БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

LEARN HIGHER MATHEMATICS IN ENGLISH

ИЗУЧАЕМ ВЫСШУЮ МАТЕМАТИКУ НА АНГЛИЙСКОМ ЯЗЫКЕ

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для студентов, обучающихся по специальностям
1-31 03 01 «Математика (по направлениям)»,
1-31 03 08 «Математика и информационные технологии»

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В пособии содержатся тексты и упражнения для совершенствования навыков чтения и перевода профессионально ориентированной литературы.

Предназначено для студентов, обучающихся по специальностям 1-31 03 01 «Математика (по направлениям)», 1-31 03 08 «Математика и информационные технологии».

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ПРЕДИСЛОВИЕ

Данное пособие представляет собой комплексный интенсивный курс английского языка. Интенсификация обучения достигается путем отбора минимального объема лексики и грамматики, необходимого для чтения математической литературы и овладения устной речью; параллельного и взаимосвязанного обучения устной и письменной речи на одном и том же языковом и учебном материале.

Цель пособия – выработать у обучаемых навыки и умения, необходимые для практического использования английского языка в профессиональной деятельности.

Книга состоит из семи разделов, включающих грамматический комментарий и комплекс упражнений по изучаемой теме. В каждом разделе содержатся текст А для чтения, лексические упражнения с заданиями на перевод терминов и словосочетаний.

В конце раздела представлен текст В, который предназначен для совершенствования навыков чтения и перевода литературы по специальности.

UNIT I

THE PARTICIPLE. ITS FORMS AND FUNCTIONS

	Active	Passive
Participle I	expressing	being expressed
Perfect Participle	having expressed	having been expressed
Participle II	–	expressed

Ex. 1. ***Read and translate the following sentences in which:***

a) ***Participle I Active*** *is used as an* ***Attribute***:
1. The line passing through these two points is a diameter. 2. The scientist working at this method is well known. 3. Maths is a science requiring professionals, not amateurs. 4. Yesterday the President of the company received an e-mail informing him about the arrival of his business partners. 5. Most mathematicians are men of genius having extraordinary mental abilities.

b) ***Participle I Passive*** *is used as an* ***Attribute***:
1. The examples being given justify what has already been proved. 2. The solution of the problem being considered can be expressed in other ways. 3. The methods being applied seem rather complicated. 4. The quantity being defined is related to the volume of this container. 5. The device being used in our laboratory should be improved.

c) ***Participle II*** *is used as an* ***Attribute***:
1. A number written in front of an algebraic expression is a coefficient. 2. A dot placed between any two numbers is sometimes used as a sign of multiplication. 3. Some questions asked by the teacher are to be stated more precisely. 4. The results so far received do not satisfy us. 5. The methods used in solving the problem were the same.

d) ***Participle I (Active, Passive)*** *is used as an* ***Adverbial Modifier***:
1. Being close to the solution of the problem, he published the results received. 2. When using mathematical language, we avoid vagueness and unwanted extra meanings of our statements. 3. Being spread throughout the world, the Internet provides the connection of networks to enable computers and software to communicate. 4. While considering the group concept, the students must remember four axioms. 5. Being reduced the fraction does not change its value.

e) ***Participle II*** *is used as an* ***Adverbial Modifier***:
1. Expressed in terms of symbols, these relations produce a formula. 2. When used as scientific terms, these concepts have different meanings. 3. If designed and devised in a proper way, the symbolic language becomes universal. 4. When given the information, they were able to complete the research. 5. When asked about the measure of the circumference, the pupil could not say anything.

f) ***Perfect Participle (Active, Passive)*** *is used as an* ***Adverbial Modifier***:
1. Having obtained the necessary devices, we could finish our experiment. 2. Having become familiar with the main laws of statics, we can study the laws of dynamics. 3. Having made a lot of experiments, Faraday discovered the electromagnetic induction. 4. Having picked out the product corresponding to these tables, we obtained a coordinate system for the space. 5. Having been properly approached, the problem appeared easy to solve. 6. Having been invited to the conference, the scientist started preparing his plenary report.

Ex. 2. ***Translate the following into Russian. Pay attention to the difference in translating Participle I and II in the function of an Attribute. Keep in mind several meanings of the verb "to follow".***

to follow	следовать за кем-либо, чем-либо следить за кем-либо, чем-либо придерживаться чего-либо

Model: The film **following** the report showed ... Фильм, последовавший за докладом, показал ...	**Model:** The film **followed** by a discussion showed ... Фильм, за которым последовала дискуссия, показал ...
1. The concert following the meeting showed ...	1. The experiment followed by a lecture showed ...
2. The discussion following the report demonstrated ...	2. His research work followed by an article demonstrated ...

3. The statement of President following the events in the country was ...	3. The theory followed by our professor was ...
4. A number of works following the experiments proved ...	4. The method followed by us proved ...
5. The lecture following the demonstration of the experiment was ...	5. The lecture followed by a demonstration of a film was ...

Ex. 3. ***Answer the following questions using the model.***

Model 1: **Sp.** They used some new method of definition. Did it give good results?

St. Yes, the method used gave good results.

1. They made some measurements. Are the measurements accurate?
2. They received good results. Do these results satisfy them?
3. They demonstrated some experiments. Do these experiments relate to your work?
4. He has measured the volume of the container. Is the volume very large?
5. You have found this relation. Does it remain constant under ordinary conditions?
6. You have determined the length of this line segment. Does this length equal 10 centimeters?

Model 2: **Sp.** He works at the Bureau of Standards. Does he deal with the units of measurement there?

St. You are right. Working at the Bureau of Standards, he deals with the units of measurements.

1. They follow that new method. Will they achieve good results?
2. We know length and time. Can we define velocity?
3. You were interested in these results. Did you discuss them?
4. The experiment was very important. Was it made accurately?

Model 3: **Sp.** Now, you have found the relation between these three lengths. Can you define the volume?

St. Yes, having found this relation, we can define the volume.

1. Now, you have found the length in meters. Can you convert it into English yards?
2. They have just tested the new equipment. Will they put it into operation?

3. They have just measured the area. Can they show us the results?
4. They have calculated the sum of the areas of the two smaller squares. Will they find the area of the largest square?

Ex. 4. ***Match the left and the right parts of the sentences.***

1. Geometry is a branch of mathematics	a) formed of successive sections or segments of straight lines.
2. A sequence is a succession of quantities	b) accepted without any proof.
3. A chord is a line segment	c) extending indefinitely in all directions.
4. A matrix is a set of quantities	d) investigating the relations, properties and measurements of solids, surfaces, lines and angles.
5. A broken line is a line	e) arranged in rows and columns to form a rectangular array.
6. A plane is a flat surface	f) joining any two points on the circumference of a circle.
7. Axioms are the facts	g) formed according to some fixed law.

Ex. 5. ***Complete each of the sentences below by choosing one of the Participles given in parentheses.***

1. An algebraic expression is one in which several numbers ... (representing, represented, having been represented) are connected by signs ... (being indicated, indicated, indicating) the operations and their order.
2. ... (Defining, Having defined, Being defined) the first and the second lengths, we may find the area.
3. We explained the binary system ... (being given, giving, having been given) a specific example.
4. We can always get a closer approximation ... (adding, added, having been added) digits at the right.
5. Unless otherwise ... (stating, stated, being stated), the values used are taken in the decimal system.
6. No number exists which has a negative value when ... (multiplied, multiplying, having been multiplied) by itself.
7. ... (Leaving, Having left, Being left) alone, they kept silence for some time and didn't know what to speak about.
8. Maths is a device ... (designing, having designed, designed) to enlarge human power.

Ex. 6. ***Mixed Bag. State the forms and functions of Participles in the following sentences and translate them.***

1. Discussing some interesting problems, they didn't notice when somebody came in.
2. Being interested in mathematics, Tom spends more time on it than on any other subject.
3. Having informed her before, I was sure that she was waiting for me.
4. Don't forget to put the lights out when going to bed.
5. Being given two more days, the student could complete his term paper.
6. Having been considerably improved, the device was widely used in research.
7. When asked about his experiments, the scientist refused to give any details.
8. The substance obtained was thoroughly investigated.
9. The phenomenon was rather complicated and the processes involved are not yet clear.
10. A graph is given showing the dependence of pressure on temperature.
11. When crossing the street, first look left and then right.
12. The method followed by Professor Webster A.G. was accurate.

Ex. 7. ***Translate the following sentences into English using the necessary Participles.***

1. Собрав весь необходимый материал, студент смог хорошо написать курсовую работу. 2. Не зная, как перевести это предложение, я попросил моего друга помочь мне. 3. Тщательно изучив всю информацию, ученый ответил на все поставленные вопросы. 4. Машина, которая следовала за нами, была зеленого цвета. 5. Деревья, растущие перед нашим домом, были посажены моим дедом. 6. Язык, на котором говорят в США, называется американским вариантом английского языка. 7. Участники конференции заслушали пленарный доклад, после которого последовало много вопросов. 8. Повернув направо, мы увидели новое здание с большими окнами. 9. Будучи очень рассеянным, он сделал много ошибок, переписывая текст. 10. Взглянув на него с удивлением, она вышла из комнаты не сказав ни слова. 11. Зная английский хорошо, он сделал прекрасный доклад на конференции. 12. Проблема, которую он затронул в своем докладе, заслуживает внимания.

Pre-Reading Activity

Guess the meaning of the followed words.

concept	[ˊkɔnsept]	element	[ˊelɪmənt]
group	[gru:p]	sign	[ˊsaɪn]

axion	[ˊæksɪəm]	individually	[ˌɪndɪˊvɪdjuəlɪ]
distributive	[dɪsˊtrɪbjutɪv]	vector	[ˊvektə]
addition	[əˊdɪʃən]	abstraction	[æbˊstrækʃn]
multiplication	[ˌmʌltɪplɪˊkeɪʃn]	abstract	[ˊæbstrækt]
logical	[ˊlɔdʒɪkəl]	component	[kəmˊpounənt]
Egyptians	[ɪˊdʒɪpʃənz]		

Read and learn the basic vocabulary terms.

concern (n)	[kənˊsəːn]	интерес; отношение; забота
contemporary (a)	[kənˊtempərərɪ]	современный
evolve (v)	[ɪˊvɔlv]	развиваться, эволюционировать
consequence (n)	[ˊkɔnsɪkwəns]	значение, важность; следствие
abstract (v)	[æbˊstrækt]	рассматривать отвлеченно, абстрагировать; отделять
curious (a)	[ˊkjuərɪəs]	удивительный, странный
awareness (n)	[əˊwɜənɪs]	осведомленность, информированность
explicitly (ad)	[ɪksˊplɪsɪtlɪ]	явно, в явном виде
implicitly (ad)	[ɪmˊplɪsɪtlɪ]	неявно, в неявном виде (форме)
entity (n)	[ˊentɪtɪ]	объект, данность; сущность; нечто объективно существующее
unique (a)	[juˊnɪːk]	единственный, однозначный; особый, исключительный
closure (n)	[ˊklouʒə]	замыкание
associative (a)	[əˊsouʃɪətɪv]	ассоциативный; сочетательный
identity (a)	[aɪˊdentɪtɪ]	тождество; единица, единичный элемент
inverse (n)	[ˊɪnˊvəːs]	обратная величина (функция, оператор); инверсия, обращение
fascination (n)	[ˌfæsɪˊneɪʃən]	волшебная сила; очарование
infinite (a)	[ˊɪnfɪnɪt]	бесконечный, бесконечно большой; неограниченный
finite(a)	[ˊfaɪnaɪt]	конечный, ограниченный

select (v)	[sɪ´lekt]	выбирать, отбирать
verify (v)	[´verɪfaɪ]	проверять, сличать
minor (a)	[´maɪnə]	малый; незначительный
property (n)	[´prɔpətɪ]	свойство, качество
sense (v)	[´sens]	осознавать; понимать; чувствовать
structure (n)	[´strʌktʃə]	структура; форма, вид; система

Memorise the following word combinations

the distributive law	распределительный (дистрибутивный) закон
to combine together	сочетать, объединять, комбинировать
the law of combination	закон комбинаций (сочетания)
a scalar product	скалярное произведение
a progression of abstractions	последовательность абстракций
to belong to the group	принадлежать группе
to make no difference	не иметь значения
i. e. (id est) that is	то есть

Notes to be paid attention to

I have not asked for help, neither do I desire it.	Помощи я не прошу и в помощи не нуждаюсь.
The first attempt was not successful and neither was the second.	Первая попытка была неудачной, вторая также.
a matter of great consequence	дело большой важности
It is of no consequence.	Это неважно. Это не имеет значения.
It follows as a logical consequence that ...	Логическим выводом из этого является то, что ... . Отсюда следует, что ... .
Time and space are entities.	Время и пространство реально существуют.
an inverse of number	обратная величина числа
an inverse of point	инверсия точки
of minor interest	не представляющий большого интереса

TEXT A

GROUP THEORY

The theory of groups, a central concern of contemporary maths, has evolved through a progression of abstractions. A group is one of the simplest and the most important algebraic structures of consequence.

Some of the components of the group concept (i. e. those essential properties that were later abstracted and formulated as axioms) were recognized as early as 1650 B.C., when the Egyptians showed a curious awareness that something was involved in assuming that $ab = ba$. The Egyptians also freely used the distributive law, namely, $a(b + c) = ab + ac$.

The group concept was not recognized as explicitly as were some of its axioms, but even so it was implicitly sensed and used before Abel and Galois brought it into focus and before Cayley (1854) defined a general abstract group.

A group in its most abstract form consists of a number of entities known as the elements of a group which can be combined together according to various axioms. The form of combination is one in which two elements combine together to give a unique third element. This generalizes the familar operations of addition in which two numbers are added to form a third number or of multiplication in which they are multiplied to give their product. For convenience, this abstract operation is called multiplication and denoted by writing the elements close together. The order in which the elements are written is usually important.

The general definition of a group. A collection of elements, ***G***, will be called ***a group*** if its elements *A, B, C* ... can be combined together (multiplied) in a way which satisfies the four axioms:

I. ***Closure***. The product of any two elements of the group is a unique element which also belongs to the group.

II. ***Associative***. When three or more elements are multiplied, the order of the multiplications makes no difference, i. e.

$$A(BC) = (AB)C = ABC.$$

III. ***Identity***. Among the elements there is **an identity element** denoted by ***I***, with the property of leaving the elements unchanged on multiplication, i. e.

$$AI = IA = A.$$

IV. ***Inverses***. Each element, ***A***, in the group has an inverse (or reciprocal)

$$A^{-1} \text{ such that } AA^{-1} = A^{-1}A = I.$$

A group does not need to have an infinite number of elements, the four elements 1, ***i***, -1, ***-i*** form **a finite group** under multiplication. Neither does it need to use ordinary multiplication as the form combination. The positive and negative integers form a group with addition as the law of combination, the number 0 as the identity and a change of sign to denote the inverse.

Subgroups. When a number of elements selected from a group do themselves form a group it is known as a subgroup. The identity element for the group is also the identity element for the subgroup. The closure and existence of inverses are, therefore, the only laws that need to be verified individually. Every group has at least one subgroup, namely, the minor one consisting of the identity element alone.

Post-Reading Activity

Ex. 8. ***Answer the following questions.***

1. What is the central concern of contemporary maths? 2. Where can we find the traces of the components of the group concept? 3. Who was the first to bring the notion of a group concept info focus? 4. When and by whom was the definition of a general abstract group given? 5. What is a group? 6. According to what axioms can the elements of a group be combined? 7. The order in which the elements are written is not important, is it? 8. What is the main property of an identity element? 9. How many elements may a group have? 10. What is a subgroup? 11. What is the minor subgroup of a group?

Ex. 9. ***Match the English words and word combinations with the Russian equivalents.***

1) a curious awareness	a) принадлежать группе
2) a group concert	b) конечная группа
3) to sense implicitly	c) проверить закон
4) a number of entities	d) ряд (некоторое количество) объектов
5) a unique element	e) современная математика
6) to belong to a group	f) понятие группы
7) an identity element	g) удивительная осведомленность
8) a finite group	h) это не имеет значения
9) to verify a law	i) неявно осознавать
10) an infinite number of elements	j) неограниченное количество элементов
11) the contemporary maths	k) однозначный элемент
12) it makes no difference	l) единичный элемент

Ex. 10. ***Find out whether the statements are True or False. Use the introductory phrases:***

I think that it right.
Quite so. Exactly.
I quite agree with you.
I'm afraid I can't agree with you.
I think you are mistaken.
On the contrary. Far from it.

1. A group consists a number of entities which can be combined together according to one axiom. 2. In the operation of multiplication two numbers are multiplied to give their quotient. 3. In the abstract operation of multiplication the elements are written close together. 4. In a closure axiom the product of any two elements of the group is a unique element which does not belong to the group. 5. When multiplying the elements of a group by an identity element, the former ones are changed. 6. A finite group under multiplication consists of seven elements. 7. There is no need to verify individually the laws of closure and existence of inverses in subgroups.

Ex. 11. ***Read and translate the following sentences paying attention to the word One used as:***

a) an **Indefinite Pronoun** to talk about people in general, including the speaker and hearer or in very general statements, when we are speaking about "anyone, at any time".

One can omit this condition.	Можно опустить это условие.
One should knock before going into somebody's room.	Следует постучать в дверь, прежде чем войти в чью-либо комнату.
One believes that	Считаю, что ... .
One knows that	Известно, что ... .

1. One must emphasize here a basic difference between linear and non-linear systems.
2. Analytic geometry is a branch of mathematics in which one studies geometry by means of algebra.
3. To interpret this phenomenon, one must know the structure of the atom.
4. One can simply determine the location of the point in space.
5. One must have the same denominators when adding two fractions.
6. In fact, one can in theory prove any theorem directly from the axioms.
7. One believes that the procedure described above will simplify the experiment.
8. When making experiments of this kind, one is faced with still another difficulty.

b) a **Substitute word** instead of repeating a singular countable noun. One has a plural ones.

This property is not so essential as that **one**.	Это свойство не является таким важным, как то свойство.
Green apples often taste better than red **ones**.	Зеленые яблоки на вкус часто лучше, чем красные.

1. The result, like the one just described, is in no way surprising.
2. The procedure is straight-forward and is the one followed throughout the experiment.
3. If a mathematical problem is a strict expression of a physical one, it has a unique solution.
4. This equation essentially differs from the one which we solved at the last lesson.
5. The numerical set is one whose members have numerical values.
6. These are easy questions to answer and those are difficult ones.
7. We shall replace the old equations by new ones.

Ex. 12. ***In which of the following English sentences the italicized group of words will be translated as:***

1. Достигнув успеха ...
a) *Having been achieved* the success did not prevent the scientist from working hard and developing the problem.
b) *Achieving* success and recognition some scientists stop working hard.
c) *Having achieved* success and recognition, the scientist went on working hard over his problem.

2. Используя алгебру ...
a) *Having used* algebra, we can reduce complex problems to simple formulas.
b) *Using* algebra, we can reduce complex problems to simple formulas.
c) *Being used,* algebra helped us to reduce complex problems to simple formulas.

3. Поняв идею ...
a) *Having understood* the idea, we can simplify our notation.
b) *Understanding* the idea, we can simplify our notation.
c) *Having been understood*, the idea turned out to be a simple one.

Ex. 13. ***Read and translate the following sentences paying attention to the inversion of the verb.*** *Certain adverbs and adverbial phrases, mostly with a negative sense, can for emphasis be placed first in a sentence and are then followed by the inverted (i. e. interrogative) form of the verb. The most important of these are given below:* ***never*** *никогда,* ***seldom*** *редко,* ***neither***, ***nor*** *а также не,* ***hardly***, ***scarcely*** *...* ***when*** *едва (только) ... как,* ***no sooner*** *...* ***than*** *как только, не успел ... как,* ***not only*** *...* ***but*** *не только ... но,* ***not until (till)*** *и только когда.*

Nor should we forget the importance of this argument.	*А также мы не должны забывать о весомости этого аргумента.*
Never before had I been asked to accept a bribe.	*Никогда раньше мне не предлагали взятку.*

1. Not till he got home did he realize that he had lost that important document.
2. He had no money, nor did he know anyone he could borrow from.
3. Hardly had I arrived when trouble started.
4. No sooner had she agreed to marry him than she started to have doubts.
5. Never in my life have I seen such a proof.
6. The ancients had no knowledge of stellar distances, neither was there then any means by which they could determine them.
7. Since the Moon has no atmosphere, there can be no wind, neither can there be any noise, for sound is carried by the air.
8. Scarcely had the professor started his lecture when the lights in the room went off.

Ex. 14. ***Translate the following word combinations into English using either Participle I or Participle II.***

1) решения, отвечающие нашим требованиям; 2) наука, обеспечивающая высокий уровень жизни общества; 3) приборы, изобретенные нашими инженерами; 4) методы исследований, хорошо известные ученым; 5) мир, созданный наукой; 6) функция, определенная посредством формулы; 7) прямая, соединяющая две точки; 8) угол, делящий плоскость; 9) разделенная диагональ; 10) примененный метод; 11) работа, продолженная на следующий день; 12) предмет, взятый в качестве модели.

Ex. 15. ***Read and translate the following sentences. Write out (in row) the numbers of sentences in which the Participle is used as:***

a) ***an Attribute*** *b)* ***an Adverbial Modifier***

1. Given two sets *X* and *Y*, there is a set whose elements are those which belong only to one of the two given sets.
2. Expressed in math terms, this theorem gives a general method of calculating the area.
3. The sense implied in this statement is not clear.
4. Certain properties of the real world can be described using numbers.
5. When finding the product of multinomials, we make use of the distributive law.
6. The group of integers under addition has subgroups comprising all even integers.
7. Lobachevsky wrote a new geometry asserting that there could be several parallels.
8. Having calculated the area, we can say now that the formula is exact.
9. Parallel lines are lines extending in the same direction and being the same distance apart no matter how far extended.

