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CONVERTIBLE BONDS: VALUATION AND OPTIMAL STRATEGIES FOR CALL AND CONVERSION

M. J. BRENNAN AND E. S. SCHWARTZ*

I

THE THEORY OF OPTION and warrant pricing has only of late been placed on a sound theoretical basis in a context of security market equilibrium [1, 6]; closed form expressions have been derived by Black-Scholes [1] and Merton [6] for the value of an option when the underlying stock pays no dividend or the option is protected against dividends, and when the stock pays a continuous dividend which is proportional to the market value of the stock. Further research has extended this option pricing model to take account of jumps in security returns [3, 8], and the basic option pricing model has been shown to obtain under certain assumptions, even in the absence of continuous trading opportunities [11]. More recently, algorithms have been developed [12] to solve the relevant dynamic programming problem when the stock does pay dividends and the option is not protected against dividend payments, so that the possibility of exercise prior to maturity must be considered for an American type option.

As yet however, little attempt has been made to apply the principles of the option pricing model to the most common type of convertible security, namely the convertible bond.[1] This security is considerably more complex than the warrant, not only because it pays a periodic coupon, but also because it involves a dual option: on the one hand, the bondholder possesses the option to convert the bond into common stock at his discretion, and on the other hand, the firm possesses the option to call the bond for redemption, the bondholder retaining the right to convert the bond or to redeem it. This call option is usually subject to some kind of restriction, a common one being that the bond may not be called for five years. The investor's optimal conversion strategy then depends on the firm's call strategy, and it appears at first sight that the optimal call strategy must also depend on the investor's conversion strategy, so that both optimal strategies must be solved for simultaneously. Fortunately, as we shall show below, the optimal call strategy is simply to call the bond as soon as the value of the bond if called is equal to the value if not called, so that the problem is considerably simplified.

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1. In an independent and contemporaneous paper Ingersoll [4] considers the valuation of convertible bonds within the same framework as this paper. Several of his results correspond to ours. The major difference between the papers is that Ingersoll concentrates on deriving "closed form" solutions for the value of a bond in a variety of special cases, whereas we offer a general algorithm for determining the value of a convertible bond.

Merton [6] has considered the related problem of valuing callable warrants on non-dividend paying stocks: callable warrants differ from convertible bonds in having no coupon payments, and in possessing no ultimate value save their conversion value; additionally, the assumption that the stock pays no dividends avoids the need to consider the possibility of voluntary conversion prior to expiration. Merton's analysis relies on the proposition that the value of a callable warrant is equal to the difference between the value of an equivalent non-callable warrant and the value of the firm's call option on the warrant. Under reasonable assumptions, a differential equation can be derived governing the value of the non-callable warrant and of the call provision, and hence of the callable warrant itself. The major difficulty appears to arise in the specification of appropriate boundary conditions; in particular in deriving the critical stock price above which the company should optimally exercise its call option: Merton shows how the critical stock price may be determined in the case of a perpetual warrant on a non-dividend paying stock.

This paper extends the work of Black-Scholes [1] and Merton [6] to the pricing of convertible bonds. The differential equation and boundary conditions governing the value of the bond are derived, and an algorithm is presented for solving the differential equation. The paper concludes with some examples designed to show the effect on the bond value of varying selected parameters.

Since numerical methods are employed to solve the differential equation, the valuation procedure is extremely flexible, and the model permits:

(i) discrete coupon payments on the bond;

(ii) discrete dividend payments on the firm's common shares which may be a function of the value of the firm and time;

(iii) the investor's right to convert the bond into common shares at any point in time; simple changes in the appropriate boundary conditions allow for the possibility of changing conversion terms over time.

(iv) the corporation's right to call the bond, the investor having the right, if the bond is called, either to convert the bond or to redeem it at the call price. The right to call the bond may be restricted; for example the bond may not be callable for five years, or may not be callable until the stock is at a certain premium above the conversion price. These restrictions may be taken into account by appropriate modification of the boundary conditions;

(v) the possibility that the firm will default on the bond by bankruptcy either prior to, or at maturity. The model thus extends Merton's analysis of risky corporate discount bonds [7] to risky coupon bonds.

The model development also allows us to dispense with the "incipient" assumption used in the above-mentioned papers [1, 6], though not in Merton's [7] paper on risky debt; this is the assumption that the net supply of the risky security is zero.

In the interest of clarity and computational convenience, the simplifying assumption is made that the firm's outstanding securities consist solely of common stock and convertible securities; this assumption could be relaxed by further modification of the boundary conditions. Additionally, the risk free rate of interest is assumed to be not only known but also to be constant through time.

In the following section we consider the problem of optimal call and conversion strategies. Solution of this problem yields certain boundary conditions which must be satisfied by the solution to the differential equation governing the value of the convertible bond. This differential equation and the boundary conditions are considered futher in Section III. Section IV describes the solution algorithm, and Section V discusses some numerical results.

II

A convertible bond can be valued only if the call and conversion strategies to be followed by the corporation and the investor respectively can be determined; for example, the bond value would in general clearly be affected if it were known that the firm would never exercise its call option, or that the investors would never convert prior to maturity. Thus, in deriving the call (conversion) strategy it is necessary to make an assumption about the strategy to be followed by the other party. The assumption we make is that each party, firm and investor, pursues an optimal strategy (to be defined below) and expects the other party to do the same. This assumptiom corresponds to the Miller-Modigliani [9] assumption of "symmetric market rationality," and results in a pair of conversion-call strategies which are equilibrium in the sense that neither party could improve his position by adopting any other strategy.

Then define:

$V(t)$ = the aggregate market value at time t of the firm's outstanding securities including the convertible bonds.

$W(V,t)$ = the market value at time t of one convertible bond with par value of \$1,000.

l = the number of convertible bonds outstanding.

