# 1 General problem statement

## 1.1 Notation

$A_i, i = 1, ..., k$- aerobases,
$a_i$ - number of UAVs located in $A_i$,
$B_j, j = 1, ..., l$- areas of operations,
$b_j$- numbers of UAVs for service of $B_j$
$d_{ij}$- distance from $A_i$ to $B_j$,
$v_{ij}$- UAVs speed
$x_{ij}$-number of UAVs from $A_i$ to $B_j$,
$h_i$- UAVs endurance located on $A_i$ aerobase

## 1.2 Problem statement

$$T^{service} = \sum_{i=1}^{k} h_i - 2\sum_{i=1}^{k}\sum_{j=1}^{l} \frac{d_{ij}}{v_{ij}} x_{ij} \to \max_{x_{ij}} \tag{1}$$

subject to

$$\sum_{i=1}^{k} x_{ij} = b_j, \quad j = 1, 2, ..., l \tag{2}$$

$$\sum_{j=1}^{l} x_{ij} = a_i, \quad i = 1, 2, ..., k$$

$$\sum_{i=1}^{k} a_i = \sum_{j=1}^{l} b_j$$

$$\frac{d_{ij}}{v_{ij}} \operatorname{sign}(x_{ij}) \geq t_j^{first}, \ i = 1, 2, ..., k$$

$$\frac{d_{ij}}{v_{ij}} \operatorname{sign}(x_{ij}) \leq t_j^{last}, \ i = 1, 2, ..., k$$

$$x_{ij} \geq 0, \qquad x_{ij} \ \text{ are integer numbers.}$$

Here the cost function presents the total service time performed by all UAVs used for mission. The first two constraints means that we are need to satisfy all service request from each area of operation and to use for that purpose all available UAVs. The last inequalities image the fact that the period of the start of service of $j - th$ zone by $i - th$ UAV is restricted by the pre-assigned time interval $[t_j^{first}, t_j^{last}]$. The cost function presents the total service time performed by all UAVs used for mission.

# 2 Illustrative example

Assume that we have 3 airbases locate at Changi $A_1$ with 3 UAVs ($a_1 = 3$) , Jurong West $A_2$ with 3 UAVs ($a_2 = 3$) , and Woodland $A_3$ with 1 UAV ($a_1 = 1$). Now 7 UAVs are requested from $B_1$-Raffles Place ($b_1 = 2$), $B_2$-Jurong Island ($b_2 = 2$), and $B_3$- Sentosa Island ($b_3 = 3$). The distances between $A_i$ and $B_j$ given below in kilometers:

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 13 | 30 | 18 |
| $A_2$ | 16 | 9 | 17 |
| $A_3$ | 21 | 20 | 23 |

(3)

Table 1: Distances between aerobases $A_i$ and area of operations $B_j$

The speed of UAVs are fixed $v_{ij} = 30\frac{m}{sec}$.

Next, for all $i$ and $j$ denote by $c_{ij} = \frac{d_{ij}}{v_{ij}}$ the benefit of sending the UAV from $i$-th aerobase to $j$-th zone of area of operation. The benefit means the flight time from $A_i \rightarrow B_j$.

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | 433 | 1000 | 600 |
| $A_2$ | 533 | 300 | 566 |
| $A_3$ | 700 | 666 | 766 |

(4)

Table 2: UAVs flight time from $A_i \rightarrow B_j$ (5)

Then using our notation we can formulate the problem statement as the following integer programming problem: To find $x_{ij}, (i = 1, 2, 3; j = 1, 2, 3)$ such that, the total service time performed by all UAVs takes a maximal value

$$T^{service} = \sum_{i=1}^{3} h_i - 2 \min_{x_{ij}} \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{d_{ij}}{v_{ij}} x_{ij} \rightarrow \max_{x_{ij}} \tag{6}$$