10. Having supposed the inequality, we obtained the necessary results.
11. Considering specific physical phenomena, we may see that one and the same quantity in one phenomenon is a constant while in another it is a variable.
12. Having started from a system of axioms, we then could make certain logical deductions.
13. The statements followed by some illustrations were rather convincing.

Ex. 16. ***Ask special questions using the words in parentheses.***

1. There are really two types of problems involved here. (How many?)
2. Having understood the ideas, we can simplify our notation. (When?)
3. Being interested in set theory, he never missed his special course. (Why?)
4. Rational functions are functions involving an additional operation of division. (What?)
5. A point representing a variable is called a variable point. (How?)
6. The students studying the theory of sets find this statement interesting. (Who?)
7. Equations containing one or more variables to the first power only are linear in one or two variables. (What?)
8. We can find that some elements form a smaller group inside the big one. (What?)
9. Groups can arise in many quite distinct situations. (In what cases?)
10. When speaking of quantities, we shall have in view their numerical values. (What?)
11. The meanings of these words are often confused in speech. (Where?)

Ex. 17. ***Translate into English the following sentences.***

1. Сейчас теория группы разрабатывается абстрактно, так что ее можно применять во многих различных ситуациях.
2. Чтобы суметь исследовать структуру группы более детально, необходимо ввести сложение как еще одну операцию между элементами группы.
3. Феликс Кляйн (Felix Klein) продемонстрировал, что понятие группы может оказаться полезным при классификации многих областей математики.
4. Развитие этих двух ветвей алгебры привело к качественно новым проблемам науки, связанным с возникновением теории Галуа и теории групп.
5. Группа – это математическая система, элементы которой удовлетворяют четырем основным правилам.
6. Таким образом, предмет «Алгебра» определился в XVIII в., превратившись в науку об алгебраических уравнениях.

7. Термин «группа» обозначает особый вид математической системы, и он не имеет ничего общего с разговорным значением, приписываемым слову «группа».

Ex. 18. ***Topics for discussion.***

1. The history of developing the group concert.
2. The working mechanism of such a structure as a group.
3. The four rules characterizing a group.
4. Dwell on the notion of a subgroup.

Ex. 19. ***Read the text and find the answers to the following questions.***

1. What was the main problem concerning algebraic equations in the XVI–XVIII centuries? 2. How could equations of degree 3 and 4 be solved? 3. In those years the scientists considered the idea of solving equations of degree 5 and higher by using roots of higher order absolutely natural, didn't they? 4. Who settled the problem once and for all? 5. What method did he offer? 6. What is the criterion for solvability of equations?

TEXT B

GALOIS'S CONTRIBUTION TO GROUP THEORY

Unsolved problems gave rise to the most remarkable and novel developments in mathematics, when, after centuries of futile (тщетный) search for solutions, the suspicion drew that these problems might be definitely insoluble. Thus, mathematicians were challenged to investigate the question: "How is it possible to prove that certain problems cannot be solved?"

In algebra, it was the problem of solving equations of degree 5 and higher which led to this new way of thinking. The equations of degree 3 or 4 could be solved by a process similar to the elementary method for solving quadratic equations. One says that algebraic equations up to the fourth degree can be solved "by radicals". Nothing seemed more natural than to extend this procedure to equations of degree 5 and higher by using roots of higher order. All such attempts failed.

It was not until early in the nineteenth century that the Norwegian genius N. H. Abel conceived the then revolutionary idea of proving the impossibility of the solution of the general algebraic equation of degree n by means of radicals. Using the group concept E. Galois settled once and for all the matter of solving algebraic equations. He proved that it is impossible to solve algebraically the general equation of degree greater than four. For centuries before his time, the greatest algebraists had been struggling to solve equations of the fifth degree and higher. His youthful contemporary, Abel, established the same important fact but not by *'so neat a method'*.

Galois showed that every equation could be associated with a characteristic group and that the properties of this group could be used to determine whether the equation could be solved by radicals. He developed the theory of groups and applied it to the question in hand. His theorem established a criterion for solvability of an equation: “An algebraic equation is algebraically solvable if and only if its group is solvable”. So, it was Galois who finally closed the subject. On the basis of the four rules characterising a group, he derived a wealth of mathematical theory. He showed that the solution cannot be affected if we insist on expressing the unknown by any combinations of the coefficients in the equation, using only a finite number of additions, subtractions, multiplications, divisions and root extractions. If we permit an infinite number of steps, however, Galois' theorem no longer holds true.

UNIT II

THE ABSOLUTE PARTICIPLE CONSTRUCTION

NOMINATIVE ABSOLUTE PARTICIPLE CONSTRUCTION (САМОСТОЯТЕЛЬНЫЙ ПРИЧАСТНЫЙ ОБОРОТ)

Самостоятельный причастный оборот – это такой оборот, в котором причастие имеет свое собственное подлежащее, выраженное **существительным в общем падеже или местоимением в именительном падеже**. В конструкции может использоваться как причастие I, так и причастие II.

Самостоятельный причастный оборот может стоять в начале или в конце предложения. Если независимый причастный оборот стоит **в начале предложения**, он переводится на русский язык придаточным предложением времени, причины, условия с союзами **когда, так как, если, поскольку**. Независимый причастный оборот **в конце предложения** обычно переводится самостоятельным предложением (иногда со словами **при этом, причем** и при помощи союзов **а, и, но**).

Иногда предлог "with" вводит самостоятельный причастный оборот, что не влияет на перевод. В предложении независимый причастный оборот **выделяется запятой** и выполняет функцию обстоятельства.

Самостоятельные причастные обороты широко употребляются в литературных произведениях и в научной литературе, но редки в обычной речи.

The classroom being occupied, they had to wait a little.	**Поскольку** аудитория была занята, им пришлось немного подождать.
The article having been translated, the student showed it to the teacher.	**Когда** статья была переведена, студент показал её преподавателю.
Two of them headed toward the director's office**, the other three staying in the hall.**	Двое из них направились к кабинету директора, **а** трое других остались в коридоре.

Sydney is the largest city in Australia, **with Melbourne being the second largest**.	Сидней – самый большой город в Австралии, **а** Мельбурн – второй по величине.
She listened to his story quietly, **with her eyes closed.**	Она слушала его рассказ спокойно, **при этом** её глаза были закрыты.

Ex. 1. ***Read the following sentences, in which the Absolute Participle Construction is used at the beginning of the sentence. Translate them into Russian.***

1. The dictionaries being brought, we were given texts for translating. 2. All the preparations completed, we could start the experiment. 3. The first part of the work having been finished, the results were published in a journal. 4. The report being well written, it was read with pleasure. 5. Most of the work having been done, we decided to have a rest. 6 The article being difficult, I couldn't translate it without your help. 7. The advantages of the new machine being obvious, we decided to make use of it.

Ex. 2. ***Read the following sentences in which the Absolute Participle Construction is used at the end of the sentence. Translate them into Russian.***

1. They finished the experiment, the result being quite satisfactory. 2. My friend was asked many questions, some of them being very difficult. 3. The teacher gave me two abstracts yesterday, both containing interesting facts. 4. The professor spoke about the latest achievements in the national economy, his lecture being illustrated with diagrams. 5. The plan was discussed in details, many scientists taking part in this discussion. 6. Many results used before remain true for our case, the proofs being similar. 7. We may use two different methods, the first being a more general one.

Ex. 3. ***Read the following sentences, translate them paying attention to the place of the Absolute Participle Construction.***

The examinations being over, most of the students left the city. 2. Cybernetics is a well known branch of science, it being still developed in the 21st century. 3. Most of the students of our group go in for sports, many of them being fond of gymnastics. 4. With the value of x being given, the velocity of a body can easily be computed. 5. The sides of the triangle having the same measure, the angles opposite these sides have the same degree measure. 6. The area of a circle is given by the formula $A = \pi r^2$, r representing the radius. 7. Every measurement can be named in many different ways, the most convenient name being chosen in every case. 8. With the distance having been defined, you can expect to find the speed.

Ex. 4. ***Change the following according to the model. Use the Absolute Participle Construction.***

Models: a) Since (as) numbers 4.12 and 3.5 are names for fractional numbers, we may write them as complex fractions.
Numbers 4.12 and 3.5 being names for fractional numbers, we may write them as complex fractions.
b) After the experimental work had been completed, they could publish the results obtained.
The experimental work having been completed, they could publish the results obtained.

1. As the speed of light is extremely great, we cannot measure it by ordinary methods. 2. After the first question had been considered, we could pass over to the next one. 3. Since other conditions are equal, the acceleration will be the same. 4. After the set of axioms had been accepted, we could predict many new properties of the system involved. 5. After the theorem had been stated, the students could begin solving it. 6. After all the necessary changes had been made, the experiment showed different results. 7. Since *a* and *b* are real numbers, the general expression $a + bi$ represents a complex number. 8. As the measure of the circumference is given, you can find the area of the interior of the circle.

Ex. 5. ***Translate the sentences into Russian paying attention to the Objective Participial Constructions and Participle II in the construction "have something done".***

I saw him entering the University building.	Я видел, как он входил в здание университета.
She wants to have several copies of this article made.	Она хочет, чтобы ей сделали несколько копий этой статьи.

1. We saw her reading a book in the library. 2. The students had their tests checked a few days ago. 3. She heard him making a speech at the conference. 4. The teacher listened to the text being translated from Russian into English by the student. 5. The supervisor had the report presented last week. 6. We are having the results of the test announced at the moment. 7. I saw her talking to the lecturer. 8. We didn't notice him entering the classroom. 9. I've never heard him speaking English. 10. Can I have my documents typed immediately?

Ex. 6. ***Translate the sentences into English, use the Absolute Participle Construction.***

1. Так как температура постоянно изменяется, мы не можем сделать точных измерений. 2. Когда занятия окончились, студенты начали

обсуждать свои планы на выходной день. 3. Если дана длина окружности, можно найти площадь внутренней части круга. 4. Существует много методик обучения иностранным языкам, причем данная методика – самая эффективная. 5. В предыдущей главе рассматривались многоугольники, этот вид геометрических фигур является очень важным в изучении геометрии. 6. Поскольку квадрат любого числа положительный, то квадратный корень от отрицательного числа является мнимым. 7. Если даны две точки *A* и *B*, мы можем начертить прямую, соединяющую их.

Pre-Reading Activity

Guess the meaning of the following words:

collection (n)	[kə´lekʃən]	algebra (n)	[´ældʒıbrə]
object (n)	[´ɔbdʒıkt]	phrase (n)	[´freız]
element (n)	[´elımənt]	symbol (n)	[´sımbəl]
theory (n)	[´θıərı]	standard (a)	[´stændəd]
concrete (a)	[´kɔnkri:t]	season (n)	[´si:zən]
real (a)	[´rıəl]	variation (n)	[vɛrı´eıʃən]
interest (n)	[´ıntrıst]	idea (n)	[aı´dıə]
abstract (a)	[´æbstrəkt]	vertical (a)	[´və:tıkəl]
sphere (n)	[´sfıə]	fact (n)	[´fækt]
special (a)	[´speʃəl]	mathematics (n)	[mæθı´mætıks]

Read and learn the basic vocabulary terms:

set (n)	[set]	множество
belong (v)	[bı´lɔŋ]	принадлежать
introduce (v)	[ıntrə´dju:s]	вводить, представить
capital (a)	[´kæpıtəl]	заглавный
convention (n)	[kən´venʃən]	договоренность, условие
keep (to) (v)	[ki:p]	придерживаться, держаться (чего-либо)
rigidly (adv)	[´rıdʒıdlı]	строго
occur (v)	[ə´kə:]	встречаться, происходить
currently (adv)	[´kʌrəntlı]	в настоящее время
find out (v)	[´faınd´aut]	узнать, выяснить
specify (v)	[´spesıʃaı]	точно определять

list (v)	[ˊlɪst]	перечислять
notation (n)	[nouˊteɪʃən]	обозначение, запись
enclose (v)	[ɪnˊkləuz]	заключать
curly (a)	[ˊkə:lɪ]	фигурный
mention (v)	[ˊmenʃən]	упоминать, ссылаться
denote (v)	[dɪˊnəut]	обозначать
allow (v)	[əˊlau]	позволять
property (n)	[ˊprɔpətɪ]	свойство
precisely (adv)	[prɪˊsaɪslɪ]	точно, определенно
exact (a)	[ɪgˊzækt]	точный
infinite (a)	[ˊɪnfɪnət]	бесконечный
finite (a)	[ˊfaɪnaɪt]	конечный
notion (n)	[ˊnəuʃən]	понятие
turn out (v)	[tə:nˊɑut]	оказываться, выпускать, выводить
confuse (v)	[kənˊfju:z]	путать, смешивать
empty (a)	[ˊemptɪ]	пустой
distinguish (v)	[dɪˊstɪŋgwɪʃ]	различать, выделять, распознавать
establish (v)	[ɪsˊtæblɪʃ]	установить
concisely (adv)	[kənˊsaɪslɪ]	кратко, сжато

Notes:

none at all – ни один вообще
to tell them apart – отличить друг от друга
instead of a list – вместо списка (перечня)
at any rate – во всяком случае, по меньшей мере

TEXT A

SETS

A set is a collection of objects. The objects belonging to the set are the elements or members of the set. Although in introducing set theory it is helpful to work with concrete sets, whose members are real objects, the sets of interest in mathematics always have members which are abstract mathematical objects: the set of all circles in the plane, the set of points on a sphere, the set of all numbers.

As in ordinary algebra we shall use letters to represent sets and elements, small letters being used for elements and capital letters for sets. But it is impossible to keep rigidly to this convention because sets can themselves be elements of other sets. The phrase “is a member of” occurs so often that it

is convenient to have a symbol, the one currently in use is $\in$. So $x \in S$ means "x is a member of S".

A set is considered to be known if we know what its elements are – or at any rate if in theory we can find out. There are many ways of specifying a set, of which the simplest is to list all the members. The standard notation for this is to enclose the list in curly brackets. So {1, 2, 3, 4} is the set whose members are 1, 2, 3, 4 and only these, while {spring, summer, autumn, winter} is the set of seasons.

Two sets are equal if they have the same elements. We can easily write things like {1, 2, 3, 4, 4}. Despite being mentioned several times, there is only one 4 in the set, which being thus equal to {1, 2, 3, 4}. When using the curly bracket notation, elements listed more than once are thought of as occurring once in the set. The order inside the brackets makes no difference. The set {1, 2, 3, 4, 4} has the same elements as {1, 2, 3, 4}, so is the same set.

More generally, a symbol such as {all epic poems} denotes the set of all epic poems. A variation of this idea allows us to write {x|x is an epic poem} for the same set. The vertical bar may be read as "such that", and the set of all x such that x is an epic poem is the same as the set of all epic poems. The set {n|n is an integer and $1 \leq n \leq 4$} is the same as the set {1, 2, 3, 4}.

Instead of a list, we give a property which specifies precisely the elements we wish to be included in the set. If we are careful with our definitions, making sure that we specify the exact property we want, this is as good as a list, and is usually more convenient. For sets with infinitely many members, such as {all whole numbers}, it is in any case impossible to give a complete list. The same is true for sets with a sufficiently large finite set of elements.

The mathematical notion of a set allows sets with only one member or even no members at all. If you specify a set by some property it may turn out later that there is only one object with that property or none at all. Sets with one element must not be confused with the element itself. It is not true that x and {x} are equal; {x} has just one member, namely x, while x may have any number of members depending on whether or not it is a set, and if it is, which set.

For exactly the same reasons that we allow sets with just one element, we have to allow sets with no elements at all. A set with no elements is called an empty set. A fact now emerges which many people find surprising: there is only one empty set. All empty sets are equal. Any two empty sets are equal because, in the absence of any members to distinguish them by, there is no way to tell them apart. Having established that there is just one empty set we can give it a symbol, the current one being Ø (which is a special symbol). The empty set is not "nothing"..., nor does it fail to exist. It is just as much in existence as any other set. It is its members that do not exist. It must not be confused with the number 0: for 0 is a number, whereas Ø is a set. Ø is one

of the most useful sets in mathematics. One of its uses is to express concisely that something does not happen.

Post-Reading Activity

Ex. 7. ***Answer the following questions.***

1. What is a set? 2. What are the elements of the set? 3. What sets are of interest in mathematics? 4. What do we use to represent sets and elements? 5. What set is considered to be known? 6. What is the simplest way of specifying a set? 7. What is the standard notation for a set? 8. What sets are equal? 9. How can we specify the elements of a set? 10. How many members may a set have? 11. What is an empty set? 12. How is an empty set represented? 13. Does an empty set exist at all?

Ex. 8. ***Match the English words and word combinations with the Russian equivalents.***

1) exact property	a) члены множества
2) to enclose the list in curly brackets	b) множества, вызывающие интерес в математике
3) to keep rigidly to the convention	c) конкретные множества
4) set theory	d) абстрактные объекты
5) just one element	e) способ определять множество
6) a finite set of elements	f) обычно принятое обозначение
7) elements listed	g) заключить перечень в фигурные скобки
8) concrete sets	h) одни и те же элементы
9) a way of specifying a set	i) перечисленные элементы
10) the same elements	j) точное свойство
11) infinitely many elements	k) бесконечное число элементов
12) standard notation	l) полный перечень
13) abstract objects	m) конечное число элементов
14) a complete list	n) только один элемент
15) it makes no difference	o) не имеет значения
16) the sets of interest in mathematics	p) теория множества
17) members of the set	q) строго придерживаться условия

Ex. 9. ***Fill in the blanks with the words from the box.***

empty, members, to list, notation, specifying, difference, precisely, the same

1. The objects belonging to the set are the elements or ... of the set. 2. There are many ways of ... a set. 3. The simplest way of specifying a set is ... all the members. 4. The standard ... is to enclose the list in curly brackets. 5. Two sets

are equal if they have ... elements. 6. The order inside the brackets makes no ... 7. Instead of a list, we give a property which specifies ... the elements. 8. A set with no elements is called an ... set.

Ex. 10. ***Ask questions for which the given sentences are answers.***

1. A set is a collection of objects. (What?) 2. The objects belonging to the set are the elements or members of the set. (Which?) 3. The sets of interest in mathematics always have members which are abstract mathematical objects. (What?) 4. In the algebra of sets we use letters to represent sets and elements. (Where?) 5. A set is considered to be known if we know what its elements are. (When?) 6. There are many ways of specifying a set. (How many?) 7. The standard notation is to enclose the list in curly brackets. (What?) 8. Two sets are equal if they have the same elements. (When?) 9. Instead of a list, we give a property which specifies precisely the elements of the set. (What?) 10. For sets with infinitely many members, it is impossible to give a complete list. (Which?) 11. The mathematical notion of a set allows sets with only one member or even no members at all. (What?) 12. A set with no elements is called an empty set. (What?) 13. All empty sets are equal. (What?)

Ex. 11. ***Find out whether the statements are true or false. Use introductory phrases.***

Exactly. Quite so.	***Quite the contrary.***
I fully agree to it.	***Not quite. It's unlikely.***
I don't think this is the case.	***Just the reverse.***

1. In ordinary algebra we use small letters to represent sets and capital letters to represent elements. 2. Sets can themselves be elements of other sets. 3. Two sets are equal if they have different elements. 4. The order inside the brackets makes difference. 5. A set is considered to be known if we know what its elements are. 6. The mathematical notion of a set doesn't allow sets with one member. 7. The empty set is "nothing". 8. The empty set must not be confused with the number 0.

Ex. 12. ***Read the following sentences, find the Absolute Participle Construction and translate them into Russian.***

1. Each major concept embraces not one but many diverse objects, all having some common property. 2. The theorem having been stated, the students began proving it. 3. We may use two different methods, the first being the more general one. 4. This system consists only of one equation, the other two being its consequences. 5. The theorem being true, we cannot assume that its converse must be true. 6. A function being continuous at every point of the set, it is continuous throughout the set. 7. No sign preceding a term, the plus sign is

understood. 8. The coordinates being given, we can specify the position of any point in the plane. 9. The set is bounded above and below, there being numbers greater than and numbers smaller than all the numbers in the set.

Ex. 13. ***Read and translate the following sentences paying attention to the translation of ONE as the subject.***

Model:	*One must ...*	*Нужно ...*
	One can ...	*Можно ...*
	One should ...	*Следует ...*
	One needs ...	*Необходимо ...*
	One knows ...	*Знаешь ...*

1. Another place where **one** must be careful about logic is when proving something impossible. 2. In elementary mathematics **one** comes across various objects designated by the term "function". 3. **One** must learn how to draw graphs. 4. In this case **one** needs to consider all possible proofs. 5. Similarly **one** can prove the other laws of arithmetic. 6. **One** should not be surprised at this. 7. In order to apply group theory to a branch of mathematics, **one** must check that the relevant objects are groups. 8. For practical purposes **one** needs good approximate constructions. 9. **One** must realize that the development of mathematics was by no means the product of one individual's efforts.

Ex. 14. ***Say this in English.***

1. Множество – это набор каких-либо объектов, называемых его элементами и обладающих общим для всех них характерным свойством. 2. Обычно множества обозначаются заглавными буквами, а члены множества строчными буквами. 3. Объекты, которые принадлежат множеству, являются элементами или членами множества. 4. Множества, представляющие интерес в математике, содержат абстрактные математические объекты. 5. Считают, что множество известно, если мы знаем его элементы. 6. Существует много способов точного определения множества. 7. Два множества равны, если они содержат одинаковые элементы. 8. Множества с одним элементом не нужно путать с самим элементом. 9. Множество, не содержащее элементов, называется пустым. 10. Пустое множество нельзя путать с нулем, так как 0 – это число, а пустое множество – это множество.

Ex. 15. ***Topics for discussion:***

1. Give the definition and properties of a set.
2. Dwell on the sets in algebra.
3. Describe sets in everyday life.
4. Give the ways of specifying a set.
5. Speak on the notion of an empty set.