$n(t)$ = the number of shares of common stock into which each bond is convertible at time t.

m = the number of shares of common stock outstanding before conversion takes place.

I = the aggregate coupon payment on the outstanding convertible bonds at each periodic coupon date.

$i = I/l$ = the periodic coupon payment per bond.

$CP(t)$ = the price at which the bonds may be called for redemption at time t, including any accrued interest.

$B(V,t)$ = the straight debt value of the bond; that is, the value of an otherwise identical bond with no conversion privilege.

$D(V,t)$ = the aggregate dividend payment on the common stock at each dividend date.

Definition 1. *The optimal conversion strategy is one which maximizes the value of the convertible bond at each instant in time.*

Definition 2. *The optimal call strategy is one which minimizes the value of the convertible bonds at each instant in time.*

The aggregate market value of the firm's securities, $V(t)$ is assumed to be determined exogenously and by the Modigliani-Miller [10] theorem to be independent of the particular call and conversion strategies followed. Hence, by minimizing the value of the outstanding convertible bonds, the management will be maximizing the value of the firm's equity, which is equal to the difference between the aggregate market value of the firm, $V(t)$, and the value of the convertible bonds.

The conversion value of a bond is equal to the number of shares into which it is convertible times the value of a share if the bond is converted. Since, by the Modigliani-Miller theorem, conversion of the bonds cannot alter the aggregate value of the firm's securities, $V(t)$, the value of a share after conversion is given by the pre-conversion value of the firm, $V(t)$, divided by the number of shares outstanding after conversion has taken place. Since each bond is convertible into $n(t)$ shares at time t, the conversion value, $C(V,t)$ is given by

$$\begin{aligned} C(V,t) &= n(t)V(t)/(m+ln(t)) \\ &= z(t)V(t), \qquad \text{where} \quad z(t)=n(t)/(m+ln(t)). \end{aligned} \tag{1}$$

Since, from Definition 1, it is optimal for the investor to convert should the value of the bond unconverted fall below the conversion value, we have the arbitrage condition:

$$W(V,t) \geqslant C(V,t). \tag{2}$$

A stronger condition on the value of the bond may be derived from the following Lemma.

LEMMA 1. *It will never be optimal to convert an uncalled convertible bond except immediately prior either to a dividend date or to an adverse change in the conversion terms, or at maturity.*

Proof. From Definition 1 it is never optimal to convert the bond if its market value exceeds its conversion value. But an uncalled bond can never sell at a price as low as conversion value except immediately prior to a dividend date or to an adverse change in conversion terms, since, if it did, the return on the bond up to the next dividend date or change in conversion terms would exhibit first degree stochastic dominance over the return on the underlying common stock. Therefore the bond will always sell above conversion value under these conditions, and the investor will never find it optimal to convert.

The stochastic dominance arises from the following consideration: if the bond is currently selling at conversion value, its rate of return up to the next dividend or conversion change date can never fall below the rate of return on the stock since the bond value can never fall below the conversion value by condition (2). However, if the value of the firm drops sufficiently, the priority of claim of the bond will cause its value to exceed the conversion value, so that the return on the bond will exceed the return on the stock under these conditions.[2]

2. Cf. Merton's [6] demonstration that a warrant will never be exercised except immediately prior to a dividend.

Hence between dividend dates (2) holds as a strict inequality if there is no adverse change in conversion terms. This Lemma serves to simplify the computational algorithm described below, since the possibility of conversion must only be considered at the discrete dividend dates, or when the conversion terms change adversely for the investor, or at call.

If a bond is called for redemption by the firm, the investor retains the option either of redeeming the bond at the current call price, $CP(t)$, or converting it and receiving an amount of shares equal in value to the conversion value, $C(V,t)$. Since by Definition 1, the investor will always select the more valuable option, the value of the bond if called, $VIC(V,t)$ is given by:

$$VIC(V,t) = \max[CP(t), C(V,t)]. \tag{3}$$

The firm's optimal call strategy is given by the following Lemma.

LEMMA 2. *The firm's optimal call strategy is to call the bond as soon as its value if it is not called is equal to the call price.*

Proof. By Definition 2, the optimal call strategy is chosen to minimize the value of the convertible bonds. This is accomplished by calling the bonds as soon as their value if not called equals their value if called. For if the bonds were left uncalled at a market value exceeding their value if called, their value would clearly not be minimized; on the other hand, if the bonds were called when their value uncalled was below their value if called, calling would confer a needless gain on the bondholders and the value of the bonds would again not be minimized. Therefore, calling the bonds when their value if not called equals their value if called is indeed the strategy which minimizes the value of the bonds. Since the minimum value if called is the call price, the bonds will be called as soon as their value if not called is equal to the call price.

In the event that the conversion value exceeds the call price at the time the bond first becomes callable, the bond will be called immediately since by the proof of Lemma 1 the uncalled bond would sell for more than the conversion value which is then the value if called.[3]

An implication of this equilibrium call strategy in an efficient market is that the bond will sell at a price equal to its value if called when the call is exercised. Were this not the case, investors, knowing the optimal call strategy to be followed by the firm, could reap arbitrage profits from the difference between the pre-call bond price and the value if called. Note however that the firm's optimal call strategy cannot be inferred from the observation that the bond will sell at its value if called when the call is exercised, since this would be true of any, not necessarily optimal, call strategy, so long as the strategy is known to investors.

3. It was implicitly assumed in Lemma 2 that the bonds may be instantaneously called for conversion. A more typical arrangement requires the firm to give notice of its intention to call the bonds, introducing the risk to the firm that between the time notice is given and the time the bonds are actually called, the conversion value of the bonds will have fallen below the call price, so that the bonds will be redeemed rather than converted.

