1 General problem statement

1.1 Notation

$A_i, i = 1, ..., k$- aerobases,
a_i - number of UAVs located in A_i,
$B_j, j = 1, ..., l$- areas of operations,
b_j- numbers of UAVs for service of B_j
d_{ij}- distance from A_i to B_j,
v_{ij}- UAVs speed
x_{ij}-number of UAVs from A_i to B_j,
h_i- UAVs endurance located on A_i aerobase

1.2 Problem statement

$$T^{service} = \sum_{i=1}^{k} h_i - 2\sum_{i=1}^{k}\sum_{j=1}^{l} \frac{d_{ij}}{v_{ij}} x_{ij} \to \max_{x_{ij}} \tag{1}$$

subject to

$$\sum_{i=1}^{k} x_{ij} = b_j, \quad j = 1, 2, ..., l \tag{2}$$
$$\sum_{j=1}^{l} x_{ij} = a_i, \quad i = 1, 2, ..., k$$
$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j$$
$$\frac{d_{ij}}{v_{ij}} \operatorname{sign}(x_{ij}) \geq t_j^{first}, \; i = 1, 2, ..., k$$
$$\frac{d_{ij}}{v_{ij}} \operatorname{sign}(x_{ij}) \leq t_j^{last}, \; i = 1, 2, ..., k$$
$$x_{ij} \geq 0, \qquad x_{ij} \;\; \text{are integer numbers.}$$

Here the cost function presents the total service time performed by all UAVs used for mission. The first two constraints means that we are need to satisfy all service request from each area of operation and to use for that purpose all available UAVs. The last inequalities image the fact that the period of the start of service of $j-th$ zone by $i-th$ UAV is restricted by the pre-assigned time interval $[t_j^{first}, t_j^{last}]$. The cost function presents the total service time performed by all UAVs used for mission.

2 Illustrative example

Assume that we have 3 airbases locate at Changi A_1 with 3 UAVs ($a_1 = 3$) , Jurong West A_2 with 3 UAVs ($a_2 = 3$) , and Woodland A_3 with 1 UAV ($a_1 = 1$). Now 7 UAVs are requested from B_1-Raffles Place ($b_1 = 2$), B_2-Jurong Island ($b_2 = 2$), and B_3- Sentosa Island ($b_3 = 3$). The distances between A_i and B_j given below in kilometers:

	B_1	B_2	B_3
A_1	13	30	18
A_2	16	9	17
A_3	21	20	23

(3)

Table 1: Distances between aerobases A_i and area of operations B_j

The speed of UAVs are fixed $v_{ij} = 30\frac{m}{sec}$.

Next, for all i and j denote by $c_{ij} = \frac{d_{ij}}{v_{ij}}$ the benefit of sending the UAV from i-th aerobase to j-th zone of area of operation. The benefit means the flight time from $A_i \to B_j$.

	B_1	B_2	B_3
A_1	433	1000	600
A_2	533	300	566
A_3	700	666	766

(4)

Table 2: UAVs flight time from $A_i \to B_j$ (5)

Then using our notation we can formulate the problem statement as the following integer programming problem: To find $x_{ij}, (i = 1, 2, 3; j = 1, 2, 3)$ such that, the total service time performed by all UAVs takes a maximal value

$$T^{service} = \sum_{i=1}^{3} h_i - 2 \min_{x_{ij}} \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{d_{ij}}{v_{ij}} x_{ij} \to \max_{x_{ij}} \tag{6}$$

Remark l:

The service time for each UAVs is equal to their endurance h_i minus the time needed to reach the preassigned zone and come back to the base. Thus the total service time of the group of UAVs involved in the mission is given by (6). Hence the total service time of the group of UAVs involved in the mission will be maximal if the total flight time to reach the preassigned zones is minimal

$$F = \sum_{i=1}^{3} \sum_{j=1}^{3} t_{ij} = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij} \to \min_{x_{ij}} \quad \text{where } c_{ij} = \frac{d_{ij}}{v_{ij}} \tag{7}$$

