### Knowledge Graph Completion

高桢 2020年10月15日

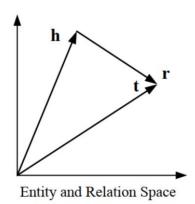
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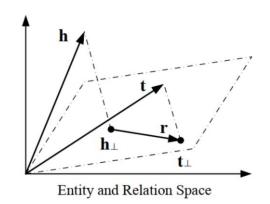
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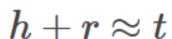
#### INTRODUCTION

- Knowledge
  - Triple (h, r, t)
  - h: head entity r: relation t: tail entity
  - Score (h, r, t)
    - Predict head entity
    - Predict relation
    - Predict tail entity
- Classify
  - Static KGC
  - Dynamic KGC

#### Translation Model







$$f_r(h,t) = \left|\left|h + r - t
ight|
ight|_2^2$$

$$egin{aligned} f_r(h,t) &= ||h_{ot} + r - t_{ot}|| \ h_{ot} &= h - w_r^T h w_r \ t_{ot} &= t - w_r^T t w_r \end{aligned}$$

Relation Space of r**Entity Space** 

 $|f_r(h,t)=||h_\perp+r-t_\perp||_2^2$ 

$$f_r(h,t) = ||h_\perp + r - t_\perp||_2^2$$

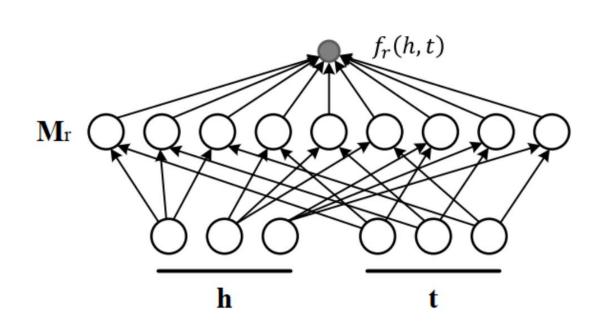
$$h_{\perp} = h M_{r} \hspace{0.5cm} t_{\perp} \!\! = t M_{r}$$

