Lecture 4: OLS Computation Intro To Scraping

Big Data and Machine Learning for Applied Economics Econ 4676

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Agenda

Class' Repos

- Syllabus Repo
- Lectures Repo
- Problem Set Repo
- Problem Set Template Repo
- e-TAs

Recap

- Least Square Estimator
- Quick Review of Statistical Properties
- Numerical Properties
- FWL
  - Fixed Effects
  - Leverage
  - Goodness of Fit
  - Updating

Recap

- The goal here is to solve something which looks like

$$f^{\star} = \underset{f \in \mathcal{F}}{\operatorname{argmin}} \left\{ E[L(Y, f(\mathbf{X}))] \right\} \tag{1}$$

- for some loss function L, and for some set of predictors $\mathcal{F}$.
- This is an optimization problem.

QR decomposition

- Linear Model: Min Risk $\iff$ Min SSR

$$\hat{\beta} = (X'X)^{-1}X'y \tag{2}$$

- Involves inverting a $k \times k$ matrix $X'X$
- requires allocating $O(nk + k^2)$ if n is "big" we cannot store this in memory

Solving directly

```
beta<-solve(t(X)%*%X)%*%t(X)%*%y
```

may not be the smartest move

QR decomposition

Most software use a QR decomposition

Theorem If $A \in \mathbb{R}^{n \times k}$ then there exists an orthogonal $Q \in \mathbb{R}^{n \times n}$ and an upper triangular $R \in \mathbb{R}^{n \times k}$ so that $A = QR$

- Orthogonal Matrices:
  - Def: $Q'Q = QQ' = I$ and $Q' = Q^{-1}$
  - Prop: product of orthogonal is orthogonal, e.g $A'A = I$ and $B'B = I$ then $(AB)'(AB) = B'(A'A)B = B'B = I$
- **(Thin QR)** If $A \in \mathbb{R}^{n \times k}$ has full column rank then $A = Q_1 R_1$ the QR factorization is unique, where $Q_1 \in \mathbb{R}^{n \times k}$ and R is upper triangular with positive diagonal entries

Can use it these to get $\hat{\beta}$

$$(X'X)\hat{\beta} = X'y \tag{3}$$

$$(R'Q'QR)\hat{\beta} = R'Q'y \tag{4}$$

$$(R'R)\hat{\beta} = R'Q'y \tag{5}$$

$$R\hat{\beta} = Q'y \tag{6}$$

Solve by back substitution

QR decomposition

$$X = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \; y = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \tag{7}$$

1. QR factorization X=QR

$$Q = \begin{bmatrix} -0.57 & -0.41 \\ -0.57 & -0.41 \\ -0.57 & 0.82 \end{bmatrix} \; R = \begin{bmatrix} -1.73 & -4.04 \\ 0 & 0.81 \end{bmatrix} \tag{8}$$

2. Calculate $Q'y = [-4.04, -0.41]'$
3. Solve

$$\begin{bmatrix} -1.73 & -4.04 \\ 0 & 0.81 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} -4.04 \\ -0.41 \end{bmatrix} \tag{9}$$

Solution is $(3.5, -0.5)$

QR decomposition

This is actually what `R` does under the hood

obj	list [12] (S3: lm)	List of length 12
coefficients	double [2]	-1.71e+08 3.01e+08
residuals	double [207607]	1.17e+09 -2.38e+08 -5.21e+08 -1.96e+08 -5.12e+07 -1.91e+08 ...
effects	double [207607]	-3.15e+11 2.10e+11 -5.24e+08 -1.98e+08 -5.34e+07 -1.93e+08 ...
rank	integer [1]	2
fitted.values	double [207607]	4.31e+08 4.31e+08 7.32e+08 4.31e+08 4.31e+08 4.31e+08 ...
assign	integer [2]	0 1
qr	list [5] (S3: qr)	List of length 5
df.residual	integer [1]	207605
xlevels	list [0]	List of length 0
call	language	lm(formula = price ~ bathrooms, data = dta0)
terms	formula	price ~ bathrooms
model	list [207607 x 2] (S3: data.fra	A data.frame with 207607 rows and 2 columns

Note that `R's lm` also returns many objects that have the same size as X and Y

Gradient-Based Optimization

- Suppose we have a function $y = f(x)$, where both x and y are real numbers.
- The derivative $f'(x)$ gives the slope of $f(x)$ at the point x
- It specifies how to scale a small change in the input to obtain the corresponding change in the output

$$f(x+\epsilon) \approx f(x) + \epsilon f'(x) \tag{10}$$

- The derivative is therefore useful for minimizing a function because it tells us how to change x in order to make a small improvement in y
- We can thus reduce $f(x)$ by moving x in small steps with the opposite sign of the derivative.
- This technique is called gradient descent (Cauchy, 1847).