Then we can consider the following optimization problem:

$$F = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij} \to \min_{x_{ij}} \tag{8}$$

$$\text{subject to} \tag{9}$$

$$\sum_{i=1}^{3} x_{ij} = b_j, \quad j = 1, 2, 3$$

$$\sum_{j=1}^{3} x_{ij} = a_i, \quad i = 1, 2, 3 \tag{10}$$

$$\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j \tag{11}$$

$$x_{ij} \geq 0, \quad x_{ij} \in \mathbb{N}. \tag{12}$$

where $A_i, i = 1, 2, 3$- number of aerobases,
$a_1 = 3, a_2 = 3, a_3 = 1$ - number of UAVs located in A_i,
$B_j, j = 1, 2, 3$- areas of operations,
$b_1 = 2, b_2 = 2, a_3 = 3$- numbers of UAVs for service of B_j
d_{ij}- distances from A_i to B_j given in table 1 , $v_{ij} = 30\frac{m}{sec}$
x_{ij}-number of UAVs from A_i to B_j
c_{ij}- given in table 2.

The first two constraints means that we are need to satisfy all service request from each area of operation and to use for that purpose all available UAVs. The last constraint means that the sum of all requests are equal to the total number of available UAVs.

The condition of that problem can be represented in table form:

	B_1	B_2	B_3	a_i
A_1	x_{11}	x_{12}	x_{13}	$a_1 = 3$
A_2	x_{21}	x_{22}	x_{23}	$a_2 = 3$
A_3	x_{31}	x_{32}	x_{33}	$a_3 = 1$
b_j	$b_1 = 2$	$b_2 = 2$	$b_3 = 3$	$\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 7$

Below we give the detailed step-by-step procedure to determine the optimal solution.

2.1 Initial feasible solution

To construct the initial feasible solution we will use "North-West corner" method. The construction of the initial supporting feasible solution consist from the several steps on each of them are filled either a row or a table column. The procedure begins with the left top ("northwest") element $x_{11} = \min(a_1; b_1)$ of the plan. If $a_1 < b_1$, i.e. $x_{11} = a_1$, than from the further consideration we eliminate all elements from the first row. If $a_1 \geq b_1$, i.e. $x_{11} = b_1$, than all elements from the first column are eliminated. In the case $a_1 < b_1$ the next element of feasible solution will be chosen from the second row by the rule $x_{21} = \min(a_2; b_1 - a_1)$. Next, if $a_2 < b_1$, i.e. $x_{21} = a_2$, and in this case we eliminated from our further consideration all elements from the second row. If $a_2 \geq b_1 - a_1$, i.e. $x_{21} = b_1 - a_1$, and further we will not consider the elements from the first column. The next assignment will be made on the intersection of the second column and second row as follows: $x_{22} = \min(a_1 + a_2 - b_1; b_2)$. Then repeated this procedure we will find all elements of the initial supporting feasible solution.
In our case we have the following:

$$\begin{aligned}
x_{11} &= \min(a_1; b_1) = \min(3; 2) = 2 \\
\Downarrow & \\
x_{12} &= \min(a_1 - b_1; a_2) = \min(1; 2) = 1 \\
\Downarrow & \\
x_{22} &= \min(a_2; b_2 + b_1 - a_!) = \min(3; 1) = 1 \\
\Downarrow & \\
x_{23} &= \min(a_2 + a_1 - b_1 - b_2; b_3) = \min(2; 3) = 2 \\
\Downarrow & \\
x_{33} &= \min(a_3; b_3 - (a_1 + a_2 - b_1 - b_2)) = \min(1; 1) = 1
\end{aligned} \tag{13}$$

	B_1	B_2	B_3	a_i
A_1	$x_{11} = 2$	$x_{12} = 1$		$a_1 = 3$
A_2		$x_{22} = 1$	$x_{23} = 2$	$a_2 = 3$
A_3			$x_{33} = 1$	$a_3 = 1$
b_j	$b_1 = 2$	$b_2 = 2$	$b_3 = 3$	$\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 7$

