

LONGITUDINAL AND TRANSVERSE DISPERSION IN POROUS MEDIA

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Abstract: In the present work, we have analysed the experimental data presented in literature to characterize dispersion in porous media, at different dispersion regimes. The vast amount of data obtained by our group, together with the extensive data available from other sources, mostly for air and water at room temperature, provide a very detailed representation of the functions $Pe_T = f_1(Pe_m, Sc)$ and $Pe_L = f_2(Pe_m, Sc)$. Empirical correlations are presented for the prediction of the dispersion coefficients (D_T and D_L) over the entire range of practical values of Schmidt number and Peclet number. The simple mathematical expressions represent the data available, in literature, with good accuracy and they are shown to be a significant improvement over previous correlations.

Keywords: longitudinal dispersion; transverse dispersion; porous media; diffusion; mass transfer; schmidt number.

INTRODUCTION

The problem of solute dispersion during underground water movement has attracted interest from the early days of this century. Since the early experiments of Slichter (1905) and particularly since the analysis of dispersion during solute transport in capillary tubes, developed by Taylor (1953) and Aris (1956, 1959), a lot of work has been done on the description of the principles of solute transport in porous media of inert particles and in packed bed reactors (see Bear, 1972; Dullien, 1979).

When a fluid is flowing through a bed of inert particles, one observes the dispersion of the fluid in consequence of the combined effects of molecular diffusion and convection in the spaces between particles. Generally, the dispersion coefficient in longitudinal direction is superior to the dispersion coefficient in radial direction by a factor of 5, for values of Reynolds number larger than 10. For low values of the Reynolds number (say, $Re < 1$), the two dispersion coefficients are approximately the same and equal to effective molecular diffusion coefficient.

The detailed structure of a porous medium is greatly irregular and just some statistical-properties are known. An exact solution to characterize flowing fluid through one of these structures is basically impossible. However, by the method of volume or spatial averaging it is possible to obtain the transport

equation for the average concentration of solute in a porous medium (Bear, 1972; Whitaker, 1967).

At a 'macroscopic' level, the quantitative treatment of dispersion is currently based on Fick's law, with the appropriate dispersion coefficients; cross stream dispersion is related to the transverse dispersion coefficient, D_T , whereas streamwise dispersion is related to the longitudinal dispersion coefficient, D_L .

Macroscopic modelling of dispersion processes in isotropic porous media is usually based on the convective-diffusion equation:

$$D_L \frac{\partial^2 C}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(D_T r \frac{\partial C}{\partial r} \right) = u \frac{\partial C}{\partial z} + \frac{\partial C}{\partial t} \quad (1)$$

where C is the mean solute concentration, u ($=U/\varepsilon$, where U is the superficial velocity and ε the porosity of the porous media of inert particles with diameter d) the mean interstitial velocity of fluid and t the time.

A large number of theories, namely the theories based on a probabilistic approach, have been proposed to explain dispersion in porous media; however the theory of Saffman (1959, 1960), who modelled the microstructure of a porous media as a network of capillary tubes of random orientation, and Koch and Brady (1985) were the most referred.

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STATE OF THE ART

Flow velocities and hydrodynamic dispersion coefficients are key parameters for description of fluid and solute transport in porous media. Dispersion topic has interested a vast scientific community, namely hydrology and contaminant studies, and for some time now it is treated at length in books on flow through porous media (e.g. Delleur, 1999; Fetter, 1999; Sahimi, 1995; Grathwohl, 1998; Bear and Verrijt, 1987; Marsily, 1986; Koch and Brady, 1985; Scheidegger, 1974; Bear, 1972; Fried and Combarnous, 1971).

Hydrodynamic dispersion includes both mechanical (convective) dispersion and molecular diffusion. For low fluid velocities values, namely in the limiting case where $u \rightarrow 0$, solute dispersion is determined by molecular diffusion ($D_L \cong D_T \cong D_m$); for high values of u convection becomes dominant but the contribution of diffusion cannot be neglected.

The experimental dispersion data are most frequently presented in logarithmic plots of D_T/D_m (or D_L/D_m) versus Pe_m ($=ud/D_m$), spanning six or more orders of magnitude in D/D_m (or D_L/D_m). The result may give the impression of a narrow spread in the data [see Figures 1(a) and 2(a)], but that is only an illusion.

Indeed, if for gas flow the experimental data reported in literature are very concordant, in the case of liquid flow, at near ambient temperature, corresponding to values of Schmidt

number, Sc , in the range $500 < Sc < 2000$, the data available are very 'disperse'.

