Controller design for USV path planning, tracking, and enemy-block processes

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Abstract

In this project some proposals of the controller design problem for USV are discussed. It is assumed that the USV maneuvering can be described in the framework of dynamical system theory. The optimization problem for piecewise linear systems in the class of discrete controls is considered. On the basis of methods for solving linear optimal control problems supplemented with optimization methods with respect to parameters, a dual method is developed for calculating open loop optimal controls. The method is also used in the synthesis of closed loop optimal controls such that the optimal controller designed on this base will be able to generate the values of an optimal feedback in real time. This eliminates the main disadvantage of dynamic programming, i.e., the need to calculate the optimal feedback for all possible states of the system in advance (before the control process begins), which results in the so-called curse of dimensionality. The results obtained are illustrated by considering problems of optimal excitation and displacement for a one-mass oscillatory system driven by a piecewise elastic force.

Keywords:

1 Introduction

The aim of this project proposal is to find an adequate problem statement of mutual interests for USV path planning, tracking, and enemy-block processes processes. As an starting position we can consider the following subproblems:

1. Open Loop and Closed Loop optimization for piecewise linear systems

2. Optimal Control based on imperfect measurements of input and output profiles

3. Centralized and decentralized control problem

4. Control synthesis under uncertainty

2 Open Loop and Closed Loop optimization

In the mathematical theory of optimal processes, to analyze nonlinear systems of the form

$$\dot{x}_i = f(x) + bu, \ (x \in \mathbb{R}^n, \ u \in \mathbb{R}) \tag{1}$$

it is often sufficient to use linear approximations

$$\dot{x}_i = A(t)x + bu, \ A(t) = \frac{\partial f(x(t))}{\partial x} \tag{2}$$

constructed along certain trajectories $x(t)(t \geq 0)$ of the nonlinear system (1). Linear approximations (2) are frequently used to develop approximate methods for solving optimal control problems. It is clear that they can only provide satisfactory descriptions of local behavior of non- linear systems in the neighborhoods of reference trajectories. Therefore, these approximations have a limited scope.

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One natural way to expand the scope of linear optimization methods is to use piecewise linear approximations (first-order splines) of the nonlinear elements of a problem. Even though the model remains non- linear in this approximation, effective optimization methods can be developed by taking into account specific properties of the piecewise linear model.

Let $T = [0, t*]$ be the control time interval, $h = t * /N$ be the quantization interval, Nbe a positive integer, and $Th = \{0, h, ..., t * -h\}$. The function $u(t)(t \in T)$ is called the discrete control (with the quantization interval h) if $u(t) = u(kh)$ for $t \in [kh, (k + 1)h[(k = 0, ..., N - 1)$.. We consider the optimal control problem for the piecewise linear system

$$c^{'}x(t^*) \to \max_u, \; \dot{x} = A(t)x + bu, \; x(0) = x_0 \tag{3}$$

$$Hx(t^*) = g, \; |u(t)| \leq 1. \; t \in T = [0, t^*]$$

in the class of discrete controls. Here, $x = x(t)$ is the state vector of the dynamical system at an instant t, and $u = u(t)$ is the value of a scalar control, $b \in \mathbb{R}^n$, $g \in \mathbb{R}^m$. The discrete control $u(\cdot) = (u(t), t \in T)$ is called feasible for the problem if it satisfies the condition $|u(t)| \leq 1$ for $t \in T$ and the corresponding trajectory $x(t)(t \in T)$ satisfies the terminal constraint $Hx(t*) = g$. A feasible control $u(\cdot)$ is called the optimal open loop control for problem if the corresponding trajectory $x(t)(t \in T)$ maximizes the objective functional of problem.

To introduce the concept of the closed loop optimal control for problem (3), we embed problem (3) in the family of problems

$$c^{'}x(t^*) \to \max_u, \; \dot{x} = A(t)x + bu, \; x(\tau) = z \tag{4}$$

$$Hx(t^*) = g \; |u(t)| \leq 1, \; t \in T(\tau) = [\tau, t^*]$$

which depends on a scalar $\tau \in T_h$ and an $n-$dimensional vector z. Let $u^0(t|\tau, z)(t \in T(\tau))$ be the optimal open loop control for problem (4) for (τ, z) and X^τ be the set of states z for which problem (4) has a solution. The function

$$u^0(\tau, z) = u^0(t|\tau, z), \; z \in X^\tau, \; \tau \in T_h \tag{5}$$

is called the closed loop (discrete) optimal control for problem (3).

