Assignment problem for MAS

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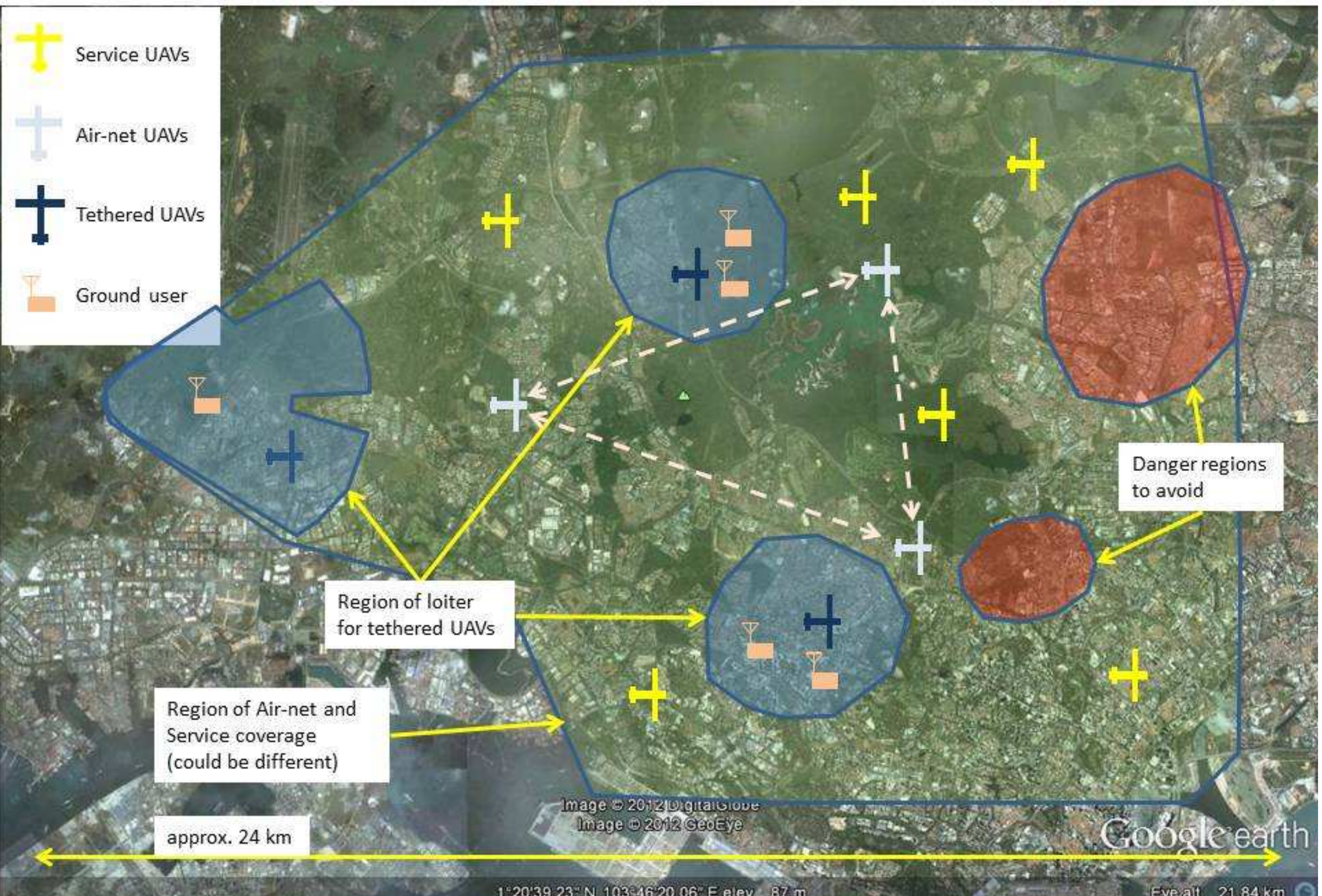
Introduction

The pool of UAVs would loiter above the area of operations and await service requests. Upon receiving a request, the best-suited UAV is deployed to provide the service to the user. At the end of serving the request, the UAV would return to the pool to serve another request:



Request info:

- Locations
- Number UAVs
- Time windows
- Duration

Introduction

Starting conditions:The MAS is located at airbases and receives multiple requests for service including:

- Location to visit;
- Number of air-vehicles required;
- Earliest time of 1-st visit;
- Latest time of 1-st visit;
- Minimum duration per visit;
- Maximum interval between visits.

Mission Objective:

Air-vehicle(s) to be assigned to request and the corresponding paths to take to the service request; (variations to requests with minimal change if it cannot be met.)

That:

Maximize the number of service requests that can be serviced.

Constraints:

Air-vehicle performance and dynamics.

Principal schema

$Benchmark \quad problem$

Principal schema

Benchmark problem

Assignment problem

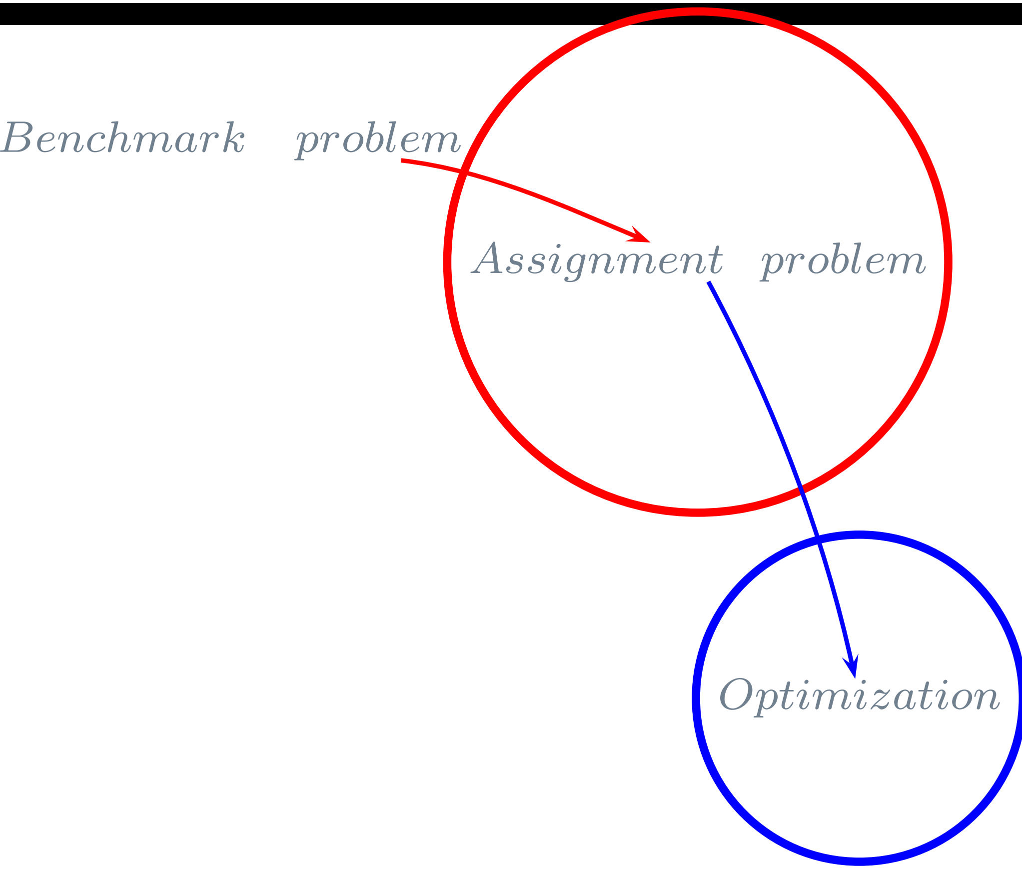
Principal schema

Benchmark problem

Assignment problem

- Heuristics approach
- Optimization approach
  LP methods,
  MIP methods,
  Graph network methods,
  Dynamical programming methods,
  and other.

Principal schema



Dynamical assignment (objectives)

$$\boxed{J_1(x) = \sum_{t=0}^{\nu} x_{ij}(t)(h_i - 2t_{ij}) \to \max_{x_{ij}(t),t=0,\Delta,2\Delta,\ldots,\nu}}$$

$$a_i(t+\Delta) = a_i(t) - \sum_{j=1}^{l} x_{ij}(t) + \sum_{j=1}^{l} x_{ij}(t+\Delta-h_i),\ \ i = 1,...,k;$$

$$b_j(t+\Delta) = b_j(t) - \sum_{i=1}^{k} x_{ij}(t-h_i+t_{ij}) + \sum_{i=1}^{k} x_{ij}(t-t_{ij}),\ \ j = 1,...,l;$$

$$a_i(t) + \sum_{j=1}^{l} x_{ij}(t) = a_i,\ \ i = 1,...,k; \qquad b_j(t) + \sum_{i=1}^{k} x_{ij}(t-t_{ij}) = b_j,\ \ j = 1,...,l.$$

1. *The total service time for multiple zones*

2. *The total number of UAVs "circles"* : $J_2(x) = \sum_{t=0}^{\nu} x_{ij}(t)$

3. *The total unobservable time for multiple zones* : $J_3(x) = H - J_1(x)$

Dynamical assignment (constraints)

$$J_1(x) = \sum_{t=0}^{\nu} x_{ij}(t)(h_i - 2t_{ij}) \to \max_{x_{ij}(t),t=0,\Delta,2\Delta,\ldots,\nu}$$

$$\boxed{a_i(t+\Delta) = a_i(t) - \sum_{j=1}^{l} x_{ij}(t) + \sum_{j=1}^{l} x_{ij}(t+\Delta-h_i),\ i=1,\ldots,k;}$$

$$\boxed{b_j(t+\Delta) = b_j(t) - \sum_{i=1}^{k} x_{ij}(t-h_i+t_{ij}) + \sum_{i=1}^{k} x_{ij}(t-t_{ij}),\ j=1,\ldots,l;}$$

