Assignment problem for MAS

Siarhei Dymkou

Temasek Laboratories

National University of Singapore

T-Lab Building 5A, Engineering Drive 1,05-02 Singapore 117411

Introduction

Starting conditions:The MAS is located at airbases and receives multiple requests for service including:

- Location to visit;
- Number of air-vehicles required;
- Earliest time of 1-st visit;
- Latest time of 1-st visit;
- Minimum duration per visit;
- Maximum interval between visits.

Mission Objective:

Air-vehicle(s) to be assigned to request and the corresponding paths to take to the service request; (variations to requests with minimal change if it cannot be met.)

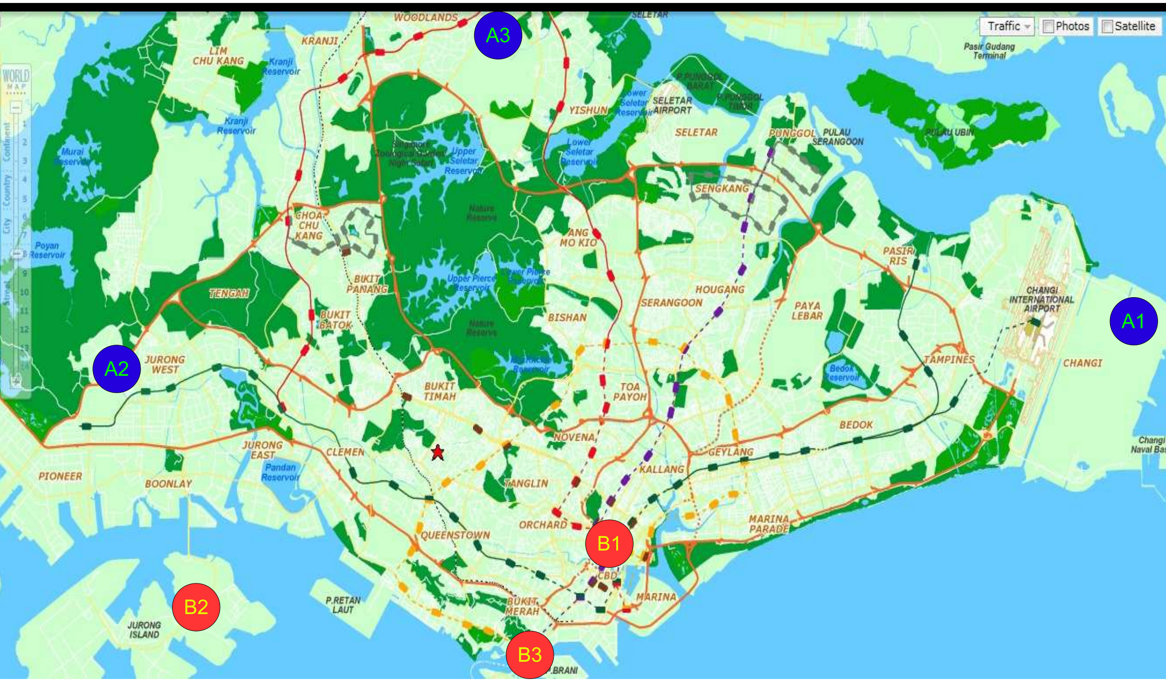
That:

Maximize the number of service requests that can be serviced.

Constraints

- Air-vehicle performance and dynamics.

Formal problem statement



3 airbases located: Changi(3), Jurong West(3), Woodland(1).

Service requests from: Raffles Place(2), Jurong Island(2), Sentosa(3).

Out task is to complete all requests in order to maximaze the total service time in the zones and satisfies all timing constraints.

Notations:

$A_i, i = 1,2,3$- number of aerobases,

$a_1 = 3, a_2 = 3, a_3 = 1$ - number of UAVs located in A_i,

$B_j, j = 1,2,3$- areas of operations,

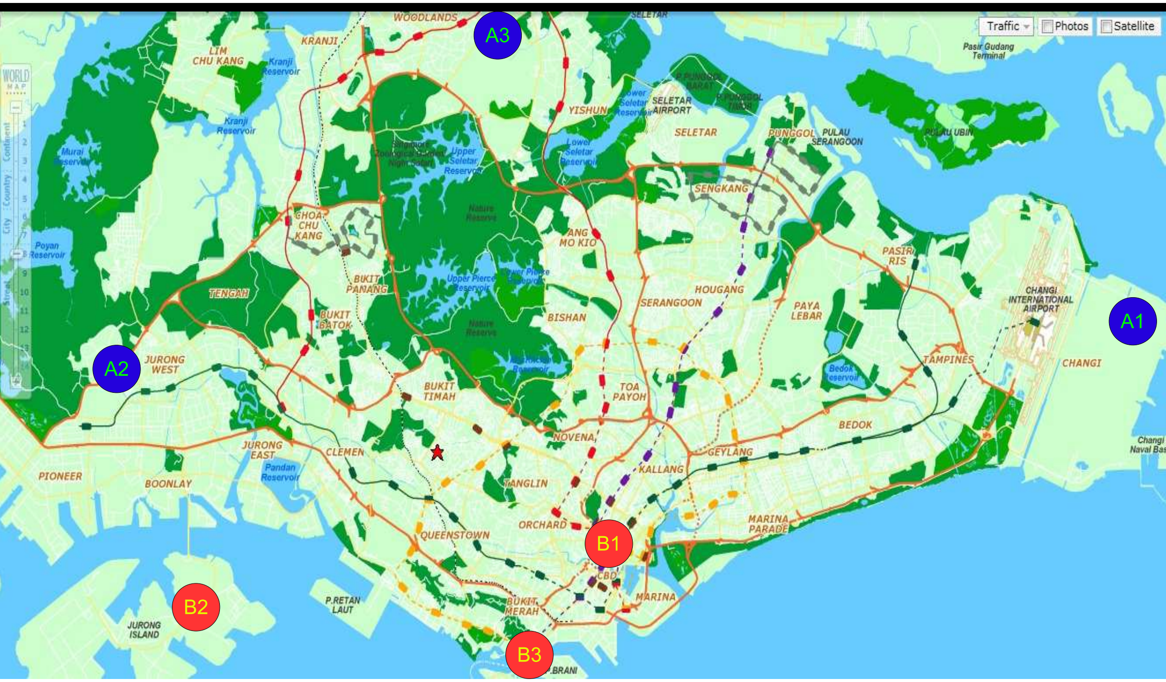
$b_1 = 2, b_2 = 2, a_3 = 3$- numbers of UAVs for service of B_j

x_{ij}-number of UAVs from A_i to B_j

$h_i = 3600 sec$- UAVs endurance located on A_i aerobase;

$v_{ij} = 30 \frac{m}{sec}$ - speed of UAVs;

Formal problem statement



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Out task is to complete all requests in order to maximaze the total service time in the zones and satisfies all timing constraints.

