016	30163800	6086400	5278485	4899853	4527454	4182164	4211690	977754
2017	31170700	6335200	5782080	5014561	4654860	4301082	3973056	1109862
2018	31466200	6363200	6018440	5492976	4763833	4422117	4086028	319606
2019	31690700	6108200	6045040	5717518	5218327	4525641	4201011	125037
2020	31338700	5884900	5802790	5742788	5431642	4957411	4299359	780190

Note: Here K represents beef cows total inventory (publicly available), k_3 represents the replacement heifers (publicly available), and k_4 to k_9 represents the cows of ages (constructed by the model) from 4 to 9 years.

In addition to the domestic quantities, we also incorporate animal imports and exports in the model. The USDA Foreign Agricultural Service (FAS) Production, Supply and Distribution (PSD) [USDA-PSD (2022)] compiles annual cattle numbers that include production, imports, exports, and more. We utilize the import and export animal numbers for accuracy purposes. In Tables 2.2 and 2.3 the annual supply of the fed cattle and cull cows are listed in the number of heads and in pounds of meat, which are constructed based on age distribution, imports, and exports. Table 2.4 contains the dressing weights with which the supply is converted from the number of heads to the pounds of meat.

Our model also includes the prices received by the producers for fed cattle (steers & heifers) and cows. Prices play a key role in the model. From USDA NASS [USDA-NASS (2022a)] monthly prices, we calculate the average annual price of cows, steers and heifers. Table 2.5 lists the calculated annual average prices. These prices are further converted into \$/pound.

Table 2.2 Annual Supply of Fed Cattle

Year	Number of head	Meat in billion pounds
2010	27589986	23.04
2011	27275103	22.93
2012	26796622	23.01
2013	25816243	22.23
2014	25436061	22.19
2015	23912308	21.33
2016	23755937	21.17
2017	24509686	21.52
2018	25782379	22.70
2019	26373314	23.18
2020	26724079	24.24

Calf crop data are used extensively in the model. Specifically, we use annual calf crop data available from USDA NASS [USDA-NASS (2022b)] in the model projection framework. Table 2.6 lists annual calf crop data in the number of head utilized in fitting the model.

Fixed parameters in the model are compiled from the literature [Aadland (2004); Aadland and Bailey (2001); Baak (1999); Rosen et al. (1994)] and are set constant. Table 2.7 lists the fixed model parameters $(\beta, \delta, g, \gamma_0, \gamma_1, \phi)$ that are set constant for model fitting.

Table 2.3 Annual Supply of Cull Cows

Year	Number of head	Meat in billion pounds
2010	4909893	2.98
2011	4724556	2.82
2012	4923412	2.99
2013	5112631	3.17
2014	4193278	2.63
2015	4260597	2.74
2016	4288690	2.76
2017	4978365	3.20
2018	4626740	2.99
2019	4427294	2.83
2020	4547230	2.92

Table 2.4 Annual Average Dressed Weight (in pounds)

Year	Fed Cattle	Cows
2010	835.17	607.25
2011	840.83	596.58
2012	858.75	608.08
2013	863.50	619.83
2014	872.50	627.33
2015	892.00	644.25
2016	891.00	642.75
2017	878.00	642.58
2018	880.50	645.33
2019	879.00	639.50
2020	907.17	641.08

Table 2.5 Annual Average Price (in $/CWT)

Year	Fed Cattle	Cows
2010	97.23	54.93
2011	117.17	71.58
2012	123.08	81.97
2013	126.33	82.29
2014	154.33	108.22
2015	149.33	104.11
2016	121.67	74.89
2017	121.92	69.56
2018	117.83	62.56
2019	118.08	62.29
2020	110.67	65.27

Table 2.6 Annual Calf Crop (in number of head)

Year	Calf Crop
2010	35739800
2011	35357200
2012	34469000
2013	33630000
2014	33522000
2015	34086700
2016	35062700
2017	35758200
2018	36312700
2019	35591600
2020	35495500

Table 2.7 Fixed Model Parameters

Parameter	Value
β	0.98
δ	0.95
g	0.97
γ_0	0.90
γ_1	0.95
ϕ	0.63

2.5 Numerical solution of the model

Using the constructed and available data along with the fixed parameters, the model is calibrated to capture the dynamics of U.S. beef cattle. The algorithm is written in R programming language and uses a simple sum squared error as the loss function. Since the system of equations is non-linear, a non-linear least squares method is employed in the iterative algorithm 2.2.2.6. The estimated parameters, fitted prices, and quantities are obtained for the model under both naïve price expectations and rational price expectations. The solution of the model under rational expectations is used to project the prices and quantities into the future. Consequently, we present the numerical solution of the model under rational price expectations. In what follows, we present various tables and figures that include estimated parameters ($\tilde{\mu}$ and $\tilde{s}$), fitted prices ($\hat{p_s}$ and $\hat{p_c}$), and fitted quantities ($\hat{sl}$ and $\hat{cl}$). We present the median of the results of the iterative algorithm 2.2.2.6 in the tables.

The following Table 2.8 lists the estimated parameters of the model. Instead of fixing the parameters to a single value, we keep them dynamic and calibrate them for each year. This is intended to reflect the changes observed in the data in consumer preferences for the fed cattle meat and cull cow meat, and reflect that in the supply of the corresponding meat. Perhaps this is also one of the novel features of the model. Tables 2.9 and 2.10 provide the model fitted and observed prices and quantities, respectively. Figures 2.1 and 2.2 illustrate the observed and the model fitted (both the mean and the median of the iteration results) prices of fed cattle and cull cows, respectively. Using the fitted quantities, the cattle inventories are replicated and illustrated in Figures 2.3 and 2.4.

Table 2.8 Estimated Parameters

Year	$\tilde{\mu}$	$\tilde{s}$
2010	2.237	0.820
2011	2.105	0.738
2012	2.155	0.750
2013	2.118	0.761
2014	2.092	0.729
2015	2.137	0.760
2016	2.055	0.726
2017	2.044	0.670
2018	2.108	0.665
2019	2.121	0.666
2020	2.112	0.743

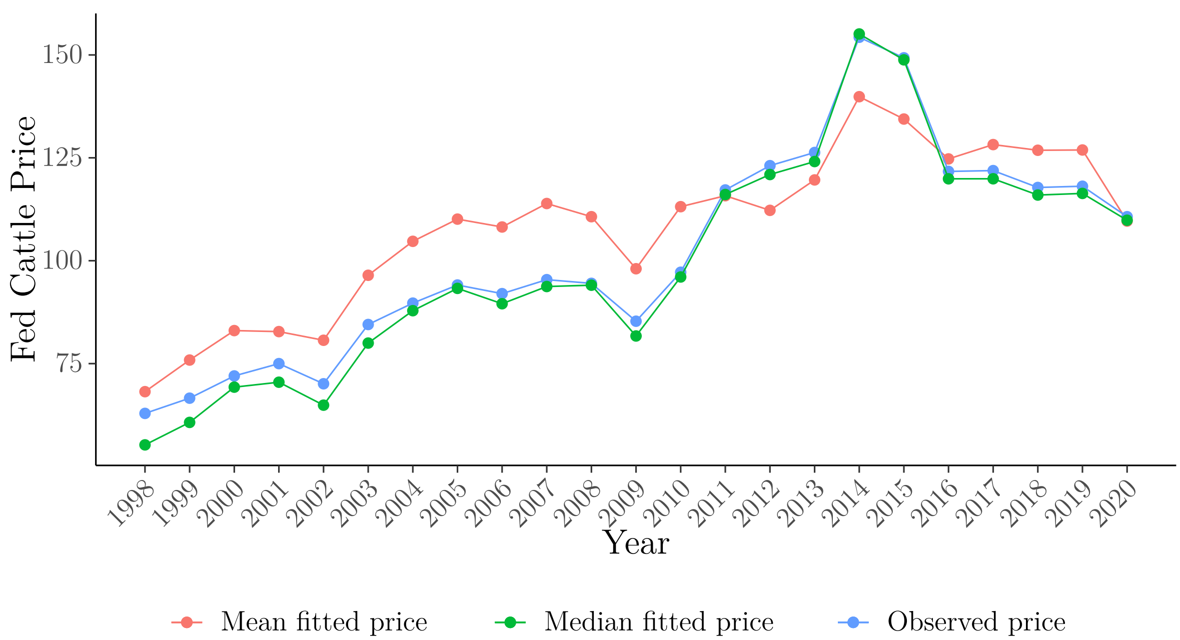

Figure 2.1 Observed and fitted fed cattle price ($/CWT)