Then the real trajectory of the control object can be presented as the following system

$$\dot{x} = A(t)x + bu^0(t, x) + w(t), \; x(0) = x_0 \tag{6}$$

closed by the optimal feedback (5) and subjected to a piecewise continuous perturbation $w(t)(t \in T)$. The purpose of this study is to develop effective algorithms for constructing the optimal open loop control and synthesize the optimal closed loop control for problem (3)

2.1 Optimal Controller

The main result of this study is the synthesis of closed loop optimal controls. Optimal synthesis is the key problem in control theory since open loop solutions are not used in actual controls; they are required only to reveal the potential capabilities of control systems. The approach described in this project essentially relies on the dynamic nature of the problem under consideration. The optimal controller generates the values of an optimal feedback in real time. This eliminates the main disadvantage of dynamic programming, i.e., the need to calculate the optimal feedback for all possible states of the system in advance (before the control process begins), which results in the so-called curse of dimensionality .

This approach will be based on the use of an optimal feedback (5) in a control process for system (3). Assume that the optimal feedback (5) has already been determined and the behavior of the closed loop system is described by the equation (6). The function $w(t)(t \in T)$ represents the influence of the perturbations neglected in the mathematical model. When piecewise linear approximations are used to optimize the nonlinear system (1), the function $w(t)(t \in T)$ includes, in addition to external perturbations, the deviation of the piecewise linear system (3) from the original nonlinear system (1). Assume that a perturbation $w^*(t)(t \in T)$ occurs in a certain control process of system (6). Driven by this perturbation and function (5), system (6) moves along a trajectory $x^*(t)(t \in T)$, while the control $u^*(t) = u^0(t, x^*(t))(t \in T)$ is applied to system (6). The function $u^*(t)(t \in T)$ is called the realization of the optimal feedback in a particular control process, and the device that calculates the values of this function in real time is called the optimal controller.

The testing examples demonstrated that the developed algorithm of the optimal controller is efficient and can therefore be implemented on modern computers for relatively complex control systems. This is illustrated by the numerical results obtained by solving two optimal control problems for a piecewise linear oscillatory system with one degree of freedom. In particular, the frictionless motion of a one-mass oscillatory system along a horizontal line was considered. On different parts of the line, the system is driven by forces exerted by different elastic elements (springs). We seek a control that maximizes the velocity gained by the mass in a given time. The mathematical model of the system is formulated as follows:

$$\dot{x}(t^*) \to \max_u, \tag{7}$$

$$\begin{cases} \ddot{x} + k_1 x = u, & \text{if } \ x \geq \alpha, \\ \\ \ddot{x} + k_1 x + k_2(x+\alpha) = u, & \text{if } \ x < \alpha, \end{cases} \tag{8}$$

$$x(0) = 0, \ \dot{x}(0) = 0, \ |u(t)| \leq 1, \ t \in T = [0, t^*] \tag{9}$$

where $x = x(t)$ is the deviation of the mass from the equilibrium point $x = 0$ at the instant t,$u = u(t)$ is the control (force), and α is the distance from the equilibrium point to the right end of the second spring.

The calculated results expose the high efficiency of the method in constructing optimal open loop controls and the possibility of implementing closed loop controls on modern computers.

3 Optimal Control based on imperfect measurements of input and output

In this part of the project can be considered the optimal guaranteed control problems for linear nonstationary dynamical systems under set-membership uncertainties. It is supposed that in the course of control process states of control object are unknown and signals of two measurement devices are only available for use. The first of them implements incomplete and inexact measurements of input signals, the second one makes imperfect measurements of control object states (output signals). By preposterior analysis an optimal output (combined) closable loop is defined. Realization of this loop (forming current values of control actions) is carried out by optimal estimators and optimal regulator. According to the separation principle of control and observation processes, optimal estimators generate in real time estimates of uncertainty using signals of measurement devices. By obtained estimates the optimal regulator produces current values of optimal loop in the same mode.

4 Adaptive decentralized control problem

Let $T = [t_*, t^*]$ be the control interval; $T_h = \{t_*, t_* + h, ..., t^* - h\}; h = \frac{t^* - t_*}{N}$; N be a natural number; $I = \{1, 2, ..., q\}$, $I_i = I \setminus i; A_{ij}(t) \in \mathbb{R}^{n_i \times n_j}; B_{ij}(t) \in \mathbb{R}^{n_i \times r_j}(t \in T, i, j \in I)$ be piecewise continuous matrix functions; $A_i(t) = A_{ii}(t)$ and $B_i = B_{ii}(t), t \in T, i \in I; H_i \in \mathbb{R}^{m \times n_i}, g_0 \in \mathbb{R}^m, c_i \in \mathbb{R}^{n_i}, u_{i*}, u_i^* \in \mathbb{R}^{r_i}$ for $i \in I$ be given matrices and vectors; and $n = \sum_{i \in I} n_i$.