Notations:

	A_1	A_2	A_3
A_1	0	32	22
A_2	32	0	17
A_3	22	17	0

Distances between A_i

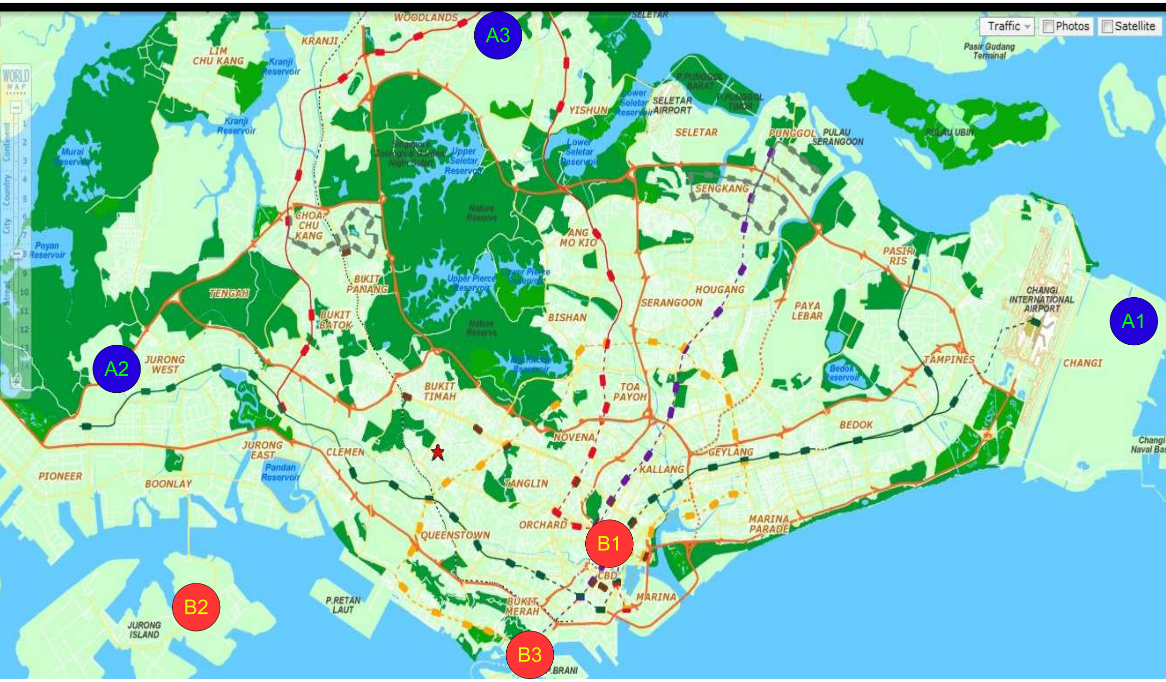
	B_1	B_2	B_3
B_1	0	17	6
B_2	17	0	14
B_3	6	14	0

Distances between B_j

	B_1	B_2	B_3
A_1	13	30	18
A_2	16	9	17
A_3	21	20	23

Distances between A_i and B_j

Formal problem statement



3 airbases located: Changi(3), Jurong West(3), Woodland(1). Service requests from: Raffles Place(2), Jurong Island(2), Sentosa(3). Out task is to complete all requests in order to maximaze the total service time in the zones and satisfies all timing constraints.

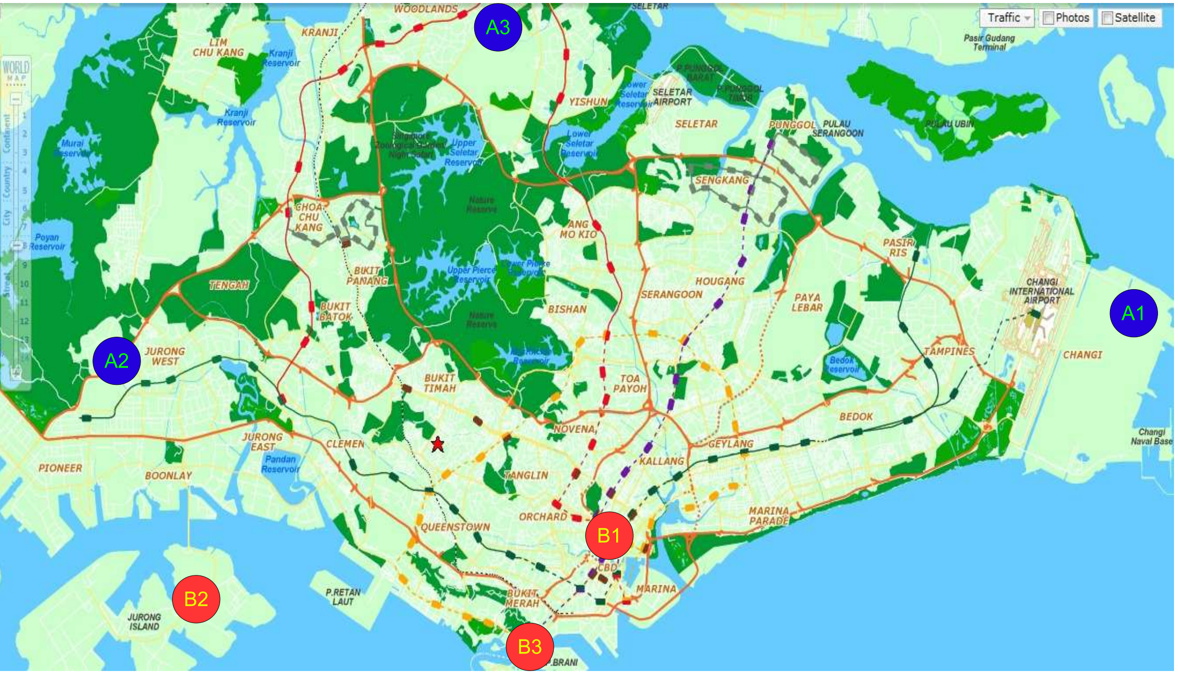
Notations:

$t^f_{B_i}; t^l_{B_i}$ - earliest and latest time for visit zone $B_i, i = 1, 2, 3$

c_{ij} -the benefit of sending the UAV from i-th aerobase to j-th zone. In particular, this benefit can be given in the form $c_{ij} = \frac{d_{ij}}{v_{ij}}$ that means the flight time from $A_i \to B_j$:

	B_1	B_2	B_3
A_1	433	1000	600
A_2	533	300	566
A_3	700	666	766

LP assignment problem



$$F = \sum_{i=1}^{k} \sum_{j=1}^{l} c_{ij} x_{ij} \to \min_{x_{ij}}$$

$$\sum_{i=1}^{3} x_{ij} = b_j, \quad j = 1, 2, 3$$

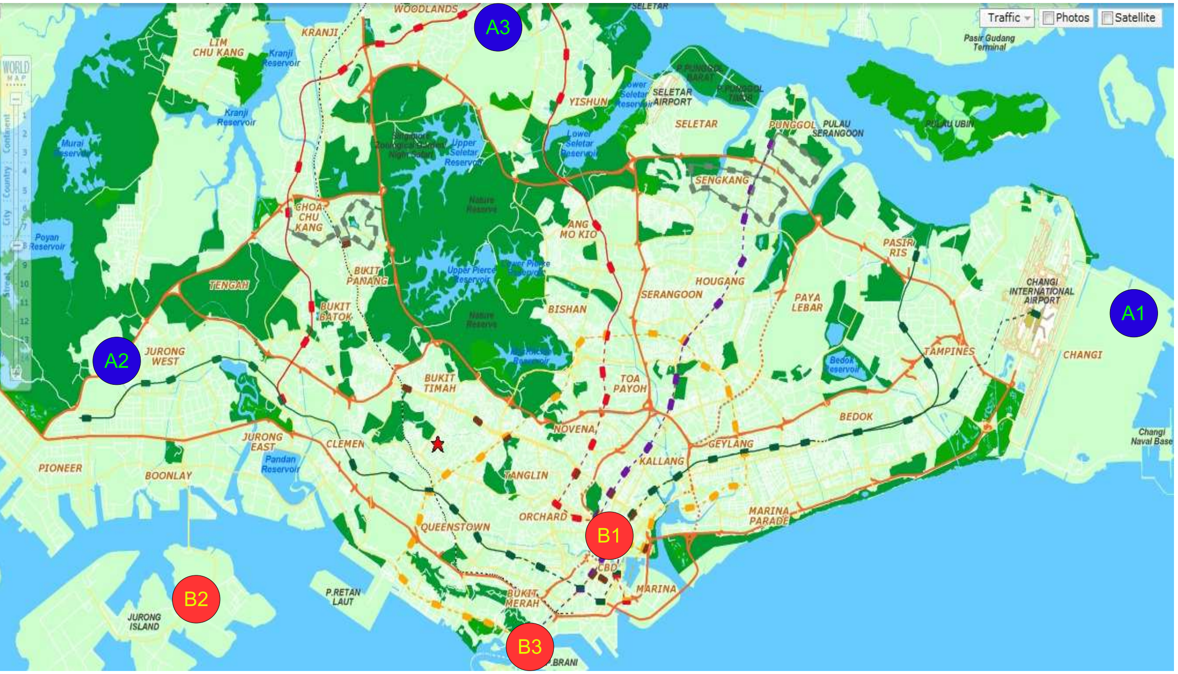
$$\sum_{j=1}^{3} x_{ij} = a_i, \quad i = 1, 2, 3$$

$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j, x_{ij} \geq 0, x_{ij} \in \mathbb{N}.$$

The most of methods include the following basic steps:

- To find initial plan x_{ij};
- Check optimality condition for that plan;
- Construct the improved plan in case of nonoptimality.

