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The use and pricing of convertible bonds

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This paper provides an overview of the main results of the literature on pricing convertible bonds. It covers simple convertible bonds which are non-callable and can be converted only at maturity as well as more complicated callable and puttable convertible bonds under stochastic interest rates. The paper also reviews the main results in the literature on why firms issue convertible bonds. The two most often cited rationales for issuing convertible bonds – as delayed equity, and to sweeten debt – are discussed in the context of both asymmetric information and agency models of capital structure. Finally, the paper provides some thoughts on incorporating strategic issues into the pricing of convertible bonds.

Keywords: risky/risk-free assets, call and put features, debt pricing, convertible debt, adverse selection, moral hazard

1. Introduction

Convertible bonds are hybrid instruments, having characteristics of both debt and equity. Like straight bonds, convertible bonds are entitled to receive coupons and principal payments. However, convertible bondholders have the option to forgo these rights by converting their bonds into stock at a prespecified rate. In its simplest form, a convertible bond can be decomposed into a straight bond and a warrant. The value of the convertible bond can therefore be found by pricing its two constituent parts separately. Many convertible bonds have features whereby the issuer can call the convertibles before maturity. This adds some complexity to pricing convertible bonds as well as to understanding why they are issued.

The purpose of this paper is to discuss both how to price convertible bonds and why firms issue them. The pricing literature has developed in the standard context of contingent claims pricing with perfect markets. This has yielded powerful pricing results, but does not address why firms issue convertibles in the first place. Although convertibles are far from the most commonly issued corporate security, from 1973 to 1995 convertibles accounted for 5.52% of money raised by UK and Irish companies on the London Stock Exchange. In 1995, £3479.5 m of convertibles were issued by UK and Irish companies on the LSE.[1] While the perfect markets assumption utilized in the pricing literature would suggest that stock prices of firms that issue convertibles should be unaffected by the announcement of the convertible issue, the empirical evidence documents average abnormal returns (AAR) of −2.07% when firms announce issues of convertibles. In comparison, firms that announce issues of equity experience abnormal returns of −3.14% on the average; while firms that announce straight debt issues have a statistically insignificant AAR of −0.26% (Smith,

[1] London Stock Exchange, Quality of Markets Faxback Service, Table B4.

1986). Given that share prices react differently to the choice of external financing, one might reasonably expect that understanding why firms issue convertibles might yield additional insights into their pricing. In Section 4, I will suggest ways in which the pricing of convertibles could incorporate recent insights about why firms issue them.

The remainder of this paper is organized as follows. Section 2 discusses pricing of simple convertible bonds. Section 3 addresses pricing of callable and puttable bonds under stochastic interest rates. Section 4 examines why firms issue convertibles, and Section 5 concludes.

2. Pricing simple convertible bonds

In this section, I will address how to price the most basic convertible bonds: ones that are non-callable and can be converted only at maturity. I will provide conditions for which there is a closed form solution for the price of a convertible bond when there is senior debt and the common stock pays dividends. Interest rates are non-stochastic. I consider a variety of cases and the final pricing formula is given by equation (18) at the end of this section.

I will start by developing the familiar result that a convertible bond can be decomposed into a straight bond plus a warrant (see Ingersoll, 1977a or Cox and Rubinstein, 1985). This insight is useful in pricing convertible bonds and also for understanding why firms might issue them. For pricing purposes, throughout this section, I will assume that markets are perfect and dynamically complete so that the standard contingent claims pricing approach can be used. This involves an assumption that markets are extremely liquid – there are no transactions costs – and there are no short-sale restrictions.

2.1 *Basic issues*

To illustrate the basic issues of convertible bond valuation, I will begin by considering a firm whose capital structure consists of common shares and non-callable convertibles. Later, I will allow for other debt in the capital structure.

The number of common shares will be denoted N. In terms of valuing the convertibles, the key parameters are the face value and coupons of the convertibles and the fraction of shares that convertible bondholders will own upon conversion, i.e. the *dilution*.

2.1.1 *Face value and coupons*

The number of convertibles will be denoted by M, and the face value per convertible by f. The total face value of the convertible issue is $F \equiv Mf$. The coupons of the convertible could either be fixed or floating. In general, one can write the coupon paid at time t as a function $c(t, \boldsymbol{X}_t)$, where $\boldsymbol{X}_t$ is a vector of state variables. In the case of fixed coupons, $c(\cdot)$ would be a function of t only. In the case of floating coupons, $c(\cdot)$ would be a function of both t and $\boldsymbol{X}_t$. In this section, I will assume that the term structure of interest rates is flat and non-stochastic so that coupons are fixed. Floating coupons will be considered in Section 3.

2.1.2 *Dilution*

Bondholders' right to convert into common stock can be expressed in terms of the *conversion ratio*,

r, the number of shares that each convertible bond can be converted into. Equivalently, the conversion right could be represented by the *conversion price*, the price at which the bonds are convertible into common stock. The conversion price can be expressed in terms of the conversion ratio as follows:

$$\text{conversion price} = \frac{\text{face value}}{\text{conversion ratio}} = \frac{F}{r}$$

If all the convertibles are converted, the total number of new shares will be Mr. The fraction of equity convertible bondholders possess if they convert is then

$$\lambda \equiv \frac{Mr}{N + Mr}$$

The parameter λ is the *dilution factor* of the convertible issue.[2] Convertible bonds are typically protected against reductions in their claim on equity which, in the absence of antidilution clauses, could result from issuings of new shares below market prices, for example through a rights issue.

2.1.3 *Value at maturity*

At maturity, if convertible bondholders convert, they forgo the principal repayment. Hence, convertible bondholders will only wish to convert if the *conversion value* of the convertibles is larger than their face value. Letting V_T denote the value of the firm at maturity after the last coupon has been paid, it will thus be in convertible bondholders' interest to convert if and only if

$$\lambda V_T > F$$

Hence, letting C_T denote the value of the convertible bond at maturity,

$$C_T = \begin{cases} V_T & \text{if } V_T < F \\ F & \text{if } F < V_T < F/\lambda \\ \lambda V_T & \text{if } V_T > F/\lambda \end{cases} \tag{1}$$

This is illustrated in Fig. 1. From the figure it can be seen that the payoff to the convertible bond can be decomposed into: (i) a straight bond with face value F and coupons $c(t)$; and (ii) a warrant component – specifically, the fraction λ of a call option on the assets of the firm less the coupons of the convertible. The call has exercise price F/λ and time to maturity, T.

2.1.4 *Current value*

It follows that the current price of the convertible, C_0, is given by

$$\begin{aligned} C_0 &= \text{value of straight bond} + \text{value of warrant} \\ &= B(V, F, T; c(t)) + \lambda C(V, T, F/\lambda; c(t)) \end{aligned} \tag{2}$$

where V denotes the current value of the firm's assets, $B(V, F, T; c(t))$ refers to the value of a

[2] Some authors call N/M the dilution of the convertibles.