TransE TransH TransR

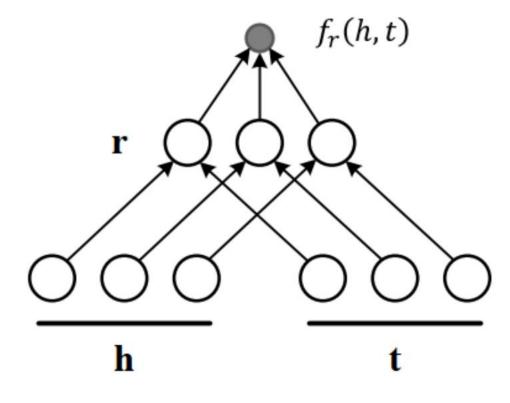
#### More Translation Model

Method	Ent. embedding	Rel. embedding	Scoring function $f_r(h,t)$	Constraints/Regularization
TransE [14]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d$	$-\ \mathbf{h}+\mathbf{r}-\mathbf{t}\ _{1/2}$	$\ \mathbf{h}\ _2 = 1, \ \mathbf{t}\ _2 = 1$
TransH [15]	$\mathbf{h},\mathbf{t} \in \mathbb{R}^d$	$\mathbf{r},\mathbf{w}_r \in \mathbb{R}^d$	$-\ (\mathbf{h} - \mathbf{w}_r^\top \mathbf{h} \mathbf{w}_r) + \mathbf{r} - (\mathbf{t} - \mathbf{w}_r^\top \mathbf{t} \mathbf{w}_r)\ _2^2$	$\ \mathbf{h}\ _{2} \leq 1, \ \mathbf{t}\ _{2} \leq 1$ $\ \mathbf{w}_{r}^{\top}\mathbf{r} /\ \mathbf{r}\ _{2} \leq \epsilon, \ \mathbf{w}_{r}\ _{2} = 1$
TransR [16]	$\mathbf{h},\mathbf{t} \in \mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^k, \mathbf{M}_r \in \mathbb{R}^{k  imes d}$	$-\ \mathbf{M}_r\mathbf{h}+\mathbf{r}-\mathbf{M}_r\mathbf{t}\ _2^2$	$\ \mathbf{h}\ _{2} \le 1, \ \mathbf{t}\ _{2} \le 1, \ \mathbf{r}\ _{2} \le 1$ $\ \mathbf{M}_{r}\mathbf{h}\ _{2} \le 1, \ \mathbf{M}_{r}\mathbf{t}\ _{2} \le 1$
TransD [50]	$egin{aligned} \mathbf{h}, \mathbf{w}_h &\in \mathbb{R}^d \ \mathbf{t}, \mathbf{w}_t &\in \mathbb{R}^d \end{aligned}$	$\mathbf{r},\mathbf{w}_r \in \mathbb{R}^k$	$-\ (\mathbf{w}_r\mathbf{w}_h^\top + \mathbf{I})\mathbf{h} + \mathbf{r} - (\mathbf{w}_r\mathbf{w}_t^\top + \mathbf{I})\mathbf{t}\ _2^2$	$\begin{aligned} &\ \mathbf{h}\ _{2} \leq 1, \ \mathbf{t}\ _{2} \leq 1, \ \mathbf{r}\ _{2} \leq 1 \\ &\ (\mathbf{w}_{r}\mathbf{w}_{h}^{\top} + \mathbf{I})\mathbf{h}\ _{2} \leq 1 \\ &\ (\mathbf{w}_{r}\mathbf{w}_{t}^{\top} + \mathbf{I})\mathbf{t}\ _{2} \leq 1 \end{aligned}$
TranSparse [51]	$\mathbf{h},\mathbf{t} \in \mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^k, \mathbf{M}_r(\theta_r) \in \mathbb{R}^{k \times d}$ $\mathbf{M}_r^1(\theta_r^1), \mathbf{M}_r^2(\theta_r^2) \in \mathbb{R}^{k \times d}$	$-\ \mathbf{M}_r(\mathbf{ heta}_r)\mathbf{h} + \mathbf{r} - \mathbf{M}_r(\mathbf{ heta}_r)\mathbf{t}\ _{1/2}^2 \ -\ \mathbf{M}_r^1(\mathbf{ heta}_r^1)\mathbf{h} + \mathbf{r} - \mathbf{M}_r^2(\mathbf{ heta}_r^2)\mathbf{t}\ _{1/2}^2$	$\begin{aligned} &\ \mathbf{h}\ _{2} \leq 1, \ \mathbf{t}\ _{2} \leq 1, \ \mathbf{r}\ _{2} \leq 1 \\ &\ \mathbf{M}_{r}(\theta_{r})\mathbf{h}\ _{2} \leq 1, \ \mathbf{M}_{r}(\theta_{r})\mathbf{t}\ _{2} \leq 1 \\ &\ \mathbf{M}_{r}^{1}(\theta_{r}^{1})\mathbf{h}\ _{2} \leq 1, \ \mathbf{M}_{r}^{2}(\theta_{r}^{2})\mathbf{t}\ _{2} \leq 1 \end{aligned}$