For longitudinal dispersion, most of the experimental values of Pe_L ($=ud/D_L$) reported in the literature, for values of Sc in the range $500 < Sc < 2000$, are shown in Figure 1(b), and they form a 'thick cloud' at approximately, $0.3 < Pe_L < 2$. However, if the data in Figure 1(b) are cleared of the points obtained in columns that were either, too narrow ($D/d < 15$), in comparison with the size of the particles in the packing, or too short ($L/D < 10$), the picture in Figure 3 emerges.

For transverse dispersion, some considerations in Figure 2(b) were made. The data of Grane and Gardner (1961) present significant scatter about all data. Many of the points reported by Simpson (1962) were obtained in beds of sand bonded by resin and this will lead to the formation of particle agglomerates, with a resulting increase in values of D_T . Blackwell (1962) had already observed that in a bed of sand particles of 74 μm to 840 μm , values of D_T were considerably higher than expected as a result of particle agglomeration (which gives a larger 'apparent particle diameter'). At the lower end of Pe_m , only the points of Hiby (1962) seem to fall consistently above our data; however it is not to be excluded that Hiby data (at low Pe_m) are subject to wall effect, for the width of the test channel was only eight

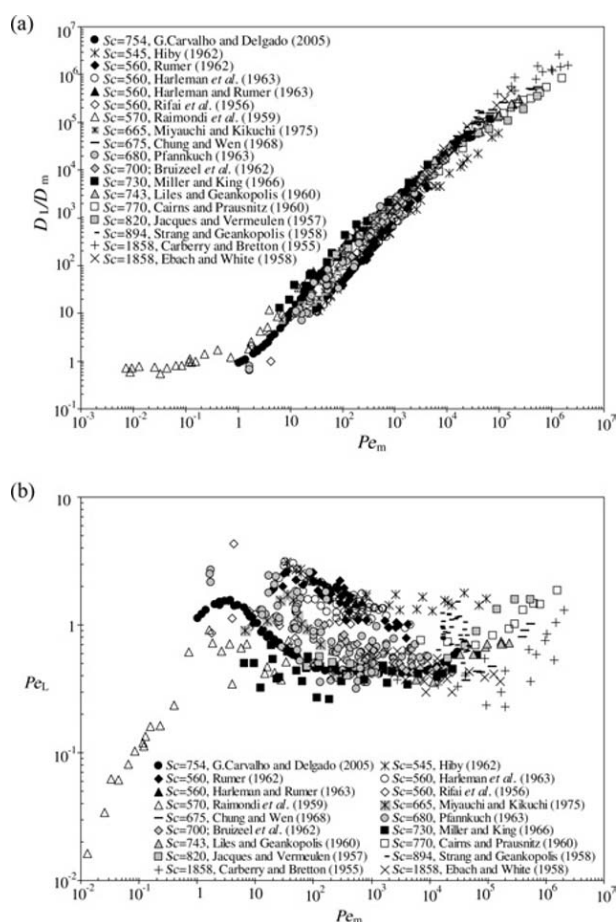


Figure 1. Longitudinal dispersion in porous media (a) D_L/D_m versus Pe_m and (b) Pe_L versus Pe_m .