Table 2.9 Observed & Fitted Prices (in \$/CWT)

Year	p_s	$\hat{p}_s$	p_c	$\hat{p}_c$
2010	97.2	96.05	54.9	61.89
2011	117.2	116.09	71.6	78.24
2012	123.1	120.94	82.0	86.05
2013	126.3	124.10	82.3	87.50
2014	154.3	155.10	108.2	112.30
2015	149.3	148.81	104.1	107.39
2016	121.7	119.90	74.9	76.39
2017	121.9	119.91	69.6	73.41
2018	117.8	115.96	62.6	68.83
2019	118.1	116.35	62.3	68.69
2020	110.7	109.84	65.3	72.18

Note: $\hat{p}_s$ and $\hat{p}_c$ denote the fitted fed cattle price and cull cow price respectively. p_s and p_c denote the observed.

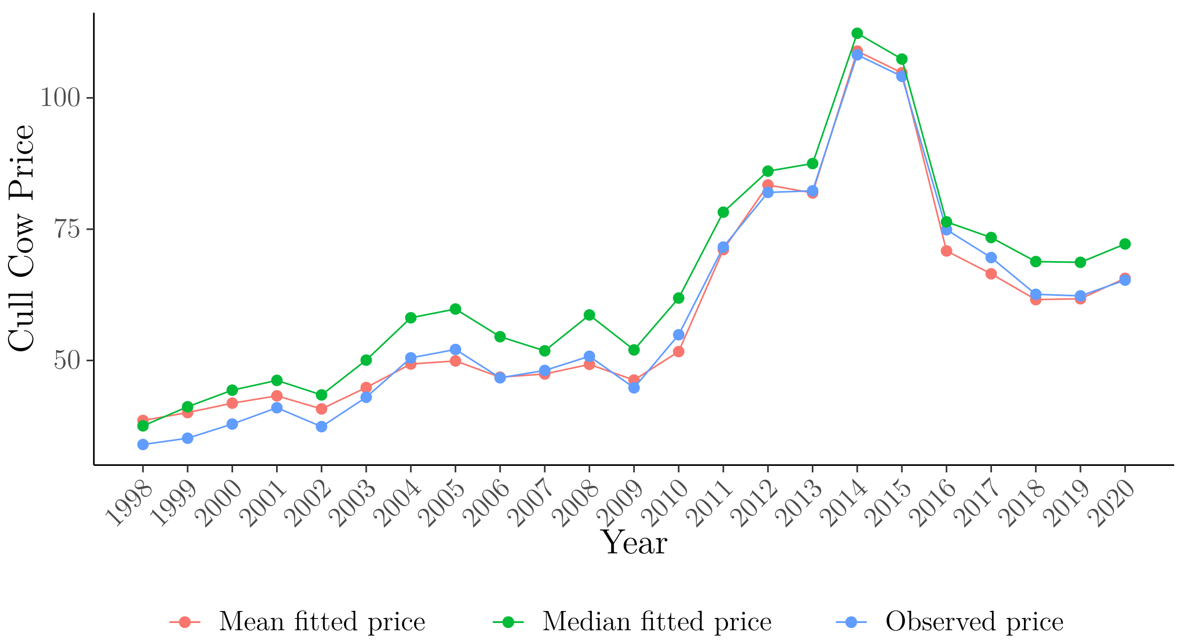

Figure 2.2 Observed and fitted cull cow price (\$/CWT)

Table 2.10 Observed & Fitted Quantities (in billion pounds)

Year	sl	$\hat{sl}$	cl	$\hat{cl}$
2010	21.85	22.04	2.83	2.82
2011	22.46	21.99	2.76	2.82
2012	22.73	21.74	2.96	2.78
2013	21.55	21.57	3.06	2.76
2014	21.74	20.33	2.58	2.59
2015	20.19	19.91	2.60	2.54
2016	20.23	21.19	2.63	2.71
2017	22.05	21.98	3.28	2.81
2018	23.34	22.59	2.99	2.89
2019	23.65	22.84	2.46	2.92
2020	25.02	22.76	2.60	2.91

Note: $\hat{sl}$ and $\hat{cl}$ denote the fitted fed cattle and cull cow meat respectively. sl and cl denote the observed.

