Task Assignment Problem for Multi Agent System

Siarhei Dymkou, Control Science Group

T-LAB Control Science Group, Aeronautical Division

Motivation

The use of unmanned aerial vehicles (UAVs) has increased dramatically in the last two decades, both in military and civilian applications. Civilian applications include geological surveying, fire monitoring, and search and rescue missions. Military applications include reconnaissance and tracking, providing a communication relay, and conducting search and destroy missions. Their extensive use was mainly promoted to reduce the risk to human life, lower cost, and increase operational capabilities.
The main purpose of our research is to develop the new approach to solve dynamical tasking problem for multiagent systems (MAS). Namely, we will consider the dynamical assignment problem for allocation of MAS, which will be reduced to static problem. The challenge is to maximize the number of service requests that can be serviced. Besides the location to visit, the service request would include further requirements such as time to visit and duration of visit.
The reduction of this problem to static one leads to the huge dimensions of the variables involved, and this together with the specific structure of the considered problem are a serious obstacle for classic methods to get a suitable solution for reasonable time. By this reason we will present the special method and design on this base the fast numerical method for assignment problems of MAS. For design the effective numerical realization we are developing the new optimality and sub-optimality conditions that are more suitable for the design of the quick numerical methods and further applications. We propose to use the idea of constructive approach and extend this setting to produce new results and constructive elements of optimization theory for the considered MAS systems, in particular, for the so-called decentralized control MAS.
The specific problem formulation for dynamical tasking is summarized in following:

Problem Formulation

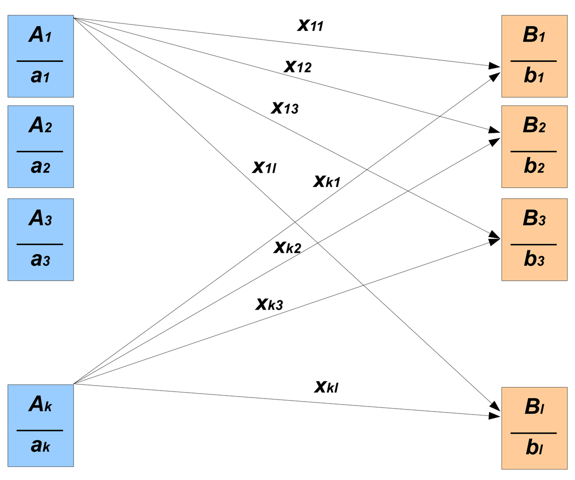
Item	Description
General Description	To assign air-vehicles to perform as many simultaneous service requests as possible.
Starting Condition	**Given:** The MAS is performing coverage and receives multiple requests for service with the following information: ◇ Number of air-vehicles required ◇ Location where air vehicles need to visit ◇ Earliest time of 1-st visit ◇ Latest time of 1-st visit ◇ Minimum duration per visit ◇ Maximum interval between visits
Mission Objectives	**Find:** ◇ Air-vehicle(s) to be assigned to request and the corresponding paths to take to the service request ◇ Variations to requests with minimal change if the request cannot be met **That:** ◇ Maximises the number of service requests that can be serviced
Constraints	**Subject to:** ◇ UAVs performance and dynamics ◇ LOS occlusion in area of operations

Scientific Originality

There are a lot of methods to solve such kind of problem. The formulated integer optimization problem will be extended to search service schedular for dynamical case of service requests. For this purpose a new supporting optimization method will be developed on the base of the so-called constructive approach by Gabasov R. and Kirillova F.M. [?]. For linear programming problem this method was compared with classical simplex method in the paper [?], also was successfully applied in gas industry [?]. The proposed method allows to use both initial information and current one produced during the solution process. This together with developed for this case duality theory leads to high efficiency of the associated numerical algorithms.

Dynamical assignment of UAVs for multiple areas of operation:

Let $[0, H]$ is the given period for service of $B_1, B_2, ..., B_j, ..., B_l$ zones of area of operation. It is assumed that each onetime service of each zone B_j requests includes at least b_j numbers of UAVs, $j = 1, \dots, l$. Also, assume that we have k aerobases $A_1, A_2, ..., A_i, ..., A_k$ with $a_1, a_2, ..., a_i, ..., a_k$ number of homogenous UAVs, respectively.