When notice of call is given the bonds will become non-callable bonds with maturity equal to the call notice period. This new bond value, which may be readily computed using the methods of Section IV, then becomes the "effective call price" which should be used in Lemma 2.

Lemma 2 gives rise to the following restrictions on the value of the bond:

(i) At time $t=t^*$ when the bond first becomes callable, its value satisfies

$$W(V,t^*)=C(V,t^*), \quad \text{if } C(V,t^*)\geqslant CP(t^*). \tag{4}$$

(ii) At any time when the bond is callable we have the call price constraint:

$$W(V,t)\leqslant CP(t). \tag{5}$$

Lemma 2 does not directly determine the values of $V(t)$ at which the bond will be called. These must be determined as part of the solution procedure.

III

Since W is a function only of V and t, it is readily shown (Cf. Black-Scholes [1] and Merton [6]) that if V follows the stochastic process

$$\frac{dV}{V}=\mu\,dt+\sigma\,dz, \tag{6}$$

where dz is a Gauss-Wiener process, then W must satisfy the stochastic differential equation

$$\tfrac{1}{2}\sigma^2V^2W_{vv}+rVW_v-rW+W_t=0, \tag{7}$$

where r is the risk free rate of interest, and the subscript denotes partial differentiation.

Additionally, W must satisfy the following boundary conditions:

(i) At any time the aggregate value of the bonds outstanding cannot exceed the total value of the firm yielding the arbitrage condition:

$$lW(V,t)\leqslant V. \tag{8}$$

Since the common shares must have a non-negative value, the value of the outstanding bonds cannot exceed the aggregate value of the outstanding bonds and stocks. Setting $V=0$ in (8), and recognizing that the bond value must also be non-negative, the bond value corresponding to a zero firm value is

$$W(0,t)=0. \tag{9}$$

A further upper bound on W may be obtained by noting that the returns on the convertible bond are stochastically dominated in the first degree by the returns on a portfolio consisting of an equivalent straight bond and the maximum number of shares into which the bond may be converted over its remaining life. Hence W must satisfy

$$W(V,t)\leqslant B(V,t)+z^*(t)V, \tag{10}$$

where $z^*(t)$ is the maximum value of $z(\tau)$ for $\tau=(t,T)$.

The investor's conversion option ensures that

$$W(V,t) \geqslant C(V,t) = z(t)V. \tag{11}$$

(ii) The maturity value condition, corresponding to $t = T$, is:

$$W(V,t) = \begin{cases} z(T)V, & z(T)V \geqslant 1000 \\ 1000, & 1000l \leqslant V \leqslant 1000/z(T) \\ V/l, & V \leqslant 1000l \end{cases} \tag{12}$$

The above boundary condition reflects the fact that at maturity the bondholder receives the conversion value of the bond, $z(T)V$, if this exceeds the par value of the bond; he receives the par value if this exceeds the conversion value and if the par value of the outstanding bonds is less than the aggregate value of the firm; he receives a proportionate share of the value of the firm if this falls short of the par value.

(iii) When the bond is callable, the call price constraint is

$$W(V,t) \leqslant CP(t). \tag{13}$$

This follows from Lemma 2.

(iv) When the bond is not currently callable, the limiting firm value condition is:

$$\lim_{V \to \infty} W_v(V,t) = z(t). \tag{14}$$

For sufficiently high values of V the risk of default in the bond payments becomes negligible. The bond may then be regarded as a warrant to buy a fraction $z(t)$ of the firm with an exercise price equal to the present value of the riskless debt payments. (14) then follows from Merton's [6] demonstration of the corresponding proposition for a warrant.

(v) On the date of a dividend or an adverse change in conversion terms, the conversion condition is

$$W(V,t^-) = \max[W(V-D,t^+), z(t^-)V] \tag{15}$$

Where t^- denotes the time immediately before the event and t^+ the following instant. Equation (15) allows for the investor's right to convert immediately prior to the event.

(vi) On a coupon date when the bond is not currently callable

$$W(V,t^-) = W(V-I,t^+) + i. \tag{16}$$

The pre-coupon value is equal to the post-coupon value plus the value of the coupon.