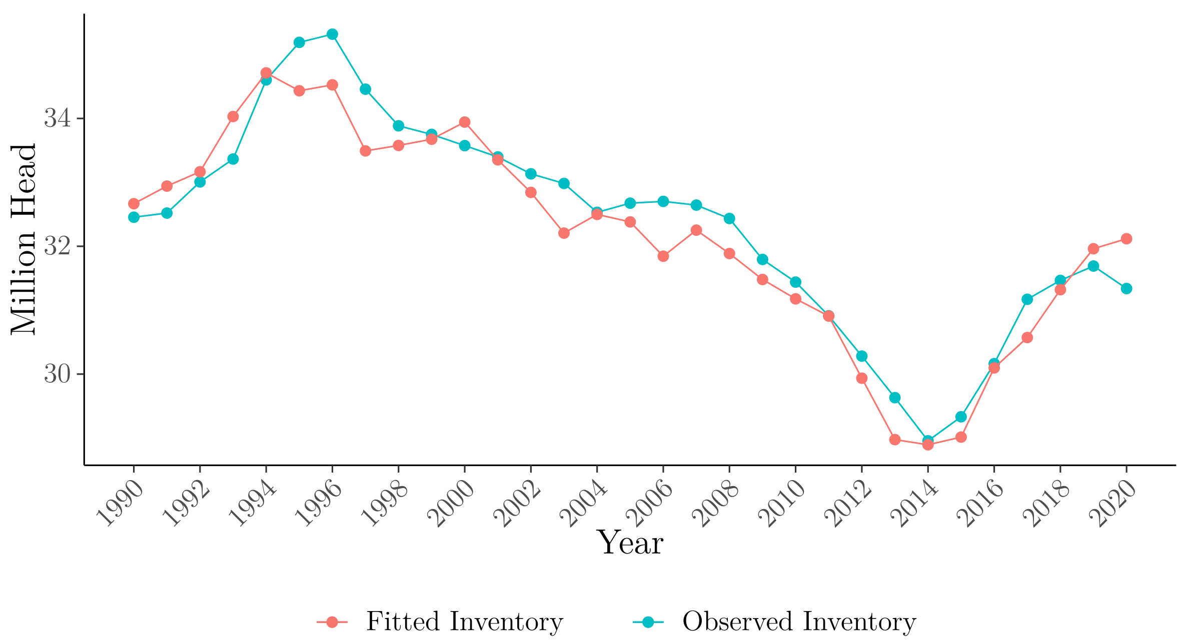

Figure 2.3 U.S. beef cow inventories and model fitted inventories

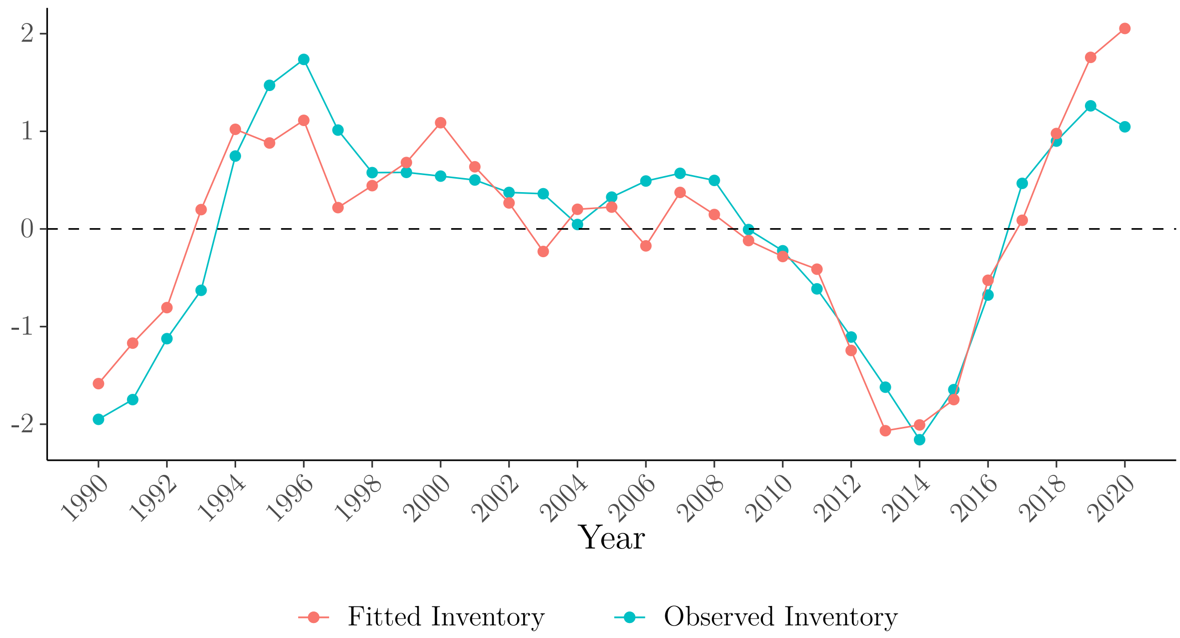

Figure 2.4 U.S. beef cow inventories and model fitted inventories - Detrended

From the above results, it can be seen that the dynamic model fits the observed data adequately. In particular, the median of the prices of the iterative algorithm is in the close neighborhood of the observed prices. Although the algorithm's fitted quantities are not that close to the observed prices, they follow the trend of the observed quantities. To support the claim, the model fits the observed data, a percent error (in unit-free from) is computed using $e = (O - \hat{M})/O$, where O and $\hat{M}$ are observed and fitted respectively. The model fits the data with a median (maximum) error of 2.66% (12.16%), -10.44% (9.70%), 3.40% (9.45%), and 1.79% (14.16%) for fed cattle price, cull cow price, fed cattle supply, and cull cow supply, respectively, strongly supporting the model fitness to the observed data. Our replication of the cattle inventories from the fitted results follows the observed inventories and demonstrates the ability of the model to capture the observed dynamics in beef cattle inventories.

2.6 Model projections

The estimated parameters, fitted prices, and fitted quantities of the model are used to project the prices and supply into the future. We use the 2020 estimates from the fitted model to perform our multi-year cyclical algorithm 2.3.1. But first, we present the regression estimates of the linear model 2.130 in the following Table 2.11. The projections (with 95% confidence interval) of total breeding stock into the future using the regression estimates of 2.130 are presented in Table 2.12.

Table 2.11 Regression Estimates of 2.130

Parameter	Estimate	Standard Error
ψ_1	0.6097	0.2162
ψ_2	-0.4947	0.1723
θ_1	0.1403	0.2169
θ_2	0.6389	0.0937
θ_3	0.3400	0.1199

Note: $\psi_j \forall j \in [1, 2]$ and $\theta_j \forall j \in [1, 2, 3]$ denote the AR and MA parameters

Table 2.12 Projections of total breeding stock with a 95% confidence interval

Year	$K_{L,95}$	K	$K_{U,95}$
2021	29609589	31051044	32492498
2022	28157618	31063066	33968514
2023	26693220	31157824	35622427
2024	25298713	31209654	37120596
2025	24199626	31194386	38189146
2026	23373813	31159438	38945062
2027	22693681	31145681	39597680
2028	22055784	31154580	40253376
2029	21424517	31166811	40909104
2030	20814139	31169867	41525595

Note: K,$K_{L,95}$, and $K_{U,95}$ denote the projected, lower 95%, and upper 95% breeding stock respectively.

As stated in the model projection framework, the projections of stock in Table 2.12 along with the model estimates from 2020 are used in the multi-year cyclical algorithm 2.3.1 to project the fed cattle price, cull cow price, fed cattle supply, cull cow supply, and total supply into the future. Some assumptions about the supply are made in the simulation. A subsistence level is assumed for the production of fed cattle and cull cows. Upon analyzing the historical supply of the fed cattle and cull cow meat in the United States, 19.01 and 1.01 billion pounds of meat are set as a floor for fed cattle and cull cow production respectively. This assumption is made to project realistic prices and quantities.[17] Thus, in our cyclical algorithm, if the algorithm determines a supply below the subsistence level, we increase the production to the above-assumed level.[18]

To validate the dynamic model, we compare our model projections to the well-recognized USDA long-term projections [USDA (2022)] and FAPRI projections [FAPRI (2022)]. In particular, we compare the total beef production and the fed cattle price projections. In addition, we present the projected cull cow prices, and the production of fed cattle meat and cull cow meat (both types of meat eventually contribute to total production, but the projection of each meat type is exclusive of the model). Tables 2.13, 2.14, 2.15, and 2.15 list the projected fed cattle price, cull cow price, fed cattle production, and cull cow production, respectively, through 2031. The corresponding plots of the projected fed cattle price, cull cow price, fed cattle production, and cull cow production are illustrated in Figures 2.5, 2.6, 2.7, and 2.8, respectively. Simple plots of the comparable model projections along with USDA and FAPRI projections are also presented. Figures 2.9 and 2.10 illustrate the fed cattle price and the total supply of the dynamic model, USDA, and FAPRI projections.

[17] Our subsistence level assumption is applied specifically when projecting lower 95% bound.

[18] One can think of this assumption as increasing imports so that they reach the subsistence level of production.

Table 2.13 Projections of fed cattle price (in $/CWT) with 95% confidence interval

Year	$p_{s_{l,95}}$	p_s	$p_{s_{u,95}}$
2021	126.40	126.40	126.40
2022	130.37	133.62	136.96
2023	127.00	134.39	140.14
2024	124.36	135.05	143.17
2025	121.41	135.58	145.76
2026	118.85	136.00	147.70
2027	116.52	136.37	149.34
2028	115.60	136.79	150.71
2029	115.61	137.21	152.02
2030	115.57	137.60	153.29
2031	115.56	138.01	154.50

Note: p_s,$p_{s_{l,95}}$, and $p_{s_{u,95}}$ denote the projected, lower 95%, and upper 95% of the fed cattle price respectively.

