OPTIMUMG OPTIMUM TIRE - FORMULA STUDENT

Wheel configuration choice for the vehicle Invictus (2020 season)

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1 Introduction

In this document a dynamic single track model is implemented to study possible wheel configurations for the Formula Student 2020 competition. This year budget limitation had to be applied in order to counter fit for the 2019 season. Rim choice was therefore limited to already available rims from the past seasons.

A mathematical model that allowed a comparison between different tire models and rims in a dynamic fashion was needed. Experience from past competitions proved that the static hypothesis is unrealistic.

A first section deals with possible wheel configuration generations. The section 3 provides a description of the concept selection method by analyzing the chosen dynamic model and tire models. A final section presents simulation results.

2 Concept generation

The table below presents different tires and rims easily available in the Formula Student competition. Each tyre and rim was considered as a thick-wall cylindrical tube so that overall mass, rolling inertia and yaw inertia could be estimated as in fig.1. The same variables have been added in order to calculate the properties of each possible configuration. A digit code was given to each tire whereas a letter was chosen for each rim. A given configuration was therefore identified by a letter-digit code.

Tx	Ty	p
mm	mm	mm
1650	1250	1035

usd2€	0,9
in2mm	25,4
lbs2g	453,592

tyre thickness [mm]	5

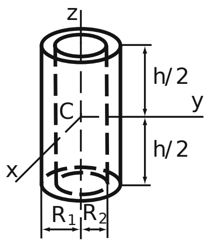
tyre		model	outer radiu	width	mass	Iy	Iz	price	TTC round	
code		str	mm	mm	g	kg m^2	kg m^2	€ HT sans sped.	cornering	drive/brake
1		Hoosier 20,5/7 - 13	260	178	4990	0,33	0,18	205	5	5
2		Hoosier 16/7.5 -10	203	191	3402	0,14	0,08	180	8	x
3		Hoosier 18/7.5 - 10	229	191	4536	0,23	0,13	180	5, 6	5, 6
4		C19 205/470 r13	235	205	3924	0,21	0,12	183	5, 8	5, 8
5		Avon 16/7-10	203	178	3266	0,13	0,07	104	7	x

rim thickness [mm]	5

rim		model	radius	width	mass	Iy	Iz	price	notes
code		str	mm	mm	g	kg m^2	kg m^2	€ HT sans sped.	
a		OZ 13 Mg	165	178	2450	0,06	0,04	250	not center lo
b		OZ 10 Mg	127	178	1660	0,03	0,02	250	center lock
c		keizer 10i Al	127	178	2041	0,03	0,02	248	$265 8 piece
d		keizer 13 Al	165	178	2835	0,07	0,04	338	
e		Oz 13 Al	165	178	3400	0,09	0,05	260	sa coute au n

config	outer radius	mass	Iyy	Izz	Iz_G	price
rim tyre	m	g	kg m^2	kg m^2	kg m^2	€ HT
a1*	0,260	7440	0,40	0,22	8,19	205
a4 *	0,235	6374	0,28	0,16	6,99	183
d1	0,260	7824	0,41	0,22	8,61	543
d4	0,165	6759	0,29	0,16	7,40	521
b2	0,203	5062	0,16	0,10	5,52	430
b3	0,229	6196	0,26	0,15	6,78	430
b5	0,203	4926	0,16	0,09	5,37	354
c2 *	0,203	5443	0,17	0,10	5,93	180
c3 *	0,229	6577	0,26	0,15	7,20	180
c5	0,203	5307	0,16	0,10	5,78	351
c5 *	0,203	5307	0,16	0,10	5,78	104
e1	0,260	8390	0,42	0,23	9,22	465
e4	0,235	7324	0,30	0,17	8,02	443

* use old rims — configuration to analyze



$$m = \pi\rho h(R_1^2 - R_2^2)$$

$$I_{xx} = I_{yy} = \frac{1}{12}m(3R_1^2 + 3R_2^2 + h^2)$$

$$I_{zz} = \frac{1}{2}m(R_1^2 + R_2^2)$$

Figure 1: Inertia calculation of a thick-wall cylindrical tube from [1]

3 Concept selection: a dynamic simulation

3.1 A nonlinear single track model

A nonlinear single track dynamic model was implemented in MATLAB Simulink following the structure proposed in section 10.2 of [2] whose schema is presented in fig.2. This model presented no suspension system and an infinitely rigid chassis. To allow the wheel configuration choice, rim (and tire) mass and inertia were included as a parameter.

