# CS 730/730W/830: Intro AI

Adversarial Search

2 handouts: slides, asst 1 solution asst 1 due

# **EOLQ**s

#### Adversarial Search

- Another Type
- Minimax
- Tic-tac-toe
- **■** Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- $\blacksquare$  EOLQs

### **Another Twist on Search**

#### Adversarial Search

#### ■ Another Type

- Minimax
- Tic-tac-toe
- **■** Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

- Shortest-path (M&C, vacuum, tile puzzle)
  - want least-cost path to goal at unkown depth
- $\blacksquare$  Constraint satisfaction (map coloring, n-queens)
  - any goal that satisfies constraints (fixed depth)
- Combinatorial optimization (TSP, max-CSP)
  - want least-cost goal (fixed depth)
- Decisions with an adversary (chess, tic-tac-toe)
  - adversary might prevent path to best goal
  - want best assured outcome

## **Adversarial Search: Minimax**

Adversarial Search

Another Type

Minimax

Tic-tac-toe

Improvements

Break  $\alpha$ - $\beta$  Pruning  $\alpha$ - $\beta$  Pseudo-code

Why  $\alpha$ - $\beta$ ?

Progress

EOLQs

Each *ply* corresponds to half a *move*.

Terminal states are labeled with value.

Can also bound depth and use a *static evaluation function* on non-terminal states.

### **Evaluation for Tic-tac-toe**

#### Adversarial Search

- Another Type
- Minimax

#### ■ Tic-tac-toe

- **■** Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

```
A 3-length is a complete row, column, or diagonal.
```

value of position  $= \infty$  if win for me,

or  $= -\infty$  if a win for you,

otherwise = # 3-lengths open for me -

# 3-lengths open for you

## Tic-tac-toe: two-ply search

- Another Type
- Minimax
- Tic-tac-toe
- **■** Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- **■** EOLQs

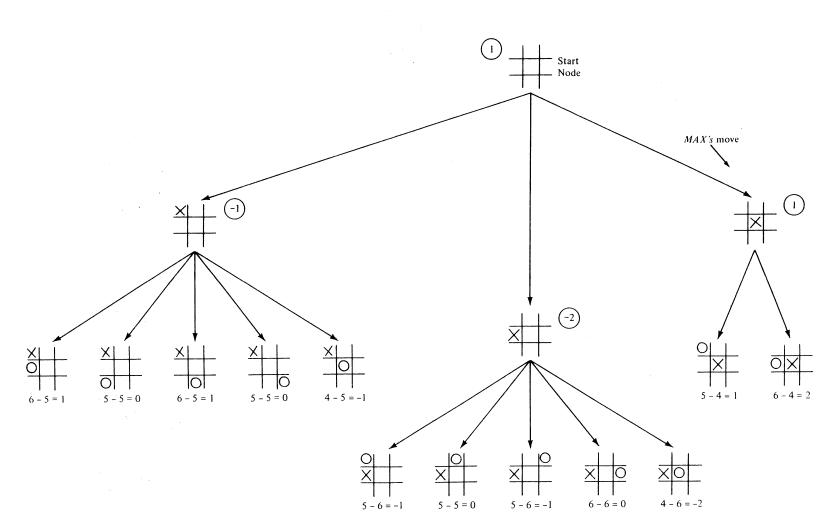


Fig. 3.8 Minimax applied to tic-tac-toe (stage 1).

