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Author(s): John J. McConnell and Eduardo S. Schwartz
Reviewed work(s):
Source: *The Journal of Finance*, Vol. 41, No. 3, Papers and Proceedings of the Forty-Fourth Annual Meeting of the America Finance Association, New York, New York, December 28-30, 1985 (Jul., 1986), pp. 561-576
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328484
Accessed: 01/08/2012 07:19

THE JOURNAL OF FINANCE • VOL. XLI, NO. 3 • JULY 1986

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JOHN J. McCONNELL and EDUARDO S. SCHWARTZ*

ABSTRACT

A Liquid Yield Option Note (LYON) is a zero coupon, convertible, callable, puttable bond. This paper presents a simple contingent claims pricing model for valuing LYONS and uses the model to analyze a specific LYON issue.

A LIQUID YIELD OPTION NOTE (LYON) is a complex security. It is a zero-coupon, convertible, callable, redeemable bond. The complexity of this security is further increased because the prices at which the issuer may call the bond and the prices at which the investor may redeem (or put) the bond escalate through time. Additionally, the bond contains call protection for the investor because the bond may not be called for a prespecified period of time after issuance unless the issuer's stock prices rises above a predesignated level.

This fascinating security was created by Merrill Lynch White Weld Capital Markets Groups in 1985. In the spring of 1985 Waste Management, Inc. and Staley Continental, Inc. were the first two issuers of this security, with Merrill Lynch acting as the underwriter.[1] Because of its novelty and complexity, potential issuers find this security difficult to analyze. Two issues are of paramount concern to LYON issuers. First, is the security "correctly" priced at the initial offering? The issuer is concerned that the security not be underpriced at the initial offering and the underwriter is concerned that the security not be overpriced. Second, the issuer is concerned that the security not be converted "too soon" after issuance. Issuers are concerned that premature conversion will dilute the issuer's earning per share and that the valuable tax savings associated with the LYON will be dissipated.

To address these concerns (and others) we were engaged to analyze the Liquid Yield Option Note. To do so, we developed a LYON pricing model using modern contingent claims pricing techniques. In developing the model we were especially concerned that it be commercially useable. Thus, our goal was to develop a model that is both rich enough to capture that salient ingredients of this complex security and simple enough to be implemented with an enhanced personal computer. Because of the complexity of the security, the final pricing equation can be solved only with numerical techniques. Thus, the focus of this paper is on the practical application of contingent claim pricing models that can be solved only with numerical techniques. The contribution of this paper is that it reports

* Purdue University and University of British Columbia, respectively. Thomas Patrick, Lynne Dinzole, Lee Coles, and Robert Moulton-Ely of Merrill Lynch White Weld Capital Markets Group were especially helpful to us in developing the ideas presented in this paper.

[1] Subsequently, LYONs were issued by the G. Heileman Brewing Co., Merrill Lynch & Co. and Joseph E. Seagram & Son, Inc. and others.

on an actual case situation in which numerical solution techniques were used to analyze a security pricing problem.

We first describe in some detail a specific LYON issue. We then present a pricing model which we shall refer to as the commercially-useable LYON pricing model. As will be quite evident, this simplified pricing model takes a number of liberties with the "state-of-the-art" in contingent claim pricing analysis.[2] Following our presentation of the commercially-useable LYON pricing model we discuss its limitations and simplifications and suggest ways in which the various limiting assumptions could be relaxed so as to yield a theoretically more sophisticated model. The benefit of a more sophisticated model is that it would likely increase the accuracy of the resulting analysis. The cost is that it would increase the difficulty of implementing and using the model. As it turns out, the commercially-useable LYON pricing model, although quite simple in comparison with a theoretically more sophisticated model, appears to work well in practice, in that the theoretical LYON prices generated with the simplified model closely tracked the reported market closing prices for both the Waste Management and the Staley Continental LYONs over the first several weeks following their issuance. Whether the accuracy of the simple model is sufficient for all commercial uses depends, of course, on the needs of the user.

Following our presentation of the simplified LYON pricing model we present our application of the model to the valuation of the Waste Management LYON. We then investigate the sensitivity of theoretical LYON values to changes in the characteristics of the issuer, the economic environment and the security. Finally, using the same data, we illustrate the way in which the model can be used to calculate the LYON's optimal conversion price. We end the paper with a brief summary and some concluding remarks.

I. The LYON

An appreciation of the LYON pricing model can perhaps best be gained by considering a specific issue. The one that we consider here was issued by Waste Management, Inc. on April 12, 1985.