**Remark 1:**

The service time for each UAVs is equal to their endurance $h_i$ minus the time needed to reach the preassigned zone and come back to the base. Thus the total service time of the group of UAVs involved in the mission is given by (6). Hence the total service time of the group of UAVs involved in the mission will be maximal if the total flight time to reach the preassigned zones is minimal

$$F = \sum_{i=1}^{3} \sum_{j=1}^{3} t_{ij} = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij} \rightarrow \min_{x_{ij}} \quad \text{where } c_{ij} = \frac{d_{ij}}{v_{ij}} \tag{7}$$

Then we can consider the following optimization problem:

$$F = \sum_{i=1}^{3} \sum_{j=1}^{3} c_{ij} x_{ij} \rightarrow \min_{x_{ij}} \tag{8}$$

$$\text{subject to} \tag{9}$$

$$\sum_{i=1}^{3} x_{ij} = b_j, \quad j = 1, 2, 3$$

$$\sum_{j=1}^{3} x_{ij} = a_i, \quad i = 1, 2, 3 \tag{10}$$

$$\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j \tag{11}$$

$$x_{ij} \geq 0, \quad x_{ij} \in \mathbb{N}. \tag{12}$$

where $A_i, i = 1, 2, 3$- number of aerobases,
$a_1 = 3, a_2 = 3, a_3 = 1$ - number of UAVs located in $A_i$,
$B_j, j = 1, 2, 3$- areas of operations,
$b_1 = 2, b_2 = 2, a_3 = 3$- numbers of UAVs for service of $B_j$
$d_{ij}$- distances from $A_i$ to $B_j$ given in table 1 , $v_{ij} = 30\frac{m}{sec}$
$x_{ij}$-number of UAVs from $A_i$ to $B_j$
$c_{ij}$- given in table 2.

The first two constraints means that we are need to satisfy all service request from each area of operation and to use for that purpose all available UAVs. The last constraint means that the sum of all requests are equal to the total number of available UAVs.

The condition of that problem can be represented in table form:

| | $B_1$ | $B_2$ | $B_3$ | $a_i$ |
|---|---|---|---|---|
| $A_1$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $a_1 = 3$ |
| $A_2$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $a_2 = 3$ |
| $A_3$ | $x_{31}$ | $x_{32}$ | $x_{33}$ | $a_3 = 1$ |
| $b_j$ | $b_1 = 2$ | $b_2 = 2$ | $b_3 = 3$ | $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 7$ |

Below we give the detailed step-by-step procedure to determine the optimal solution.

## 2.1 Initial feasible solution

To construct the initial feasible solution we will use ”North-West corner” method. The construction of the initial supporting feasible solution consist from the several steps on each of them are filled either a row or a table column. The procedure begins with the left top (”northwest”) element $x_{11} = \min(a_1; b_1)$ of the plan. If $a_1 < b_1$, i.e. $x_{11} = a_1$, than from the further consideration we eliminate all elements from the first row. If $a_1 \geq b_1$, i.e. $x_{11} = b_1$, than all elements from the first column are eliminated. In the case $a_1 < b_1$ the next element of feasible solution will be chosen from the second row by the rule $x_{21} = \min(a_2; b_1 - a_1)$. Next, if $a_2 < b_1$, i.e. $x_{21} = a_2$, and in this case we eliminated from our further consideration all elements from the second row. If $a_2 \geq b_1 - a_1$, i.e. $x_{21} = b_1 - a_1$, and further we will not consider the elements from the first column. The next assignment will be made on the intersection of the second column and second row as follows: $x_{22} = \min(a_1 + a_2 - b_1; b_2)$. Then repeated this procedure we will find all elements of the initial supporting feasible solution.
In our case we have the following:

$$
\begin{aligned}
x_{11} &= \min(a_1; b_1) = \min(3; 2) = 2 \\
\Downarrow & \\
x_{12} &= \min(a_1 - b_1; a_2) = \min(1; 2) = 1 \\
\Downarrow & \\
x_{22} &= \min(a_2; b_2 + b_1 - a_!) = \min(3; 1) = 1 \\
\Downarrow & \\
x_{23} &= \min(a_2 + a_1 - b_1 - b_2; b_3) = \min(2; 3) = 2 \\
\Downarrow & \\
x_{33} &= \min(a_3; b_3 - (a_1 + a_2 - b_1 - b_2)) = \min(1; 1) = 1
\end{aligned}
\tag{13}
$$

| | $B_1$ | $B_2$ | $B_3$ | $a_i$ |
|---|---|---|---|---|
| $A_1$ | $x_{11} = 2$ | $x_{12} = 1$ | | $a_1 = 3$ |
| $A_2$ | | $x_{22} = 1$ | $x_{23} = 2$ | $a_2 = 3$ |
| $A_3$ | | | $x_{33} = 1$ | $a_3 = 1$ |
| $b_j$ | $b_1 = 2$ | $b_2 = 2$ | $b_3 = 3$ | $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 7$ |

Next calculate the value of our cost function:

$$F = 433 \times 2 + 1000 \times 1 + 300 \times 1 + 566 \times 2 + 766 \times 1 = 4061 \; seconds \approx 67.7 \; minutes$$

## 2.2 Optimality condition

We will use the so called method of potentials, also known as "$u - \nu$" method . Consider auxiliary numbers $u_1, u_2, ..., u_k$ and $\nu_1, \nu_2, ..., \nu_l$. For any admissible solution the value $\sum\limits_{i=1}^{k}\sum\limits_{j=l}^{l}(u_i + \nu_j)x_{ij}$ is the same and constant:

$$\sum_{i=1}^{k}\sum_{j=1}^{l}(u_i+\nu_j)x_{ij} = \sum_{i=1}^{k} u_i \sum_{j=1}^{l} x_{ij} + \sum_{j=1}^{l} \nu_j \sum_{i=1}^{k} x_{ij} = \sum_{i=1}^{k} u_i a_i + \sum_{j=1}^{l} \nu_j b_j = C$$

Next, assume that for some admissible solution we found the numbers $u_i$ and $\nu_j$ such that the following conditions

$$\begin{aligned} u_i + \nu_j &= c_{ij},\ for\ x_{ij} > 0, \\ u_i + \nu_j &\leq c_{ij},\ for\ x_{ij} = 0 \end{aligned} \tag{14}$$

hold.

The solution is called potential solution if it satisfies to condition (14) and the sum $u_i + \nu_j = \bar{c_{ij}}$ called pseudocost. Then the condition for potential solution can be rewritten (14) as

$$\begin{aligned} \bar{c_{ij}} - c_{ij} &= 0,\ for\ x_{ij} > 0, \\ \bar{c_{ij}} - c_{ij} &\leq 0,\ for\ x_{ij} = 0 \end{aligned} \tag{15}$$

Let us check the optimality condition for our problem. Consider the following table

| | $B_1$ | $B_2$ | $B_3$ | $a_i$ | $u_i$ |
|---|---|---|---|---|---|
| $A_1$ | $\bar{c_{11}} = 433$ $c_{11} = 433$ $x_{11} = 2$ | 1000 1000 1 | | 3 | |
| $A_2$ | | 330 330 1 | 566 566 2 | 3 | |
| $A_3$ | | | 766 766 1 | 1 | |
| $b_j$ | 2 | 2 | 3 | F=4064 | |
| $\nu_j$ | | | | | |

Then we should find potential $u_i$ and $\nu_j$ such that for $x_{ij} > 0$ the condition $c_{ij} = u_i + \nu_j$ hold. One of the potentials can be chosen arbitrary.