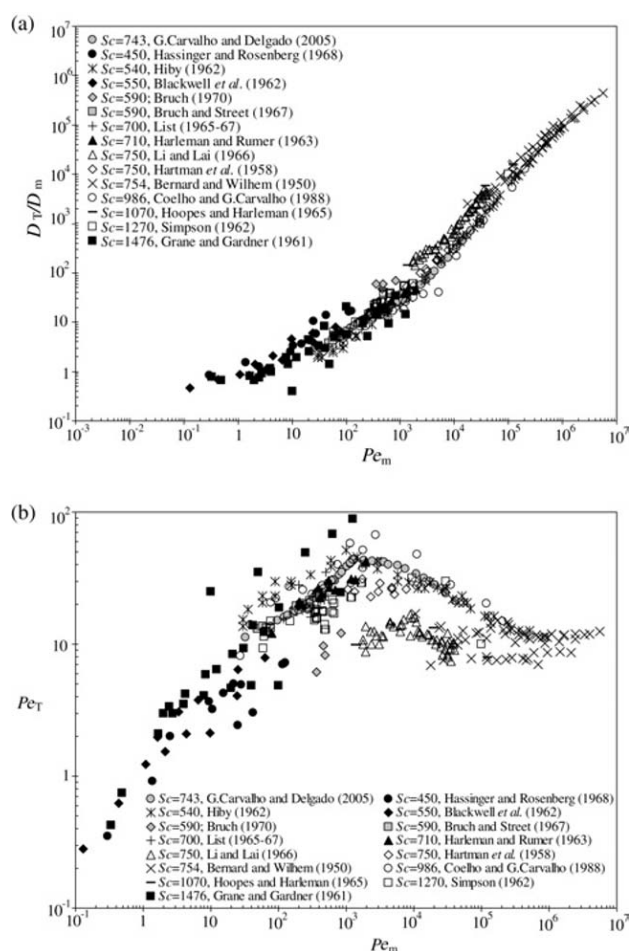


Figure 2. Transverse dispersion in porous media (a) D_T/D_m versus Pe_m and (b) Pe_T versus Pe_m .

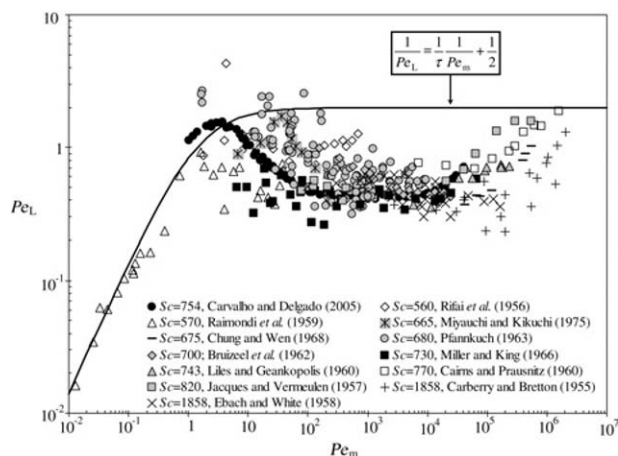


Figure 3. Selected set of published data, for $Sc > 550$, plotted as Pe_L versus Pe_m .

times the diameter of the particles. If the data in Figure 2(b) are cleared of these points, the picture in Figure 4 emerges (all for $Sc > 550$) in a plot of $Pe_T (=ud/D_T)$ versus Pe_m .

A large number of equations have been proposed to correlate dispersion experimental data in porous media, and the most used are the equations proposed by Bear and Verruijt (1987) in groundwater works and the equations proposed by Koch and Brady (1985).

The coefficients of transverse and longitudinal dispersion are not linear function of velocity and expressions with the form $D_L = D'_m + \alpha_L u^n$ or $D_T = D'_m + \alpha_T u^n$ are suggested. The coefficients α_T and α_L are the transverse and longitudinal dispersivities, respectively, of the porous medium in the direction of transport and n is an empirically constant, $n = 1-2$ (Freeze and Cherry, 1979). In most applications, the exponent, n , is assumed to be unity, i.e., dispersion coefficient is assumed to be a linear function of fluid velocity. However, n may be greater than unity in many situations (Bear and Verruijt, 1987). Also, the dispersivity, α_L , is probably scale dependent with larger values for α_L being associated with greater transport distances (Pickens and Grisak, 1981). For example, values of α_L reported from the results of field

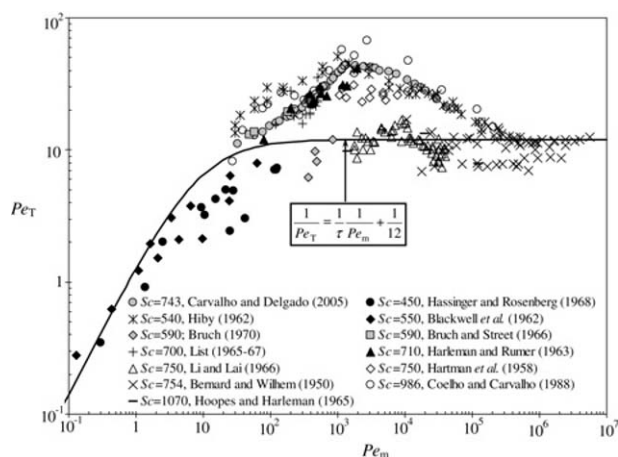


Figure 4. Selected set of published data, for $Sc > 550$, plotted as Pe_T versus Pe_m .

studies may be as much as four or six orders of magnitude greater than the corresponding laboratory measured values which commonly are found to range between 0.1 and 10 mm (Freeze and Cherry, 1979). Ratios of α_L/α_T of 5:1 to 100:1 have been reported in literature (Bear and Verruijt, 1987).