Gradient-Based Optimization

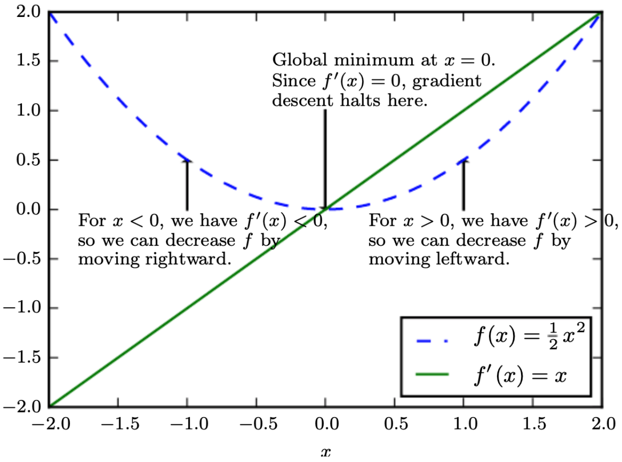


Figure 4.1: Gradient descent. An illustration of how the gradient descent algorithm uses the derivatives of a function to follow the function downhill to a minimum.

Gradient-Based Optimization

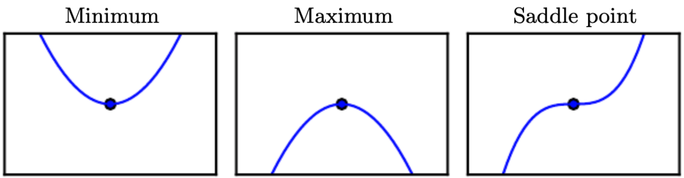


Figure 4.2: Types of critical points. Examples of the three types of critical points in one dimension. A critical point is a point with zero slope. Such a point can either be a local minimum, which is lower than the neighboring points; a local maximum, which is higher than the neighboring points; or a saddle point, which has neighbors that are both higher and lower than the point itself.

Gradient-Based Optimization

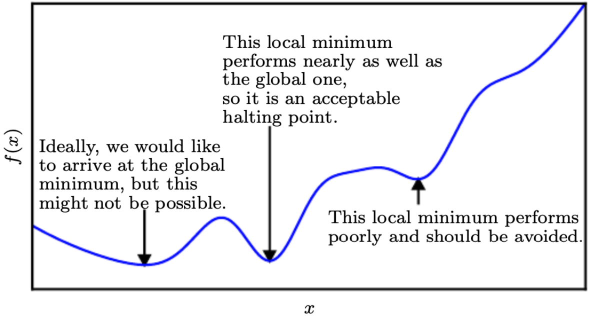


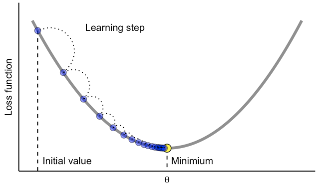
Figure 4.3: Approximate minimization. Optimization algorithms may fail to find a global minimum when there are multiple local minima or plateaus present. In the context of deep learning, we generally accept such solutions even though they are not truly minimal, so long as they correspond to significantly low values of the cost function.

Gradient-Based Optimization

- **Steepest descent**, or **gradient descent** proposes a new point

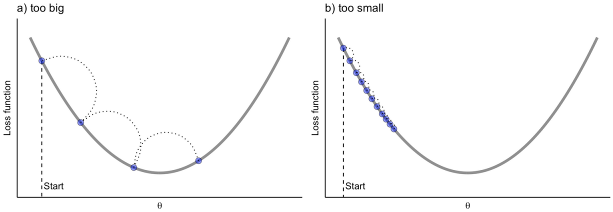
$$\theta' = \theta - \epsilon \nabla_\theta f(\theta) \tag{11}$$

- where ϵ is the learning rate, a positive scalar determining the size of the step.



