Assignment problem for MAS

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Introduction

Starting conditions:The MAS is located at airbases and receives multiple requests for service including:

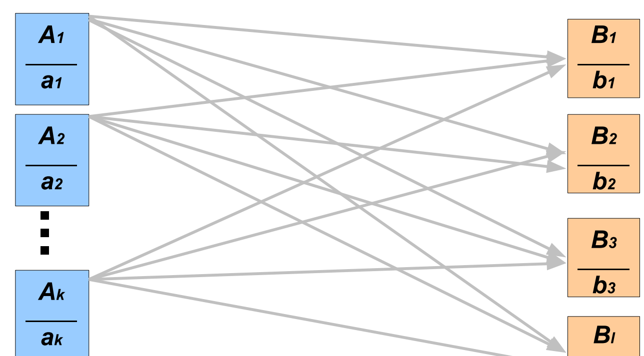
- Location to visit;
- Number of air-vehicles required;
- Earliest time of 1-st visit;
- Latest time of 1-st visit;
- Minimum duration per visit;
- Maximum interval between visits.

Mission Objective:
Air-vehicle(s) to be assigned to request and the corresponding paths to take to the service request; (variations to requests with minimal change if it cannot be met.)

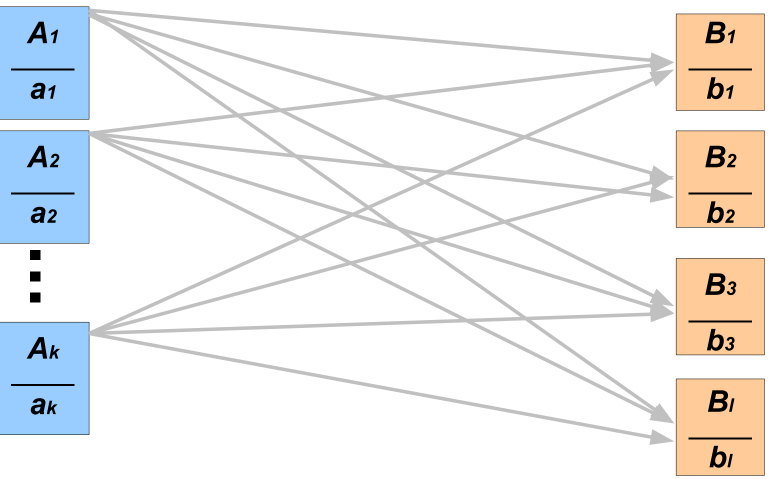
That:
Maximize the number of service requests that can be serviced.

Constraints

- Air-vehicle performance and dynamics;
- LOS occlusion in area of operations.



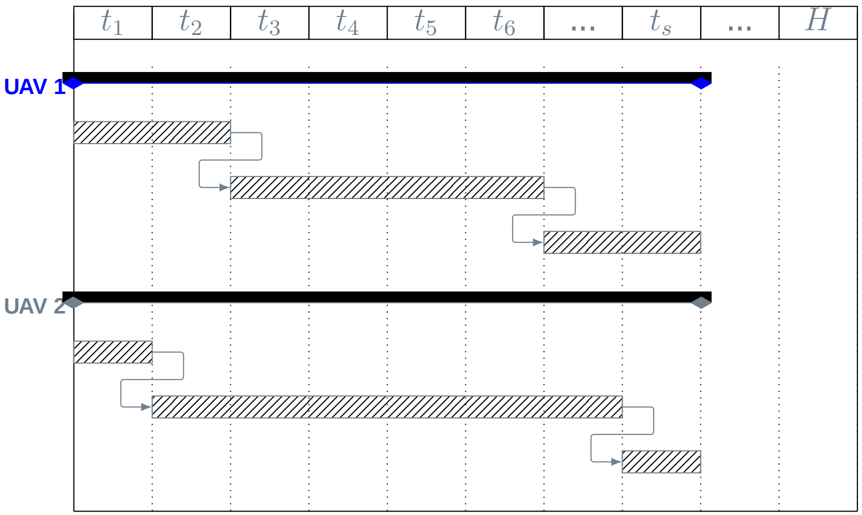
Variables and constraints



Divide the interval $[0, H]$ by the moments $t_s = s\Delta,\ s = 1, 2, \ldots.., \nu$ where $\nu = \left[\dfrac{H}{\Delta}\right]$