Some of most referred works were developed by Fried and Combarnous (1971) and Bear and Verruijt (1987, p. 166); the authors showed the existence of five dispersion regimes, in unconsolidated porous media. Sahimi (1995) and Marsily (1986) analyse the data compiled by Fried and Combarnous (1971) to characterize longitudinal dispersion in five dispersion regimes and transverse dispersion in four dispersion regimes and a holdup dispersion. The (1) pure molecular diffusion regime, for very low fluid velocity, is represented by (Koplik *et al.*, 1988)

$$\frac{D_L}{D_m} = \frac{D_T}{D_m} = \frac{1}{\tau} \quad (2)$$

The (2) superposition regime ($0.3 < Pe_m < 5$) not quantify by an equation. In this zone the effect of molecular diffusion and dispersion are of the same order of magnitude. The (3) predominant mechanical dispersion ($5 < Pe_m < 300$):

$$\frac{D_L}{D_m} = \frac{1}{\tau} + 0.5Pe_m^{1.2} \quad (3a)$$

$$\frac{D_T}{D_m} = \frac{1}{\tau} + 0.025Pe_m^{1.1} \quad (3b)$$

The (4) pure mechanical dispersion ($300 < Pe_m < 10^5$) represented by

$$\frac{D_L}{D_m} = \frac{1}{\tau} + (1.8 \pm 0.4)Pe_m \quad (4a)$$

$$\frac{D_T}{D_m} = \frac{1}{\tau} + 0.025Pe_m \quad (4b)$$

In this zone (yet in the range of validity of Darcy's law) the effect of molecular diffusion is negligible. The (5) dispersion out of Darcy domain ($Pe_m > 10^5$) not quantify by an equation. This is the region in which the effects of inertia and turbulence cannot be neglected.

The (6) holdup dispersion (Koch and Brady, 1985):

$$\frac{D_L}{D_m} = \frac{D_T}{D_m} \approx Pe_m^2 \quad (5)$$

Probably, one of the most important studies in dispersion topic was presented by Saffman (1959, 1960). These works are the most detailed analysis of dispersion in porous media, but the correlation proposed for axial and radial dispersion diverge slightly from the existing experimental data, in the literature (see Figures 5 and 6). The author proposed the following equation, for longitudinal dispersion:

$$\frac{D_L}{D_m} = \frac{Pe'_m}{6} \left[\ln \left(\frac{3}{2} \tau Pe'_m \right) - \frac{1}{4} \right] \quad \text{valid for } Pe'_m \gg 1 \quad (6)$$

Another important study was presented by Koch and Brady (1985) who showed that the ratio of the dispersion coefficient to the molecular diffusivity is only a function of the product $ReSc$. However, experimental measurements show that the dispersion Peclet group is a function of both Reynolds and

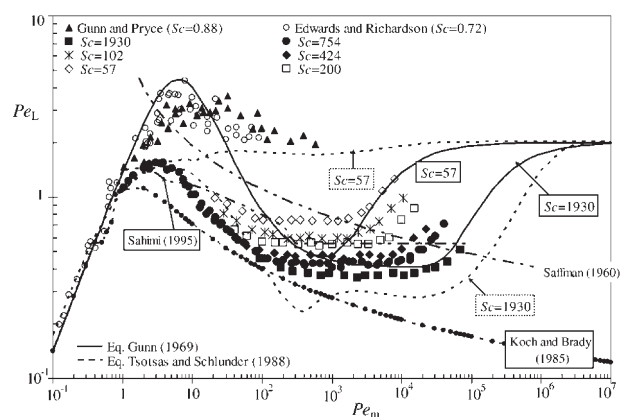


Figure 5. Comparison between experimental data and correlations of other authors, for longitudinal dispersion.

Schmidt groups as $Pe = f(Re, Sc)$ and not $Pe = f(ReSc)$. For Stokes flow through a random packed bed of spheres, Koch and Brady (1985) derived analytical expressions for the longitudinal and transverse dispersion coefficients. Since the solid phase is not permeable to the tracer, for $Pe_m > 1$, D_L/D_m and D_T/D_m are given by

$$\frac{D_L}{D_m} = 1 + \frac{3Pe'_m}{4} + \frac{\pi^2}{6}(1-\varepsilon)\frac{Pe'_m}{2} \ln\left(\frac{Pe'_m}{2}\right) \quad (7)$$

$$\frac{D_T}{D_m} = 1 + \frac{63\sqrt{2}}{320}\sqrt{1-\varepsilon}\frac{Pe'_m}{2} \quad (8)$$