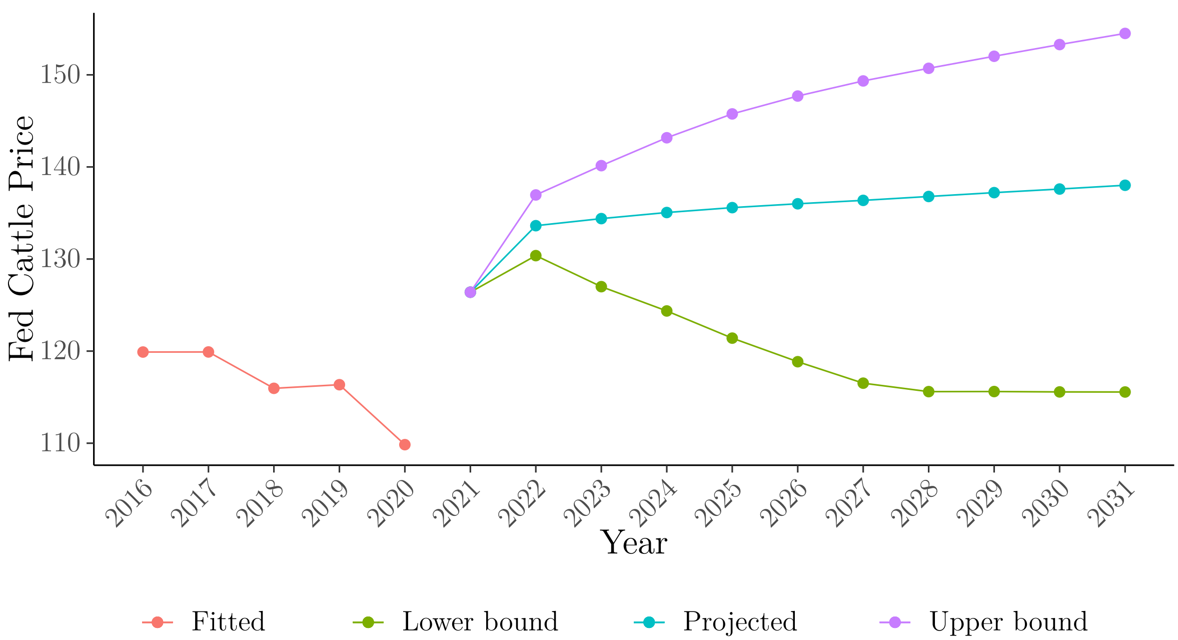

Figure 2.5 Projected fed cattle price ($/CWT)

Table 2.14 Projections of cull cow price (in $/CWT) with 95% confidence interval

Year	$p_{c_{l,95}}$	p_c	$p_{c_{u,95}}$
2021	74.46	74.46	74.46
2022	80.56	81.22	82.66
2023	78.92	80.53	83.09
2024	77.35	79.56	85.12
2025	76.14	78.61	87.43
2026	75.28	77.76	89.37
2027	74.71	76.98	91.11
2028	74.15	76.14	91.89
2029	73.59	75.31	91.72
2030	73.04	74.51	91.76
2031	72.54	73.69	91.76

Note: p_c,$p_{c_{l,95}}$, and $p_{c_{u,95}}$ denote the projected, lower 95%, and upper 95% of the cull cow price respectively.

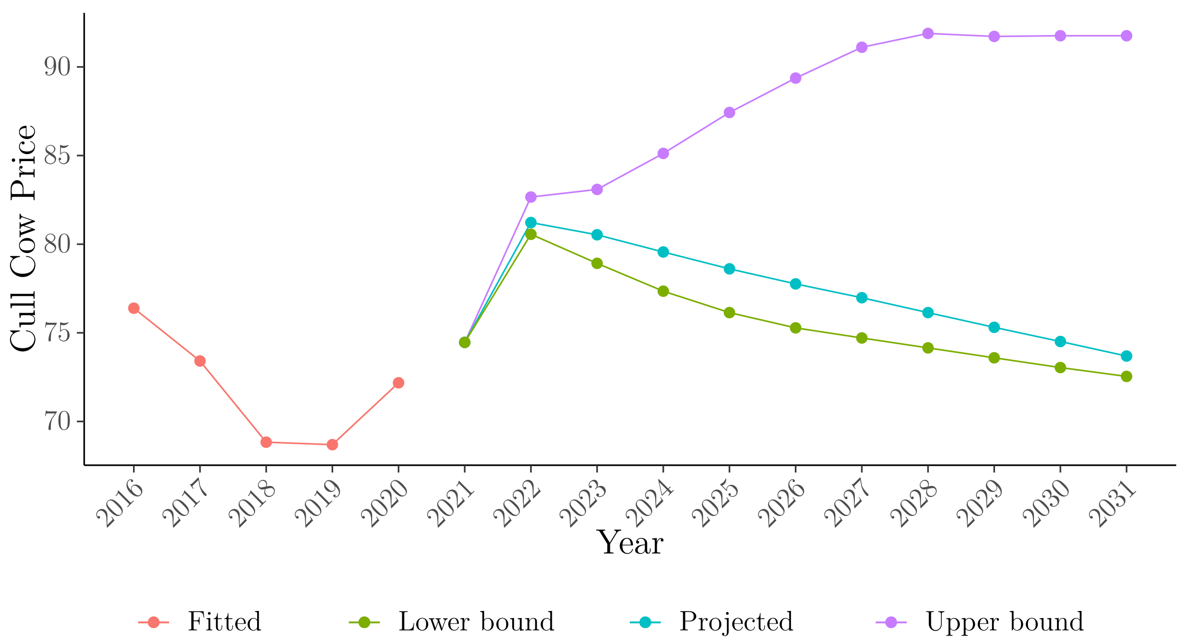

Figure 2.6 Projected cull cow price ($/CWT)

Table 2.15 Projections of fed cattle production (in billion pounds of meat) with 95% confidence interval

Year	$sl_{l,95}$	sl	$sl_{u,95}$
2021	23.28	23.28	23.28
2022	22.48	23.05	23.64
2023	21.76	22.91	24.07
2024	21.04	22.79	24.57
2025	20.34	22.65	24.99
2026	19.78	22.48	25.23
2027	19.78	22.32	25.33
2028	19.77	22.15	25.39
2029	19.77	22.00	25.45
2030	19.77	21.86	25.50
2031	19.78	21.70	25.53

Note: sl,$sl_{l,95}$, and $sl_{u,95}$ denote the projected, lower 95%, and upper 95% of the fed cattle production respectively.

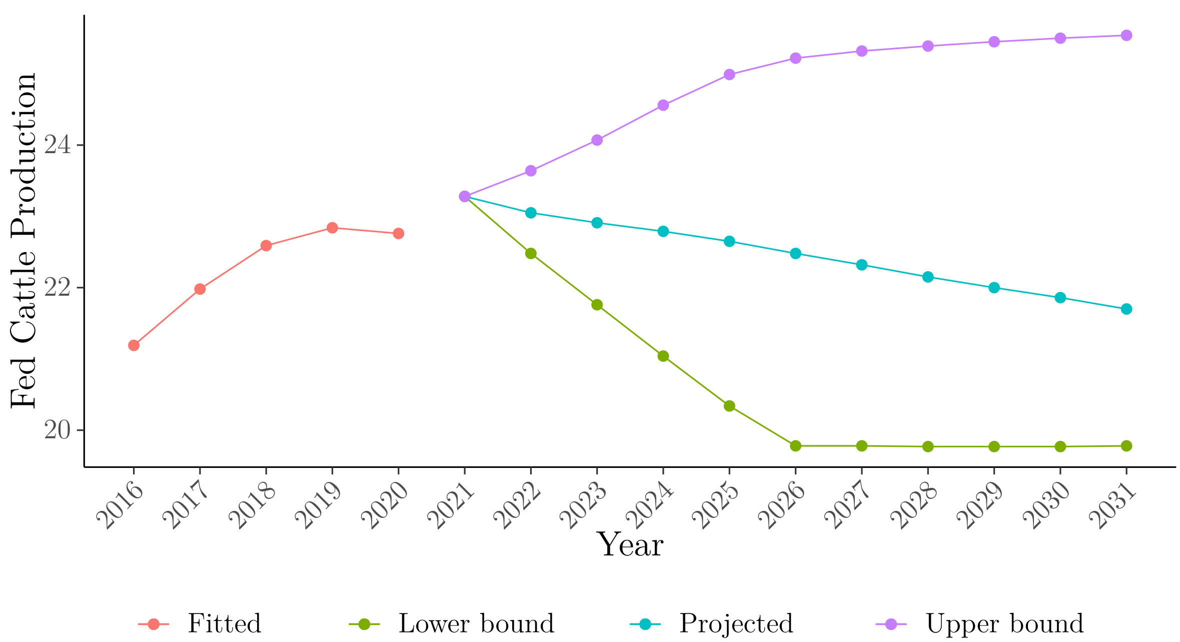

Figure 2.7 Projected fed cattle production (billion pounds)

Table 2.16 Projections of cull cow production (in billion pounds of meat) with 95% confidence interval

Year	$cl_{l,95}$	cl	$cl_{u,95}$
2021	3.05	3.05	3.05
2022	2.58	3.03	3.47
2023	2.21	3.12	4.02
2024	1.85	3.25	4.63
2025	1.51	3.36	5.20
2026	1.25	3.46	5.65
2027	1.08	3.54	6.00
2028	1.01	3.64	6.32
2029	1.02	3.74	6.64
2030	1.02	3.83	6.97
2031	1.02	3.93	7.28

Note: cl,$cl_{l,95}$, and $cl_{u,95}$ denote the projected, lower 95%, and upper 95% of the cull cow production respectively.