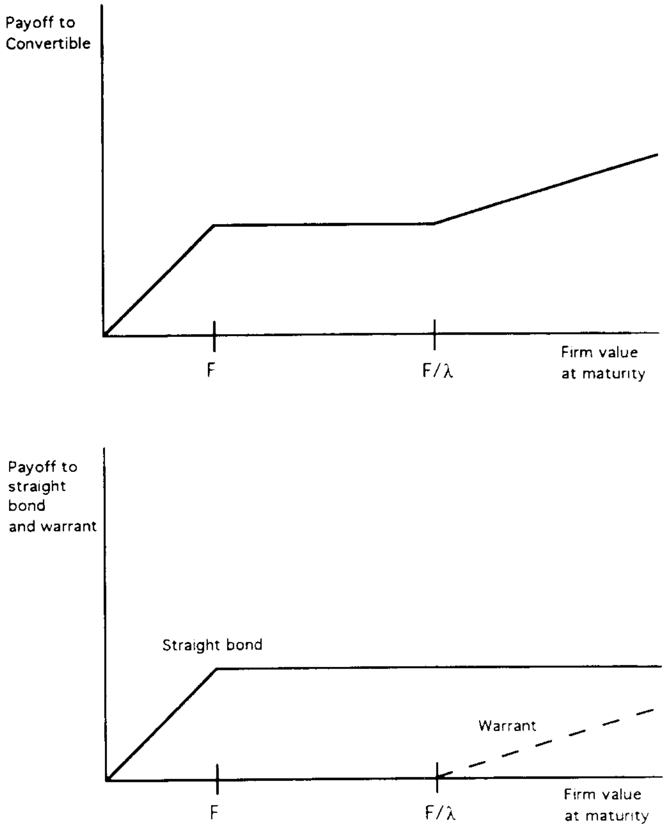

Fig. 1. Payoff and decomposition of convertible bond.

straight bond with face value F, coupons $c(t)$, and time to maturity T; and $C(V, T, F/\lambda; c(t))$ refers to a call on the firm's assets less the coupons that will be paid out between now and maturity, essentially a call on the firm's equity. The call has time to maturity T and exercise price F/λ.

Equation (2) forms the basis of valuing a European, non-callable convertible. However, there is still scope for different valuations depending on the assumptions one makes about the stochastic processes governing interest rates and the value of the firm.

I will assume in this section that the risk-free rate is non-stochastic and equals r (continuously compounded), but this will be relaxed in Section 3. With respect to the value of the firm, I will work within the standard confines of geometric Brownian motion. Within this framework, different valuations can arise depending on how coupon payments are financed. This is part of a more general issue regarding how to treat cash outflows from the firm. If coupons are financed out of the firm's assets, it is critical whether these assets are risk-free or risky. In the former case, the Black–Scholes (1973) model can be used to price the convertible. In the latter case, one can use standard binomial methods. I discuss these two cases below. I also consider the case that coupons are financed by contributions from common stock holders.

2.2 *Pricing when cash outflows are financed by selling risk-free assets*

Here I will assume that the coupons are financed by selling risk-free assets.[3] One can think of this as the firm setting aside a sum in escrow which will be just big enough to finance coupon payments. The remaining portion of the firm's assets will be referred to for convenience as the equity of the firm (despite the fact that this includes the present value of the principal of the convertible). Thus defined, the equity is assumed to follow geometric Brownian motion with volatility σ. Letting V_r denote the value of the risky assets (the equity) and V_f denote the value of the risk-free assets (used to pay coupons), the value of the firm equals $V = V_r + V_f$. The critical assumption here is that coupon payments do not affect the distribution of firm value at maturity since the stochastic process does not work on the assets used to pay coupons. Formally, the value of the risky assets evolves according to the process

$$\mathrm{d}V_r = \mu V_r \mathrm{d}t + \sigma V_r \mathrm{d}z \tag{3}$$

where dz is a Wiener process. The value of the risk-free assets evolves according to

$$\mathrm{d}V_f = (r - c(t))V_f \mathrm{d}t \tag{4}$$

where the initial value of the risk-free asset is $V_f(0) = \mathrm{PV}$ (coupons). Since the coupons are risk-free, $V_f = \mathrm{PV}$ (coupons) at all times.

Since the risk-free asset is just big enough to pay off the coupons of the convertible, the Black–Scholes model can be used to value the warrant component of the convertible as follows:

$$W = \lambda\left[(V - \mathrm{PV(coupons)})N(d_1) - \frac{F}{\lambda}\mathrm{e}^{-rT}N(d_2)\right] \tag{5}$$

where

$$d_1 = \frac{\log[(V - \mathrm{PV(coupons)})/(\frac{F}{\lambda}\mathrm{e}^{-rT})}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t} \tag{6}$$

and

$$d_2 = \frac{\log[(V - \mathrm{PV(coupons)})/(\frac{F}{\lambda}\mathrm{e}^{-rT})}{\sigma\sqrt{t}} - \frac{1}{2}\sigma\sqrt{t} \tag{7}$$

and $N(\cdot)$ is the cumulative normal distribution function.

As seen in (5), to value the warrant, it is necessary to value the coupons. Since they have been assumed to be risk-free, they should be valued by discounting at the risk-free rate.[4]

To be consistent, the coupons should also be discounted at the risk-free rate when valuing the straight bond component of the convertible. The principal repayment can be valued by discounting at the appropriate rate. Alternatively, valuation can be based on the observation that the principal being long is equivalent to being long the firm's equity and short a call on the equity, with exercise

[3] This is based on Rubinstein's (1983) diffusion model.

[4] I have assumed here that the term structure is flat. If it were not, coupons should be discounted at the appropriate risk-free rates as calculated from the term structure.

price F and time to maturity T. Thus the value of the straight bond component of the convertible equals

$$\begin{aligned}
B &= \mathrm{PV}(\text{coupons}) + \mathrm{PV}(\text{principal}) \\
&= \mathrm{PV}(\text{coupons}) + V - \mathrm{PV}(\text{coupons}) - C(V - \mathrm{PV}(\text{coupons}), T, F) \\
&= V - C(V - \mathrm{PV}(\text{coupons}), T, F) \\
&= V - [(V - \mathrm{PV}(\text{coupons}))N(d_3) - F\mathrm{e}^{-rT}N(d_4)] \qquad (8)
\end{aligned}$$

where

$$d_3 = \frac{\log[(V - \mathrm{PV}(\text{coupons}))/(F\mathrm{e}^{-rT})]}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t} \qquad (9)$$

and

$$d_4 = \frac{\log[(V - \mathrm{PV}(\text{coupons}))/F\mathrm{e}^{-rT})]}{\sigma\sqrt{t}} - \frac{1}{2}\sigma\sqrt{t} \qquad (10)$$

In practice, a criticism of this approach is that it might be difficult to estimate the volatility, which is necessary in order to value the call. However, it should be no more difficult to estimate the volatility than finding an appropriate discount rate since these are just two sides of the same coin. Once one knows the volatility, one can impute the discount rate, or vice versa. If, in practice, it is easier to calculate a correct discount rate to value the principal repayment, then one could use the implied volatility from (8) in valuing the warrant component of the convertible (assuming that the repayment of the principal is uncertain).

Combining the above valuations of the warrant and straight bond components results in the following value for the convertible bond:

$$C_0 = V - C_{BS}(V - \mathrm{PV}(\text{coupons}), T, F; \sigma) + \lambda C_{BS}(V - \mathrm{PV}(\text{coupons}), T, F/\lambda; \sigma) \qquad (11)$$

where C_{BS} is the Black–Scholes value for a call. Equation (11) expresses the value of the convertible as a function of the assets of the firm. However, this is not usually observable directly. It can be calculated by adding the market values of the firm's various securities, including the convertible bond which we want to value in the first place. Nevertheless, (11) can be used to verify the observed market values of the common stock and convertibles. Assuming that the combined value is correct, one can check whether stock and convertible prices are correct. Another view would be that the market price of the common stock is always correct since markets are efficient, but since convertibles are usually not as liquid as common stock, it might be difficult to observe the true value of the convertibles. In this case, (11) could be used to infer the value of the convertibles by substituting in $V = S + C$, where S is the market capitalization of the common stock. The value of the convertible bond can then be solved for iteratively.

