# Convertible Bonds: Model, Value Attribution, and Analytics

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*Convertible bonds provide investors an option to convert the bond into the underlying equity. For this reason, a convertible bond is exposed to both equity and interest rate risk. Incorporating these two sources of risk into the model is particularly important for callable issues. In this study, a two-factor model is used to analyze a sample of bonds. The model shows that the correlation of stock risk and interest rate risk may affect convertible bond prices significantly. The bond pricing model also provides portfolio analytics and can decompose a convertible bond into its basic components—the stock and bonds with different maturities. This approach enables an investor to implement a more precise hedging strategy than is possible using only delta.*

A convertible bond is a corporate bond or a structured product that offers investors the right to convert a bond to either a specified number of shares of stock of the issuing firm or an equitylike index. This basic structure has many variations that have been adapted for cross-currency bonds, preferred stock, and high-yield and investment-grade bonds. Also, these bonds have many special features, including call and put provisions, mandatory conversion, and restrictions on conversions. In essence, however, a convertible bond is a hybrid security that offers the market special investment characteristics and opportunities.

The hybrid feature of convertible bonds provides investors with the downside protection of bonds and the upside return of equities. For this reason, convertible bonds are an efficient way to implement some asset allocation strategies that take advantage of both fixed-income and equity markets. For equity investment, convertible bonds offer protection against losses, enabling portfolio managers to invest in riskier firms without exceeding their risk-exposure limits. For fixed-income investors, the equity kicker offers upside returns that offset the firm's credit risk. For these reasons, convertible bonds provide managers with investment choices not often available in equity or fixed-income markets.

A valuation model is an important tool for implementing these investment strategies. The model can be used to determine the fair value and isolate the equity and fixed-income components of convertible bonds. It can also establish the worth of the special features of the bonds. The result is an appropriate procedure for comparing the values of the convertible bonds through their corresponding building blocks of values. From this comparison, investors can formulate a variety of approaches, including arbitrage, hedging, and asset allocation strategies.

Because convertible bonds are hybrid securities, the valuation model must capture all the salient features of a vanilla corporate bond valuation model, as well as those of an equity model. Breakeven analysis, yield advantage, and other non-model-based analyses are inadequate for managing these complex securities, because those methods do not capture the intricate relationships of such market parameters as stock and interest rate volatilities, stock return, and rate correlations, nor do they capture the bond's features such as the call provisions.

What combination of stocks and bonds is needed to hedge a convertible bond? What is the bond's price sensitivity along the yield curve? What is the value of each of the special features of the bond? This article presents a convertible bond valuation model that can deal with such issues. This model is based on a two-factor, arbitrage-free framework. Ingersoll (1977a, 1977b) proposed a convertible bond model based on a stock option model. This perspective, however, does not account for the fact that proper bond valuation requires a more precise interest rate evaluation process than do stock options. Merton (1973) and Black and Cox

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(1976) took a similar approach to Ingersoll but applied this perspective to securities in general. Brennen and Schwartz (1980) and Longstaff and Schwartz (1993) presented a two-factor model that includes both stock and interest rate risks. That model, however, does not take the market yield curve as given and therefore may not ensure that the bond valuation is consistent with the market time value of money. Ho (1990) described a one-factor, arbitrage-free model that takes the spot curve as given so that the convertible bond pricing, isolated from the equity value, is ensured to be consistent with the market. Also, the model assumes away interest rate risk and thus fails to identify the impacts of cash call value when the interest rate falls and of the correlation of stock and interest rate returns on the bond's price.

The two-factor, arbitrage-free model presented here builds upon the prior attempts to evaluate convertible bonds. Because this structure incorporates both stock and interest rate risks, the model can take into account the embedded stock and interest rate options among the convertible's special features. Furthermore, the model is arbitrage free in that it prices the convertible bond relative to the observed Treasury and corporate bond market as well as to the stock price. The model takes the observed Treasury and stock prices as given. The fair convertible bond price is defined as the cost of replicating the convertible bond by using stocks and straight bonds and following the basic principle of the Black–Scholes model of relative valuation. Relative valuation does not assume that the stocks and bonds are correctly priced; in fact, they could be mispriced. The importance of the arbitrage-free model (or relative valuation) is its ability to isolate the analysis of the value of the underlying securities from the contingent claim, which in this case is the convertible bond.

## VALUATION FRAMEWORK

This section describes the valuation model and presents a selection of convertible bonds, showing how the model can analyze them.

### Bond Descriptions

Table 1 uses seven bonds to illustrate the analysis of convertible bonds. Because the bonds are used for purposes of comparison, they differ in parity value and credit risk, but they are all U.S. dollar based, have analogous call provisions, and have no other special features.

The bonds are standard convertible bonds with maturities of less than 10 years. The coupon rates are fixed, and the issuers are U.S. corporations.