Ex. 16. ***Read the text and find the answers to the following questions.***

1. What does set theory study?
2. What objects is set theory applied to?
3. Where can the language of set theory be used?
4. When was the modern study of set theory initiated?
5. What does set theory begin with?
6. What is the subset relation?
7. How many full derivations of complex mathematical theorems from set theory have been formally verified?

TEXT B

SET THEORY

Set theory is the branch of mathematical logic that studies sets, which are collections of objects. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics. The language of set theory can be used in the definitions of nearly all mathematical objects.

The modern study of set theory was initiated by George Cantor and Richard Dedekind in the 1870s.

Set theory begins with a fundamental binary relation between an object o and a set A. If o is a member (or an element) of A, write $o \in A$. Since sets are objects, the membership relation can relate sets as well.

A derived binary relation between two sets is the subset relation, also called set inclusion. If all the members of set A are also members of set B, then A is a subset of B, denoted $A \subseteq B$. For example, {1,2} is a subset of {1,2,3}, but {1,4} is not. From this definition, it is clear that a set is a subset of itself; for cases where one wishes to rule out this, the term 'proper subset' is defined. A is called a proper subset of B if and only if A is a subset of B, but B is not a subset of A.

Just as arithmetic features binary operations on numbers, set theory features binary operations on sets.

Set theory as a foundation for mathematical analysis, topology, abstract algebra, and discrete mathematics is likewise uncontroversial; mathematicians accept that (in principle) theorems in these areas can be derived from the relevant definitions and the axioms of set theory. Few full derivations of complex mathematical theorems from set theory have been formally verified, however, because such formal derivations are often much longer than the natural language proofs mathematicians commonly present.

Set theory is a major area of research in mathematics, with many interrelated subfields.

UNIT III

THE GERUND. ITS FORMS AND FUNCTIONS

FORMS OF THE GERUND

	Indefenite	Perfect
Active	asking writing	having asked having written
Passive	being asked being written	having been asked having been written

Ex. 1. ***Study the functions of the Gerund. State their forms. Translate the sentences into Russian.***

a) *the Gerund is used as a* ***Subject****.*
1. Writing a sentence in algebraic form, as we have seen, involves two steps. 2. Drawing a straight line in one direction gives you a one way extension. 3. Reducing a fraction means bringing it to lower terms. 4. Reading slowly is useful for beginners. 5. Locating the point on the y-axis gives you the first point on the line. 6. Knowing the properties of equality will help you decide whether a sentence is true or false.

b) *the Gerund is used as a part of a* ***Predicate****.*
1. Our task is proving the correctness of the given statement. 2. The young scientist began experimenting. 3. We expected being given further assistance. 4. This terminology needs improving. 5. The scientist expected being included in the experimental group. 6. She stopped investigating the problem as her approach was wrong.

c) *the Gerund is used as a* ***Direct object****.*
1. We discussed improving the shape of the model. 2. Do you mind being examined first? 3. I don't remember speaking to him about this fact. 4. Avoid

making such bad mistakes. 5. They are busy now reading the text. 6. He suggested taking part in this conference.

d) *the Gerund is used as a* ***Prepositional object****.*
1. He was prevented from finishing his work. 2. We succeeded in accomplishing our task. 3. He insisted on writing the thesis as soon as possible. 4. These computers are capable of solving systems with a hundred or more unknowns, if necessary. 5. They are concerned with applying their knowledge of the subject to solving these problems. 6. We cannot agree to testing the new method without being given additional time.

e) *the Gerund is used as an* ***Attribute****.*
1. What ways of learning words do you find most effective? 2. This is the method of doing such tasks. 3. I can't improve my English because I don't have any opportunities of speaking it. 4. The idea of using symbols instead of words proved very helpful. 5. There exists a very efficient algorithm for solving most linear programming problems. 6. The procedure of reducing a fraction to its lowest terms is not complicated.

f) *the Gerund is used as an* ***Adverbial modifier****.*
1. In considering the problem we have to deal with the laws of motion. 2. The product may be found by multiplying the factors contained in the given mathematical sentence. 3. We can't agree to testing the new method without being given additional time. 4. In naming geometric objects we often use capital letters. 5. By applying the knowledge of geometry you can locate the point in the plane. 6. After discussing the problem in detail they found the best solution.

Ex. 2. ***State the form and the function of the Gerund. Translate the sentences into Russian.***

1. We insisted on carrying out another experiment to check the results.
2. The absolutely new contribution made by Descartes was in importing the idea of motion into geometry.
3. This is the basic method of solving problems of statics.
4. It is worth noting that the work of the early Arab mathematicians makes no clear division between arithmetic and algebra.
5. Since the equation is linear and has constant coefficients it can be easily solved by using classical differential equation theory.
6. He also improved the notation for representing the extraction of roots.
7. Combining the integrals gives the following equation.
8. The preceding definitions have laid the foundation for considering the variation of a functional.
9. Leonardo's solution is worth quoting for its elegance.

Ex. 3. ***Put the Gerund in the correct form. Use prepositions where necessary.***

1. She continued (to translate) the text from English into Russian.
2. He found the product (to multiply) the numerals.
3. She is afraid (to take) the exam.
4. He used a ruler (to draw) a straight line.
5. One must be very careful (to measure) the volume of an object.
6. They insist on the question (to reconsider).
7. She stopped (to investigate) the problem as her approach was wrong.
8. The problem (to discuss) various points of view was a very useful exercise.
9. Measurement is a process (to associate) numbers with certain objects.

Ex. 4. ***Insert prepositions (in, of, to, from, by). You can use the same preposition in more than one sentence.***

1. His mathematical power, which never failed him to the end of his life, was employed at this period __ originating the calculus of probabilities, and __ inventing the arithmetical triangle.
2. The mathematician who came nearest __ solving the challenge questions issued by Pascal on the cycloid (циклоид) was John Wallis.
3. But he differs from B. F. Cavalieri (an Italian mathematician) __ regarding lines as made up of infinitely small lines, surfaces of infinitely small surfaces, and volumes of infinitely small volumes.
4. Leonardo's favorite method __ solving many problems is by the method of 'false assumption', which consists __ assuming a solution and then altering (изменение) it by simple proportion as in the rule of three (вычислительный метод в математике).
5. We have succeeded __ verifying that the increment (приращение) can be written in the form of the following equation.
6. His famous experiment __ dropping bodies of different weights from the tower of Pisa enabled him to demonstrate that all bodies undergo the same acceleration __ falling towards the earth, a result which his experiment with light and heavy pendulums (маятники) also proved.
7. He also discusses solids generated __ revolving a curve about an axis, and in the last section deals with the problems of maxima and minima.
8. The intellectual trend of that time was such as to prevent mathematics __ becoming a popular subject.
9. Since Euler's equations usually cannot be solved analytically, one naturally thinks __ using numerical integration.
10. F. Viet (a French mathematician) succeeded __ finding 23 of the 45 roots.

Ex. 5. ***Change the time clause into the 'in + Gerund' structure.***

Model: *He made a mistake when he was proving the theorem.*
He made a mistake in proving the theorem.

1. The solution of the cubic was, of course, unknown at that time, but Leonardo showed great ability when he obtained a very close approximate solution.
2. The middle years of the seventeenth century constitute the greatest period of mathematical activity, and when we describe the work of this period it must be remembered that mathematicians no longer worked in comparative isolation.
3. The fundamental theorem used when one finds extreme values of functions is the necessary condition that the differential vanishes at an extreme point.
4. When one considers the mathematical work of J. Kepler, one is chiefly impressed by the quality of his imagination.
5. The detailed investigation of this historical question is lengthy, but it is of sufficient interest and importance when we give the main facts.
6. We have already said that Henry IV was much struck by the ability shown by Viet when he was solving a certain problem.
7. Diophantus (Диофант) was satisfied with a single solution of such equations, and his amazing ingenuity (мастерство) was shown when he dealt with each equation as a particular case.
8. The variation plays the same role when one determines extreme values of functionals as the differential does when one finds maxima and minima of functions.

Ex. 6. ***Answer the following questions using 'by + Gerund' structure.***

Model: *How can you solve this problem? (to use Pythagoras' Theorem)*
We can solve this problem by using Pythagoras' Theorem.

1. How will the author introduce new concepts concerning functionals in this section? (to apply some familiar results from the theory of functionals)
2. How does Napier (a Scottish mathematician) obtain the notation of a logarithm? (to compare two motions)
3. How does Wallis (an English mathematician) reach several remarkable results in this work? (often to deduce general propositions from a number of particular cases)
4. How did he verify this result? (to extract the square root of $(1 - x^2)$, and also to multiply the above series by itself)
5. How can we find six unknowns? (to solve the system of six equations)
6. How can we obtain the unknown quantities? (to solve the triangle)
7. How did mathematicians find a way out of the dilemma? (to change their conception of what a number is to what we nowadays call the real number)
8. How did he obtain a root of several cubics? (to use conic sections)
9. How does the search begin? (to find the curves that satisfy the fundamental theorem)

Ex. 7. ***Translate the sentences with the Complex Gerundial Construction.***

Вспомните! Герундиальный оборот при переводе на русский язык всегда вводится словами: то, что; тем, что; как; после того, как.

Model: His having failed the exam was a great disappointment.
То, что он не сдал экзамен, было большим разочарованием.

1. Our being invited to take part in the conference is very important for us.
2. We heard of those equations having been solved by the students at the previous seminar.
3. Do you mind my being examined first?
4. Newton's having discovered the law of gravitation contributed much to the world science.
5. We knew of his having translated the text a week ago.
6. On lecturer's appearing in the hall, there was loud applause.
7. Their not having done the task was quite a surprise.
8. They know of his presenting a paper at the seminar.
9. His having proved the theorem was very important for him.

Ex. 8. ***State whether the -ing form is the Gerund, the Participle or the Verbal Noun. Give your reasons.***

1. He was not a professional mathematician, **being** an engineer and an architect, but he came into contact with the greatest mathematicians of his time.
2. We started the **reading** of English books last year.
3. He repeated Torricelli's experiments, and showed that barometric **readings** really did depend on atmospheric pressure by **obtaining**, at the same moment, readings at different heights on the slope of the hill of the volcano.
4. By **considering** a number of special cases Cavalieri finally arrived at a theorem.
5. In the last section of the book various theorems are proved **relating** to the centrifugal force on a body **moving** in a circle.
6. Even today, despite the simple picture of the real numbers as the points on a line, university students of mathematics always have trouble **understanding** the formal (and highly abstract) development of the real numbers.
7. Similarly, most people have difficulty in **coming** to terms with complex numbers – numbers that involve the square root of negative quantities, such as $i = \sqrt{} - 1$.
8. Intuitively **speaking**, the norm of the difference of two functions should be zero if the functions are identical, small if the functions are "close", and large if the functions are "far apart".
9. John Pell was a minor seventeenth-century English mathematician who was mistakenly credited by Euler with **having** investigated this equation.

Ex. 9. ***Translate the following word combinations and make up sentences using them:***

1. On hearing about the conference, on arriving at the university, on entering the classroom, on learning about the lecture, on hearing the question.
2. The habit of getting up early, the way of doing it, the necessity of organizing it, the hope of meeting them, the chance of winning the prize, the way of travelling, the pleasure of reading this book.

Ex. 10. ***Translate into English using the following models:***

1. *I can't help* doing the task. – *Я не могу не* сделать задание. (Не принять участия в этой конференции; не перевести этот текст с русского на английский, не доказать эти теоремы; не прочитать книгу.)
2. *It's no use* solving the problem. – *Бесполезно* решать эту задачу. (Пытаться найти эту книгу; объяснять это правило ему ещё раз; говорить с ними на эту тему; искать решение этого уравнения.)
3. *We enjoyed* reading this book. – *Нам понравилось* читать эту книгу. (Слушать лекцию этого профессора; участвовать в конференции; делать этот доклад; посещать семинарские занятия.)
4. *Do you mind* our attending your lecture? – *Вы не возражаете*, если мы посетим вашу лекцию? (Если он представит свою работу первым; если я немного опоздаю на занятие; если она попробует перевести это предложение ещё раз; если они сдадут экзамен раньше.)
5. *I remember* proving the theorem last week. – *Помню*, что доказывал эту теорему на прошлой неделе. (Читал этот текст на английском, учил эти слова, обсуждал эту книгу с ним, переводил эти статьи.)

Pre-Reading Activity

Guess the meaning of the following words:

differential adj.	[dɪfə́renʃəl]	original adj.	[ɔ́rɪdʒənl]
general adj.	[́dʒenərəl]	primitive adj.	[́prɪmɪtɪv]
constant adj.	[́kɔnstənt]	integration n.	[ɪntɪ́greɪʃən]
algebraical adj.	[ældʒɪ́breɪkəl]	theoretical adj.	[θɪə́retɪkəl]
transcendental adj.	[trænseńdentl]	theorem n.	[́θɪərəm]
coefficient n.	[kouɪ́fɪʃənt]	unique adj.	[juː́niːk]
classify v.	[́klæsɪfaɪ]	initial adj.	[ɪ́nɪʃəl]
linear adj.	[́lɪnɪə]	integrable adj.	[ɪńtɪgrəbl]
rationalize v.	[́ræʃnəlaɪz]	integral adj.	[́ɪntɪgrəl]
total adj.	[toutl]		

Read and learn the basic vocabulary terms:

to denote	[dɪ´nout]	означать, обозначать, указывать на связь
relationship	[rɪ´leɪʃənʃɪp]	отношение, взаимоотношение
explicitly	[ɪks´plɪsɪtlɪ]	явно
to involve	[ɪn´vɔlv]	включать в себя, подразумевать
to abandon	[ə´bændən]	оставлять, отказываться от
single	[sɪŋgl]	один, единственный, единый
to enter	[´entə]	входить, вносить
to occur	[ə´kə:]	случаться, происходить, иметь место
order	[´ɔ:də]	порядок
degree	[dɪ´gri:]	степень
auxiliary	[ɔ:g´zɪljərɪ]	вспомогательный, дополнительный
to regard	[rɪga:d]	считать, рассматривать
to derive	[dɪ´raɪv]	выводить, получать, производить
to admit	[əd´mɪt]	допускать, признавать
to follow	[´fɔlou]	следовать, вытекать
to assume	[ə´sju:m]	предполагать, допускать, принимать
continuity	[kɔnti´njuɪtɪ]	непрерывность, неразрывность
derivative	[dɪ´rɪvətɪv]	производная
likewise	[´laɪkwaɪz]	аналогично, так же, подобно
to proceed	[prə´si:d]	продолжать, возобновлять
to obtain	[əb´teɪn]	получать, достигать
to ensure	[ɪn´ʃuə]	обеспечивать, гарантировать
namely	[´neɪmlɪ]	а именно, то есть
comprehensive	[¸kɔmprɪ´hensɪv]	всесторонний, всеобъемлющий
mere	[mɪə]	простой
to eliminate	[ɪ´lɪmɪneɪt]	исключать, ликвидировать
to vary	[´vɛərɪ]	изменять
remote	[rɪ´mout]	отдаленный

Notes:

a priori [´əpraɪ´ɔ:raɪ]	(*лат.* априори), не опираясь на изучение фактов, до опыта, предварительно
apart from [ə´pa:t]	не говоря уже о, кроме, не считая
in the first place	сперва, прежде всего, в первую очередь
a step-by-step process	поэтапный процесс
it is evident	очевидно

$\frac{d^2y}{dx} + y = x^3$ — the second derivative of y with respect to x, plus y, equals x cubed;

$(x+y)^2 \frac{dy}{dx} = 1$ – the quantity x plus y, squared, multiplied by the first derivative of y with respect to x, equals 1;

$\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{2}{3}$ – square brackets, 1 plus parenthesis, the first derivative of y with respect to x, close parenthesis, squared, close square brackets, to the power two thirds;

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ – x multiplied by the first partial derivative of z with respect to x, plus y multiplied by the first partial derivative of z with respect to y, equals zero;

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y} + \frac{\partial^2 u}{\partial z^2}\partial^2 = 0$ – the second partial derivative of u with respect to x, plus the second partial derivative of u with respect to y, plus the second partial derivative of u with respect to z, equals zero;

$\frac{\partial^2 z}{\partial x^2} \times \frac{\partial^2 z}{\partial y} - \frac{\partial^2 z}{\partial x \partial y} = 0$ – the second partial derivative of z with respect to x, multiplied by the second partial derivative of z with respect to y, minus the second partial derivative of z with respect to x, y, equals zero.

TEXT A

ORDINARY DIFFERENTIAL EQUATIONS

Definitions. The term *"differential equation"* was first used by Leibniz in 1676 to denote a relationship between the differentials *dx* and *dy* of two variables *x* and *y*. Such a relationship, in general, explicitly involves the variables *x* and *y* together with other symbols *a, b, c* ... which represent constants.

This restricted use of the term was soon abandoned; differential equations are now understood to include any algebraical or transcendental equalities which involve either differentials or differential coefficients. It is to be understood, however, that the differential equation is not an identity.

Differential equations are classified, in the first place according to the number of variables which they involve. An ordinary differential equation expresses a relation between an independent variable, a dependent variable and one or more differential coefficients of the dependent with respect to the independent variable. A partial differential equation involves one dependent and two or more independent variables, together with partial differential coefficients of the dependent with respect to the independent variables. A total differential equation contains two or more dependent variables together with

their differentials or differential coefficients with respect to a single independent variable which may, or may not, enter explicitly into the equation.

The order of the equation is that of the highest derivative contained in it, so that the general differential equation of order n can be written in the form

$F\left\{y^{(n)}, y^{(n-r)}, \ldots, y^{(r)}, y, x\right\} = 0$, the symbol y^r denoting $d^r y / dx^r$.

The degree of the equation is defined mathematically to be that of its highest order derivative, when the equation has been made rational as far as the derivatives are concerned. When, in an ordinary or partial differential equation, the dependent variable and its derivatives occur to the first degree only, the equation is said to be linear. The coefficients of a linear equation are therefore either constants or functions of the independent variable or variables.
Thus, for example:

$\frac{d^2y}{dx} + y = x^3$ — is an ordinary equation of the second order;

$(x+y)^2 \frac{dy}{dx} = 1$ — is an ordinary non-linear equation of the first order and the first degree;

$\left[1 + \left(\frac{dy}{dx}\right)^2\right]\frac{2}{3}$ — is an ordinary equation of the second order which when rationalised by squaring both members is of the second degree;

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ — is a linear partial differential equation of the first order in two independent variables;

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ — is a partial differential equation of the second order in three independent variables;

$\frac{\partial^2 z}{\partial x^2} \times \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} = 0$ — is a non-linear partial differential equation of the second order and the second degree in two independent variables;

$Udx + Vdy + Wdz = 0$, where U, V, W are functions of x, y and z, is a total differential equation of the first order and the first degree and

$x^2dx^2 + 2xydxdy + y^2dy^2 - z^2dz^2 = 0$ — is a total differential equation of the first order and the second degree.

In the case of a total differential equation any of the variables may be regarded as independent and the remainder as dependent, thus, taking x as an independent variable, the equation $Udx + Vdy + Wdz = 0$ may be written

$U + V\frac{dy}{dx} + W\frac{dz}{dx} = 0$ or an auxiliary variable t may be introduced and the original variables regarded as functions of t, thus $U\frac{dx}{dt} + V\frac{dy}{dt} + W\frac{dz}{dt} = 0$.

The solutions of an ordinary differential equation.

When an ordinary differential equation is known to have been derived by the process of elimination from a primitive containing *n* arbitrary constants, it is evident that it admits a solution dependent upon *n* arbitrary constants. But since it is not evident that any ordinary differential equation of order *n* can be derived from such a primitive, it does not follow that if the differential equation is given a priori it possesses a general solution which depends upon *n* arbitrary constants. In the formation of a differential equation from a given primitive it is necessary to assume certain conditions of differentiability and continuity of derivatives. Likewise in the inverse problem of integration, or proceeding from a given differential equation to its primitive corresponding conditions must be assumed to be satisfied. From the purely theoretical point of view the first problem which arises is that of obtaining a set of conditions, as simple as possible, which when satisfied ensure the existence of a solution. This problem will be considered later, where an existence theorem, which for the moment is assumed, will be proved, namely, that when a set of conditions of a comprehensive nature is satisfied, an equation of order n does admit of a unique solution dependent upon *n* arbitrary initial conditions. From this theorem, it follows that the most general solution of an ordinary equation of order *n* involves *n*, and only *n*, arbitrary constants.

It must not, however, be concluded that no solution exists which is not a mere particular case of the general solution. To make this point clear, let us consider, for instance, the differential equation of the following form $c^2 + 2cy + a^2 - x^2 = 0$, where *c* is an arbitrary, and *a* is a definite constant.

Let the primitive be solved for *c* and let this value of *c* be substituted into the derived equation. Then the derived equation becomes the differential equation $cdy - xdy = 0$, which, on eliminating *c,* becomes the differential equation $(-y + (x^2 + y^2) - a^2)\frac{1}{2}\, dy - xdx = 0.$

The total differential equation obtained by varying *x, y* and *c* simultaneously is $(c + y)\, dc + cdy - xdx = 0$ or, on eliminating c

$$(x^2 + y^2 - a^2)^{1/2}\, dc + (-y + x^2 + y^2 - a^2)^{1/2}\, dy - xdx = 0.$$

Thus, apart from the general solution there exists the singular solution $x^2 + y^2 = a^2$, which obviously satisfies the differential equation. A differential equation of the first order may be regarded as being but one stage removed from its primitive. An equation of higher order is more remote from its primitive and therefore its integration is in general a step-by-step process in which the order is successively reduced, each reduction of the order by unity being accompanied

by the introduction of an arbitrary constant. When the given equation is of order n, and by a process of integration an equation of order n^{-1} involving an arbitrary constant is obtained, the latter is known as the first integral of the given equation.

Thus, when the given equation is $y'' = f(y)$, where $f(y)$ is independent of x, the equation becomes integrable, when both members are multiplied by $2y'$, thus $2y'$(*prime*) y'' (*double prime*) $= 2f(y)\ y'$, and its first integral is $y'2 = (c + 2)\ f(y)dy$, where c is the arbitrary constant of integration.