The works of Saffman (1959, 1960) and Koch and Brady (1985) have shown that $D_L/D_m \sim Pe'_m \ln(Pe'_m)$ and $D_T/D_m \sim Pe'_m$. The logarithmic dependence results from the fact that velocity of fluid at the walls is zero. Different Peclet number contributions to dispersion were identified: hold-up in a closed circulation region (Pe'_m), particle boundary layer [$Pe'_m \ln(Pe'_m)$] and random velocity field contribution (Pe'_m). The agreement with the experimental results of Fried and Combarnous (1971) was very good for both longitudinal and transverse dispersivities. Saffman (1959) modelled random porous media with randomly oriented capillary tubes and found Fickian dispersion at long times with dispersivity approaching a $Pe'_m \ln(Pe'_m)$ behaviour.

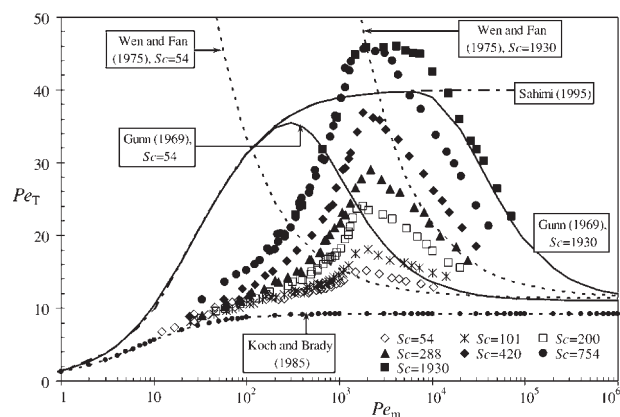


Figure 6. Comparison between experimental data and correlations of other authors, for transverse dispersion.

It is important to enhance the fact that in literature, it was possible to find a large number of empirical correlations. For example, Fetter (1999) used the experimental data obtained by Carberry and Bretton (1958), Raimondi *et al.* (1959), Blackwell *et al.* (1959) and Von Rosenberg (1956) to conclude that longitudinal dispersion coefficient is well represented by the fitted curve:

$$\frac{D_L}{D'_m} = 1 + 1.75Pe'_m \quad \text{valid for } Pe'_m < 50\tau \quad (9)$$

and the data of Blackwell (1962) and Grane and Gardner (1961), to suggest the following equation for transverse dispersion:

$$\frac{D_T}{D_m} = 1 + 0.055Pe'_m \quad (10)$$

In conclusion, we can say that in literature, it was possible to find a large number of equations that have been proposed to correlate dispersion experimental data in porous media. However, the vast amount of data available is obtained mostly for air and water at room temperature (the influence of Schmidt number is not taking in account).

In Figure 5, the lines corresponding to the correlations of Gunn (1969) and of Tsotsas and Schlunder (1988) are represented, for the higher and lower values of Sc in our experiments ($Sc = 57$ and $Sc = 1930$), as well as for gas flow ($Sc = 1$). It may be seen that the correlation of Gunn (1969) is not sensitive to changes in Sc , for $Pe_m < 10^3$, and the correlation of Tsotsas and Schlunder (1988) is much too sensitive to variations in Sc ; however, this correlation describes dispersion in gas flow with good accuracy.

In Figure 6, the lines corresponding Gunn (1969) and Wen and Fan (1975) correlations are represented for the two extreme values of Schmidt observed in our experiments ($Sc = 54$ and $Sc = 1930$); comparison with the experimental points shows that the correlations are very inadequate over significant ranges of Pe_m .

NEW CORRELATIONS

In the present paper attention is restricted to those situations for which dispersion data have been obtained in laboratory investigations, i.e., we excluded the results obtained in groundwater, which depends on the architecture of the sand and gravel sedimentary deposits. For example, the dispersivity values reported from the results of field studies may be as much as six orders of magnitude greater than the corresponding laboratory measured values.

After an exhaustive compilation and a critical analysis of the dispersion data, for beds of mono-sized particles of constant voidage, the results obtained suggest that our experimental data, more than five hundred values (Guedes de Carvalho and Delgado, 2003, 2005; Delgado and Guedes de Carvalho, 2001), are consistent and accurate with the vast amount of data available in literature. And, since they span very wide ranges of the parameters involved, they were taken as the reference data to help identify simple mathematical expressions for the prediction of transverse and longitudinal dispersion coefficients.