Bond price is the observed bond price, including the accrued interest based on March 8, 1995. Parity is the stock price multiplied by the number of shares to which the bond may convert. Yield to maturity is the internal rate of return of the bond, given the bond price and the stated coupon and principal. Because a bondholder can always convert the bond to parity, the arbitrage condition ensures that the bond price must exceed the parity value. The percentage the bond trades above the parity is given by $Pm/P$.

$Pm/P$ shows that Allegheny Ludlum is trading very close to parity. The bond therefore is behaving more like a stock. Because a significant portion of the bond value is equity, the yield to maturity is relatively low. By contrast, WMS Industries is trading significantly above parity. Indeed, the bond is primarily a fixed-income entity. For this reason, the yield to maturity is comparable to that of a vanilla corporate bond.

### Assumptions

A number of assumptions are critical to this model.

▒ *Binomial Lattice*. The market is assumed to trade at discrete times. At each interval, the stock can move up or down, as can interest rates (the yield curve). Therefore, at each state (node point),

**Table 1. Convertible Bond Descriptions**

| ID | Name | Amount Outstanding (millions) | Coupon | Maturity Date | Bond Price | Parity | Yield to Maturity | $Pm/P$[a] |
|---|---|---|---|---|---|---|---|---|
| ALS | Allegheny Ludlum Corporation | $100 | 5.875% | 03/15/02 | $105.13 | $101.85 | 5.00% | 3.2% |
| AMB | American Brands Inc. | 229 | 5.750 | 04/11/05 | 114.50 | 94.63 | 4.00 | 21.0 |
| GSX | General Signal Corporation | 102 | 5.750 | 06/01/02 | 102.13 | 89.89 | 5.34 | 13.6 |
| TREN | Trenwick Group | 105 | 6.000 | 12/15/99 | 101.00 | 90.73 | 5.70 | 11.3 |
| PIR | Pier 1 Imports | 69 | 6.875 | 04/01/02 | 98.25 | 81.25 | 7.19 | 20.9 |
| AES | AES Corporation | 46 | 6.500 | 03/15/02 | 92.13 | 68.81 | 7.99 | 33.9 |
| WMS | WMS Industries Inc. | 51 | 5.750 | 11/30/02 | 88.13 | 67.67 | 7.82 | 30.2 |

[a] $\dfrac{\text{Bond price} - \text{Parity}}{\text{Parity}} \times 100.$

four possible outcomes could result from the recombination of the stock and rate movements. That is, over two periods, for a given interest rate movement, the stock that rises after falling would reach a node point that is the same as that reached by the stock that falls after rising. An analogous recombining requirement applies to the bond movements. Therefore, this two-dimensional binomial lattice has 4 node points after one period, followed by 9 node points in the second period, 16 node points in the third period, and a continuing pattern of increase in subsequent periods.

*Arbitrage-Free Interest Rate.* The two-dimensional binomial lattice is constructed so that when the stock movement is ignored, the two-dimensional lattice is identical to the one-factor, arbitrage-free model described by Ho and Lee (1986) and Black, Derman, and Toy (1990). Other one-factor models such as those of Dothan (1978) and Cox, Ingersoll, and Ross (1985) can also be adapted to agree with this model.

When this two-dimensional binomial model is used to price interest rate contingent claims, it ensures that the bond values are consistent with the observed Treasury yield curve. As defined in Ho and Lee, the yield curve movements at each node point are arbitrage free over each period. Furthermore, the projected yield curves exhibit mean reversion with a declining term structure of volatilities as input.

The credit risk of the bond is captured by the option-adjusted spread (OAS). OAS is a constant spread that adds to the Treasury rate at each node point. The sum of the OAS and the one-period Treasury rate is used to discount the bond's cash flow.

*Stock Movements.* At each node point, the expected return of the stock is the same as the one-period Treasury rate, thus ensuring that the arbitrage-free assumption is satisfied. This assumption is identical to that used by Ingersoll (1977a and 1977b), Brennan and Schwartz (1980), and Ho (1990). There are two kinds of stock volatilities. At each node point, the stock return may rise or fall. The standard deviation of the stock return is the idiosyncratic stock risk.

Because interest rates also rise and fall and the expectation of the stock return must also rise and fall in step with such movements, interest rate risk induces additional stock risk. Therefore, even though the stock has no idiosyncratic risk, the stock return over a certain time horizon is still unpredictable.

We define stock volatility to be only the idiosyncratic stock risk. Similarly, at each node, we define the correlation between the stock price and short-term interest rate movement. We assume that the stock volatility and the correlation are the same for all the node points.

*Optimal Decision of Forced Conversion.* When a bond is called "efficiently," the issuer exercises the call option whenever the convertible bond price exceeds the call price. Empirically, such is not the case with convertible bonds. Many convertible bonds trade significantly above the call price. One reason for such "inefficiency" is that the issuer wants to make sure that the convertible bondholders will all convert to stock and not redeem the bonds at the (hard) call price.

Therefore, we assume that the issuer will call the bonds only when the parity exceeds the call price by such an amount that the probability for the parity to fall below the call price at the end of the month is less than $X$ percent. "Call shift" is the standard deviation that gives such a probability. For example, a call shift of 0.5 is equivalent to a 30 percent probability of the parity falling below the call price over a one-month period. In this study, we assumed that the call shift is 0.5.