### Tic-tac-toe: second move

#### Adversarial Search

- Another Type
- Minimax

#### ■ Tic-tac-toe

- **■** Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

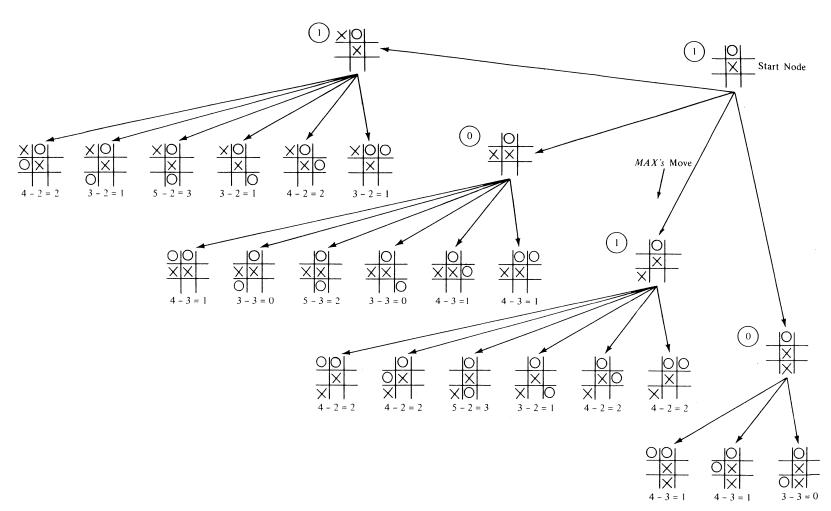


Fig. 3.9 Minimax applied to tic-tac-toe (stage 2).

## Tic-tac-toe: third move

#### Adversarial Search

- Another Type
- Minimax

#### ■ Tic-tac-toe

- **■** Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

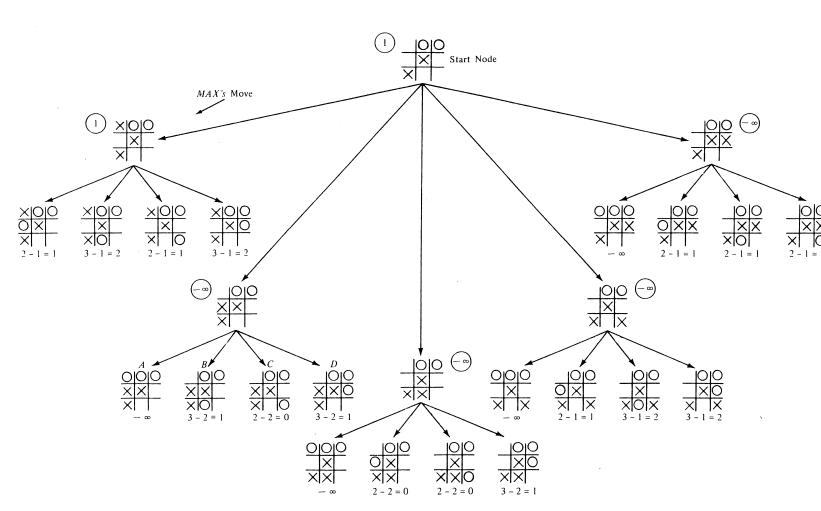


Fig. 3.10 Minimax applied to tic-tac-toe (stage 3).

## **Improving the Search**

- Another Type
- Minimax
- Tic-tac-toe
- Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

- partial expansion, SEF
- symmetry ('transposition tables')
- search more ply as we have time (De Groot figure)
- avoid unnecessary evaluations

### **Break**

- Another Type
- Minimax
- Tic-tac-toe
- **■** Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

- asst 1 was due
- book
- asst 2 (theorem prover) going out on Wed. parse simple CFG.
- exams are during common exam time
- have web access? a clicker?

## Which Values are Necessary?

- Another Type
- Minimax
- Tic-tac-toe
- **■** Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- $\blacksquare$  EOLQs

# $\alpha$ - $\beta$ Pruning

#### Adversarial Search

- Another Type
- Minimax
- Tic-tac-toe
- **■** Improvements
- Break

#### $\square$ $\alpha$ - $\beta$ Pruning

- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

- $\alpha$  best outcome Max can force at previous decision on this path (init to  $-\infty$ )
- eta best outcome Min can force at previous decision on this path (init to  $\infty$ )

 $\alpha$  and  $\beta$  values are copied down the tree (but not up). Minmax values are passed up the tree, as usual.