(vii) On a coupon date when the bond is currently callable

$$W(V,t^-) = \min[W(V-I,t^+) + i, CP(t^-)]. \tag{17}$$

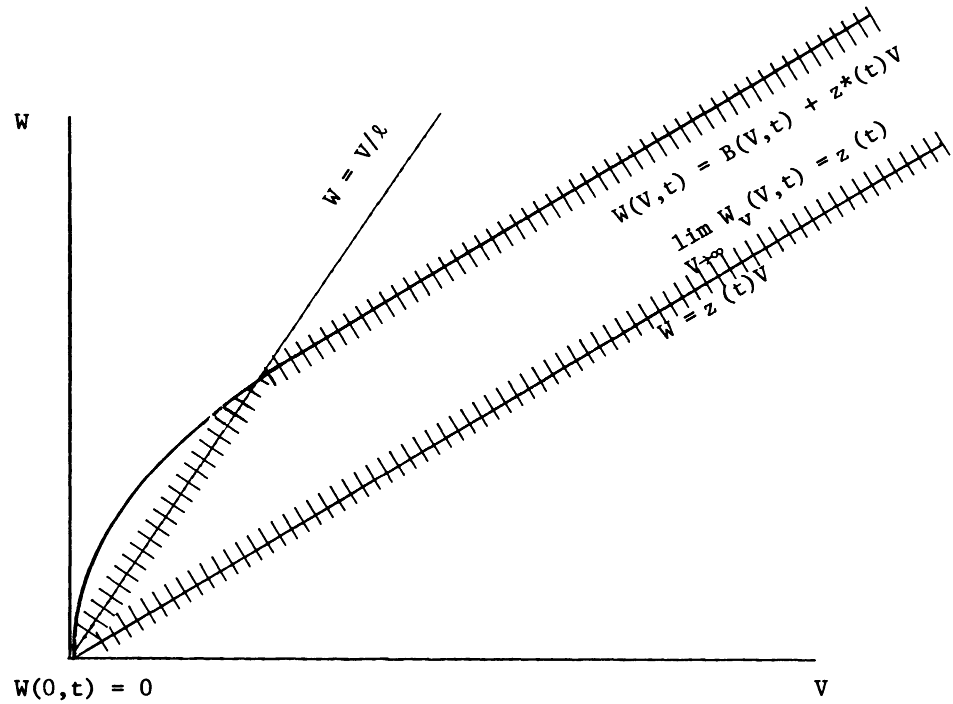

FIGURE 1. Bond Not Currently Callable

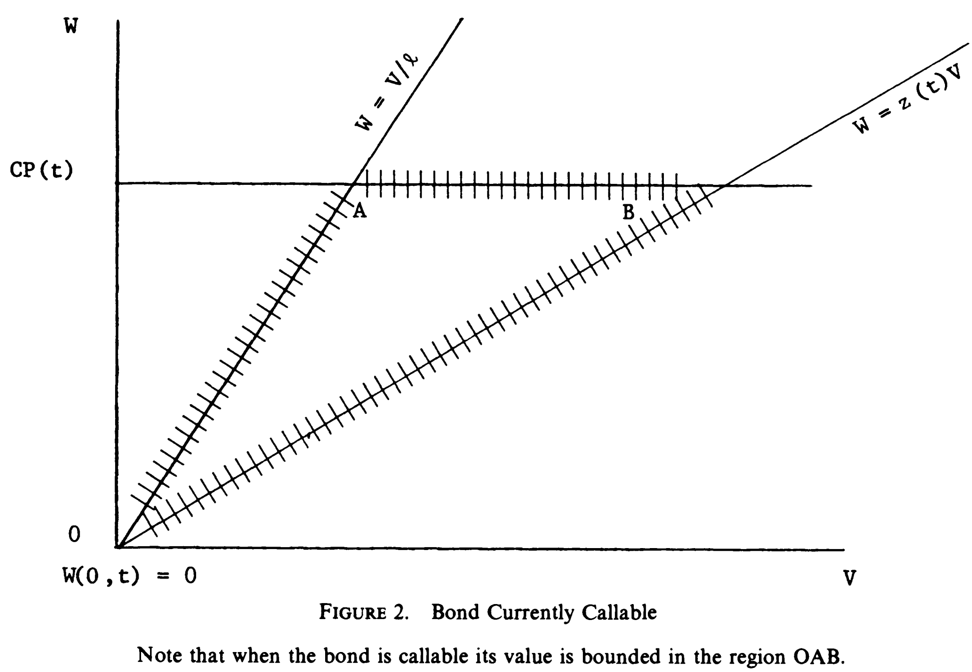

FIGURE 2. Bond Currently Callable

Note that when the bond is callable its value is bounded in the region OAB. The upper bound (10) may cross OA above or below A.

This follows from the condition

$$W(V,t^-)=\min[W(V-I,t^+)+I, VIC(V,t^-)], \tag{18}$$

together with the implication of Lemma 2 that $VIC(V,t)=CP(t)$ when the bond is called if an optimal call strategy is followed. Equation (18) itself follows from the observation that the firm's optimal strategy is to call the bond if the post-coupon uncalled value plus the coupon exceeds the value if called; by thus minimizing the value of the bond, the value of the equity is maximized, since the sum of the values is equal to the exogenously determined value of the firm.

Not all of the above boundary conditions need to be explicitly taken into account in the solution of the differential equation. The arbitrage condition (8) is automatically satisfied by the model. By Lemma 1, the conversion option condition (11) is automatically satisfied except at dividend or conversion term change dates when it is incorporated in (15).

The boundary conditions which must always be satisfied by the value of the bond when it is non-callable and callable are shown in Figs. 1 and 2 respectively.

IV. The Solution Algorithm

Since there exists no known analytical solution to the differential equation (7) subject to the boundary conditions discussed above, it is necessary to resort to numerical methods to solve the equation. For this purpose it is more convenient to employ the variable τ, time to maturity, instead of the variable t, calendar time. (7) can then be re-written as

$$\tfrac{1}{2}\sigma^2V^2W_{vv}+rVW_v-rW-W_\tau=0. \tag{7'}$$

Then, by writing finite differences instead of partial derivatives in (7′), the differential equation can be approximated by[4]

$$a_iW_{i-1,j}+b_iW_{i,j}+c_iW_{i+1,j}=W_{i,j-1}, \qquad i=1,\ldots,(n-1), \quad j=1,\ldots,m, \tag{19}$$

where $a_i=\frac{1}{2}rki-\frac{1}{2}\sigma^2ki^2$, $b_i=1+rk+\sigma^2ki^2$, $c_i=-\frac{1}{2}rki-\frac{1}{2}\sigma^2ki^2$

$$W(V,\tau)=W(V_i,\tau_j)=W(ih,jk)=W_{i,j}$$

The symbols h and k are the discrete increments in the value of the firm and in time to maturity respectively. By reducing these step sizes, any desired degree of accuracy in the solution can be achieved, but at the expense of increased computational cost. The symbols n and m represent the number of steps in the time dimension and the firm value dimension respectively; the former is chosen to correspond to the maturity of the bond under consideration, while the latter must be sufficiently large for the limiting firm value condition (14) to be well approximated at the maximum firm value considered.