According to the indenture agreement, each Waste Management LYON has a face value of $1,000 and matures on January 21, 2001. If the security has not been called, converted, or redeemed (i.e., put to the issuer) prior to that date, and if the issuer does not default, the investor receives $1,000 per bond. At any time prior to maturity (or on the maturity date), the investor may elect to convert the bond into 4.36 shares of Waste Management common stock. Additionally, however, the investor can elect to put the bond to Waste Management beginning on June 30, 1988, and on each subsequent anniversary date, at fixed exercise prices that escalate through time.[3] The put exercise prices are:

[2] An excellent survey of recent applications of contingent claims pricing analysis in corporate finance is provided by Mason and Merton [6].

[3] The investor must give Waste Management at least 30 days' notice and not more than 90 day's notice prior to exercising the put option.

Date	Put Price	Date	Put Price
June 30, 1988	$301.87	June 30, 1995	613.04
June 30, 1989	333.51	June 30, 1996	669.45
June 30, 1990	375.58	June 30, 1997	731.06
June 30, 1991	431.08	June 30, 1998	798.34
June 30, 1992	470.75	June 30, 1999	871.80
June 30, 1993	514.07	June 30, 2000	952.03
June 30, 1994	561.38		

Finally, Waste Management can elect to call the LYON at fixed exercise prices that escalate through time. Although the issuer may call the LYON immediately after issuance, the investor does receive some call protection because Waste Management may not call the bond prior to June 30, 1987 unless the price of the Waste Management common stock rises above $86.01.[4] On the LYON issue date, the Waste Management stock price was $52.125. The LYON call prices are:

Date	Call Price	Date	Call Price
At Issuance	$272.50	June 30, 1994	563.63
June 30, 1986	297.83	June 30, 1995	613.04
June 30, 1987	321.13	June 30, 1996	669.45
June 30, 1988	346.77	June 30, 1997	731.06
June 30, 1989	374.99	June 30, 1998	798.34
June 30, 1990	406.00	June 30, 1999	871.80
June 30, 1991	440.08	June 30, 2000	952.03
June 30, 1992	477.50	At maturity	1,000.00
June 30, 1993	518.57		

Additionally, if the LYON is called between the dates shown above, the call price is adjusted to reflect the "interest" accrued since the immediately preceding call date shown in the schedule.[5]

As our brief description indicates, analysis of a LYON is not a simple matter. To value a LYON it is necessary to take into account the unique characteristics of the security, the issuer, and the economic environment in which the security is issued. Furthermore, the security can be valued only if it is possible to identify the conversion and redemption strategies to be followed by investors and the call strategy to be followed by the issuer. In the spirit of Brennan and Schwartz [2], [4] and Ingersoll [5], we assume that the issuer follows a call policy that minimizes the value of the LYON at each point in time and that the investor follows conversion and redemption strategies that maximize the value of the LYON at each point in time. We refer to these as the optimal call, the optimal conversion, and the optimal redemption strategies, respectively. The optimal call, conversion, and redemption strategies depend upon, among other things, the bond's conver-

[4] Waste Management must give the investor at least 15 days' notice prior to exercising the call option.

[5] The imputed interest is computed by increasing the call prices at a rate of 9.0% per year compounded semiannually.

sion ratio and upon the call and redemption schedules specified in the bond's indenture agreement.

A. Optimal Conversion Strategy

Because the investor seeks to maximize the value of the LYON, the investor will never convert if the market value of the LYON is greater than the value of the stock into which the LYON can be converted. That is, the LYON will never be converted as long as its market value exceeds its conversion value. Contrarily, because the investor would receive an immediate gain from conversion, the investor would always convert if the value of the LYON were less than its conversion value. Thus, investors will optimally convert the LYON when the value of the security just equals its conversion value. As a consequence, the value of an outstanding LYON must be greater than its conversion value.

B. Optimal Redemption Strategy

On each redemption date the investor must choose between holding the LYON and putting it to the issuer for the prespecified redemption value. However, because the investor seeks to maximize the market value of the security, on any anniversary date the investor will not put the LYON to the issuer if the security's value is greater than its redemption price at that time. Contrarily, because the investor would receive an immediate gain from redemption, the investor would always redeem the LYON if the LYON value were less than its redemption price on any redemption date. Thus, investors optimally will redeem the LYON when the LYON's market value just equals its redemption value. At no time will the value of the LYON be less than its redemption value.

The redemption value, of course, is the exercise price of a put option. The twist here is that unlike a conventional put option, the exercise price of the put option imbedded in a LYON changes through time.