Next calculate the value of our cost function:

$$F = 433 \times 2 + 1000 \times 1 + 300 \times 1 + 566 \times 2 + 766 \times 1 = 4061 \; seconds \approx 67.7 \; minutes$$

2.2 Optimality condition

We will use the so called method of potentials, also known as "$u-\nu$" method . Consider auxiliary numbers $u_1, u_2, ..., u_k$ and $\nu_1, \nu_2, ..., \nu_l$. For any admissible solution the value $\sum_{i=1}^{k}\sum_{j=l}^{l}(u_i+\nu_j)x_{ij}$ is the same and constant:

$$\sum_{i=1}^{k}\sum_{j=1}^{l}(u_i+\nu_j)x_{ij} = \sum_{i=1}^{k} u_i \sum_{j=1}^{l} x_{ij} + \sum_{j=1}^{l} \nu_j \sum_{i=1}^{k} x_{ij} = \sum_{i=1}^{k} u_i a_i + \sum_{j=1}^{l} \nu_j b_j = C$$

Next, assume that for some admissible solution we found the numbers u_i and ν_j such that the following conditions

$$\begin{aligned} u_i + \nu_j &= c_{ij},\ for\ x_{ij} > 0, \\ u_i + \nu_j &\le c_{ij},\ for\ x_{ij} = 0 \end{aligned} \tag{14}$$

hold.

The solution is called potential solution if it satisfies to condition (14) and the sum $u_i + \nu_j = \bar{c_{ij}}$ called pseudocost. Then the condition for potential solution can be rewritten (14) as

$$\begin{aligned} \bar{c_{ij}} - c_{ij} &= 0,\ for\ x_{ij} > 0, \\ \bar{c_{ij}} - c_{ij} &\le 0,\ for\ x_{ij} = 0 \end{aligned} \tag{15}$$

Let us check the optimality condition for our problem. Consider the following table

	B_1	B_2	B_3	a_i	u_i
A_1	$\bar{c_{11}} = 433$ $c_{11} = 433$ $x_{11} = 2$	1000 1000 1		3	
A_2		330 330 1	566 566 2	3	
A_3			766 766 1	1	
b_j	2	2	3	F=4064	
ν_j					

Then we should find potential u_i and ν_j such that for $x_{ij} > 0$ the condition $c_{ij} = u_i + \nu_j$ hold. One of the potentials can be chosen arbitrary.

Let $\nu_1 = 0$, since $u_1 + \nu_1 = 433$ then $u_1 = 433$. Next following this logic we found step by step:

$$\nu_2 + u_1 = 1000 \longrightarrow \nu_2 = 1000 - 433 = 567,$$

$$\nu_2 + u_2 = 300 \longrightarrow u_2 = 300 - 567 = -267,$$

$$u_2 + \nu_3 = 566 \longrightarrow \nu_3 = 566 + 267 = 833,$$

$$\nu_3 + u_2 = 766 \longrightarrow u_3 = 766 - 833 = -67$$

Than we will have the following table:

	B_1	B_2	B_3	a_i	u_i
A_1	$\bar{c_{11}} = 433$ $c_{11} = 433$ $x_{11} = 2$	1000 1000 1		3	433
A_2		330 330 1	566 566 2	3	-267
A_3			766 766 1	1	-67
b_j	2	2	3	F=4064	
ν_j	0	567	833		

Now we are ready to check our initial supporting feasible solution for optimality. Namely to check the condition $\bar{c_{ij}} - c_{ij} \leq 0$ for $x_{ij} = 0$.