4. See McCracken and Dorn [5] for a detailed explanation of the solution procedure.

With no loss of generality, it is assumed that $W(V,\tau)$ is the value of all the convertible bonds outstanding, convertible into a fraction z of the firm's shares.[5] Then the maturity value condition, (12), may be written in finite difference form as

$$W_{i,0}=\begin{cases} zV=zhi, & \text{for} \quad zhi \geqslant P \\ P & \text{for} \quad P \leqslant hi \leqslant P/z \\ V=hi & \text{for} \quad hi \leqslant P \end{cases} \tag{20}$$

where P is the par value plus accrued interest at maturity of the convertible bonds.

At any time prior to maturity, the zero firm value condition (9) applies and is written as:

$$W_{0,j}=0, \qquad j=0,1,\ldots,m. \tag{21}$$

When the bond is not currently callable, the limiting firm value condition (14) applies and is approximated by

$$\frac{W_{n,j}-W_{n-1,j}}{h}=z. \tag{22}$$

For any given value of j, (19) constitutes a set of $(n-1)$ linear equations in the $(n+1)$ unknowns, $W_{i,j}$ $(i=0,1,\ldots,n)$. The remaining two equations come from the boundary conditions (21) and (22). The resulting set of $(n+1)$ linear equations enable us to solve for $W_{i,j}$ in terms of $W_{i,j-1}$. Since $W_{i,0}$ $(i=0,1,\ldots,n)$ is given by (20), the whole set of $W_{i,j}$ may be generated by repeated solution of this set of equations, taking into account the boundary conditions imposed by the call and conversion options to be discussed below.

When the bond is currently callable, the limiting firm value condition (14) is replaced by the call price constraint (13), which, in the notation of this section, is written as

$$W_{i,j} \leqslant CP_j. \tag{23}$$

This boundary condition is taken into account by an iterative procedure described below. First, observe that since $W_{0,j}=0$ from (21), the matrix of coefficients in the system (19) is tridiagonal, having zeros everywhere except on the main diagonal and the two adjacent diagonals. Then, by successive subtraction of each equation from a suitable multiple of the succeeding one, the system may be transformed into the simpler one

$$e_i W_{i,j}+f_i W_{i+1,j}=g_i, \qquad i=1,\ldots,(n-1), \tag{24}$$

where e_i, f_i, g_i are the coefficients of the transformed system.

Note that on account of the boundary conditions provided by call and conversion $W_{i,j}$ is undefined for $i>q$ where

$$q=CP_j/zh.$$

The symbol q corresponds to the value of the firm for which the conversion value of the bonds is equal to the call price. Therefore, when the bond is callable, the system (24) is reduced to $(q-1)$ equations $(i=1,\ldots,q-1)$. The iterative procedure

5. For clarity of presentation we omit the dependence of z on time to maturity, τ.

is as follows. Set $W_{q,j}=CP_j$. Solve equation $(q-1)$ of (24) for $W_{q-1,j}$. If $W_{q-1,j} > CP_j$ set $W_{q-1,j}$ equal to CP_j and solve equation $(q-2)$ for $W_{q-2,j}$. If $W_{q-2,j} > CP_j$ set $W_{q-2,j}=CP_j$. This process is continued until a set of $W_{i,j}$ is obtained which satisfy the boundary condition $W_{i,j} \leqslant CP_j$, and the differential equation. The value of $i(i=p)$ for which $W_{p,j}=CP_j$ corresponds to the value of the firm at which the bonds should be called, and $W_{i,j}$ is undefined for $i>p$.

On a dividend date, j_D, the conversion option gives rise to the boundary condition (15), which can be written as

$$W_{i,j_D}=\begin{cases} W_{i-D/h,j_D} & \text{for} \quad W_{i-D/h,j_D} \geqslant zV=zih \\ zih & \text{for} \quad W_{i-D/h,j_D} \leqslant zih \end{cases}$$

On a coupon date, j_c, the boundary condition (16) can be written as

$$W_{i,j_c}=W_{i-I/h,j_c}+I \tag{26}$$

if the bond is not currently callable, while if it is callable, (17) applies, and this can be written as

$$W_{i,j_c}=\begin{cases} W_{i-I/h,j_c}+I & \text{for} \quad W_{i-I/h,j_c}+I \leqslant CP_{j_c} \\ CP_{j_c} & \text{for} \quad W_{i-I/h,j_c}+I > CP_{j_c} \end{cases}$$

V. Comparative Statics: Some Numerical Results

This section reports the effects of variation in selected parameters on the relationship between the value of the convertible bond and the value of the firm. The parameters of the basic example, from which deviations are considered in the following examples, are given in Table 1.

TABLE 1

Data for Basic Example

Par Value of Bond	40
Semi-Annual Coupon	1.0
Quarterly Dividend	1.0
Convertible into 10% of the shares outstanding after conversion	
Firm Variance Rate	.001 per month
Risk Free Rate	.005 per month
Call terms:	non callable for 5 years
	callable at 43 for next 5 years*
	callable at 42 for next 5 years*
	callable at 41 for last 5 years*

*plus accrued interest.

(i) *Time to Maturity*

Figure 3 shows the relationship that exists at the time the bond is issued ($T=20$). At issue, for firm values above 150, the relationship corresponds closely to that derived from intuition and casual empiricism [2]; that is to say, the bond value reflects a premium above conversion value, and for lower values of the firm shows the influence of the "bond value floor" or straight debt value. As many investors in convertible bonds have discovered to their chagrin, this floor value is itself variable, and for sufficiently low values of the firm declines rapidly, reflecting the possibility that the firm will actually default on the bond. As the bond approaches closer to maturity, the left hand section of the curve shifts further to the left, approaching the dotted line along which the bond value is equal to the firm value; this is a consequence of the fact that at low firm values the probability of default is high, and in the event of default the bondholders acquire the assets of the firm. On the other hand, with decreasing time to maturity the right hand section of the curve shifts further to the right, approaching the dotted line which represents the conversion value of the bond. This reduction in the conversion premium corresponds to that observed with warrants as time to maturity decreases.