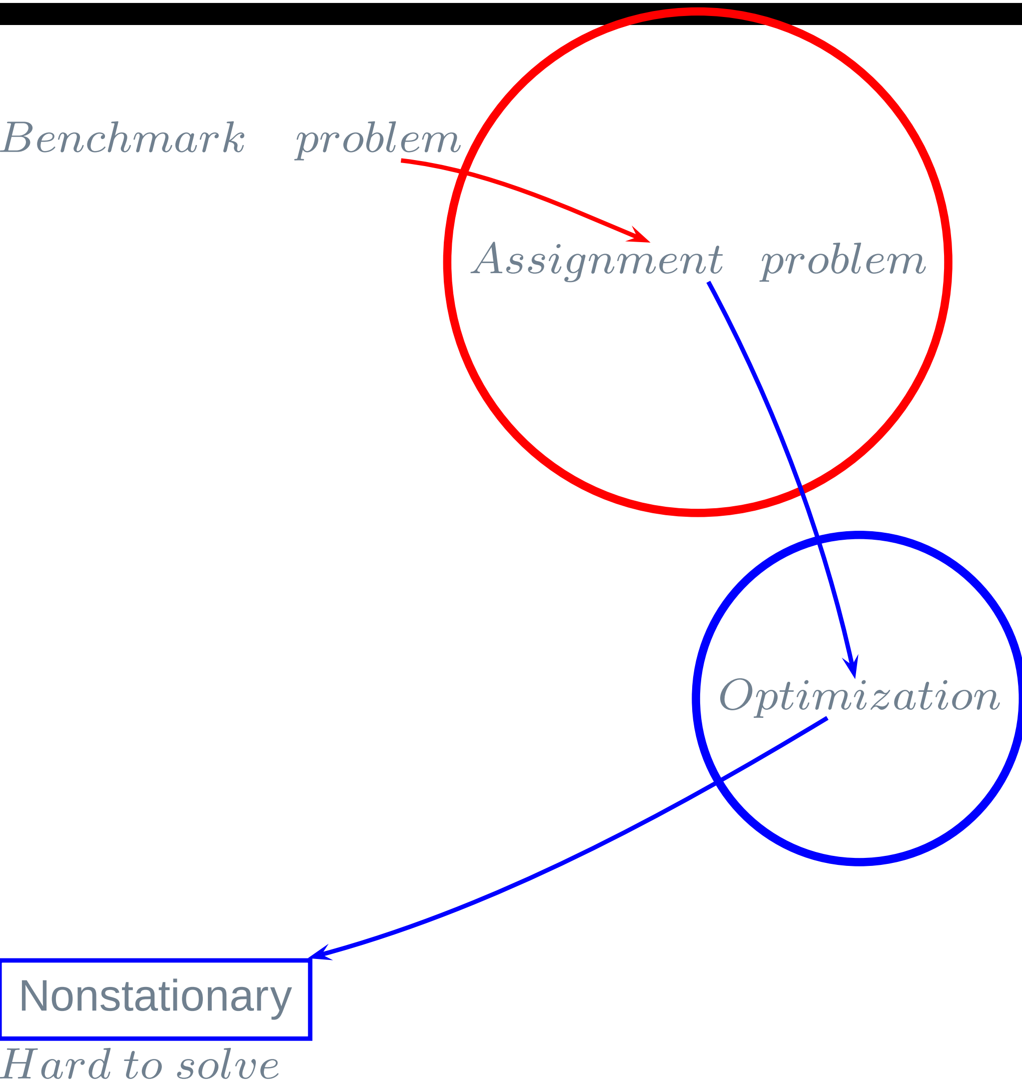
$$a_i(t) + \sum_{j=1}^{l} x_{ij}(t) = a_i,\ i=1,\ldots,k; \qquad b_j(t) + \sum_{i=1}^{k} x_{ij}(t-t_{ij}) = b_j,\ j=1,\ldots,l.$$

The number of UAVs at A_i and at B_j at the next moment $t+\Delta$ is composed by :

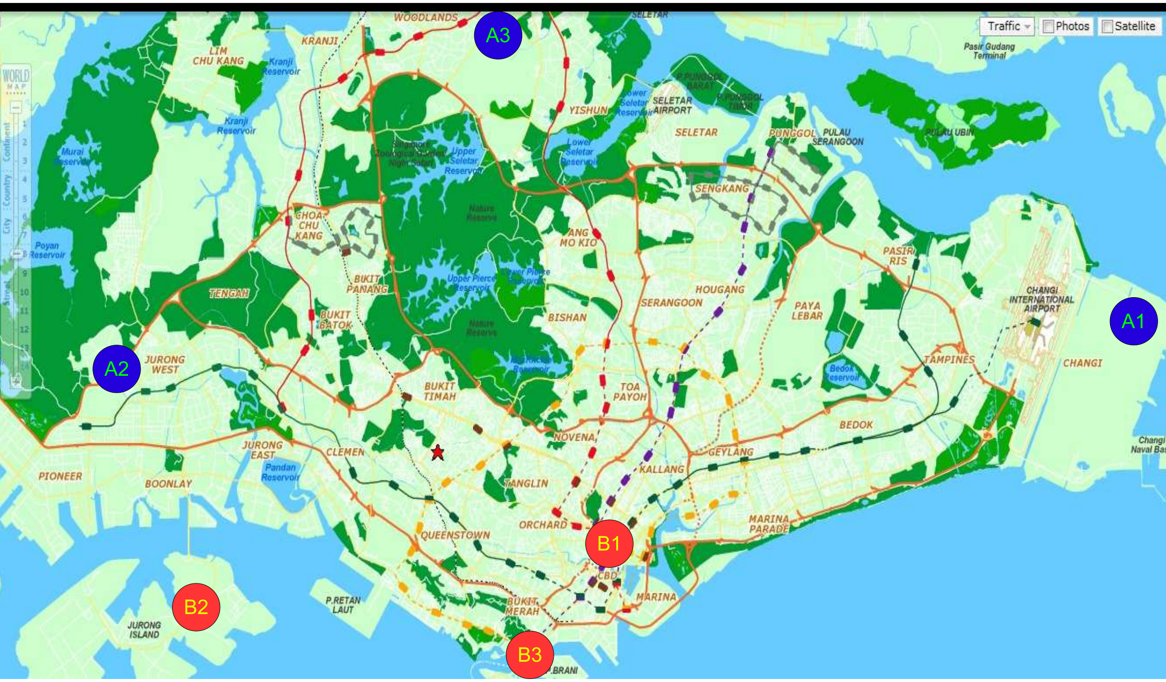
- UAVs at the previous moment t
- UAVs that are returned during the period $[t, t+\Delta]$
- UAVs were send to zones at the moment t

- UAVs working at zone at moment t with sufficient flight h;
- UAVs that are reached zone in $[t, t+\Delta]$;
- UAVs that should leave $j-th$ zone due out-of-fuel

Principal schema



Formal problem statement



3 airbases located:

- Changi(3);
- Jurong West(3);
- Woodlands(1).

Service requests from:

- Raffles Place(2);
- Jurong Island(2);
- Sentosa(3).

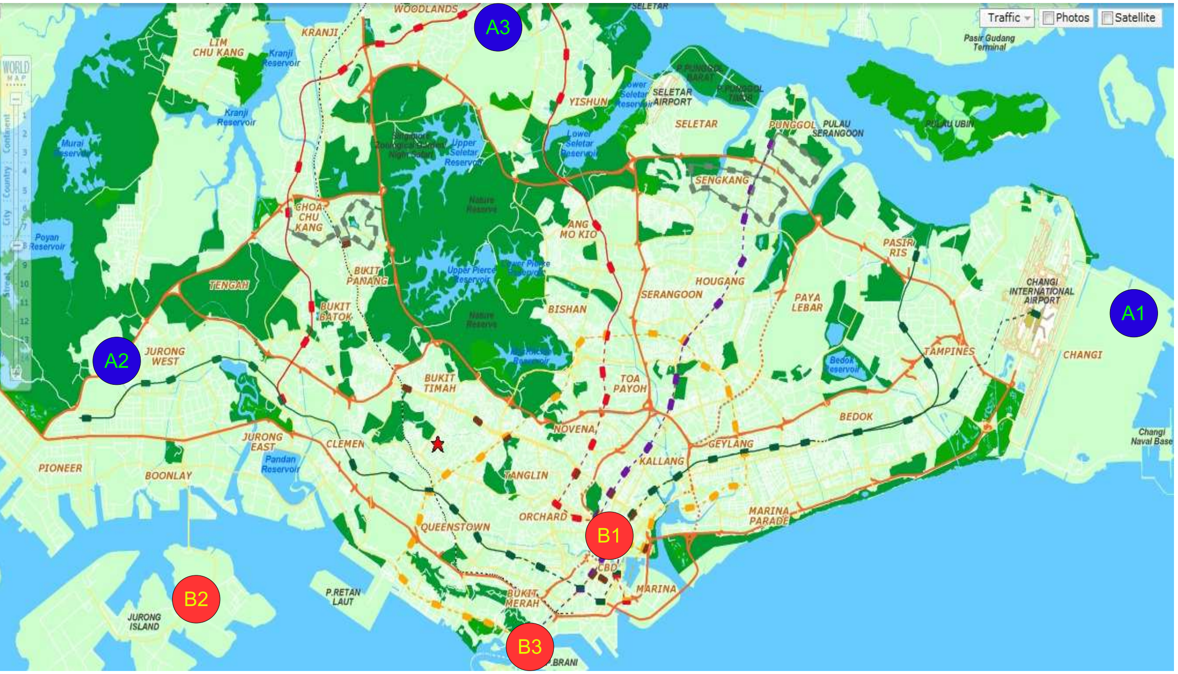
Notations:

$t^f_{B_i}$; $t^l_{B_i}$ - earliest and latest time for visit zone $B_i, i = 1, 2, 3$

c_{ij} -the benefit of sending the UAV from i-th aerobase to j-th zone. In particular, this benefit can be given in the form $c_{ij} = \frac{d_{ij}}{v_{ij}}$ that means the flight time from $A_i \rightarrow B_j$:

	B_1	B_2	B_3
A_1	433	1000	600
A_2	533	300	566
A_3	700	666	766

LP assignment problem



$$F = \sum_{i=1}^{k} \sum_{j=1}^{l} c_{ij} x_{ij} \to \min_{x_{ij}}$$

$$\sum_{i=1}^{3} x_{ij} = b_j, \quad j = 1, 2, 3$$

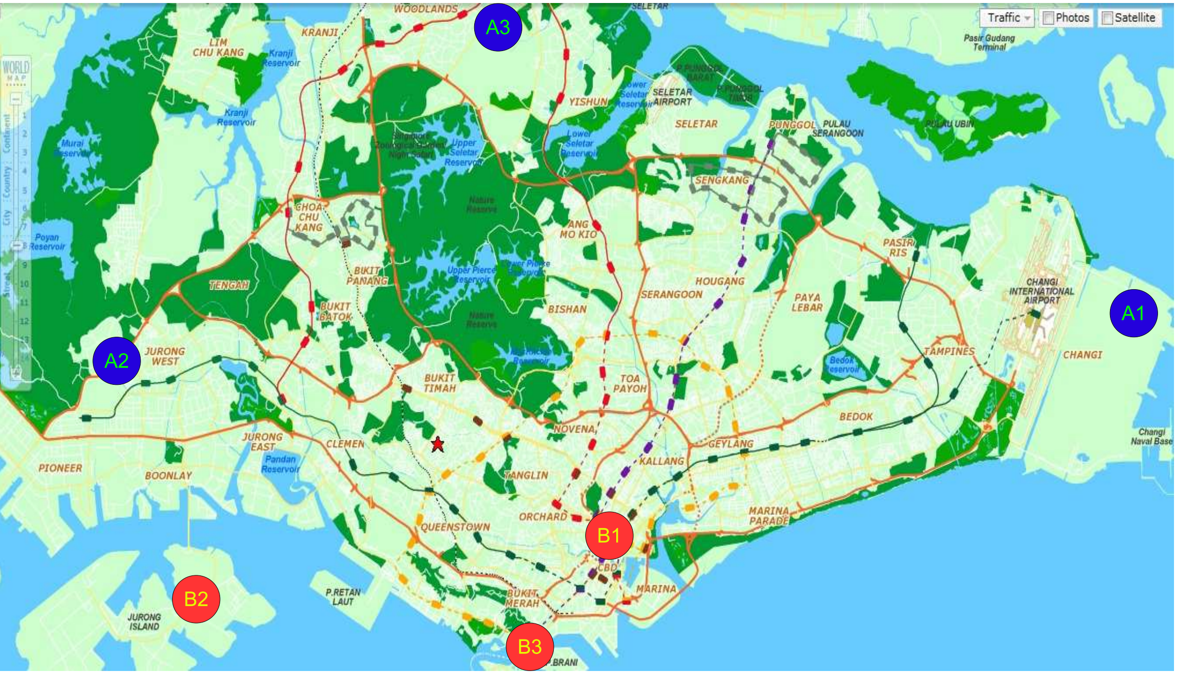
$$\sum_{j=1}^{3} x_{ij} = a_i, \quad i = 1, 2, 3$$

$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j, x_{ij} \geq 0, x_{ij} \in \mathbb{N}.$$

The most of methods include the following basic steps:

- To find initial plan x_{ij};
- Check optimality condition for that plan;
- Construct the improved plan in case of nonoptimality.

Optimal solution



$$F=\sum_{i=1}^{k}\sum_{j=1}^{l}c_{ij}x_{ij}\to\min_{x_{ij}}$$

$$\sum_{i=1}^{3}x_{ij}=b_j,\quad j=1,2,3$$

$$\sum_{j=1}^{3}x_{ij}=a_i,\quad i=1,2,3$$

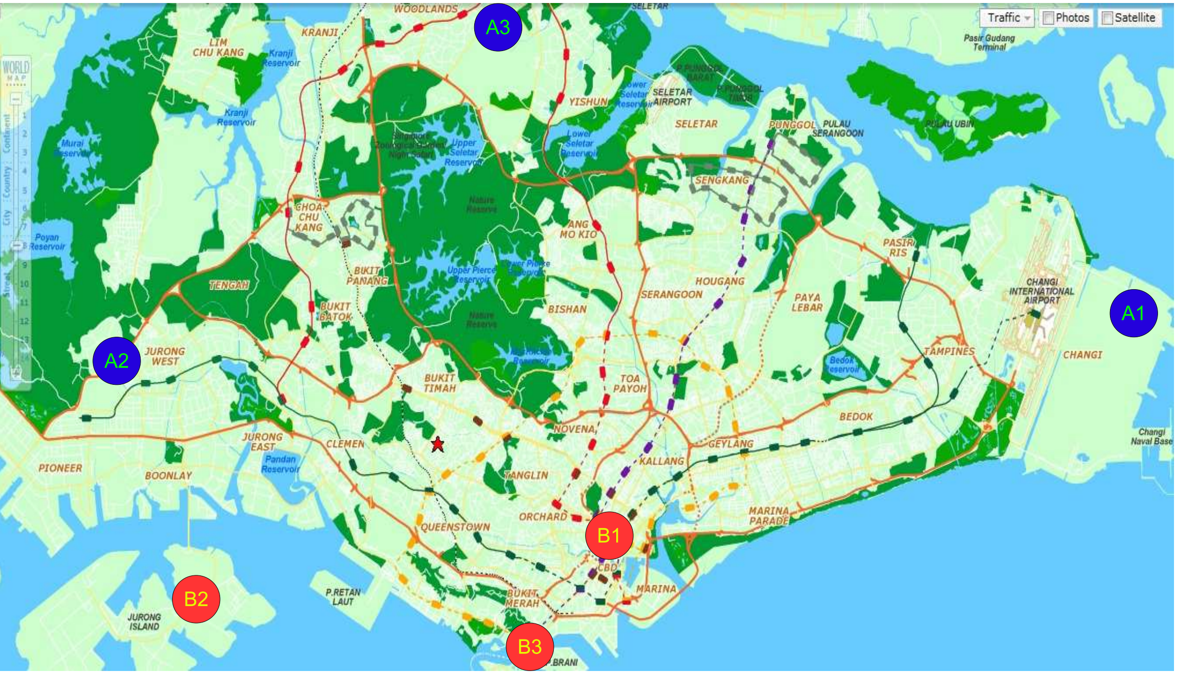
$$\sum_{i=1}^{k}a_i=\sum_{j=1}^{l}b_j,\, x_{ij}\geq 0,\, x_{ij}\in\mathbb{N}.$$

	B_1	B_2	B_3	a_i
A_1	2	$\checkmark$ 0	1	3
A_2	$\checkmark$ 0	2	1	3
A_3	$\checkmark$ 0	$\checkmark$ 0	1	1
b_j	2	2	3	

Optimal solution ($F=56,6\ minutes$):

$$x_{11}=2;\ x_{13}=1;$$
$$x_{22}=2;\ x_{23}=1;$$
$$x_{33}=1.$$

Timing constraints(case 1)