## VALUE ATTRIBUTION PROCEDURES

This section proposes a systematic procedure to evaluate the building blocks that compose the convertible bond's value. First, we examined the most basic part of the corporate bond value, referred to as the "investment value." Then, we examined the warrant embedded in all convertible bonds, the "latent warrant." Finally, we calculated the impact of the forced conversion and the value of the call option.

### Investment Value

Investment value is the present value of the bond's coupons and principal. More precisely, investment value is the value of the convertible bond, assuming that the bondholder will not convert the bond any time in the future. This investment value can be computed from the convertible bond model by taking the parity value as zero.

In essence, the investment value can be derived as follows, beginning with the Treasury spot yield curve. The spot yield curve for March 8, 1995, is given below.

| Term | 3mo | 1yr | 2 | 3 | 4 | 5 | 7 | 10 |
|---|---|---|---|---|---|---|---|---|
| Yield (%) | 6.348 | 6.775 | 6.972 | 7.069 | 7.145 | 7.214 | 7.345 | 7.532 |

For each bond, we determined the OAS of that bond by deriving it from similar corporate bonds. For example, for investment grade, we used the average OAS of the sector and rating group. For high yields, we evaluated the spread of each bond. Then, we used the spot curve and the OAS as inputs to the Ho and Lee (1986) one-factor interest rate model, using the term structure of volatilities derived for the pricing date, which are 13.49 percent for the 1-year rate and 11.32 percent for the 10-year rate. The hard call provision is also taken into account in the valuation. The Ho and Lee model was used to price this callable corporate bond and find the investment value. The results obtained for the selected bonds are presented in Table 2.

The OAS of the investment-grade bonds was estimated based upon a bond-pricing data base. OAS data for the high yields were not available, so the OAS numbers used here are for illustrative purposes only. These bonds are callable within six months at the first call price. Duration (*Dur*) is the price sensitivity to interest rate changes. *Dur* represents the percentage change in bond value for a 100 basis point shift in the spot yield curve. The call option value is calculated in basis points given by the call spread, *CallSpr*. These option values are small because we have ignored the equity component in the convertible bond. Indeed, interest rate volatility significantly affects the call option value for Pier 1 Imports.

Pier 1's bond price is \$98.25, and its call price is \$104.3. The bond has a duration of 4.99 years. If interest rates fall 121 basis points, and even if the parity remains at \$81.25, the convertible bond price would exceed the call price. In this case, the issuer may exercise the cash call. Because *CallSpr* ignores that equity component, its value understates the impact of the interest rate risk on the cash call option value.

*Prm* is the bond value net of the investment value as a percentage of the investment value. The analysis shows that Allegheny Ludlum, American Brands, and General Signal have more equity, exceeding the investment value, and Pier 1, AES Corporation, and WMS Industries are trading more similarly to vanilla corporate bonds.

## Latent Warrant

Consider a convertible bond for which the issuer does not use the call provision to force the bondholder to convert to stocks. In this case, Ho (1990) has shown that the bondholder will only convert the bond at the maturity date (assuming that the dividend yield is low). Therefore, the convertible bond can be viewed as a portfolio of a bond whose value is the investment value. This bond has a warrant with exercise price of \$100 (the par value), an expiration date that is the same as the bond maturity date, and an underlying security value that is equivalent to the parity.

Typically, the latent warrant is a deep in-the-money option whenever parity—the product of the stock price and conversion ratio—is close to or above par (face amount). In these cases, the likelihood of the parity exceeding par at maturity is very high. Option pricing theory shows that the value of these warrants should be the parity minus the present value of par plus a small premium. Consider Allegheny Ludlum, American Brands, and General Signal. Using the convertible bond model to derive the warrant value produces results that are similar to the net values of parity and the present value of par.

To the extent that the parity trades at discount, the warrant valuation model is important. The warrant model depends crucially on the stock's volatility, the stock and rate correlation, and dividend assumptions. Warrant value is related positively to the stock volatility and negatively to the dividends. Percentage warrant value is defined as the warrant value as a percentage of the investment value.

The results, shown in Table 3, show that the warrant values are highly significant, even for those cases in which the parity is relatively low.