Longitudinal Dispersion

In the limit of very low fluid velocity, dispersion is determined solely by molecular diffusion, with $D_L = D_m/\tau$ (τ being the tortuosity factor for diffusion). At high fluid velocities, dispersion is purely 'fluid mechanical' (see Wilhelm, 1962), with $D_L = ud/Pe_L(\infty)$, where u is the interstitial fluid velocity and $Pe_L(\infty) \cong 2$ for gas or liquid flow through beds of (approximately) isometric particles, with diameter d . A common approximation for the 10 intermediate range of fluid velocities is to assume that the effects of molecular diffusion and fluid mechanical dispersion are additive and the resulting expression is given by

$$D_L = D'_m + ud/Pe_L(\infty) \quad (11)$$

where $D'_m = D_m/\tau$. This equation is expected to give the correct asymptotic behaviour at high and low values of Pe_m . In the case of gas flow, see Figure 7, significant deviations are observed in the range $0.6 < Pe_m < 60$; the experimental values of Pe_L are generally higher than predicted by equation (11), with $Pe_L(\infty) \cong 2$. Several equations have been proposed to represent the data in this intermediate range and the equation presented by Hiby (1962),

$$\frac{D_L}{D'_m} = 1 + \frac{0.65Pe'_m}{1 + 7\sqrt{\tau/Pe'_m}} \quad (\text{valid for } Re < 100) \quad (12)$$

is shown to fit the data points reasonably well.

For most gaseous mixtures, far from the critical point, the value of Sc is close to unity, whereas for most solutes in cold water, it is $550 < Sc < 2000$. In recent years, data on longitudinal dispersion have been made available for values of Sc between these ranges, such data were obtained with either supercritical carbon dioxide ($1.5 < Sc < 20$) or heated water ($55 < Sc < 550$).

Some workers have measured axial dispersion for the flow of supercritical carbon dioxide through fixed beds and this provides important new data in the range $1.5 < Sc < 20$ (see Figure 8). The experiments of Catchpole *et al.* (1996) were performed in the range $8 < Sc < 20$, but the individual values of Sc , for the data points represented, are not given by

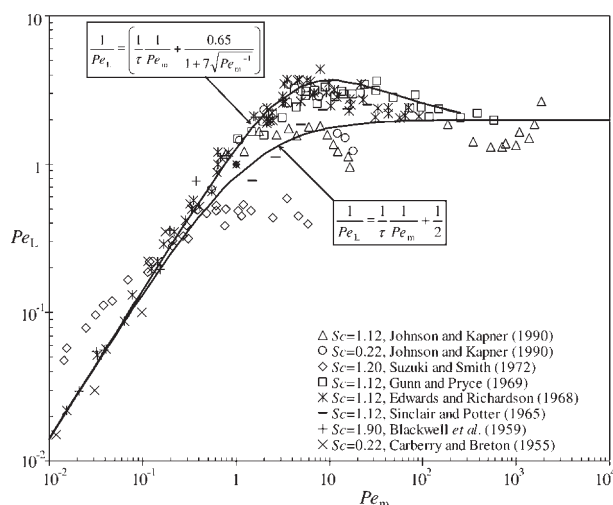


Figure 7. Longitudinal dispersion in gas flow.

the authors. One would expect that the data of Catchpole *et al.* (1996) would lie somewhere between the series of points for $Sc = 57$ and the line corresponding to equation (12). It turns out that values of Pe_L are a little lower than expected, possibly as a result of low accuracy; the scatter is certainly very pronounced. The data of Tan and Liou (1989) represents some 90 data points by those authors, for $0.3 < Re < 135$ and $1.5 < Sc < 3$; the points are more or less evenly distributed, meaning that scatter is very significant.

The data of Yu *et al.* (1999) are for $0.01 < Re < 2$ and $2.0 < Sc < 9.1$. Unfortunately they are not very consistent, particularly in the range $1 < Pe_m < 20$, where the scatter is high and the values of Pe_L are much too low. The experiments of Ghoreishi and Akgermanb (2004), again for the flow of supercritical carbon dioxide, are for $0.1 < Re < 0.3$ and $3.3 < Sc < 5.8$. No influence of Sc on Pe_L is detected, but this is what might be expected, considering that the values of Re are generally very low, with the consequence that the points are partly in the range where dispersion is totally determined by molecular diffusion.

For the case of liquid flow in a porous media, our group used the division in five dispersion regimes to obtain the expressions presented below.