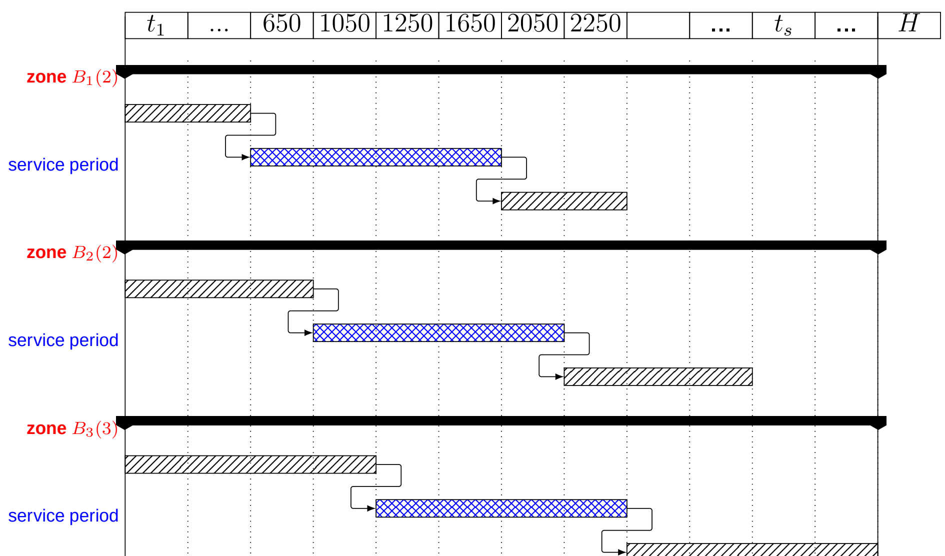


$$F=\sum_{i=1}^{k}\sum_{j=1}^{l}c_{ij}x_{ij}\rightarrow\min_{x_{ij}}$$

$$\sum_{i=1}^{3}x_{ij}=b_j,\quad j=1,2,3$$

$$\sum_{j=1}^{3}x_{ij}=a_i,\quad i=1,2,3$$

$$\sum_{i=1}^{k}a_i=\sum_{j=1}^{l}b_j, x_{ij}\geq 0, x_{ij}\in\mathbb{N}.$$



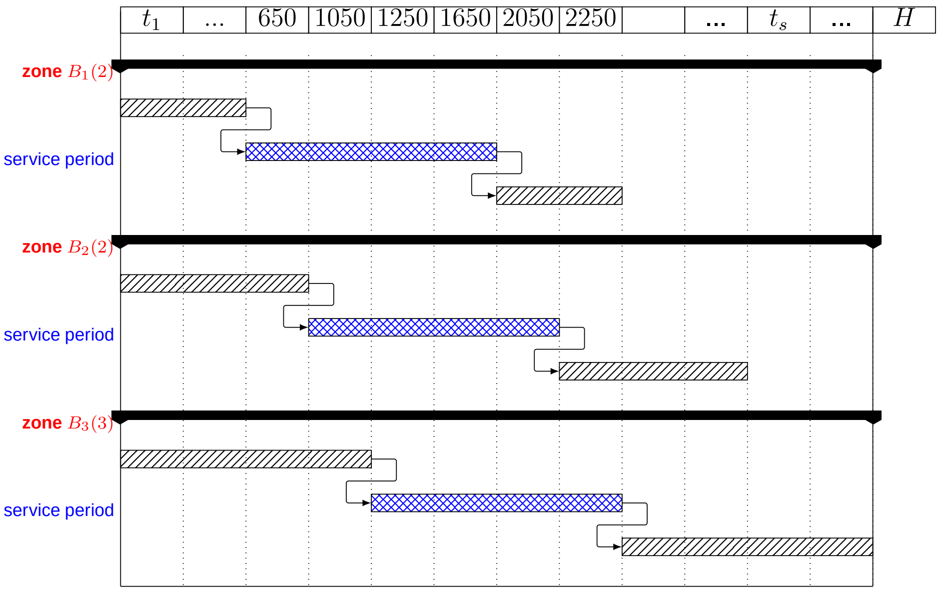
Time "windows":

$$t^f_{B_1}=650sec,\quad t^l_{B_1}=1650sec;$$

$$t^f_{B_2}=1050sec,\quad t^l_{B_2}=2050sec;$$

$$t^f_{B_3}=1250sec,\quad t^l_{B_3}=2250sec.$$

Solution procedure



Time "windows":

$$t^f_{B_1} = 650sec, \quad t^l_{B_1} = 1650sec;$$
$$t^f_{B_2} = 1050sec, \quad t^l_{B_2} = 2050sec;$$
$$t^f_{B_3} = 1250sec, \quad t^l_{B_3} = 2250sec.$$

Divide our problem by considering the assignments problem on the following 5 periods:

$Period1 : [650, 1050]$ - 1 problem for B_1 to assign 2 UAVs (i.e. $B_1(2)$);

$Period2 : [1050, 1250]$ - 2 problems for B_1 and B_2 (4 UAVs – $B_1(2)$, $B_2(2)$);

$Period3 : [1250, 1650]$ - 3 problems for $B_1(2)$, $B_2(2)$ and $B_3(3)$;

$Period4 : [1650, 2050]$ - 2 problems for $B_2(2)$ and $B_3(3)$;

$Period5 : [2050, 2250]$ - 1 problem for zone $B_3(3)$.

Solution

Time schedular for each UAV in the table form:

		B_1		B_2		B_3	
		Departure time (D/T)	Arrival time (A/T)	D/T	A/T	D/T	A/T
A_1	UAV 1	217	2083	-	-	-	-
	UAV 2	217	2083	-	-	-	-
	UAV 3	-	-	-	-	650	2850
A_2	UAV 4	-	-	750	2350	-	-
	UAV 5	-	-	750	2350	-	-
	UAV 6	-	-	-	-	684	2816
A_3	UAV 7	-	-	-	-	484	3016

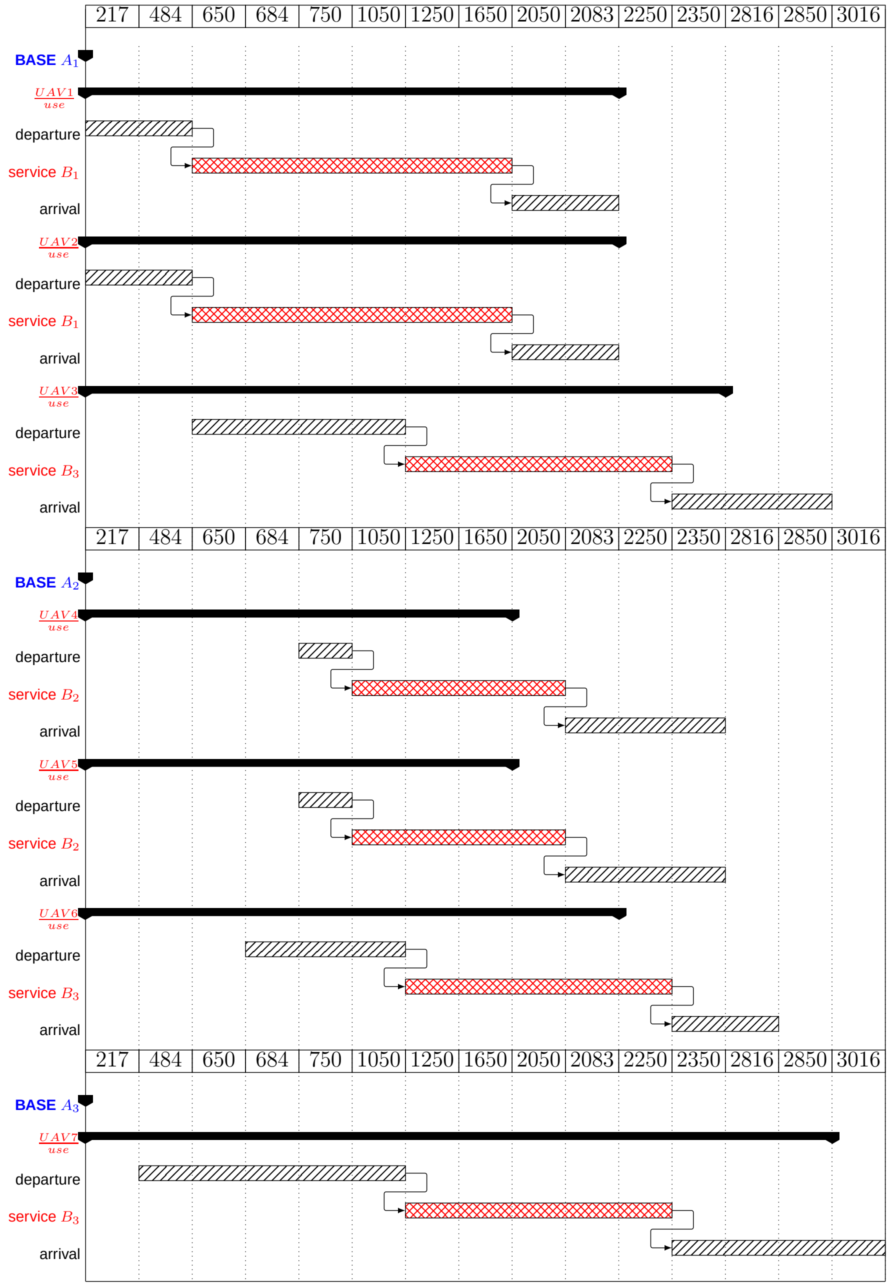
The total service time performed by all UAVs takes an optimal value

$$T^{service} = \sum_{i=1}^{7} h_i - 2 \min_{x_{ij}} \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{d_{ij}}{v_{ij}} x_{ij} - \sum_{i=1}^{7} T_i^{zone} = \tag{1}$$