Let $\nu_1 = 0$, since $u_1 + \nu_1 = 433$ then $u_1 = 433$. Next following this logic we found step by step:

$$\nu_2 + u_1 = 1000 \longrightarrow \nu_2 = 1000 - 433 = 567,$$

$$\nu_2 + u_2 = 300 \longrightarrow u_2 = 300 - 567 = -267,$$

$$u_2 + \nu_3 = 566 \longrightarrow \nu_3 = 566 + 267 = 833,$$

$$\nu_3 + u_2 = 766 \longrightarrow u_3 = 766 - 833 = -67$$

Than we will have the following table:

| | $B_1$ | $B_2$ | $B_3$ | $a_i$ | $u_i$ |
|---|---|---|---|---|---|
| $A_1$ | $\bar{c_{11}} = 433$ $c_{11} = 433$ $x_{11} = 2$ | 1000 1000 1 | | 3 | 433 |
| $A_2$ | | 330 330 1 | 566 566 2 | 3 | -267 |
| $A_3$ | | | 766 766 1 | 1 | -67 |
| $b_j$ | 2 | 2 | 3 | F=4064 | |
| $\nu_j$ | 0 | 567 | 833 | | |

Now we are ready to check our initial supporting feasible solution for optimality. Namely to check the condition $\bar{c_{ij}} - c_{ij} \leq 0$ for $x_{ij} = 0$.

$\bar{c_{ij}} = u_i + \nu_j \longrightarrow \bar{c_{21}} = -267$
$\bar{c_{13}} = 433 + 833 = 1266,$
$\bar{c_{31}} = 0 - 67 = -67,$
$\bar{c_{32}} = 567 - 67 = 500$

Then in matrix of estimates $\Delta = c_{ij} - \bar{c_{ij}} = \begin{pmatrix} 0 & 0 & -666 \\ 800 & 0 & 0 \\ 767 & 166 & 0 \end{pmatrix}$ find a minimal element $\Delta_{13} = -666 = \min_{i,j} \Delta_{ij}$.

In our case for one zero component of our feasible solution this conditions are not satisfied. Hence our solution is not optimal.

## 2.3 Improvement of the feasible solution

Change the initial feasible solution by adding the value *theta* to element $x_{13}$ with some corrections of other elements too.

| | $B_1$ | $B_2$ | $B_3$ | $a_i$ |
|---|---|---|---|---|
| $A_1$ | 2 | $1-\theta$ | $\theta$ | $a_1 = 3$ |
| $A_2$ | | $1+\theta$ | $2-\theta$ | $a_2 = 3$ |
| $A_3$ | | | 1 | $a_3 = 1$ |
| $b_j$ | $b_1 = 2$ | $b_2 = 2$ | $b_3 = 3$ | $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 7$ |

Find the value $\theta = \min(2-\theta, 1-\theta) = 0 \longrightarrow \theta = 1$. Then we will have the following new feasible solution:

| | $B_1$ | $B_2$ | $B_3$ | $a_i$ |
|---|---|---|---|---|
| $A_1$ | 2 | | 1 | $a_1 = 3$ |
| $A_2$ | | 2 | 1 | $a_2 = 3$ |
| $A_3$ | | | 1 | $a_3 = 1$ |
| $b_j$ | $b_1 = 2$ | $b_2 = 2$ | $b_3 = 3$ | $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 7$ |

Next calculate the value of our cost function:

$$F = 433 \times 2 + 600 \times 1 + 300 \times 2 + 566 \times 1 + 766 \times 1 = 3398 \ \textit{seconds} \approx 56.6 \ \textit{minutes}$$

Now, we are need to repeat the described procedure again, namely we will need to calculate new potentials:

Let $\nu_1 = 0$, since $u_1 + \nu_1 = 433$ then $u_1 = 433$. Next following this logic we found step by step:

$$\nu_3 + u_1 = 600 \longrightarrow \nu_3 = 600 - 433 = 167,$$
$$\nu_3 + u_2 = 566 \longrightarrow u_2 = 567 - 167 = 389,$$
$$\nu_2 + u_2 = 300 \longrightarrow \nu_2 = 300 + 399 = -99,$$
$$\nu_3 + u_3 = 766 \longrightarrow u_3 = 766 - 167 = 599$$