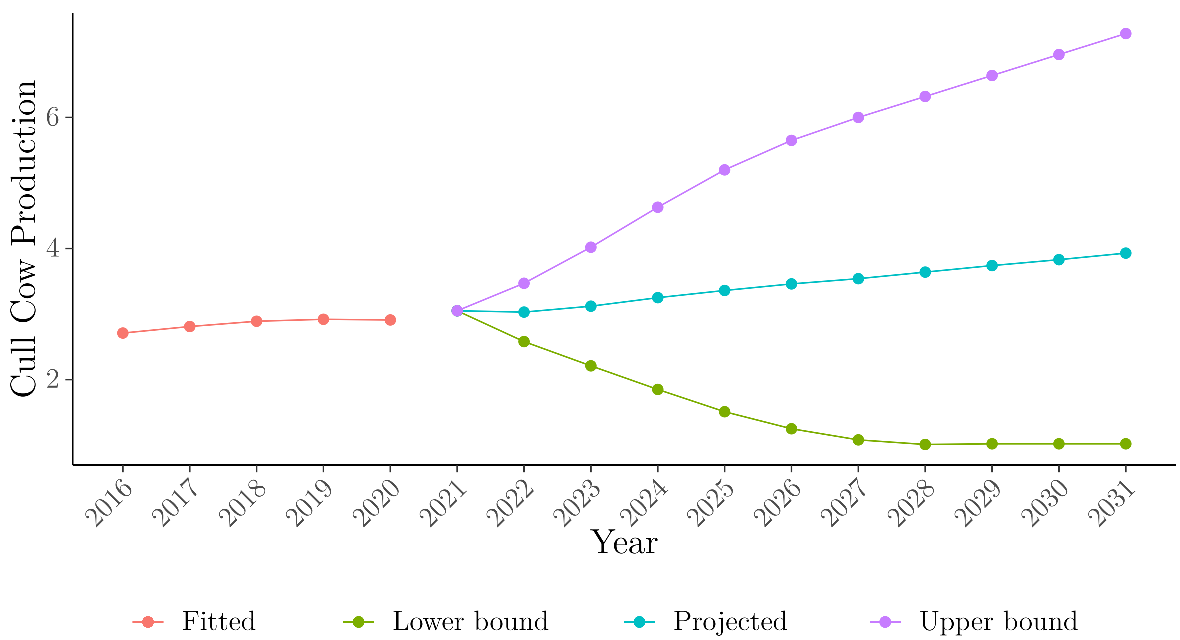

Figure 2.8 Projected cull cow production (billion pounds)