## Forced Conversion Value

Because of the arbitrage condition, the convertible bond must trade above the maximum of the investment value and the parity. It also must trade

**Table 2. Investment Values of Convertible Bonds**

| ID | Rating | Sector | Option-Adjusted Spread | Call Price | Call Spread (bps) | Investment Value | Duration (years) | *Prm*[a] |
|---|---|---|---|---|---|---|---|---|
| ALS | A– | Industrial | 84 | \$104.1 | 2 | \$90.726 | 5.24 | 15.87% |
| AMB | A | Industrial | 75 | 100.0 | 1 | 85.538 | 6.96 | 33.86 |
| GSX | A– | Industrial | 80 | 103.5 | 2 | 88.640 | 5.48 | 15.21 |
| TREN | BBB | Finance | 91 | 103.4 | 1 | 93.173 | 3.91 | 8.40 |
| PIR | B | Retail | 115 | 104.3 | 4 | 94.471 | 4.99 | 4.00 |
| AES | B+ | Industrial | 145 | 105.0 | 3 | 91.207 | 5.10 | 1.01 |
| WMS | B | Entertainment | 155 | 103.6 | 1 | 83.654 | 5.76 | 5.35 |

[a] $\dfrac{\text{Bond price} - \text{Investment value}}{\text{Investment value}} \times 100.$

**Table 3. Percentage Warrant Value**

| ID | Industry | Dividend Yield | Volatility | Correlation[a] | Warrant Value | Warrant Value |
|---|---|---|---|---|---|---|
| ALS | Steel | 2.32 | 35% | –16% | $50.79 | 57% |
| AMB | Tobacco | 5.35 | 35 | –18 | 54.28 | 55 |
| GSX | Electrical Equipment | 2.70 | 17 | –29 | 32.02 | 32 |
| TREN | Insurance | 2.54 | 25 | –21 | 27.42 | 28 |
| PIR | Retail | 1.23 | 27 | –14 | 29.74 | 30 |
| AES | Energy | 0.00 | 25 | –19 | 19.28 | 19 |
| WMS | Entertainment | 0.00 | 32 | –5 | 24.26 | 24 |

[a]Correlation between the stock return and rate change.

below the sum of the latent warrant value and the investment value. The forced conversion value determines how the bond should be traded within this range. More precisely,

Bond price = Investment value + Warrant – Forced conversion value,

or

$$B = I + W - C. \tag{1}$$

Rewriting gives

$$(B - I)/I = (W - I)/I - (C - I)/I. \tag{2}$$

Thus,

$$\%B = \%W - \%C, \tag{3}$$

where

$\%B = 100\,(B - I)/I$
$\%W = 100\,(W - I)/I$
$\%C = 100\,(C - I)/I$

The value attribution diagram, Figure 1, is a plot of $\%B$ and $\%W$ against $\%P$, where

$\%P = 100\,(P - I)/I$
$\%Q = 100\,(Q - I)/I$
$Q$ = quoted price
$P$ = parity

The value attribution diagram provides a framework to compare convertible bond values by comparing the worth of each of their components.

The significant magnitude of %C in Table 4 demonstrates the importance of forced conversion. The callability of convertible bonds contributes more than 10 percent of investment value in all the cases. Comparing %B and %Q, we see that the fair value is similar to the quoted price, except for WMS Industries and American Brands. WMS Industries is a high-yield bond, and the discrepancy between its listed fair value and the quoted price may simply arise from the assumed credit spread, which is not observable. For American Brands, the discrepancy is too significant to be explained by any of the market assumptions. In this case, further investigation into the bond is necessary.

**Table 4. Value Attribution**

| ID | %P | %W | %C | %B | %Q |
|---|---|---|---|---|---|
| ALS | 17.3 | 57.8 | 30.2 | 23.6 | 21.1 |
| AMB | 12.3 | 63.7 | 34.2 | 23.8 | 35.9 |
| GSX | 2.9 | 32.4 | 18.3 | 15.7 | 16.9 |
| TREN | –1.4 | 27.6 | 13.5 | 15.1 | 9.7 |
| PIR | –12.7 | 29.9 | 20.4 | 10.0 | 5.5 |
| AES | –20.9 | 19.5 | 11.4 | 9.0 | 5.8 |
| WMS | –18.8 | 24.3 | 12.5 | 14.1 | 5.6 |

## Relative Valuation by Components

The analytics of the model enable us to break the bond value down into the components shown in Table 5. The breakdown can be represented by the following equation:

Bond quoted price = Investment value + Warrant – Conversion + Cheap/rich.

This decomposition can assist investors in identifying whether a convertible bond fits their investment needs.

# ANALYTICS

This section considers some of the standard and model-driven analysis of convertible bonds.

## Value Analytics

Breakeven yield (*BE*) is the period of time that is required for the yield advantage of a bond to pay for its premium:

$BE$ = Premium/Yield pick up, or

$$\frac{\text{Convertible bond price} - \text{Parity}}{\text{Coupon rate} - \text{Dividend yield}}.$$