Figure 4 shows the relationship which obtains at $T=15$ when the bond is callable at 43; in this example the curve passes through the point at which the conversion value is equal to the call price. Since the bond is called as soon as it reaches the call price, this is the maximum value it attains.

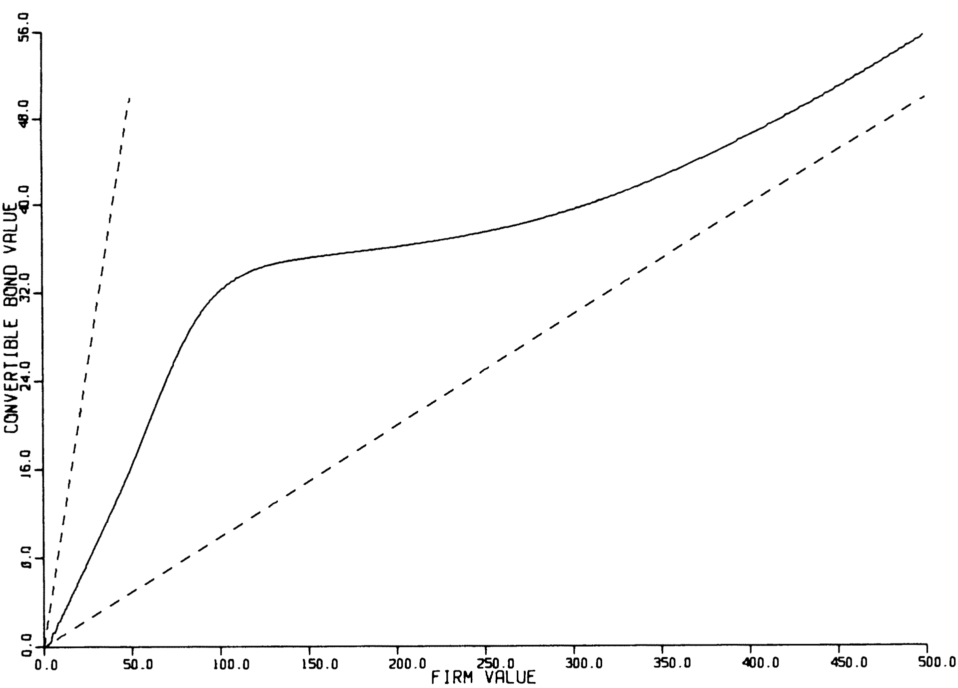

FIGURE 3. Bond Value at Time of Issue

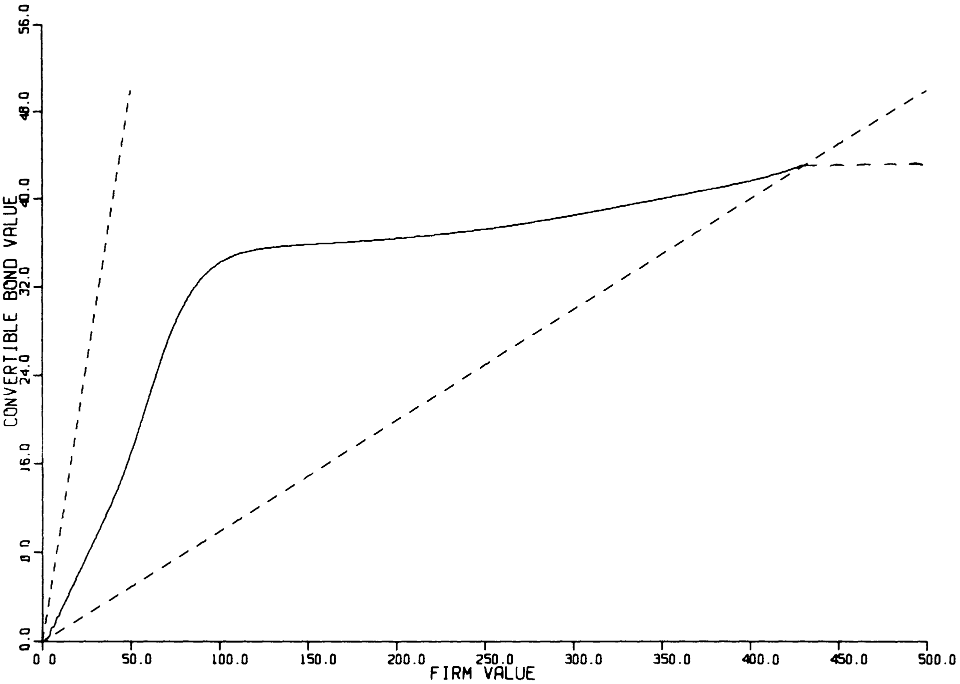

FIGURE 4. Bond Value at Time of First Call

(ii) *Dividend Payments*

Since convertible bonds, like warrants, are not protected against dividend payments by the firm, the effect of a higher dividend payment is to reduce the value of the bond. Figure 5 illustrates this effect at $T=20$ for three different values of the dividend. The dividend actually has two distinct effects on the value of the convertible bond. First, it affects the straight debt value of the bond by increasing the probability of default and by reducing the assets available for the bondholders in the event of default. This is clearly visible in the relationships on the left hand side of the figure. Secondly, when the probability of default is small (i.e. for large values of V) the conversion premium is reduced. This latter effect is much smaller for two reasons: since the bond is callable at $T=15$ the convertible bondholder is foregoing only 5 years of dividends; in addition, the right of the bondholder to convert limits the losses than can be imposed upon him by more generous dividend payments.