Source: Boehmke, B., & Greenwell, B. (2019)

Gradient-Based Optimization

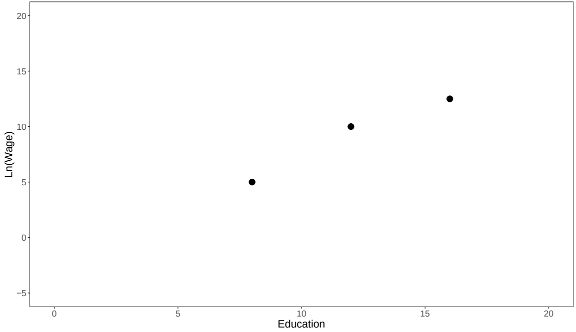


Source: Boehmke, B., & Greenwell, B. (2019)

- We can choose ϵ in several different ways:
  - A popular approach is to set ϵ to a small constant.
  - Sometimes, we can solve for the step size that makes the directional derivative vanish.
  - Another approach is to evaluate $f(\theta - \epsilon\nabla_\theta f(\theta))$ for several values of ϵ and choose the one that results in the smallest objective function value. This is called a line search.
- Steepest descent converges when every element of the gradient is zero (or, in practice, very close to zero).
- In some cases, we may be able to avoid running this iterative algorithm and just jump directly to the critical point by solving the equation $\nabla_\theta f(\theta) = 0$ for x.

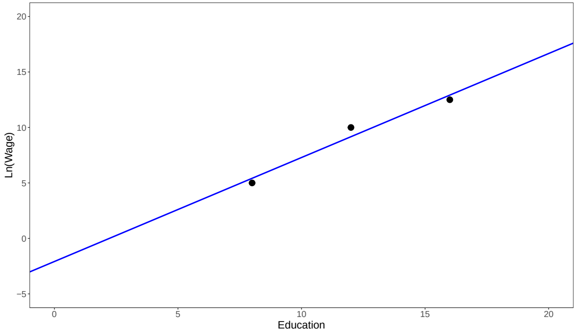
Gradient-Based Optimization

log(wage)	Education (years)
5	8
10	12
12.5	16



Gradient-Based Optimization

log(wage)	Education (years)
5	8
10	12
12.5	16



$$\hat{\beta} = (X'X)^{-1}X'y$$

```
beta<-solve(t(X)%*%X)%*%t(X)%*%y

lm(y~x,data)
```

$$y = -2.0833 + 0.9375 \times Educ$$

Gradient-Based Optimization

The Loss Function

$$SSR = f(\theta) = \sum_{i=1}^{n}(y_i - (\alpha + \beta x_i))^2$$

The Gradient

$$\nabla f_\theta(\theta) = \begin{pmatrix} \frac{\partial f}{\partial \alpha} \\ \frac{\partial f}{\partial \beta} \end{pmatrix} = \begin{pmatrix} -2\sum_{i=1}^{n}(y_i - \alpha - \beta x_i) \\ -2\sum_{i=1}^{n} x_i(y_i - \alpha - \beta x_i) \end{pmatrix}$$

Updating

$$\alpha' = \alpha - \epsilon \frac{\partial f}{\partial \alpha}$$

$$\beta' = \beta - \epsilon \frac{\partial f}{\partial \beta}$$

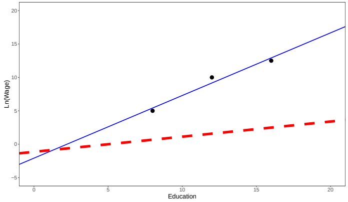
Gradient-Based Optimization

First Iteration

log(wage)	Education (years)
5	8
10	12
12.5	16

Start with an initial guess: $\alpha = -1; \beta = 2$, and a learning rate ($\epsilon = 0.005$). Then we have

$$
\begin{aligned}
\alpha' &= (-1) - 0.005\left(-2\left((5 - (-1) - 2 \times 8) + (10 - (-1) - 2 \times 12) + (12.5 - (-1) - 2 \times 16\right)\right) \\
\beta' &= 2 + 0.005\left(-2\left(8(5 - (-1) - 2 \times 8) + 12(10 - (-1) - 2 \times 12) + 16(12.5 - (-1) - 2 \times 16\right)\right) \\
\alpha' &= -1.1384 \\
\beta' &= 0.2266
\end{aligned}
$$