2.2.1 *A variation where the principal is financed from risk-free asset sales*

It is worth recalling that in deriving the pricing equation (11), I have assumed that there is a stochastic process which works on all the firm's assets less the coupons. In some cases, the principal repayment may be considered risk-free and the stochastic process may be deemed to work only on the portion of assets which remain after the coupons and the principal have been taken out. In this case, all promised payments to the straight bond should be discounted at the risk-free rate. The warrant would now be a call on the firm's assets less the future coupons and principal repayment, with exercise price $F/\lambda - F$. This can be written

$$W = \lambda\left[(V - (\mathrm{PV(coupons)} + \mathrm{PV(principal)}))N(d_1) - \left(\frac{F}{\lambda} - F\right)e^{-rt}N(d_2)\right] \tag{12}$$

where d_1 and d_2 are defined in the obvious manner.

2.2.2 *Other debt in the capital structure*

If there is other debt, the value of the convertible will depend on when that other debt matures and its seniority. The simplest case is when the other debt matures at the same time as the convertible. As before, the warrant component of the convertible can be viewed as a call on the assets of the firm less coupons between now and maturity. Irrespective of the seniority of the other debt, convertible bondholders will convert if and only if

$$\lambda(V_T - P) > F$$

where P is the principal of the other debt. Hence the exercise price of the call is $F/\lambda + P$, and the value of the warrant component can be written as

$$W = \lambda C(V - \mathrm{PV(all\ coupons)}, T, F/\lambda + P) \tag{13}$$

where PV(all coupons) refers to all coupons paid between now and maturity.[5]

Unlike the value of the warrant component of the convertible, the value of the straight debt component depends on seniority. For example, if the other debt is senior, the straight debt component of the convertible has the value

$$B = \mathrm{PV(coupons)} + C(V - \mathrm{PV(all\ coupons)}, T, P) - C(V - \mathrm{PV(all\ coupons)}, T, P + F) \tag{14}$$

In contrast, if the other debt is junior, the straight debt component of the convertible has the value

$$B = V - \mathrm{PV(coupons\ of\ other\ debt)} - C(V - \mathrm{PV(all\ coupons)}, T, F) \tag{15}$$

If the other debt is short-term, so that it matures before the convertibles and will not be rolled over, the critical issue is how repayment of the short-term debt is financed. The issues are the same as with coupon payments. The general result that has been illustrated above is that if cash outgoings are financed by selling risk-free assets, the Black–Scholes model can be used to value convertible bonds (when the remainder of the assets follow geometric Brownian motion).

[5] If the other debt is risk-free, the stochastic process would only work on the assets less all coupons and the principal of the other debt. In this case the warrant component would be worth:

$$W = \lambda C(V - \mathrm{PV(all\ coupons)} - \mathrm{PV(principal\ of\ the\ other\ debt)}, T, F/\lambda).$$

If the other debt is long-term, so that it matures after the convertibles, then the convertibles will be converted if

$$\lambda E_T > F$$

where E_T is the value of the firm less the value of the long-term debt at maturity. In principle, valuing the convertibles now requires valuing the other debt also. As always, exact valuation depends on assumptions regarding underlying stochastic process(es). The most straightforward valuation would involve assuming that the equity of the firm (assets less coupons and the other debt) follows geometric Brownian motion. This will allow a valuation of the convertibles using the Black–Scholes model.

2.3 *Pricing when cash outflows are financed by selling risky assets*

In the previous subsection, the assets of the firm could be divided into two categories: risk-free assets from which coupons were paid and risky assets whose value followed geometric Brownian motion. In this subsection, all asset values follow geometric Brownian motion so that cash outgoings will affect the scale of the firm's risky investments. Formally, the value of the firm evolves according to the process

$$\mathrm{d}V = (\mu V - c(t))\mathrm{d}t + \sigma V \mathrm{d}z \tag{16}$$

Under this representation, the value of the firm at maturity of the convertible is not lognormally distributed and the Black–Scholes model does not apply. To value the convertible now, one can build a binomial tree.

Figures 2 and 3 illustrate the differences between the stochastic process governing firm value in this subsection (Fig. 3) from that in the previous subsection (Fig. 2) in terms of binomial trees. The figures are built around the following two-period binomial example. A firm has outstanding common stock and convertibles. The face value of the convertibles is 100, coupons of 10 are paid at dates 1 and 2, the dilution factor is 0.455. Thus the warrant component of the convertible is 0.455 of a call with exercise price 219.78 on the firm's assets after coupons have been paid. The current value of the firm is 240.

In Fig. 2, the firm's value is made up of 223.743 in a risky asset and 16.257 in a risk-free asset. The risky asset either moves up in value by 25% or down by 20%, and the risk-free rate is 15%. The firm's holding of the risk-free asset will be used to finance coupon payments. As seen in the figure, the warrant component of the convertible is in-the-money in the two highest states at date 2. Discounting the expected payoffs to the warrant under the risk-neutral measure gives a warrant value of 27.490. Also as seen in the figure, all promised payments to the straight bond component of the convertible will be paid with certainty and should therefore be discounted at the risk-free rate. Hence, the value of the straight debt component is 91.871. Adding the values of the warrant and straight bond components gives a total value of the convertible of 119.361.

In Fig. 3, the firm's value is wholly made up of 240 in a risky asset, which has the same volatility as the risky asset in Fig. 2. Coupon payments now reduce the scale of the investment in risky assets. As seen, the binomial tree in Fig. 3 does not recombine. It remains true, of course, that the convertible bond can be decomposed into a straight bond and a warrant component. Thus the basic

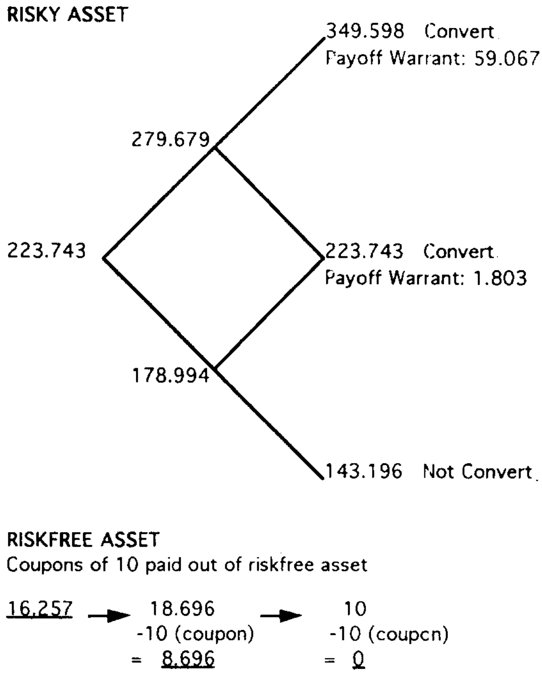

Fig. 2. Coupons are paid out of risk-free assets.