C. Optimal Call Strategy

On the one hand, because the issuer seeks to minimize the value of the LYON, the issuer will never allow the market value of the security to exceed its call price. On the other hand, the issuer will never call the LYON when its value is less than the call price because this would convey an immediate windfall gain to the investor. Thus, the issuer will optimally call the LYON when the LYON's market value just equals its call price. When the issuer calls the LYON, the investor can elect to receive either the cash call price or the conversion value of the security, whichever is greater. As a consequence, at any point in time, the value of a callable LYON will not exceed the greater of its call price or conversion value.

To determine the equilibrium value of the LYON, we assume that investors and issuers follow the optimal conversion, redemption and call policies and that each party expects the other also to follow the optimal strategy. Under the optimal strategies, the value of the LYON is bounded from above by the maximum of its call price and conversion value and it is bounded from below by the maximum of its redemption price and conversion value.

II. The LYON Pricing Model

To derive the LYON pricing model we assume that the value of the LYON depends upon the issuer's stock price (S) and that instantaneous changes in the issuer's stock price follow a diffusion process with constant variance (σ_s). That is,

$$dS = [S\mu - D(S, t)]dt + S\sigma_s dz_s \tag{1}$$

where $S(t)$ is the issuer's stock price at time t; μ is the (possibly stochastic) instantaneous total expected return on the issuer's common stock; σ_s is the standard deviation of the rate of return on the issuer's common stock; and $D(\mathrm{s}, t)$ is the total rate of dividends paid to stockholders at time t. In applications of the model, we allow dividend payments to take the general form

$$D(S, t) = d_y S + de^{g(t-t_0)} \tag{2}$$

where d_y is the issuer's dividend yield; d is the issuer's dividend rate; g is the constant growth rate of dividends; and t_0 is the issue date of the LYON. This general form for dividend payments permits either a constant dividend yield (when $d = 0$) or a constant dividend growth rate (when $d_y = 0$).

We further assume that capital markets are perfect, that investors and issuers have costless access to all relevant information, and that the term structure of interest rates if flat and known with certainty. Then, given the usual arbitrage arguments, the value of the LYON must satisfy the partial differential equation

$$\tfrac{1}{2}\sigma_s^2 S^2 L_{ss} + [rS - D(S, t)]L_s + L_t - rL = 0 \tag{3}$$

where r is the known, constant interest rate and subscripts represent partial derivatives.

Solution of (3) subject to four boundary conditions gives the theoretical value of the LYON. The boundary conditions follow from the optimal conversion, redemption and call strategies and from the maturity condition specified in the LYON contract.

A. The Maturity Condition

At the maturity date of the contract, the value of the LYON will be the greater of the conversion value or the face value of the contract:

$$L(S, T) = \mathrm{Max}(C_r S, F) \tag{4}$$

where C_r is the number of shares of the issuer's common stock into which the LYON can be converted (i.e., C_r is the conversion ratio and $C_r S$ is the conversion value of the LYON); F is the face value of the LYON at maturity (typically specified to be \$1,000); and T is the maturity date of the contract.

B. The Conversion Condition

At any point in time, the value of the LYON must be greater than or equal to its conversion value:

$$L(S, t) \geq C_r S \tag{5}$$

C. The Redemption (or Put) Condition

At any redemption date the value of the LYON must be greater than or equal to the then prevailing redemption price:

$$L(S, t_p) \geq P(t_p) \tag{6}$$

where $P(t_p)$ is the redemption (or put) price at time t_p.

D. The Call Condition

At every point in time, the value of the LYON must be less than or equal to the greater of the call price and the conversion value:

$$L(S, t) \leq \text{Max}\{C(t), C_r S\} \tag{7}$$

where $C(t)$ is the call price of the LYON at time t.

Partial differential equation (3) subject to the boundary conditions (4), (5), (6) and (7) gives the value of the LYON under our set of assumptions. Although there is no known closed form solution to this equation, the virtue of this simplified model is that it can be solved easily by means of numerical methods with an enhanced personal computer. In our applications of the model, the method of finite differences was used to solve (3) on an IBM personal computer. Solution of a typical problem required less than 10 minutes.

III. Discussion of the Simplified LYON Pricing Model

It is readily apparent that the commercially-useable LYON pricing model embodies a number of simplifying assumptions. These assumptions were dictated largely by the circumstances under which the model was developed. For the most part, however, the assumptions seem justifiable given the requirements of the model. In this section we discuss some of these assumptions in more detail, suggest ways in which the assumptions can be relaxed, and consider the costs and benefits of relaxing these assumptions.

Perhaps the most egregious assumptions are that the value of the LYON depends upon the value of the issuer's common stock rather than the total market value of the firm and that the term structure of interest rates is flat and known with certainty.