$\bar{c_{ij}} = u_i + \nu_j \longrightarrow \bar{c_{21}} = -267$
$\bar{c_{13}} = 433 + 833 = 1266$,
$\bar{c_{31}} = 0 - 67 = -67$,
$\bar{c_{32}} = 567 - 67 = 500$

Then in matrix of estimates $\Delta = c_{ij} - \bar{c_{ij}} = \begin{pmatrix} 0 & 0 & -666 \\ 800 & 0 & 0 \\ 767 & 166 & 0 \end{pmatrix}$ find a minimal element $\Delta_{13} = -666 = \min\limits_{i,j} \Delta_{ij}$.

In our case for one zero component of our feasible solution this conditions are not satisfied. Hence our solution is not optimal.

2.3 Improvement of the feasible solution

Change the initial feasible solution by adding the value *theta* to element x_{13} with some corrections of other elements too.

	B_1	B_2	B_3	a_i
A_1	2	$1-\theta$	θ	$a_1 = 3$
A_2		$1+\theta$	$2-\theta$	$a_2 = 3$
A_3			1	$a_3 = 1$
b_j	$b_1 = 2$	$b_2 = 2$	$b_3 = 3$	$\sum\limits_{i=1}^{3} a_i = \sum\limits_{j=1}^{3} b_j = 7$

Find the value $\theta = \min(2-\theta, 1-\theta) = 0 \longrightarrow \theta = 1$. Then we will have the following new feasible solution:

	B_1	B_2	B_3	a_i
A_1	2		1	$a_1 = 3$
A_2		2	1	$a_2 = 3$
A_3			1	$a_3 = 1$
b_j	$b_1 = 2$	$b_2 = 2$	$b_3 = 3$	$\sum\limits_{i=1}^{3} a_i = \sum\limits_{j=1}^{3} b_j = 7$

Next calculate the value of our cost function:

$$F = 433 \times 2 + 600 \times 1 + 300 \times 2 + 566 \times 1 + 766 \times 1 = 3398 \ \textit{seconds} \approx 56.6 \ \textit{minutes}$$

Now, we are need to repeat the described procedure again, namely we will need to calculate new potentials:

Let $\nu_1 = 0$, since $u_1 + \nu_1 = 433$ then $u_1 = 433$. Next following this logic we found step by step:

$$\nu_3 + u_1 = 600 \longrightarrow \nu_3 = 600 - 433 = 167,$$
$$\nu_3 + u_2 = 566 \longrightarrow u_2 = 567 - 167 = 389,$$
$$\nu_2 + u_2 = 300 \longrightarrow \nu_2 = 300 + 399 = -99,$$
$$\nu_3 + u_3 = 766 \longrightarrow u_3 = 766 - 167 = 599$$

Than we will have the following table:

	B_1	B_2	B_3	a_i	u_i
A_1	$\bar{c_{11}} = 433$ $c_{11} = 433$ $x_{11} = 2$		600 600 1	3	433
A_2		330 330 2	566 566 1	3	399
A_3			766 766 1	1	599
b_j	2	2	3	F=3398	
ν_j	0	-99	167		

Now we are ready to check our supporting feasible solution for optimality. Namely to check the condition $\bar{c_{ij}} - c_{ij} \leq 0$ for $x_{ij} = 0$.

$\bar{c_{ij}} = u_i + \nu_j \longrightarrow \bar{c_{12}} = u_1 + \nu_2 = 433 - 99 = 334$
$\bar{c_{21}} = 399 + 0 = 399,$
$\bar{c_{31}} = 599 + 0 = 599,$
$\bar{c_{32}} = 599 - 99 = 500$

Then in matrix of estimates $\Delta = c_{ij} - \bar{c_{ij}} = \begin{pmatrix} 0 & 666 & 0 \\ 134 & 0 & 0 \\ 101 & 166 & 0 \end{pmatrix}$

The optimality conditions are satisfied, since $\forall \Delta_{ij} \geq 0$.