Gradient-Based Optimization

Second Iteration

log(wage)	Education (years)
5	8
10	12
12.5	16

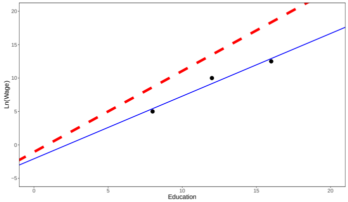
Start with an initial guess: $\alpha = -1; \beta = 2$, and a learning rate ($\epsilon = 0.005$). Then we have

$$\alpha^2 = (-1.1384) - 0.005\,(-2\,((5 - (-1.1384) - (0.2266) \times 8) + (10 - (-1.1384) - (0.2266) \times 12) + (12.5 - (-1.1384) - (0.2266) \times 16))$$

$$\beta^2 = (0.2266) + 0.005\,(-2\,(8(5 - (-1.1384) - (0.2266) \times 8) + 12(10 - (-1.1384) - (0.2266) \times 12) + 16(12.5 - (-1.1384) - (0.2266) \times 16))$$

$$\alpha^2 = -1.0624$$

$$\beta^2 = 1.212689$$



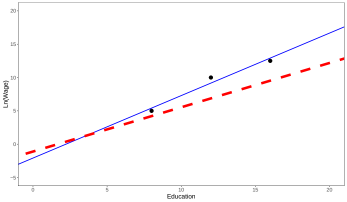
Gradient-Based Optimization

Third Iteration

log(wage)	Education (years)
5	8
10	12
12.5	16

$$\alpha^3 = -1.0624$$

$$\beta^3 = 1.212689$$



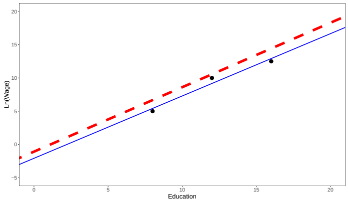
Gradient-Based Optimization

Fourth Iteration

log(wage)	Education (years)
5	8
10	12
12.5	16

$$\alpha^4 = -1.082738$$

$$\beta^4 = 0.9693922$$



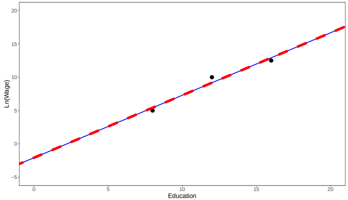
Gradient-Based Optimization

7211 Iteration

log(wage)	Education (years)
5	8
10	12
12.5	16

$$\alpha^{7211} = -2.076246$$
$$\beta^{7211} = 0.9369499$$

$$y^{ols} = -2.0833 + 0.9375 \times Educ$$



Stochastic Gradient-Based Optimization

- Computing the gradient can be very time consuming.
- However, often it is possible to find a “cheap” approximation of the gradient.

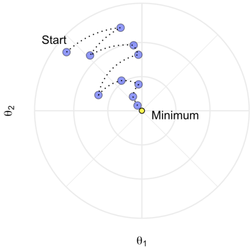
$$SSR = f(\theta) = \sum_{i=1}^{n}(y_i - (\alpha + \beta x_i))^2$$

$$\theta_{j+1} = \theta_j - \epsilon_j \sum_{i=1}^{n}(\nabla_\theta f_i(\theta_j))$$

- Approximating the gradient is still useful as long as it points in roughly the same direction as the true gradient.
- We randomly typically without replacement) choose a subset of $\nabla_\theta f_i(\theta)$ for mini-batch gradient descent (mini-batch can be one)

Stochastic Gradient-Based Optimization

- The key insight we only need taht $E(\nabla_\theta f_i(\theta_j)) = \nabla_\theta f(\theta)$
- The word stochastic here refers to the fact that we acknowledge that we do not know the gradient precisely, but instead only know a noisy approximation to it.

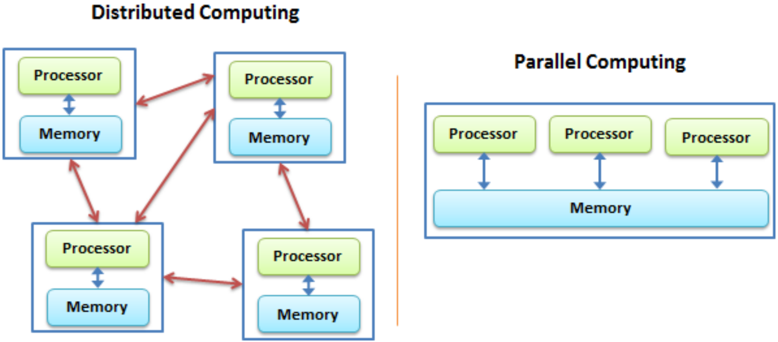


Source: Boehmke, B., & Greenwell, B. (2019)

- This makes the algorithm faster but
- Adds some random nature in descending the loss function's gradient.
- Although this randomness does not allow the algorithm to find the absolute global minimum, it can actually help the algorithm jump out of local minima and off plateaus to get sufficiently near the global minimum.