On interval T, consider the group of q objects to be controlled assuming that the i-th object ($i \in I$) is governed by the equation

$$\dot{x}_i \quad = \quad A_i(t)x_i + \sum_{j\in I_i} A_{ij}(t)x_j + B_i(t)u_i + \sum_{j\in I_i} B_{ij}(t)u_j, \quad x_i(t_*) = x_{i0} \tag{10}$$

Here, $x_i = x_i(t) \in \mathbb{R}^{n_i}$ is the state of the ith object at the time t, $u_i = u_i(t) \in U_i$ is the value of the discrete control at the time t, and $U_i = \{u \in \mathbb{R}^{r_i} : u_{i*} \leq u \leq u_i^*\}$ is a bounded set of available values of the i-th control. If $u(t) \equiv u(s)$ for $t \in [s, s+h[, (s \in T_h)$, then the function $u(t), (t \in T)$ is said to be discrete with sampling period h. In (10), the function $A_i(t), t \in T$ characterizes the self-dynamics of ith object; the function $(A_{ij}(t), t \in T, j \in I_i)$ describes the influence of other objects on object i; the function $B_i(t), t \in T$ characterizes the input properties of the object i; and the functions $B_{ij}(t), (t \in T, j \in I_i)$ describes the influence of the controls of other objects on object i.

A group of dynamical objects can be controlled in two different ways- in a centralized or decentralized fashion. In the first case, there is a common control center that, given perfect (complete and accurate) information about the current state $x^*(\tau) = (x_i^*(\tau), i \in I)$ of the group, produces for each time interval $[\tau, \tau+h[, \tau \in T_h$, a control $u^*(t) = (u_i^*(t), i \in I), t \in [\tau, \tau+h[$ for all the objects. In the second case, each ith object of the group has a particular (local) control center that produces at each time interval $[\tau, \tau+h[, (\tau \in T_h)$ a control action $u_i^*(t)(t \in [\tau, \tau+h[, \tau \in T_h)$ based on the perfect information about its own current state $x_i^*(\tau)$ and about the states $x_k^*(\tau-h)(k \in I_i)$ of other objects. Also assume that delay of the information about the state of the other objects coincides with the sampling period h. The aim of the control is follows:
(1) Steer the group to a given(common) terminal set at the time t^*:

$$x(t^*) \in X^* = \{x = (x_i, i \in I) : \sum_{i\in I} H_i x_i = g_0, \} \tag{11}$$

(2) Achieve the maximum value of the terminal objective function

$$J(u) = \sum_{i\in I} c_i' x_i(t^*) \rightarrow \max \tag{12}$$

Depending on the way used to control the group, we have two optimal control problems a centralized and a decentralized problem. Assume that centralized real-time control of the group of objects is impossible for some reason.

4.1 Decentralized close-loop control in classical form

Before starting the control, the set of functions

$$\begin{aligned} &u_i(t_*, x), x = (x_i, i \in I) \in \mathbb{R}^n, \\ &u_i(\tau, x_i; x_k, k \in I_i), \qquad \tau \in T_h \setminus t_*,\ x_i \in \mathbb{R}^{n_i},\ x_k \in \mathbb{R}^{n_k},\ k \in I_i,\ i \in I \end{aligned} \tag{13}$$

is chosen that is called the (discrete) decentralized feedback. Mathematical models (10)($i \in I$) are closed by feedback (13):

$$\begin{aligned} \dot{x}_i(t) &= A_i(t)x_i(t) + \sum_{j\in I_i} A_{ij}(t)x_j(t) + B_i(t)u_i(t, x(t)) + \sum_{j\in I_i} B_{ij}(t)u_j(t, x(t)), \\ &\qquad x_i(t_*) = x_{i0},\ t \in [t_*, t_*+h[, \\ \dot{x}_i(t) &= A_i(t)x_i(t) + \sum_{j\in I_i} A_{ij}(t)x_j(t) + B_i(t)u_i(t, x_i(t); x_k(t-h), k \in I_i) + \\ &+ \sum_{j\in I_i} B_{ij}(t)u_j(t, x_j(t); x_k(t-h), k \in I_j), t \in [\tau, \tau+h[, \tau \in T_h \setminus t_*,\ i \in I. \end{aligned} \tag{14}$$