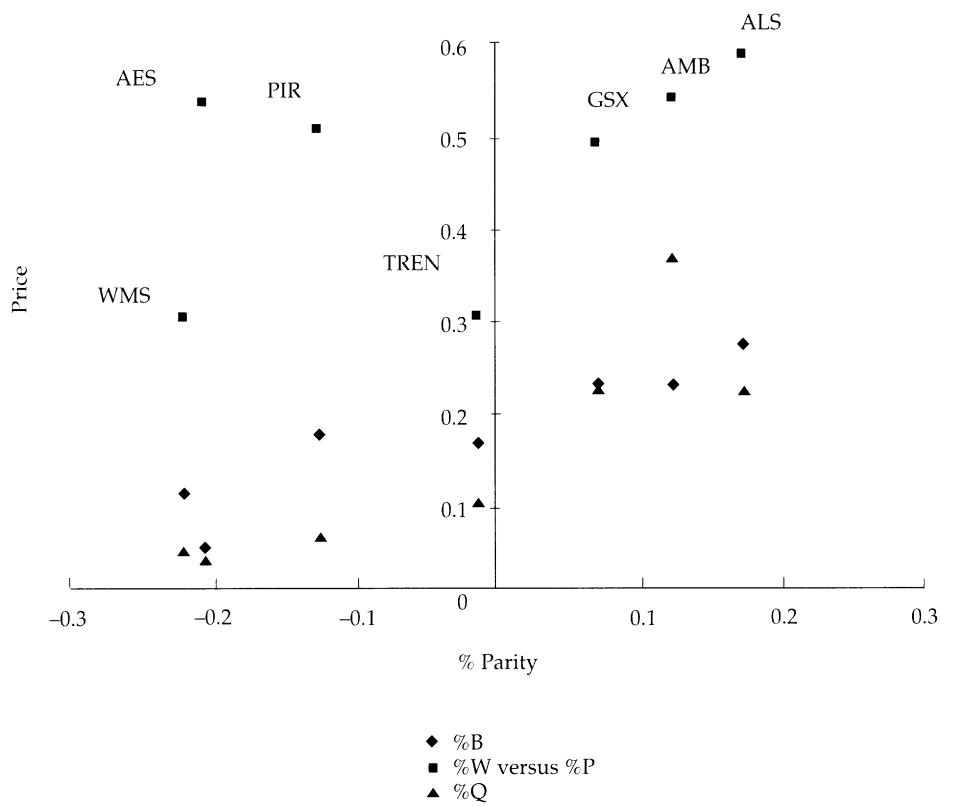
For valuation purposes, a short breakeven is preferable.

Cheap/rich is the observed price or quote net of the fair value. Positive cheap/rich means the bond is rich (expensive) and less preferred. Conversely, a negative cheap/rich value means that the bond is of good value.

## Sensitivity Analytics

Duration, as mentioned earlier, is the percentage change in the bond price when the spot yield

**Figure 1. Value Attribution Diagram**




curve undergoes a 100 basis point parallel shift. Extending this definition to the sensitivity along the yield curve provides the key rate duration. A key rate duration is the percentage change of the bond price when there is a 100-basis-point change for one key rate. The sum of all the key rate durations is equal to the duration number. Details are explained in Ho (1990).

Convexity is the incremental price change not attributable to effective duration. Delta is the unit change in bond price for one unit change in parity. Gamma is the second derivative of the bond value for the stock price divided by the bond value. Vega is the unit change in the bond price for a 1 percentage point change in stock volatility.

Table 6 shows that Allegheny Ludlum is not sensitive to interest rate shifts. When the interest rate falls by 1 percentage point, the convertible bond value only changes by 0.23 percent. This insensitivity is not a result of the length of the bond maturity; it is because of the relatively high parity value. Delta demonstrates how the convertible bond reacts to the change in parity value. For Allegheny Ludlum, the convertible bond value rises or falls at a 75 percent proportion of the change in parity value. Vega shows the sensitivity of the bond to changes in stock volatility. For example, if the stock volatility goes up 1 percentage point, Allegheny Ludlum's value will rise by $0.137.

**Table 5. Breakdown of Bond Value**

| ID | Investment Value (a) | Warrant (b) | Call Free (a+b) | Forced Conversion (c) | Bond Price (Fair) (a+b–c) | Bond Price (Observed) (d) | Cheap/ rich |
|---|---|---|---|---|---|---|---|
| ALS | $87.90 | $50.79 | $138.69 | $30.25 | $107.35 | $105.13 | –2.22% |
| AMB | 85.20 | 54.26 | 139.48 | 34.24 | 104.32 | 114.50 | +10.18 |
| GSX | 88.42 | 32.02 | 120.44 | 18.28 | 101.10 | 102.13 | +1.03 |
| TREN | 93.33 | 27.42 | 120.75 | 13.53 | 105.94 | 101.00 | –4.94 |
| PIR | 94.40 | 29.74 | 124.14 | 20.43 | 102.39 | 98.25 | –4.14 |
| AES | 88.21 | 19.28 | 107.49 | 11.43 | 94.90 | 92.13 | –2.77 |
| WMS | 84.48 | 24.26 | 108.74 | 12.47 | 95.21 | 88.13 | –7.08 |

