Functional Programming

Functions

Prof. Dr. Peter Thiemann

Albert-Ludwigs-Universität Freiburg, Germany

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Function definition by cases

Example: Absolute value

Find the absolute value of a number

- if `x` is positive, result is `x`
- if `x` is negative, result is `-x`

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Definition

```
-- returns the absolute value of x
absolute :: Integer -> Integer
absolute x | x >= 0 = x
absolute x | x < 0  = - x
```

Alternative styles of definition

One equation

```
absolute' x | x >= 0 = x
            | x < 0  = -x
```

Using if-then-else in an expression

```
absolute'' x = if x >= 0 then x else -x
```

Recursion

Standard approach to define functions in functional languages (**no loops!**)

- Reduce a problem (e.g., `power x n`) to a smaller problem of the same kind
- Eventually reach a base case that can be solved immediately
- Build up solutions from smaller solutions

Example: power

Compute `x^n` without using the built-in operator

```
-- compute x to n-th power
power x 0         = 1
power x n | n > 0 = x * power x (n - 1)
```

Example: Counting intersections

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- One line can intersect with the remaining lines at most $n - 1$ times.
- Remove this line. The remaining lines can intersect at most $I(n - 1)$ times
- Combine the above to $I(n) = I(n - 1) + n - 1$

Definition

Counting intersections

```
-- max number of intersections of n lines
nisect :: Integer -> Integer
nisect 0   = 0
nisect n | n > 0 = nisect (n - 1) + n - 1
```

Questions?

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