The problem is to assign UAVs between areas of operations $B_j, j = 1, \dots, l$ in a such way that the total service time will be maximal.
Thus, the optimal schedule problem of UAVs for multiple zones can be formulated as, for example, the following special integer dynamical linear programming problem (we change t_s by $t_s = s\Delta,\ s = 1, 2, ..., \nu$): maximize the cost value function

$$J_1(x) = \sum_{i=1}^{k}\sum_{j=1}^{l}\sum_{s=0}^{\nu} x_{ij}(s\Delta)(h_i - 2t_{ij}) \to \max_{x_{ij}(s\Delta), s=0,1,....,\nu} \tag{1}$$

subject to

$$\begin{aligned} a_i(s\Delta + \Delta) = a_i(s\Delta) - \sum_{j=1}^{l} x_{ij}(s\Delta) + \sum_{j=1}^{l} x_{ij}(s\Delta + \Delta - h_i),& \\ i = 1, ..., k.& \\ b_j(s\Delta + \Delta) = b_j(s\Delta) - \sum_{i=1}^{k} x_{ij}(s\Delta - h_i + t_{ij}) + \sum_{i=1}^{k} x_{ij}(s\Delta - t_{ij}),& \\ j = 1, ..., l.& \\ a_i(s\Delta) + \sum_{j=1}^{l} x_{ij}(s\Delta) = a_i,& \\ i = 1, ..., k.& \\ b_j(s\Delta) + \sum_{i=1}^{k} x_{ij}(s\Delta - t_{ij}) = b_j,& \\ j = 1, ..., l,& \\ s = 1, 2, ..., \nu,& \end{aligned} \tag{2}$$

where $\nu = \left[\frac{H}{\Delta}\right]$ denotes the integer part of the fraction $\frac{H}{\Delta}$.
The proposed partition of the planing horizon $[0, H]$ with small step Δ yields an ability to produce optimal schedule for UAVs, in fact, in regime of real time. The realization of this idea demands the development of some fast numerical algorithms for solution of the special classes of linear programming problems. Some new approaches to accelerate the solution of general linear programming problem is discussed in the paper [?]

Types of objective function:

Various optimization problems can be considered. For example the cost value function can be determined as follows:

a) the total service time for multiple zones

$$J_1(x) = \sum_{i=1}^{k}\sum_{j=1}^{l}\sum_{s=0}^{\nu} x_{ij}(t_s)(h_i - 2t_{ij}). \tag{3}$$

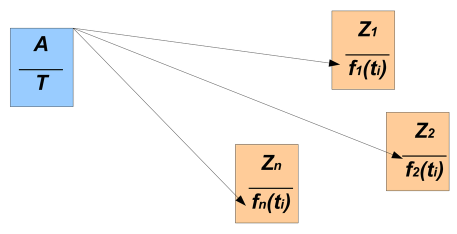
b) the total number of UAVs "circles"

$$J_2(x) = \sum_{i=1}^{k}\sum_{j=1}^{l}\sum_{s=0}^{\nu} x_{ij}(t_s) \tag{4}$$

c) the total unobservable time for multiple zones

$$J_3(x) = \sum_{i=1}^{k}\sum_{j=1}^{l}\sum_{s=0}^{\nu} x_{ij}(t_s)(H - h_i - 2t_{ij}) \tag{5}$$

Objectives with general form



- Invariant embedding of the problem into $P(k, y), k \in [1; n]; y \in [0; T]$,
then define the Bellman function;
- Construct the Bellman equation;

$$\begin{matrix} \displaystyle\sum_{i=1}^{n} f_i(t_i) \to \max & & P(k, y) : \displaystyle\sum_{i=1}^{k} f_i(t_i) \to \max \\ & \Longrightarrow & \\ \displaystyle\sum_{i=1}^{n} t_i \le T,\ t_i \ge 0, i = 1, ..., n & & \displaystyle\sum_{i=1}^{k} t_i \le y,\ t_i \ge 0, i = \overline{1, k} \end{matrix}$$

Bellman equation

$$\begin{matrix} B_k(y) = & & B_k(y) = \max_{t_i} \displaystyle\sum_{i=1}^{k} f_i(t_i), \\ \max_{0 \le z \le y} \left[f_k(z) + B_{k-1}(y - z)\right] & \Longleftarrow & \displaystyle\sum_{i=1}^{k} t_i \le y,\ t_i \ge 0, i = 1, ..., k \end{matrix}$$