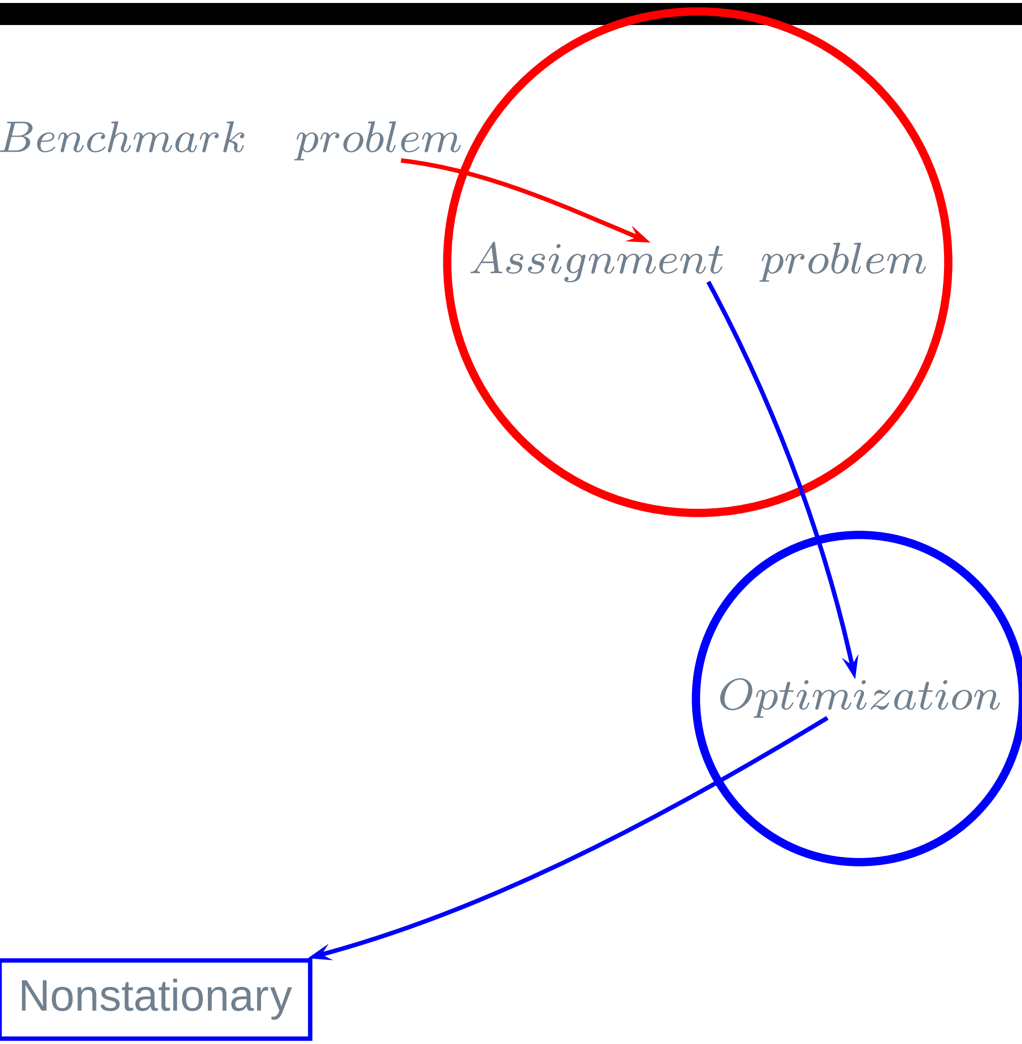
$$= 7 * 3600 - 2 * 3398 - 7 * 1000 \ sec.$$