Optimal solution



$$F=\sum_{i=1}^{k}\sum_{j=1}^{l} c_{ij}x_{ij} \rightarrow \min_{x_{ij}}$$

$$\sum_{i=1}^{3} x_{ij} = b_j, \quad j=1,2,3$$

$$\sum_{j=1}^{3} x_{ij} = a_i, \quad i=1,2,3$$

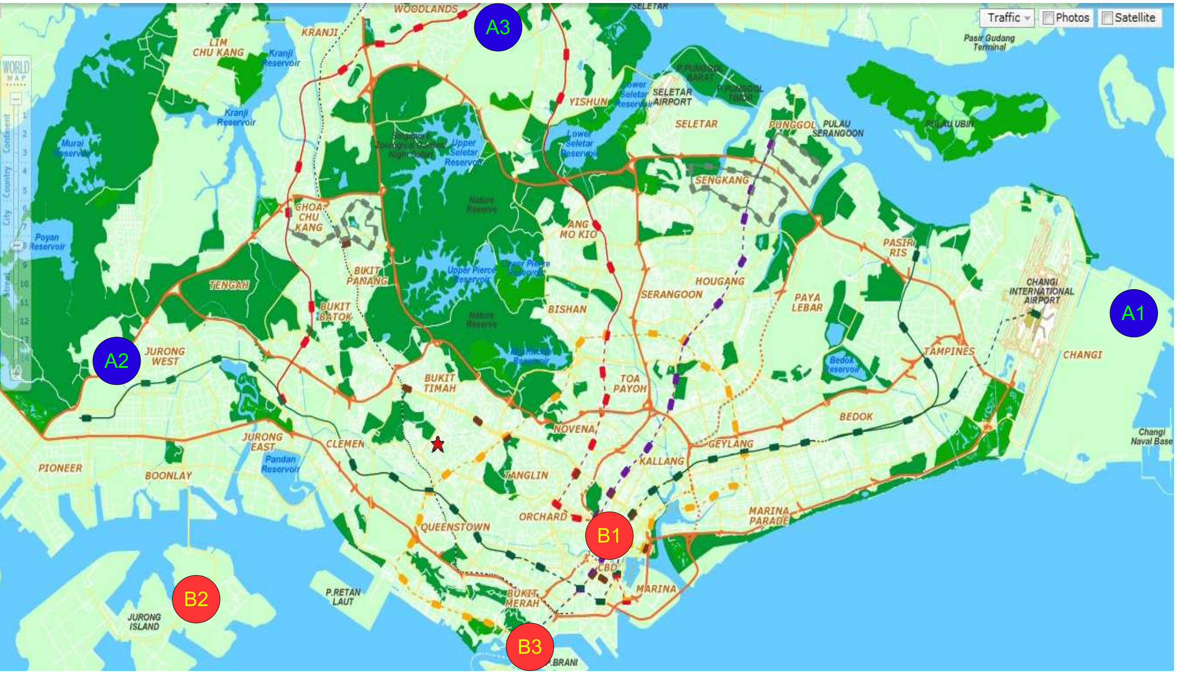
$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j, x_{ij} \geq 0, x_{ij} \in \mathbb{N}.$$

	B_1	B_2	B_3	a_i
A_1	2	✓ 0	1	3
A_2	✓ 0	2	1	3
A_3	✓ 0	✓ 0	1	1
b_j	2	2	3	

Optimal solution
($F = 56,6\ minutes$):

$$x_{11} = 2;\ x_{13} = 1;$$
$$x_{22} = 2;\ x_{23} = 1;$$
$$x_{33} = 1.$$

Timing constraints(case 1)



$$F = \sum_{i=1}^{k} \sum_{j=1}^{l} c_{ij} x_{ij} \to \min_{x_{ij}}$$

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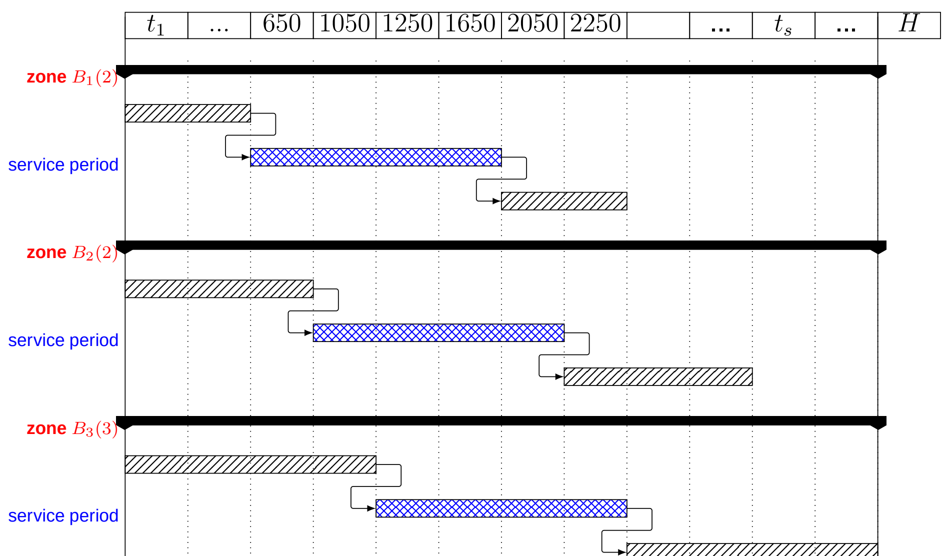


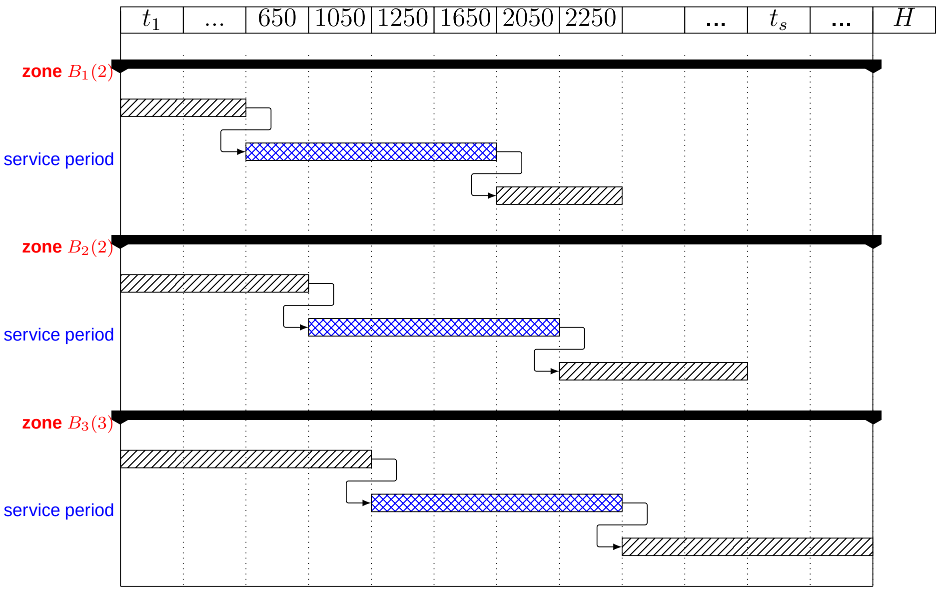
Time "windows":

$$t^f_{B_1} = 650sec, \quad t^l_{B_1} = 1650sec;$$

$$t^f_{B_2} = 1050sec, \quad t^l_{B_2} = 2050sec;$$

$$t^f_{B_3} = 1250sec, \quad t^l_{B_3} = 2250sec.$$

Solution procedure



Time "windows":