Than we will have the following table:

| | $B_1$ | $B_2$ | $B_3$ | $a_i$ | $u_i$ |
|---|---|---|---|---|---|
| $A_1$ | $\bar{c_{11}} = 433$ $c_{11} = 433$<br>$x_{11} = 2$ | | 600 600<br>1 | 3 | 433 |
| $A_2$ | | 330 330<br>2 | 566 566<br>1 | 3 | 399 |
| $A_3$ | | | 766 766<br>1 | 1 | 599 |
| $b_j$ | 2 | 2 | 3 | F=3398 | |
| $\nu_j$ | 0 | -99 | 167 | | |

Now we are ready to check our supporting feasible solution for optimality. Namely to check the condition $\bar{c_{ij}} - c_{ij} \leq 0$ for $x_{ij} = 0$.

$\bar{c_{ij}} = u_i + \nu_j \longrightarrow \bar{c_{12}} = u_1 + \nu_2 = 433 - 99 = 334$
$\bar{c_{21}} = 399 + 0 = 399$,
$\bar{c_{31}} = 599 + 0 = 599$,
$\bar{c_{32}} = 599 - 99 = 500$

Then in matrix of estimates $\Delta = c_{ij} - \bar{c_{ij}} = \begin{pmatrix} 0 & 666 & 0 \\ 134 & 0 & 0 \\ 101 & 166 & 0 \end{pmatrix}$

The optimality conditions are satisfied, since $\forall \Delta_{ij} \geq 0$.

Optimal solution are

$$\begin{aligned} &x_{11} = 2; \quad x_{13} = 1; \\ &x_{22} = 2; \quad x_{23} = 1; \\ &x_{33} = 1. \end{aligned} \tag{16}$$

Thus we will need to send our UAVs as follows: from Changi to Raffles Place: 2 UAVs;
from Changi to Sentosa Island: 1 UAV;
from Jurong West to Jurong Island: 2 UAVs;
from Jurong West to Sentosa Island : 1 UAV;
from Woodland to Sentosa Island: 1 UAV.

# 3 Assignment problem with timing constraints

## 3.1 Notation

$n$- number of aerobases,
$K$ - number of zones for service,
$m$- number of different types of UAVs,
$f_{ij}, i = 1, ..., m; j = 1, ..., n$- number of UAVs of $i$-th types located at $j$-th aerobase,
$V_{ik}$-number of UAVs of $i$-th types which are required for service of $k$-th zone,
$[\underline{T_k}, \overline{T_k}]$- "time window" (i.e. earliest $\underline{T_k}$ and latest $\overline{T_k}$ time for service $k$-th zone),
$r_{jk}$-distance from $j$-th aerobase to $k$-th zone,
$d_{ij}$- distance from $i$-th zone to $j$-th zone.

Introduce the network of aerobases and zones as a pair $(S, U)$.
Here $S = \{1, 2, ..., n, n+1, ..., n+K\}$- the set of numbered nodes- aerobases and zones, such that to each node corresponds aerobase or zone.
$U$-set of edges, which are connect the pair of nodes. The set $S$ consist from two subsets $S_A$ (set of aerobases) and $S_Z$ (set of zones). For each pair $(i, j), i \in S, j \in S$ corresponds the edge $U_{ij}$ which is connected the node $i$ and node $j$. The edge $U_{ij}$ have the characteristic $\rho_{ij}$ -the distance between node $i$ and $j$, i.e.
if $i \in S_A$ and $j \in S_Z$ then $\rho_{ij} = r_{ij}$;
if $i \in S_Z$ and $j \in S_Z$ then $\rho_{ij} = d_{ij}$.
Denote by $x_{ij}^k$ - number of UAVs of $i$-th type (should be defined), which must be send from $j$-th aerobase to $k$-th zone.