As an alternative to the assumption that the LYON value depends upon the value of the issuer's common stock which follows a diffusion process with constant variance, a theoretically more palatable assumption is that the total value of the firm follows a diffusion process with constant variance and that the LYON and the issuer's stock are both contingent claims that depend upon the total value of the firm. This assumption is theoretically more desirable because it would more appropriately capture the default risk of the LYON. The assumption that the value of the LYON depends upon the value of the issuer's common stock precludes the possibility of bankruptcy. Under this assumption, at the maturity date, the investor receives either the face value of the LYON or the conversion value, whichever is greater. Under the alternative assumption, the maturity condition would be altered such that the investor would receive either the greater of the conversion value of the security or the lesser of the face value of the bond or the total value of the firm.[6] Our simplifying assumption, by precluding bankruptcy, means that the simplified model overstates the value of the LYON. Quite clearly, the lower the probability of bankruptcy, the smaller the overstatement of value.

In actual applications of the model, we do, however, compensate for this overstatement of value. Rather than using the risk-free rate of interest as the discount rate, we use an intermediate-term interest rate that is grossed up to capture the default risk of the issuer. This higher discount rate tends to reduce the value of the LYON.[7]

The more vexing assumption is that the term structure of interest rates is flat and known with certainty. This assumption is vexing for two opposing reasons. On the one hand, one of the features of the LYON is the ability of the investor to put the LYON to the issuer at prespecified redemption prices. The redemption feature will be especially valuable if interest rates rise dramatically (and unexpectedly) during the life of the LYON. In that case, the investor would elect to cash in the LYON for the redemption price and invest the proceeds elsewhere. The assumption that future interest rates are known with certainty reduces the value of the put option and, consequently, tends to understate the value of the LYON.

On the other hand, the call option is especially valuable to the issuer if interest rates fall dramatically (and unexpectedly) in the future. In that case, the issuer would call the LYON and issue an alternative security with a lower "cost." For this reason, ignoring interest rate uncertainty tends to overstate the value of the LYON. Which of the two opposing interest rate effects is of greater importance in pricing the LYON depends, among other things, upon the call and redemption schedules specified in a specific indenture agreement. Of course, there are ways in which the model could be expanded to account for interest rate uncertainty. One possible approach, which has been successful in other contexts, is the two factor model of interest rate uncertainty developed by Brennan and Schwartz [3]. We should note that the simplified model does take into account the level of interest rates through the term, r, in equation (3) and changes in r do permit sensitivity analysis with respect to changes in this variable.

The disadvantage of the simplified LYON pricing model is that it may contain

[6] This assumes that the issuer has only two securities outstanding—common stock and the LYON. A model could be developed (as in Brennan and Schwartz [4]) which would allow for multiple senior securities.

[7] A second desirable feature of the alternative assumption is that it is more reasonable to assume that the value of the firm follows a process with constant variance than to assume that the value of the stock follows a process with constant variance. This is because the equity of the firm and all of the firm's senior securities can be considered contingent claims on the total value of the firm. If the total return on the firm follows a process with constant variance, the variance of return on equity must be stochastic because the existence of the firms' senior securities (including the LYON) will affect the stochastic process followed by the stock price.

errors in valuing the LYON and, because of the various opposing effects, the direction of the errors is unknown. The benefit of the simplified model is that it reduces substantially the difficulty of implementing the model. A theoretically more elegant model would encompass three stochastic variables—the value of the issuing firm and the two interest rate factors. Solution of a partial differential equation with three stochastic variables is substantially more difficult than solving a single variable model. Perhaps more importantly, though, are the reduced estimation demands of the simplified model. Implementation of a theoretically more complete model would require estimation of the total value of the firm and of the volatility of the total value of the firm and it would require estimation of the market price of interest rate risk and the parameters of the two factor interest rate process.

The degree to which a theoretically more sophisticated model would enhance the analysis is, of course, an empirical issue for which we do not have a ready answer. For most reasonably secure issuers a more appropriate accounting for default risk would probably have little effect on the theoretical LYON values. Additionally, as regards the question of introducing a stochastic interest rate, we can take comfort from the conclusions of Brennan and Schwartz [4]. They compare traditional convertible bond prices generated by means of a nonstochastic interest rate model with prices generated by means of a single factor stochastic interest rate process and conclude that "... for a reasonable range of interest rates the errors from the certain interest rate model are likely to be slight, and, therefore, for practical purposes it may be preferable to use this simpler model for valuing convertible bonds" (pp. 925–926). Thus, although the commercially-useable LYON pricing model is relatively simple, for most practical purposes it may well be more than adequate given the costs of implementing a more sophisticated model.