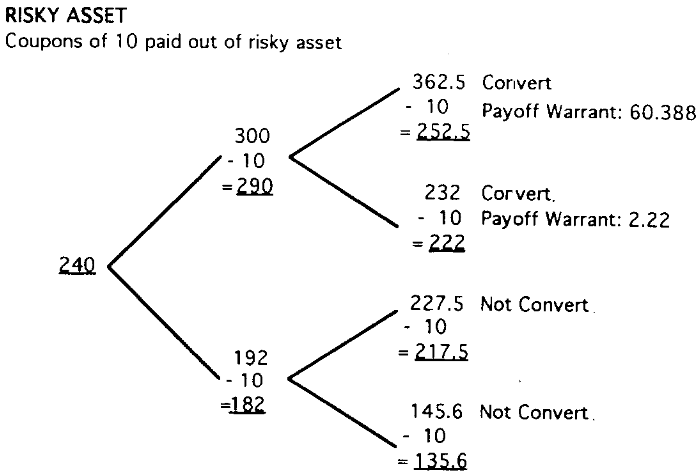

Fig. 3. Coupons are paid out of risky assets.

approach to valuation is the same as before. The straight bond component has a value of 91.871 as in Fig. 2. The warrant has a value of 27.754, yielding a total value for the convertible of 119.626.

The convertible is worth about 0.2% more in Fig. 3 than in Fig. 2 since a larger proportion of the firm's assets is invested in the risk assets (without affecting the riskiness of the straight debt component). This may not seem large when one considers estimation errors in computing the volatility. Obviously, keeping to the assumption that cash outflows can be financed out of risk-free assets has computational advantages. However, if the cash outgoings are likely to be large relative to the size of the firm, the choice of model may be critical. For example, the firm may have short-term debt which will not be rolled over.

If there is other debt in the capital structure, the same comments as earlier apply. If short-term debt will not be rolled over, the effect is the same as a large coupon payment.

2.4 *Pricing when cash outflows are financed by raising new capital*

When we write down a stochastic process for the value of the firm, we implicitly assume something about the firm's investment policy. If the firm has a policy of taking all positive NPV projects then the resulting stochastic process of the firm will be different than if the firm had another investment policy. For the valuation of convertible bonds, we are not so much interested in the stochastic process that the firm value follows as in the stochastic process followed by the firm's assets in place and growth opportunities net of new financing. It is this stochastic process that I referred to when saying that firm value follows geometric Brownian motion with volatility σ. New financing will raise the scale of the firm, but if new securities are sold at their fair market price, the NPV of new investments will accrue to old securityholders. Thus if the NPV is zero, new financing has no effect on the value of existing securities, assuming perfect markets where the Miller and Modigliani theorem holds. In perfect markets, it is therefore not necessary to consider explicitly the possibility of new financing between now and the maturity date of the convertibles. This is true for convertible bonds even though the dilution factor may fall as a consequence of a new equity issue.[6]

Similarly, if new shares will be issued to pay for coupons, we do not need explicitly to consider the increase in the scale of the firm resulting from the new equity if we assume that assets in place plus growth opportunities follow a given stochastic process, after deducting future coupons. For example, if we assume that this process is geometric Brownian motion, then the value of the convertibles is the same as if coupons were paid out of risk-free assets. The implicit assumption is that the firm will not default on coupon payments. This is not unreasonable if coupons are small relative to firm value. Valuation thus follows (11).

2.5 *Hedging*

Above I have run through various scenarios in which convertible bonds can be priced using

[6] Convertible bonds are typically protected against dilution caused by rights issues or other warrant issues. However, if new shares are sold at fair market prices, the dilution factor will be allowed to fall.

standard option pricing methodologies. The Black–Scholes model as well as the binomial model assumes that markets are dynamically complete. In practice, this means that options can be hedged without cost. There is a growing literature on options pricing under transaction costs, but here I will assume that there are no transaction costs.

Suppose that there is only common stock and one issue of convertible bonds in the capital structure so that $V = C + S$. Suppose further that the coupons of the convertible are financed by selling risk-free assets. Then, (11) gives the value of the convertibles.

Thus, in terms of common stock, the hedge ratio of the convertible is:

$$\frac{\partial C}{\partial S} = \frac{\partial V}{\partial S} - 1 = \frac{1 - N(d_3) + \lambda N(d_1)}{N(d_3) - \lambda N(d_1)} \tag{17}$$

where d_1 is given by (6) and d_3 is given by (9).[7]

2.6 *Dividends*

Dividends to common stock can be handled in the same way as coupon payments; they can be financed out of risk-free or risky assets. Both assumptions are commonly used in option pricing, obviously depending on which approach makes the most sense in the situation one is dealing with. Over a horizon of a few months, it is in most cases not unreasonable to take the dividend as a fixed, risk-free sum. However, over a horizon of a few years, this is less likely to be a good approximation to reality since dividend payments will tend to reflect the fortunes of a firm (although firms tend to smooth dividend payments).

Within the framework of the model with a risky and a risk-free asset discussed in Section 2.2, a computationally convenient approach for dealing with uncertain dividend payments is as follows. Let the dividend payment be a constant fraction, y, of the value of the firm's risky assets. Denote by d the number of dividend payments between now and maturity of the convertibles. Then the value of the convertible is given by

$$\begin{aligned} C_0 = {} & \mathrm{PV(coupons)} + C_{BS}((V - \mathrm{PV(coupons)})(1-y)^d, T, P; \sigma) \\ & - C_{BS}((V - \mathrm{PV(coupons)})(1-y)^d, T, P + F; \sigma) \\ & + \lambda C_{BS}((V - \mathrm{PV(coupons)}(1-y)^d, T, P + F/\lambda; \sigma)) \end{aligned} \tag{18}$$

[7] Equation (17) can also be derived by first calculating the hedge ratio of the convertibles in terms of the assets of the firm:

$$\frac{\partial C}{\partial V} = 1 - N(d_3) + \lambda N(d_1)$$

where $N(d_1)$ is given by (6) and $N(d_3)$ is given by (9). Hence shorting 1% of the convertibles can be hedged by (a) buying both 1% of the convertibles and the common stock, (b) selling 1% $\times$ $N(d_3)$ of both the convertibles and the common stock, and (c) buying $\lambda \times 1\% \times N(d_1)$ of both the convertibles and the common stock. This implies that the hedge ratio of the convertible in terms of common stock is as given in (17).

where coupons are discounted at the risk-free rate and P denotes the principal of senior debt maturing at time T.[8] If the 'dividend yield', y, is likely to change over the life of the convertible, it can be replaced in (18) by the (forecasted) average 'dividend yield'.

It is useful to restate the conditions under which the convertible bond pricing formula in (18) is valid.

2.6.1 *Assumptions*

(i) The firm's capital structure consists of: an issue of convertibles with face value F and dilution λ, an issue of senior, straight bonds with principal P, and common stock. Both bonds mature in T years.
(ii) The convertible is non-callable, non-putable, and can only be converted at maturity.
(iii) The term structure is flat and non-stochastic; the continuously compounded rate is r.
(iv) The assets of the firm consist of a risk-free component, which is just sufficient to pay all coupons, and a risky asset whose value follows geometric Brownian motion with volatility σ.
(v) The firm will pay d dividends to the common stock between now and maturity of the bonds. Dividends are always a constant fraction y of the value of the risky assets of the firm.