Parallel vs Distributed

- An algorithm is parallel if it does many computations at once.
  - It needs to see all of the data
- It is distributed if you can work with subsets of data
  - `Stata-mp` is parallel. (license charges by core)
  - `R` and `Python` can be parallel **and** distributed

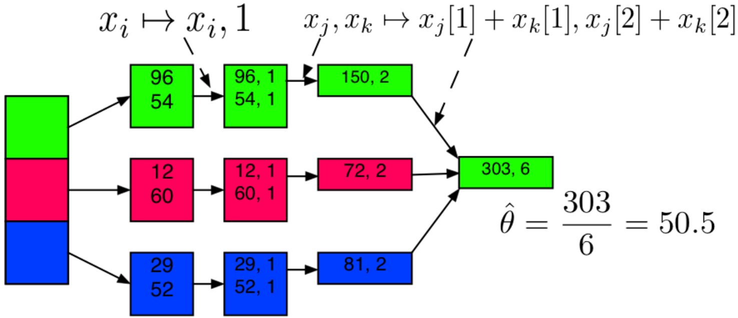


https://tinyurl.com/y3nzvkwh

Map Reduce

- Original Paper *MapReduce: Simplified data processing on large clusters (2004) Dean and Ghemawat*
- It is of the most popular frameworks
- Basic Idea:
  1. You need to be able to specify a key that indexes subgroups of data that can be analyzed in isolation.
  2. Map: Calculate and sort relevant statistics by key
  3. Partition and pipe the outcome of map so that outcomes with the same key end up on the same machine
  4. Reduce: Apply a summarization operation within the subgroup defined by each key.

Example: Mean by groups



https://datascienceguide.github.io/map-reduce

QR decomposition for block matrices

Idea on how to distribute OLS (Constantine & Gleich, 2011)

$$X_{8n\times k} = \begin{bmatrix} X^1_{2n\times k} \\ X^2_{2n\times k} \\ X^3_{2n\times k} \\ X^4_{2n\times k} \end{bmatrix} \tag{12}$$

QR to each block

$$X_{8n\times k} = \underbrace{\begin{bmatrix} Q^1_{2n\times k} & & & \\ & Q^2_{2n\times k} & & \\ & & Q^3_{2n\times k} & \\ & & & Q^4_{2n\times k} \end{bmatrix}}_{8n\times 4k} \underbrace{\begin{bmatrix} R^1_{k\times k} \\ R^2_{k\times k} \\ R^3_{k\times k} \\ R^4_{k\times k} \end{bmatrix}}_{4k\times k} \tag{13}$$

$$X_{8n\times k} = \underbrace{\underbrace{\begin{bmatrix} Q^1_{2n\times k} & & & \\ & Q^2_{2n\times k} & & \\ & & Q^3_{2n\times k} & \\ & & & Q^4_{2n\times k} \end{bmatrix}}_{8n\times 4k} \underbrace{Q_2}_{4k\times k}}_{Q} \underbrace{R_2}_{k\times k} \tag{14}$$

Spark

- The tools facilitating distributed computing are rapidly improving.
- One prominent system is Spark, that is quickly replacing MapReduce
- Seamlessly integration with `R` and `Python` and has it's own `MLlib`
  - E.g. Spark uses distributed version of stochastic gradient descent to compute OLS
- One of the key differences with MapReduce is how they load data
  - MapReduce has to read from and write to a disk
  - Spark loads it in-memory (can get 100x faster)

Motiviation Web Scraping

Are Online and Offline Prices Similar? Evidence from Large Multi-Channel Retailers†