IV. Application of the LYON Pricing Model to Waste Management, Inc.

On April 12, 1985 the Waste Management LYON was issued at a price of $250.00 per bond. On April 11, 1985 the closing price of the Waste Management common stock was $52.125. On the issue day, the closing price of the Waste Management LYON was $258.75.

To apply the LYON pricing model to Waste Management, Inc. it was necessary to estimate the volatility of the company's common stock and to specify an appropriate interest rate. The common stock volatility used was the standard deviation of daily returns over the 100 trading days prior to issuance of the LYON. The estimated volatility is 30% per year. Whether this is the appropriate estimation period or technique is an open question—which we cannot resolve here—but sensitivity analysis does allow us to determine the likely impact of errors in the estimate of the stock price volatility.

The interest rate used is 11.21% per year. The rate was chosen because on the issue date this was the approximate yield of intermediate term bonds of the same risk rating as the Waste Management bond.

Table I

Waste Management Common Stock Prices, Theoretical LYON Prices, And Reported LYON Market Prices from April 12, 1985 Through May 10, 1985

Date	Closing Stock Price	Closing LYON Market Price	High LYON Market Price	Low LYON Market Price	LYON Theoretical Price
April 12, 1985	$52¼	$258.75			$262.7
15	53	258.75	$260.0	$258.75	264.6
16	52⅝	257.5	257.5	257.5	263.7
17	52	—	—	—	262.1
18	52⅜	257.5	275.5	255.0	263.0
19	52¾	257.5	257.5	257.5	264.0
22	52½	257.5	257.5	257.5	263.3
23	53¼	260.0	260.0	257.5	265.3
24	54¼	265.0	265.0	262.5	267.9
25	54¼	265.0	265.0	262.5	267.9
26	54	265.0	265.0	265.0	267.2
29	53¾	260.0	265.0	260.0	266.6
30	52⅛	260.0	260.0	257.5	262.4
May 1, 1985	49¾	252.5	257.5	252.5	256.7
2	50½	250.0	252.5	250.0	258.4
3	50¾	252.5	252.5	252.5	259.0
6	50½	252.5	255.5	251.25	258.4
7	50⅞	255.0	256.25	252.5	259.3
8	50¾	253.75	257.5	253.75	259.0
9	51¼	255.0	255.0	253.75	260.3
10	53⅛	260.0	260.0	255.0	265.0

Finally, the dividend yield of the Waste Management common stock was specified as a constant 1.6% per year. This yield was chosen because the company's previous quarterly dividend payment was $.20 per share. With recent stock prices of approximately $50.00 per share, this dividend payment provides an annual yield of 1.6% (i.e., 4 × $.20/$50.00).

With these parameters, and the data given in the Waste Management prospectus, the theoretical LYON price on the issue date was $262.70. As the data in Table I indicate, over the first four weeks following issuance, the theoretical LYON prices closely track the reported market closing prices, although there is a tendency for the model prices to overstate slightly the reported closing prices. Whether this slight overstatement in prices is due to the simplicity of the model or due to an error in the estimation of the stock volatility is not known. Apparently, though, the model is sufficiently accurate to provide a rough guideline for the pricing of new LYON issues. Other LYON issuers would, of course, have different characteristics than Waste Management and would be issuing the security in other interest rate environments. For that reason it is interesting to investigate the sensitivity of the theoretical LYON value to changes in the values of the parameters used in the base case example.

Panel A of Table II illustrates the sensitivity of the LYON price to changes in the level of the issuer's stock price and to changes in the issuer's stock price volatility. It should come as no surprise that the LYON value increases monotonically with increases in the issuer's stock price and with increases in the volatility of the issuer's stock price. Additionally, as is the case with other stock price contingent claims, the LYON value is highly sensitive to changes in the volatility of the underlying stock. The result emphasizes the importance of accurate stock volatility measurement procedures—an area in which the volume of research now approaches that of a small cottage industry.

Panel B of Table II illustrates the sensitivity of LYON values to changes in the issuer's dividend yield. The table indicates that the LYON value declines monotonically with increases in the issuer's dividend yield. This occurs because a higher dividend yield implies a lower expected rate of stock price appreciation. Additionally, the value of dividends is not impounded in the LYON price because the LYON investor does not receive dividend payments. Perhaps somewhat surprisingly, the LYON values are not terribly sensitive to changes in the dividend yield. For example, for the base case stock price of $52.125, an increase in the dividend yield from 1.6% to 3.0% reduces the LYON value by only about $3.00 per bond.