$$\approx 3,18 \ hours$$

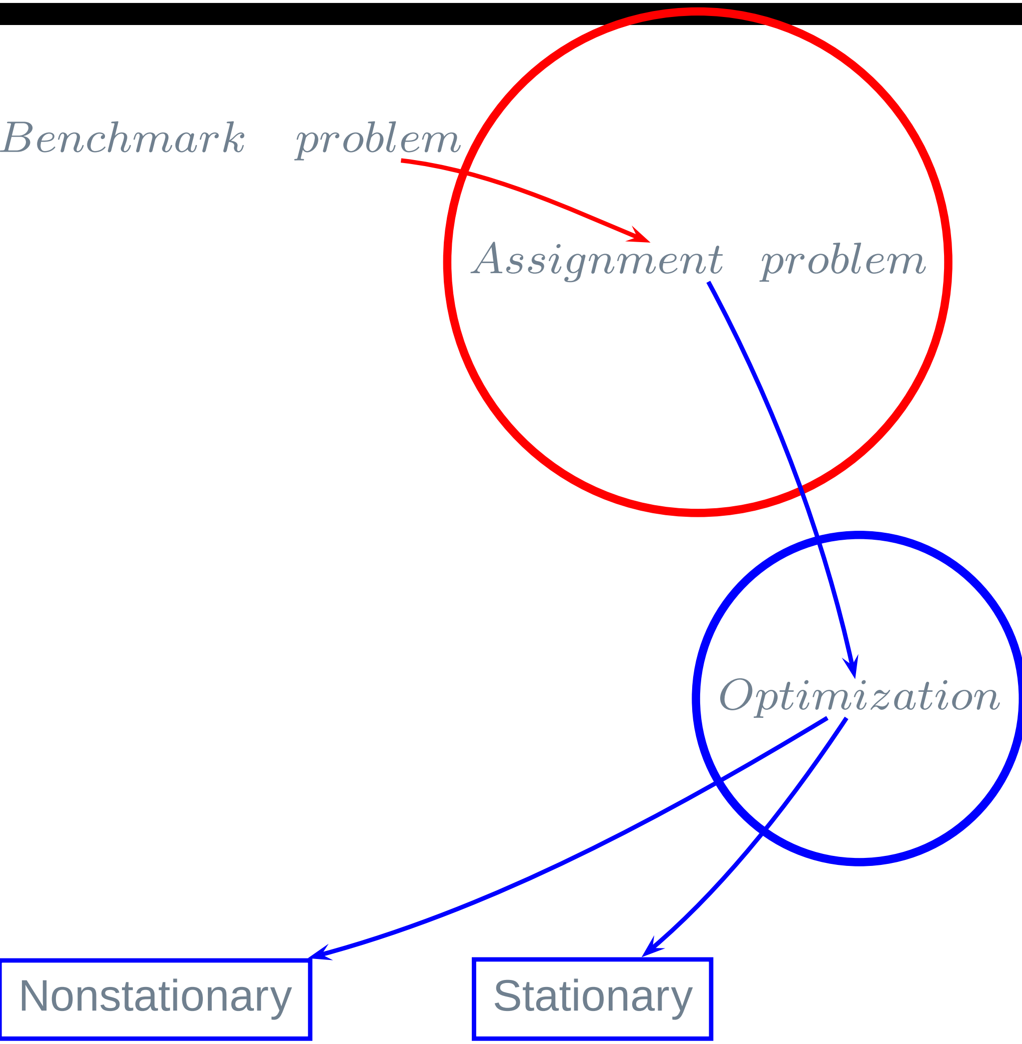
Flight schedular plan



Principal schema

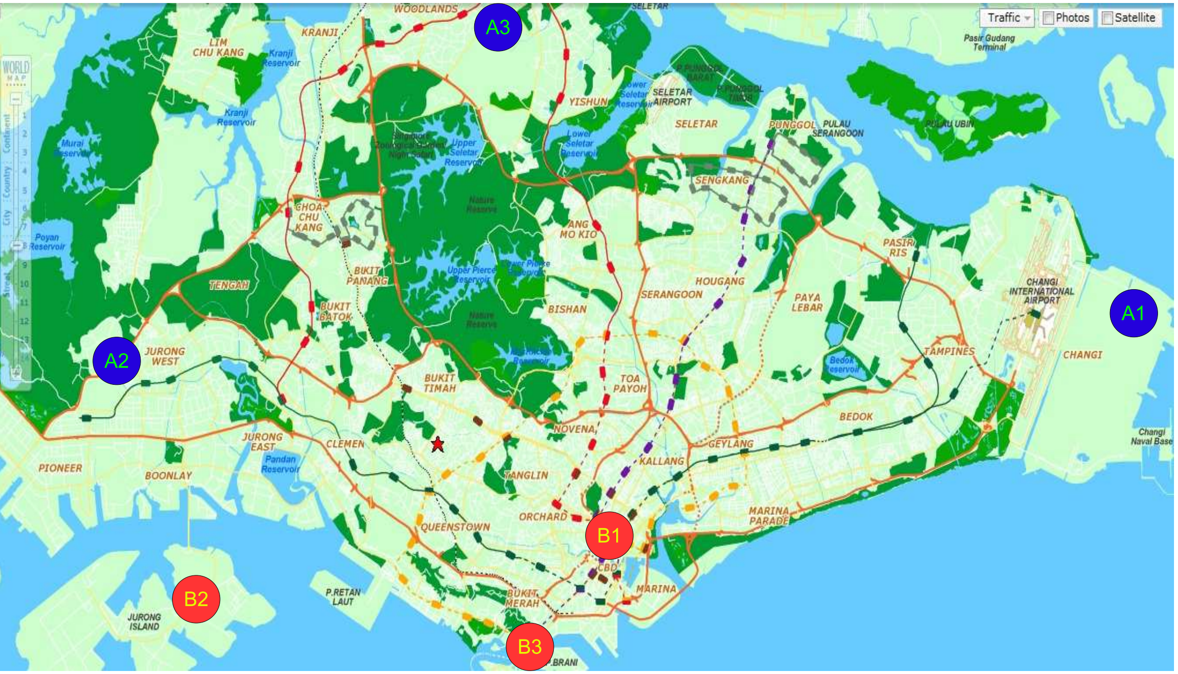


Principal schema



- optimality condition ✓

Matrix model



$$F=\sum_{i=1}^{k}\sum_{j=1}^{l}c_{ij}x_{ij}\to\min_{x_{ij}}$$

$$\sum_{i=1}^{3}x_{ij}=b_j,\quad j=1,2,3$$

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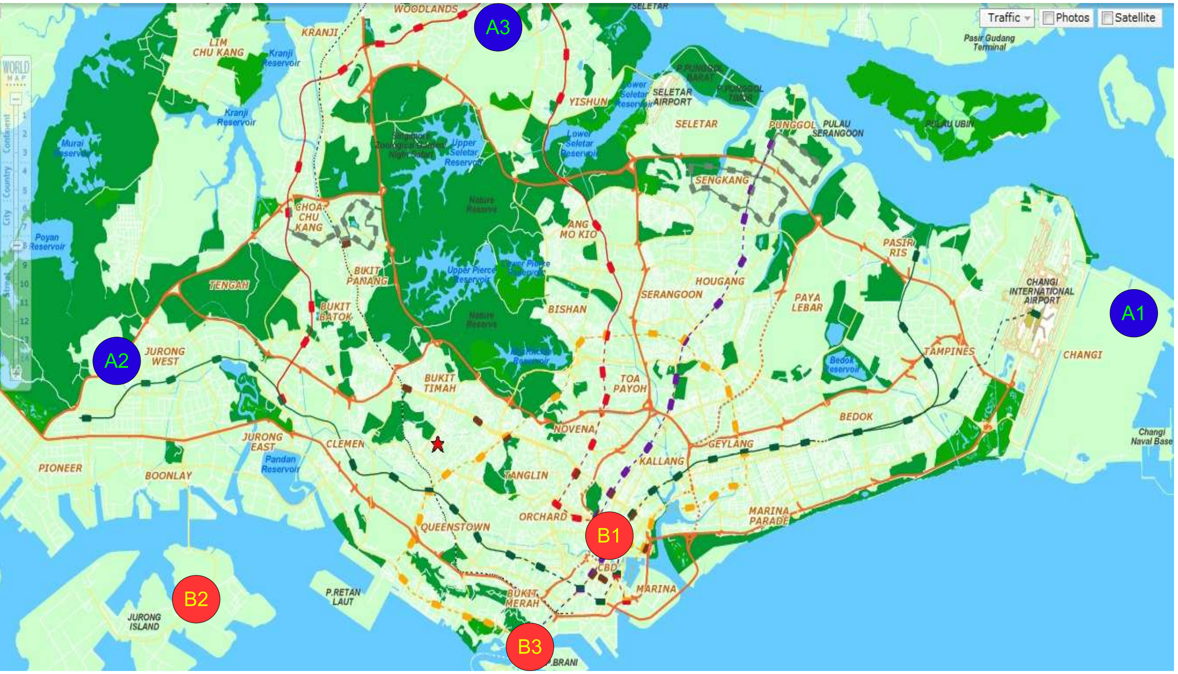
$$\sum_{i=1}^{k}a_i=\sum_{j=1}^{l}b_j,\, x_{ij}\geq 0,\, x_{ij}\in\mathbb{N}.$$

$$B=\begin{pmatrix}1&1&1&0&0&0&0&0&0\\0&0&0&1&1&1&0&0&0\\0&0&0&0&0&0&1&1&1\\1&0&0&1&0&0&1&0&0\\0&1&0&0&1&0&0&1&0\\0&0&1&0&0&1&0&0&1\end{pmatrix}$$

- $x=(x_{11},x_{12},x_{13},...,x_{32},x_{33})$
- $c=(c_{11},c_{12},c_{13},...,c_{32},c_{33})$
- $d=(a_1,a_2,a_3,b_1,b_2,b_3)$

Matrix model



$$c^T x \to \min x$$

$$Bx = d$$

$$x \geq 0$$

$$B = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

- $x = (x_{11}, x_{12}, x_{13}, \ldots, x_{32}, x_{33})$
- $c = (c_{11}, c_{12}, c_{13}, \ldots, c_{32}, c_{33})$
- $d = (a_1, a_2, a_3, b_1, b_2, b_3)$

Matrix model

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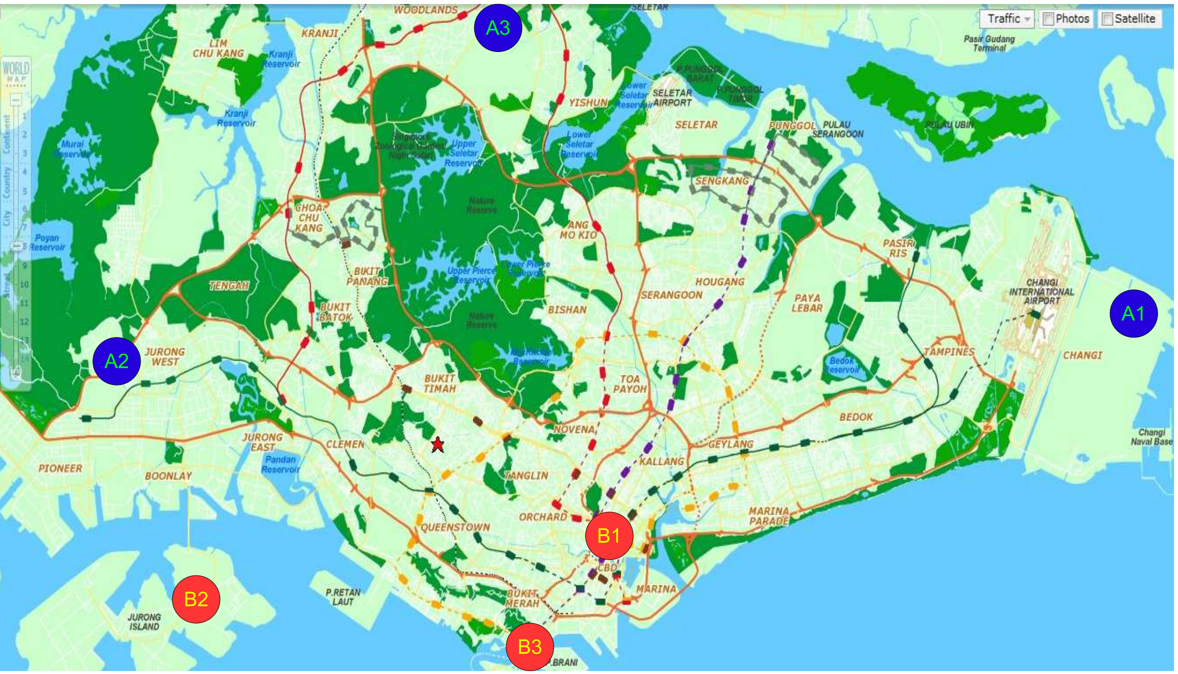


$$F = c^T x \to \max$$

$$Bx = a$$

$$f_{*i} \le x_i \le f_i^*,$$

$$i = 1, 2, ..., k, k+1, ..., k+r.$$

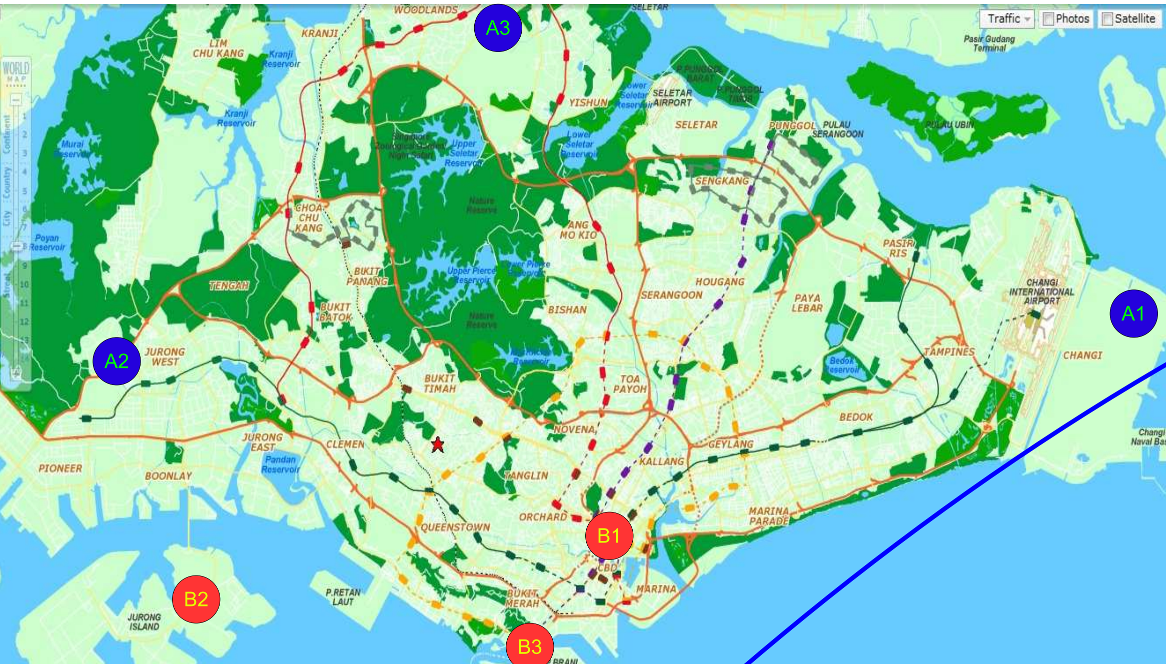
$$I = \{1, 2, ..., m\}, \;\; J = \{1, 2, ..., k+r\}, \;\; x = (x_1, ..., x_{k+r}) = x(J) = (x_j, j \in J),$$

$$c = (c_1, ..., c_{k+r}) = c(J) = (c_j, \; j \in J), \quad a = \{a_i, ..., a_m\} = a(I) = (a_i, i \in I)$$

$$f^* = (d_1^*, ..., d_{k+r}^*) = f^*(J) = (f_j^*, j \in J), \;\; f_* = (d_{1*}, ..., d_{(k+r)*}) = f_*(J) = (f_{j*}, j \in J),$$

$$B = \begin{pmatrix} b_{11} & \dots & b_{1(k+r)} \\ \dots & \dots & \dots \\ b_{m1} & \dots & b_{m(k+r)} \end{pmatrix} = B(I, J) = \begin{pmatrix} b_{i,j}, & i \in I, \\ & j \in J \end{pmatrix}$$

Matrix model



$$F = c^T x \to \max$$

$$Bx = a$$

$$f_{*i} \le x_i \le f_i^*,$$

$$i = 1, 2, ..., k, k+1, ..., k+r.$$

Denote by $X = \{x \in R^{k+r} : Bx = a, f_* \le x \le f^*\}$

$\forall x \in X$ are called the feasible points, and the set X- is admissible set.

$x^o \in X$ optimal solution if the objective function achieves the maximal value at this point.