(1) Diffusion regime (valid for $Pe_m < 0.1$):

$$\frac{D_L}{D'_m} = 1 \quad (13)$$

(2) Predominant diffusional regime (valid for $0.1 < Pe_m < 4$):

$$\frac{D_L}{D'_m} = \frac{Pe'_m}{0.8/Pe'_m + 0.4} \quad (14)$$

with an average relative deviation lesser than 14%.

(3) Predominant mechanical dispersion (valid for $4 < Pe_m$ and $Re < 10$):

$$\frac{D_L}{D'_m} = \frac{Pe'_m}{\sqrt{18Pe'_m}^{-1.2} + 2.35Sc^{-0.38}} \quad (15)$$

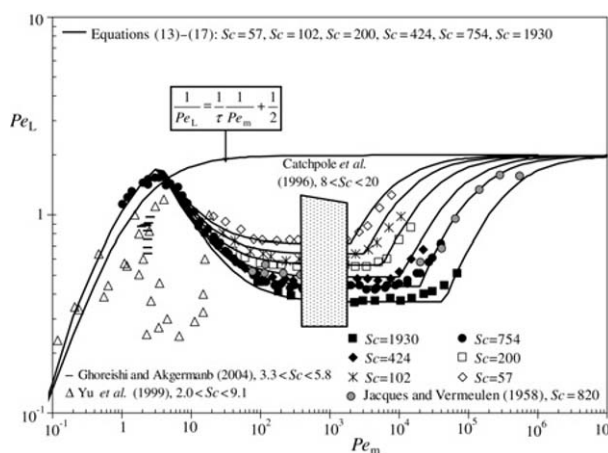


Figure 8. Comparison between experimental data and our correlations, for longitudinal dispersion.

with a deviation lesser than 11%, over the entire range of Pe'_m and Sc .

- (4) Pure mechanical dispersion (valid for $10 < Re$ and $Pe_m < 10^6$):

$$\frac{D_L}{D_m} = \frac{Pe'_m}{25Sc^{1.14}/Pe'_m + 0.5} \quad (16)$$

with an average relative deviation lesser than 16%, over the entire range of Pe'_m and Sc .

- (5) Dispersion out of Darcy domain (valid for $Pe_m > 10^6$):

$$\frac{D_L}{D_m} = \frac{Pe'_m}{2} \quad (17)$$

The correlations proposed are shown (see Figure 8) to be significantly more accurate than previous correlations (see Figure 5) and they cover the entire spectrum of values of Pe_m and Sc expected to be useful. It is important to bear in mind that equations (13)–(17) are recommended only for random packings of approximately 'isometric' particles.

Transverse Dispersion

For gas flow, by simply adding the contributions of molecular diffusion and turbulent dispersion, we suggested the well-known equation (in dimensionless form)

$$\frac{D_T}{D_m} = 1 + \frac{Pe'_m}{12} \quad (18)$$

with an average relative deviation lesser than 12%. Equation (18) give the correct asymptotic behaviour (both for very high and very low Pe_m) for both gases and liquids, as reported by several workers (see Gunn, 1969; Wilhelm, 1962). In the intermediate range of Pe_m they are still a reasonable approximation for gases, the wider deviation being observed in the intervals $3 < Pe_m < 300$, as shown in Figure 9.

For the case of liquid flow in a porous media, our group used the division in four dispersion regimes to obtain the expressions presented below.

- (1) Diffusion regime (valid for $Pe_m < 1$):

$$\frac{D_T}{D_m} = 1 \quad (19)$$

- (2) Predominant mechanical dispersion (valid for $1 < Pe_m < 1600$):

$$\frac{D_T}{D_m} = 1 + \frac{1}{2.7 \times 10^{-5} Sc + 12/Pe'_m} \quad \text{for } Sc < 550 \quad (20a)$$

$$\frac{D_T}{D_m} = 1 + \frac{1}{0.017 + 14/Pe'_m} \quad \text{for } Sc \geq 550 \quad (20b)$$

with a deviation lesser than 8% and 5%, respectively, over the entire range of Pe'_m and Sc .