- Solve the Bellman equation, and apply the solution of it to initial problem

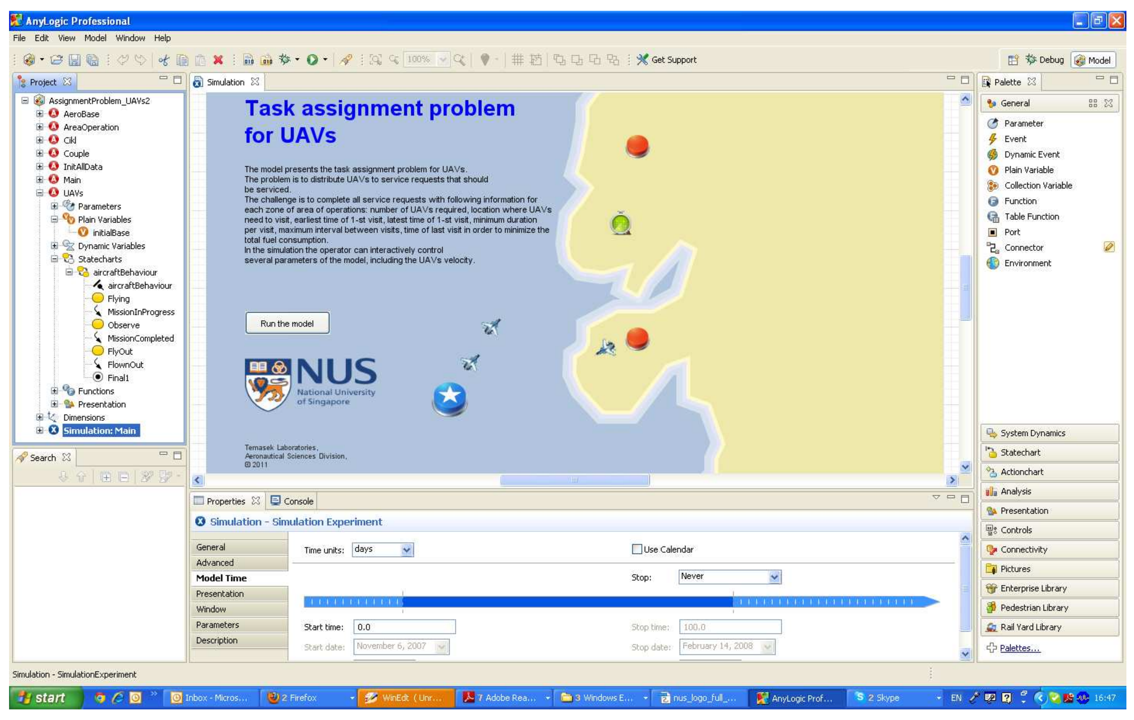
Put $k = n,\ y = T$ and
find the value $t_n^0 \doteq z^0(T)$ for
the zone Z_n

$$\begin{matrix} f_n(z_n^0) + B_{n-1}(T - z_n^0) = & \Longrightarrow & k = n - 1, y = T - t_n^0 \text{ and} \\ \max_{0 \le z \le T} \left[f_n(z) + B_{n-1}(T - z)\right] & & \text{find the value } t_{n-1}^0 \doteq z^0(T - t_n^0) \end{matrix}$$

Continue this procedure we will find the optimal solution:

$$t_n^0,\ t_{n-1}^0,\ \dots,\ t_2^0,\ t_1^0$$

Simulation model



The simulation model for two dimensional case will be created on "AnyLogic" simulation software using the agent-based modeling approach, since it is the only simulation tool, which allows creating flexible models with agents, interacting with each other and their environment. "AnyLogic" supports all known ways of specifying the agent behavior such as statecharts, synchronous and asynchronous event scheduling.

Publications

Conference publications:

[C1] Dymkou S., Jank G.: *Time optimal control problem for a class of linear differential-algebraic systems with delay* (accepted for Sixteenth International Symposium on MTNS 2004, Katholieke Universiteit Leuven, Belgium July 5-9, 2004)

[C2] Dymkou S., Rogers E, Dymkov M. P., Galkowski K., Owens D. H.: *Controllability and Optimization for Differential Linear Repetitive Processes* (42-th IEEE Conference on Decision and Control (CDC-2003), December 9–12, 2003, Hawai, USA.

Journal publications:

[J0] Dymkou S, Kai Yew Lum, Jian Xin Xu, *Comparison of the adaptive method with classical simplex method for linear programming.*(2011) (in preparation)

[J1] Dymkou S., Dymkov M.P. *Controllability and Optimization of Composed Systems* "Avtomatika and Telemechanika", 2003 (Russian); English version "Automation and Remote Control"