### 3.2 Cost functions

With each plan of distribution of UAVs between zones $X = \{x_{ij}^k, i = 1, ..., m; j = 1, ..., n; k = 1, ..., K\}$ we can consider the following effectiveness criteria:
1) number of used UAVs to complete all service requests:

$$C_1(X) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ij}^k \tag{17}$$

2) total flying distance:

$$C_2(X) = \sum_{j=1}^{n} \sum_{k=1}^{K} (\rho_{jk} \sum_{i=1}^{m} x_{ij}^k), \tag{18}$$

where $\sum_{i=1}^{m} x_{ij}^k$- denotes the number of UAVs of all types, which was sent from $j$-th base to $k$-th zone.
3)and many others...

### 3.3 Constraints

Each plan $X$ should satisfy to the following conditions:
a) the $x_{ij}^k$ should not be more then available UAVs of $i$-th type located at the corresponding aerobases:

$$\sum_{k=1}^{K} x_{ij}^k \leq f_{ij}, i = 1, ..., m; j = 1, ..., n \tag{19}$$

b) For each $s$-aerobase ($s = 1, ..., n$ ) introduce the collection of visited nodes (i.e. zones) as follows:

$$\eta^{(s)} = \{\eta_{A_s}, \eta_1^{(s)}, \eta_2^{(s)}, ..., \eta_{q_s}, \eta_{A_s}\} \tag{20}$$

where
$\eta_{A_s}$- node ($s$-th base);
$\eta_i^{(s)} \in S_Z$- nodes (zones) which should be visited;
$i \in \{1, ..., q_s\}$, where $q_s$- number of zones which should be visit).

Next consider the time diagram of flying route $\eta^{(s)}$. In order to analyze this time diagram assume the following: Firstly, on each $s$-th base the all UAVs located there should be the same type. (Otherwise introduce an auxiliary base with homogenous UAVs.) Secondly, if UAV once was assigned to the zone, then this UAV will not reassigned to another zone.
Since in the considered collection $\eta^{(s)}$, the zone-node $\eta_1^{(s)}$ is a first one, and for this zone we have the time-window for service as $[\underline{T_{\eta_1}^{(s)}}, \overline{T_{\eta_1}^{(s)}}]$, then the time of first departure from $s$-th base is:

$$t_0^{(s)} = \underline{T_{\eta_1^{(s)}}} - t_{fly}^{s \to \eta_1^{(s)}} \tag{21}$$

where $t_{fly}^{s \to \eta_1^{(s)}} = \frac{\rho_{s\eta_1^{(s)}}}{v_s}$.

Also it should be noted that it is not possible to start service of the node $\eta_1^{(s)}$ at the moment $\underline{T_{\eta_1^{(s)}}}$ if $t_0^{(s)} < 0$. But it is possible partially service if $h_s > t_{fly}^{s \to \eta_1^{(s)}} + t_{fly}^{\eta_1^{(s)} \to s}$, where $h_s$-endurance of UAVs located at $s$-th base. If $t_0^{(s)} > 0$, then the service time of the first zone $\eta_1^{(s)}$ in a collection (or considered route) $\eta^{(s)}$ is:

$$T_{service}^{\eta_1^{(s)}} = \begin{cases} a)0, if\ t_0^{(s)} < 0 \\ b)\overline{T_{\eta_1}^{(s)}} - \underline{T_{\eta_1}^{(s)}}, if\ t_0^{(s)} > 0\ and\ h_s > 2t_{fly}^{s\leftrightarrow\eta_1^{(s)}} + (\overline{T_{\eta_1}^{(s)}} - \underline{T_{\eta_1}^{(s)}}) \\ c)h_s - 2t_{fly}^{s\leftrightarrow\eta_1^{(s)}}, if\ t_0^{(s)} > 0\ and\ h_s < 2t_{fly}^{s\leftrightarrow\eta_1^{(s)}} + (\overline{T_{\eta_1}^{(s)}} - \underline{T_{\eta_1}^{(s)}}) \end{cases} \tag{22}$$