Support

From index set I of matrix $B(I,J)$ select $\forall$ subset I_{supp},

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$\Downarrow$ *the number* $|I_{supp}| = |J_{supp}|$

Then by pair

$$K_{supp} = \{I_{supp}, J_{supp}\}$$

$\Downarrow$ construct the quadratic matrix

$$B_{supp} = B(I_{supp}, J_{supp})$$

Support

From index set I of matrix $B(I,J)$ select $\forall$ subset I_{supp},
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Then by pair

$$K_{supp} = \{I_{supp}, J_{supp}\} \ - \ \mathit{Support}$$

$\Downarrow$ construct the quadratic matrix

$$B_{supp} = B(I_{supp}, J_{supp}) \ - \ \mathit{Support\ matrix}(\det B_{supp} \neq 0)$$

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From index set J of matrix $B(I,J)$ select $\forall$ subset J_{supp}.
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What does the support means physically ?

Then by pair

$$K_{supp} = \{I_{supp}, J_{supp}\} \; - \; \textit{Support}$$

$\Downarrow$ construct the quadratic matrix

$$B_{supp} = B(I_{supp}, J_{supp}) \; - \; \textit{Support matrix} (\det B_{supp} \neq 0)$$

Support

Introduce the vector ("the intensity of pump") $z = Bx$, where

$$B = \begin{pmatrix} B_{supp} & B_{supp,N} \\ B_{N,supp} & B_{N,N} \end{pmatrix}$$

$$\begin{aligned} B_{supp} = B(I_{supp}, J_{supp}), \\ B_{supp,N} = B(I_{supp}, J_N), B_{N,supp} = B(I_N, J_{supp}) \\ B_{N,N} = B(I_N, J_N), \;\; I_N = I \setminus I_{supp}. \end{aligned}$$

then support components $z(I_{supp})$ of vector z:

$$z_{supp} = B_{supp} x_{supp} + B_{supp,N}\, x_N$$

$$\Downarrow \;\; since \;\; \det B_{supp} \neq 0$$

$$x_{supp} = B_{supp}^{-1} z_{supp} + B_{supp}^{-1} B_{supp,N}\, x_N$$

Support

support component of the feasible point $x_{supp} = x(J_{supp})$

$$B = \begin{pmatrix} B_{supp} & B_{supp,N} \\ B_{N,supp} & B_{N,N} \end{pmatrix}$$

$$B_{supp} = B(I_{supp}, J_{supp}),$$
$$B_{supp,N} = B(I_{supp}, J_N), B_{N,supp} = B(I_N, J_{supp})$$
$$B_{N,N} = B(I_N, J_N), \quad I_N = I \setminus I_{supp}.$$

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$$\Downarrow \quad since \quad \det B_{supp} \neq 0$$

$$x_{supp} = B_{supp}^{-1} z_{supp} + B_{supp}^{-1} B_{supp,N}\, x_N$$

non-support component of the feasible point $x_N = x(J_N)$

Support

Support- is pair of indices sets $K(I_{supp}, J_{supp})$, such that:

$\forall z_{supp}$

$\forall x_N$

$\Downarrow$we can find the support component x_{supp} of $\forall$ feasible point$x = (x_{supp}, x_N)$ such that :

$$z_{supp} = B_{supp} x_{supp} + B_{supp,N}\ x_N$$

holds.

Support

$$We\ can\ realize\ the\ support\ general\ constraints\ Bx = z$$

$$using\ Support\ (namely\ support\ component\ x_{supp}\ of\ feasible\ point)$$

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$$z_{supp} = B_{supp} x_{supp} + B_{supp,N}\, x_N$$

holds.

Support

$$The\ \ pair\ \ (x, K_{supp})\ -\ \ support\ \ feasible\,(SF)\ point$$

$$\forall x, \qquad\qquad \forall K_{supp}$$

Support- is pair of indices sets $K(I_{supp}, J_{supp})$, such that:

$\forall z_{supp}$

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holds.

Increment formula

Let (x, K_{supp}) arbitrary SF point.
Consider another feasible point $\bar{x}$.

We set $\Delta x = \bar{x} - x$ and calculate the increment of the objective value:

$$\Delta F(x) = c^T \bar{x} - c^T x = c^T \Delta x$$

Increment formula

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$\Downarrow$ construct the convenient formula for increment calculation.
Consider the support components of the "intensity vector" z_{supp} on two feasible points:

$$x$$

$$\bar{x}$$

Increment formula

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$$z_{supp} = B_{supp} x_{supp} + B_{supp,N} x_N$$

$$\bar{z}_{supp} = B_{supp} \bar{x}_{supp} + B_{supp,N} \bar{x}_N$$

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$$since \qquad \det B_{supp} \neq 0 \Downarrow$$

$$\Delta x_{supp} = B_{supp}^{-1} \Delta z_{supp} - B_{supp}^{-1} B_{supp,N} \Delta x_N$$

Increment formula

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$\Downarrow$ component form of formula for increment calculation.
Consider the support components of the "intensity vector" z_{supp} on two feasible points:

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Consider the support components of the "intensity vector" z_{supp} on two feasible points:

$$\Delta z_{supp} = \bar{z}_{supp} - z_{supp} = B_{supp} \Delta x_{supp} + B_{supp,N} \Delta x_N$$

$$since \qquad \det B_{supp} \neq 0 \Downarrow$$

$$\Delta x_{supp} = B_{supp}^{-1} \Delta z_{supp} - B_{supp}^{-1} B_{supp,N} \Delta x_N$$

$$\boxed{\Delta F(x) = c^T x = c_{supp}^T \Delta x_{supp} + c_N^T x_N}$$

Increment formula

Let (x, K_{supp}) arbitrary SF point.
Consider another feasible point $\bar{x}$.

We set $\Delta x = \bar{x} - x$ and calculate the increment of the objective value:

$$\Delta F(x) = c^T \bar{x} - c^T x = c^T \Delta x$$

$\Downarrow$ component form of formula for increment calculation.
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Increment formula

$$\begin{aligned} c^T \Delta x &= c^T_{supp}(B^{-1}_{supp}\Delta z_{supp} - B^{-1}_{supp}B_{supp,N}\Delta x_N) + c^T_N \Delta x_N = \\ &= c^T_{supp}B^{-1}_{supp}\Delta z_{supp} + (c^T_N - c^T_{supp}B^{-1}_{supp}B_{supp,N})\Delta x_N \end{aligned}$$

Increment formula

⊗

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\begin{aligned}
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Increment formula

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$$\Delta F(x) = u^T_{supp}\Delta z_{supp} + \Delta^T_N\Delta x_N = \sum_{i\in I_{supp}} u_i\Delta z_i + \sum_{j\in J_N}\Delta_j\Delta x_j$$

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Complete vectors u^T_{supp} and Δ^T_N by components:

$$u_N = u(J_N) = 0$$

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Vector of potentials   Vector of estimates

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\end{aligned}
$$

$$u(I) = (u(I_{supp}), u(I_N)) = (u_{supp}, 0) \qquad \Delta(J) = (\Delta(J_{supp}, \Delta(J_N))) = (0, \Delta_N)$$

$$\Delta F(x) = u^T_{supp}\Delta z_{supp} + \Delta^T_N \Delta x_N = \sum_{i \in I_{supp}} u_i \Delta z_i + \sum_{j \in J_N} \Delta_j \Delta x_j$$

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reduced gradient of the cost function by z

$$\Delta F(x) = u^T_{supp}\Delta z_{supp} + \Delta^T_N\Delta x_N = \sum_{i\in I_{supp}} u_i\Delta z_i + \sum_{j\in J_N}\Delta_j\Delta x_j$$

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Increment formula

$\otimes$

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reduced gradient of the cost function by x

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Since $\Delta = c - B^T u \quad \Rrightarrow$, it easy to see that we can get Δ from c by correction $B^T u$ which depends on general constraints. And correcting multiplier u constructed with Support help.

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Since $\Delta = c - B^T u \;\; \Rrightarrow$, it easy to see that we can get Δ from c by correction $B^T u$ which depends on general constraints. And correcting multiplier u constructed with help of support.

$$\Delta \;\; - \;\; is \;\; support \;\; gradient \;\; of \;\; the \;\; cost \;\; function$$

Optimality criteria

Let x be a feasible point.

Question: is it optimal point?

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Let x be a feasible point.

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Use the support K_{supp} and calculate the support gradient.

Optimality criteria

Let x be a feasible point.

For the optimality of a feasible point x it is sufficient and, in the case of non-degeneracy of SF-point $\{x, K_{supp}\}$, also necessary, that the following conditions:

$$\begin{cases} u_i \leq 0 \ \text{ for } z_i = a_{*i}, \\ u_i \geq 0 \ \text{ for } z_i = a_i^*, \\ u_i = 0 \ \text{ for } \ a_{*i} \leq x_i \leq a_i^*, \ \ i \in I_{supp}; \end{cases}$$

$$\begin{cases} \Delta_j \geq 0 \ \text{ for } x_j = f_{*j}, \\ \Delta_j \leq 0 \ \text{ for } x_j = f_j^*, \\ \Delta_j = 0 \ \text{ for } \ f_{*j} \leq x_j \leq f_j^*, \ \ j \in J_N \end{cases}$$

holds.

SF- $\{x, K_{supp}\}$ and vector z

Independent variables :

$$x_j, \qquad j \in I_N$$

Dependent variables :

$$x_j, \qquad j \in J_{supp}$$

SF- $\{x, K_{supp}\}$ and vector z

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Then for any $\bar{z} = z + \Delta z : \Delta z_i \geq 0$

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Dependent variables :

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Then for any $\bar{z} = z + \Delta z : \Delta z_i \geq 0$

In this case by optimality criteria $\Rightarrow u_i \leq 0$ holds.