The analysis in this section has focused on an admittedly simple setting in order to highlight the fundamental feature of a convertible bond as consisting of a straight bond and a warrant component. This section has also shown that a simple convertible bond can be priced in a straightforward manner using the standard European option pricing framework due to Black and Scholes.

3. A two-factor model of callable convertible bonds

This section incorporates call and put features into the valuation of the convertible bonds as well as allowing for voluntary conversion prior to maturity. With these features, a closed form solution for the price of a convertible does not exist. However, the price can be characterized in terms of a partial differential equation which can be solved numerically.

The basic model in this section is due to Brennan and Schwartz (1980). As in the previous section, the value of the convertible bond will depend on the interest rate and on the value, risk, capital structure, and cash distribution policy of the firm. Additionally, in Brennan and Schwartz's model, the convertible bond depends on the call and conversion strategies of the firm and investors,

[8] If the common stock pays a continuous 'dividend yield' of y, (18) can be rewritten as follows:

$$\begin{aligned}C &= \text{PV(coupons)} + C_{BS}((V - \text{PV(coupons)})e^{-yT}, T, P; \sigma)\\ &\quad - C_{BS}((V - \text{PV(coupons)}e^{-yT}, T, P + F; \sigma)\\ &\quad + \lambda C_{BS}((V - \text{PV(coupons)})e^{-yT}, T, P + F/\lambda; \sigma))\end{aligned}$$

See Merton (1973), Cox and Rubinstein (1985), or Hull (1993).

respectively. The model presented in Section 2 was a one-factor model where the convertible bond value was driven by the stochastic process governing the value of the firm. The Brennan and Schwartz model adds to this by allowing the interest rate to be stochastic.[9] I have added two minor innovations to the analysis: I allow for a put feature as well as for floating coupons.

The capital structure of the firm consists of convertible bonds, senior straight bonds, and common stock. The parameters of the common stock and convertible bond are as described in Section 2. Additionally, the convertibles can be converted voluntarily at any time and called at a price of $CP(t)$. A call protection period can be incorporated by setting the call price infinitely large for t below some cutoff date. The convertibles can also be put back to the firm by investors at a price of $PP(t)$. A put protection period can be incorporated by setting $PP(t) = 0$ for t below some cutoff date. The senior straight bond matures at $\hat{T} > T$ and has a face value equal to P.

The value of the firm evolves according to the following process:

$$\mathrm{d}V = (\mu_v V - Q(V, r, t))\mathrm{d}t + \sigma_v \mathrm{d}z_v \tag{19}$$

where $\mathrm{d}z_v$ is a Wiener process and $Q(V, r, t)$ are the cash distributions to the firm's securityholders. Thus the firm can be thought of as holding a risky asset only and cash distributions represent reductions in the scale of investments in the risky assets. The cash distributions are composed of coupon payments to the convertibles, $c(r, t)$, and to the straight bond, $b(r, t)$, and dividends to the common, $d(V, t)$. Short-term debt repayments could also be accommodated by defining $b(r, t)$ as coupon payments to the straight debt plus short-term debt repayments.

$$Q(V, r, t) = c(r, t) + b(r, t) + d(V, t) \tag{20}$$

Since coupons can be a function of both the short rate and time, both fixed and floating coupons can be accommodated.

The short rate follows the process

$$\mathrm{d}r = \alpha(\mu_r - r)\mathrm{d}t + r\sigma_r \mathrm{d}z_r \tag{21}$$

where $\mathrm{d}z_r$ is a Wiener process. Thus interest rates exhibit mean reversion.

Other variations of the stochastic processes can obviously be accommodated but would lead to a different partial differential equation (PDE) for the value of the convertible bonds. It can be shown that the convertible bond value satisfies the following PDE:

$$\begin{aligned}&\frac{1}{2}\frac{\partial^2 C}{\partial V^2}V^2\sigma_v^2 + \frac{\partial^2 C}{\partial V \partial r}Vr\rho\sigma_v\sigma_r + \frac{1}{2}\frac{\partial^2 C}{\partial r^2}r^2\sigma_r^2 + \frac{\partial C}{\partial r}[\sigma(\mu_r - r) - \lambda r\sigma_r]\\ &+ \frac{\partial C}{\partial V}(rV - Q(V, r, t)) - rC + c(r, t) + \frac{\partial C}{\partial t} = 0\end{aligned} \tag{22}$$

where $\rho = \mathrm{corr}(\mathrm{d}z_v, \mathrm{d}z_r)$, and λ is the market price of interest rate risk; it is well known that for any security whose value is dependent only on r and time (any default-free bond), the reward to variability ratio is the same (i.e. λ; see Hull, 1993 or Brennan and Schwartz, 1980). While in general λ is a function of r and t, under the pure expectations theory of the term structure, $\lambda = 0$. When solving

[9] Brennan and Schwartz (1980) extends Brennan and Schwartz (1977) and Ingersoll (1977a) where the interest rate is non-stochastic.

for this PDE, one must estimate the drift and volatility terms of the underlying stochastic processes as well as ρ and λ. Notice that while the growth rate of firm value drops out of the equation (as usual), the growth rate of the short rate does not drop out. This is because the short rate is not a tradable security. In deriving the PDE, hedging out interest rate risk is done via trading in bonds, whose values depend on the interest rate.

The PDE has boundary conditions as discussed below.

3.1 *Boundary conditions*

3.1.1 *Conversion condition*

The conversion value of the convertibles is defined as the value of the convertibles if converted into common stock. Letting, $B(V, r, t)$ denote the value of the straight bond, the conversion value of the convertible bond is given by $\lambda[V - B(V, r, t)]$. If the value of the convertibles fell below the conversion value, it would be optimal for the convertible bondholders to convert. Hence the value of the convertibles is bounded below by the conversion value:

$$C(V, r, t) \geqslant \lambda[V - B^*(V, r, t)] \tag{23}$$

where $B^*(V, r, t)$ denotes the value of the straight bond if the convertibles are converted. Note here that it is generally not optimal to convert voluntarily before maturity except possibly immediately before a dividend payment (see Brennan and Schwartz, 1977 or Ingersoll, 1977).[10]

3.1.2 *Call condition*

Assuming that the objective of the firm's management is to maximize the value of the common stock, it is optimal to call the convertibles as soon as their value if not called is equal to the call price. Hence,

$$C(V, r, t) \leqslant CP(t) \tag{24}$$

When convertibles are called, convertible bondholders must decide whether to convert or take the call price. It is optimal to convert if the conversion value is larger that the call price. Since the value of convertibles is bounded below by the conversion value, this means that under an optimal call policy, one should not observe non-call-protected convertibles with a conversion value which is larger than the call price. The optimal call policy stated above is roughly the same as a policy of forcing conversion as soon as possible. This is intuitive, since by forcing conversion the firm

[10] It has been shown by Constantinides (1984) that for a large warrantholder, it may be optimal to exercise warrants strategically before maturity in the absence of dividends. This involves exercising some warrants and retaining others. The reason is that the new capital that flows into the firm from the warrants that are exercised increases the scale of the firm and thus the total variability of firm value (assuming that the new cash is invested in sufficiently risky assets; see Spatt and Sterbenz, 1988). This increase in firm value variability increases the value of the outstanding warrants. In some cases, this increase can offset the option value lost by early exercise. This argument does not apply to zero coupon convertibles since conversion does not bring a cash inflow to the firm (Lewis, 1991). However, for coupon convertibles, sequential exercise may be optimal for a large convertible bondholder if the coupons are sufficiently large relative to the size of the firm and are financed by selling risky assets.

extinguishes convertible bondholders' option not to convert. Stated slightly differently, as explained in Section 2, a convertible bond can be viewed as consisting of a straight bond component and a warrant. One can also view holding a convertible bond as holding the equity while having the right to sell it back to the firm at a prespecified price. Forcing convertible bondholders to convert is equivalent to extinguishing the put option held by the convertible bondholders.

