Honours Project Proposal:
Valuation of Convertible Bonds with Credit Risk

David Naylor

Supervisor: Dr. Thomas McWalter

Programme in Advanced Mathematics of Finance,
University of the Witwatersrand,
Johannesburg.

August 14, 2012

Abstract

A convertible bond is a complex derivative that cannot be priced as a simple combination of bond and stock components. Convertible bonds can be broken down as a bond with two embedded options (a put option for the investor and a call option for the issuer) and an option to convert the bond into stock. Due to the multiple continuous options, the pricing of the convertible bond is path dependent.

This research project explores and implements a binary tree and finite difference scheme to price the convertible bond, taking into account credit risk.

1 Introduction

This research project aims to price convertible bonds based on the paper by Ayache et al. [3].

Convertible bonds are a hybrid instrument available on financial markets. The convertible bond is an instrument that is similar to a normal bond, except the holder has the option to convert the bond into a specified number of shares. The convertible bond typically also has embedded options whereby the issuer may buy back the convertible bond for a specified price and whereby the investor can force the issue to repurchase the convertible bond. In the event of a default by the issuer the bond could have partial recovery or total default.

More rigorously, as described by Ammann et al. [1], the convertible bond can be specified as having:

- A nominal value of N. Without loss of generality it is assumed the convertible bond always matures at par.
- A maturity at time T.
- A continuous conversion provision held by the investor to convert the bond into κ_t shares (for a value of $\kappa_t S_t$). Conversion may only happen in time set Ω_c (typically $T \in \Omega_c$).
- A continuous call provision held by the issuer to buy-back the bond for K_t. The investor may opt to convert the bond instead of received the buy-back value. The call may only happen in the time set Ω_k.
- A continuous put provision held by the investor to force early conversion of the bond for P_t. The put may only happen in the time set Ω_p.

As assumed by Ayache et al. [3], the probably of default in time set $[t, t + dt]$ is $p\,dt$ where $p(S, t)$ is a deterministic hazard rate. The following assumptions will be made about the behaviour of the bond in the event of default:

- The bond has recovery ratio of R, for a recovery value of RN.
- The share price reduces in value by η so that the share price after default is $S^+ = S^-(1 - \eta)$ where S^- is the share price immediately before default.
- The investor has an option of either receiving:
    - RN, or
    - $\kappa_t S_t(1 - \eta)$

The stock price is assumed to follow a standard Wiener process of:

$$dS = \mu S dt + \sigma S dz$$

with drift rate μ, volatility of σ and Wiener process increment of dz [3].

2 Literature Review

Ingersoll [10] started the literature of pricing a convertible bond with extensions from Brennan and Schwartz [8] and Brennan and Schwartz [7]. The original approach was to treat the bond and equity as components of the issuer's value and to treat default as when the issuer's value drops below a point where it can no longer meet its financial obligations. An overview of this type of approach is provided by Nyborg [13] and criticisms are addressed by Jarrow and Turnbull [11]. The main problems with this model is that the issuer's value is not directly observable, difficult to parameterise and all senior debt to the convertible bond also needs to be priced.

A second approach was to price the convertible bond based on the issuer's stock price [12]. A refined method, called "reduced form", treats default as a discrete jump in time. The probability of the loss jump over a short period of time is described by a hazard rate.

Also some Monte Carlo based pricing methods have been considered as proposed by Bossaerts [6] with Garcia [9] providing an optimisation approach to handle optimal early exercise of the American options. Further improvements to the pricing methods were done by Ammann et al. [1].

Other methods used for pricing include a finite element method by Barone-Adesi et al. [4] and a binomial tree method by Takahashi et al. [15] and Ayache et al. [2].

3 Research Method and Aims

This research project aims to price convertible bonds with credit risk using a finite difference scheme to solve the difference equation numerically.

Initially a binomial lattice will be implemented then a finite difference scheme will be implemented, in Python 2.7. Ayache et al. [3] used a Crank-Nicolson scheme with modifications as described in Rannacher [14] for the finite difference scheme. The BDF finite difference scheme [5] was proposed as an alternative finite difference scheme.

A best effort will include pricing subject to credit risk and implementing a Monte Carlo simulation to compare pricing models. Ammann et al. [1] describes one such Monte Carlo based model.

The use of Octave (a near Matlab compatible language), as a language of implementing the numerical solution, was rejected as it does not support object-orientation via classes and Matlab was rejected as it is a proprietary platform.

References

[1] M. Ammann, A. Kind, and C. Wilde, *Simulation-based pricing of convertible bonds*, Journal of Empirical Finance **15** (2007), 310–331.

[2] E. Ayache, P. A. Forsyth, and K. R. Vertzal, *Next generation models for convertible bonds with credit risk*, Wilmott Magazine (2002), 68–77.

[3] E. Ayache, P. A. Forsyth, and K. R. Vertzal, *The valuation of convertible bonds with credit risk*, Journal of Derivatives **11** (2003), 9–44.

[4] G. Barone-Adesi, A. Bermudez, and J. Hatgioannides, *Two-factor convertible bonds valuation using the method of characteristics/finite elements*, Journal of Economic Dynamics and Control **27** (2003), 1801–1831.

[5] J. Becker, *A second order backward difference method with variable steps for a parabolic problem*, BIT Numerical Mathematics **38** (1998), 644–662.

[6] P. Bossaerts, *Simulation estimator of optimal early exercise*, 1989.

[7] M. J. Brennan and E. S. Schwartz, *Analyzing convertible bonds*, Journal of Financial and Quantitative Analysis **32** (1977), 1699–1715.

[8] M. J. Brennan and E. S. Schwartz, *Convertible bonds: Valuation and optimal strategies for call and conversion*, Journal of Finance **15** (1980), 907–929.

[9] D. Garcia, *Convergence and biases of monte carlo estimates of American option pricing using a parametric exercise rule*, Journal of Economic Dynamics and Control **26** (2003), 1855–1879.

[10] J. E. Ingersoll, *A contingent-claims valuation of convertible securities*, Journal of Financial Economics **4** (1977), 289–322.

[11] R. A. Jarrow and S. M. Turnbull, *Pricing derivatives on financial securities subject to credit risk*, Journal of Finance **50** (1995), 789–819.

[12] J. J. McConnell and E. S. Schwartz, *Lyon taming*, Journal of Finance **41** (1986), 561–576.

[13] K. G. Nyborg, *The use and pricing of convertible bonds*, Applied Mathematical Finance **3** (1996), 167–190.

[14] R. Rannacher, *Finite element solution of diffusion problems with irregular data*, Numerische Mathematik **43** (1984), 309–327.

[15] A. Takahashi, T. Kobayashi, and N. Nakagawa, *Pricing convertible bonds with default risk*, Journal of Fixed Income **11** (2001), 20–29.