

# Tutorial of Orthogonal Matching Pursuit

A tutorial for Beginners and Dummies

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- ② The Orthogonal Matching Pursuit
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# 1. Problem Statement

Consider the following very simple example: Given the following sparse signals

$$x = \begin{pmatrix} -1.2 \\ 1 \\ 0 \end{pmatrix}$$

The following is the measurement matrix  $A$  ( $2 \times 3$ ):

$$A = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix}$$

Therefore  $y = A \cdot x$  gives:

$$y = \begin{pmatrix} -1.65 \\ -0.25 \end{pmatrix}$$

Now, Given that :  $y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$  and  $A = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix}$

How to find original  $x$ ?

## 2. Orthogonal Matching Pursuit

**BASIS** Previous example: Given :  $y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$  and

$$A = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix}$$

Columns in matrix A are called BASIS (CHEN and DONOHO : **ATOMS**). In the example, we have the following *atoms*:

$$b_1 = \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix} \quad b_2 = \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix} \quad b_3 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

## 2. Orthogonal Matching Pursuit

Interpretation of equation  $Ax = y$

Since  $A = [b_1 \ b_2 \ b_3]$ ; and if we let :  $x = [a \ b \ c]$ , then

$A \cdot x = a \cdot b_1 + b \cdot b_2 + c \cdot b_3$   $A \cdot x$  is the linear combination of  $b_1$ ,  $b_2$ ,  $b_3$ .

$$A \cdot x = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1.2 \\ 1 \\ 0 \end{pmatrix}$$

$$= -1.2 \cdot \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix} + 1 \cdot \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix} = y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$$

## 2. Orthogonal Matching Pursuit

$$A \cdot x = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1.2 \\ 0 \\ 1 \end{pmatrix}$$

$$= -1.2 \cdot \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix} + 1 \cdot \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix} = y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$$

From the equation above, it is clear that atom  $b_1$  contribute the biggest influence in  $y$ , next is  $b_2$ , dan last is  $b_3$ . ORTHOGONAL MATCHING PURSUIT works reversely: we start finding which of  $b_1$ ,  $b_2$ ,  $b_3$  that will influence the STRONGEST to  $y$ . Then the SECOND STRONGEST from the residual, and so on.

## 2. Orthogonal Matching Pursuit

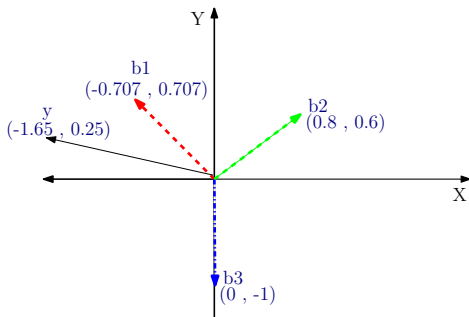
STRONGEST influence is measured using DOT PRODUCT / INNER PRODUCT OMP Algorithm:

- 1 find atom with has the biggest inner product with  $y$

$$p_i = \max_j \langle b_j, y \rangle$$

- 2 calculate the residue  $r_i = p_i - p_i \cdot \langle p_i, y \rangle$
- 3 find atom with has the biggest inner product with  $r_i$
- 4 repeat step 2 and 3 until residue achieve a certain threshold

Geometrically:



## 2. Orthogonal Matching Pursuit

Here the dot product of  $y$  to any of  $b_1, b_2, b_3$ :

$$\langle y, b_1 \rangle = -1.34$$

$$\langle y, b_2 \rangle = 1.17$$

and

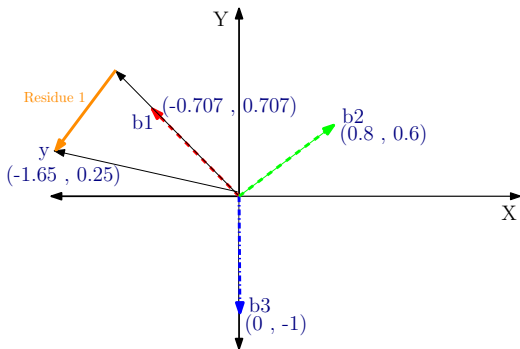
$$\langle y, b_3 \rangle = 0.25$$

Taking the absolute value, we see  $b_1$  gives the biggest inner product. Then,  $b_1$  is chosen as the atom in first step, DOT PRODUCT  $-1.34$ . We next count the residue:

$$r_1 = y - b_1 \cdot \langle y, b_1 \rangle = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix} - (-1.34) \cdot \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$$



## 2. Orthogonal Matching Pursuit



Next we count the DOT PRODUCT of this residue to  $b_2$  and  $b_3$  (no need to count with  $b_1$ , since this residue must perpendicular to  $b_1$ ).

$$\langle r_1, b_1 \rangle = 1$$

$$\langle r_1, b_3 \rangle = -0.7$$

Taking the absolute value, we get  $b_2$  as the next strongest influence.

## 2. Orthogonal Matching Pursuit

Next we count again the residue:

$$\begin{aligned} r_2 &= r_1 - \langle r_1, b_2 \rangle b_2 = \begin{pmatrix} -0.7 \\ 0.7 \end{pmatrix} - (1) \cdot \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix} \\ &= \begin{pmatrix} -0.099 \\ 0.099 \end{pmatrix} \end{aligned}$$

From residue  $r_2$ , we finally count the final DOT PRODUCT, between  $r_2$  with the last  $b_3$  :

$$\langle r_2, b_3 \rangle = -0.099$$

## 2. Orthogonal Matching Pursuit

The following base is in STRONGEST influence in order:  $b_1$ ,  $b_2$ ,  $b_3$ , with influence measure from dot product is :  $-1.34$ ,  $1$ , dan  $-0.099$

Therefore, the reconstructed  $x$  is  $\begin{pmatrix} -1.34 \\ 1 \\ -0.099 \end{pmatrix}$

Original was :  $\begin{pmatrix} -1.2 \\ 1 \\ 0 \end{pmatrix}$

### 3. Base Coherency

Base Coherency indicate how close one base to the others.

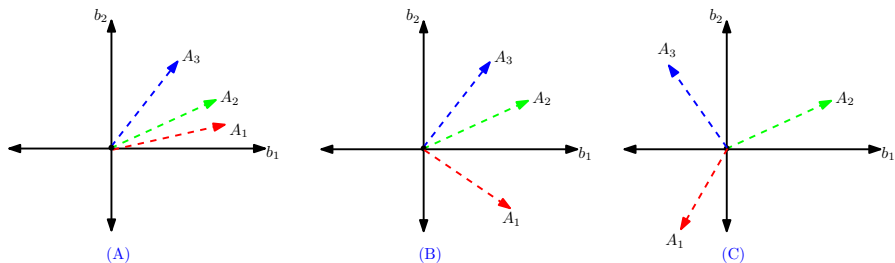


Figure : Ilustrasi Koherensi Basis. (A). Koherensi tinggi. (B). Koherensi Sedang. (C). Koherensi Rendah

### 3. Base Coherency

A simple measure for base coherency is INNER PRODUCT.

$$\mu = \max_{(i,j), i \neq j} (< A_i, A_j >)$$

- Range of Coherency Value  $\mu$  is :  $0 \leq |\mu| \leq 1$
- 0 : least coherency
- 1 : full coherent

We expect least coherency in our base in order to success in MP.