TransM [52]	$\mathbf{h},\mathbf{t} \in \mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d$	$- heta_r \ \mathbf{h} + \mathbf{r} - \mathbf{t}\ _{1/2}$	$\ \mathbf{h}\ _2 = 1, \ \mathbf{t}\ _2 = 1$
ManifoldE [53]	$\mathbf{h},\mathbf{t} \in \mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d$	$-(\ \mathbf{h}+\mathbf{r}-\mathbf{t}\ _2^2-\theta_r^2)^2$	$\ \mathbf{h}\ _2 \le 1, \ \mathbf{t}\ _2 \le 1, \ \mathbf{r}\ _2 \le 1$
TransF [54]	$\mathbf{h},\mathbf{t} \in \mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d$	$(\mathbf{h} + \mathbf{r})^{\top} \mathbf{t} + (\mathbf{t} - \mathbf{r})^{\top} \mathbf{h}$	$\ \mathbf{h}\ _2 \le 1, \ \mathbf{t}\ _2 \le 1, \ \mathbf{r}\ _2 \le 1$
TransA [55]	$\mathbf{h},\mathbf{t} \in \mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d, \mathbf{M}_r \in \mathbb{R}^{d  imes d}$	$-( \mathbf{h}+\mathbf{r}-\mathbf{t} )^{\top}\mathbf{M}_r( \mathbf{h}+\mathbf{r}-\mathbf{t} )$	$\ \mathbf{h}\ _{2} \le 1, \ \mathbf{t}\ _{2} \le 1, \ \mathbf{r}\ _{2} \le 1$ $\ \mathbf{M}_{r}\ _{F} \le 1, [\mathbf{M}_{r}]_{ij} = [\mathbf{M}_{r}]_{ji} \ge 0$
KG2E [45]	$egin{aligned} \mathbf{h} \! \sim \! \mathcal{N}(oldsymbol{\mu}_h, & oldsymbol{\Sigma}_h) \ \mathbf{t} \! \sim \! \mathcal{N}(oldsymbol{\mu}_t, & oldsymbol{\Sigma}_t) \ oldsymbol{\mu}_h, & oldsymbol{\mu}_t \in \mathbb{R}^d \ oldsymbol{\Sigma}_h, & oldsymbol{\Sigma}_t \in \mathbb{R}^{d  imes d} \end{aligned}$	$egin{aligned} \mathbf{r} &\sim \mathcal{N}(oldsymbol{\mu}_r, oldsymbol{\Sigma}_r) \ oldsymbol{\mu}_r &\in \mathbb{R}^d, oldsymbol{\Sigma}_r \in \mathbb{R}^{d  imes d} \end{aligned}$	$-\operatorname{tr}(\boldsymbol{\Sigma}_{r}^{-1}(\boldsymbol{\Sigma}_{h} + \boldsymbol{\Sigma}_{t})) - \boldsymbol{\mu}^{\top} \boldsymbol{\Sigma}_{r}^{-1} \boldsymbol{\mu} - \ln \frac{\det(\boldsymbol{\Sigma}_{r})}{\det(\boldsymbol{\Sigma}_{h} + \boldsymbol{\Sigma}_{t})}$ $-\boldsymbol{\mu}^{\top} \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} - \ln (\det(\boldsymbol{\Sigma}))$ $\boldsymbol{\mu} = \boldsymbol{\mu}_{h} + \boldsymbol{\mu}_{r} - \boldsymbol{\mu}_{t}$ $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_{h} + \boldsymbol{\Sigma}_{r} + \boldsymbol{\Sigma}_{t}$	$\begin{split} &\ \boldsymbol{\mu}_h\ _2 \leq 1, \ \boldsymbol{\mu}_t\ _2 \leq 1, \ \boldsymbol{\mu}_r\ _2 \leq 1 \\ &c_{min}\mathbf{I} \leq \boldsymbol{\Sigma}_h \leq c_{max}\mathbf{I} \\ &c_{min}\mathbf{I} \leq \boldsymbol{\Sigma}_t \leq c_{max}\mathbf{I} \\ &c_{min}\mathbf{I} \leq \boldsymbol{\Sigma}_r \leq c_{max}\mathbf{I} \end{split}$

