Functional Programming

GADT: Generalize Algebraic DataType

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Interpreters, again

Language definition

```haskell
data Term = I Integer
          | Add Term Term
            deriving (Eq, Show)
```

Interpreters, again

Evaluation

```
eval :: Term -> Integer
eval (I n) = n
eval (Add t op u) = eval t + eval u
```

A language with multiple types

Language definition

```haskell
data Term = I Integer
          | B Bool
          | Add Term Term
          | Eq Term Term
            deriving (Eq, Show)
```

A language with multiple types

Evaluation

```
type Value = Int Integer | Bool Bool
           deriving Show

eval :: Term -> Value
eval (I n) = Int n
eval (B b) = Bool b
eval (Add t t') = case (eval t, eval t') of
               (Int i, Int i2) -> Int (i + i2)
eval (Eq t t') = case (eval t, eval t') of
               (Int i, Int i2) -> Bool (i == i2)
               (Bool i, Bool i2) -> Bool (i == i2)
```

Issues with our interpreter

- The interpreter can fail.
- We need to consider failures manually.
- The more value with have in our language, the more complicated it becomes.
- The Haskell type system does not help us.

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GADTs to the rescue!

Algebraic Data Type

```
data Maybe a =
    Nothing
  | Just a
```

GADTs to the rescue!

Generalized Algebraic Data Type

```haskell
{-# LANGUAGE GADTs #-}

data Maybe a where
  Nothing :: Maybe a
  Just :: a -> Maybe a
```

We now also specify the return type!

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Language definition with GADTs

```
data Term =
    I Integer
  | B Bool
  | Add Term Term
  | Eq Term Term
```

Language definition with GADTs

```
data Term where
  I :: Integer -> Term
  B :: Bool -> Term
  Add :: Term -> Term -> Term
  Eq :: Term -> Term -> Term
```

Language definition with GADTs

```
data Term a where
  I :: Integer -> Term Integer
  B :: Bool -> Term Bool
  Add :: Term (?) -> Term (?) -> Term (?)
  Eq :: Term (?) -> Term (?) -> Term (?)
```

Language definition with GADTs

```
data Term a where
  I :: Integer -> Term Integer
  B :: Bool -> Term Bool
  Add :: Term Integer -> Term Integer -> Term Integer
  Eq :: Term (?) -> Term (?) -> Term (?)
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Language definition with GADTs

```haskell
data Term a where
  I :: Integer -> Term Integer
  B :: Bool -> Term Bool
  Add :: Term Integer -> Term Integer -> Term Integer
  Eq :: (Eq a) => Term a -> Term a -> Term Bool
```

Evaluation for GADTs

```
eval :: Term a -> a -- This type annotation is mandatory
eval (I i) = i
eval (B b) = b
eval (Add t t') = eval t + eval t'
eval (Eq t t') = eval t == eval t'
```

We call this kind of interpreters "tag-less".

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What about functions?

We want to add functions to our language.

First try

```
data FExp a where
  Var :: FExp a
  Lam :: FExp b -> FExp (a -> b)
  App :: FExp (a -> b) -> FExp a -> FExp b
```

This doesn't work: not enough type information for variables and lambdas.

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Existential Types

In the definition of App, a is present inside the types, but not in the result. This is an *existential* type.

```
App :: FExp (a -> b) -> FExp a -> FExp b
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Demo!

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App :: FExp (a -> b) -> FExp a -> FExp b
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Demo!

Back to functions!

Type definition

```
data FExp e a where
  App :: FExp e (a -> b) -> FExp e a -> FExp e b
  Lam :: FExp (a, e) b -> FExp e (a -> b)
  Var :: Nat e a -> FExp e a

data Nat e a where
  Zero :: Nat (a, b) a
  Succ :: Nat e a -> Nat (b, e) a
```

Demo!

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Demo!

Origin of GADTs

- Young extension to HM type systems.
  Invented by 3 different groups:
  - Augustsson & Petersson (1994): Silly Type Families
  - Cheney & Hinze (2003): First-Class Phantom Types.
  - Xi, Chen & Chen (2003): Guarded Recursive Datatype Constructors.
- Type *checking* is decidable.
- Type *inference* is undecidable.
- Pattern matching is quite more complicated.

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Wrapping up

- GADTs allows to express more properties in types:
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- We leverage Haskell's type system.
- GADTs do not solve *all* the problems. For example, you can try to write a function of type

```
parse :: String -> Expr a
```

  Fortunately, we can combine GADTs with other Haskell features such as Type classes and Type families.
- GADTs become very complex when the domain grows!

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