Here $x_0 = (x_{i0}, i \in I)$; $u_i(t, x(t)) \equiv u_i(t_*, x_0)$ for $t \in [t_*, t_*+h[$, and $u_i(t, x_i(t); x_k(t-h), k \in I_i) \equiv u_i(\tau, x_i(\tau); x_k(\tau-h), k \in I_i)$ for $t \in [\tau, \tau+h[, \tau \in T_h \setminus t_*, i \in I$. The trajectory of nonlinear system (14) is defined as the unique function $x(t \mid x_0; u_i, i \in I), (t \in T)$ composed of the continuously connected solutions to the linear differential equations

$$\dot{x}_i = A_i(t)x_i + \sum_{j\in I_i} A_{ij}(t)x_j + B_i(t)u_i + \sum_{j\in I_i} B_{ij}(t)u_j, \quad x_i(t_*) = x_{i0}$$
$$u_i(t) = u_i(t_*, x_0), \quad t \in [t_*, t_* + h[,$$
$$u_i(t) = u_i(\tau, x_i(\tau); x_k(\tau - h), k \in I_i), t \in [\tau, tau + h[, \tau \in T_h \setminus t_*,$$
$$i \in I.$$

Feedback (13) is said to be admissible for the state x_0 if
1) $u_i(t) \in U_i$ for $t \in T$ and $i \in I$ and
2) $x(t^* \mid x_0; u_i, i \in I) \in X^*$.

Let $X(t_*)$ be the set of initial states x_0 for which there exists an admissible feedback, and let $X_i(\tau)$ be the set of states $x_i; x_k, k \in I_i$ for which function (13) is define at the time $\tau \in T_h \setminus t^*$. The quality of an admissible feedback for the state $x_0 \in X(t_*)$ is evaluated using the functional

$$J(u, x_0) = \sum_{i\in I} c_i' x_i(t^* \mid x_0; u_i, i \in I).$$

The admissible feedback

$$u_i(t_*, x), x \in X(t_*); \quad u_i^0(\tau, x_i; x_k, k \in I_i), (x_i; x_k, k \in I) \in X_i(\tau), \quad \tau \in T_h \setminus t_*, i \in I, \tag{15}$$

is said to be optimal for $x_0 \in X_{t_*}$ if $J(u^0, x_0) = \max_u J(u, x_0), x_0 \in X(t_*)$.

Similarly to the classical centralized optimal feedback, the decentralized optimal feedback is determined on the basis of the mathematical model but is designed for controlling its physical prototype. The decentralized optimal control of a group of dynamical objects designed using the classical closed-loop principle assumes that feedback (15) is preliminary synthesized, and group (10) $i \in I$ is closed by this feedback, which yields the optimal automatic control system governed by the equations

$$\dot{x}_i(t) = A_i(t)x_i(t) + \sum_{j\in I_i} A_{ij}(t)x_j(t) + B_i(t)u_i(t, x(t)) + \sum_{j\in I_i} B_{ij}(t)u_j(t, x(t)) + w_i,$$
$$x_i(t_*) = x_{i0}, \ t \in [t_*, t_* + h[,$$

$$\dot{x}_i(t) = A_i(t)x_i(t) + \sum_{j\in I_i} A_{ij}(t)x_j(t) + B_i(t)u_i(t, x_i(t); x_k(t - h), k \in I_i) + \tag{16}$$
$$+ \sum_{j\in I_i} B_{ij}(t)u_j(t, x_j(t); x_k(t - h), k \in I_j) + w_i,$$
$$t \in [\tau, \tau + h[, \tau \in T_h \setminus t_*, \ i \in I.$$

where $w = (w_i \in \mathbb{R}^{n_i}, i \in I)$ is a collection of terms describing inaccuracies of the mathematical modeling and of the implementation of the optimal feedback and the perturbations that affect the objects in the course of control. For simplicity, we will call w the perturbation, and we will assume that it is realized as unknown bounded piecewise continuous functions $w_i(t), (t \in T, i \in I)$. The problem of synthesizing system (16) is a very difficult one and has not yet been solved even for centralized control.

The purpose of our investigation is to propose algorithms for synthesis of decentralized automatic control systems using decentralized real-time optimal control of a group of objects; in this case, the control function among individual systems of which each solves an individual autonomous problem (performs self-control) taking into account the actions of the other members of the group. Our approach will be based on a fast implementation of the dual method and the learning algorithm from information obtained by each controller from the other controllers.