Post-Reading Activity

Ex. 11. ***Answer the following questions.***

1. When was the term "differential equation" first used? 2. What did this term denote at that time? 3. What does this term denote now? 4. In what ways are differential equations classified? 5. What kinds of differential equations do you know? 6. What does an ordinary differential equation express? 7. What does a partial differential equation involve? 8. What does a total differential equation contain? 9. What is the order of a differential equation? 10. What is the degree of a differential equation? 11. When is an ordinary or a partial differential equation linear? 12. What does the most general solution of an ordinary equation of order n involve?

Ex. 12. ***Match the English words and word combinations with the Russian equivalents.***

1) an ordinary differential equation	a) дифференцируемость и непрерывность
2) a partial differential equation	b) произвольные константы
3) a total differential equation	c) обыкновенное дифференциальное уравнение
4) an auxiliary variable	d) точка зрения
5) a primitive	e) одновременно
6) differentiability and continuity	f) дифференциальное уравнение в частных производных
7) point of view	g) первый интеграл уравнения
8) an existence theorem	h) теорема существования
9) arbitrary constants	i) уравнение в полных дифференциалах
10) simultaneously	j) кроме, не говоря о, не считая
11) the first integral of the equation	k) поэтапный процесс
12) a step-by-step process	l) вспомогательная переменная
13) differential coefficients	m) дифференциальные коэффициенты; первообразная
14) apart from	

Ex. 13. ***Translate the following sentences into Russian.***

1. Differential equations are now understood to include any algebraical or transcendental equalities which involve either differentials or differential coefficients. 2. When an equation is polynomial in all the differential coefficients involved, the power to which the highest differential coefficient is raised is known as the degree of the equation. 3. The coefficients of the linear equation are either constants or functions of the independent variable or variables. 4. In the formation of a differential equation from a given primitive it is necessary to assume certain conditions of differentiability and continuity of derivatives. 5. From this theorem, it follows that the most general solution of an ordinary equation of order n involves n, and only n, arbitrary constants. 6. Let the primitive be solved for c and this value of c be substituted into the derived equation. 7. When the given equation is of order n, and by a process of integration an equation of order involving an arbitrary constant is obtained, the latter is known as the first integral of the given equation. 8. To make this point clear, let us consider, for instance, the differential equation of the following form $c^2 + 2cy + a^2 - x^2 = 0$.

Ex. 14. ***Find out whether the statements are true or false. Use introductory phrases.***

Exactly. Quite so.	***Quite the contrary.***
I fully agree to it.	***Not quite. It's unlikely.***
I don't think this is the case.	***Just the reverse.***

1. The term "differential equation" was first used by Leibniz in the 16th century. 2. Differential equations are now understood to include only algebraical equalities which involve differential coefficients. 3. It is important to remember that the differential equation isn't an identity. 4. A partial differential equation involves two dependent and three independent variables. 5. The order of the equation is that of the lowest derivative contained in it. 6. The most general solution of an ordinary equation of order n involves n, and only n, arbitrary constants. 7. A differential equation of the first order may not be regarded as being one stage removed from its primitive. 8. An equation of higher order is less remote from its primitive.

Ex. 15. ***State the function of the Gerund and translate into Russian.***

1. Instead of representing the position of a point in a plane in terms of its horizontal and vertical distances along two standard lines of reference, it is sometimes more convenient to define the position of the point by length and direction. 2. The new parametric net is therefore the net whose equation in the old parameters u, v is written by setting the right member of this equation equal to zero. 3. Certain notions from analytic projective geometry are quite useful in interpreting some of the formulas of metric differential geometry. 4. In writing

"the sine of the angle PON" in an equation or formula, it would be abbreviated sin PON. 5. By solving a triangle we mean that we have some of the sides and angles given and proceed to calculate the rest. 6. We found the solutions of this system of equations by eliminating unknowns, that is, by multiplying equations by scalars and then adding to produce equations in which some of the xy were not present. 7. Then we indicate that g is an element of G by writing $g \in G$. 8. The method of solving by successive eliminations may perhaps be known to the reader. 9. For the purpose of formulating a precise definition of the angle between two tangents at a point of a surface, a positive sense of rotation in the tangent plane of the surface at the point is assigned by this convention. 10. Our purpose in considering two separate problems is one of convenience. 11. The method of illustrating the variation of functions by the use of graphs is well-known to the reader. 12. The problem of subtracting a number from a smaller number is considered impossible in arithmetic.

Ex. 16. ***Translate into English.***

1. Обыкновенное дифференциальное уравнение выражает связь между зависимой и независимой переменными, а также между одной или несколькими производными зависимой переменной и независимой переменной. 2. Дифференциальные уравнения классифицируются согласно числу переменных, которые они включают. 3. Из теоремы существования следует, что общее решение обыкновенного дифференциального уравнения порядка n содержит n и только n произвольных постоянных. 4. Дифференциальное уравнение в частных производных содержит одну или несколько независимых переменных с частными производными зависимых производных по независимым переменным. 5. Коэффициенты линейного уравнения являются либо постоянными, либо функциями независимых переменных. 6. При образовании дифференциального уравнения из данной первообразной необходимо допускать некоторые условия дифференцируемости и непрерывности производных. 7. Пусть первообразная решается для c, и пусть это значение c подставляется в выведенное уравнение. 8. Когда множество условий выполняется, уравнение порядка n имеет единственное решение, зависящее от исходных условий.

Ex. 17. ***Topics for discussion.***

1. The nature of the term "differential equation".
2. The classification of differential equations according to the number of variables.
3. The classification of differential equations according to the order and the degree of the equation.

Ex. 18. ***Read the text and find the answers to the following questions.***

1. Where do differential equations arise?
2. What is given as an example of modeling a real world problem using differential equations?
3. What does finding the velocity as a function of time involve?
4. Where is the study of differential equations a wide field?
5. What does pure mathematics focus on?
6. What does applied mathematics emphasize?
7. Where do differential equations play an important role?

TEXT B

THE APPLICATION OF DIFFERENTIAL EQUATIONS

Differential equations arise in many areas of science and technology, specifically whenever a deterministic relation involving some continuously varying quantities (modeled by functions) and their rates of change in space and/or time (expressed as derivatives) is known or postulated. This is illustrated in classical mechanics, where the motion of a body is described by its position and velocity as the time value varies. Newton's laws allow one (given the position, velocity, acceleration and various forces acting on the body) to express these variables dynamically as a differential equation for the unknown position of the body as a function of time. In some cases, this differential equation (called an equation of motion) may be solved explicitly.

An example of modelling a real world problem using differential equations is the determination of the velocity of a ball falling through the air, considering only gravity and air resistance. The ball's acceleration towards the ground is the acceleration due to gravity minus the acceleration due to air resistance. Gravity is considered constant, and air resistance may be modeled as proportional to the ball's velocity. This means that the ball's acceleration, which is a derivative of its velocity, depends on the velocity (and the velocity depends on time). Finding the velocity as a function of time involves solving a differential equation.

The study of differential equations is a wide field in pure and applied mathematics, physics, and engineering. All of these disciplines are concerned with the properties of differential equations of various types. Pure mathematics focuses on the existence and uniqueness of solutions, while applied mathematics emphasizes the rigorous justification of the methods for approximating solutions. Differential equations play an important role in modelling virtually every physical, technical, or biological process, from celestial motion, to bridge design, to interactions between neurons. Differential equations such as those used to solve real-life problems may not necessarily be directly solvable, i.e. do not have closed form solutions. Instead, solutions can be approximated using numerical methods.

The theory of differential equations is well developed and the methods used to study them vary significantly with the type of the equation.

UNIT IV

THE INFINITIVE. ITS FORMS AND FUNCTIONS

THE FORMS OF THE INFINITIVE

	Active	Passive	
Indefinite	to translate	to be translated	Выражает действие, одновременное с действием глагола-сказуемого
Continuous	to be translating	–	Выражает одновременное длительное действие
Perfect	to have translated	to have been ranslated	Выражает действие, предшествующее действию глагола сказуемого (переводится прошедшим временем)
Perfect Continuous	to have been translating	–	Выражает предшествующее длительное действие

THE FUNCTIONS OF THE INFINITIVE

Ex. 1. ***Translate the following sentences paying attention to the function of the Infinitive as:***

I. ***Subject.***

Models: a) '*To'* + *Infinitive* before the predicate (It's formal)
To make mistakes is easy. *Совершать* ошибки легко.

b) *It* + *the Infinitive clause* ('*It*' is a preparatory subject).
It's easy *to make* mistakes. Легко *совершать* ошибки.

1. To solve the equation is to find the numerical values of the unknowns. 2. It's not difficult to define which of the numbers is less. 3. To evaluate an expression means to substitute the numerical equivalents for the letters. 4. To find the logarithm of

the given number means to find its exponent. 5. To prove a theorem in a deductive system is to show that it is a necessary logical consequence of some previous proposition. 6. It's necessary to determine this distance for further research. 7. It's difficult to refuse his invitation. 8. It's nice to be sitting here with you.

II. ***Object.***

Model: You have the right *to remain* silent.
У вас есть право *хранить* молчание.

1. They expected to be given complete information. 2. This force caused the body to move. 3. The use of algebra in geometry permits one to speak of a space of more than three dimensions. 4. We were asked to solve the equation in three variables. 5. The students wanted to investigate the properties of square matrices. 6. The word "smooth" is used to suggest that the motion of a point has no abrupt changes of direction. 7. The teacher told me to speak loudly in order that everyone could hear me. 8. She wants to visit a lot of exciting places in Europe.

III. ***Adverbial modifier of purpose or result.***

Models: a) He went to the florist *to buy* a bouquet of flowers.
Он пошел в цветочный магазин *купить* букет цветов.
(adverbial modifier of purpose).

b) I woke one morning *to find* myself famous.
Однажды утром я проснулся *и обнаружил*, что я знаменит.
(adverbial modifier of result).

c) after *too* + adjective *(слишком ..., чтобы)*
after adjective /adverb + *enough (достаточно ..., чтобы)*
He is too tired *to answer* my questions.
Он *слишком* уставший, чтобы *отвечать* на мои вопросы.
The text is difficult enough to understand it at once.
Текст *достаточно* сложный, чтобы *понять* его сразу же.

1. To understand some formulas, she used the text-book of mathematics. 2. To raise a power to a power, it is sufficient to multiply their exponents. 3. To simplify an expression in two unknowns, one transforms it in the same way as an equation in one unknown. 4. To keep the number unchanged in value, we must multiply it by the same power of ten. 5. He checked the results of the experiment only to find it didn't agree with the expected one. 6. He looked around to see them coming up. 7. She went to see her old friend only to find out he was away. 8. She is clever enough to win the competition. 9. He is too young to stay out so late.

IV. ***Predicative or Part of Predicate.***

Models: a) His course paper is *to be submitted* in May.
Его курсовая работа *должна быть представлена* на рассмотрение в мае.

b) Her dream is to become a good programmer.
Ее мечта – стать хорошим программистом.
(The Infinitive follows the subject expressed by the words: *task, method, plan, problem, aim, purpose and function*).

c) *part of a modal predicate*
He must have studied several proofs of this theorem.
Он, вероятно, изучил несколько доказательств этой теоремы.

1. Fortunately, people can visualize or picture two- or three-dimensional ideas by means of drawing on paper. 2. They should have used some graphs. 3. They might have permitted them to reason from visual pictures. 4. A more general treatment of the whole subject is to be found in the monograph. 5. Our aim is to master this subject. 6. We are to use a similar device in triple integrals. 7. They had to change the date of the conference. 8. Their intention was to win the first prize in the competition.

V. ***Attribute (in post-position)***

Models: a) after abstract nouns and concrete nouns.
There is every reason t*o believe* it.
Есть все основания *поверить* этому.
I want some paper to write on.
Мне нужна бумага *для письма.*

b) after pronouns.
I have nothing t*o tell* you.
Мне нечего тебе *сказать*.

c) after adjectives.
I'm sorry *to leave* you.
Мне жаль *уходить* от вас.
I was glad to meet him.
Я был рад *познакомиться* с ним.

d) after *the last, the first, the only*
She was the last *to leave* the room.
Она *последней вышла* из комнаты.

1. The terms to be insisted on are as follows. 2. The first issue to be considered is one about the coming examinations. 3. All operations to be performed in succession will give the expected result. 4. This student was the first to prove this law. 5. There will be six independent elements to be determined. 6. The proof to be tested concerns the conditions of function series convergence.

Ex. 2. ***Read the sentences and state the form and the function of the Infinitive. Translate into Russian.***

1. Einstein was to develop the theory of relativity which replaced Newton's theories of gravity. 2. It's important to save your work before you switch off

the computer. 3. Students are under enormous pressure to learn huge amounts of vocabulary. 4. It's easy to be wise after the event, as the saying goes. 5. We assembled to discuss and arrange our plans. 6. He was the first to claim to have solved the problem. 7. I'm glad to have finished the translation of this article. 8. The plane to be constructed should contain two intersecting lines. 9. We need to introduce a function in order to solve the equation. 10. They must be determining now whether a given equation is or is not solvable algebraically. 11. Jacobi was the first to apply elliptic functions to the theory of numbers. 12. To create one of the great masterpieces of all mathematics Lobachevsky had to have been working for twenty years or even more.

Ex. 3. ***Open the parentheses and give the correct form of the Infinitive.***

1. The methods (to describe) above are widely used. 2. In order (to find) the square of a number you are to multiply it by itself. 3. Had Gauss found Clerk Maxwell's equations of the electromagnetic field, he might (to be satisfied). 4. Differential geometry may roughly (to consider) as the study of properties of curves, surfaces, and their generalizations by means of the calculus. 5. He was happy (to work) weeks or even months over his research and (to obtain) expected results. 6. Likewise on the surface of a sphere the coordinates in this case can (to think) as latitude and longitude. 7. The scientist had to show his mathematical genius (to make) discoveries in astronomy surpassing those of all his predecessors. 8. To show the way in which common logarithms are used to facilitate calculation attempts should (to make). 9. He pretended (to look) for necessary information via the internet.

Ex. 4. ***Change the verb in parentheses to a to-infinitive or -ing-form as appropriate.***

1. Using algebra I like (reduce) complex problems to simple formulas, but I dislike (spend) too much time on such work. 2. We were meant (give) the definition of a curve but that would have meant (consider) two fundamental problems of analytic geometry. 3. As you're so keen on (get) to know more about education in Oxford University, you'll probably be keen (visit) it. 4. I regret (inform) you that you have not been enrolled for this course. 5. Fortunately I remembered (hand) in the abstract by the end of the week, but to be honest I don't remember (do) so. 6. She stopped (make) a speech about outstanding mathematicians and he decided (ask) her some questions. 7. To stop (think) about things for a while is something we all need to do. 8. Have you ever considered (study) mathematical analysis; it is considered (be) one of the most difficult subjects of mathematics.

Ex. 5. ***Choose the correct form of the Infinitive or the Gerund.***

1. I'll never forget ... four more or less happy years in Cambridge.
   a) to have spent; b) having spent
   c) spending d) to spend

2. Only to enumerate all the fields of mathematics he enriched would take more space in the book than can ... to one man.
   a) have been devoted b) to be devoted
   c) to devote d) be devoted
3. This time next week I'll ... to the lecture on the applications of the calculus to curved surfaces (twisted curves).
   a) be listening b) being listened
   c) listen d) have listened
4. Some of the mathematicians in the 19th century, notably Leibniz and De Morgan, went on ... of adding logic itself to the domain of algebra.
   a) to be dreaming b) to dream
   c) being dreamt d) dreaming
5. I'm sorry ... the problem of determining the equation of a locus.
   a)have not studied b) not studying
   c) not to have studied d) being not studied
6. Teachers do not allow ... and ... in the exams
   a) to speak / cheat b) to be spoken to / be cheated
   c) speaking / cheating d) speaking / to cheat
7. The accuracy of the root needs ... Why not ... the number to the power?
   a) being checked /raising b) be checked /be raised
   c) to be checked /raise d) checking /to raise
8. The theory of analytic functions of a complex variable ... one of the greatest fields of mathematical triumphs in the nineteenth century.
   a) should have been b) were to have been
   c) must have been d) might have been

Ex. 6. ***Fill each of the gaps with a suitable word from the list. Entitle the story. Mind there is an extra word.***

to pocket, to be done, to be shown, to meet, say, did not discourage, taking, to offer, liking, to understand, doing, take, to create, have been, go, to get, do, explaining, getting

Even the new dignity of his rectorship (1) ... Lobachevsky from (2) ... the work in the library and museum. 'In order (3) ... a thing done to your (4) ..., you must (5) ... it yourself', he used to say. Sometimes he would (6) ... off his collar and coat and (7) ... to work. Once a distinguished foreign visitor, (8) ... the coatless Rector for a workman, asked (9) ... through the libraries and museum collections. Lobachevsky showed him the chosen treasures, (10) ... as he exhibited. The visitor was greatly impressed and charmed by the superior intelligence of that Russian workman and decided (11) ... a handsome tip. Lobachevsky couldn't help (12) ... angry and indignantly threw the coin

to the floor. Though the visitor was a bit bewildered he had (13) ... his money and (14) ... good-bye. On his way home he was thinking about the workman and was not able (15) ... him. "It might (16) ... one more strangeness of his behavior", he thought.

That evening the visitor had a chance (17) ... Lobachevsky again at the Governor's table, where apologies were offered and accepted on both sides.

It seems incredible that Lobachevsky so burdened with teaching and administration had managed (18) ... one of the great masterpieces of our mathematics.

Ex 7. ***Translate the sentences into English:***

1. Этот метод недостаточно точный, чтобы дать хороший результат.
2. Он даже не попытался найти общее решение для этой системы уравнений.
3. Цель науки – найти допущения, логическая последовательность которых соответствует опыту.
4. Необходимо точно определить, что подразумевается под словом *множество*.
5. Он единственный, кто ответил на этот сложный вопрос.
6. Полученные данные могут рассматриваться только как приближенные.
7. Чтобы понять эту процедуру, рассмотрим аналогичную ситуацию.
8. Он предпочитает читать книги в оригинале.
9. Трудно ли сдавать экзамен по математическому анализу?
10. Должно быть, их пригласили на конференцию.

Pre-Reading Activity

Guess the meaning of the following words.

fundamental (a)	[ˌfʌndəˈmentl]	object (n)	[ˈɔbdʒɪkt]
interpretation (n)	[ɪntəːprɪˈteɪʃən]	unique (a)	[juː ˈnɪːk]
remark (n)	[rɪˈmaːk]	analytic (a)	[ˌænəˈlɪtɪk]
problem (n)	[ˈprɔbləm]	figure (n)	[ˈfɪgə]
converse (a)	[ˈkɔnvəːs]	pair (n)	[pɛə]
coordinate (n)	[koəˈɔːdnɪt]	nature (n)	[ˈneɪtʃə]

Read and learn the following words.

preliminary (a)	[prɪˈlɪmɪnərɪ]	предварительный
determine (v)	[dɪˈtəːmɪn]	определять
essentially (adv)	[ɪˈsenʃəlɪ]	*зд.* по существу
essential (a)	[ɪˈsenʃəl]	существенный
strictly (adv)	[ˈstrɪktlɪ]	строго, точно

closely (adv)	[ˊkləuslı]	*зд.* тесно, близко
to constitute (v)	[ˊkɔnstıtju:t]	составлять
characteristic (n)	[ˌkærıktəˊrıstık]	свойство, особенность
further (a)	[ˊfə:ðə]	другой, дальнейший
initially (adv)	[ıˊnıʃəlı]	вначале, с самого начала
rather (adv)	[ˊra:ðə]	скорее
convenience (n)	[kənˊvi:njəns]	удобство
convention (v)	[kənˊvenʃən]	условие
to emphasize (v)	[ˊemfəsaız]	подчеркивать
strongly (adv)	[ˊstrɔŋlı]	сильно, строго
to locate (v)	[ləuˊkeıt]	определить местоположение
implication (n)	[ˌımplıˊkeıʃen]	смысл, значение
to imply (v)	[ımˊplaı]	значить, означать, подразумевать
carefully (adv)	[ˊkɛəfəlı]	осторожно
to note (v)	[nout]	отмечать, обращать внимание
familiar (a)	[fəˊmılјə]	хорошо известный, знакомый
accordingly (adv)	[əˊkɔ:dıŋlı]	таким образом, поэтому
path (n)	[pa:θ]	траектория
to trace (v)	[treıs]	чертить, начертать
necessarily (adv)	[ˊnesısərılı]	обязательно, непременно
to summarize (v)	[ˊsʌməraız]	суммировать, подводить итог
to enable (v)	[ı'neıbl]	давать возможность, право

Notes:

to focus attention	сосредоточить внимание
totality of points	совокупность точек

Reading Activity

TEXT A

EQUATION AND LOCUS

Two Fundamental Problems of analytic geometry. In this chapter we shall make a preliminary study of the following two fundamental problems of analytic geometry:

I. Given an equation, to determine its geometric interpretation or representation.

II. Given a geometric figure or condition, to determine its equation or analytic representation.

The students will note that these problems are essentially converses of each other. Strictly speaking, however, both problems are so closely related that together they constitute the fundamental problem of all analytic geometry. For example, we shall see later that, after obtaining the equation for a given geometric condition, it is often possible by a study of this equation to determine further geometric characteristics and properties for the given condition. Our purpose in initially considering two separate problems is not one of necessity, but rather one of convenience; we are thus enabled to focus our attention on fewer ideas at a time.

First Fundamental Problem. The Locus of an Equation.

Assume that we are given an equation in the two variables x and y, which we may write briefly in the form

$$f(x, y) = 0. \tag{1}$$

In general there are infinitely many pairs of values of x and y which satisfy this equation. Each such pair of real values will be taken as the coordinates (x, y) of a point in the plane.