3.1.3 *Put condition*

It is optimal to put the convertibles back to the firm as their value if not put falls below the put price. Hence,

$$C(V,r,t) \geqslant PP(t) \tag{25}$$

3.1.4 *Maturity condition*

Since the straight bond is senior, the convertible bond will only be paid off in full if the value of the firm is larger than the principal of the straight bond. Thus the payoff to the convertible at maturity, if not yet converted, is:

$$C(V,r,T) = \begin{cases} \lambda(V - B^*(V,r,T)) & \text{if } V \geqslant F/\lambda + B^*(V,r,T) \\ F & \text{if } F + P < V < F/\lambda + B^*(V,r,T) \\ V - P & \text{if } P < V < F + P \\ 0 & \text{if } V < P. \end{cases} \tag{26}$$

3.1.5 *Bankruptcy condition*

The convertible bond is to receive the fraction k of its par value in case of default. Hence the firm is bankrupt if its value falls to $kF + P$:

$$C(V,r,t) = kF \quad \text{if } V = P + kF \tag{27}$$

3.2 *Straight bond value if convertibles are converted*

As seen in 3.1.1 and 3.1.4, the value of the convertibles depends on the value of the straight bond if the convertibles are not converted. This value, $B^*(V,r,t)$, must satisfy the PDE (22) with $b(r,t)$ in place of $c(r,t)$ and $Q^*(V,r,t) \equiv b(r,t) + d^*(V,t)$ in place of $Q(V,r,t)$. This reflects that as soon as the convertibles are converted, coupon payments (to convertibles) stop, while total dividend payments may change to d^* to reflect the additional number of shares.

The maturity conditions for B^* are:

$$B^*(V,r,\hat{T}) = \begin{cases} P & \text{if } V > P \\ V & \text{if } V < P \end{cases} \tag{28}$$

The bankruptcy condition is assumed to be similar to that of the convertible. In particular, bankruptcy is supposed to be triggered if the value of the firm drops to the fraction $0 \leqslant h < 1$ of firm value. Thus the second boundary condition for the straight bond PDE is

$$B^*(V, r, t) = hP \quad \text{if } V = hP \tag{29}$$

3.3 *Solving for the convertible value*

To solve for the convertible, it is necessary to first solve for B^* as described above. Once B^* is obtained, the PDE (22) with boundary conditions (23), (24), (25), (26), and (27) can be solved to give a value for the convertible. The PDEs can be solved using finite difference methods. See, for example, Brennan and Schwartz (1977) where a similar problem was solved in the absence of senior debt and with a constant and non-stochastic term structure.

4. Why do firms issue convertible bonds?

The most direct way to find out why firms issue convertible bonds is to ask the managers who made the decision to issue them. Table 1 reports the findings of questionnaire studies by Pilcher (1955), Brigham (1966), and Hoffmeister (1977).

The most commonly cited reason for issuing convertibles is that it is an indirect means to add equity to the capital structure. Brigham reports that firms viewed issuing convertibles as preferable to issuing equity outright because managers '... believed [their] stock's price would rise over time, and convertibles provide a way of selling common stock at a price above the existing market' (Brigham, 1966, p.51). Essentially, if managers expect the convertibles to be converted, the advantage of convertibles to an outright issuing of equity is that convertibles create less dilution. In this view, convertibles can be thought of as a form of delayed equity. Indeed, Van Horne (1986), writes: 'Convertible securities, in many cases, are employed as deferred common-stock financing. Technically these securities represent debt or preferred stock, but in essence they are delayed common stocks' (p.650).

Convertible bonds can also be an alternative to straight bonds. The second most cited reason why firms issue convertibles is that convertible debt allows a firm to borrow cheaply relative to straight debt. The equity 'sweetener' implicit in convertibles means that their coupons are lower than on comparable straight bonds.

Table 1. Why do firms issue convertibles?

	Pilcher (1955)	*Brigham* (1966)	*Hoffmeister* (1977)
Delayed equity	82%	68%	40%
Sweeten debt	9%	27%	37%
Other	9%	5%	23%

However, we know from the Miller and Modigliani theorem that in perfect markets, convertible debt cannot be a cheaper source of financing than either straight debt or equity. As discussed earlier, convertible debt can be viewed as a package of straight debt and warrants. The lower coupon on convertible bonds relative to straight bonds merely reflects the value of the warrant. Convertible debt can also be viewed as a package of equity and a put (to sell the shares back to the firm). The lower dilution created by convertible bonds *if* they are converted reflects the value of the put.

The cited rationales for issuing convertibles are best understood in an imperfect markets setting. The two main strands of the literature focus on (1) information asymmetry concerning firms' *mean* returns, and (2) information asymmetry about, or management's incentives to influence, the *riskiness* of returns. The first type of model generally addresses the issue of convertible debt as delayed equity, while the second is better suited to explain the debt sweetener argument.

4.1. *Convertible debt as delayed equity*

The literature on convertible debt as delayed equity has focused on callable convertibles. The vast majority of convertibles issued in the United States are callable: out of a total of 491 US convertibles listed in *Moody's Bond Record* (July 1994), only 25 are non-callable. Jalan and Barone-Adesi (1995) present a cooperative game theoretic model where the advantage of convertible debt as delayed equity arises from the tax deductibility of coupon payments. Stein (1992) and Nyborg (1995) develop signalling models with callable convertible debt where the choice of financing signals management's private information about the firm's prospects (mean return). Constantinides and Grundy (1990) do not explicitly address the idea of convertible debt as delayed equity, but develop a signalling model where managers are better informed about the mean return of the firm and signal that information through a combination of repurchasing stock and the terms of convertible debt.

The driving force in both Stein and Nyborg's models is a trade off between adverse selection and insurance against adverse stock prices in the future. In both models, the existence of bankruptcy costs implies that equity is the preferred security in the absence of proprietary information. However, under asymmetric information, it is well known from the work of Myers and Majluf (1984) that riskier securities such as equity have worse adverse selection properties than less risky securities such as straight debt. The advantage of equity as a source of financing is that it has excellent insurance properties against financial distress, while straight debt is less good on this dimension. Hence equity will be issued by the more pessimistic firms, while straight debt will be issued by the more optimistic firms. Convertible debt will be issued by medium-quality firms. Both Stein and Nyborg emphasize the importance of the call feature of convertible debt as an insurance device. If a firm's stock price has risen sufficiently that the conversion value is larger than the call price, conversion into equity can be guaranteed by calling the convertibles, thus lowering the expected costs of financial distress. This would not be possible with non-callable convertible debt; hence the callable version is preferred.