4.2 Control problem with moving targets

Two optimal control problems of approaching and aiming with moving targets are under consideration.

Let $T = [t_*, t^*]$ be the control interval.

Object of control:

$$\dot{x} = A(t)x + b(t)u + M(t)w, t \in T; x(t_*) = x_0 \tag{17}$$

and moving target:

$$\dot{\tilde{x}} = \tilde{A}(t)\tilde{x} + \tilde{b}(t)v + \tilde{M}(t)w, t \in T; \tilde{x}(t_*) = \tilde{x}_0 \tag{18}$$

Here

$x = x(t) \in \mathbb{R}^n, \tilde{x} = \tilde{x}(t) \in \mathbb{R}^n$ are the state of control object and state of the target.

$u = u(t) \in \mathbb{R}$ - control input; $v = v(t) \in \mathbb{R}$ - maneuvering effort of target; $w = w(t) \in \mathbb{R}^{n_w}$ - disturbances.

If $\tilde{b}(t) = 0, t \in T$ then target is *not maneuvering*, otherwise it is *maneuvering target.*

Admissible discrete control function:

$$u(\cdot) = (u(t) \in U, t \in T)$$

Unknown maneuvering efforts $v(t), t \in T$ and disturbances $w(t), t \in T$ represented in the following form:

$$v(t) = v_1(t) + v_2(t),\ w(t) = w_1(t) + w_2(t),\ t \in T$$

where

$$v_1(t) = K_v(t)v, v \in V_1;\ w_1(t) = K_w(t)w, w \in W_1, t \in T$$

- regular components,

$$v_2(t) \in V_2;\ w_2(t) \in W_2, t \in T$$

not regular components $(V_1, W_1, V_2, W_2; K_v(t), K_w(t), t \in T)$ - are known; $v, w; v_2(t), w_2(t), t \in T$ - are arbitrary.

In case,when $M(t) = 0, \tilde{M}(t) = 0, \tilde{b}(t) = 0, t \in T; x_0, \tilde{x}_0$ are fixed vectors, the models (17)-(18) *deterministic*,otherwise *indeterministic.*

Also let $X_\rho(\tilde{x}) = \{x \in \mathbb{R}^n : \rho g_* \leq H(x - \tilde{x}) \leq \rho g^*\}$ - ρ-neighborhood of a point $\tilde{x}$, where

$$H \in \mathbb{R}^{m \times n}; g_*, g^* \in \mathbb{R}^m, -\infty < g_* < 0 < g^* < \infty; \rho \geq 0.$$

The problem of optimal approaching consist in the way of choosing $u^0(\cdot)$,such that (17) at final moment t^* will be at the set $X_{\rho^0(\tilde{x}(t^*))}$ with minimal ρ^0.

The problem of aiming- to be on $X^* = X_1(\tilde{x}(t^*))$ with minimal $J(u) = \int_{t_*}^{t^*} |u(t)|dt.$

For the problems above can be investigated the following situations:

- Deterministic problem statement;
- Maneuvering target;
- Not maneuvering target with noncompletely defined motion, imperfect measuring;
- Maneuvering target,imperfect measuring;
- Indeterministic models, imperfect measuring.

5 Control synthesis under uncertainty

A considerable amount of work was done by the control community in coping with uncertainty and incomplete information in the field of control. The adopted scheme is based on constructing superpositions of value functions for open-loop control problems. In the limit these relations reflect the Principle of Optimality under set-membership uncertainty. This principle then allows one to describe the closed loop reach set as a level set for the solution to the forward HJBI (Hamilton JacobiBellmanIsaacs) equation. The final results are then presented either in terms of value functions for this equation or in terms of setvalued relations. Schemes of such type have been used in synthesizing solution strategies for guaranteed control,dynamic games and related problems of feedback control under realistic data, including those of control under incomplete information and measurement feedback control. The control schemes were constructed in backward time in more or less equivalent forms of solvability sets, stable bridges of N. N. Krasovski (1968; 1998), the alternated integrals of L. S. Pontryagin (1980), the scheme of B. N. Pschenichniy in Kiev (Pschenichniy and Ostapenko, 1992). Effective and original dynamic programming-type constructions were developed in USA by R. Isaacs, P. Varaiya, G.Leitmann, R. J. Elliot and N. J. Kalton, T.Basar and others, as well as in France, by A. Blaquiere and later, through the notion of H^{∞}-control by P.Bernhard and the idea of capture basins by J. P. Aubin and his associates P.Saint-Pierre, M.Quincampoix and others.