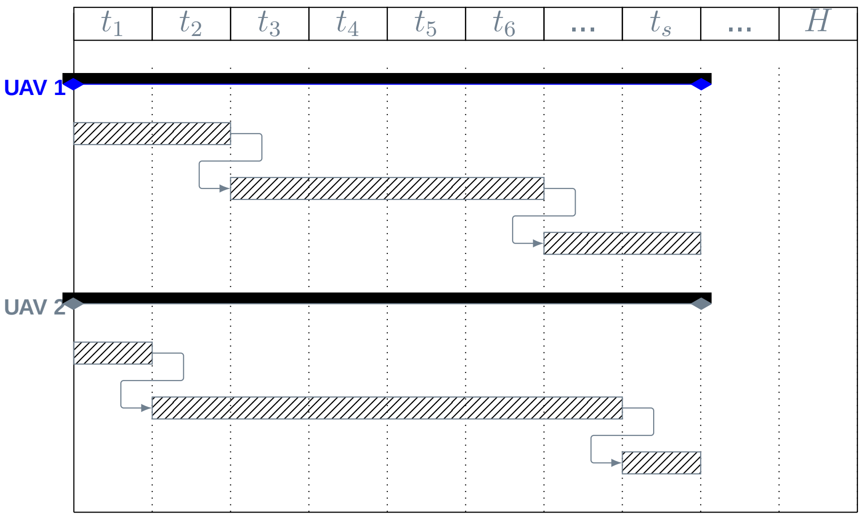
- $B_j, j = 1, ..., l$ is the locations to visit;
- $x_{ij}(t_s)$ is the number of UAVs from i-th aerobase send to j-th zone at the moment t_s;
- $b_j(t_s)$ is the number of UAVs that are serving the $j - th$ zone at the moment t_s;
- $A_i, i = 1, ..., k$ is the locations of aerobases;
- $a_i(t_s)$ is the number of UAVs at the $i - th$ aerobase at the moment t_s;
- k and l are the number of aerobases and zones for service, respectively;

Variables and constraints



- h_i is the flight endurance for UAVs from $i - th$ aerobase.
- t_{ij} is the flight time from $i - th$ aerobase to $j - th$ zone;
- $h_i - 2t_{ij}$- maximal possible service time of the zone j;

Variables and constraints



- h_i is the flight endurance for UAVs from $i-th$ aerobase.
- t_{ij} is the flight time from $i-th$ aerobase to $j-th$ zone;
- $h_i - 2t_{ij}$- maximal possible service time of the zone j;

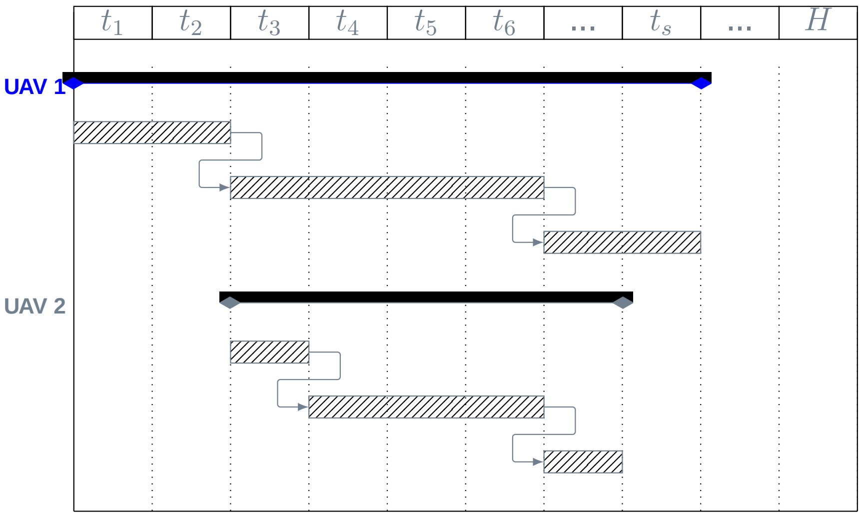
$$a_i(t_s+\Delta) = a_i(t_s) - \sum_{j=1}^{l} x_{ij}(t_s) + \sum_{j=1}^{l} x_{ij}(t_s+\Delta-h_i)$$

$$b_j(t_s+\Delta) = b_j(t_s) - \sum_{i=1}^{k} x_{ij}(t_s-h_i+t_{ij}) + \sum_{i=1}^{k} x_{ij}(t_s-t_{ij})$$

$$a_i(t_s) + \sum_{j=1}^{l} x_{ij}(t_s) = a_i; \quad b_j(t_s) + \sum_{i=1}^{k} x_{ij}(t_s-t_{ij}) = b_j,$$

$$(t_s-h_i+t_{ij}>0); (t_s-t_{ij}>0);\ i=1,...,k; j=1,...,l.$$

Types of objective function



- the total service time
$$J_1(x) = \sum_{i=1}^{k}\sum_{j=1}^{l}\sum_{s=0}^{\nu} x_{ij}(t_s)(h_i - 2t_{ij})$$
- the total number of UAVs "circles"
$$J_2(x) = \sum_{i=1}^{k}\sum_{j=1}^{l}\sum_{s=0}^{\nu} x_{ij}(t_s)$$
- the total unobservable time for multiple zones $J_3(x) = H - J_1(x)$