Optimal solution are

$$\begin{aligned} x_{11} &= 2; \quad x_{13} = 1; \\ x_{22} &= 2; \quad x_{23} = 1; \\ x_{33} &= 1. \end{aligned} \tag{16}$$

Thus we will need to send our UAVs as follows: from Changi to Raffles Place: 2 UAVs;
from Changi to Sentosa Island: 1 UAV;
from Jurong West to Jurong Island: 2 UAVs;
from Jurong West to Sentosa Island : 1 UAV;
from Woodland to Sentosa Island: 1 UAV.

3 Assignment problem with timing constraints

3.1 A case with single UAVs at aerobases

To simplify at this stage our calculations we suppose that every aerobase has one UAV. Otherwise, the aerobases where there are several UAVs can be formally divided onto the collection of several aerobases with alone UAV at every one. In the next chapter we will consider the general case, too.

3.1.1 Notation

Introduce the following notations:
n— number of aerobases,
K — number of zones for service,
V_k— number of UAVs which are required for service of k-th zone, $k = 1, ..., K$
$[\underline{T}_k, \overline{T}_k]$— "time window" for k-th zone where $\underline{T}_k$ and $\overline{T}_k$ is the earliest and latest time for service of k-th zone),
r_{jk}— distance from j-th aerobase to k-th zone,
d_{ij}— distance from i-th zone to j-th zone.

Introduce the network of aerobases and zones as a pair (S, U). Here $S = \{1, 2, ..., n, n+1, ..., n+K\}$- the set of numbered nodes- aerobases and zones, such that to each node corresponds aerobase or zone. U-set of edges, which are connect the pair of nodes. The set S can be divided onto two subsets: S_A (set of aerobases) and S_Z (set of zones). Each node pair $(i, j), i \in S, j \in S$ corresponds the edge U_{ij} connecting the node i and node j. The edge U_{ij} have the characteristic ρ_{ij} — the distance between node i and j, i.e.

if $i \in S_A$ and $j \in S_Z$ then $\rho_{ij} = r_{ij}$;
if $i \in S_Z$ and $j \in S_Z$ then $\rho_{ij} = d_{ij}$.
Denote by
$\alpha_s, (s = 1, ..., n)$ — boolean variable where $\alpha_s = 1$ means that the $s-$ th aerobase (their UAV) involve into asked service, and $\alpha_s = 0$ — otherwise.
$\eta_i^{(s)}, (s = 1, ..., n;\ i = 1, ..., K$ — boolean variable where $\eta_i^{(s)} = 1$ means that the $s-$ th aerobase (their UAV) involve into service of $i-$ th zone, and $\eta_i^{(s)} = 0$ — otherwise.

3.2 Cost functions

Obviously, each assignment plan $\eta^{(s)} = \left(\eta_1^{(s)}, \eta_2^{(s)}, ..., \eta_K^{(s)}\right)$, $s = 1, ..., n$ of UAVs generates the boolean values α_s as follows

$$\alpha_s = \begin{cases} 1, \ if \ z^s > 0 \\ 0, \ if \ z^s = 0, \quad (s = 1, ..., n) \end{cases} \tag{17}$$

where $z^s = \sum\limits_{k=1}^{K} \eta_i^{(s)}$.

Then we can consider the cost functions

$$C_1(\eta) = \sum_{s=1}^{n} \alpha_s \tag{18}$$

that denotes the total number of UAVs used for service requests.

Next we introduce some other cost functions where it will be determined:
i) how many times each UAV is used in service
ii) total time service subject to constraints in the form of "time windows" for zone service.

To this aim we need to analyze some details of assignment plans in details.

3.3 Service logic and Constraints

Let $\eta^{(s)} = \left(\eta_1^{(s)}, \eta_2^{(s)}, ..., \eta_K^{(s)}\right)$ be an assignment plan for $s-th$ UAV (aerobase). Note, that the total number of all assignment plans for every aerobase is equal $K!$ (the number of all permutation of K elements). The value of $K!$ can be huge. By this reason, we can suppose that for each aerobase there exists some service order for considered zones. For example, this order can be determined in accordance with order of the assigned zone "time windows" such that the first for service is the zone with the smallest beginning of " window time". Some other ideas can be put to fix this order, also.