In a separate analysis not shown here, LYON values were computed with the dividend specified to grow at a constant rate (rather than being specified as a constant yield). That analysis indicated that the theoretical LYON values are even less sensitive to major changes in the assumed dividend growth rate.

Panel C of Table II illustrates the sensitivity of the theoretical LYON values

Table II

Sensitivity Of The Theoretical LYON Values To Changes In The Issuer's Stock Price, Stock Price Volatility, And Dividend Yield And To Changes In The Interest Rate

A. Sensitivity of LYON Values To Changes In The Issuer's Stock Price Volatility

Stock Price[a] (per share)	Stock Price Volatility[b] (per year) 0.10	0.20	0.30	0.40	0.50
$46.00	$223.23	$236.01	$247.34	$257.22	$265.10
47.00	224.67	237.92	249.48	259.44	267.33
48.00	226.26	239.92	251.69	261.71	269.59
49.00	228.02	242.03	253.96	264.03	271.90
50.00	229.94	244.24	256.30	266.39	274.23
51.00	232.03	246.54	258.71	268.80	276.60
52.00	234.28	248.94	261.18	271.26	279.00
53.00	236.71	251.44	263.71	273.76	281.44
54.00	239.29	254.04	266.31	276.30	283.91
55.00	242.04	256.73	268.97	278.88	286.40
56.00	244.94	259.51	271.68	281.51	288.93
57.00	247.99	262.38	274.46	284.17	291.49
58.00	251.19	265.34	277.29	286.87	294.08
59.00	254.52	268.39	280.18	289.62	296.69

Table II—*continued*

B. Sensitivity of LYON Values To Changes In The Issuer's Dividend Yield

Stock Price[a] (per share)	Dividend Yield[c] (per year) 0.0%	1.6%	3.0%	5.0%
$46.00	$250.50	$247.34	$244.84	$241.34
47.00	252.72	249.48	246.90	243.28
48.00	255.00	251.69	249.03	245.31
49.00	257.34	253.96	251.24	247.40
50.00	259.74	256.30	253.52	249.58
51.00	262.20	258.71	255.87	251.83
52.00	264.72	261.18	258.29	254.16
53.00	267.30	263.71	260.78	256.57
54.00	269.93	266.31	263.34	259.05
55.00	272.62	268.97	265.96	261.61
56.00	275.36	271.68	268.65	264.25
57.00	278.14	274.46	271.40	266.96
58.00	280.98	277.29	274.22	269.74
59.00	283.86	280.18	277.10	272.60

C. Sensitivity of LYON Values To Changes In The Interest Rate

Stock Price[a] (per share)	Interest Rate[d] (per year) 7.21%	9.21%	11.21%	13.21%	15.21%
$46.00	$301.36	$264.73	$247.34	$235.80	$228.43
47.00	302.19	266.38	249.48	238.27	231.13
48.00	303.07	268.10	251.69	240.82	233.91
49.00	304.01	269.89	253.96	243.43	236.75
50.00	305.00	271.74	256.30	246.10	239.66
51.00	306.04	273.66	258.71	248.85	242.63
52.00	307.14	275.64	261.18	251.66	245.66
53.00	308.29	277.69	263.71	254.53	248.75
54.00	309.49	279.81	266.31	257.46	251.90
55.00	310.76	281.98	268.97	260.45	255.11
56.00	312.07	284.22	271.68	263.50	258.38
57.00	313.44	286.51	274.46	266.60	261.70
58.00	314.87	288.87	277.29	269.77	265.07
59.00	316.35	291.29	280.18	272.98	268.49

[a] Base case stock price is $52.125 per share.
[b] Base case stock price volatility is 0.20 per year.
[c] Base case dividend yield is 1.6% per year.
[d] Base case interst rate is 11.21% per year.

to changes in the discount rate. As we would anticipate, the LYON value declines monotonically as the interest rate increases.

In evaluating our example LYON we have proceeded as if the terms of the contract were given and have analyzed the sensitivity of the LYON value to the issuer's stock price volatility and dividend payment policy and the level of interest rates. However, the more likely situation is one in which these parameters are given and the issuer wishes to analyze the effect of changes in the terms of the contract on the LYON price. The LYON pricing model permits an analysis of the various tradeoffs between the terms of the contract and the LYON price. For example, the issuer may wish to examine the effect on the LYON price of changes in the conversion ratio or of changes in the schedules of put prices and call prices specified in the LYON indenture.