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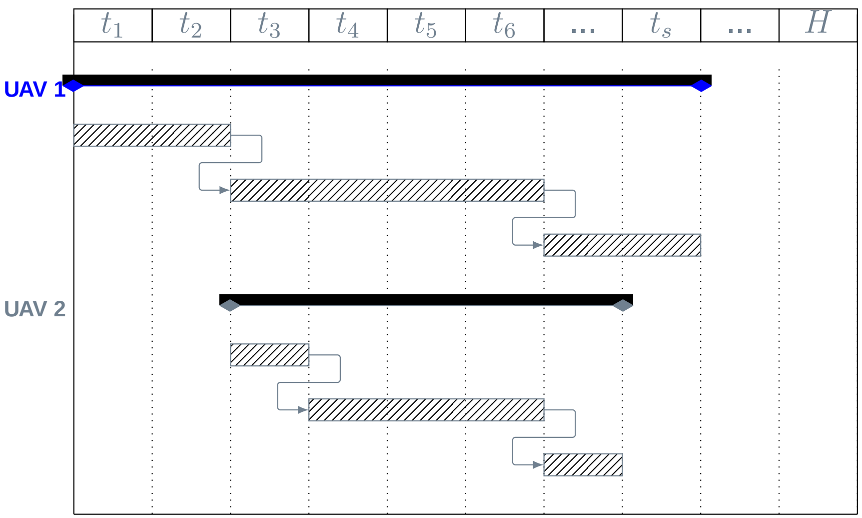
Problem statement



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$$(t_s-h_i+t_{ij}>0); (t_s-t_{ij}>0);\ i=1,...,k; j=1,...,l.$$

The proposed model can be extended by additional conditions (constraints) followed from the formal description.

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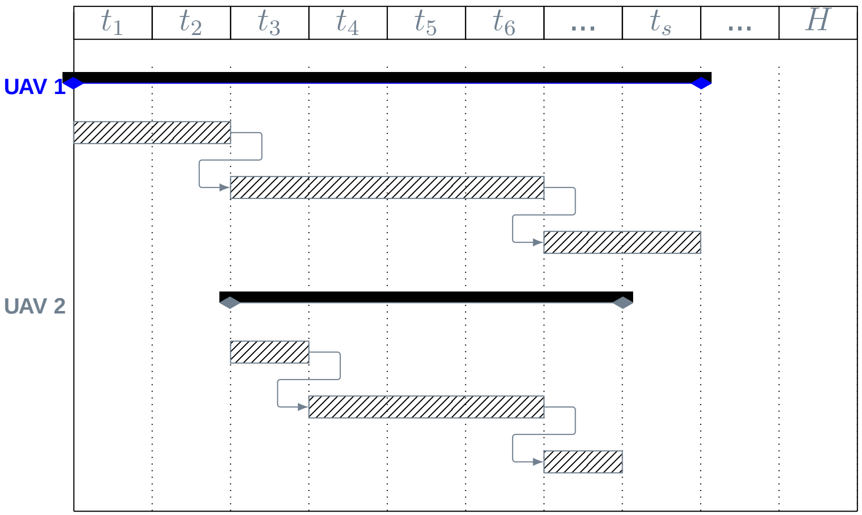

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$$a_i(t_s + \Delta) = a_i(t_s) - \sum_{j=1}^{l} x_{ij}(t_s) + \sum_{j=1}^{l} x_{ij}(t_s + \Delta - h_i)$$