Next consider the time diagram of the considered flying route $\eta^{(s)}$.

Since in the considered route the zone-node $\eta_1^{(s)}$ is the first, and for this zone we have the time-window for service as $[\underline{T}_{\eta_1^{(s)}}, \overline{T}_{\eta_1^{(s)}}]$, then the time of the first departure from s-th base is:

$$t_1^{(s)} = \underline{T}_{\eta_1^{(s)}} - t_{fly}^{s \to \eta_1^{(s)}} \tag{19}$$

where $t_{fly}^{s \to \eta_1^{(s)}} = \dfrac{\rho_{s\eta_1^{(s)}}}{v_s}$ denotes the flying time from $s-th$ base to zone $\eta_1^{(s)}$.

Also it should be noted that it is not possible to start service of zone $\eta_1^{(s)}$ at the moment $\underline{T}_{\eta_1^{(s)}}$ if $t_1^{(s)} < 0$. But it is possible partially service if $h_s > t_{fly}^{s \to \eta_1^{(s)}} + t_{fly}^{\eta_1^{(s)} \to s}$, where h_s means the endurance of UAVs located at s-th base. If $t_1^{(s)} > 0$, then the service time of the first zone $\eta_1^{(s)}$ in the considered route $\eta^{(s)}$ is equal

$$T_{service}^{\eta_1^{(s)}} = \begin{cases} 0, \ if \ t_1^{(s)} < 0 \\ \overline{T}_{\eta_1}^{(s)} - \underline{T}_{\eta_1^{(s)}}, if \ t_1^{(s)} > 0 \ \text{ and } \ h_s > 2t_{fly}^{s \leftrightarrow \eta_1^{(s)}} + (\overline{T_{\eta_1}^{(s)}} - \underline{T_{\eta_1}^{(s)}}) \\ h_s - 2t_{fly}^{s \leftrightarrow \eta_1^{(s)}}, if \ t_0^{(s)} > 0 \quad \text{and} \quad h_s < 2t_{fly}^{s \leftrightarrow \eta_1^{(s)}} + (\overline{T_{\eta_1}^{(s)}} - \underline{T_{\eta_1}^{(s)}}) \end{cases} \tag{20}$$

Thus, after analysis of the first node $\eta_1^{(s)}$ we can define the time of ending service for the first zone by s-th UAVs located at s-th base as follows:

$$t_{1,final}^{(s)} = \begin{cases} 0, \begin{cases} a) if\ t_1^{(s)} < 0 \text{(i.e. UAVs was not used for service of the first node)} \\ b) if\ t_1^{(s)} > 0\ and\ h_s < 2t_{fly}^{s\leftrightarrow\eta_1^{(s)}} + (\overline{T_{\eta_1}^{(s)}} - \underline{T_{\eta_1}^{(s)}}) \\ \text{(i.e. UAVs was used at first zone and then it returned to base due to restricted endurance)} \end{cases} \\ (t_{fly}^{s\to\eta_1^{(s)}} + T_{service}^{s\to\eta_1^{(s)}}), \text{(i.e. when endurance of UAV was more then required for service zone } \eta_1^{(s)} \\ \text{and UAV can fly for service from zone } \eta_1^{(s)} \text{ to next zone } \eta_2^{(s)}) \end{cases} \tag{21}$$

Now consider how we can to start the service of the next zone from our route $\eta^{(s)}$ taking into account the previous analysis and (21). Find the starting moment

$$t_{start}^{\eta_2^{(s)}} = \begin{cases} \underline{T_{\eta_2}^{(s)}} - t_{fly}^{s\to\eta_2^{(s)}}, \text{ if } t_{1,final}^{(s)} = 0 \text{(i.e. this is the case, when we are "start" from the base)} \\ t_{fly}^{s\to\eta_1^{(s)}} + T_{service}^{\eta_1^{(s)}}, \text{ otherwise (namely we are starting from the first zone } \eta_1^{(s)}) \end{cases} \tag{22}$$