While Stein and Nyborg's models are centred around the same trade off, they offer different views on callable convertible debt as delayed equity. Stein argues that it is the call feature that allows convertibles to be an advantageous method for some firms to get equity into their capital structure. In his model, firms that issue convertible debt do so with the intention of forcing conversion at the earliest opportunity. This is because in his model, information asymmetry between management and investors is short-lived and, as shown by Brennan and Schwartz (1977) and Ingersoll (1977a), in perfect markets it is optimal to force conversion as soon as possible. However, this is not consistent with the empirical evidence. Firms that force conversion experience negative abnormal returns, suggesting that information asymmetry is persistent. Mikkelson (1981) finds an AAR of -2.08% around the announcement date of calls that force conversion. Perhaps for this reason, firms also tend to delay forcing conversion. Ingersoll (1977b) finds that in his sample the median firm delayed forcing conversion until the conversion value exceeded the call price by 43.9%.

Nyborg (1995) suggests that the benefits of convertible debt as delayed equity are preserved only if conversion is voluntary. In his model, management continue to receive updated signals about the prospects of the firm after the initial financing decision. Following Harris and Raviv (1985), call policy then transmits these updated signals to the market. Forcing conversion is a bad signal since it indicates a desire to insure against a deterioration in the stock price and the risk of ending up with unconverted convertibles.[11] On the other hand, not forcing conversion is a good signal, informing the market that management are confident that the convertible bonds will be converted voluntarily in the future. In Nyborg's model, the combined negative effect from issuing convertibles and later forcing conversion is greater than the initial negative effect from issuing equity outright. This seems to be consistent with the empirical evidence, although there is no proper test of this proposition in the literature. Using the numbers cited above, the evidence suggests that issuing equity by first issuing convertibles and later forcing conversion leads to a larger negative abnormal return (-4.15%) than issuing equity outright (-3.14%).

A strategy of issuing convertibles with the intention of forcing conversion as soon as possible does not seem to be optimal for the average firm. Indeed, Brigham (1966) reports that among firms with convertibles outstanding, only 23% of firms planned to force conversion as soon as possible, another 23% planned to stimulate conversion by raising dividends, and the remainder had no clear plans.

In Nyborg's model, callable convertible debt is not generally synonymous with delayed equity because firms do not follow a strategy of forcing conversion at the earliest opportunity. But firms issue callable convertible debt in the hope that it will be converted. The call feature allows firms to force conversion when the conversion value is above the call price, if the chance of voluntary conversion, as assessed by managers, is not good. While firms' tendencies to delay forcing conversion is an integral aspect of Nyborg's model, Asquith (1995), Asquith and Mullins (1991), and Campbell *et al.* (1991) cast doubt on the view that firms tend to delay forcing conversion. These papers argue that many firms with conversion values larger than call price have cash flow incentives not to force conversion. Additionally, the call notice period (typically around 30 days) means that firms will let the conversion value rise above the call price before calling. In addition, call protection periods can obviously explain why some firms do not call before the conversion value is considerably larger than the call price. Below I will focus on the cash flow incentive arguments.

[11] Although I am focusing on the costs of financial distress, the costs of unconverted convertibles could also rise from management losing their jobs, bonuses, etc. if the stock price declines 'too much' (Nyborg, 1995). Firms that wish to raise additional financing may also wish to reduce their leverage. For example, Myers and Majluf (1984) have emphasized the importance of financial slack.

Campbell *et al.* (1991) argue that management would call only when the anticipated dividend to the convertibles if converted would be smaller than the after-tax coupons. Asquith (1995) shows that

> [F]irms call convertible bonds as soon as conversion value is greater than call price unless: 1) there is a significant risk that the conversion value will fall before the call notice period expires or 2) the present value of after-tax coupon payments is less than the present value of the dividend payments on the converted bond by an amount greater than the option value extinguished in a call. (p.1276)

While these arguments take the anticipated dividends to convertibles after conversion as given, management typically have discretion over dividend payments. The cash flow advantage of not forcing conversion could easily disappear if dividends were reduced. The puzzle seems to be why firms do not simultaneously reduce dividends and call outstanding convertibles when the conversion value is greater than the call price. The answer might be that both of these actions are interpreted as bad news by the market. Campbell *et al.* (1991) find that two-day abnormal returns around the announcement of conversion-forcing calls are significantly negative only in the case in which the dividend to be received on conversion is smaller than the coupon on the bond. When the dividend is larger than the coupon, a conversion-forcing call would, in effect, increase net dividends (coupons plus dividends) and as such the call should be good news. But Campbell *et al.* (1991) find negative but statistically insignificant abnormal returns upon a conversion-forcing call that increases net dividends. This suggests that while a call may have net dividend implications, the (negative) information contained in extinguishing the put option element of the convertibles is also important empirically.

The finding that for some firms there is an apparent cash flow advantage not to force conversion may also be understood in light of Brigham's (1966) finding that many firms prefer to stimulate conversion through increasing dividends rather than forcing conversion (see also Van Horne, 1986, p.651). This would avoid the negative stock price reaction typically associated with forced conversion. The benefits from paying 'high' dividends would be fourfold: (i) increased likelihood of conversion; (ii) a 'bird in the hand' effect: if convertible bond holders do not convert, more of the firm's earnings would accrue to current equityholders (retained earnings will eventually be shared with convertible holders when and if they convert); (iii) high dividends would presumably be a positive signal; and, related to the last point, (iv) if the convertibles are voluntarily converted, the negative signalling effect associated with a conversion-forcing call would presumably be avoided. Where the dividend is internally financed, a cost of increasing dividends to stimulate conversion may be underinvestment (see e.g. Miller and Rock, 1985) and a loss of financial slack.[12] In some jurisdictions, dividends also result in double taxation of corporate earnings.

The cash flow advantage hypothesis advanced in the empirical literature cited above suggests that we do not yet fully understand convertible bond call policy. The evidence suggests that call policy is related to dividend policy, and further theoretical and empirical investigations into this issue are needed.

Pricing callable convertible bonds will be sensitive to the optimal call policy. Brennan and Schwartz's model, presented in Section 3, assumes that the value of callable convertible debt cannot rise above the call price. This is roughly the same as saying that firms force conversion

[12] Dividend increases could also be financed by a rights issue. Since most convertible issues have anti-dilution clauses, a dividend financed by a rights issue would typically not have a conversion stimulating effect. Myers and Majluf (1984) and Huberman (1984) have emphasized that financial slack is valuable.

at the earliest opportunity. Buf if firms delay conversion, this means that the Brennan and Schwartz model tends to undervalue convertible bonds. An *ad hoc* solution would be to assume that conversion would be forced as soon as the conversion value rose to a certain percentage above the call price. Asquith and Mullins (1991) report that managers tend to delay forcing conversion until the conversion value is 20% larger than the call price, because of the risk that the stock price will fall during the call notice period. Allowing for a call notice period in Brennan and Schwartz's model might therefore be a useful first step in achieving more accurate prices for convertible bonds (but see Ingersoll 1977d). Perhaps a greater challenge for the pricing literature is to allow for strategic call policies, for example where call policy is determined by management's private information.

As argued above, management force conversion when they receive unfavourable information regarding the firms' future prospects in order to minimize the costs of financial distress, or more generally, any costs associated with unconverted convertibles. When the conversion value is below the call price, the preference for having the convertibles converted into equity should still be there, but it may seem impossible to force conversion. However, in practice, there are strategies that firms can employ to force conversion even when the conversion value is below the call price. Conversion into equity may be achieved by simultaneously calling and offering to swap the convertibles for shares having a larger value than the call price. This has the same effect as increasing the dilution factor of the convertibles (decreasing the conversion price) and then calling. Brennan and Her (1993) report that the average dilution factor in their sample of convertible bonds is 16.3%, suggesting that there is ample scope for a convertible to equity swap as described here. A consistent model of convertible debt call policy should also include the incentive to seek conversion when the conversion value is below the call price. This will also have implications for pricing convertible debt.