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$$t^f_{B_3} = 1250sec, \quad t^l_{B_3} = 2250sec.$$

Divide our problem by considering the assignments problem on the following 5 periods:

$Period1 : [650, 1050]$ - 1 problem for B_1 to assign 2 UAVs (i.e. $B_1(2)$);

$Period2 : [1050, 1250]$ - 2 problems for B_1 and B_2 (4 UAVs – $B_1(2)$, $B_2(2)$);

$Period3 : [1250, 1650]$ - 3 problems for $B_1(2)$, $B_2(2)$ and $B_3(3)$;

$Period4 : [1650, 2050]$ - 2 problems for $B_2(2)$ and $B_3(3)$;

$Period5 : [2050, 2250]$ - 1 problem for zone $B_3(3)$.

Solution procedure

Time schedular for each UAV in the table form:

		B_1		B_2		B_3	
		Departure time (D/T)	Arrival time (A/T)	D/T	A/T	D/T	A/T
A_1	UAV 1	217	2083	-	-	-	-
	UAV 2	217	2083	-	-	-	-
	UAV 3	-	-	-	-	650	2850
A_2	UAV 4	-	-	750	2350	-	-
	UAV 5	-	-	750	2350	-	-
	UAV 6	-	-	-	-	684	2816
A_3	UAV 7	-	-	-	-	484	3016

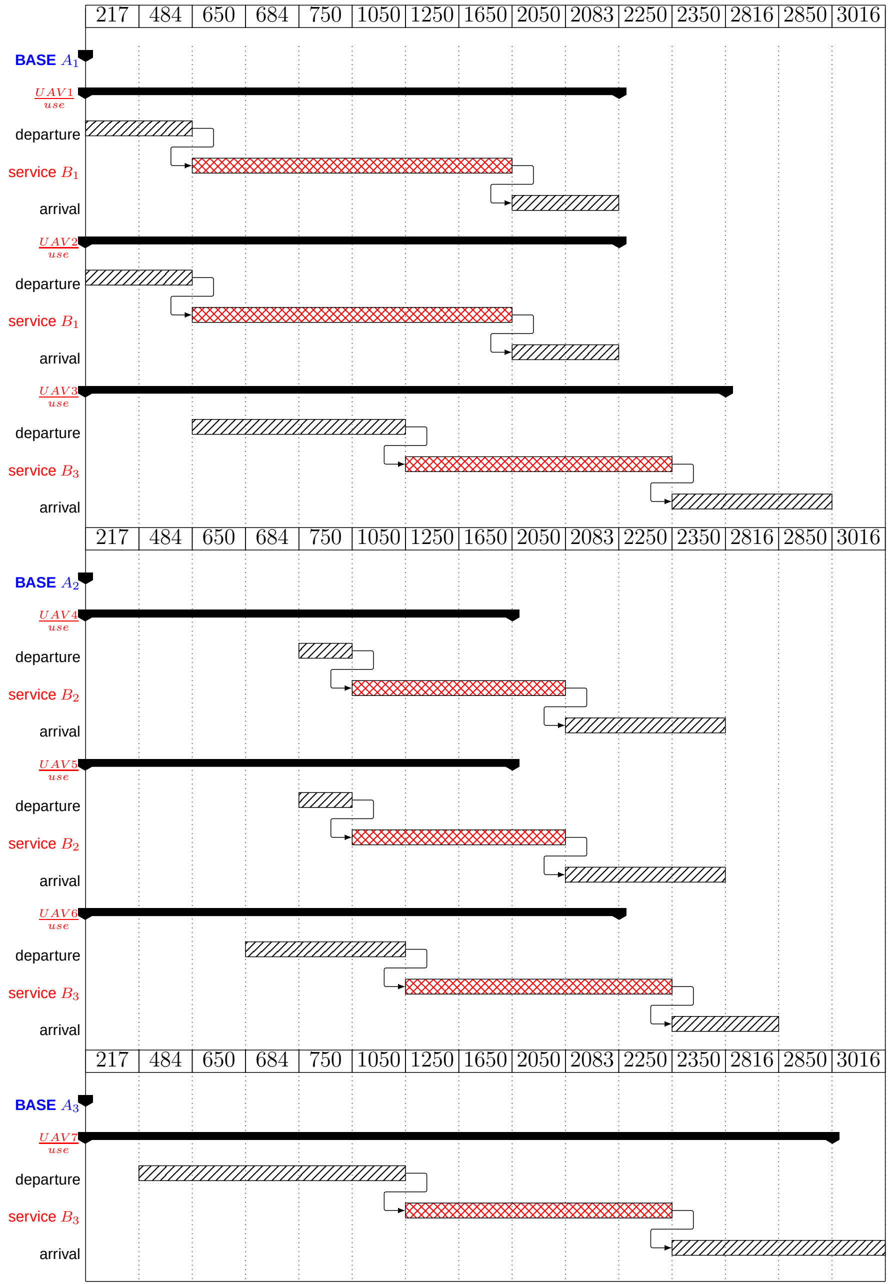
The total service time performed by all UAVs takes an optimal value

$$T^{service} = \sum_{i=1}^{7} h_i - 2\min_{x_{ij}} \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{d_{ij}}{v_{ij}} x_{ij} - \sum_{i=1}^{7} T_i^{zone} = \quad (1)$$

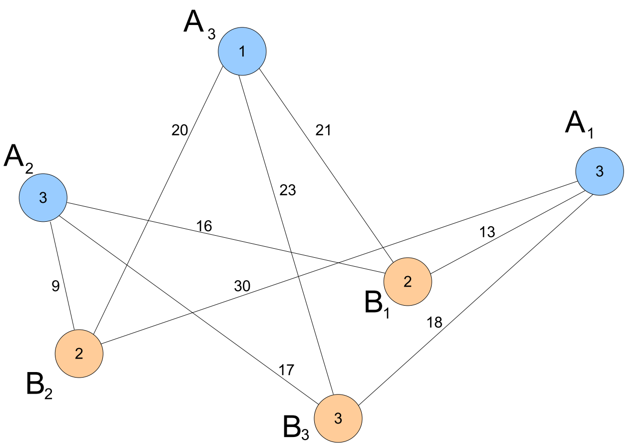
$$= 7 * 3600 - 2 * 3398 - 7 * 1000 \ sec.$$

$$\approx 3,18 \ hours$$

Flight schedular plan



Timing constraints(case 2)