5.1 Uncertain dynamics. The Standard Model.

This is given by differential equation

$$\dot{x} = f(t, x, u, v), \tag{19}$$

with properties of continuous function f defined as in control theory, with inputs representing controls u to be specified and unknown disturbances (v). Here $x \in \mathbb{R}^n$ as always is the state and $u \in \mathbb{R}^p$is the control that may be selected either as an open loop control - a measurable function of time t, restricted by the inclusion

$$u(t) \in \mathbb{P}(t), a.e.,$$

or as a closed-loop control a feedback strategy which is either sought for either as a multivalued map

$$u = \mathbb{U}(t, x) \subseteq \mathbb{P}(t).$$

or as a single-valued function $u(t, x) \in U_c$ which ensure existence in some appropriate sense of solutions to differential inclusion

$$\dot{x} \in f(t, x, U(t, x), v), \tag{20}$$

or to differential equation

$$\dot{x} = f(t, x, u(t, x), v), \tag{21}$$

respectively.

Here $v \in \mathbb{R}^q$ is the unknown input disturbance with values

$$v(t) \in (\mathbb{Q}(t), a.e., \tag{22}$$

$\mathbb{P}(t), \mathbb{Q}(t)$ are set-valued continuous functions with compact values, ($\mathbb{P} \in comp\mathbb{R}^p, \mathbb{Q} \in comp\mathbb{R}^q$). Given also is a closed target set $\mathbb{M}$.

The Problem of Control Synthesis under Uncertainty is to find an admissible feedback control strategy $U = u(t,x)$ or $U = U(t,x)$ which steers system (19) to reach the target set $\mathbb{M}$ despite the unknown disturbances v. Such problems are usually treated within the notions of game-type dynamics introduced by R. Isaacs.

Typical admissible classes of feedback controls and trajectory solutions involved for the given problem are due to N. N. Krasovski in the singlevalued case (Krasovski, 1968; Krasovski and Subbotin, 1998) and A. F. Filippov in the multivalued case (Filippov, 1959). It is N. N. Krasovski who introduced the most developed formalized and integrated solution theory for problems in "gametype" controlled dynamics, which was developed further by him and his associates for a broad class of control problems under uncertainty or conflict.

Continuing with the last problem, we are to find the value function

$$V(t,x) = \min_{\mathbb{U}} \max_{x(\cdot)} \{d^2(x[t_1], \mathbb{M}) \mid \mathbb{U} \in U_c, x(\cdot) \in \mathbb{X}_{\mathbb{U}}\}.$$

Here $\mathbb{X}_{\mathbb{U}}$ is the variety of all trajectories of equation (20) or (21).

The formal solution equation for the problem is the HamiltonJacobiBellmanIsaacs (HJBI) equation

$$V_t = \min_u \max_v (V_x, f(t,x,u,v)) = 0, \tag{23}$$

with minmax often assumed interchangeable and with control $u(t,x)$ to be found from the solution to the minmax problem in (23). However in reality this is just a symbolic relation as the function $V(t,x)$ in general turns out to be nondifferentiable.

The solutions to the control synthesis problem under uncertainty are then found, for example, through procedures of constructing solvability tubes in the form of stable bridges

$$W[t] = \{x : V(t,x) \leq 0\},$$

as introduced by N. N. Krasovski with control strategy further found from conditions of "extremal aiming":

$$u(t,x) = arg \min_u \{\max_v d(x, W[t])\},$$

and trajectories interpreted as "constructive motions" of (Krasovski and Subbotin, 1998). Here $V(t,x)$ is a generalized solution to equation (23).

Along with the theory, the numerical methods of constructing solutions were developed. Several global successive approximation numerical schemes were proposed as well schemes to approximate the deterministic control problem or game by a stochastic discrete-time process. The game-theoretic approaches in conjunction with set-valued techniques and new results in nonlinear analysis allowed to formalize basic problems of control under measurement feedback and unknown but bounded disturbances. Various formalizations and applications of the theory of control under incomplete information may be found in books and papers by N. N. and A. N. Krasovski (1994), A. B. Kurzhanski (1977), Yu. S. Osipov and A. V. Kryazhimski (1995), F. L. Chernousko and A. A. Melikyan (1978), V. M. and A. V. Kuntsevich (2002), B. N. Pschenichniy and V. V. Ostapenko (1992).