This convention is the basis of **Definition 1.** ***The totality of points, and only those points, whose coordinates satisfy an equation* (1)*, is called the locus or graph of the equation.***

Another convenient expression is given by **Definition 2.** ***Any point whose coordinates satisfy an equation* (1)** ***is said to lie on the locus of the equation.***

It cannot be emphasized too strongly that only those points whose coordinates satisfy an equation lie on its locus. That is, if the coordinates of a point satisfy an equation, that point lies on the locus of the equation; and conversely, if a point lies on the locus of an equation, its coordinates satisfy the equation. Since the coordinates of the point of a locus are restricted by its equation, such points will in general be located in positions which, taken together, form a definite path called a curve as well as a graph or locus.

Second fundamental problem. We will now consider the second fundamental problem of analytic geometry.

A geometric figure, such as a curve, is generally given by its definition. By the definition of an object is meant a description of that object of such a nature that it is possible to identify it definitely among all other objects of its class. The implication of this statement should be carefully noted: it expresses a necessary and sufficient condition for the existence of the object defined. Thus, let us consider that we are defining a plane curve of type C by means of a unique property P which C possesses. Then, in the entire class of all plane curves, a curve is of type C if and only if it possesses property P.

As a specific example, let us consider that familiar plane curve, the circle. We define a circle as a plane curve possessing the unique property P that all its points are equally distant from a fixed point in its plane. This means that

every circle has property *P*; and conversely, every plane curve having property *P* is a circle.

For a curve, a geometric condition is a law which the curve must obey. This means that every point on the curve must satisfy the particular law for the curve. Accordingly a curve is often defined as the locus or path traced by a point moving in accordance with a specified law. Thus, a circle may be defined as the locus moving in a plane so that it is always at a constant distance from a fixed point in that plane. A locus need not necessarily satisfy a single condition; it may satisfy two or more conditions. Thus, we may have a curve which is the locus of a point moving so that it passes through a given point, and it is always at a constant distance from a given line. We may then summarize the preceding remarks in the following definition:

A curve is the locus of all those points, and only those points, which satisfy one or more given geometric conditions.

The student should note that this definition implies that the given condition or conditions are both necessary and sufficient for the existence of the curve.

Post-Reading Activity

Ex. 8. ***Answer the following questions.***

1. What are the two fundamental problems of analytic geometry? 2. How many pairs of values of x and y satisfy the equation $F(x, y) = 0$? 3. What is the locus or graph of the equation? 4. What point lies on the locus of the equation $F(x, y) = 0$? 5. What are the coordinates of the point of a locus restricted by? 6. By what is a geometric figure generally given? 7. How may a circle be defined? 8. How many conditions may a locus satisfy? 9. What is the difference between the first and the second problems?

Ex. 9. ***Match the English words and word combinations with their Russian equivalents.***

1) a familiar plane curve	a) совокупность точек
2) to lie on the locus	b) весь класс
3) strictly speaking	c) как необходимые, так и достаточные условия
4) two fundamental problems	d) известная плоская кривая
5) to write briefly	e) обязательно удовлетворять
6) the totality of points	f) точно говоря
7) the implication of a statement	g) единственное условие

8) both necessary and sufficient conditions	h) предварительное изучение
9) a preliminary study	i) тесно связанный
10) necessarily satisfy	j) находиться на графике
11) the entire class	k) записывать кратко
12) a single condition	l) две основные задачи
13) to pass through a given point	m) проходить через данную точку
14) closely related	n) смысл утверждения

Ex. 10. ***Give the corresponding plural forms of the following nouns.***

a) us [əs] → i [aɪ] focus → foci
Calculus, genius, locus, modulus, nucleus, radius.

b) is [ɪs] → es [ɪːz] axis → axes
Thesis, emphasis, analysis, basis, hypothesis, crisis, phasis, parenthesis.

c) ix [ɪks] → es [ɪːz] matrix → matrices
ex [əks] → es [ɪːz] vertex → vertices
Directrix, bisectrix, index.

d) on [ɔn] → a[ə] polyhedron → polyhedra
um [əm] → a [ə] datum → data
Continuum, medium, spectrum, minimum, maximum, phenomenon, criterion.

e) a [ə] → ae [iː] formula → formulae (formulas)
Abscissa, hyperbola, lacuna, corona.

Ex. 11. ***Use the plural and singular forms of the nouns given in Ex.10. The first letters of the words are given.***

1. The area of an ellipse equals ϖ times the product of the long and the short *r* ... . 2. If a curve is symmetric with respect to both *a* ..., is it symmetric with respect to the origin? 3. Analytic methods give us a means of finding the equations of *l* ... . 4. The notion of a four-dimensional geometry is a very helpful one in studying physical *p* ... . 5. Find the equation of the ellipse with *f* ... at the points (0, 4). 6. In each of the following *h* ..., locate the vertices and *f* ... . 7. All these facts may serve as reference *d* ... . 8. *C* ... is a branch of mathematics divided into two parts differential calculus and mathematical calculus. 9. Circular area is measured by its *r* ... . 10. Einstein was a mathematical *g* ... .

Ex. 12. ***Ask disjunctive questions (tag-questions).***

1. To draw the graph of a function isn't difficult, ...? 2. Don't try to obtain the equation of a locus quickly, ...? 3. Every student must understand the implication of the equation of a locus, ...? 4. Let's do a preliminary study of the function, ...? 5. This theorem constitutes a common property of fields, ...? 6. For convenience we had to focus attention on the characteristic of the locus, ...? 7. They have studied the entire class of plane curves, ...? 8. You should know the coordinates of a point to determine its position in a plane, ...? 9. There is an analytic interpretation of the equation of a locus, ...? 10. I'm to study the course of analytic geometry, ...?

Ex. 13. ***Find out whether the statements are true or false. Use introductory phrases.***

Exactly. Quite so.	***Quite the contrary.***
I fully agree to it.	***Not quite. It's unlikely.***
I don't think this is the case.	***Just the reverse.***

1. There exists a close relationship between two fundamental problems of analytic geometry.
2. After obtaining the equation for a given geometric condition it is impossible to determine further geometric properties for the given condition.
3. There is only one pair of values x and y satisfying the equation $F(x, y) = 0$.
4. The totality of points satisfying Equation (1) is called the locus or graph of the equation.
5. Any point whose coordinates satisfy Equation (1) is said not to lie on the locus of the equation.
6. A geometric figure, such as a curve is generally given by its formula.
7. A circle possesses a unique property that all its points are equally distant from the points in its plane.
8. For the curve, a geometric condition is a law which the curve must obey.
9. A curve is the locus of the points which satisfy one and only one condition.
10. There is some difference between the first and the second problems of analytic geometry.

Ex. 14. ***Say these sentences in English.***

1. Нам предстоит рассмотреть методы анализа линейных уравнений.
2. Во многих текстах чертеж уравнения называется кривой, даже если это прямая линия.
3. Такая кривая называется геометрическим местом точек уравнения.
4. Геометрическим местом точек уравнения (его графиком) является кривая, содержащая точки и только те точки, координаты которых удовлетворяют этому уравнению.

5. Иногда кривая может быть определена множеством условий, а не уравнением, хотя уравнение может быть получено из данных условий.
6. В этом случае рассматриваемая кривая являлась бы графиком всех точек на плоскости, которые соответствовали бы этим уравнениям.
7. Например, можно сказать, что кривая – это геометрическое место всех точек на плоскости, расположенных на фиксированном расстоянии от одной фиксированной точки, называемой центром окружности.
8. Прямая линия может быть определена как геометрическое место всех точек на плоскости, равноудалённых от двух фиксированных точек.
9. Метод выражения множества условий в аналитической форме даёт уравнения.
10. Основные понятия геометрического места точки в геометрии тесно связаны с понятием уравнения в алгебре.

Ex.15. ***Read and translate the following sentences. Group them according to the models.***

Models: a) He ***should*** help us (obligation). Он должен нам помочь.

b) You ***should*** have helped us. Вам следовало бы нам помочь.
(negative probabilities; unwanted things)

c) We answered that we ***should*** help him. Мы ответили, что поможем ему.
(reported speech; expresses future action)

1. I should have indicated the directions in which the distances were to be measured. 2. We answered that we should explain the methods by which the results had been obtained. 3. The professor said we should determine the equation of that geometric figure. 4. I replied that I should have written the thesis by the end of the year. 5. A circle is a plane curve which should satisfy a unique property. 6. This locus should be investigated by the students. 7. You should have solved more difficult puzzles, because your intelligence is above average. 8. Students should know more about transcendental functions. 9. Any point on the curve should possess the unique property.

Ex. 16. ***Topics for discussion.***

1. Dwell on the fundamental problems of all analytic geometry.
2. Speak on the locus of an equation.
3. Describe the second fundamental problem of analytic geometry.

Ex. 17. ***Read the text and answer the following questions.***

1. What is the parabola? (The ellipse, the hyperbola).
2. What does the equation of the parabola depend on?
3. How do we call the points in which the ellipse cuts the principal axis?

4. Can we consider the notion of correspondence between a geometric locus and an equation as a general concept?
5. Is there any relationship between the parabola, the ellipse and the hyperbola?

TEXT B

PARTICULAR SPECIES OF LOCI

We shall proceed to the discussion of particular species of loci, namely, a parabola, an ellipse and a hyperbola.

The ***parabola*** is the locus of points which are equidistant from a fixed point and a fixed straight line.

The fixed point is the locus, the fixed line is the directrix. The line perpendicular to the directrix and passing through the focus is the axis of the parabola. The axis of the parabola is, obviously, a line of symmetry. The point on the axis halfway between the focus and the directrix on the parabola is the vertex of the parabola. The parabola is fixed when the focus and the directrix are fixed. The equation of the parabola, however, depends on the choice of the coordinate system. If the vertex of the parabola is at the origin and the focus is at the point (*O, P*), its equation is $X^2 = 2PY$ *or* $Y^2 = 2PX$.

The ellipse is the locus of a point which moves so that the sum of its distances from two fixed points called the foci is constant. This constant will be denoted by $2a$, which is necessarily greater than the distance between the foci (the focal distance). The line through the foci is the principal axis of the ellipse; the points in which the ellipse cuts the principal axis are called the vertices of the ellipse. If the centre of the ellipse is at the origin but the foci are on the y-axis its equation is

$$\frac{Y^2}{a^2} + \frac{X^2}{b^2} = 1.$$

where a and b represent the lengths of its semimajor and semiminor axes (большая и малая полуоси).

The ***hyperbola*** is the locus of a point which moves so that the difference of its distances from two fixed points is a constant $2a$. Its equation is

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1.$$

This equation shows that the hyperbola is symmetric with respect to both coordinate axes and also the origin. It intersects the X-axis but does not cut the Y-axis. Hence, the curve is not contained in a bounded portion of a plane. The curve consists of two branches. The line segment joining the vertices is called the transverse axis of the hyperbola; its length is $2a$. The point midway between the vertices is a geometrical centre and is called the centre of the hyperbola.

UNIT V

THE INFINITIVE CONSTRUCTIONS. THE OBJECTIVE WITH THE INFINITIVE. CONSTRUCTION (COMPLEX OBJECT). THE NOMINATIVE WITH THE INFINITIVE. CONSTRUCTION (COMPLEX SUBJECT). FOR + INFINITIVE

Ex. 1. ***The Objective with the Infinitive Construction (Complex Object)*:**

a) **With the verbs:** ***to want, to wish, to like, to love, to prefer and to desire.***

Example. I want ***you to participate*** in the experiment.
Я хочу, чтобы вы участвовали в этом эксперименте.

1. The teacher wanted his *students to learn* as quickly as possible.
2. I prefer *you to call* me by my first name.
3. He would like *me to change* my mind.
4. They wished *the article to be published* next month.
5. We want *one of her best students to be granted* a scholarship.

b) **With the verbs:** ***to order, to ask, to tell, to advise, to allow, to forbid and to cause.***

Example. Did he allow ***you to take*** the dictionary?
Он разрешил тебе взять словарь?

1. Professor permitted *us to be helped* with the solution of the problem.
2. We don't allow *students to cheat or talk* to each other during exams.
3. I advise *you to look for* a new job.
4. They ordered *us to finish* our projects until Tuesday.
5. She asked *everybody not to use* the office photocopier.

c) **With the verbs:** ***to think, to believe, to mean, to admit, to expect, to suppose, to consider, to know and to prove.***

Example. She knows York to be a historic city in England with many ancient buildings.
Она знает, что Йорк исторический город в Англии, где много древних зданий.

1. In the future they suppose *students to use* computers instead of text books.
2. We expect you *to be able to account* for this mistake.
3. I suppose *the sense* of beauty *to be* very personal and subjective.
4. We consider the algebraic *language* of analytic geometry *to have* both advantages and disadvantages.
5. They know *maths to provide* basic tools for computer science.

Complex Object + with Infinitive without 'to'

d) **With the verbs:** ***to see, to feel, to watch, to notice and to hear.***

Example. Do you feel the Earth move?
Вы чувствуете, как Земля движется?

1. We cannot see a *point move* along the line.
2. We both heard *him say* that I was leaving.
3. The students watched their *teacher draw* two perpendicular lines.
4. Nobody heard *him complain* about being overworked.
5. He noticed *them walk* out of the building together laughing and joking.

e) **With the verbs:** ***to let, to make (заставлять), to have.***

Example. Let him stay up late tonight.
Разрешите ему позже лечь спать сегодня вечером.

1. She made *him accept* full responsibility for the accident.
2. Shall I have my *secretary send* you the details?
3. Let *him solve* this cubic equation in terms of substitution.
4. You can't make *her come* on time. She is not punctual.
5. The teacher let his *students use* a calculator in the test.

Ex. 2. ***Translate these sentences into English using Complex Object.***

1. Мне бы хотелось, чтобы эта задача была решена другим способом.
2. Декарт (R. Descartes) хотел, чтобы методы, используемые математиками, были обобщены и расширены.
3. Гений и трудолюбие Эйлера (L. Euler) позволили ему стать самым плодовитым (prolific) математиком за всю историю ее существования.
4. Ученые полагают, что эта теория может стать основой для получения многих новых идей в науке.
5. В соответствии с постулатом геометрии мы знаем, что точка M и направляющий вектор q определяют прямую линию L.
6. Сегодня мы заставляем компьютер решать сложные задачи, запоминать непростые цепочки операций, рисовать графики и выполнять другую полезную и нужную работу.

Ex. 3. ***The Nominative with the Infinitive Construction (Complex Subject)***
Subject + */predicate/* + ***to – Infinitive***.

a) **With the predicate in the passive voice:** ***to see, to believe, to suppose, to expect, to think, to know, to announce, to report and to say.***

Example. ***He*** is believed ***to be*** a great authority on the subject.
Считают, что он крупный специалист по этой теме.

1. Unless stated otherwise, curves are always assumed to be simple, that is they do not intersect.
2. The binary system is expected to be appropriate to the nature of an electronic machine.
3. The Earth is known to rotate and as a result it has the shape of a ball.
4. When the limits of integration to be performed are stated, the integral is said to be definite.
5. Lack of oxygen is believed to make people lose health.

b) **With the predicate in the active voice:** ***to seem, to appear, to happen, to chance, to prove, to turn out.***

Example. His method proved to be the only possible one. *Оказалось*, что его метод является единственно возможным методом.

1. I chanced to be passing when she fell and hurt herself.
2. Computers do not seem to be able to use their memories in the way human beings use theirs.
3. The fundamental concepts of many branches prove to have been suggested by physical experiments.
4. This function, whose graph is a line, appears to be classified as a linear function.
5. The graph turned out to be drawn without picking up the pencil.

c) **The predicate** ***to be + adjective (likely, unlikely, sure, certain).***

Example. The computer is not likely to make such mistakes.
Маловероятно, что компьютер делает такие ошибки

1. When the quantity is a constant multiple of another quantity, they are certain to be directly proportional to one another.
2. He is unlikely to know what a discontinuous function is.
3. Such an effect is likely to be the result of an experimental error.
4. The traditional role of science seems to make people healthy and the world a brighter place to live in.
5. Any quantity associated with a curve (e.g. the coordinates of a point on a curve, the length of a tangent to the curve) are sure to represent a function.

Ex. 4. ***Translate these sentences into English using Complex Subject.***

1. Маловероятно, что этот план утвердят на собрании.
2. Несомненно, что они выбрали другую систему координат.

3. Оказывается, что если значение одной тригонометрической функции *A* будет дано, то все остальные функции будут однозначно определены.
4. Предполагали, что полученные данные будут точными.
5. Говорят, что такое уравнение $(x + 1)^2 + y^2 = -4$ не имеет решения и описывает мнимую окружность.
6. Известно, что такие поверхности, как сфера или эллипс, характеризуются квадратными уравнениями.
7. Несомненно, что значение математики непрерывно возрастает.

Ex. 5. ***Replace the Object Clause with Complex Object. See the models.***

Example 1. I would like to draw both axes (she).
She would like me to draw both axes.

1. The astronomers wish to locate this distant object in the sky. (they)
2. We expect to find the equation of the ellipse with foci at the points (0,4). (he)
3. We want to use analytic methods to find the equation of loci. (she)
4. I expect to prove the advantages of this system of notation in this particular case. (you)
5. He wants to interpret these facts correctly. (they)

Example 2. Scientists suppose that non-Euclidean geometries liberated the science.
Scientists suppose ***non-Euclidean geometries to have liberated*** the science.

1. We know that P. Fermat and R. Descartes came to develop analytic geometry almost simultaneously.
2. We believe that Cybernetics is the twentieth century Queen of sciences.
3. I know that more and more prospective employees take lie-detector tests.
4. Programmers assume that community networks, like public libraries, serve citizens and society.
5. He believed that concepts of importance in science had come to men with great difficulty.

Ex. 6. ***Change the sentences with Complex Object to the sentences with Complex Subject.***

Example. We know **the concepts** of gravity, of energy and of limitless space **to have taken** years to develop.
The concepts of gravity, of energy and of limitless space are known **to have taken** years to develop.

1. They believe men of genius to be required to express the concepts of importance in science.

2. We expect the best mathematical proofs to be short and direct.
3. I believe CD-ROM to be going to replace printed books and newspapers.
4. She supposes him to understand such simple sentences.
5. My science adviser expects me to take part in the international conference.

For + Infinitive Construction
For + object + to – Infinitive
(*Can be used in different functions*).

a) ***The Subject*** (подлежащее).
It's for you to decide what university to choose.
Тебе решать, какой университет выбрать.

b) ***The Predicative*** (именная часть сказуемого).
The best thing to do is for us to come to an agreement.
Лучшее, что мы можем сделать, это прийти к соглашению.

c) ***The Object*** (дополнение).
The conference called for the government to take measures against growing unemployment.
Конференция потребовала, чтобы правительство приняло меры против растущей безработицы.

d) ***The Adverbial modifier of purpose*** (обстоятельство цели).
For the expected results to be obtained, we have to use the computations.
Чтобы получить ожидаемые результаты, мы должны использовать эти вычисления.

e) ***The Attribute*** (определение).
The speaker proposed a new method for the experiment to be carried out successfully.
Выступающий предложил новый метод для успешного выполнения эксперимента.

Ex. 7. ***Translate these sentences into Russian.***

1. I must find somewhere for him to practice his English.
2. It's important for the meeting to start at eight a.m.
3. It seems unnecessary for him to start his work this week.
4. It was a real shame for them not to have won after all their attempts.
5. For us to fail the experiment now would be a disaster.
6. Two conditions must be met for the phenomenon to occur.

Ex. 8. ***Translate these sentences into English using the for + Infinitive Construction.***

1. Математикам важно помнить, что строгий метод легче понять.
2. Возможно, что фигура будет иметь больше, чем одну ось симметрии.

3. Чтобы этот закон выполнялся, необходимо соблюдать два условия.
4. Потребуется больше экспериментальной работы, чтобы мы смогли объяснить это явление.
5. Необходимо, чтобы финансовые сделки (transactions) в интернете были безопасны. Однако существует определенная проблема.
6. Жизненно важно (vital), чтобы существовала хорошая система общественного транспорта в больших городах.

Ex. 9. ***Make up sentences about the mathematicians using the same models with any of the given verbs:*** *alleged, believed, claimed, considered, known, reported, said, supposed, thought and understood.*

Models: 1. ***Marie Curie*** was thought ***to be*** the greatest scientist of her generation.
2. ***She*** is known t***o have succeeded*** where all others had failed.
3. ***It used to be said*** that she owed her success to her husband Pierre.

L. Euler – a great geometer of 18th century;
– worked at the St. Petersburg Academy;
– dealt with analytic functions;
– was the first to solve the equation in three variables;
– his greatest discovery was the calculus of quaternion (кватернион).

P. Fermat – the greatest "amateur in the history of mathematics";
– enjoyed classical literature, wrote verses;
– inventor and discoverer of coordinate geometry;
– the proofs of his famous theorems are not obtained;
– his books were not lost after his death.

W. P. Hamilton – one of the greatest scientists;
– at the age of 21 submitted to the Royal Irish Academy a paper "A Theory of Systems of Rays";
– when an undergraduate was elected to the chair of Professor of Astronomy at Trinity College;
– devised math tools for the exploration of physical space;
– thanks to his mathematical tools the subject of vector analysis appeared.