**Table 6. Convertible Bond Analytics**

| ID | Breakeven (years) | Cheap/ rich | Duration (years) | Convexity | Delta | Gamma | Vega |
|---|---|---|---|---|---|---|---|
| ALS | 0.92 | –2.22% | 0.23 | –1.06 | 0.75 | 0.20 | 13.70 |
| AMB | 49.67 | +10.18 | 0.40 | 0.45 | 0.40 | 1.49 | 14.32 |
| GSX | 4.01 | –1.03 | 1.34 | –5.73 | 0.45 | 0.09 | 11.33 |
| TREN | 2.96 | –4.94 | 0.85 | –2.66 | 0.52 | 0.06 | 22.57 |
| PIR | 3.01 | –4.14 | 1.13 | 5.83 | 0.22 | 0.17 | 12.97 |
| AES | 3.58 | –2.77 | 2.48 | 0.98 | 0.26 | 0.02 | 15.16 |
| WMS | 3.55 | –7.08 | 2.31 | –4.57 | 0.38 | –0.02 | 16.67 |

## Hedging Decomposition

Given that convertible bonds are hybrids of stocks and bonds, we should be able to determine a portfolio of stock and bonds such that we can immunize the convertible bond from stock and interest rate risks. This portfolio is called the hedging portfolio. The hedging portfolio can clearly identify the basic components of the convertible bond for an instant of time. As the stock price changes and the yield curve shifts, the hedging portfolio must also change. Therefore, the hedging portfolio is the replicating portfolio used in arbitrage-free analysis. Table 7 shows the hedging portfolio using only the stock and Treasury par bonds. The credit risks are not hedged.

This hedging portfolio is uniquely determined by selecting the positions in stocks and bonds such that the hedging portfolio has the same market value, delta, and key rate durations as the convertible bond.

The hedging decomposition precisely provides the appropriate hedging position for the bond. The Treasury par bond position has the same key rate duration as that of the convertible bonds.[1] The hedging position changes with varying market conditions. This method of decomposition gives traders access to dynamic hedging strategies.

## Breakeven Valuation

Convertible bond value is related to the stock and yield curve levels in a complex manner. A convertible bond with a low parity value would behave like a straight bond. As the interest rate falls, the convertible bond's value rises. The bond value does not rise in step with the falling rate, however, because of the cash call provision. Because the parity value is low, the convertible bond's value would be limited to the call price and would not increase further even though interest rates continue to decline.

Similarly, a convertible bond's price does not relate to the parity value in a simple fashion. When the parity value is low, the convertible bond's price is close to the investment value and its price is insensitive to the changes in parity value. As the parity value rises, the convertible bond's value and its sensitivity to the stock price will both increase. As the likelihood of a forced conversion grows, the convertible bond's price will become less sensitive to the parity value.

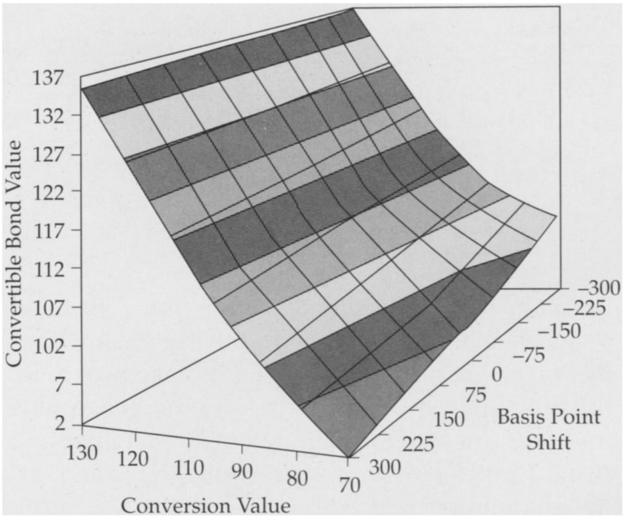
A convertible bond's value relates both to variations of parity and to interest rate levels. Therefore, the convertible bond's value should be represented as a "surface" that is related to stock and yield curve changes. The price surface of Trenwick Group is shown in Figure 2.

The price surface enables managers to more effectively consider an investment in a convertible as an alternative to the stock or the underlying bond. When compared with the stock, the convertible bond requires an additional premium but offers downside protection and higher yields. The crux of the analysis is therefore to compare the likelihood of forced conversion and the importance of downside protection. The model can find the breakeven points by determining all the combinations of stock

**Table 7. Hedging Decomposition**

| | | | Par Bond Maturities (years) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ID | Stock Value | Cash | 1/4 | 1/2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| ALS | 77.2 | 9.7 | 11.0 | 3.5 | 2.3 | 1.1 | 0.4 | 0.2 | 0.2 | 0.3 | 1.4 | 0.0 | 0.0 |
| AMB | 37.6 | 22.4 | 29.7 | 6.7 | 1.0 | 1.7 | 0.8 | 0.5 | 0.3 | 0.2 | 0.2 | 0.1 | 3.0 |
| GSX | 40.5 | –143.0 | 137.0 | 24.2 | 20.0 | 8.5 | 3.7 | 1.9 | 1.6 | 1.2 | 5.0 | 0.8 | 0.0 |
| TREN | 47.4 | 4.5 | 2.7 | 8.7 | 18.8 | 5.2 | 4.3 | 6.7 | 7.6 | 0.0 | 0.0 | 0.0 | 0.0 |
| PIR | 18.0 | 53.1 | 19.7 | –31.0 | 18.2 | 7.0 | 3.1 | 1.6 | 1.3 | 1.1 | 10.4 | 0.0 | 0.0 |
| AES | 18.4 | 8.8 | 1.7 | –5.6 | 17.3 | 9.9 | 6.8 | 5.5 | 3.8 | 22.7 | 0.0 | 0.0 | 0.0 |
| WMS | 25.9 | 8.3 | 1.9 | 3.0 | 6.3 | 9.9 | 7.0 | 3.0 | 2.1 | 2.6 | 11.9 | 13.3 | 0.0 |