Illustrating the sensitivity of the theoretical LYON value to changes in the conversion ratio and the redemption and call schedules is a somewhat more complicated procedure because there exists an infinite number of possible ratios and schedules. However, to give some indication of the sensitivity of the LYON price to changes in the redemption and call schedules, Table III presents values of the LYON with and without the call and redemption features. Column 1 gives the issuer's stock price, Column 2 presents the value of the LYON with the redemption and call schedules as specified in the Waste Management prospectus, Column 3 gives the value of the LYON without the call option (but with the redemption option), Column 4 gives the value of the LYON without the redemption option (but with the call option), and Column 5 gives the value of the LYON without the call option and without the redemption option. Thus, Column 5 gives the value of a zero-coupon convertible bond.

As the table indicates, the call option is valuable to the issuer. When the call option is removed, the LYON value increases. Similarly, the redemption option is valuable to the investor. When the redemption option is removed, the LYON value declines. The two effects are not symmetric. Removal of the call feature in the base case increases the value of the LYON by about \$20.00, whereas removal of the redemption option reduces the value of the LYON by almost \$50.00. Nevertheless, when both features are removed (in Column 5) the LYON value is almost the same as when the LYON contains both features. Obviously, the value of the LYON is not merely the sum of the values of its individual components. Each of the features of this complex security interacts with the others to determine the security's value.

V. The Optimal Stock Price to Convert a LYON

An important feature of the LYON is that issuers may deduct the imputed interest costs of the security without any offsetting cash outflow to investors. This tax shelter may be valuable to LYON issuers. Once the LYON is converted, however, this tax shield disappears. For this reason, LYON issuers may be concerned that investors will convert their LYON prematurely.

At any point in time, the investor can choose to convert the LYON. In deciding whether to convert, the investor weighs the value of the dividends he gives up by continuing to hold the LYON against the value of the downside risk protection that he gives up by converting the LYON to the issuer's common stock. The downside risk protection is provided by the redemption option held by the investor.

In general, when the dividend yield of the issuer's stock is relatively low, the benefits of conversion (to obtain the dividend) also will be relatively low. In the

Table III

Analysis Of The Value Of A LYON With And Without The Call And Redemption Options

Stock Price[a] (per share)	Callable[b] Redeemable LYON (per bond)	Noncallable Redeemable LYON (per bond)	Callable Nonredeemable LYON (per bond)	Noncallable Nonredeemable LYON (per bond)
$45.00	$245.28	$264.85	$181.94	$244.08
46.00	247.34	267.26	186.48	246.92
47.00	249.48	269.72	191.02	249.79
48.00	251.69	272.22	195.58	252.69
49.00	253.96	274.76	200.14	255.63
50.00	256.30	277.34	204.72	258.60
51.00	258.71	279.96	209.30	261.60
52.00	261.18	282.62	213.89	264.63
53.00	263.71	285.31	218.49	267.69
54.00	266.31	288.04	223.10	270.78
55.00	268.97	290.81	227.72	273.90
56.00	271.68	293.61	232.34	277.05
57.00	274.46	296.44	236.98	280.22
58.00	277.29	299.31	241.62	283.43
59.00	280.18	302.22	246.27	286.66
60.00	283.13	305.15	250.93	289.92

[a] Base case stock price is $52.125 per share.

[b] This column represents the base case. The call and redemption schedules in the base case are taken from the Waste Management LYON prospectus (see Section I).

extreme, when the underlying common stock pays no dividend, there is no incentive for the investor ever to convert the LYON into common stock. Similarly, for low dividend paying stocks there is relatively little incentive for the investor to convert the LYON into common stock. However, even for low dividend paying stocks, if the stock price rises high enough, it will be so far above the put price that the downside protection provided by the investor's put option become negligible. In that case, the investor will decide optimally to convert to common stock.

The LYON pricing model can be used to calculate the stock price at which it is optimal to convert a LYON. The optimal conversion stock price is the price at which the investor is just indifferent between holding the LYON and converting to common stock. At any stock price above this critical point, the investor is better off to convert to common stock. At any stock price below this critical point, the investor is better off holding the LYON.

The critical conversion stock price is that price at which the present value of the future dividends forgone by continuing to hold the LYON just equals the present value of the downside protection forgone if the investor converts to common stock. The present value of the downside protection forgone is the expected loss to the investor if he converts now and the conversion value of the LYON at maturity (if the investor had held the LYON) turns out to be less than the security's face value at that date.