SF- $\{x, K_{supp}\}$ and vector z

Independent variables :

$$x_j, \qquad j \in I_N$$

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$$z_i = a_{*i} \qquad z_i = a_i^* \qquad a_{*i} < z_i < a_i^*$$

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Then for any $\bar{z} = z + \Delta z : \Delta z_i \geq 0$

$\Downarrow$ Hence the variation of the z_i can not increase the value of the cost function (since $u_i \Delta z_i \leq 0$)

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Then for any $\bar{z} = z + \Delta z : \Delta z_i \geq 0$

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By analogy can be explained the signs of other variables in optimality criteria.

Maximum principle

The optimality condition can be rewritten in maximum principle form.

by intensity vector z:

by feasible point x:

Maximum principle

The optimality condition can be rewritten in maximum principle form.

by intensity vector z:

$$u_i z_i = \max_{a_{*i} \le \zeta_i \le a_i^*} u_i \zeta_i \qquad i \in I_{supp}$$

by feasible point x:

$$\Delta_j x_j = \max_{f_{*j} \le \omega_j \le f_j^*} \Delta_j \omega_j \qquad j \in J_N$$

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ε - maximum principle

Consider the SF point $\{x, K_{supp}\}$ and corresponding vector z, u, Δ then the $\varepsilon - maximum$ condition is:

by vector z:

by feasible point x:

ε - maximum principle

Consider the SF point $\{x, K_{supp}\}$ and corresponding vector z, u, Δ then the $\varepsilon - maximum$ condition is:

by vector z:

$$u_i z_i = \max_{a_{*i} \leq \zeta_i \leq a_i^*} u_i \zeta_i - \varepsilon_{zi} \qquad i \in I_{supp}$$

by feasible point x:

$$\Delta_j \chi_j = \max_{f_{*j} \leq \omega_j \leq f_j^*} \Delta_j \omega_j - \varepsilon_{xj} \qquad j \in J_N$$

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$$\varepsilon_{zi} = u_i(\xi_i - z_i) \qquad i \in I_{supp}$$

by feasible point x:

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$$\varepsilon_{xj} = \Delta_j(\chi_j - x_j) \quad j \in J_N$$

$$c^T x^o - c^T x^\varepsilon \le \max_{\substack{a_* \le \bar{z}_{supp} \le a^*, \\ f_* \le x_N \le f^*}} c^T(\bar{x} - x^\varepsilon) =$$

$$= \beta(x^\varepsilon, K_{supp}) \le \varepsilon = \sum_{i \in I_{supp}} \varepsilon_{zi} + \sum_{j \in J_N} \varepsilon_{xj}.$$

ε - maximum principle

Consider the SF point $\{x, K_{supp}\}$ and corresponding vector z, u, Δ then the $\varepsilon - maximum$ condition is:

For a feasible point x to be ϵ-optimal it is sufficient that there exists a support K_{supp} such that

$$\beta(x, K_{supp}) \leq \epsilon.$$

by vector z:

$$u_i z_i = \max_{a_{*i} \leq \zeta_i \leq a_i^*} u_i \zeta_i - \varepsilon_{zi} \quad i \in I_{supp}$$

$$\varepsilon_{zi} = u_i(\xi_i - z_i) \quad i \in I_{supp}$$

by feasible point x:

$$\Delta_j \chi_j = \max_{f_{*j} \leq \omega_j \leq f_j^*} \Delta_j \omega_j - \varepsilon_{xj} \quad j \in J_N$$

$$\varepsilon_{xj} = \Delta_j(\chi_j - x_j) \quad j \in J_N$$

Optimization method

- Select the initial support Q_{supp}

  a)Calculate the coefficients μ_i;

  b) Construct the matrix and check the support criteria:

$$G_{supp} = \left(g_{si}, s \in I_{\Sigma supp}, \; i \in I_{\Delta supp} \right)$$

$$g_{si} = \sum_{k \in I_{\Delta(i)} \cup I_{\Sigma(s)}} \mu_k a_{ik}, \; s \in I_{\Sigma supp}, \; i \in_{\Delta supp} .$$

  The collection $Q_{supp} = \{I_{\Delta supp}, I_{\Sigma supp}\}$ is called the support of the network S if $|I_{\Delta supp}| = |I_{\Sigma supp}|$ and $\det G_{supp} \neq 0$.

- Support flow $\{z, Q_{supp}\}$
- Verify the optimality criteria for the support flow
- Changing the flow $z \to \bar{z}$
- The second part of the iteration $Q_{supp} \to \overline{Q}_{supp}$
- Complete iteration $\{z, Q_{supp}\} \to \{\bar{z}, \bar{Q}_{supp}\}$
- The suboptimality estimation

Optimization method

- Select the initial support Q_{supp}
- Support flow $\{z, Q_{supp}\}$

  formed by the flow z, corresponding to the initial admissible input flow

$$z = \{x_i, i \in I_\Delta; x_{ij}, (i,j) \in U_*; f\}.$$

- Verify the optimality criteria for the support flow
- Changing the flow $z \to \bar{z}$
- The second part of the iteration $Q_{supp} \to \overline{Q}_{supp}$
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Optimization method

- Select the initial support Q_{supp}
- Support flow $\{z, Q_{supp}\}$
- Verify the optimality criteria for the support flow

  a)Calculate the potencials y_i and estimation Δ;

  The nondegenerate support flow $\{z, Q_{supp}\}$ is optimal iff:

$$\Delta_i \geq 0 \quad at \quad x_i = d_{*i}; \quad \Delta_i \leq 0 \quad at \quad x_i = d_i^*$$

$$\Delta_i = 0 \quad at \quad d_{*i} < x_i < d_i^*, \; i \in I_{\Delta nsupp}$$

$$\Delta_{ij} \geq 0 \quad at \quad x_{ij} = d_{*ij}, \; \Delta_{ij} \leq 0 \quad at \quad x_{ij} = d_{ij}^*;$$

$$\Delta_{ij} = 0 \quad at \quad d_{*ij} < x_{ij} < d_{ij}^*, \; (i,j) \in U_{nsupp}.$$

- Changing the flow $z \rightarrow \bar{z}$
- The second part of the iteration $Q_{supp} \rightarrow \overline{Q}_{supp}$
- Complete iteration $\{z, Q_{supp}\} \rightarrow \{\bar{z}, \bar{Q}_{supp}\}$
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Optimization method

- Select the initial support Q_{supp}
- Support flow $\{z, Q_{supp}\}$
- Verify the optimality criteria for the support flow
- Changing the flow $z \to \bar{z}$

$$\overline{z} = z + \theta_0 \Delta z,$$

where the improvement direction is θ_0, and

$$\Delta z = (\Delta x_i,\ i \in I_\Delta;\ \Delta x_{ij},\ (i,j) \in U_*;\ \Delta f)$$

- The second part of the iteration $Q_{supp} \to \overline{Q}_{supp}$
- Complete iteration $\{z, Q_{supp}\} \to \{\bar{z}, \bar{Q}_{supp}\}$
- The suboptimality estimation

Optimization method

- Select the initial support Q_{supp}
- Support flow $\{z, Q_{supp}\}$
- Verify the optimality criteria for the support flow
- Changing the flow $z \rightarrow \bar{z}$
- The second part of the iteration $Q_{supp} \rightarrow \overline{Q}_{supp}$

  This are realized on the basis of dual theory.
- Complete iteration $\{z, Q_{supp}\} \rightarrow \{\bar{z}, \bar{Q}_{supp}\}$
- The suboptimality estimation

Optimization method

- Select the initial support Q_{supp}
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- Verify the optimality criteria for the support flow
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- The second part of the iteration $Q_{supp} \to \overline{Q}_{supp}$
- Complete iteration $\{z, Q_{supp}\} \to \{\bar{z}, \bar{Q}_{supp}\}$

  $\Downarrow$

  Here the iteration of the optimization method is complete.
- The suboptimality estimation

Optimization method

- Select the initial support Q_{supp}
- Support flow $\{z, Q_{supp}\}$
- Verify the optimality criteria for the support flow
- Changing the flow $z \to \bar{z}$
- The second part of the iteration $Q_{supp} \to \overline{Q}_{supp}$
- Complete iteration $\{z, Q_{supp}\} \to \{\bar{z}, \bar{Q}_{supp}\}$
- The suboptimality estimation

The suboptimality estimation of the new support flow is

$$\beta(\overline{z}, \overline{Q}_{supp}) =$$

$$(1-\theta_0)\beta(z, Q_{supp}) + v_0\sigma_{i_{(1)}} + \sum_{k=1}^{\nu-1} v_k(\sigma_{i_{k+1}} - \sigma_{i_{(k)}})$$

$$\leq \beta(z, Q_{supp}).$$

Short summary

Comparison of the proposed adaptive method with classical LP methods.

- The basic "instrument" of the adaptive method - support - quite flexible react on a different situation during the solution process.
- Simplex methods start from a specified basis. The support lets us satisfy the general constraints initially and later.
- Nonsupport (nonbasic) variables need not be zero—they may have any value satisfying the bounds.
- The adaptive method allows to use any priory information about feasible solution.
- The new principle used on iteration of the adaptive method.
- The method equipped with stop criteria.
- The primal adaptive method significantly uses the ideas of the dual theory.(dual step in second procedure)
- The dual adaptive method is much more effective then traditional dual simplex methods due to the long step rule.
- ....provide sensitivity analisys

By these reasons the method called adaptive since its properties of using the all the initial and current information for effective construction of suboptimal feasible solution.

The end

Thank you!