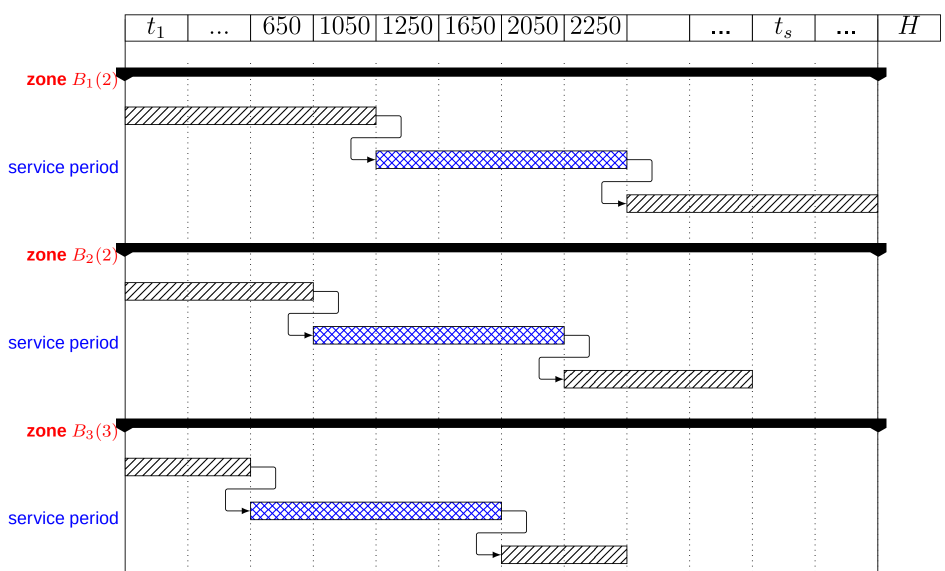


$$F = \sum_{i=1}^{k}\sum_{j=1}^{l} c_{ij}x_{ij} \to \min_{x_{ij}}$$

$$\sum_{i=1}^{3} x_{ij} = b_j, \quad j = 1,2,3$$

$$\sum_{j=1}^{3} x_{ij} = a_i, \quad i = 1,2,3$$

$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j, x_{ij} \geq 0, x_{ij} \in \mathbb{N}.$$



Time "windows":

$$t^f_{B_1} = 1250sec, \quad t^l_{B_1} = 2250sec;$$

$$t^f_{B_2} = 1050sec, \quad t^l_{B_2} = 2050sec;$$

$$t^f_{B_3} = 650sec, \quad t^l_{B_3} = 1650sec.$$

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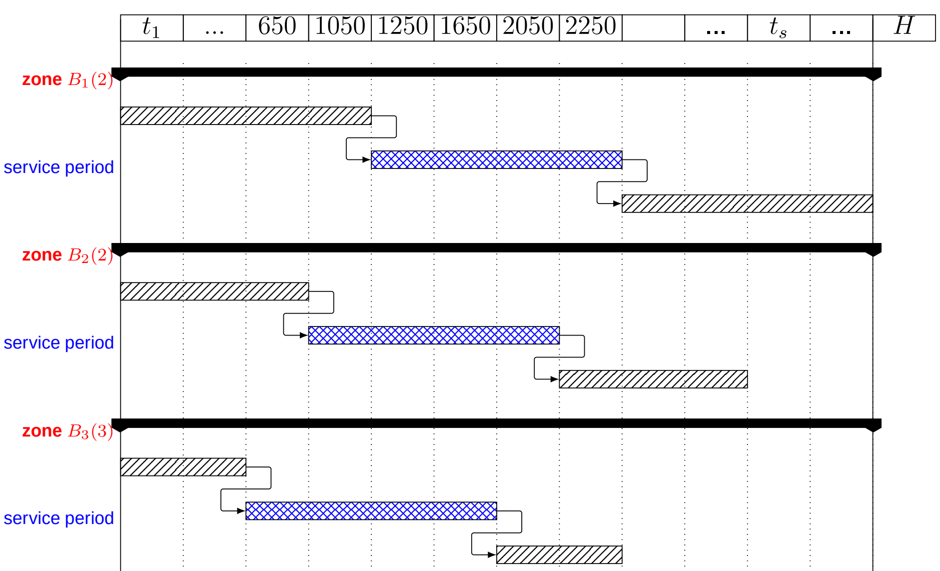
$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j, x_{ij} \geq 0, x_{ij} \in \mathbb{N}.$$

$$X^{flight} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$



Time "windows":

$$t^f_{B_1} = 1250sec, \quad t^l_{B_1} = 2250sec;$$

$$t^f_{B_2} = 1050sec, \quad t^l_{B_2} = 2050sec;$$

$$t^f_{B_3} = 650sec, \quad t^l_{B_3} = 1650sec.$$

Timing constraints(case 2)

$$X^{flight} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}, A_1 \xrightarrow{600} B_3, A_2 \xrightarrow{566} B_3,$$

$$A_3 \xrightarrow{766} B_3 > 650$$

$$F = \sum_{i=1}^{k} \sum_{j=1}^{l} c_{ij} x_{ij} \to \min_{x_{ij}}$$

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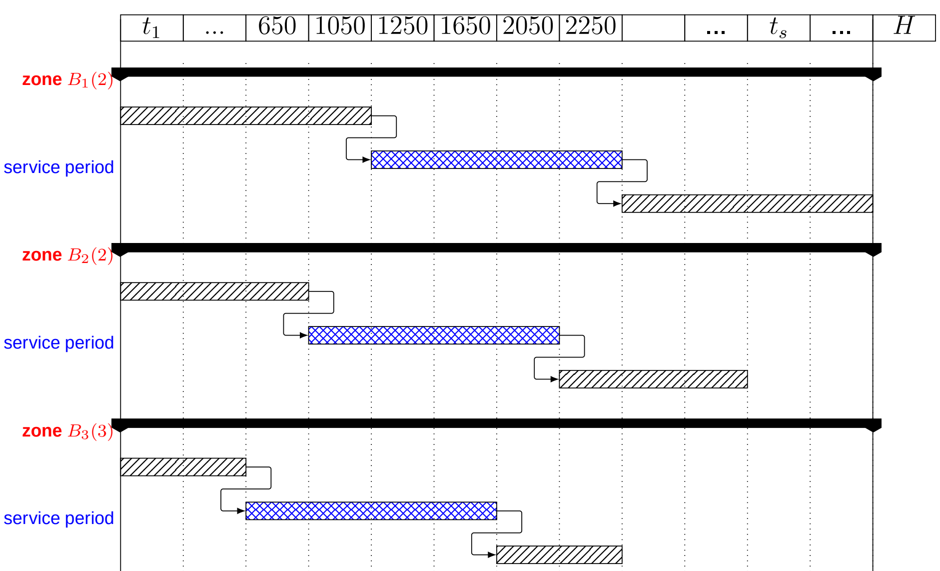

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Timing constraints(case 2)

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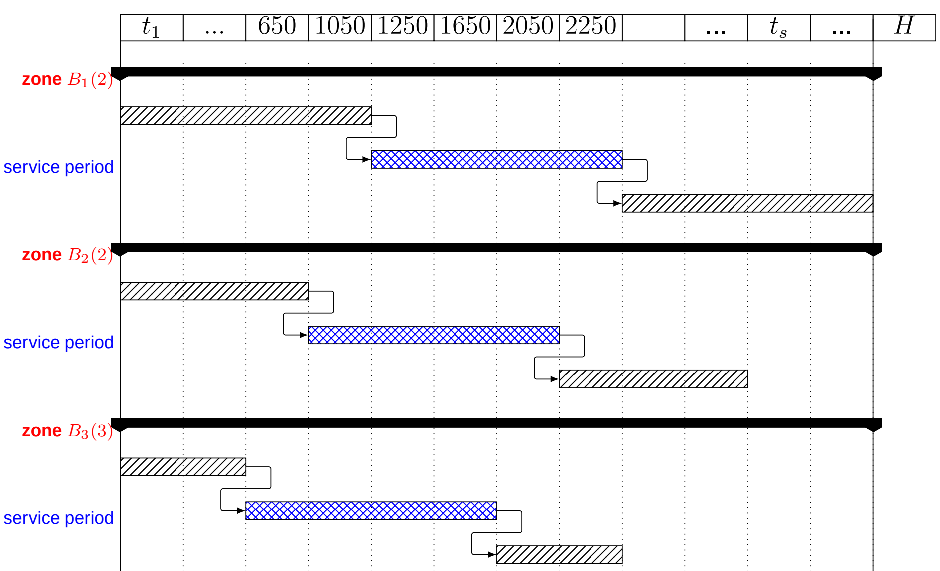
$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j, x_{ij} \geq 0, x_{ij} \in \mathbb{N}.$$

$$X^{flight} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} - not\ feasible\ solution$$