Ex. 10. ***Rewrite each sentence, beginning as shown. Do not change the meaning and use the word given.***

1. Some people can't remember historical dates.
   For some people ... . (impossible)
2. It goes without saying, we'll be very happy to accept your invitation.
   We are sure ... . (delighted)
3. You must remember to check your work through for mistakes.
   It is ... . (essential)

4. I think they were very much pleased with my work.
   I expect them ... . (extremely)
5. There's no doubt, her business was successful, as she was a very capable person.
   Her business is certain ... . (gifted)
6. To make notes before you start writing is very important.
   For you ... . (essential)
7. My teacher has checked my answers for slips of the pen.
   I've had ... . (carefully)
8. He normally stays up late on Saturdays.
   To stay up late ... . (normal)
9. I had to write my name in block capitals.
   They made ... . (clearly)
10. It's hard for her to live up to her mother's expectations.
    She finds ... . (difficult)

Ex. 11. ***Complete the answers to the questions. For some of them use the words or phrases suggested.***

1. What's the Latin version of the name R. Descartes?
   – We know ... . (Renatus Cartesius)
2. Who is considered to be the founder of analytic geometry?
   – Descartes and Fermat ... .
3. What does the name analytic mean?
   – Analytic geometry is certain to be ... .
   (general method of geometry; the basis of all modern applied maths)
4. What is the heart of Descartes' and Fermat's idea concerning studying geometric figures and curves?
   – To each curve there belongs an equation, and it is expected ... .
   (points of the curve, no other points, to describe)
5. Was Descartes the original creator of rectangular coordinates?
   – No, he wasn't. But it often ... . (to refer to as, Cartesian)
   – It's due to his ... . (to introduce, his coordinate geometry, innovations, to improve algebraic notation)
6. Who was the first to introduce the concepts of a variable and a function?
   – Everybody knows ... .
7. What allowed negative numbers to become legitimate in maths as directed ordinates?
   – The rule of signs and his coordinate geometry appear ... .
8. Who devised the method of plotting graphs with x and y coordinates?
   – French mathematicians Descartes and Fermat are supposed ... .
9. To Descartes maths served to solve philosophical and scientific problems and master nature, didn't it?
   – Yes, he expected maths ... .

10. How can you characterize his life and his philosophy?
– In contrast to Fermat's life his life is said ... .
(to devote, thinking about the nature of truth and the physical structure of the universe)

Pre-Reading Activity

Guess the meaning of the following words.

function (n)	[ˊfʌŋkʃn]	formula (n)	[ˊfɔ:mjulə]
graph (n)	[ˊgræf]	distance (n)	[ˊdıstəns]
real (a)	[ˊrıəl]	temperature (n)	[ˊtemprıʧə]
condition (n)	[kənˊdıʃn]	reflect (v)	[rıˊflekt]
qualify (v)	[ˊkwɔlıfaı]	assistance (n)	[əˊsıstəns]
physical (a)	[ˊfızıkəl]	special (a)	[ˊspeʃəl]
coordinate (n)	[kouˊɔ:dnıt]		

Read and learn the following words.

approach (v,n)	[əˊprouʧ]	приближаться, подходить; подход
assign (v)	[əˊsaın]	ставить в соответствие
assume (v)	[əˊsju:m]	предполагать, допускать
concept (n)	[ˊkɔnsept]	логическое понятие
convex (a)	[ˊkɔnˊveks]	выпуклый
correspond (v)	[ˌkɔrısˊpɔnd]	соответствовать
describe (v)	[dısˊkraıb]	описывать, изображать, начертить
domain (n)	[dəˊmeın]	область (определения)
dimension (n)	[dıˊmenʃn]	размерность
emerge (v)	[ıˊmə:dʒ]	появляться, выходить, выяснять
feed (v)	[ˊfi:d]	подавать, питать
graph (n)	[ˊgræf]	диаграмма, график
instance (n)	[ˊınstəns]	случай, пример
mapping (n)	[ˊmæpıŋ]	отображение
modify (v)	[ˊmɔdıfaı]	(видо)изменять, модифицировать
notion (n)	[ˊnouʃn]	понятие, определение
original (a)	[ɔˊrıdʒınəl]	первоначальный
obey (v)	[əˊbeı]	удовлетворять условиям
qualify (v)	[ˊkwɔlıfaı]	определять, квалифицировать

regardless (a)	[rɪˊgaːdlɪs]	независимо от, несмотря на
require (v)	[rɪˊkwaɪə]	нуждаться, требовать
restrict (v)	[rɪˊstrɪkt]	ограничивать, заключать
satisfy (v)	[ˊsætɪsfaɪ]	выполнять, удовлетворять
statement (n)	[ˊsteɪtmənt]	утверждение, формулировка
suitably (adj)	[ˊsjuːtəblɪ]	соответственно, подходяще
whereas (cj)	[wɛərˊæz]	тогда как, поскольку
unique (a)	[juːˊnɪk]	единственный, однозначный
yield (v)	[ˊjiːld]	производить, вырабатывать

Notes:

to bear in mind	помнить
according to	согласно
to refer to...as	называть
side by side	рядом
by analogy	по аналогии
an ordered pair	упорядоченная пара
at least	по крайней мере

TEXT A

FUNCTIONS AND GRAPHS

The notion of function is essentially the same as that of correspondence. A numerical-valued function f assigns to each point p in its **domain of definition** a single real number $f(p)$ called the **value** of f at p. The rule of correspondence may be described by a formula such as

$$f(p) = x^2 - 3xy, \text{ when } p = (x, y)$$

or by several formulas, such as

$$\begin{array}{ll} f(p) = x & \text{when } x > y \\ x^2 + y & y \le x \end{array}$$

or by geometric description.

$F(p)$ is the distance from p to the point (4,7)

or even by an assumed physical relationship:

$f(p)$ is the temperature at the point p.

In all of these instances, it is important to bear in mind that the rule of correspondence is the function f, whereas $f(p)$ is the numerical value which

f assigns to p. A function may be thought of as a machine into which specific points may be fed, while the corresponding values emerge at the other end.

Real-valued functions are often classified according to the dimension of their domain of definition. If $f(p)$ is defined for all $p \in S$ and S is a subset of the plane, then we may write p as (x, y) and $f(p)$ as $f(x, y)$ and refer to f as a function of **two real variables**. Similarly, when S is a set in 3-space, we may write $f(x, y, z)$ for $f(p)$ and say that f is a function of **three real variables**. When S is a set on the line, we usually write $f(x)$ and call f a function of one real variable. In all these cases, however, f can still be thought of as a function defined for the single variable point p.

Other cases also arise. A function f may be defined only for points p which lie on a certain curve C in space.

Side by side with the notion of a function as a correspondence or mapping between two sets (e.g. points and numbers), we have the concept of **graph**. If f is a function of one real variable, the graph of f is the set f points (x, y) in the plane for which $y = f(x)$. If f is a function of two real variables, the graph of f is the set of points (x, y, z) in 3-space for which $z = F(x, y)$. Conversely, it is possible to base the notion of function on that of graph. Let A and B be any two sets, and let E be any set composed of ordered pairs (a, b) with $a \in A$ and $b \in B$. By analogy, (a, b) may be called the "point" in a $A \times B$ space having coordinates a and b, regardless of the nature of the sets A and B. Any such set E can be called a graph or relation, and those that have the special property of being single-valued are called functions.

Many special properties of a function are reflected in simple geometrical properties of its graph. A function f defined on the line is said to be monotonic increasing if $f(x) \leq f(x')$ whenever $x < x'$; this means that the graph of f "rises" as we move along it from left to right. Again, a function of two variables is said to be convex if it obeys the condition

$$f(p_1) + f(p_2) \leq 2f;$$

this says that Σ, the graph of f, is a surface with the property that if A and B are any two points on Σ, their mid – point lies on or below Σ.

Sometimes it is said that equation in x and y defines y as a function of x. This must both be explained and qualified. What is meant is that, given an equation $E(x, y) = 0$, one is generally able (at least in theory) to "solve for y", getting $y = f(x)$. Again, solution of the equation for y seldom gives a unique answer, while in writing $y = f(x)$, we require that exactly one value of y corresponds to a given value of x. We must therefore modify the original statement and say that if the function E is suitably restricted, the equation $E(x, y) = 0$ defines a set of functions (possibly just one) such that if f is one of these, then $\boldsymbol{E(x, F(x)) = 0}$ for all x in the domain of definition of f. The equation $x^2 + y^2 - 16 = 0$ yields two functions,

$$f(x) = \sqrt{16 - x^2} \text{ and } g(x) = -\sqrt{16 - x^2}.$$

Post-Reading Activity

Ex. 12. ***Answer the questions on the text:***

1. What may the notion of function be compared with? 2. What can the rule of correspondence be described by? 3. In what way are real-valued functions related to the concept of graph? 4. How are the special properties of a function reflected in geometrical properties of its graph? 5. Can it be explained that any equation in x and y defines y as a function of x?

Ex. 13. ***Match the English words and word combinations with their Russian equivalents.***

1) a mapping	a) называться
2) to assign to	b) область определения
3) a domain of definition	c) правило соответствия
4) the rule of correspondence	d) быть монотонно возрастающей
5) to bear in mind	e) помнить
6) to refer to ... as	f) допускаемая величина
7) a value assumed	g) упорядоченная пара
8) a function defined	h) определяемая функция
9) to lie on a curve	i) лежать на кривой
10) the concept of graph	j) подчиняться условию
11) an ordered pair	k) без ограничений
12) to be monotonic increasing	l) однозначный ответ
13) to obey a condition	m) первоначальное утверждение
14) without restrictions	n) отображение
15) a unique answer	o) соответственно ограниченный
16) the original statement	p) понятие графика
17) suitably restricted	q) ставить соответствие

Ex. 14. ***Replace the Russian words by their English equivalents according to the text.***

1. Describe (область определения) of each function f defined below.
2. (Правило соответствия) may be described by a formula.
3. A function f (может быть определена) only for points p, which lie on a certain curve C in space.
4. We must modify (первоначальное утверждение).
5. Real-valued functions are often classified (в соответствии с размером) of their domain of definition.
6. It is important (помнить) that the rule of correspondence is the function f.

7. Many (особые свойства) of a function are reflected in simple geometrical (свойства) of its graph.
8. A function of two variables, (говорят, выпуклая) if it obeys the certain conditions.

Ex. 15. ***Complete the sentences using the required forms of the verb "to participate" (Infinitive or Gerund):***

1) to participate
2) participate
3) participating

} *in the scientific research*

a) I had no desire ... .
b) In fact I was fully against ... .
c) But my colleagues wanted me ... .
d) Of course, my plans for future wouldn't let me ... .
e) But then my boss insisted on me ... .
f) I wasn't too keen on ... .
g) But he made me ... .
h) And finally I came round to the idea of ... .
i) And I must say I don't regret ... .

Ex. 16. ***Translate the following sentences with the emphatic construction*** **"It is (was)... that (who, which, when...)".**

Model: ***It is*** the method of elimination ***that*** is used for solving such equations.
Именно этот метод исключения используется для решения таких уравнений.

1. It was the Arabs who preserved the Greek and Hindu scientific writings through the Dark Ages of Europe.
2. It was Italy that produced the greatest algebraists during 1200–1620 period.
3. It was the Greeks' mathematics rigour that forced them to use line segments to express numbers as in their geometrical algebra.
4. It was not until the next century that L. Euler supplied a proof for the case $n = 3$.
5. It was principally through the Arabs that algebra entered Europe.
6. It was Euler who introduced the new standard notation $f(x)$.

Ex. 17. ***Translate the sentences with the Infinitive Constructions into Russian.***

1. We consider one of the axes of the ellipse to be the major axis, while the other – the minor axis.
2. The scientists think the orbital path of a planet around the sun to be an ellipse such that the sun is located at a focus.
3. Professor watched his student draw the number lines making the axes of a coordinate system.

4. He made us plot the ordered pairs above and connect the points with a smooth curve.
5. Let us remove the parentheses on the right side of the equation and combine similar terms.
6. I want you to make conclusion about the validity of the formula.
7. If $n = -1$, then we expect the equation $y = \frac{1}{2}$ to describe the hyperbola.
8. One should understand all functions to be relations, but not all relations are functions.
9. Functions are considered to be parts of our everyday thinking, converting from degree Celsius to degree Fahrenheit, DNA testing, determining stock values, and the sale price of a shirt.
10. The comprehension of limits seems to create the necessity for understanding the other concepts in calculus.
11. In order to demonstrate the unity of various approaches to this concept, we are supposed to combine intuitive arguments with rigorous proofs of propositions.
12. The above definition proved to be easily translated into the rigorous mathematical language.
13. The graph of the even function is believed to be symmetric with respect to the *y-axis* and that of the odd function is known to be symmetric with respect to the origin.
14. The method of math induction turns out to be very helpful in proving many statements about integers.
15. A geometric progression is certain to be a sequence in which each term is determined by multiplying the preceding term by a constant.
16. When we try to prove the truth of some general statement, it's quite natural for us to check its validity in a particular case.
17. It's for you to decide how to determine this function: by means of tables or graphically.
18. You must factor a polynomial expression, i.e. represent it as a product of the irreducible polynomial for the equation to be solved.
19. It was essential for us to understand that the sign of the discriminate is an important characteristic of the quadratic equation.
20. I asked for the function to be determined on the interval.
21. His idea is for us to understand that a coordinate system locates points in a plane by means of numbers.

Ex. 18. ***Topics for discussion.***

1. Dwell on the notion of a function as a correspondence.
2. Give the classification of real-valued functions.
3. Speak about the relation between the concept of a function and that of a graph.

4. Show some special properties of a function using the properties of a graph.
5. Prove or disprove the statement that equation in x and y defines y as a function of x.

Ex. 19. ***Read the text and answer the following questions.***

1. What did a word 'function' mean to mathematicians a century ago?
2. When and why did sufficient grounds appear to extend the notion of a function?
3. What is the key idea of the function definition used today?

TEXT B

FUNCTIONS

We now turn to the discussion of the fundamental notion of a function or mapping. It will be seen that a function is a special kind of a set, although there are other visualizations which are often suggestive.

To the mathematician of a century ago the word "function" ordinarily means a definite formula, such as $f(x) = x^2 + 3x + 5$, which associates to each real number x another real number $f(x)$. The fact that certain formulas, such as $g(x) = \sqrt{x - 5}$, do not give rise to real numbers for all real values of x was, of course, well-know but was not regarded as sufficient grounds to require an extension of the notion of function. Probably one could arouse controversy among those mathematicians as to whether the absolute value $h(x) = |x|$ of a real number is an "honest function" or not. For, after all, the definition of $|x|$ is given "in pieces" by

$$|x| = x, \text{ if } x \geq 0, \ |x| = -x, \text{ if } x < 0.$$

As mathematics developed, it became increasingly clear that the requirement that a function be a formula was unduly restrictive and that a more general definition would be useful. It also became evident that it is important to make a clear distinction between the function itself and the values of the function.

Our first revised definition of a function would be:

A function f from a set A to a set B is a rule of correspondence that assigns to each x in a certain subset D of A, a uniquely determined element $f(x)$ of B.

Certainly, the explicit formulas of the type mentioned above are included in this definition. The proposed definition allows the possibility that the function might not be defined for certain elements of A and also allows the consideration of functions for which the set A and B are not necessarily real numbers.

However suggestive the proposed definitions may be, it has a significant defect: it is not clear. There remains the difficulty of interpreting the phrase "rule of correspondence". The most satisfactory solution seems to define "a function" entirely in terms of sets and the notions introduced above.

The key idea is to think of the graph of the function, that is, a collection of the ordered pairs.

Definition. Let A and B be sets. A function from A to B is a set f of ordered pairs in A x B with the property that if (a, b) and (a', b') are elements of f, then $b = b'$. The set of all elements of A that can occur as first members of elements in f is called the domain of f and will be denoted $D(f)$. The set of all elements of B that can occur as second members of elements f is called the range of f (or the set of values of f) and will be denoted by $R(f)$. In case $D(f) = A$, we often say that f maps A into B (or is a mapping of A into B) and write $f: A \to B$.

If (a, b) is an element of a function f, then it is customary to write $b = f(a)$ or $f: a \to b$ instead of $(a, b) \in f$. We often refer to the element b as the value of f at the point a, or the image under f of the point a.

Ex. 20. ***Say these sentences in English.***

Основные понятия функции

Если каждому значению x из некоторой области D поставлено в соответствие значение переменной y, то говорят, что в области D задана функция y аргумента x:

$$y = f(x).$$

Это типичное обозначение функции. Область D называется **областью определения** функции, а совокупность значений переменной y – **областью ее изменения**. Уравнение $y = f(x)$ можно интерпретировать графически как уравнение кривой в x, y – плоскости. Говорят, что функция f задает отображение множества X на множество Y, если для любого $y \in Y$ существует такое $x \in X$, что $f(x) = y$. Это отображение является взаимно однозначным, если из равенства $f(x) = f(z)$ следует, что

$$x = z.$$

Функции можно также задавать с помощью таблиц. Например:

x	–3	–2	–1	0	1	2	3
$f(x)$	5	2	0	–1	3	4	5

Кроме того, зависимость между переменными x и y можно задавать графически. Каждая пара чисел в таблице задает точку в плоскости $x\,0\,y$. Если нанести эти точки и соединить их плавной кривой, то мы получим график функции $y = f(x)$.

UNIT VI

CONDITIONAL SENTENCES. MIXED CONDITIONALS. INVERSION

Types of Conditional Sentences

	If-clause (hypothesis)	Main clause (result clause)	Use
Type 1	If + any present form (Present Simple, Present Continuous or Present Perfect)	Future /Imperative, can, may, might, must, should + Present Simple	*Real Present* True or likely to happen in the present or the future
	If we **catch** the 10 o'clock train, we **shall (can, may,** etc.**) get** there by lunch time. *Если мы сядем на поезд в 10 часов, мы приедем (можем приехать) туда к обеду.* If you **wake** up before me, **give** me a call. *Если вы проснетесь раньше меня, позвоните мне.* We can use **should** after **if** to express something which is possible, but not likely to happen. If he **should come** earlier, **tell** him to wait for me. *Если он **все же** придет раньше, скажите ему подождать меня.*		
Type 2	If + Past Simple or Past Continuous	would/could, might + Infinitive without **to**	*Unreal Present* Untrue in the present; also is used *to give advice*

	If we **caught** the 10 o'clock train, we **would (could**, **might**, **etc.) get** there by lunch time. *Если бы мы сели на поезд в 10 часов, мы бы приехали (могли бы приехать) туда к обеду.* If I **knew** how the device worked, I **could tell** you what to do. *Если бы я знал, как это устройство работает, я мог бы сказать вам, что делать.* If I **were** you, I **would follow** his advice. *На вашем месте я бы последовал его совету.* After **if** we normally use **were** instead of **was** in all persons in Type 2 conditionals.		
Type 3	If + Past Perfect or Past Perfect Continuous	would/could/ might + have + Past Participle	Unreal Past Imaginary situation contrary to the fact in the past; also is used to express *regret or criticism*
	If we **had caught** the 10 o'clock train, we **would (could**, **might**, etc.) **have got** there by lunch time. *Если бы мы сели на 10-часовой поезд, мы бы приехали (могли бы приехать) туда к обеду.* If he **had been studying** hard, he **could have passed** the exam. *Если бы он учился хорошо, он смог бы сдать экзамен.*		

Ex. 1. ***State the type of the conditional sentences and translate them.***

1. If you work hard, you'll be able to finish your work in time.
2. If it is not raining, we shall play football.
3. If he had time, he would do the work.
4. If she were more attentive, she would not make so many mistakes.
5. If I had known your telephone number, I should have phoned you.
6. If he had gone to the station an hour ago, he wouldn't have missed the train.
7. You will not solve this problem unless you know the Viet's theorem.
8. If it were not so late, we would continue our debates.
9. If you follow the advice of the teacher, you will save a lot of time.
10. If I had understood the importance of learning English earlier, I should have taken part in the international scientific conference for young scientists.

Ex. 2. ***Complete these sentences following one of the patterns for conditionals of Type 1.***

1. We'll just manage to find the correct solution if ... .
2. If I see him again, I ... .
3. I will accept your explanation only if ... .
4. If you tell me the truth, I ... .
5. What will happen if ... .

6. If your work harder, ...
7. If you don't reserve the ticket, ...
8. If it rains, ...
9. If you are hungry, ...

Ex. 3. ***Complete the sentences following the pattern of Type 2 conditionals.***

1. If you explained the situation to your friend, he ... .
2. Perhaps he... if you spoke to him.
3. If you changed your job, you ... .
4. If they came to see us in London, we ... .
5. If you read the book a second time, you ... .
6. If I were you, I ... .
7. If they had more money, ...

Ex. 4. ***Write sentences following the pattern of Type 3 conditional based on the given facts.***

Model: The driver was not careful enough last Sunday, the accident happened.
If the driver had been more careful last Sunday, the accident wouldn't have happened.

1. As you didn't explain your problem to me, I wasn't able to help you.
2. She didn't read the book, she couldn't discuss a new novel.
3. We didn't take a map, we didn't find the hotel quickly.
4. You didn't invite him, he didn't come to the party.
5. I didn't know you were arriving on the train, I didn't meet you.
6. He missed the seminar, he was not told about it.

Ex. 5. ***Translate into English.***

1. Если мы определим кривую, мы найдем уравнение геометрического места точек.
2. Он бы помог вам, если бы он был в городе сейчас.
3. Если бы я знала ее электронный адрес, я бы написала ей немедленно.
4. Если бы наша студенческая группа приняла участие в спортивных соревнованиях вчера, мы бы заняли первое место.
5. Я бы взял такси, если бы знал, что у нас мало времени.
6. Если бы я хорошо знал английский, я бы читал всю новую научную литературу по-английски.
7. Ваша команда обязательно выиграет следующий матч, если вы будете много тренироваться.
8. Если бы я был на вашем месте, я бы не просил его помочь, а постарался бы справиться с трудным заданием сам.
9. Если она не сможет прийти сама, она позвонит по телефону.

10. Если бы он мог получить необходимую информацию вчера, он бы сказал нам о своих планах на выходной день.

Mixed Conditionals

All types of conditionals can be mixed. Any tense combination is possible if the context permits it.

If clause	Main clause
If they were playing all day, (Type 2) *Если они играли весь день,*	they will be tired out now. (Type 1) *они будут усталыми сейчас.*
If I were you, (Type 2) *На вашем месте,*	I would have visited them. (Type 3) *я бы навестил их тогда.*
If we had brought a map with us, (Type 3) *Если бы мы тогда взяли карту с собой,*	we would know which road to take now. (Type 2) *мы бы сейчас знали, по какой дороге ехать.*

Ex. 6. ***Write mixed sentences based on the given facts, use the table given above.***

1. He failed his examination last year, so he is taking it again in June.
2. Since you didn't take my advice, you're in a difficult position now.
3. There was a sharp frost last night, so we are able to go skating now.
4. She isn't at the meeting because she wasn't told about it.
5. I didn't apply for the job as I don't want to work there.
6. He is not a fast runner, so he didn't win the race.
7. She didn't save her money, so she isn't going on holiday.