It should be noted once again that, if $t_{start}^{\eta_2^{(s)}} < 0$, then this zone will be eliminated from further consideration, since the considered $s-th$ UAV does not reach this zone. In the case when $t_{start}^{\eta_2^{(s)}} > 0$ we can to continue the analysis of possibilities of servicing node (zone) $\eta_2^{(s)}$ taking into account the "time window" constraint $[\underline{T_{\eta_2}^{(s)}}, \overline{T_{\eta_2}^{(s)}}]$.

Then

$$T_{service}^{\eta_2^{(s)}} = \begin{cases} a) 0, if\ t_{start}^{\eta_2^{(s)}} < 0 \\ b) \overline{T_{\eta_2}^{(s)}} - \underline{T_{\eta_2}^{(s)}}, if\ t_{start}^{\eta_2^{(s)}} > 0\ and\ t_{1,final}^{(s)} = 0\ and\ h_s > 2t_{fly}^{s\leftrightarrow\eta_2^{(s)}} + (\overline{T_{\eta_2}^{(s)}} - \underline{T_{\eta_2}^{(s)}}) \\ \text{i.e the case, when we will start from the base} \\ \text{and we have sufficient endurance to serve the node } \eta_2^{(s)} \text{ and coming back to base} \\ c) h_s - 2t_{fly}^{s\to\eta_1^{(s)}}, if\ t_{start}^{\eta_2^{(s)}} > 0\ and\ t_{1,final}^{(s)} = 0\ but\ h_s < 2t_{fly}^{s\to\eta_2^{(s)}} + (\overline{T_{\eta_2}^{(s)}} - \underline{T_{\eta_2}^{(s)}}) \\ \text{(i.e. the case when not completely "close" the window....)} \\ d) h_s - t_{fly}^{s\to\eta_1^{(s)}} - t_{fly}^{\eta_1\to\eta_2} - t_{fly}^{\eta_2\to s}, \text{if served } \eta_2 \text{ from } \eta_1 \text{ and then back to base } s, \\ \text{since there was not sufficient endurance to continue service} \\ e) T_{service}^{\eta_1^{(s)}} + (\overline{T_{\eta_2}^{(s)}} - \underline{T_{\eta_2}^{(s)}}), \text{if served the node } \eta_2 \text{ from } \eta_1 \text{ and have sufficient endurance.} \end{cases} \tag{23}$$

Continue by analogy with above the given analysis for the remainder zones from the considered route $\eta^{(s)}$ we find the sequence

$$T_{service}^{\eta_1^{(s)}}, T_{service}^{\eta_2^{(s)}}, ..., T_{service}^{\eta_K^{(s)}}$$

of time services of each zones. Then the total service time which generates the considered route $\eta^{(s)}$ is

$$T_{service}(\eta^{(s)}) = T_{service}^{\eta_1^{(s)}} + T_{service}^{\eta_2^{(s)}} + ... + T_{service}^{\eta_K^{(s)}} \tag{24}$$

and, hence, the total service time of the required zones is

$$T_{service} = \sum_{s=1}^{n} T_{service}(\eta^{(s)}) \tag{25}$$

To guarantee the needed number V_k of pre-assigned UAVs for $k-th$ zone we should set the following constraints for the introduced boolean variables

$$\sum_{s=1}^{n} \eta_k^{(s)} = V_k,\ k = 1.2, ..., K \tag{26}$$

Finally, the assignment problem with timing constraints can be formulated as the following boolean optimization problem; Maximize the total service time

$$\sum_{s=1}^{n} T_{service}(\eta^{(s)}) \rightarrow \max_{\eta^{(s)} \in \text{boolean}} \tag{27}$$

subject to constraints

$$\sum_{s=1}^{n} \eta_k^{(s)} = V_k, \ \ k = 1.2, ..., K \tag{28}$$