Subject panel: Mathematical Methods and Applications

Suggested title of dissertation: Dynamics on hypergraphs

Dissertation supervisor: Renaud Lambiotte

Suggested second assessors:

Description of proposal: Network science is a powerful framework for modelling interacting systems. Its strength comes from its generality in distilling connectivity into core elements, nodes and edges, that can combine to form indirect connections. Many social, natural and engineered systems can be represented as networks. However, the node-and-edge paradigm of network science places fundamental limitations on modelling capabilities. These limitations, combined with the availability of detailed network data, have led to the early development of several higher-order network models of richer interactions, including multiplex networks, non-Markovian networks and, and multiway networks. One major mathematical framework for multiway networks is the one of hypergraphs. A hypergraph is a generalization of a graph in which an edge can connect more than two vertices. In other words, an edge is a subset of vertices, whose cardinality can be fixed, as in uniform hypergraphs, or vary in a system. In many situations, the richer structure associated to the hyperedges implies that hypergraph concepts are not uniquely defined. For instance, the simplest way to define a walk is by a sequence of hyperedges with at least one node in common, but this concept may then be generalised through the notion of k-walks, requiring that two successive edges have a least k nodes in common. The main purposes of the proposed dissertations will be to investigate dynamical systems on hypergraphs, focusing on consensus dynamics and diffusive processes, and to use the processes in order to build algorithms to detect structures in hypergraphs.

Possible avenues of investigation:
Possible venues of investigation include the study of non-linear consensus on block graphs, that can be seen as generalisation fo trees for hypergraphs, the study of non-Markovian diffusion on hypergraphs in order to account for the context of each hyperedge in the spreading, and the design of appropriate approximation procedures for the study of dynamics on randomised hypergraphs.

Pre-requisite knowledge: Recommended: the student would have taken Nonlinear Systems, Graph Theory and follow Networks.

Useful reading: Battiston, Federico, et al. "Networks beyond pairwise interactions: structure and dynamics." Physics Reports (2020).
Lambiotte, Renaud, Martin Rosvall, and Ingo Scholtes. "From networks to optimal higher-order models of complex systems." Nature physics 15.4 (2019): 313-320.