In most cases, the critical conversion stock price would imply a LYON value that exceeds the specified call price. Thus, in most cases, if issuers follow the call policy that minimizes the value of the LYON, the issuer would call the bond prior to the point at which the investor would optimally convert. To calculate the stock price at which it is optimal for the investor to convert, it is necessary to assume that the issuer follows a policy of never calling the bond or, alternatively, to assume that the bond is noncallable. With this assumption, the critical stock price can be determined by solving equation (3) subject to boundary conditions (4), (5) and (6). At the critical stock price, the value of the LYON is equal to its conversion value. As an illustration, Column 2 of Table IV displays the stock price at which it would be optimal for an issuer to convert the Waste Management LYON on each anniversary date. At the issue date (or immediately thereafter) the stock price would have to increase to \$129.50 per share. As time progresses, the critical stock price increases. The critical stock price increases for two reasons. First, as time passes, the present value of the dividends forgone by holding the LYON declines. Secondly, because the redemption prices of the LYON increases through time, the value of the downside risk protection for holding the LYON increases. Both of these effects reduce the incentive to convert. However, with two years remaining to maturity, the optimal conversion price declines. This occurs because the critical conversion stock price at the maturity date of the LYON equals the bond's face value divided by the conversion ratio. In this case that critical value is \$1,000/4.36 = \$229.36. Because the optimal conversion value previously calculated is above that level, the critical price declines as the term-to-maturity of the bond becomes shorter.

Table IV presents the optimal conversion price for one set of parameters. However, the model is flexible. Issuers that are concerned about premature

Table IV

The Stock Price At Which It Is Optimal To Convert A Waste Management LYON

Anniversary Date	Optimal Conversion[a] Stock Price (per share)
At Issue	$129.50
June 30, 1985	132.00
June 30, 1986	145.00
June 30, 1987	158.50
June 30, 1988	173.50
June 30, 1989	194.50
June 30, 1990	217.00
June 30, 1991	238.50
June 30, 1992	257.00
June 30, 1993	273.00
June 30, 1994	287.00
June 30, 1995	301.50
June 30, 1996	316.00
June 30, 1997	329.50
June 30, 1998	339.00
June 30, 1999	340.00
June 30, 2000	317.50
January 21, 2001	229.36

[a] Data used to calculate the optimal conversion stock price are taken from the base case example and the Waste Management LYON prospectus.

conversion could use the LYON pricing model to test the sensitivity of the optimal conversion price to changes in the terms of the contract and to changes in dividend policy.

VI. Conclusion

Following the pathbreaking work by Black and Scholes [1] and Merton [7], contingent claims pricing methodology has been applied to the pricing and analysis of a wide variety of securities—put options, convertible bonds, warrants, forward contracts, futures contracts, mortgage-backed securities and many others. Models for analyzing some of these securities give rise to closed-form solutions. Models for many others can be solved only with numerical techniques. Those models with closed-form solutions—especially stock option pricing models—have been readily adopted by practical market participants. Those models requiring numerical solution techniques have not yet met wide acceptance, probably because of limitations imposed by the lack of availability of the computer hardware and software needed to implement the models. In this paper we report on one case in which numerical solution techniques were used in a practical situation to solve a simplified model for pricing and analyzing a complex security. Presumably, as more powerful personal computers evolve and as the availability of the software used with numerical solution techniques increases, market practitioners will find other situations in which contingent claims pricing models that can be solved only with numerical techniques can be of use in analyzing complex securities.

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DISCUSSION

SCOTT P. MASON*: This paper by John McConnell and Eduardo Schwartz (M&S) concerns the pricing of Liquid Yield Option Notes (LYONs) through an application of Contingent Claims Analysis (CCA). The paper is an excellent example of the art of striking a balance between rigor and practicality in applying CCA to real-world problems. I will try to keep my comments consistent with this "trade-off" spirit of the paper since it is easy to describe (as M&S do) more rigorous or elegant approaches to the LYON pricing problem.

The approach that M&S take is to treat the LYON as a single security which is a function of the firm's stock price. As discussed in the paper, it would be more complete to recognize the role of interest rate uncertainty but the trade-off between the practicality and increased accuracy of this approach is questionable. At this junction I might have explored the possibility of viewing the LYON as a callable/puttable unit comprised of a risky zero coupon bond and a warrant which requires the use of the bond as scrip to exercise. The merit of this approach is to underscore the fact that the LYON is made up of a warrant, not a call option, and therefore has a dilutive effect on the firm's equity. It is also true that with this approach the discount rate for the risky discount bond must be specified as opposed to a discount rate for the entire LYON.

Furthermore, with regards to M&S's specification of a single risk adjusted discount rate for the LYONs problem, I would have specified the short risk free rate of interest as time dependent in a manner consistent with the implied forward rates in the U.S. Treasury term structure. I would have then priced the

* Harvard University