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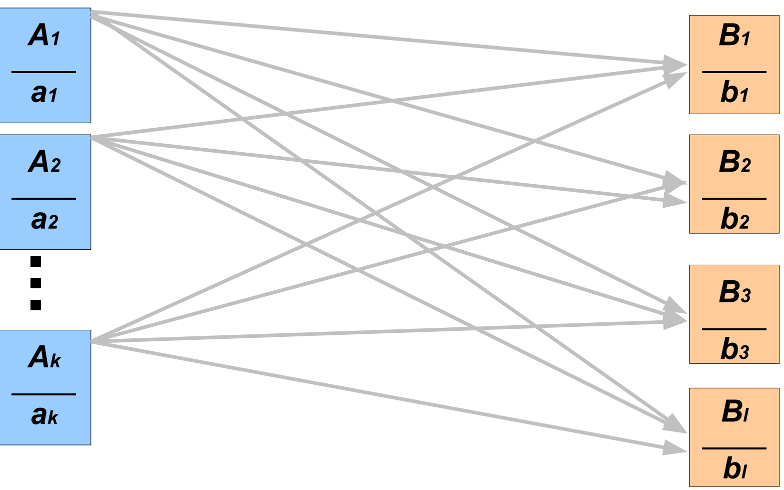
$$(t_s - h_i + t_{ij} > 0); (t_s - t_{ij} > 0); \; i = 1, ..., k; j = 1, ..., l.$$

$$\sum^{k} x_{ij}(t_s^{first}) \neq 0, s^{first}\Delta \leq t^{first} \leq (s^{first} + 1)\Delta$$

For example, if the time of earliest time of 1-st visit t_j^{first} for j zone is pre-assigned.

LP assignment problem



$$F = \sum_{i=1}^{k}\sum_{j=1}^{l} c_{ij}x_{ij} \to \min_{x_{ij}}$$

$$\sum_{i=1}^{k} x_{ij} = b_j, \quad j = 1, 2, ..., l$$

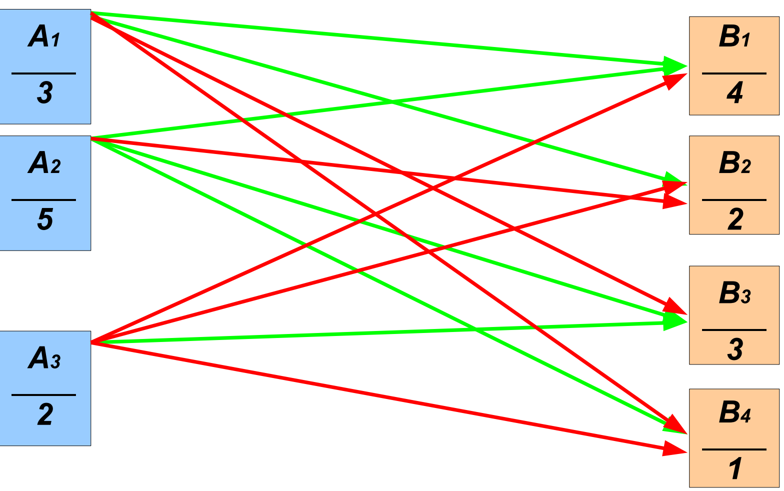
$$\sum_{j=1}^{l} x_{ij} = a_i, \quad i = 1, 2, ..., k$$

$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j, x_{ij} \geq 0, x_{ij} \in \mathbb{N}.$$

The most of methods include the following basic steps:

- To find initial plan x_{ij};
- Check optimality condition for that plan;
- Construct the improved plan in case of nonoptimality.

Optimal solution



$$F = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{ij}x_{ij} \to \min_{x_{ij}}$$

$$\sum_{i=1}^{3} x_{ij} = b_j, \quad j = 1, 2, 3, 4$$

$$\sum_{j=1}^{4} x_{ij} = a_i, \quad i = 1, 2, 3$$

$$\sum_{i=1}^{3} a_i = \sum_{j=1}^{4} b_j, x_{ij} \geq 0, x_{ij} \in \mathbb{N}.$$

	B_1	B_2	B_3	B_4	a_i	u_i
A_1	1	2	✓ 0	✓ 0	3	
A_2	3	✓ 0	1	1	5	
A_3	✓ 0	✓ 0	2	✓ 0	2	
b_j	4	2	3	1	F =46	
ν_j						

Optimal solution:

$$x_{11} = 1;\; x_{12} = 2;$$
$$x_{21} = 3;\; x_{23} = 1;$$
$$x_{24} = 1;\; x_{33} = 2.$$

Outline of the method

The method for LP assignment problem as well as simplex methods are:

- iterative,
- exact(satisfied all constraints),
- finite,
- relaxed(in a sense of the value of objective function).