#### RESCAL & DisMult



$$f_r(h,t) = h^T M_r t$$



$$f_r(h,t) = h^T diag(M_r)t$$

# A2N: Attending to Neighbors for Knowledge Graph Inference

ACL 2019

#### Abstract

- Attention-based model
- Query-dependent representation
- Calculate by graph neighborhood of entity
- Interpretable

#### Formula

$$\begin{split} &\tilde{n}_i = W_n[\tilde{r}_i; \tilde{e}_i^0] \quad W_n \in \mathbb{R}^{K \times 2K} \\ &a_i = f(s, r, n_i) = (\tilde{s}^0)^T Diag(\tilde{r}) \tilde{n}_i \\ &p_i = \frac{\exp(a_i)}{\sum_{j \leq |N_s|} \exp(a_j)} \\ &\hat{s} = \sum_{i \leq |N_s|} p_i \tilde{n}_i \\ &\tilde{s} = W_s[\hat{s}; \tilde{s}^0] \quad W_s \in \mathbb{R}^{K \times 2K} \end{split}$$
 DistMult score:

[1]A2N: Attending to Neighbors for Knowledge Graph Inference. ACL2019

#### Interpretable

```
(Burt_Young, nationality, ?)
(Bill_Payne, profession, ?)
Prediction: Musician
                                                              Prediction: US
Top Neighbors:
                                                              Top Neighbors:
(recording_contribution, Synthesizer) Prob: 0.0911
                                                              (place of birth, Queens) Prob: 0.2714
(Inverse: Instrumentalist, Keyboards) Prob: 0.0906
                                                              (places lived, Queens) Prob: 0.2039
(track_contribution, Synthesizer) Prob: 0.0878
                                                              (Inverse: ethnicity/people, Italian_American) Prob: 0.1860
(Inverse: Instrumentalist, Hammond_organ) Prob: 0.0823
                                                              (performance/film, Transamerica) Prob: 0.0445
(track_contribution, Accordion) Prob: 0.0758
                                                              (gender, Male) Prob: 0.0372
(Fantastic Four: Rise of the Silver Surfer, genre, ?)
                                                             (Armstrong County, Pennsylvania, time zones, ?)
Prediction: Fantasy
                                                             Prediction: Eastern Time Zone
                                                             Top Neighbors:
Top Neighbors:
(genre, Superhero_film) Prob: 0.0614
                                                             (Inverse: location/contains, Pittsburgh metropolitan area) Prob: 0.3219
(genre, Superhero) Prob: 0.0490
                                                             (Inverse: location/contains, Pennsylvania) Prob: 0.2994
                                                             (Inverse: location/country/second level divisions, US) Prob: 0.1092
(genre, Science fiction film) Prob: 0.0460
(genre, Action_film) Prob: 0.0443
                                                             (estimated number of mortages/source, US Department of HUD) Prob: 0.0692
(language, Arabic language) Prob: 0.0395
                                                             (currency, US dollar) Prob: 0.0309
```

#### Experiments

	FB15k-237				WN18RR					
	MRR	Hits@10	Hits@3	Hits@1	MRR	Hits@10	Hits@3	Hits@1		
DistMult	0.370	0.568	0.417	0.275	0.43	0.48	0.44	0.41		
ComplEx	0.394	0.572	0.434	0.303	0.42	0.48	0.43	0.38		
ConvE	0.410	0.600	0.457	0.313	0.44	0.52	0.45	0.40		
MINERVA	0.293	0.456	0.329	0.217	0.45	0.51	0.46	0.41		
A2N	0.422	0.608	0.464	0.328	0.49	0.55	0.50	0.45		

Table 1: Results for target-only prediction of various models. A2N performs significantly better.

		FB15	5k-237		WN18RR					
	MRR	Hits@10	Hits@3	Hits@1	MRR	Hits@10	Hits@3	Hits@1		
DistMult	0.278	0.444	0.304	0.196	0.43	0.49	0.44	0.39		
ComplEx	0.247	0.428	0.275	0.158	0.44	0.51	0.46	0.41		
R-GCN	0.249	0.417	0.264	0.151	_	_	_	_		
ConvE	0.325	0.501	0.356	0.237	0.43	0.52	0.44	0.40		
A2N	0.317	0.486	0.348	0.232	0.45	0.51	0.46	0.42		

Table 2: Results for both source and target prediction of various models. A2N performs better or competitively to most state-of-the-art models, specially on top prediction (Hits@1).