While a convertible bond pricing model that takes account of signalling through call policy has not been developed, other strategic issues are starting to be incorporated into debt pricing models. In an important contribution, Anderson and Sundaresan (1996) develop a framework for pricing straight bonds under the possibility of strategic default by the firm. This framework should also be relevant for convertible bonds.

This subsection has discussed models which look at callable convertible debt as an alternative to equity financing. Callable convertible debt is preferred to the non-callable version because it allows firms, in some circumstances, to force convertible bondholders to convert into equity. The possibility of swapping non-callable convertibles for equity in the future does not change this conclusion if information asymmetry is persistent. Management's reasons for wishing to do such a swap would be the same as their reasons for wishing to force conversion: they have received unfavourable information about future cash flows and attempt to minimize the expected costs of financial distress by forcing conversion – or making an exchange offer. However, while callable convertible debt allows the firm to force conversion at terms which have been set conditional on management's information at the time of the initial issuing, a swap of non-callable convertibles for equity will be done at terms reflecting management's updated information. Since this information is likely to be more precise than management's initial information, the terms of exchange will now be worse for equityholders (see Nyborg, 1995). For the same reason, a callable convertible bond should be preferred as a signal of intermediate quality to a mixture of straight bonds and equity. Indeed, stock for debt exchange offers are associated with substantially larger negative abnormal returns (−9.9%, Smith, 1986) than conversion forcing calls (−2.08%).

I will close this section by offering a new rationale for convertible debt as an alternative to equity. Recall that a convertible can be viewed as a package of the firm's equity and a put option to sell a fraction of that equity back. If investors undervalue the equity because of underestimating the value of the firm, they will overvalue the put. It should be possible to construct the terms of a convertible so that it is less sensitive to a firm's true value than an outright equity issuing would be. Hence, convertible debt can be an attractive source of financing for firms whose equity is undervalued. This argument relates to Stein and Nyborg's models since it is just a different way of saying that convertible debt has better adverse selection properties than equity. It relates loosely to the idea of convertible debt as delayed equity since it offers a rationale for convertible debt as an alternative to equity.

4.2 *Sweetening debt*

The argument that funding costs may be reduced by adding an equity 'sweetener' to straight debt can be related to the moral hazard problem of debt first discussed by Jensen and Meckling (1976). Firms can shift value from debtholders to equityholders by taking more risky projects. This can be seen by viewing the equity as a call option on the firm's assets. We know from option pricing theory that increasing riskiness increases the value of options, *ceteris paribus*. However, if firm value is kept constant, this means a corresponding loss in value to the firm's debt. This tends to increase the interest rates relative to a situation where the firm could truthfully promise not to increase risk, and investments will be suboptimal. Bond covenants go some way in deterring firms from increasing risk, but can hardly cover all contingencies. However, Green (1986) shows that the cost of debt can be reduced and optimal investment restored by attaching warrants to a debt issue, thus creating convertible bonds. While the straight debt component falls in value as risk is increased, the warrant component increases in value. By choosing the terms of the combined instruments judiciously, the total package can be made insensitive to changes in risk. However, Spatt and Sterbenz (1993) show that if the warrants are detachable after issuance, there may be a time inconsistency problem whereby equityholders have an incentive to buy the warrants and then increase risk. The conclusion is that convertible debt can be a cheaper source of financing than straight debt since the agency costs associated with the latter can be eliminated.

Although convertible debt can resolve the moral hazard problem of debt, the question remains: why issue straight debt in the first place? Jensen and Meckling (1976) show that agency costs of outside equity arise from managers' incentives to work less hard than needed to maximize value and to invest corporate funds in perks such as posh offices, corporate jets, etc. The fewer shares managers own, the stronger will these incentives be, since the costs are shared with other shareholders while the benefits are enjoyed solely by managers. In a rational expectations equilibrium, investors will take these costs into account when buying new shares, so that the costs are ultimately borne by the vendors. The solution to this problem is to issue debt. But this merely replaces one agency problem with another. While convertible debt can eliminate the incentives to increase risk, the equity participation implicit in convertible debt suggests that the perk consumption problem is larger with convertible debt than with straight debt, although not as large as it is with outside equity. However, as suggested in the previous subsection, since convertible debt can also be viewed as equity plus a put option to sell the equity back to the firm, convertible debt can also be made insensitive to incentives to decrease the value of the firm through perk consumption. It seems unlikely that both agency problems can be resolved simultaneously by issuing convertible debt. But since equity and straight debt can be viewed as extreme cases of convertible debt, it does not seem contentious to say that agency costs can be minimized with an appropriately chosen convertible debt issue.

Brennan and Schwartz (1988) have offered a rationale for the use of convertible debt as an alternative to straight debt which is essentially an adverse selection version of the Jensen–Meckling–Green agency story (see also Brealey and Myers, 1981). A firm whose riskiness is overestimated by investors must pay a higher coupon on straight debt than if investors knew the firm's correct volatility. This problem can be resolved by issuing convertible debt since these instruments are less sensitive to the true riskiness of the firm. As a result, convertible bonds offer cheap debt, in the sense that they allow firms for which issuing debt would be a negative NPV transactions to issue convertible debt at a fair price.

The sweetening debt rationale for convertible debt is based on either hidden action or information regarding a firm's risk. In related work, Brennan and Kraus (1987) have shown that if managers are better informed than investors about the riskiness of firm returns, costless signalling involves issuing claims that are both convex and concave in the value of the firm. Figure 1 shows that convertible debt satisfies this condition. Brennan and Kraus's model predicts that the conversion ratio is negatively related to the variance of returns, while the opposite applies to the face value of a convertible issue. In an empirical study of convertible debt, Brennan and Her (1993) find a positive relation between the announcement returns and the face value ratio (the ratio of face value to pre-announcement equity value) and a negative relation between the announcement returns and the conversion ratio. While this is consistent with Brennan and Kraus's model, Kim (1990) also shows that the conversion ratio finding is consistent with the types of model discussed in the previous subsection where management have information about the *mean* returns and there are bankruptcy costs. Additionally, Brennan and Her's face value ratio finding seems consistent with mean-based, bankruptcy cost models; intuition would suggest that a large face value signals confidence in future returns. It is unlikely that there is one model that applies to all firms that issue convertible bonds. In some cases, agency costs may be a driving force, while in other cases information asymmetry may be more important. The tax deductibility of coupons may also make convertibles more attractive than equity, but the models reviewed here best capture the spirit of the two most commonly stated reasons for issuing convertible bonds.

5. Concluding remarks

This paper has reviewed the basic elements of convertible bond pricing as well as presented Brennan and Schwartz's (1980) general framework for pricing complicated convertible bonds. The paper has also reviewed the two primary rationales for issuing convertibles as stated by firms that actually have issued them, and how these rationales can be understood in light of adverse selection and agency models of capital structure. The strategic issues explored here have not been dealt with in the pricing literature. A challenge for the next generation of convertible bond pricing models is to incorporate these strategic concerns.

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