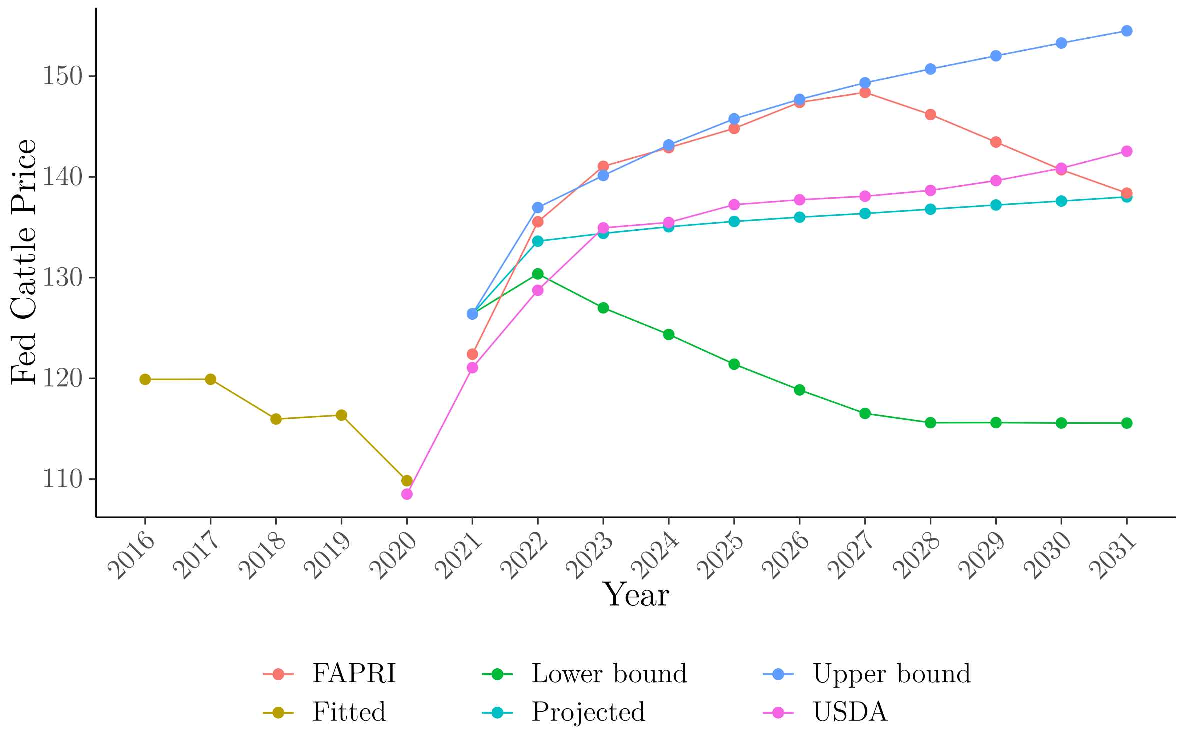

Figure 2.9 Projected fed cattle price vs USDA and FAPRI counterparts

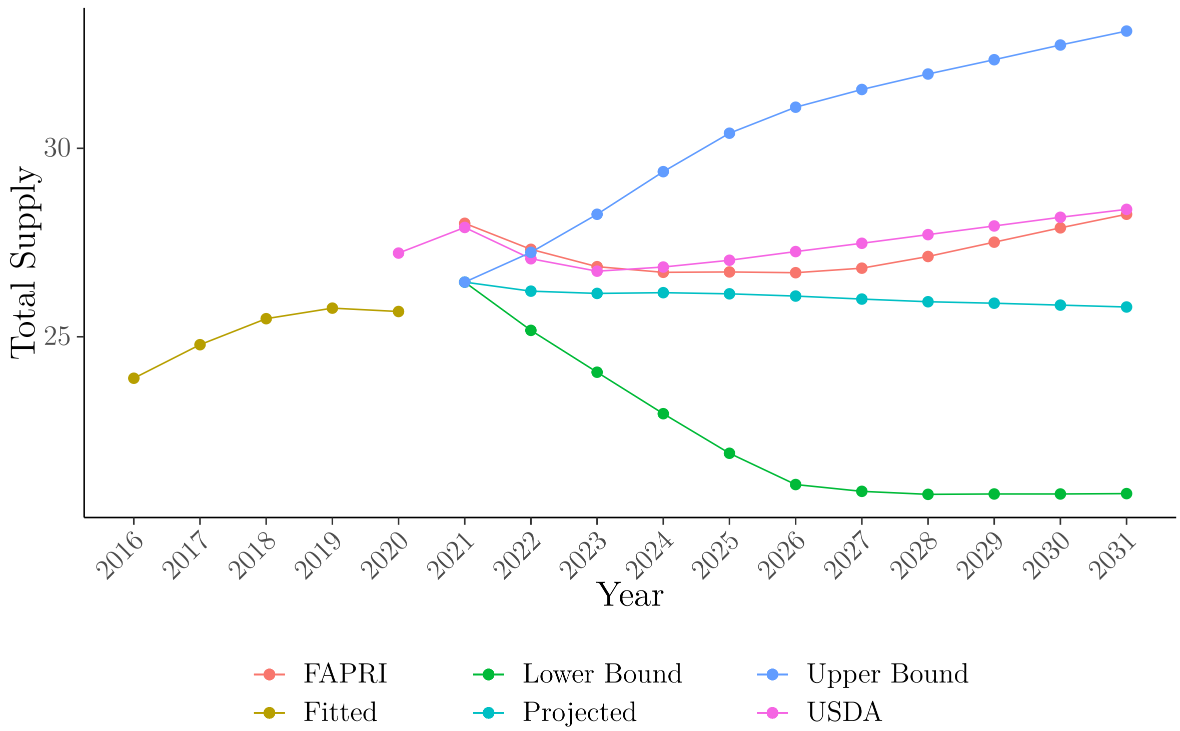

Figure 2.10 Projected total supply vs USDA and FAPRI counterparts

From the comparable projections above, our projected fed cattle price and total supply confidence interval include the analogous long-term projections from the USDA and FAPRI in general, providing strong evidence our model is adequately projecting the market. Overall, our fitted results and comparable projections further support that our model fits with US beef cattle data. The replication of the cattle inventories from the fitted model (Figures 2.3 and 2.4), also provides further evidence our model captures the dynamics of the U.S. beef cattle industry and explains the evolution of the U.S. beef cattle inventories.

2.7 Summary and implications

A dynamic model of the U.S. beef cattle industry is developed and calibrated to appropriately capture the dynamics of the U.S. beef cattle industry. The analytical solution is presented under naïve and rational price expectations. In addition, a numerical solution (iterative algorithm) under rational price expectations is developed and presented. Data from various USDA organizations were collected, compiled, and applied in the model to reflect the changes observed in the U.S. beef cattle industry. Numerical methods are used to find an equilibrium solution for the model. In particular, a collocation method is employed in the iterative algorithm to find a solution to the model with rational expectations.

A theoretical projection framework based on the dynamic model and the algorithm for the projection framework is developed. Using the estimated parameters and fitted prices and quantities, the beef market variables (prices and quantities) are projected into the future. In particular, a multi-year cyclical algorithm is employed to project the fed cattle price, cull cow price, fed cattle production, cull cow production, and total production into the future. The model projections are compared with the long-term USDA and FAPRI projections. The long-term projections of the dynamic model are consistent with those of the USDA and FAPRI counterparts, indicating the dynamic model adequately captures and projects the market variables of beef industry.

The model's ability to capture the beef cattle prices and quantities makes it valuable for policy analysis and analyzing any exogenous impact (e.g., foreign animal diseases) on the beef cattle industry. As stated in the first chapter, understanding not only the short-run impacts but also the long-run impacts is crucial to any beef cattle policy design. The ability to project the prices and quantities several years into the future demonstrates the models' potential to estimate the long-run impacts and the variance of these impacts over time to policy proposals or modifications and provides valuable assistance to policymakers.

CHAPTER 3. A DYNAMIC ASSESSMENT OF THE ECONOMIC IMPACTS OF A FOOT-AND-MOUTH DISEASE OUTBREAK ON THE U.S. BEEF CATTLE INDUSTRY

3.1 Abstract

The United States is the world's largest beef producer and has been free of foot-and-mouth disease (FMD) since 1929. However, the disease is active in two-thirds of the world and with frequent international travel and trade, there is a significant risk of the disease entering the country. Many major beef importers traditionally exclude beef imports from exporting countries whose herds show evidence of FMD. Furthermore, FMD can have destructive effects on the productivity of livestock, can lead to high mortality rates in young animals, and the cost of disease control can be enormous. As a result, a FMD outbreak in the United States would have disastrous economic consequences due to the costs directly associated with massive animal depopulation, disposal, and disinfection, loss of access to export markets, and supply chain disruptions. There would be additional costs for any vaccination or control program implemented.

The objective of this study is to determine the economic impacts of a FMD outbreak on the U.S. beef cattle industry. We specifically quantify the short-run impacts of an outbreak on the prices and quantities of fed cattle and cull cows, as well as how the impacts vary over time. Furthermore, we quantify an outbreak's net welfare impacts under various control and eradication response strategies. This will be the first study to estimate impacts under a dynamic framework and across various response strategies.

Previous studies that have estimated the economic impacts of disease outbreaks in the United States have relied heavily on static input-output and partial equilibrium models. A limitation of these models is that they fail to consider the dynamic nature of livestock inventories. Dynamics is an essential feature in cattle production and markets. Inclusion of dynamics will yield more accurate estimates and can help policymakers better understand possible impacts and design more effective response strategies.

USDA-APHIS is the Federal agency with primary responsibility and authority for animal disease control and will interface with federal, state, tribal, and local partners to control, contain, and eradicate FMD in the event of an outbreak. Possible response strategies are described in the FMD Disease Response Plan or Red Book. We treat an FMD outbreak as a one-time exogenous supply (production) shock to the cattle stock. The magnitude of other exogenous shocks, such as mortality rates in older and younger animals and vaccination costs are determined from existing studies. The exogenous change in domestic demand is determined by studying the changes in the demand from previous disease outbreaks (e.g., the 2001 FMD outbreak in the U.K. and the 2003 BSE outbreak in the U.S.). It has been documented from the previous animal disease outbreaks (the 2003 BSE outbreak), that the importing countries closed their markets for the American beef. Loss of access to international markets is treated as another exogenous shock. We incorporate this exogenous shock by studying the number of years it took for the importing countries to open their markets for the U.S. beef after the 2003 BSE outbreak.

Using the exogenous shocks, the calibrated model is simulated under a combination of outbreak and response scenarios to quantify the economic impacts of an FMD outbreak on the U.S. beef cattle industry. To validate our model, using the same time frame as in previous studies, we will compare our results to those of the literature. In addition to short-run estimates, we also provide how producer surplus, price, and cattle stocks change over time. Our short-run and overtime estimates can be used to analyze the market reaction to an FMD outbreak, the duration it will take for the market to recover, and the number of years it will take for the cattle stocks to reach equilibrium.

With the USDA's new updated FMD response strategies, disease incidents around the world, and the lack of new studies analyzing the disease impacts, it is imperative to accurately estimate the economic impacts of an FMD outbreak. Our results will be of interest to policymakers, and the model and subsequent analysis will provide guidance on the economic consequences of an FMD outbreak. Our net welfare impacts under the various scenarios can be used to assess the economic impacts of different FMD response strategies, as well as the policies that may be appropriate to implement to minimize economic losses and successfully eradicate the disease. Furthermore, with minor modifications, our dynamic model can be adapted to estimate the economic impacts of other foreign animal diseases and to design effective prevention and mitigation strategies.

BIBLIOGRAPHY

Aadland, D. (2004). Cattle cycles, heterogeneous expectations and the age distribution of capital. *Journal of Economic Dynamics and Control*, 28(10):1977–2002.

Aadland, D. and Bailey, D. (2001). Short-Run Supply Responses in the U.S. Beef-Cattle Industry. *American Journal of Agricultural Economics*, 83(4):826–839.