[1]A2N: Attending to Neighbors for Knowledge Graph Inference. ACL2019

A Capsule Network-based Embedding Model for Knowledge Graph Completion and Search Personalization

NAACL2019

#### Capsule Network

- Capsule like neuron
- Capsule output a vector rather than value
- Neuron only detects a special pattern
- Judge by the length of the output vector

#### CapsE Model

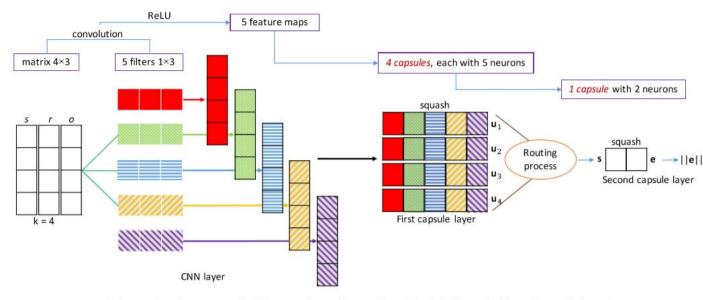


Figure 1: An example illustration of our CapsE with k = 4, N = 5, and d = 2.

squash 
$$(\mathbf{s}) = \frac{\|\mathbf{s}\|^2}{1+\|\mathbf{s}\|^2} \frac{\mathbf{s}}{\|\mathbf{s}\|}$$

# TuckER: Tensor Factorization for Knowledge Graph Completion

ICML2019

#### Tucker decomposition

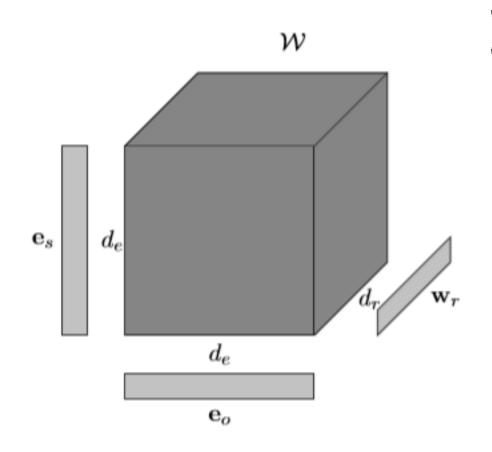
$$\mathcal{X} \in \mathbb{R}^{I \times J \times K}$$

$$\mathcal{X} \approx \mathcal{Z} \times_1 \mathbf{A} \times_2 \mathbf{B} \times_3 \mathbf{C}$$

$$\mathcal{Z} \in \mathbb{R}^{P \times Q \times R}$$
 Core tensor

$$\mathbf{A} \in \mathbb{R}^{I \times P} \quad \mathbf{B} \in \mathbb{R}^{J \times Q} \quad \mathbf{C} \in \mathbb{R}^{K \times R}$$

#### TuckER Architecture



$$\mathbf{E} = \mathbf{A} = \mathbf{C} \in \mathbb{R}^{n_e \times d_e}$$

$$\mathbf{R} = \mathbf{B} \in \mathbb{R}^{n_r \times d_r}$$

 $\mathbf{e}_s, \mathbf{e}_o \in \mathbb{R}^{d_e}$  Rows of entity embedding matrix **E** 

 $\mathbf{w}_r \in \mathbb{R}^{d_r}$  Rows of relation embedding matrix **R** 

$$\mathcal{W} \in \mathbb{R}^{d_e imes d_r imes d_e}$$
 Core tensor

$$\phi(e_s, r, e_o) = \mathcal{W} \times_1 \mathbf{e}_s \times_2 \mathbf{w}_r \times_3 \mathbf{e}_o$$