Inversion

We can omit **if** in conditional sentences. When we do that, **should**, were, **had** (Past Perfect) and **could** come before the subject.

If he **were** here, he **would help** us.	**Were** he here, he **would help** us.
If I **should see** him today, I'll tell him to call you. If he **were** here now, we could work together. If I **had known** about that matter, I would have told him yesterday.	**Should I see** him today, I'll tell him to call you. **Were he** here now, we could work together. **Had I known** about that matter, I would have told him yesterday.

Ex. 7. ***Rewrite the sentences making an inversion in the conditional clauses.***

Model: If he should come, give him my letter.
Should he come, give him my letter.

1. The talks will continue if the need should arise.

2. If you should be late again, you'll lose your job.
3. If he had taken a little more time to think, he might have acted more sensibly.
4. If it were not for the price of the ticket, I would go there by plane.
5. If he had known the news, he would have told us.
6. If I were you, I wouldn't buy such an expensive iPhone.
7. If you should drink too much coffee, you won't be able to sleep.

Ex. 8. ***Translate the following sentences.***

1. Were these words synonyms, you could use either of them.
2. Had I known the facts better, I should have made a new test.
3. Were he not so tired, he would continue his work.
4. Had you taken part in our experiment, you would have helped us a lot.
5. Were she good at mathematical analysis, she would be able to solve some of these problems.
6. Should he come to the laboratory, tell him to leave his notes there.
7. Had the students of our group attended all lectures and seminars, the results of the examination in functional analysis wouldn't have been so bad.

Ex. 9. ***Answer the questions.***

1. What will you do next Sunday if the weather is fine? 2. Where would you go if you were free now? 3. Would your favourite football team have won the last match if the football players trained more? 4. Will you study German if you have enough time? 5. Will you speak English better if you watch a lot of foreign films in English? 6. Who will you ask to help you if you can't translate the article yourself? 7. What places of interest would you visit in London if you had an opportunity to go there? 8. How long can you stay in the south if you go there in summer? 9. What present would you buy to your mother if it were her birthday tomorrow? 10. Will you go to the station by underground or will you take a taxi if you have little time? 11. Will he improve his health if he goes in for sport?

Pre-Reading Activity

Guess the meaning of the following words.

interval	[ˈɪntəv(ə)l]	function	[ˈfʌŋkʃ(ə)n]
system	[ˈsɪstɪm]	form	[fɔ:m]
distance	[ˈdɪst(ə)ns]	contrary	[ˈkɔntrərɪ]
portion	[ˈpɔ:ʃən]	section	[ˈsekʃ(ə)n]
reserve	[rɪˈzə:v]	result	[rɪˈzʌlt]
special	[ˈspeʃ(ə)l]	identity	[aɪˈdentɪtɪ]
figure	[ˈfɪgə]	complex	[ˈkɔmpleks]
family	[ˈfæmɪlɪ]	fix	[ˈfɪks]

real	[ˊrɪəl]	ordinary	[ˊɔ:dnrɪ]
hypothesis	[haɪˊpɔθɪsɪs]	linear	[ˊlɪnɪə]
proportional	[prəˊpɔ:ʃənl]	projection	[prəˊdʒekʃ(ə)n]
term	[tə:m]		

Read and learn the following words.

curve (n)	[kə:v]	кривая
dimension(n)	[dɪˊmenʃ(ə)n]	размерность, измерение
establish (v)	[ɪsˊtæblɪʃ]	устанавливать, основывать, создавать
describe (v)	[dɪsˊkraɪb]	описывать, вычерчивать
locus (loci) (n)	[ˊloukəs]	геометрическое место точек
single (adj)	[ˊsɪŋgl]	единственный, одиночный, отдельный, единый
infinity (n)	[ɪnˊfɪnɪtɪ]	бесконечность
vary (v)	[ˊvɛərɪ]	менять, изменять
variable (n)	[ˊvɛərɪəbl]	переменная
proper (adj)	[ˊpropə]	правильный, собственный
permit (v)	[pəˊmɪt]	позволять, допускать, разрешать
take on (v)	[ˊteɪkˊɔ:n]	принимать, приобретать (форму, качество и т. д.)
condition (n)	[kənˊdɪʃ(ə)n]	условие
imaginary (adj)	[ɪˊmædʒɪnərɪ]	мнимый
reduce (v)	[rɪˊdju:s]	уменьшать, превращать, приводить к, сокращать
occur (v)	[əˊkə:]	случаться, происходить, встречаться
singular (adj)	[ˊsiŋgjulə]	особый
assume (v)	[əˊsju:m]	принимать, допускать
expand (v)	[ɪksˊpænd]	расширять, разлагать
power series	[ˊpauə ˊsɪərɪ:z]	степенные ряды
converge (v)	[kənˊvə:dʒ]	сходиться в одной точке, сводить в одну точку
adduce (v)	[əˊdju:s]	приводить (в качестве доказательства)
revolve (v)	[rɪˊvɔlv]	вращаться
revolution (n)	[ˌrevəˊlu:ʃ(ə)n]	вращение
implicit (adj)	[ɪmˊplɪsɪt]	неявный
explicit (adj)	[ɪksˊplɪsɪt]	явный
point of view	[ˊpɔɪnt əvˌvju:]	точка зрения
simultaneous (adj)	[ˌsɪm(ə)lˊteɪnjəs]	одновременный, совместный
helix (n)	[ˊhi:lɪks]	спираль, винтообразная линия

twisted cubic (n)	[ˈtwɪstɪdˈkjubɪk]	неплоская кривая 3-го порядка
residual (adj)	[rɪˈzɪdjuəl]	остаточный
screw (n)	[skru:]	винт, шуруп

TEXT A

CURVES

Definition and equations of a curve. In ordinary three-dimensional space let us establish a left-handed orthogonal cartesian coordinate system with the same unit of distance for all three axes. In this system any point P has coordinates x, y, z.

A curve may be described qualitatively as the locus of a point moving with one degree of freedom. A curve is also sometimes said to be the locus of a one-parameter family of points or the locus of a single infinity of points.

Definition 1. Let the coordinates x, y, z of a point P be given as single-valued real-valued analytic functions of a real independent variable t on an interval T of t-axis by equations of the form

$$x = x(t),\ y = y(t),\ z = z(t). \tag{1}$$

Further suppose that the functions $x(t)$, $y(t)$, $z(t)$ are not all constant on T. Then the locus of the point P, as t varies on the interval T, is a real proper analytic curve C.

Some comments on the foregoing definition will perhaps clarify its meaning. Equations (1) are called the parametric equations of the curve C, the parameter being the variable t. We reserve the right to permit the parameter t to take on complex values. Moreover, one or more of the coordinates x, y, z may, under suitable conditions, be allowed to be complex. The curve C would in this case be called complex, or perhaps, on suitable conditions, imaginary. To say that a curve is proper means that it does not reduce to a single fixed point, as it would do if the coordinates x, y, z were all constant. It is clear that at an ordinary point of a real proper analytic curve, i. e. a point where nothing exceptional occurs, the inequality

$$x'^2 + y'^2 + z'^2 > 0 \left(x' = \frac{dx}{dt} \right) \tag{2}$$

holds. Any point of such a curve where this inequality fails to hold is called singular, although the singularity may belong to the parametric representation being used for the curve defined as a point-locus, or may belong to the curve itself. A curve, or portion of a curve, which is free of singular points may be called nonsingular. Furthermore, we assume that the interval T is so small that values of the parameter t on the interval T and points (x, y, z) on the curve C are in one-to-one correspondence, so that the parameter t is a coordinate of the corresponding point (x, y, z) on the curve C.

To say that the functions are analytic means, roughly, that they can be expanded into power series. More precisely, this statement means that, at each point t_0 within

the interval T, each of these functions can be expanded into a Taylor's series of power of the difference $t-t_0$ which converges when the absolute value $t-t_0$ is sufficiently small. It would be possible to study differential geometry under the hypothesis that the functions considered possess only a definite, and rather small number of derivatives; but we assume analyticity in the interests of simplicity. So the word "function" will mean for us "analytic function", and the word "curve" will mean a real proper nonsingular analytic curve unless the contrary is indicated.

Some examples of parametric equations of curves will now be adduced. First of all, the equations (1) may be linear, of the form

$$x = a + lt,\ y = b + mt,\ z = c + nt \tag{3}$$

in which a, b, c and l, m, n are constants. Then the curve C is a straight line through the fixed points (a, b, c) and with direction cosines proportional to l, m, n. If t is the algebraic distance from the fixed point (a, b, c) to the variable point (x, y, z) on the line then l, m, n are the direction cosines of the line and satisfy the equation

$$l^2 + m^2 + n^2 = l. \tag{4}$$

As a second example, equations (1) may take the form

$$x = t,\ y = t^2,\ x = t^3. \tag{5}$$

The curve C is then a cubical parabola. This is one form of a twisted cubic which can be defined as the residual intersection of two quadric surfaces that intersect elsewhere in a straight line. Finally, if equations (1) have the form

$$x = a\cos t,\ y = a\sin t,\ z = bt\ (a > 0,\ b < 0). \tag{6}$$

The curve C is a left-handed circular helix, or machine screw. This may be described as the locus of a point which revolves around the *z-axis* at a constant distance a from it and at the same time moves parallel to the *z-axis* at a rate proportional to the angle t of revolution. If we had supposed $b < 0$, then the helix would have been right-handed.

A curve can be represented analytically in other ways than by its parametric equations. For example, it is known that one equation in x, y, z represents a surface, and that two independent simultaneous equations in $x, y, z,$ say

$$F(x, y, z) = 0,\ C(x, y, z) = 0 \tag{7}$$

represent the intersection of two surfaces, which is a curve. Equations (7) are called implicit equations of this curve. Sometimes it is convenient to represent a curve by implicit equations, when really the curve under consideration is only part of the intersection of the two surfaces represented by the individual equations.

If the implicit equations (7) be solved for two of the variables in terms of the third, say for y and z in terms of $x,$ the result can be written in the form

$$y = y(x),\ z = z(x). \tag{8}$$

These equations represent the same curve as equations (7), and they, or the equations, which similarly express any two of the coordinates of a variable point on the curve as functions of the third coordinate, are called explicit equations of the curve. Each of equations (8) separately represents a cylinder projecting the curve onto one of the coordinate planes. So equations (8) are a special form of equations (7) for which the two surfaces are projecting cylinders.

If the first of the parametric equations (1) of a curve C be solved for t as a function of x, and if the result is substituted in the remaining two of these equations, the explicit equations (8) of the curve C are obtained. From one point of view the explicit equations (8) of a curve, when supplemented by identity, $x = x$, are parametric equations

$$x = x,\ y = y(x),\ z = z(x) \tag{9}$$

of the curve, the parameter now being the coordinate x.

Post-Reading Activity

Ex. 10. ***Match the English words and word combinations with their Russian equivalents.***

1) one-to-one correspondence	a) соответствующая точка
2) a real proper analytic function	b) взаимнооднозначное соответствие
3) to describe qualitatively	c) разлагать в степенные ряды
4) the inequality doesn't hold	d) направляющие косинусы
5) the above definition	e) действительная правильная аналитическая кривая
6) a suitable condition	f) параметрические уравнения
7) to adduce the examples	g) действительная независимая переменная
8) a point-locus	h) описывать качественно
9) the corresponding point	i) вышеуказанное определение
10) direction cosines	j) подходящее условие
11) parametric equations	k) неравенство не выполняется
12) to expand into power series	l) представить примеры
13) a real independent variable	m) удовлетворять уравнениям
14) to satisfy the equations	n) пересекаться на прямой линии
15) a left-handed helix	o) график точки
16) intersection of two surfaces	p) левосторонняя спираль
17) to revolve around the axis	q) независимая система уравнений

18) to substitute the result into the equations — r) пересечение двух поверхностей
19) to intersect in a straight line — s) подставить результат в уравнения
20) independent simultaneous equations — t) вращаться вокруг оси

Ex. 11. ***Find out whether the statements are true or false. Use introductory phrases.***

Exactly. Quite so. — ***Quite the contrary.***
I fully agree to it. — ***Not quite. It's unlikely.***
I don't think this is the case. — ***Just the reverse.***

1. A curve can be described qualitatively and quantitatively as the locus of a point moving with one degree of freedom.
2. In the parametric equations of the curve *C* the parameter is the variable *t*.
3. If the coordinate *x, y, z* were all constant, the proper curve would reduce to a single fixed point.
4. Analytic functions can be expanded into power series.
5. A cubical parabola is the residual intersection of two plane surfaces that intersect elsewhere in a straight line.
6. A curve is represented by the intersection of two surfaces, if we have two independent simultaneous equations in *x, y, z*.
7. If any two of the coordinates of a variable point on the curve are expressed as functions of the third coordinate, the equations are called the explicit equations of the curve.

Ex. 12. ***Learn the following word combinations.***

a) Use "**under consideration**" or "**in question**" instead of "**considered**" in order to express the same idea "рассматриваемый":

The theorem considered, the figure considered, the problem considered, the function considered, the equation considered, the point considered, the curve considered.

b) Use "**to hold**" instead of "**to be valid**", "**to be true**" meaning "иметь силу, выполняться":

1. This inequality is valid for all cases. 2. This theorem is valid in the case of the uniform convergence. 3. This formula is valid for a single-valued analytic function too. 4. For $a = b = 1$ the given property is true. 5. These relations are true under suitable conditions.

c) Use "**to fail to do something**" instead of "**do not**" meaning "не суметь, не быть в состоянии, оказаться неспособным сделать что-то":

1. I did not solve the problem given by the professor. 2. These properties do not hold for real numbers. 3. We did not expand these functions into power series.

4. He did not prove the theorem correctly. 5. We do not represent this curve by an implicit equation. 6. I did not understand your question. 7. The boy did not add these two numbers correctly.

Ex. 13. ***Replace the Russian words by their English equivalents according to the text.***

1. A curve may be defined as *геометрическое место точек* of a one-parameter family of points.
2. If one or more of the coordinates *x, y, z* are allowed to be complex, the curve C will be called complex or *мнимая*.
3. Any point of such a curve where this inequality *не выполняется* is called singular.
4. The functions are analytic if they can be expanded into *степенной ряд*.
5. We *представим* some examples of parametric equations of curves.
6. If we have supposed $b < 0$, then *спираль* would have been right-handed.
7. When the curve under consideration is only part of the intersection of two surfaces represented by the individual equations, we represent a curve by *неявные* equations.
8. In differential geometry we assume the functions considered possess only a definite number of *производные*.

Ex. 14. ***Analyze the use of the conjunctions introducing adverbial clauses of condition****: in case, provided, suppose, unless, on condition, as long as.* ***Translate the sentences into Russian.***

1. Suppose you told him the truth, what could he do about it.
2. You can borrow my notes on condition (provided) you give them back to me tomorrow.
3. You can come with us as long as you don't make too much noise.
4. He wouldn't have come unless you had invited him.
5. Unless you read the text on graphs, you won't be able to discuss it.
6. I'm taking an umbrella in case it rains later on.

Ex. 15. ***State the type of these conditional sentences and translate them.***

1. A curve is called nonsingular if it is free of singular points. 2. The curve would reduce to a single fixed point if the coordinates *x, y, z* were all constants. 3. The curve is called complex if one or more of the coordinates *x, y, z* are complex. 4. If *t* is the algebraic distance from the fixed point (*x, y, z*) on the line, then *l, m, n* in the given equations are direction cosines. 5. If the endpoints are included, the interval is called closed. 6. The result would have been written in the form $y = y(x)$, $z = z(x)$ if the implicit equations had been solved for the two of the variables in terms of the third. 7. If these implicit equations were solved for two of the variables in terms of the third, they could be written in another form. 8. If the equations $x = x(t)$, $y = y(t)$, $z = z(t)$ had taken the form $x = t$, $y = t^2$, $z = t^2$, then the curve *C* would have been a cubic parabola.

Ex. 16. ***Answer the following questions.***

1. In what way may a curve be described? 2. Can you give the definition of a real proper analytic curve? 3. What do we call the equations of the form $x = x(t)$, $y = y(t)$, $z = z(t)$? 4. What does the letter t denote in these equations? 6. May the parameter t take on complex values? 7. What do the letters x, y, z denote in these equations? 8. In what case is the curve C called complex or imaginary? 9. When do we call the curve C proper? 10. When does the curve C reduce to a single point? 11. What point is called singular? 12. What curve is called nonsingular? 13. When are the coordinates x, y, z analytic? 14. What forms can the equations $x = x(t)$, $y = y(t)$, $z = z(t)$ take? 15. In which case is the curve C a straight line, a cubic parabola and a left-handed helix? 16. Can a curve be represented analytically? 17. By what equations can it be represented analytically? 18. What equations are called implicit or explicit equations?

Ex. 17. ***Translate into English.***

1. Установим в трехмерном пространстве левостороннюю ортогональную декартову систему координат для того, чтобы вывести уравнение кривой. 2. Любая точка этой системы имеет три координаты по осям (x, y, z), имеющим одинаковый масштаб. 3. Уравнения $x = x(t)$, $y = y(t)$, $z = z(t)$ называются параметрическими уравнениями кривой C, при этом параметром является переменная t. 4. Кривую можно описать как траекторию точки, движущейся с одной степенью свободы. 5. Кривая является собственной, когда она не сводится к отдельной фиксированной точке. 6. Аналитические формулы могут быть разложены в степенные ряды. 7. Кривая может быть представлена аналитически не только параметрическими уравнениями, иногда удобно представить кривую неявными и явными уравнениями. 8. Кривая может представлять прямую линию, кубическую параболу (неплоскую кривую третьего порядка) или левостороннюю круговую спираль.

Ex. 18. ***Topics for discussion.***

1. Give the definition and equations of a curve.
2. Dwell on the analytic representation of a curve.
3. Speak on the implicit and explicit equations of a curve.

Ex. 19. ***Read the text and find the answers to the following questions.***

1. What curves are of interest in mathematics?
2. What does the precise meaning of the term “curve” depend on?
3. When is an arc called a line segment?
4. What can you say about algebraic curves?

TEXT B

CURVES

In mathematics, a **curve** (also called a **curved line** in older texts) is, generally speaking, an object similar to a line but which is not required to be straight. This means that a line is a special case of a curve, namely a curve with null curvature. Often curves in two-dimensional (plane curves) or three-dimensional (space curves) Euclidean space are of interest.

Various disciplines within mathematics have given the term different meanings depending on the area of study, so the precise meaning depends on context. However many of these meanings are special instances of the definition which follows. **A curve is a topological space which is locally homeomorphic to a line.** In everyday language, this means that a curve is a set of points which, near each of its points, looks like a line, up to a deformation. A simple example of a curve is the parabola, shown to the right.

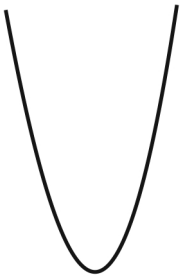
A large number of other curves have been studied in multiple mathematical fields.

The term *curve* has several meanings in non-mathematical language as well. For example, it can be almost synonymous with mathematical function, or graph of a function.

An arc or segment of a curve is a part of a curve that is bounded by two distinct end points and contains every point on the curve between its end points. Depending on how the arc is defined, either of the two end points may or may not be part of it. When the arc is straight, it is typically called a line segment.

Algebraic curves are the curves considered in algebraic geometry. A plane algebraic curve is the locus of the points of coordinates x, y such that $f(x,y) = 0$, where f is a polynomial in two variables defined over some field F. Algebraic geometry normally looks not only on points with coordinates in F but on all the points with coordinates in an algebraically closed field K. If C is a curve defined by a polynomial f with coefficients in F, the curve is said to be defined over F. The points of the curve C with the coordinates in a field G are said to be rational over G and can be denoted $C(G)$; thus the full curve $C = C(K)$.

Algebraic curves can also be space curves, or curves in even higher dimension, obtained as the intersection (common solution set) of more than one polynomial equation in more than two variables. By eliminating variables (by any tool of elimination theory), an algebraic curve may be projected onto a plane algebraic curve, which however may introduce singularities such as cusps (точка пересечения двух прямых) or double points.



A parabola, a simple example of a curve

UNIT VII
THE SUBJUNCTIVE MOOD

The Subjunctive Mood shows that the action or state expressed by the verb is presented as non-fact, as something imaginary, or desired. The Subjunctive mood is also used to express an emotional attitude of the speaker to real facts.

Subject Clause		Expresses
It is necessary *Необходимо,* It is important (recommended, advisable, requested, probable, strange, etc.) *Важно,*	that you (should) make a report. *чтобы вы сделали доклад.* that this law (should) be observed. *чтобы этот закон соблюдался.*	Supposition Necessity Probability Order Purpose Advice
Object Clause		
I demand/demanded (order, advise, insist, suggest, require, propose, etc.) *Я требую/потребовал,*	that they (should) work together. *чтобы они работали вместе.*	

Ex. 1. ***Translate the sentences into Russian.***

1. It is extremely important that you should carry out as many exercises as possible, no matter how trivial some of them may seem. 2. It is essential that the students should know all the rules. 3. I demand that they employ the differential calculus to deal with curves and surfaces. 4. He suggests that we deduce the properties of the curves involved by algebraic equations. 5. It is requested that the researcher (should) complete the calculations next week. 6. He advised that the students (should) inscribe a regular *n-gon* in a circle. 7. She proposed to her group-mates that they (should) discuss the topics of the term papers.

In Object Clause, introduced by the conjunction *lest* we use *should* for all persons.
She fears (feared) *lest* her secret *should* be discovered.
Она боится, чтобы ее секрет не был раскрыт.