Asche, F., Oglend, A., and Selland Kleppe, T. (2017). Price Dynamics in Biological Production Processes Exposed to Environmental Shocks. *American Journal of Agricultural Economics*, 99(5):1246–1264.

Azzam, A. M. and Anderson, D. G. (1996). Assessing competition in meatpacking: Economic history, theory, and evidence. *USDA*. US Department of Agriculture, Packers and Stockyards Programs, Grain Inspection, Packers and Stockyards Administration.

Baak, S. J. (1999). Tests for bounded rationality with a linear dynamic model distorted by heterogeneous expectations. *Journal of Economic Dynamics and Control*, 23(9):1517–1543.

Ball, V. E. and Chambers, R. G. (1982). An Economic Analysis of Technology in the Meat Products Industry. *American Journal of Agricultural Economics*, 64(4):699–709.

Blasi, D. A., Brester, G. W., Crosby, C. R., Dhuyvetter, K. C., Freeborn, J., Pendell, D. L., Schroeder, T. C., Smith, G., Stroade, J., and Tonsor, G. T. (2009). Benefit-Cost Analysis of the National Animal Identification System. *Final Report submitted to USDA-APHIS on January 14, 2009*. Available at : https://agmanager.info/livestock-meat/cross-subject-areas/benefit-cost-analysis-national-animal-identification-system.

Brester, G., Marsh, J., and Atwood, J. (2004). Distributional impacts of Country-of-Origin Labeling in the U.S. meat industry. *Journal of Agricultural and Resource Economics*, 29(2):206–227.

Capps, Oral, J., Farris, D. E., Byrne, P. J., Namken, J. C., and Lambert, C. D. (1994). Determinants of Wholesale Beef-Cut Prices. *Journal of Agricultural and Applied Economics*, 26(1):183–199.

Chavas, J.-P. (2000). On information and market dynamics: The case of the U.S. beef market. *Journal of Economic Dynamics and Control*, 24(5):833–853.

Cooley, T. F. and DeCanio, S. J. (1977). Rational Expectations in American Agriculture, 1867-1914. *The Review of Economics and Statistics*, 59(1):9–17.

Eckstein, Z. (1984). A Rational Expectations Model of Agricultural Supply. *Journal of Political Economy*, 92(1):1–19.

FAPRI (2022). U.S. Agricultural Market Outlook. Available at: https://www.fapri.missouri.edu/wp-content/uploads/2022/03/2022-U.S.-Agricultural-Market-Outlook.pdf.

Fetrow, J., Nordlund, K. V., and Norman, H. D. (2006). Invited Review: Culling: Nomenclature, Definitions, and Recommendations. *Journal of Dairy Science*, 89(6):1896–1905.

Foster, K. A. and Burt, O. R. (1992). A Dynamic Model of Investment in the U.S. Beef-Cattle Industry. *Journal of Business & Economic Statistics*, 10(4):419–426.

Goodwin, T. H. and Sheffrin, S. M. (1982). Testing the Rational Expectations Hypothesis in an Agricultural Market. *The Review of Economics and Statistics*, 64(4):658–667.

Gouel, C. (2013). Comparing Numerical Methods for Solving the Competitive Storage Model. *Computational Economics*, 41(2):267–295.

Jarvis, L. S. (1974). Cattle as Capital Goods and Ranchers as Portfolio Managers: An Application to the Argentine Cattle Sector. *Journal of Political Economy*, 82(3):489–520.

Judd, K. L. (1998). *Numerical Methods in Economics*. MIT press.

Kydland, F. E. and Prescott, E. C. (1982). Time to Build and Aggregate Fluctuations. *Econometrica*, 50(6):1345–1370.

Lusk, J. and Anderson, J. (2004). Effects of Country-of-Origin Labeling on Meat Producers and Consumers. *Journal of Agricultural and Resource Economics*, 29(2):185–205.

Marsh, J. M. (1991). Derived Demand Elasticities: Marketing Margin Methods versus an Inverse Demand Model for Choice Beef. *Western Journal of Agricultural Economics*, 16(2):382–391.

Miranda, M. J. (1997). Numerical Strategies for Solving the Nonlinear Rational Expectations Commodity Market Model. *Computational Economics*, 11(1):71–87.

Miranda, M. J. and Fackler, P. L. (2002). *Applied Computational Economics and Finance*. MIT press.

Miranda, M. J. and Glauber, J. W. (1995). Solving Stochastic Models of Competitive Storage and Trade by Chebychev Collocation Methods. *Agricultural and Resource Economics Review*, 24(1):70–77.

Miranda, M. J. and Glauber, J. W. (2021). A Model of Asynchronous Bi-Hemispheric Production in Global Agricultural Commodity Markets. *American Journal of Agricultural Economics*.

Miranda, M. J. and Schnitkey, G. D. (1995). An Empirical Model of Asset Replacement in Dairy Production. *Journal of Applied Econometrics*, 10:S41–55.

Mundlak, Y. and Huang, H. (1996). International Comparisons of Cattle Cycles. *American Journal of Agricultural Economics*, 78(4):855–868.

Muth, J. F. (1961). Rational Expectations and the Theory of Price Movements. *Econometrica*, 29(3):315–335.

Mutondo, J. E., Brorsen, B. W., and Henneberry, S. R. (2009). Welfare impacts of BSE-driven trade bans. *Agricultural and Resource Economics Review*, 38(3):324–329.

Nerlove, M. and Fornari, I. (1998). Quasi-rational expectations, an alternative to fully rational expectations: An application to US beef cattle supply. *Journal of Econometrics*, 83(1):129–161.

Pendell, D. L., Brester, G. W., Schroeder, T. C., Dhuyvetter, K. C., and Tonsor, G. T. (2010). Animal Identification and Tracing in the United States. *American Journal of Agricultural Economics*, 92:927–940.

Pendell, D. L., Leatherman, J. C., Schroeder, T. C., and Alward, G. (2007). The Economic Impacts of a Foot-And-Mouth Disease Outbreak: A Regional Analysis. *Journal of Agricultural and Applied Economics*, 39(s1):19–33. doi : https://doi.org/10.1017/S1074070800028911.

Pendell, D. L., Tonsor, G. T., Dhuyvetter, K. C., Brester, G. W., and Schroeder, T. C. (2013). Evolving beef export market access requirements for age and source verification. *Food Policy*, 43:332–340.

Powell, J. and Ward, H. (2009). Culling the Beef Cow Herd. *BEEF, Business Insights: Essentials*.

Ritten, J. P., Frasier, W. M., Bastian, C. T., and Gray, S. T. (2010). Optimal Rangeland Stocking Decisions Under Stochastic and Climate-Impacted Weather. *American Journal of Agricultural Economics*, 92(4):1242–1255.

Rosen, S. (1987). Dynamic Animal Economics. *American Journal of Agricultural Economics*, 69(3):547–557.

Rosen, S., Murphy, K. M., and Scheinkman, J. A. (1994). Cattle Cycles. *Journal of Political Economy*, 102(3):468–492.

Schmitz, J. D. (1997). Dynamics of Beef Cow Herd Size: An Inventory Approach. *American Journal of Agricultural Economics*, 79(2):532–542.

Schulz, L. L., Andresen, C. E., and Gunn, P. J. (2016). Factors affecting timing and intensity of calving season of beef cow-calf producers in the midwest. *The Professional Animal Scientist*, 32(4):430–437.

Shiptsova, R., Thomsen, M., and Goodwin, H. (2002). Producer Welfare Changes from Meat and Poultry Recalls. *Journal of Food Distribution Research*, 33.

Stuttgen, S. (2019). When is the Best Time to Calve Beef in Wisconsin? *University of Wisconsin-Madison Extension*.