[3] TuckER: Tensor Factorization for Knowledge Graph Completion

#### Advantages

- Fully expressive
- Entity and relation embedding dimension determines number of parameters
- RESCAL, DistMult can be viewed as a special case of TuckER
  - RESCAL:

$$I = K = n_e$$
  $P = R = d_e$   $Q = J = n_r$   $\mathbf{B} = \mathbf{I}_J$ 

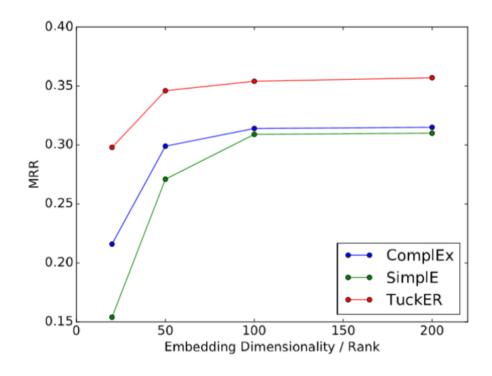
• DistMult:

$$P=Q=R=d_e$$
  $z_{pqr}=\begin{cases} 1, p=q=r \\ 0, p \neq q \neq r \end{cases}$ 

### Experiments

		WN18				FB15k			
	Linear	MRR	Hits@10	Hits@3	Hits@1	MRR	Hits@10	Hits@3	Hits@1
TransE (Bordes et al., 2013)	no	_	.892	_	_	_	.471	_	_
DistMult (Yang et al., 2015)	yes	.822	.936	.914	.728	.654	.824	.733	.546
ComplEx (Trouillon et al., 2016)	yes	.941	.947	.936	.936	.692	.840	.759	.599
ANALOGY (Liu et al., 2017)	yes	.942	.947	.944	.939	.725	.854	.785	.646
Neural LP (Yang et al., 2017)	no	.940	.945	_	_	.760	.837	_	_
R-GCN (Schlichtkrull et al., 2018)	no	.819	.964	.929	.697	.696	.842	.760	.601
TorusE (Ebisu and Ichise, 2018)	no	.947	.954	.950	.943	.733	.832	.771	.674
ConvE (Dettmers et al., 2018)	no	.943	.956	.946	.935	.657	.831	.723	.558
HypER (Balažević et al., 2019)	no	.951	958	.955	.947	.790	.885	.829	.734
SimplE (Kazemi and Poole, 2018)	yes	.942	.947	.944	.939	.727	.838	.773	.660
TuckER (ours)	yes	.953	.958	.955	.949	.795	.892	.833	.741

Table 4: Link prediction results on WN18 and FB15k.



## Anytime Bottom-Up Rule Learning for Knowledge Graph Completion

IJCAI19

### AnyBURL

#### Algorithm 1 Anytime Bottom-up Rule Learning

```
AnyBURL(\mathbb{G}, s, sat, Q, ts)
 1: n = 2
 2: R = \emptyset
 3: loop
      R_s = \emptyset
      start = currentTime()
      repeat
 6:
 7:
         p = samplePath(\mathbb{G}, n)
         R_p = generateRules(p)
         for r \in R_p do
 9:
10:
            score(r,s)
11:
            if Q(r) then
12:
              R_s = R_s \cup \{r\}
            end if
13:
         end for
14:
      until\ currentTime() > start + ts
15:
      R'_s = R_s \cap R
16:
      if |R'_s|/|R_s| > sat then
18:
         n = n + 1
      end if
19:
      R = R_s \cup R
21: end loop
22: return R
```

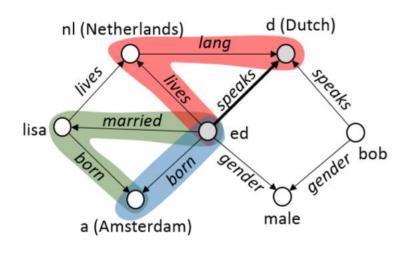


Figure 1: A small knowledge graph  $\mathbb{G}$  used for sampling paths. We marked the body of Rule 1 (blue), Rule 2 (green), and Rule 3 (red).

$$speaks(ed, d) \leftarrow born(ed, a)$$
 (1)

$$speaks(ed, d) \leftarrow married(ed, lisa), born(lisa, a)$$
 (2)

$$speaks(ed, d) \leftarrow lives(ed, nl), lang(nl, d)$$
 (3)

#### Advantages

- Candidate ranking can be explained
- Some rules can be reused
- Not require to learn dataset specific hyper parameters

#### Application

- Fact judgement
- Entity classification
- Automatically check the quality of relation extraction