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Timing constraints(case 2)

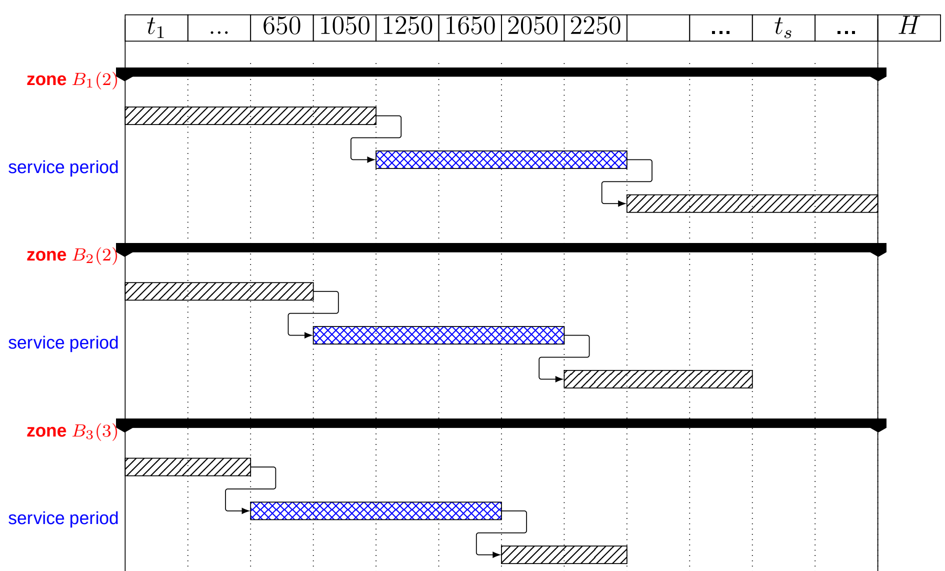
$$F = \sum_{i=1}^{k} \sum_{j=1}^{l} c_{ij} x_{ij} \rightarrow \min_{x_{ij}}$$

$Period1 : [650, 1050]$ - $B_3(3)$;
$Period2 : [1050, 1250]$ - $B_3, B_2(2)$;
$Period3 : [1250, 1650]$ - $B_3, B_2, B_1(2)$
$Period4 : [1650, 2050]$ - B_2, B_1;
$Period5 : [2050, 2250]$ - B_1.

$$\sum_{i=1}^{3} x_{ij} = b_j, \quad j = 1, 2, 3$$

$$\sum_{j=1}^{3} x_{ij} = a_i, \quad i = 1, 2, 3$$

$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j, x_{ij} \geq 0, x_{ij} \in \mathbb{N}.$$



Apllying NSW methods:

$$X_{new}^{flight} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

$$T^{service} \approx 2,06 \; hours < T_{optimal}^{service}$$

Modification

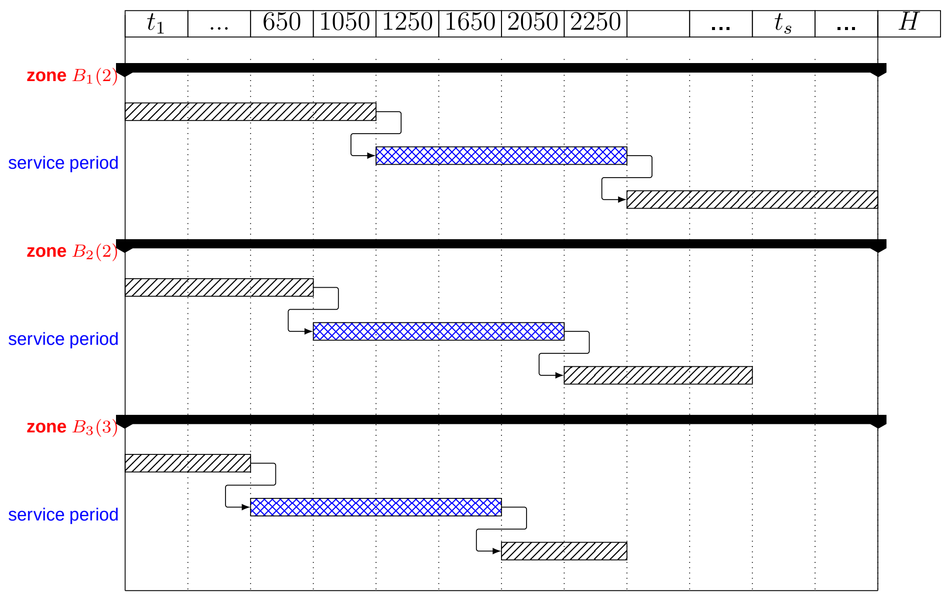
$$X^{flight} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}, A_1 \xrightarrow{600} B_3,$$

$$A_2 \xrightarrow{566} B_3,$$

$$A_3 \xrightarrow{766} B_3 > 650$$

	B_1	B_2	B_3
B_1	0	17	6
B_2	17	0	14
B_3	6	14	0

Flight time from B_3 to B_1: $t_{fly}^{B_3 \to B_1} = \frac{d_{ij}}{v_ij} = \frac{6000}{30} = 200$ ($B_3 \xrightarrow{200} B_1$).



Time "windows":

$$t_{B_1}^f = 1250sec, \quad t_{B_1}^l = 2250sec;$$
$$t_{B_2}^f = 1050sec, \quad t_{B_2}^l = 2050sec;$$
$$t_{B_3}^f = 650sec, \quad t_{B_3}^l = 1650sec.$$

Modification

$$X^{flight} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & \color{red}{1} \end{pmatrix}, A_1 \xrightarrow{600} B_3,$$

$$A_2 \xrightarrow{566} B_3,$$

$$\color{red}{A_3 \xrightarrow{766} B_3 > 650}$$

	B_1	B_2	B_3
B_1	0	17	6
B_2	17	0	14
B_3	6	14	0

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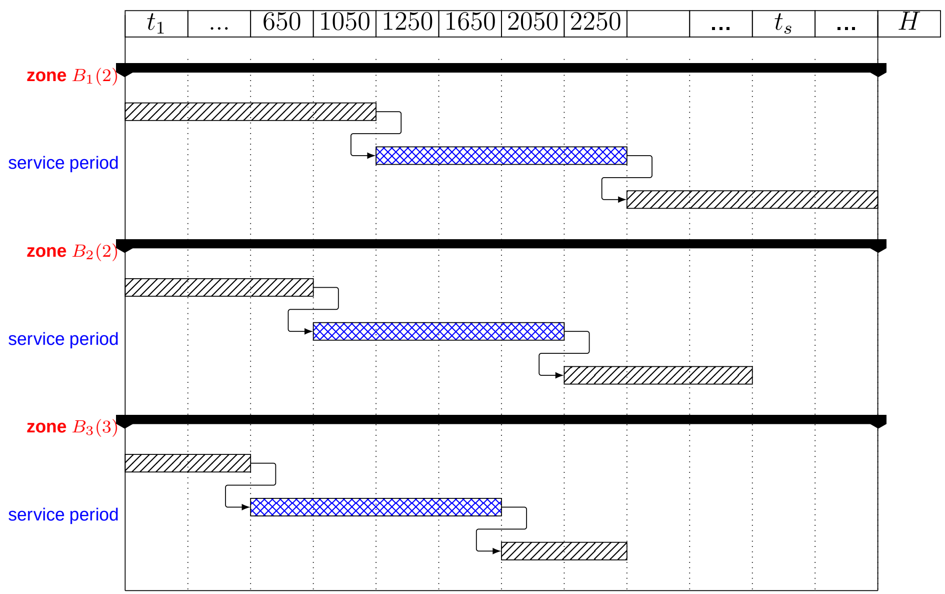


We will send 2 UAVs to the zone B_1 from base A_1 to time windows [1250,2250]. Thus we can use another UAV form base A_1 in order to serve on first period [650,1650] base B_3, since flight time $A_1 \xrightarrow{600} B_3$ allows us to do this.