The convexity of the curves for $D=1.00$ and $D=2.00$ at extremely low values of V reflects a quirk of this model, which assumes that the same dividend is paid whatever the value of the firm.[6] At sufficiently low firm values it will actually pay the bondholders to convert prior to a dividend before the assets providing security for their bonds are paid out from under them in the form of dividends. Realistically

6. The apparent discontinuity in the slope at extremely low values of V is a product of the discreteness of the solution procedure.

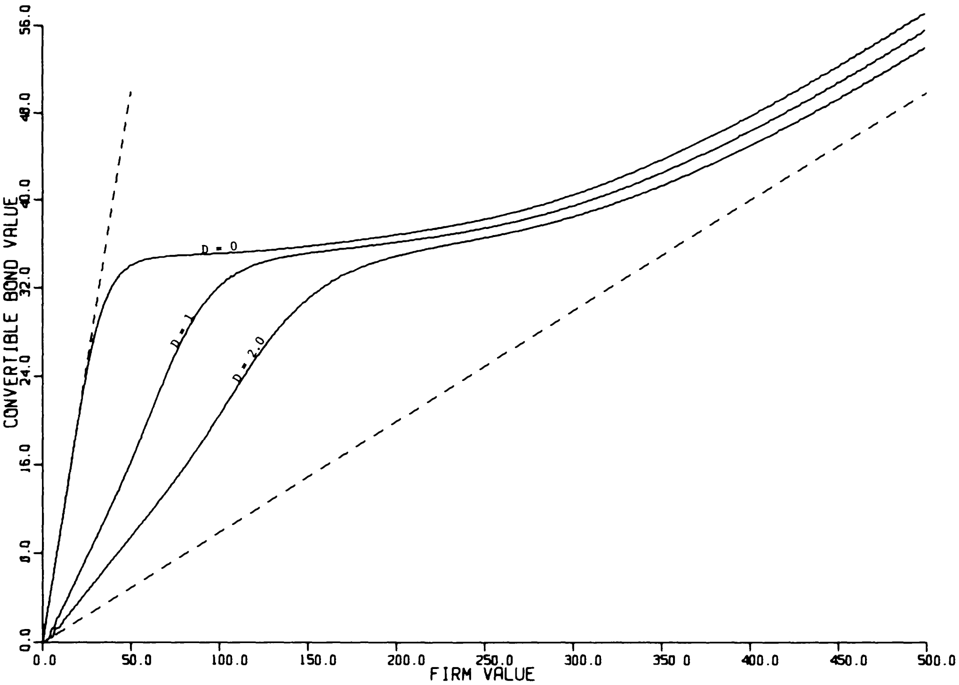

FIGURE 5. Bond Value at Time of Issue: Effect of Dividend Payment

of course, indenture provisions would force a cessation of dividend payments long before this critical stage were reached.

(iii) *Variance Rates*

Figure 6 illustrates the relationships that obtain at time of issue for three different variance rates. As the figure indicates, an increase in the variance rate may increase or decrease the value of the bond. First, at very low firm values where default is almost certain whatever the variance rate, there is no effect. At intermediate firm values, an increase in the variance rate both raises the expected loss through default as for a straight bond, and increases the expected gain from conversion. While the former effect predominates for firm values between about 50.0 and 200.0, for higher firm values the debt is almost risk free, and the convertible bond is essentially equivalent to a riskless straight bond plus a warrant with an exercise price equal to the straight bond value. It is then well known that higher variance rates will lead to higher warrant values, and this effect is apparent for high values of V.

(iv) *Call Dates*

Figure 7 illustrates the effect of varying the date of first call on the value of the bond at time of issue. As would be expected, this has no effect on the value of the bond for low values of the firm where the prospect of conversion is remote in any event. At higher firm values, the bond value declines with the call deferral period so that at the first call date the upper portion of the curve is coincident with the dotted line representing the conversion value.