***Thus, after analysis of the first node $\eta_1^{(s)}$ we can define the time of end of service of the first node by $s$-th UAVs located at $s$-th base:***

$$t_{1,final}^{(s)} = \begin{cases} 0, \begin{cases} a)if\ t_0^{(s)} < 0 (\text{i.e. UAVs was not used for service of the first node}) \\ b)if\ t_0^{(s)} > 0\ and\ h_s < 2t_{fly}^{s\leftrightarrow\eta_1^{(s)}} + (\overline{T_{\eta_1}^{(s)}} - \underline{T_{\eta_1}^{(s)}}) \end{cases} \\ t_{fly}^{s\rightarrow\eta_1^{(s)}} + T_{service}^{s\rightarrow\eta_1^{(s)}}, (\text{i.e. when endurance of UAV was more then required for service zone } \eta_1^{(s)} \\ \text{and UAV can fly for service from zone } \eta_1^{(s)} \text{ to next zone } \eta_2^{(s)}) \end{cases} \tag{23}$$

Now consider how we can service the next node of our route $\eta^{(s)}$ using information from (23):

$$t_{start}^{\eta_2^{(s)}} = \begin{cases} \underline{T_{\eta_2}^{(s)}} - t_{fly}^{s\rightarrow\eta_2^{(s)}}, if\ t_{1,final}^{(s)} = 0 (\text{i.e. this is the case, when we are "start" from the base}) \\ t_{fly}^{s\rightarrow\eta_1^{(s)}} + T_{service}^{\eta_1^{(s)}}, otherwise \text{ (namely we are start from the zone } \eta_1^{(s)}) \end{cases} \tag{24}$$

It should be noted once again that, if $t_{start}^{\eta_2^{(s)}} < 0$, then this node will be eliminated from further consideration, since it will be not possible to reach this zone. In case when $t_{start}^{\eta_2^{(s)}} > 0$ we can to continue the analysis of possibilities of servicing node (zone) $\eta_2^{(s)}$ taking into account the constraints $[\underline{T_{\eta_2}^{(s)}}, \overline{T_{\eta_2}^{(s)}}]$.

Then

$$T_{service}^{\eta_2^{(s)}} = \begin{cases} a)0, if\ t_{start}^{\eta_2^{(s)}} < 0 \\ b)\overline{T_{\eta_2}^{(s)}} - \underline{T_{\eta_2}^{(s)}}, if\ t_{start}^{\eta_2^{(s)}} > 0\ and\ t_{1,final}^{(s)} = 0\ and\ h_s > 2t_{fly}^{s\leftrightarrow\eta_2^{(s)}} + (\overline{T_{\eta_2}^{(s)}} - \underline{T_{\eta_2}^{(s)}}) \\ \text{i.e the case, when we will start from the base} \\ \text{and we have sufficient endurance to serve the node } \eta_2^{(s)} \text{ and coming back to base} \\ c)h_s - 2t_{fly}^{s\rightarrow\eta_1^{(s)}}, if\ t_{start}^{\eta_2^{(s)}} > 0\ and\ t_{1,final}^{(s)} = 0\ but\ h_s < 2t_{fly}^{s\rightarrow\eta_2^{(s)}} + (\overline{T_{\eta_2}^{(s)}} - \underline{T_{\eta_2}^{(s)}}) \\ (\text{i.e. the case when not completely "close" the window....}) \\ d)h_s - t_{fly}^{s\rightarrow\eta_1^{(s)}} - t_{fly}^{\eta_1\rightarrow\eta_2} - t_{fly}^{\eta_2\rightarrow s}, \text{if served } \eta_2 \text{ from } \eta_1 \text{ and then back to base } s, \\ \text{since there was not sufficient endurance to continue service} \\ e)T_{service}^{\eta_1^{(s)}} + (\overline{T_{\eta_2}^{(s)}} - \underline{T_{\eta_2}^{(s)}}), \text{if served the node } \eta_2 \text{ from } \eta_1 \text{ and have sufficient endurance.} \end{cases} \tag{25}$$