Modification

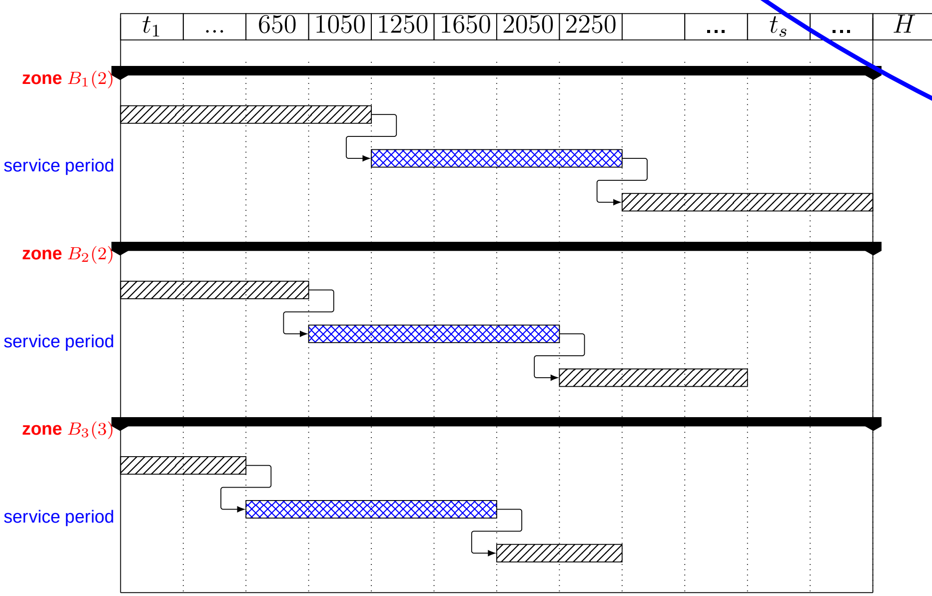
$$X^{flight} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}, A_1 \xrightarrow{600} B_3,$$

$$A_2 \xrightarrow{566} B_3,$$

$$A_1 \xrightarrow{600} B_3 < 650$$

	B_1	B_2	B_3
B_1	0	17	6
B_2	17	0	14
B_3	6	14	0

Flight time from B_3 to B_1: $t_{fly}^{B_3 \to B_1} = \frac{d_{ij}}{v_ij} = \frac{6000}{30} = 200$ ($B_3 \xrightarrow{200} B_1$).



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Modification

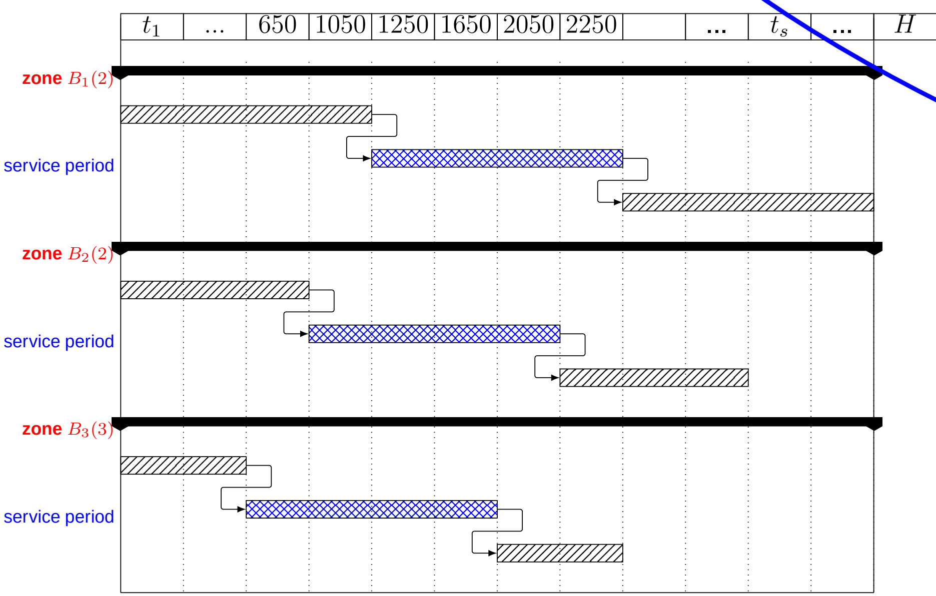
$$X^{flight} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}, A_1 \xrightarrow{600} B_3,$$

$$A_2 \xrightarrow{566} B_3,$$

$$A_1 \xrightarrow{600} B_3 < 650$$

	B_1	B_2	B_3
B_1	0	17	6
B_2	17	0	14
B_3	6	14	0

Flight time from B_3 to B_1: $t_{fly}^{B_3 \rightarrow B_1} = \frac{d_{ij}}{v_i j} = \frac{6000}{30} = 200$ ($B_3 \xrightarrow{200} B_1$).



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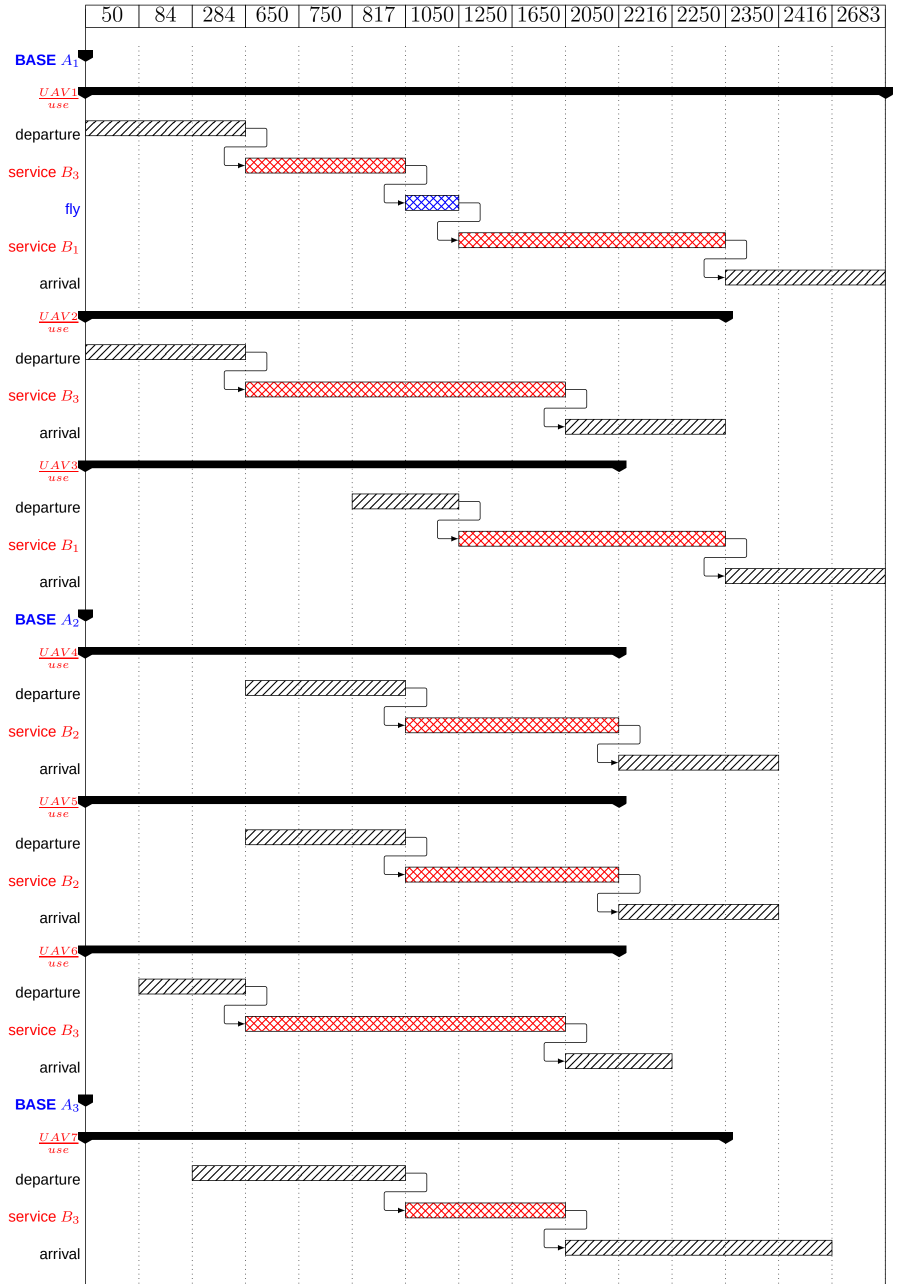
Flight schedular plan



