Viscous Control of Shallow Elastic Fracture

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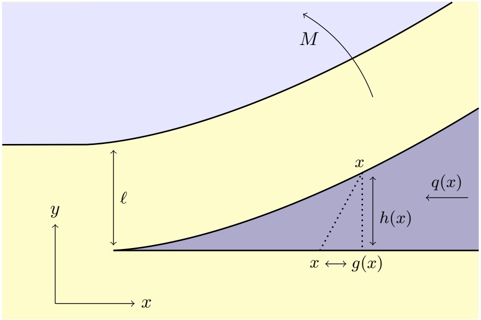
Introduction



- Set up: a semi-infinite elastic solid with a thin strip peeled off and the resulting crack filled with an incompressible fluid
- The motion is driven by a bending moment applied to the arm of the solid.

Governing Equations

Fluid Mechanics:

- Assume that the flow everywhere is in lubrication, i.e. Poiseulle flow in the crack.
- Look for travelling wave solutions with speed c. Get the equation,

$$\frac{dp}{dx} = 12\mu c/h^2$$

Solid Mechanics:

- Assume the solid is a linear elastic solid. We can use Muskhelishiveli methods to derive another equation,

$$\begin{bmatrix} p \\ 0 \end{bmatrix} = \frac{E}{4\pi\ell(1-\nu^2)} \int_0^\infty \mathbf{K}(x-\tilde{x}) \begin{bmatrix} g'(\tilde{x}) \\ h'(\tilde{x}) \end{bmatrix} d\tilde{x}.$$

- K_{ij} is the integral kernel specific to this geometry.

Boundary conditions

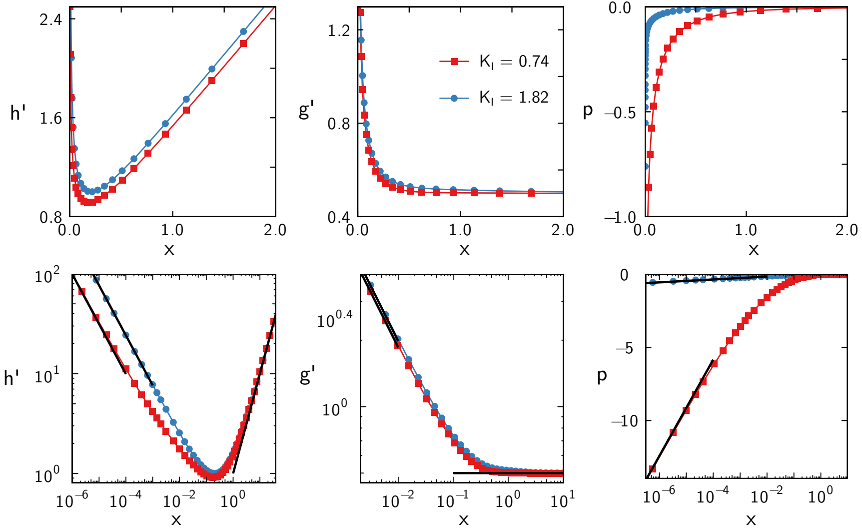
- The boundary conditions as $x \to \infty$ are governed by the bending moment. We can use beam theory to approximate the elasticity equations for x large. This gives the boundary conditions

$$M(x) = \frac{E\ell^3}{12(1-\nu^2)} \frac{d^2 h}{dx^2} \to M_0 = \text{ const.}$$

- The boundary conditions as $x \to 0$ are govened by Linear Elastic Fracture Mechanics, they are

$$K_I = \lim_{x \to 0} \frac{E}{1-\nu^2} \sqrt{\frac{\pi}{8}} \sqrt{x} h'(x), \quad K_{II} = \lim_{x \to 0} \frac{E}{1-\nu^2} \sqrt{\frac{\pi}{8}} \sqrt{x} g'(x).$$

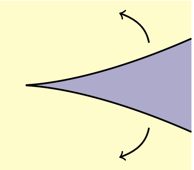
Results



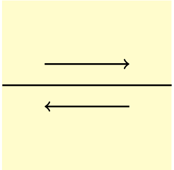
h'
2.4
1.6
0.8
0.0
1.0
2.0
x
g'
1.2
0.8
0.4
$K_I = 0.74$
$K_I = 1.82$
p
0.0
−0.5
−1.0
10^2
10^1
10^0
10^{-6}
10^{-4}
10^{-2}
$10^{0.4}$
10^{-1}
10^1
0
−5
−10

Speed vs. toughness

- For a given bending moment, the speed should depend on the two toughness constants of the rock K_I, K_{II} which correspond to mode I and II fracture respectively.
- Actually have critical values of K_I and K_{II} for a given speed, only one of K_I, K_{II} has to be equal to their critical values.
- For example, if the critical values for $c = 1$ were $(K_I, K_{II}) = (1.3, 1.5)$, then for both pairs of fracture constants: $(1.3, 20)$, $(15, 1.5)$, the speed would still be 1.

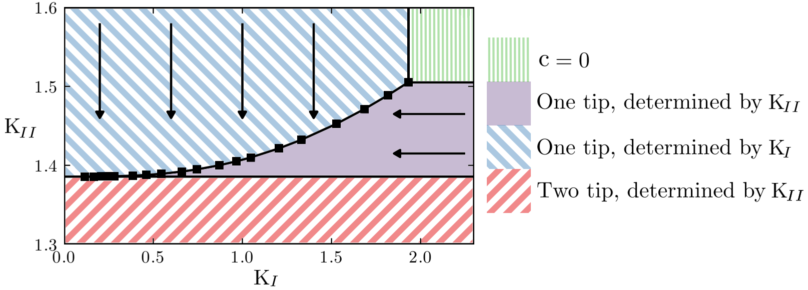


K_I, Bending moment



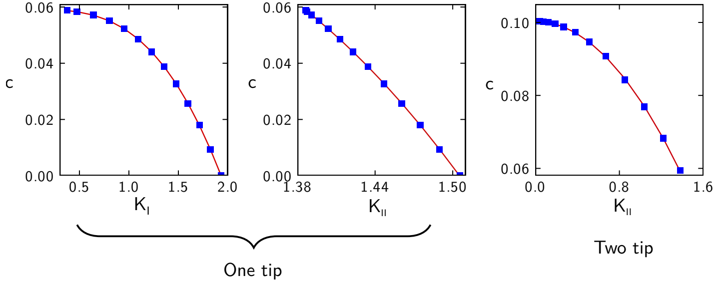
K_{II}, Shear

Catagorising the toughness constants



- There is a problem: the critical values only exist for $K_{II} > 1.4$. So does that mean no travelling wave solutions for $K_{II} < 1.4$?
- Actually something else happens, and a dry crack precedes the wet tip, the "two tip" problem.

Final Results



- Success! We can find the speed of travelling waves for all values of K_I, and K_{II}.