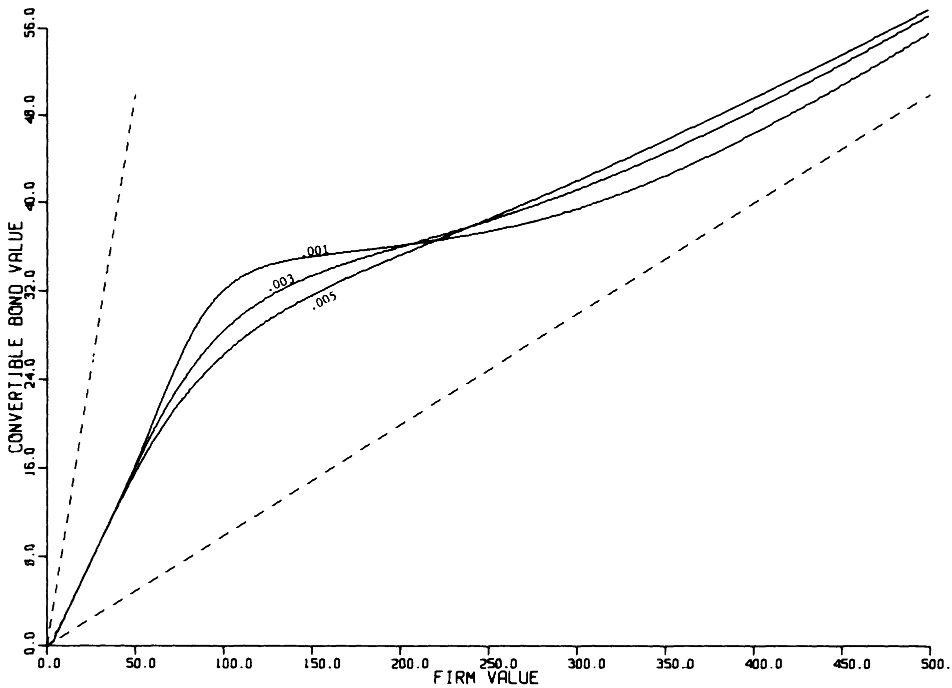

FIGURE 6. Bond Value at Time of Issue: Effect of Variance Rate

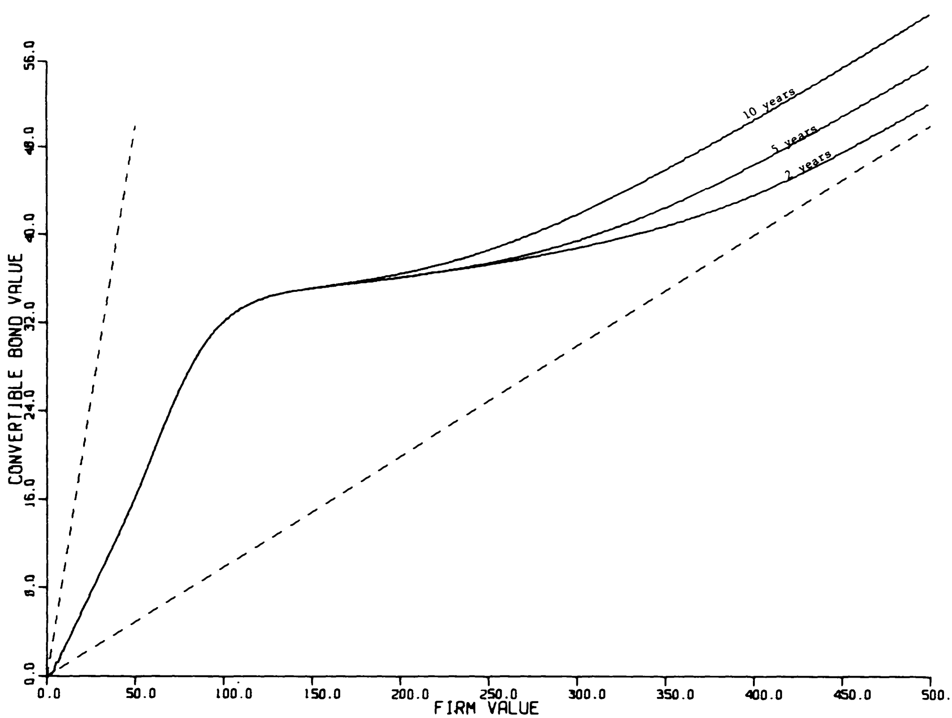

FIGURE 7. Bond Value at Time of Issue: Effect of Call Deferral Period

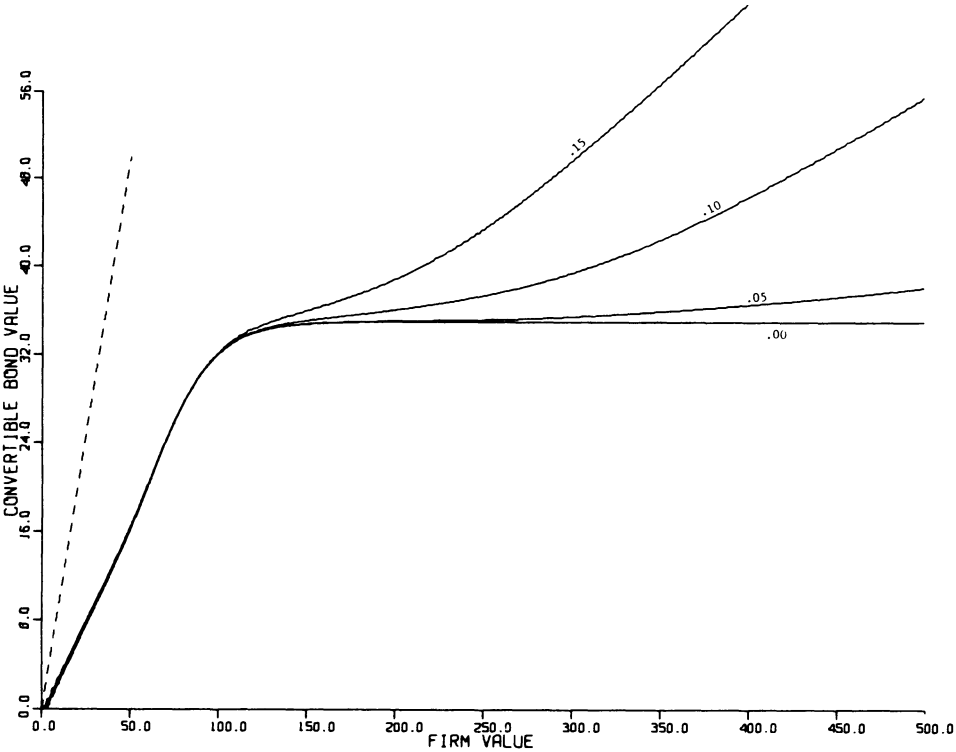

FIGURE 8. Bond Value at Time of Issue: Effect of Conversion Ratio

(v) *Conversion Terms*

In Figure 8 the fraction of the firm's shares into which the bond is convertible is varied. The effect of this is to change the limiting slope of the relationship for high values of V.

The extreme case in which the conversion ratio is zero represents of course a straight bond, so that the vertical difference between the lowest curve and any of the others corresponds to the value of the conversion privilege.

While this paper is ostensibly concerned only with convertible bonds, it should be apparent that the analysis captures many of the most important aspects of risky coupon-paying straight bonds, and thus represents a significant generalization of Merton's [6] path breaking analysis of risky bonds, which was restricted only to discount bonds. A subsequent paper will treat the problem of valuing straight coupon bonds with risk of default, and examine in detail the effects of such common provisions as sinking funds, call privileges, and indenture restrictions on dividend payments.

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