Object Clause (after **wish**)

Unreal Present *I wish (if only) + Past Tense*	Use
I wish (if only) you worked more efficiently. *Я бы хотел, чтобы вы работали более эффективно.* We wish (if only) we were not so busy now. *Если бы только мы не были так заняты сейчас.*	Regret about the **present situation**
I wish I could drive a car. *Жаль, что я не умею водить машину.*	Regret in the **present** concerning lack of ability
Unreal Past *I wish (if only) + Past Perfect*	
I wish I had attended the seminar yesterday. *Жаль, что я не пришел на семинар вчера.*	Regret that something happened or didn't happen in **the past**
I wish (if only) + subject + would + Indefinite Infinitive	
I wish he would calculate more carefully. *Хотелось бы, чтобы вы делали вычисления более внимательно.* I wish it would stop raining. *Я бы хотел, чтобы дождь прекратился.*	Wish for **a future change** unlikely to happen, or annoyance

Ex. 2. ***Translate the sentences from English into Russian.***

1. I wish I knew the dimension of the body to calculate its volume.
2. I wish they had solved the three famous construction problems under the specified condition.
3. I wish I had written the test better, I have to revise the material again.
4. I wish he would stop working, he is very tired.
5. I wish I could understand the theory better.
6. I wish a new experiment would be started in our laboratory this month.
7. If only I were as good as you at mathematics.
8. I wish I had invited more people to the party.
9. If only you had taken his advice when he gave it.

Attributive clauses modifying the now "time" in the main clause
It is (*high, about*) *time... Пора...* (*давно пора, уже пора*)

It is time we **left** for the University, our classes start at 8.15.
Пора уходить в университет, наши занятия начинаются в 8.15.

1. It's time we went there.
2. It's time you learnt you're a student of Mechanics and Mathematics Faculty.
3. It's high time you made up mind!
4. It's about time you obtained the expected result.

Comparison clause (*as if ..., as though ...*)

It **is** as cold as if it **were** winter. *Холодно, как будто сейчас зима.* He **looks** as if he **were** very tired. *Он выглядит так, как если бы он очень устал.*	Present
He **spoke** as though she **had known** about it long before that day. *Он говорил, как если бы он знал об этом задолго до этого дня.*	Past

Had better = should

I ***had better*** + Present Infinitive You had better give up smoking. *Вам бы лучше бросить курить.*	Present, Future reference
It ***would have been better*** if + Past Prefect It ***would have been better*** if you hadn't said that. *Было бы лучше, если бы вы это не говорили.*	Past reference

Would rather = I'd prefer

I'd rather smb. + Past Simple **I'd rather smb.** + Past Perfect I'd rather you studied a little harder. *Лучше бы вы учились немного усерднее.* I'd rather you had read the book last term. *Лучше бы вы прочитали эту книгу в прошлом семестре.*	Present, Future reference Past reference

Ex. 3. ***Translate the sentences from English into Russian.***

1. I wish I weren't having my final test tomorrow. 2. I'd rather you hadn't spoken so rudely to him. 3. The man speaks as if he had never heard of this country. 4. He acts as though he owned the house. 5. It's time we went to the students' canteen.

6. I had better obtain the necessary information using the Internet. 7. He looks as if he were impressed by the news. 8. The teacher advised that the students discuss the material in detail. 9. I'd rather you didn't keep asking me every time you have a problem. 10. We worked slowly lest the task should have many mistakes.

Ex. 4. ***Put the verbs into the correct form.***

1. We'd better (to take) some money for a taxi in case we (to miss) the last bus home. 2. Would you rather (to write) in ink or in pencil? 3. I'd rather you (not/mention) this anyone until next week. 4. He didn't win the prize, but the looked as though he (to win) one. 5. It's about time you (to go) on holiday. 6. Helen went to France this year, but she says she would rather (to go) to Spain. 7. You had better (not to speak) to me like that again. 8. I wouldn't argue with her if I (to be) you. 9. If only I (not to drink) so much coffee. 10. I wish I (to listen) to you but I didn't. 11 It's essential that these conditions (to be satisfied).

Ex. 5. ***Translate from Russian into English.***

1. Важно, чтобы студенты работали в компьютерных классах. 2. Странно, что он всегда опаздывает. 3. Желательно, чтобы вы выполнили это задание как можно скорее. 4. Я настаиваю, чтобы вы написали эту статью. 5. Жаль, что я не знал об этой научной конференции, я бы хотел послушать доклады по аналитической геометрии. 6. Он выглядит так, как если бы он провел свой отпуск на юге. 7. Пора бы знать такие простые истины! 8. Ты бы лучше купил билет на этот концерт заранее. 9. Я бы предпочел побыть сейчас один. 10. Ах, если бы только я тогда сделал правильный выбор!

Ex. 6. ***Complete the sentences.***

1. It is probable that the question ... .
2. If I were you ... .
3. I'll dictate slowly lest you ... .
4. It seems as if ... .
5. I shall go to the theater with pleasure unless ... .
6. It's time they ... .

Pre-Reading Activity

Guess the meaning of the following words.

differentiate	[dɪfəˈrenʃɪeɪt]	converse (adj)	[kɔnˈvəːs]
function	[ˈfʌŋkʃ(ə)n]	actual	[ˈæktjuəl]

constant	[ˊkɔnst(ə)nt]	coordinate	[kouˊɔ:d(ı)nıt]
identical	[aıˊdentik(ə)l]	associate (v)	[əˊsouʃiet]
complex	[ˊkɔmpleks]	integrate	[ˊıntıgreıt]
finally	[ˊfaınəlı]	cylinder	[ˊsılındə]
isolated	[ˊaısə`leıtıd]	distance	[ˊdıstəns]
familiar	[fəˊmılıə]		

Read and learn the following words.

surface (n)	[ˊsə:is]	поверхность
double (a)	[dʌbl]	двойной
consist (v) (of)	[kənˊsıst]	состоять
unit (n)	[ˊju:nıt]	единица
impose (v)	[imˊpouz]	налагать (условие)
homogeneous (a)	[ˌhɔməˊdʒi:nıəs]	однородный
miss (v)	[mıs]	отсутствовать
generator (n)	[ˊdʒenəreıtə]	образующая
exhibit (v)	[ıgˊzıbıt]	показывать, проявлять
transpose(v)	[trænsˊpouz]	переставлять, переносить
vanish(v)	[ˊvænıʃ]	исчезать, стремиться к нулю
degenerate(a)	[dıˊdʒen(ə)rıt]	вырожденный
rule out (v)	[ru:lˊaut]	исключать
further more (adv)	[ˊfə:ðəˊmɔ:]	кроме того, к тому же
imply (v)	[imˊplaı]	подразумевать, означать
arbitrary (a)	[ˊa:bıtrərı]	произвольный
elimination (n)	[ɪˌlımıˊneıʃ(ə)n]	удаление, исключение (неизвестного)
vice versa	[ˊvaısıˊvə:sə]	наоборот, обратно
jacobian (n)	[dʒəˊkoubıən]	якобиан
desire (v)	[dıˊzaıə]	желать, испытывать желание
infinite (a)	[ˊınfınıt]	бесконечный
infinity (n)	[ınˊfınıtı]	бесконечность
to set equal to	[ˊsetˊi:kwəl]	положить равным
respectively (adv)	[rısˊpektıvlı]	соответственно, в указанном порядке
proper	[ˊprɔpə]	собственный

TEXT A

SURFACES

A **surface** can be described as a two-parameter family, or double infinity, of points. A surface can also be said to be the locus of a point moving with two degrees of freedom.

One method of representing a surface analytically consists in first establishing the usual left-handed orthogonal cartesian coordinate system with the same unit of distance on all three axes and then imposing one condition on a variable point $P(x, y, z)$ by an equation of the form

$$F(x, y, z) = 0. \tag{1}$$

Such an equation is called the implicit equation of the surface represented by it.

Certain very simple types of surfaces are already familiar. For example, if the equation (1) is linear in the variables x, y, z the surface represented by it is a plane, which is the simplest surface of all. Perhaps the next simplest surface is the sphere. If the equation (1) is homogeneous in x, y, z it represents a cone which vertex is at the origin. Finally, if one of the variables is missing from the implicit equation of a surface, the surface is a cylinder whose generators are parallel to the axis of the missing variable.

If the implicit equation (1) be solved for one of the variables as a function of the other two, say for z as a function of x, y, the resulting equation

$$z = f(x, y), \tag{2}$$

represents the same surface as before. Such an equation is called the explicit equation of the surface represented by it. The explicit equation (2) can be exhibited as a special case of the implicit equation (1) by transposing z to the right member and placing

$$F(x, y, z) = f(x, y) - z.$$

Although for some purposes the implicit and explicit equations of surfaces are very useful, the definition of a real proper analytic surface will be based on a parametric representation.

Definition 1. Let the coordinates x, y, z of a point P be given as a single-valued analytic function of two real independent variables u, v on a rectangle T in a uv-plane by equations of the form

$$x = x(u, v), \quad y = y(u, v), \quad z = z(u, v). \tag{3}$$

Further, let the jacobians of x, y, z with respect to u, v be denoted by J_1, J_2, J_3 so that

$$J_1 = y_u z_v - y_v z_u, \; J_2 = z_u x_v - z_v x_u, \; J_3 = x_u y_v - x_v y_u \left(x_u = \frac{dx}{dt}, \; \ldots \right) \tag{4}$$

and suppose that not all of $J1, J2, J3$ vanish identically on the rectangle T. Then the locus of the point P, as u, v vary on T, is a real proper analytic surface. Equations (3) are called parametric equations of the surface S, the parameters being the variables u, v. We reserve the right to permit the parameters to take on complex values. Moreover, one or more of the coordinates x, y, z may, under suitable conditions, be allowed to be complex. To say that a surface is proper means that it does not reduce to a curve. Both of these degenerate cases are ruled out by the hypothesis that the jacobians $J_1 = (1, 2, 3)$ do not all vanish identically. For, if the locus S were to reduce to a fixed point P, the coordinates x, y, z of P would all be constant, and the jacobians J_1 would all vanish identically. Furthermore, if the locus S were to reduce to a curve, this curve could be represented parametrically by equations of the form (2). If in these equations the parameter t is set equal to any function of u, v, the result is three equations of the form (3), for which the jacobians J_1 are easily proved, by actual calculation, to vanish identically. Conversely, the identical vanishing the jacobians J_1 would imply that the locus of the point P was not a proper surface. For, if the jacobians all vanish identically, then the functions x, y, t are three solutions of a linear homogeneous partial differential equation of the form

$$a\theta_u + b\theta_v = 0 \tag{5}$$

in which the coefficients a, b are functions of u, v. The theory of linear partial differential equations of the first order tells us how to integrate this equation. First form the associated ordinary differential equation

$$bdu - adv = 0. \tag{6}$$

This equation has an integral

$$t(u, v) = \text{const} \tag{7}$$

and the most general solution of equation (5) is given by the formula

$$\theta_u = F(t(u, v)), \tag{8}$$

the function F being arbitrary. Consequently, the coordinates x, y, z are either all constant or are, at most functions of a single parameter t, so that either P is a fixed point or else has for its locus a curve.

Even if the jacobians J_1, J_2, J_3 do not all vanish identically on the rectangle T. It may happen that they vanish simultaneously for one or more isolated pairs of values of u, v or perhaps they vanish simultaneously along a curve $v = v(u)$ in T. Any point of a real proper analytic surface at which the jacobians J_1, J_2, J_3 vanish simultaneously is called singular, although the singularity may belong to the parametric representation being used for the surface defined as a point-locus, as in the case of the sphere, or else the singularity may belong to the surface itself. A surface, or portion of a surface, which is free of singular points may be called nonsingular.

Elimination of u, v from the parametric equations (3) of a surface S would lead to the implicit equation (1) of S. Vice versa if the implicit equation (1) of a surface is desired, let two of the variables, say x and y, be arbitrary functions of two parameters u, v, and then solve (1) for z as a function of u, v. In particular, we might take $z = u, y = v$. Then solution of z would lead to the explicit equation (2) of the surface, except that u and v would occur in place of x and y, respectively. Indeed, the explicit equation (2) of a surface, when supplemented by the identities $x = x, y = y$, becomes the parametric equations

$$X = x, y = y, z = f(x, y)$$

of the same surface, the parameters now being the coordinates x, y.

Post-Reading Activity

Ex. 7. ***Match the English words and word combinations with the Russian equivalents.***

1) a double infinity of points	a) тождественно обращаться в нуль
2) the locus of a point	b) две степени свободы
3) two degrees of freedom	c) геометрическое место точки
4) to impose a condition	d) сохранять право
5) a variable point	e) линейное однородное дифференциальное уравнение в частных производных
6) a missing variable	f) двойная бесконечность точек
7) the resulting equation	g) отсутствующая переменная
8) a real proper analytic surface	h) налагать условие
9) with respect to	i) полученное уравнение
10) to vanish identically	j) переменная точка
11) to reserve the right	k) сводиться к кривой
12) under suitable conditions	l) при подходящих условиях
13) to reduce to a curve	m) наоборот, обратно
14) a linear homogeneous partial differential equation	n) произвольные функции
15) vice versa	o) действительная собственная аналитическая поверхность
16) arbitrary functions	p) относительно

Ex. 8. ***Find out whether the statements are true or false. Use introductory phrases.***

Exactly. Quite so.	***Quite the contrary.***
I fully agree to it.	***Not quite. It's unlikely.***
I don't think this is the case.	***Just the reverse.***

1. To represent a surface analytically, we establish a polar coordinate system. 2. The equation $f(x, y, z) = 0$ is called an implicit equation of the surface. 3. The

simplest surface of all types of surfaces is a cone. 4. In the parametric equations of the surface $x = x(u, v)$; $y = y(u, v)$; $z = z(u, v)$ the letters x, y, z denote the parameters. 5. A surface or a portion of a surface, which is free of singular points may be called singular. 6. A surface is proper if it does not reduce to a curve. 7. If the equation $f(x, y, z) = 0$ is homogeneous in x, y, z, it represents a plane. 8. The explicit equation of the surface $Z = f(x, y)$ is a special case of the implicit equation of the form $F(x, y, z) = 0$ if we transpose z to the right member.

Ex. 9. ***Translate the sentences, point out the meaning of the words.***

1) order (n)	1) порядок, последовательность, исправность, хорошее состояние, приказ, распоряжение
2) in order that in order to	2) для того, чтобы

1. The names in this list are placed in alphabetical order. 2. Get your ideas in some kind of order before beginning to write. 3. The device has been repaired and it is now in perfect order. 4. It is the business of the police to keep order. 5. He ordered that the work should be started at once. 6. Phone him in order that he should arrive in time. 7. In order to understand the procedures, consider the following analogy.

Ex. 10. ***Complete the sentences using the given word-groups:***

at the origin, the variables, the implicit equation of the surface, a plane, the parametric equations of the surface, arbitrary, a curve, singular, nonsingular, vanish simultaneously

1. The equation $F(x, y, z) = 0$ is called ... . 2. If this equation is linear in the variables x, y, z, the surface which it represents is ..., and if it is homogeneous in x, y, z, it represents a cone which vertex is ... . 3. A surface is proper if it does not reduce to ... . 4. The function F in the equation $\theta = F(t(u, v))$ is ... . 5. Any point of a real proper analytic surface at which the jacobians ... is called ... . 6. A surface, or a portion of a surface, which is free of singular points may be called ... . 7. Equations $x = x(u, v)$; $y = y(u, v)$; $z = z(u, v)$ are called ..., whose parameters are ... .

Ex. 11. ***Complete the sentences using the right meaning of the word.***

1) to mean (meant, meant)	1) (*v*) значить, предназначаться для, подразумевать под
2) means	2) (*n*) средство
3) mean	3) (*a*) средний, (*n*) середина
4) by no means	4) никоим образом
5) by means of	5) при помощи, посредством

1. Your friendship ... a great deal to me, I value it highly.
2. A dictionary tries to tell you what words... .
3. What do you ... by saying that? What have you in mind?
4. Is this figure ... to be a 1 or a 7?
5. Does the end always justify the ... ?
6. Thoughts are expressed by ... of words.
7. These results are by ... satisfactory.
8. The ... of 3, 5 and 7 is 5.
9. The happy or golden ... is a moderate course of action.

Ex. 12. ***Translate the sentences from Russian into English, use the Subjunctive Mood.***

1. Проверьте решение этой задачи, чтобы не получить неверный результат. 2. Он предложил, чтобы они рассмотрели многоугольники, так как этот тип геометрических фигур очень важен при изучении геометрии. 3. Жаль, что мы не исключили неизвестные, мы смогли бы решить эту систему уравнений. 4. Чтобы составить уравнение данной кривой, определим геометрическое место точек. 5. Пора ввести однопараметрическое семейство кривых на поверхности *S*. 6. Нам следует представить данные кривые аналитически с помощью системы уравнений. 7. Необходимо, чтобы часть поверхности не содержала особых точек. 8. Нам бы хотелось, чтобы в этом случае якобиан стремился к нулю. 9. Было бы лучше, если бы один из параметров менялся, в то время как другой оставался постоянным.

Ex. 13. ***Answer the questions.***

1. What does this text consider? 2. What must be established in order to represent a surface analytically? 3. What types of surfaces do you know? 4. A plane and a sphere are the simplest types of surfaces, aren't they? 5. Is the equation $F(x, y, z) = 0$ an implicit or an explicit equation of a surface? 6. When does the equation $F(x, y, z) = 0$ represent a plane, a cone and a cylinder? 7. What can you say about the equation $x = x(u, v)$; $y = y(u, v)$; $z = z(u, v)$? 8. What does it mean to say that a surface is proper? 9. What is a jacobian? 10. What do we call any point of a real proper analytic surface at which the jacobians vanish simultaneously? 11. What surface is called nonsingular?

Ex. 14. ***Topics for discussion.***

1. Speak on the definition of a surface given in this text and the definitions given in the lectures on differential geometry.
2. Discuss implicit and explicit equations of a surface.

3. Speak about the cases,when a surface represented by the equations $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$ reduces to a point or to a curve.

Ex. 15. ***Say it in English.***

1. В этом тексте даётся определение поверхности и ее аналитическое представление. 2. Установим левостороннюю ортогональную декартову систему координат для того, чтобы представить поверхность аналитически. 3. Эта система координат имеет масштаб измерения для всех трех осей. 4. Затем мы налагаем одно условие на точку $P(x, y, z)$ уравнением вида $F(x, y, z) = 0$. 5. Это уравнение называется неявным уравнением поверхности. 6. Плоскость, сфера, цилиндр, конус известны как простые типы поверхностей. 7. Они могут быть представлены уравнением $(x, y, z) = 0$ при определенных условиях. 8. Уравнения $x(u, v)$, $y = y(u, v)$, $z = z(u, v)$ с переменными v, u называются параметрическими уравнениями поверхности. 9. Любая точка поверхности, в которой якобианы одновременно равны нулю, называется вырожденной, а поверхность, не имеющая вырожденных точек, называется невырожденной. 10. Поверхность является собственной поверхностью, если она не превращается в кривую.

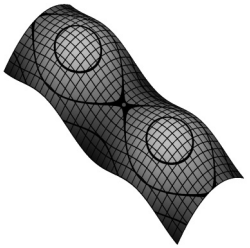
Ex. 16. ***Read the text and find the answers to the following questions.***

1. What is a surface in mathematics? 2. Where does the concept of a surface find application? 3. What is a topological surface? 4. What is a coordinate chart? 5. What is known as the boundary of the surface?

TEXT B

SURFACE

In mathematics, specifically, in topology, **a surface** is a two-dimensional, topological manifold (множество, многообразие). The most familiar examples are those that arise as the boundaries of solid objects in ordinary three-dimensional Euclidean space R^3 – for example, the surface of a ball. On the other hand, there are surfaces, such as the Klein bottle, that cannot be embedded in three-dimensional Euclidean space without introducing singularities or self-intersections.



An open surface *X*-, *Y*- and *Z*-contours shown

To say that a surface is "two-dimensional" means that, about each point, there is a *coordinate patch* (часть) on which a two-dimensional coordinate system is defined. For example, the

surface of the Earth is (ideally) a two-dimensional sphere, and latitude (широта) and longitude (долгота) provide two-dimensional coordinates on it (except at the poles and along the 180th meridian).

The concept of a surface finds application in physics, engineering, computer graphics, and many other disciplines, primarily in representing the surfaces of physical objects. For example, in analyzing the aerodynamic properties of an airplane, the central consideration is the flow of air along its surface.

A (*topological*) *surface* is a nonempty second countable Hausdorff topological space in which every point has an open neighbourhood (окрестность) homeomorphic to some open subset of the Euclidean plane E^2. Such a neighborhood, together with the corresponding homeomorphism, is known as a (*coordinate*) *chart* (координатные сетки). It is through this chart that the neighborhood inherits (наследовать) the standard coordinates on the Euclidean plane. These coordinates are known as *local coordinates* and these homeomorphisms lead us to describe surfaces as being *locally Euclidean*.

More generally, a ('*topological*) *surface with boundary* is a Hausdorff topological space in which every point has an open neighbourhood homeomorphic to some open subset of the closure of the upper half-plane H^2 in C. These homeomorphisms are also known as (*coordinate*) *charts*. The boundary of the upper half-plane is the *x-axis*. A point on the surface mapped via a chart to the x-axis is termed a *boundary point*. The collection of such points is known as the *boundary* of the surface which is necessarily a one-manifold, that is, the union of closed curves. On the other hand, a point mapped to above the x-axis is an *interior point*. The collection of interior points is the *interior* of the surface which is always non-empty. The closed disk is a simple example of a surface with boundary. The boundary of the disc is a circle.

The term *surface* used without qualification refers to surfaces without boundary. In particular, a surface with an empty boundary is a surface in the usual sense. A surface with an empty boundary which is compact is known as a 'closed' surface. The two-dimensional sphere, the two-dimensional torus (тор), and the real projective plane are examples of closed surfaces.

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