— DIVYATMIKA

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HOMEWORK -3

HWENF552 — Solutions

4.7.3. Solving the previous question first

$$p_{(k)}(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-1/2\sigma^2 k (x-\mu_k)^2\right)}{\sum_{i=1}^{K} \pi_i \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma k^2}(x-\mu_i)^2\right)} \quad \text{— 4.12}$$

$$\sigma_k = x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k) \quad \text{— 4.13}$$

$$\log(p_{(k)}(x)) = \log\left(\frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_k^2)\right)}{\sum_{i=1}^{k} \pi_i \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_i)^2\right)}\right)$$

$$\Rightarrow \log\left(\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_k)^2\right)\right) - \log\left(\sum_{l=1}^{k} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_l)^2\right)\right) \quad \to \text{eq } 1$$

let $K = \log\left(\sum_{l=1}^{k} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_l)^2\right)\right)$

then eq^n 1 becomes :—

$$\log((p_k(x)) = \log\left(\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-1}{2\sigma_k^2}(x-\mu_k)^2\right)\right) - K$$

Further simplifications made :→

$$\log(p_k(x)) = \log(\pi_k) + \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) + \log\left(\exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)\right) - K$$

$$= \frac{-1}{2\sigma^2}(x-\mu_k)^2 + \log(\pi_k) + \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - K$$

$$= \frac{-1}{2\sigma^2}(x^2 + \mu_k^2 - 2x\mu_k) + \log(\pi_k) + \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - K$$

$$= \frac{x \cdot \mu_k}{\sigma^2} + \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k) - \frac{x^2}{2\sigma^2} + \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - K$$

where since the Bayes Classifier assigns an observation to the class which $p_k(x)$ is maximized, it assigns to class j if $p_{(j)}(x) > p_i(x)$ for all $i \neq j$

~~P(j)(x)~~

$p_j(x) > p_i(x) = \log(p_j(x)) > \log(p_i(x))$ for $\forall\, i \neq j$

$$x \cdot \frac{\mu_j}{\sigma^2} + \frac{\mu_j^2}{2\sigma^2} + \log(\pi_j) - \frac{x^2}{2\sigma^2} + \log\left(\frac{1}{\sqrt{2\pi\sigma}}\right) - K$$

$$> x \cdot \frac{\mu_i}{\sigma^2} + \frac{\mu_i^2}{2\sigma^2} + \log(\pi_i) - \frac{x^2}{2\sigma^2} + \log\left(\frac{1}{\sqrt{2\pi\sigma}}\right) - K$$

where if no assumption regarding σ made then an observation is assigned to the class for which the following is true,

$$x \cdot \frac{\mu_j}{\sigma^2} + \frac{\mu_j^2}{2\sigma^2} + \log(\pi_j) - \frac{x^2}{2\sigma_j^2} + \log\left(\frac{1}{\sqrt{2\pi\sigma_j}}\right) > x \cdot \frac{\mu_i}{\sigma_i^2} + \frac{\mu_i^2}{2\sigma^2} + \log(\pi_i) - \frac{x^2}{2\sigma^2} + \log\left(\frac{1}{\sqrt{2\pi\sigma_i}}\right)$$

for all $j \neq i$

<u>4.7.7.</u> The normal density function is $f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(\frac{-1}{\sqrt{2\pi\sigma^2}}(x-\mu_k)^2\right)$

where for $k=0$, $f(4) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(\frac{-1}{2(6)^2}(4-0)^2\right) \approx 0.05568$

and $k=1$, $f(4) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(\frac{-1}{2(6)^2}(4-10)^2\right) \approx 0.0403$

then plugging into Bayes formula gives the probability <u>0.751</u>

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