#### Adversarial Search

- Another Type
- Minimax
- Tic-tac-toe
- Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning

#### $\square \alpha$ - $\beta$ Pseudo-code

- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

```
Max-value (state, \alpha, \beta): when depth-cutoff (state), return SEF(state) for each child of state \alpha \leftarrow \max(\alpha, \text{ Min-value (child, } \alpha, \beta)) when \alpha \geq \beta, return \alpha return \alpha
```

```
Min-value (state, \alpha, \beta):
when depth-cutoff (state), return SEF(state)
for each child of state
\beta \leftarrow \min(\beta, \text{Max-value (child, } \alpha, \beta))when \beta \leq \alpha, return \beta
return \beta
```



- Another Type
- Minimax
- Tic-tac-toe
- Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\square \alpha$ - $\beta$  Pseudo- $\alpha$
- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

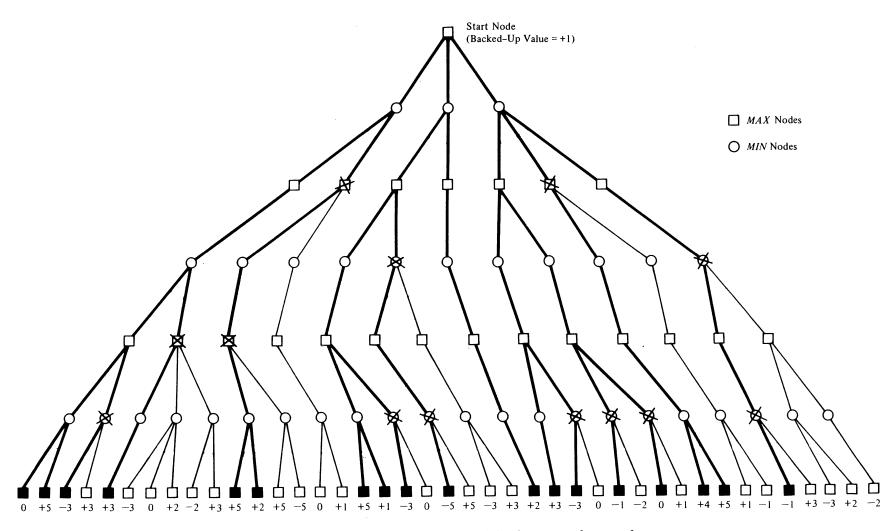


Fig. 3.12 An example illustrating the alpha-beta search procedure.

Adversarial Search

■ Another Type

■ Minimax

■ Tic-tac-toe

**■** Improvements

■ Break

 $\blacksquare \alpha$ - $\beta$  Pruning

 $\blacksquare \alpha$ - $\beta$  Pseudo-code

 $\blacksquare$  Why  $\alpha$ - $\beta$ ?

■ Progress

**■** EOLQs

Time complexity of  $\alpha$ - $\beta$  is about  $O(b^{d/2})$ 

## **Progress on Games**

Adversarial Search

■ Another Type

■ Minimax

■ Tic-tac-toe

**■** Improvements

■ Break

 $\blacksquare \alpha$ - $\beta$  Pruning

 $\blacksquare \alpha$ - $\beta$  Pseudo-code

■ Why  $\alpha$ - $\beta$ ?

■ Progress

■ EOLQs

Computers best: chess, checkers, Othello, backgammon,

Scrabble

Computers competitive: bridge, crosswords, poker, small Go

Computers amateur: full Go

# **EOLQ**s

#### Adversarial Search

- Another Type
- Minimax
- Tic-tac-toe
- **■** Improvements
- Break
- $\blacksquare \alpha$ - $\beta$  Pruning
- $\blacksquare \alpha$ - $\beta$  Pseudo-code
- Why  $\alpha$ - $\beta$ ?
- Progress
- EOLQs

Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!