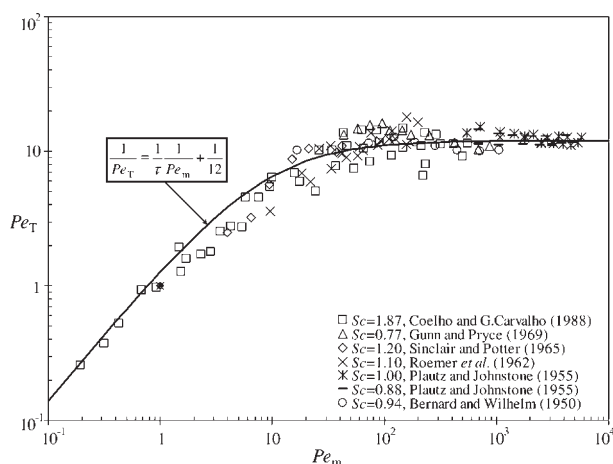


Figure 9. Transverse dispersion in gas flow.

- (3) Pure mechanical dispersion (valid for $1600 < Pe_m < 10^6$):

$$\frac{D_T}{D_m} = \frac{Pe'_m}{(0.058Sc + 14) - (0.058Sc + 2) \exp(-500Sc^{0.5}/Pe'_m)} \quad \text{for } Sc < 550 \quad (21a)$$

$$\frac{D_T}{D_m} = \frac{Pe'_m}{45.9 - 33.9 \times \exp(-21Sc/Pe'_m)} \quad \text{for } Sc < 550 \quad (21b)$$

and the experimental data do not deviate by more than 6% and 4% from the values given by equations (21a) and (21b), respectively.

- (4) Dispersion out of Darcy domain (valid for $Pe_m > 10^6$):

$$\frac{D_T}{D_m} = \frac{Pe'_m}{12} \quad (22)$$

The experimental data are shown in Figure 10, alongside the solid lines corresponding to equations (19)–(22), for the values of Sc indicated in the figure. The agreement is seen to be generally very good, even when the values of Pe_T are

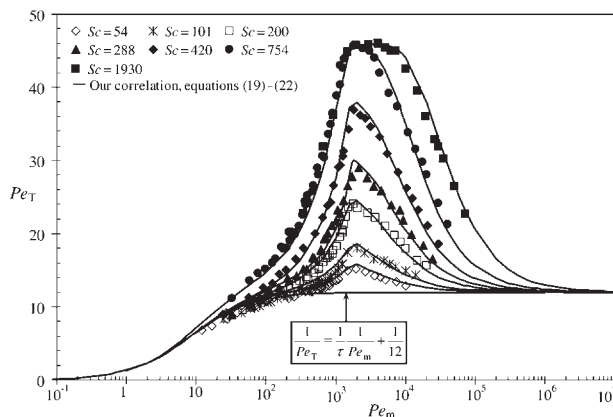


Figure 10. Comparison between experimental data and our correlations, for transverse dispersion.

represented on a linear scale. For $Sc > 550$ (see experiments with values of Sc of 754 and 1930), the above equations representing the data must take into account that Pe_T is only dependent on Pe_m , in the ascending part of the curve Pe_T versus Pe_m and that Pe_T only depends on Re ($=\varepsilon Pe_m/Sc$), in the descending part of the same curve.

CONCLUSIONS

A critical analysis of the vast data on dispersion in porous media, available in literature, is shown to pave the way for the formulation of new simple correlations for the prediction of transverse dispersion and longitudinal dispersion coefficients. The empirical correlations proposed are significantly more accurate and easy to use than previous equations and they cover the entire spectrum of values of Pe_m and Schmidt number expected to be useful.

NOMENCLATURE

C	solute concentration
d	diameter of inert particles
D	diameter of the packed bed
D_L	longitudinal (axial) dispersion coefficient
D_m	molecular diffusion coefficient
D_m'	effective molecular diffusion coefficient ($=D_m/\tau$)
D_T	transverse (radial) dispersion coefficient
L	length of the packed bed
n	empirical constant
r	radial coordinate
t	time
U	superficial fluid velocity
u	average interstitial fluid velocity
z	axial coordinate

Greek symbols

α_L	coefficient of longitudinal dispersivity
α_T	coefficient of transverse dispersivity
ε	bed voidage
μ	dynamic viscosity
ρ	density
τ	tortuosity factor

Dimensionless groups

Pe_m	Peclet number of inert particle ($=ud/D_m$)
Pe_m'	effective Peclet number of inert particle ($=ud/D_m'$)
Pe_L	Peclet number based on longitudinal dispersion coefficient ($=ud/D_L$)
$Pe_L(\infty)$	asymptotic value of Pe_L when $Re \rightarrow \infty$
Pe_T	Peclet number based on transversal dispersion coefficient ($=ud/D_T$)
Re	Reynolds number ($=\rho Ud/\mu$)
Sc	Schmidt number ($=\mu/\rho D_m$)

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