Tonsor, G. T. and Schroeder, T. (2015). Market Impacts of E. Coli Vaccination in U.S. Feedlot Cattle. *Agricultural and Food Economics*, 3(7):1–15.

Tonsor, G. T., Schroeder, T. C., and Parcell, J. (2015). Report to Congress: Economic Analysis of Country of Origin Labeling (COOL). *U.S. Department of Agriculture, Office of the Chief Economist*. Available at : http://www.agri-pulse.com/Uploaded/USDACOOLEconomicReport.pdf.

Trapp, J. N. (1986). Investment and disinvestment principles with nonconstant prices and varying firm size applied to beef-breeding herds. *American Journal of Agricultural Economics*, 68(3):691–703.

USDA (2022). USDA Agricultural Projections to 2031. Available at: https://www.ers.usda.gov/webdocs/outlooks/103310/oce-2022-01.pdf?v=7671.2.

USDA-ERS (2021). Cattle & Beef: Statistics & Information. Available at : https://www.ers.usda.gov/topics/animal-products/cattle-beef/statistics-information/.

USDA-ERS (2022). What is agriculture's share of the overall U.S. economy? Available at : https://www.ers.usda.gov/data-products/chart-gallery/gallery/chart-detail/?chartId=58270.

USDA-NASS (2016). Overview of the United States cattle industry.

USDA-NASS (2022a). USDA National Agricultural Statistics Service (NASS) - Agricultural Prices. Available at: https://usda.library.cornell.edu/concern/publications/c821gj76b?locale=en#release-items.

USDA-NASS (2022b). USDA National Agricultural Statistics Service (NASS) - Calf Crop. Available at: https://usda.library.cornell.edu/concern/publications/h702q636h?locale=en.

USDA-NASS (2022c). USDA National Agricultural Statistics Service (NASS) - January Cattle Inventory. Available at: https://usda.library.cornell.edu/concern/publications/h702q636h?locale=en.

USDA-NASS (2022d). USDA National Agricultural Statistics Service (NASS) - Livestock Slaughter. Available at: https://usda.library.cornell.edu/concern/publications/rx913p88g?locale=en#release-items.

USDA-PSD (2022). USDA Foreign Agricultural Service (FAS) - Production, Supply and Distribution (PSD) online - Animal Numbers, Cattle. Available at: https://apps.fas.usda.gov/psdonline/app/index.html#/app/advQuery.

Xia, Y. and Steven, B. (2002). Size, cost, and productivity in the meat processing industries. *Agribusiness; Hoboken*, 18(3):283–299.

APPENDIX A. Proof for equation 2.129

$$
\begin{aligned}
cl_{t+1} &= k_{9,t+1} + (k_{8,t+1} - k_{9,t+2}) + (k_{7,t+1} - k_{8,t+2}) \\
&= k_{9,t+1} + (1-\delta)k_{8,t+1} + (1-\delta)k_{7,t+1}
\end{aligned}
$$

using the relationship $k_{j+1,t+1} = \delta k_{j,t}$ we rewrite $k_{9,t+1}$, $k_{8,t+1}$, $k_{7,t+1}$ in terms of k_3 as below

$$cl_{t+1} = \delta^3 k_{3,t-5} + (1-\delta)\delta^5 k_{3,t-4} + (1-\delta)\delta^4 k_{3,t-3}. \tag{A.1}$$

Using the linear relationship $k_{3,t+1} = \gamma k_{3,t} + \eta k_{0,t-3}$, we rewrite $k_{3,t-5}$, $k_{3,t-4}$, $k_{3,t-3}$ in terms of $k_{3,t+2}$.

$$k_{3,t+2} = \gamma k_{3,t+1} + \eta k_{0,t-2} \tag{A.2}$$

$$= \gamma(\gamma k_{3,t} + \eta k_{0,t-3}) + \eta k_{0,t-2} \tag{A.3}$$

$$= \gamma^2(\gamma k_{3,t-1} + \eta k_{0,t-4}) + \gamma\eta k_{0,t-3} + \eta k_{0,t-2} \tag{A.4}$$

$$= \gamma^3(\gamma k_{3,t-2} + \eta k_{0,t-5}) + \gamma^2\eta k_{0,t-4} + \gamma\eta k_{0,t-3} + \gamma k_{0,t-2} \tag{A.5}$$

$$= \gamma^4(\gamma k_{3,t-3} + \eta k_{0,t-6}) + \gamma^3\eta k_{0,t-5} + \gamma^2\eta k_{0,t-4} + \gamma\eta k_{0,t-3} + \gamma k_{0,t-2} \tag{A.6}$$

$$= \gamma^5 k_{3,t-3} + \eta\gamma^4 k_{0,t-6} + \eta\gamma^3 k_{0,t-5} + \eta\gamma^2 k_{0,t-4} + \eta\gamma k_{0,t-3} + \eta k_{0,t-2} \tag{A.7}$$

$$= \gamma^6 k_{3,t-4} + \eta\gamma^5 k_{0,t-7} + \eta\gamma^4 k_{0,t-6} + \eta\gamma^3 k_{0,t-5} + \eta\gamma^2 k_{0,t-4} + \eta\gamma k_{0,t-3} + \eta k_{0,t-2} \tag{A.8}$$

$$= \gamma^7 k_{3,t-5} + \eta\gamma^6 k_{0,t-8} + \eta\gamma^5 k_{0,t-7} + \eta\gamma^4 k_{0,t-6} + \eta\gamma^3 k_{0,t-5} + \eta\gamma^2 k_{0,t-4} + \eta\gamma k_{0,t-3} + \eta k_{0,t-2} \tag{A.9}$$

by rewriting A.7, A.8, and A.9 in-terms of $k_{3,t-3}$, $k_{3,t-4}$, and $k_{3,t-5}$ respectively, we get

$$k_{3,t-3} = \frac{1}{\gamma^5}k_{3,t+2} - \frac{\eta}{\gamma}\left(k_{0,t-6} + \frac{1}{\gamma}k_{0,t-5} + \frac{1}{\gamma^2}k_{0,t-4} + \frac{1}{\gamma^3}k_{0,t-3} + \frac{1}{\gamma^4}k_{0,t-2}\right) \tag{A.10}$$

$$k_{3,t-4} = \frac{1}{\gamma^6}k_{3,t+2} - \frac{\eta}{\gamma}\left(k_{0,t-7} + \frac{1}{\gamma}k_{0,t-6} + \frac{1}{\gamma^2}k_{0,t-5} + \frac{1}{\gamma^3}k_{0,t-4} + \frac{1}{\gamma^4}k_{0,t-3} + \frac{1}{\gamma^5}k_{0,t-2}\right) \tag{A.11}$$

$$k_{3,t-5} = \frac{1}{\gamma^7}k_{3,t+2} - \frac{\eta}{\gamma}\left(k_{0,t-8} + \frac{1}{\gamma}k_{0,t-7} + \frac{1}{\gamma^2}k_{0,t-6} + \frac{1}{\gamma^3}k_{0,t-5} + \frac{1}{\gamma^4}k_{0,t-4} + \frac{1}{\gamma^5}k_{0,t-3} + \frac{1}{\gamma^6}k_{0,t-2}\right) \tag{A.12}$$

substituting A.10, A.11, and A.12 in A.1 and rearranging gives 2.129

$$\begin{aligned} cl_{t+1} = {} & \frac{\delta^4}{\gamma^7}\left[\delta^2 + (1-\delta)\gamma(\delta+\gamma)\right] \\ & \left[k_{3,t+2} - \eta\gamma^4 k_{0,t-6} + \gamma^3 k_{0,t-5} + \gamma^2 k_{0,t-4} + \gamma k_{0,t-3} + k_{0,t-2}\right] \\ & - \frac{\delta^5}{\gamma^2}\eta\left[\delta\gamma k_{0,t-8} + (\delta + (1-\delta)\gamma)k_{0,t-7}\right] \end{aligned}$$