**Figure 2. Price Surface of TREN**




and yield curve levels such that the investment in the stock breaks even with that for the convertible bond. Figure 3 provides an example of such a surface.

Similarly, the breakeven valuation can be compared with the underlying bonds. In this case, the convertible involves a trade-off between the cost of the premium over the investment value and the potential upside return as the stock appreciates. Figure 4 shows the breakeven points at which the two alternative strategies would be comparable.

## CONCLUSION

A two-factor arbitrage-free model gives convertible bond managers a rational way to design, implement, and assess bond strategies. The valuation model can provide fair value analysis, simulations of returns, and projected cash flows.

This model's advantage in accurate assessment of fair valuation stems from its ability to provide relative valuation. In the model, the fair value of the convertible bond is relative to the observed stock price and the yield curve. Therefore, the model can separate the fundamental analysis of the stock value and yield curve level from the optionality of the convertible bond on the stock and the yield curve.

As a result, if the stock is "undervalued," a fair-valued convertible bond would be undervalued. If the stock is "overvalued," the convertible bond also would be overvalued. The model becomes a tool to quantify the mispricing of the convertible bond relative to that of the stock.

The model can also be used to simulate convertible bond returns, a matter of importance to total return portfolio managers. Because the model provides the convertible bond price, given the stock price and the yield curve, investment managers can use the model to simulate "what if" scenarios and determine the risk and returns of the convertible bond relative to the stock returns.

For managers who need to estimate the cash flow that will be projected from holding a convertible bond, the model can produce an accurate simulation. Making the optimal decision for forcing a conversion depends on the convertible bond's simulated price. Prespecifying the conversion decision

**Figure 3. Breakeven Valuation of TREN with Stock**

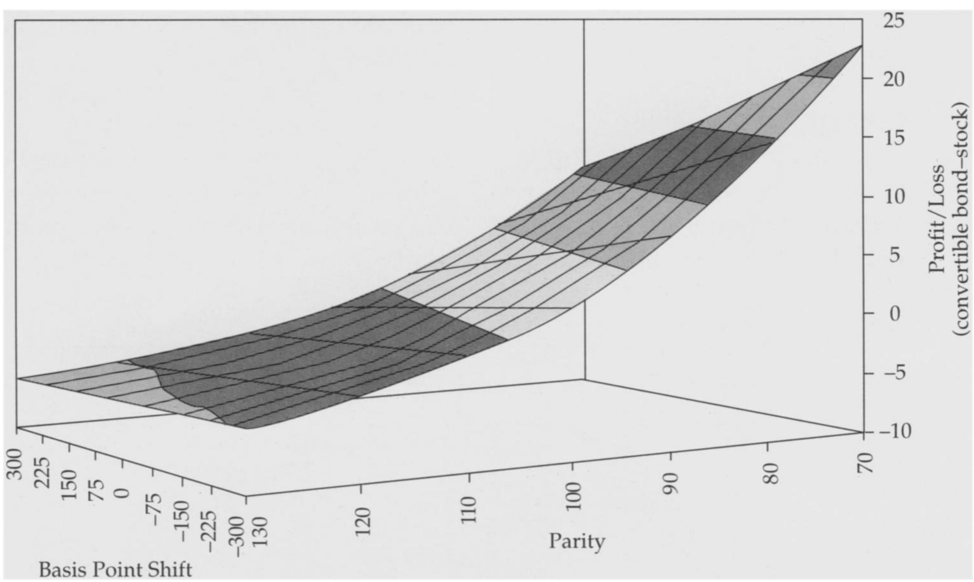
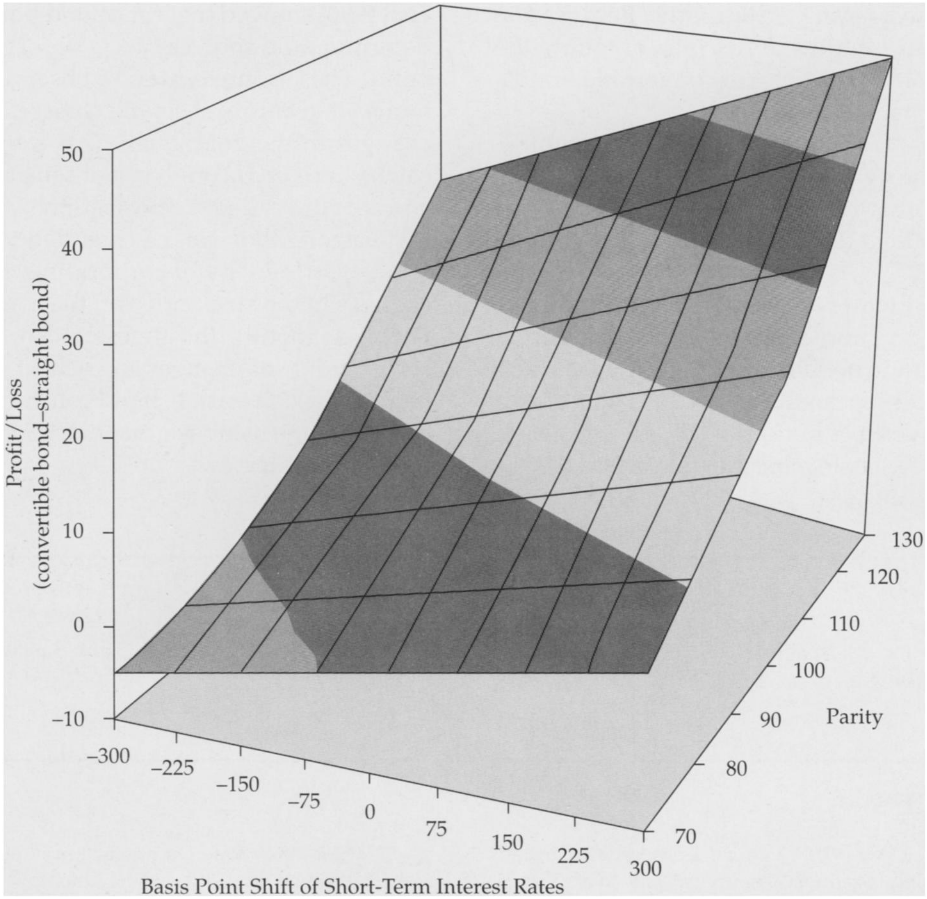