Solution procedure

Time schedular for each UAV in the table form:

		B_1		B_2		B_3	
		Departure time (D/T)	Arrival time (A/T)	D/T	A/T	D/T	A/T
A_1	UAV 1	-	2683	-	-	50	-
	UAV 2	-	-	-	-	50	2250
	UAV 3	817	2683	-	-	-	-
A_2	UAV 4	-	-	650	2350	-	-
	UAV 5	-	-	650	2350	-	-
	UAV 6	-	-	-	-	84	2216
A_3	UAV 7	-	-	-	-	284	2416

The total service time performed by all UAVs takes an near optimal value

$$T^{service} = \sum_{i=1}^{7} h_i - 2\min_{x_{ij}} \sum_{i=1}^{3}\sum_{j=1}^{3} \frac{d_{ij}}{v_{ij}} x_{ij} - \sum_{i=1}^{7} T_i^{zone} =$$

$$= 7 * 3600 - 2 * 4198 - 7 * 1000 \; sec.$$

$$\approx 2,74 \; hours$$

Outline of the method

The method for LP assignment problem as well as simplex methods are:

- iterative,
- exact(satisfied all constraints),
- finite,
- relaxed(in a sense of the value of objective function).

Thus in some sense this method is analog of simplex method, but the ideas of this method is more naturally can be applied to assignments LP problems.

Further work

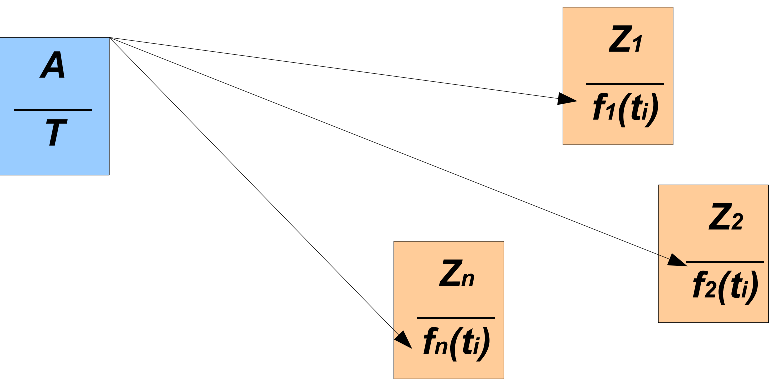
The major task is to solve the specific assignment problem for MAS by developing

- new optimality condition,
- ϵ - optimality condition,
- realize sensitivity analysis (robustness analysis),
  Siarhei Dymkou, Kai Yew Lum, Jian Xin Xu, Comparison of the adaptive method with classical simplex method for linear programming.(2011) (in preparation)
- consider objectives with very general functional forms.

The end

Thank you!

Extra slides (DP)



- Invariant embedding of the problem into $P(k, y), k \in [1; n]; y \in [0; T]$, then define the Bellman function;
- Construct the Bellman equation;

$$\sum_{i=1}^{n} f_i(t_i) \to \max \qquad \Longrightarrow \qquad P(k, y) : \sum_{i=1}^{k} f_i(t_i) \to \max,$$

$$\sum_{i=1}^{n} t_i \le T,\ t_i \ge 0, i = 1, ..., n \qquad\qquad \sum_{i=1}^{k} t_i \le y,\ t_i \ge 0, i = 1, ..., k$$

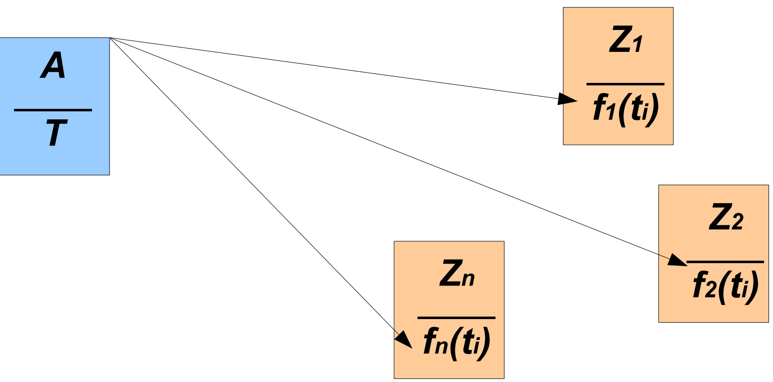
Bellman equation

$$B_k(y) = \max_{t_i} \sum_{i=1}^{k} f_i(t_i),$$

$$B_k(y) = \max_{0 \le z \le y} \left[f_k(z) + B_{k-1}(y - z) \right] \qquad \Longrightarrow \qquad \sum_{i=1}^{k} t_i \le y,\ t_i \ge 0, i = 1, ..., k$$

"back"

Extra slides (DP)



- Invariant embedding of the problem into $P(k, y), k \in [1; n]; y \in [0; T]$, then define the Bellman function;
- Construct the Bellman equation;
- Solve the Bellman equation, and apply the solution of it to initial problem

$$t_n^0,\ t_{n-1}^0,\ \ldots,\ t_2^0,\ t_1^0 \quad \Longleftarrow \quad \textit{Continue this procedure we will find the optimal solution of our problem}$$

Put $k = n,\ y = T$ and
find the value $t_n^0 \doteq z^0(T)$ for the zone Z_n

$$f_n(z_n^0) + B_{n-1}(T - z_n^0) =$$
$$\max_{0 \le z \le T} \left[f_n(z) + B_{n-1}(T - z) \right]$$

$\Longrightarrow$

$k = n - 1,\ y = T - t_n^0$ and
find the value $t_{n-1}^0 \doteq z^0(T - t_n^0)$

$$f_{n-1}(z_{n-1}^0) + B_{n-2}(T - z_{n-1}^0) =$$
$$\max_{0 \le z \le T - t_n^0} \left[f_{n-1}(z) + B_{n-2}(T - t_n^0 - z) \right]$$