Thus in some sense this method is analog of simplex method, but the ideas of this method is more naturally can be applied to assignments LP problems.

$$Further\ \ work$$

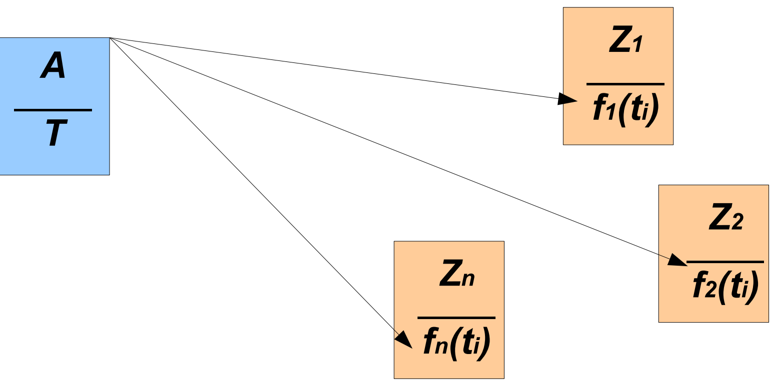
The major task is to solve the specific assignment problem for MAS by developing

- new optimality condition,
- ϵ - optimality condition,
- realize sensitivity analysis (robustness analysis),
  Siarhei Dymkou, Kai Yew Lum, Jian Xin Xu, Comparison of the adaptive method with classical simplex method for linear programming.(2011) (in preparation)
- consider objectives with very general functional forms.

The end

Thank you!

Extra slides (DP)



- Invariant embedding of the problem into $P(k,y), k \in [1;n]; y \in [0;T]$, then define the Bellman function;
- Construct the Bellman equation;

$$\sum_{i=1}^{n} f_i(t_i) \to \max \qquad \Longrightarrow \qquad P(k,y) : \sum_{i=1}^{k} f_i(t_i) \to \max,$$

$$\sum_{i=1}^{n} t_i \le T,\ t_i \ge 0, i = 1, ..., n \qquad \sum_{i=1}^{k} t_i \le y,\ t_i \ge 0, i = 1, ..., k$$

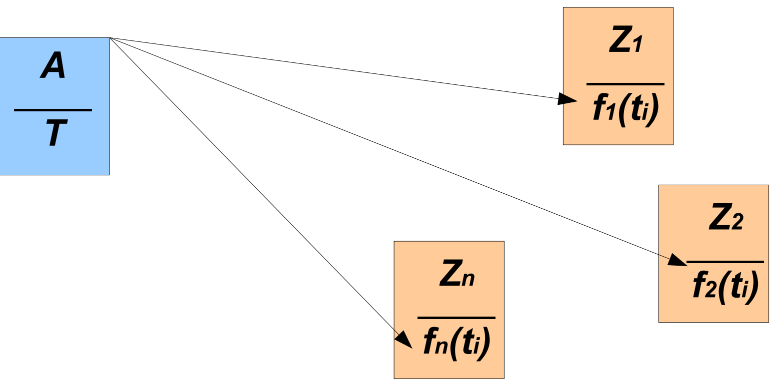
Bellman equation

$$B_k(y) = \max_{t_i} \sum_{i=1}^{k} f_i(t_i),$$

$$B_k(y) = \max_{0 \le z \le y} \Big[f_k(z) + B_{k-1}(y-z) \Big] \qquad \Longrightarrow \qquad \sum_{i=1}^{k} t_i \le y,\ t_i \ge 0, i = 1, ..., k$$

"back"

Extra slides (DP)



- Invariant embedding of the problem into $P(k, y), k \in [1; n]; y \in [0; T]$, then define the Bellman function;
- Construct the Bellman equation;
- Solve the Bellman equation, and apply the solution of it to initial problem

$$t_n^0,\ t_{n-1}^0,\ \ldots,\ t_2^0,\ t_1^0 \quad \Longleftarrow \quad \mathit{Continue\ this\ procedure\ we\ will\ find}$$

$$\mathit{the\ optimal\ solution\ of\ our\ problem}$$

Put $k = n,\ y = T$ and
find the value $t_n^0 \doteq z^0(T)$ for the zone Z_n

$$f_n(z_n^0) + B_{n-1}(T - z_n^0) =$$
$$\max_{0 \le z \le T} \Big[f_n(z) + B_{n-1}(T - z) \Big]$$

$\Longrightarrow$

$k = n - 1,\ y = T - t_n^0$ and
find the value $t_{n-1}^0 \doteq z^0(T - t_n^0)$

$$f_{n-1}(z_{n-1}^0) + B_{n-2}(T - z_{n-1}^0) =$$
$$\max_{0 \le z \le T - t_n^0} \Big[f_{n-1}(z) + B_{n-2}(T - t_n^0 - z) \Big]$$