**Figure 4. Breakeven Valuation of TREN with Bond**




without using the prevailing market value of the convertible bond is incorrect. This optimal decision rule is similar to that of a callable corporate bond or American stock option. For this reason, a pricing model is needed to provide the simulated projected cash flows.

Because the model takes as given all the market parameters, such as interest rate volatilities and the yield curve, it provides a consistent way of valuing a portfolio of convertible bonds.

The model can be used in different ways depending on a manager's objectives. Some examples follow.

*Investment Strategies.* Convertible bonds can be evaluated as an alternative to the corresponding stock investment. One way of evaluating the appropriateness of this alternative investment is through breakeven analysis. This analysis compares the yield pick up with the premium paid for the convertible bond. If the breakeven time is short, then the convertible bond could offer a higher return because beyond the breakeven time, the investor can always convert the bond to the stock.

One drawback to this investment strategy is that the convertible bond might be called prior to the breakeven date. In this case, the yield pick up would not be enough to compensate for the premium. In using breakeven analysis, therefore, understanding the impact of forced conversion is important. The model provides a consistent framework for analyzing the likelihood of a forced conversion by taking all the features of the callability into account.

A convertible bond can also be considered as an alternative investment to a corporate bond. In this case, the analysis involves separating the potential upside return from the rise of the stock versus the premium paid on the convertible bond over the investment value. The convertible bond's price, however, does not rise in step with the underlying stock price. Therefore, to analyze the upside return of the convertible bond, the model can be used to simulate the convertible bond value as the stock price rises.

Similarly, the convertible bond does not react to the changes of the OAS and the yield curve in the same way as the underlying bond. The widening of the spread could be the result of deterioration of credit or the worsening of liquidity. Because the model takes the bond's OAS into account, the bond's duration quantifies the convertible bond's sensitivity to changes in spreads.

▪ *Portfolio Strategies.* Hedging decomposition can also be aggregated to the portfolio level. Therefore, another application of this model is to determine the hedge portfolio for a convertible bond portfolio.

Decomposition enables us to identify the implicit allocation of the convertible bond portfolio between stocks and bonds. The information derived from decomposition analysis can help determine whether a convertible bond portfolio is consistent with an investor's asset allocation strategies.

Also, this analysis can facilitate a return attribution approach to a convertible bond portfolio. Ho (1990) showed that for a bond portfolio, one can determine the sources of the observed return of a bond. These sources may be observed movements in the yield curve, observed changes in the OAS, or changes in the volatilities. This return attribution can be derived through use of sensitivity analytics, a procedure called "factorization."

Factorization can be extended to a convertible bond portfolio by incorporating the impact of changes in the stock price on the convertible bond. For each month, the investor can determine the contribution of changes in each of the sources of risks to the convertible bond returns. These results give valuable feedback that helps in implementing portfolio strategies.[2]

## NOTES

1. See Ho (1992) for an explanation of the use of key rate duration for hedging.
2. The authors would like to thank Sanjay Mazumdar and Thomas Jackson for their comments and assistance.

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