By Alberto Cavallo*

Online prices are increasingly used for measurement and research applications, yet little is known about their relation to prices collected offline, where most retail transactions take place. I conduct the first large-scale comparison of prices simultaneously collected from the websites and physical stores of 56 large multi-channel retailers in 10 countries. I find that price levels are identical about 72 percent of the time. Price changes are not synchronized but have similar frequencies and average sizes. These results have implications for national statistical offices, researchers using online data, and anyone interested in the effect of the Internet on retail prices. (*JEL* D22, L11, L81, O14)

Motiviation Webscraping

Decriminalizing Indoor Prostitution: Implications for Sexual Violence and Public Health

SCOTT CUNNINGHAM

Baylor University

and

MANISHA SHAH

University of California, Los Angeles & NBER

First version received November 2015; *Editorial decision August* 2017; *Accepted November* 2017 (*Eds.*)

Most governments in the world, including the U.S., prohibit sex work. Given these types of laws rarely change and are fairly uniform across regions, our knowledge about the impact of decriminalizing sex work is largely conjectural. We exploit the fact that a Rhode Island District Court judge unexpectedly decriminalized indoor sex work to provide causal estimates of the impact of decriminalization on the composition of the sex market, reported rape offences, and sexually transmitted infections. While decriminalization increases the size of the indoor sex market, reported rape offences fall by 30% and female gonorrhoea incidence declines by over 40%.

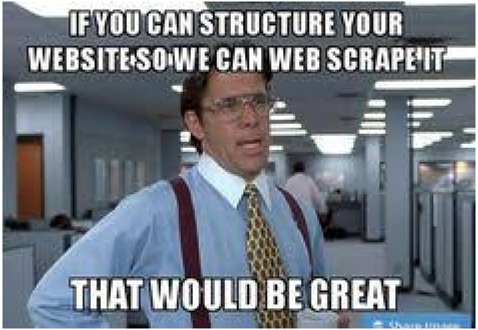
Key words: Regulation, Sex work, Public health, Crime.

JEL Codes: I18, J4, K42

Motiviation Webscraping

We also harvest data from an online review site called The Erotic Review. TER, a reputation website similar to Yelp.com, is one of the largest sex websites in the country and only covers indoor sex workers. Customers use it primarily to provide feedback on transactions with sex workers in a particular area. We collect approximately 90,000 records from TER database from 1999 to 2007 from all over the country. We identify Rhode Island-based sex workers by using phone number area codes. We primarily use the data to focus on the types of services provided, transaction prices, and provider race.

Webscraping basics



Webscraping basics

- How to get data, or ”content”, off the web and onto our computers.
- If you see it in your browser it exists somewhere
- To be “successful” one must have a working knowledge on:
  - how web pages display content (Hyper Text Markup Language or HTML)
  - where is the content “located”
    1. Server side
    2. Client side
  - The good news is that both server-side and client-side websites allow for web scraping

Caveat: ethical and legal limitations

- Just because you *can* scrape it, doesn't mean you *should*.
- Check `The Robots Exclusion Protocol` of a website, adding ``` ``/robots.txt'' ``` to the website's URL
  1. User-agent: the type of robots to which the section applies
  2. Disallow: directories/prefixes of the website not allowed to robots
  3. Allow: sections of the website allowed to robots
- `robots.txt` is de facto standard (see `http://www.robotstxt.org`)
- Also always check the terms and conditions and what they say about scraping
- Remember the immortal words of uncle Ben: "with great power comes great responsibility"

Review & Next Steps

- Computation
- QR decomposition
- Gradient Descent
- MapReduce and Spark
- Intro to `Web Scraping`
- Message: web scraping involves as much art as it does science

- **Next Class:** Problem Set Presentations

- Questions? Questions about software?

Further Readings

- Boehmke, B., & Greenwell, B. (2019). Hands-on machine learning with R. Chapman and Hall/CRC.
- Constantine, P. G., & Gleich, D. F. (2011, June). Tall and skinny QR factorizations in MapReduce architectures. In Proceedings of the second international workshop on MapReduce and its applications (pp. 43-50).
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- Goodfellow, I., Bengio, Y., Courville, A., & Bengio, Y. (2016). Deep learning (Vol. 1, No. 2). Cambridge: MIT press.
- Van Loan, C. F., Golub, G. H. (2012). Matrix Computations. United States: Johns Hopkins University Press.
- `Webscraping` tutorial from Prof. Grant McDermott.
- Web Scrapping slides from Fernandez Villaverde J., Guerrón P. & Zarruk Valencia, D.