A simplified magic formula tire model was used as suggested in [2]. Coefficients equations to link between this simplified model and the magic formula 5.2 structure were derived from [3]:

pure longitudinal (slip angle = 0)

$$\begin{aligned}
F_{x,0} &= D_x \sin[C_x \arctan(B_x \kappa)] \\
C_x &= p_{Cx1} \\
D_x &= \mu_x F_z \\
\mu_x &= (p_{Dx1} + p_{Dx2}\, df_z) \\
df_z &= \frac{F_z - F_{z,0}}{F_{z,0}} \\
K_{x\kappa} &= F_z (p_{Kx1} + p_{Kx2}\, df_z) \exp(p_{Kx3} df_z) \\
B_x &= \frac{K_{x\kappa}}{C_x D_x + \varepsilon}
\end{aligned}$$

pure lateral (slip ratio = 0)

$$\begin{aligned}
F_{y,0} &= D_y \sin[C_y \arctan(B_y \alpha)] \\
C_y &= p_{Cy1} \\
D_y &= \mu_y F_z \\
\mu_y &= p_{Dy1} + p_{Dy2}\, df_z \\
K_y &= p_{Ky1} F_{z,0} \\
B_y &= \frac{K_y}{C_y D_y + \varepsilon}
\end{aligned}$$

The model was given the steering angle δ and the rear axle thrusting torque $M_{A,h}$ as inputs. A summary of all parameters is presented in table 1. Output variables were chosen with respect to the test cases defined in sec.3.3 among the following variables:

s_v, α_v, $s_{v,a}$ front axle slip ratio, slip angle and normalized slip

s_h, α_h, $s_{h,a}$ rear axle slip ratio, slip angle and normalized slip

ψ_V, $\dot{\psi}_V$ overall vehicle angle (and its time derivative) with respect to a fixed reference system

$r\dot{\rho}_v$, $r\dot{\rho}_h$ front and rear axle wheel speed at contact point

$\dot{x}_V$, $\dot{y}_V$ longitudinal an lateral vehicle speed with respect to a fixed reference system

An overall view of the MATLAB Simulink schema is presented below with a focus on the tire block implementation.

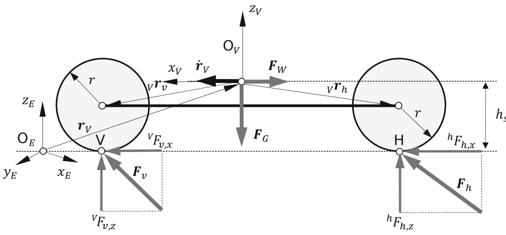


Fig. 10.8 Nonlinear single track model—side view

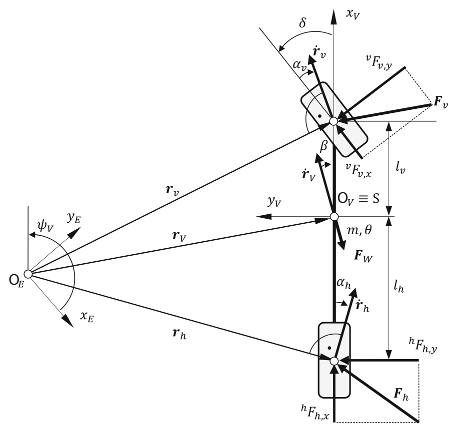
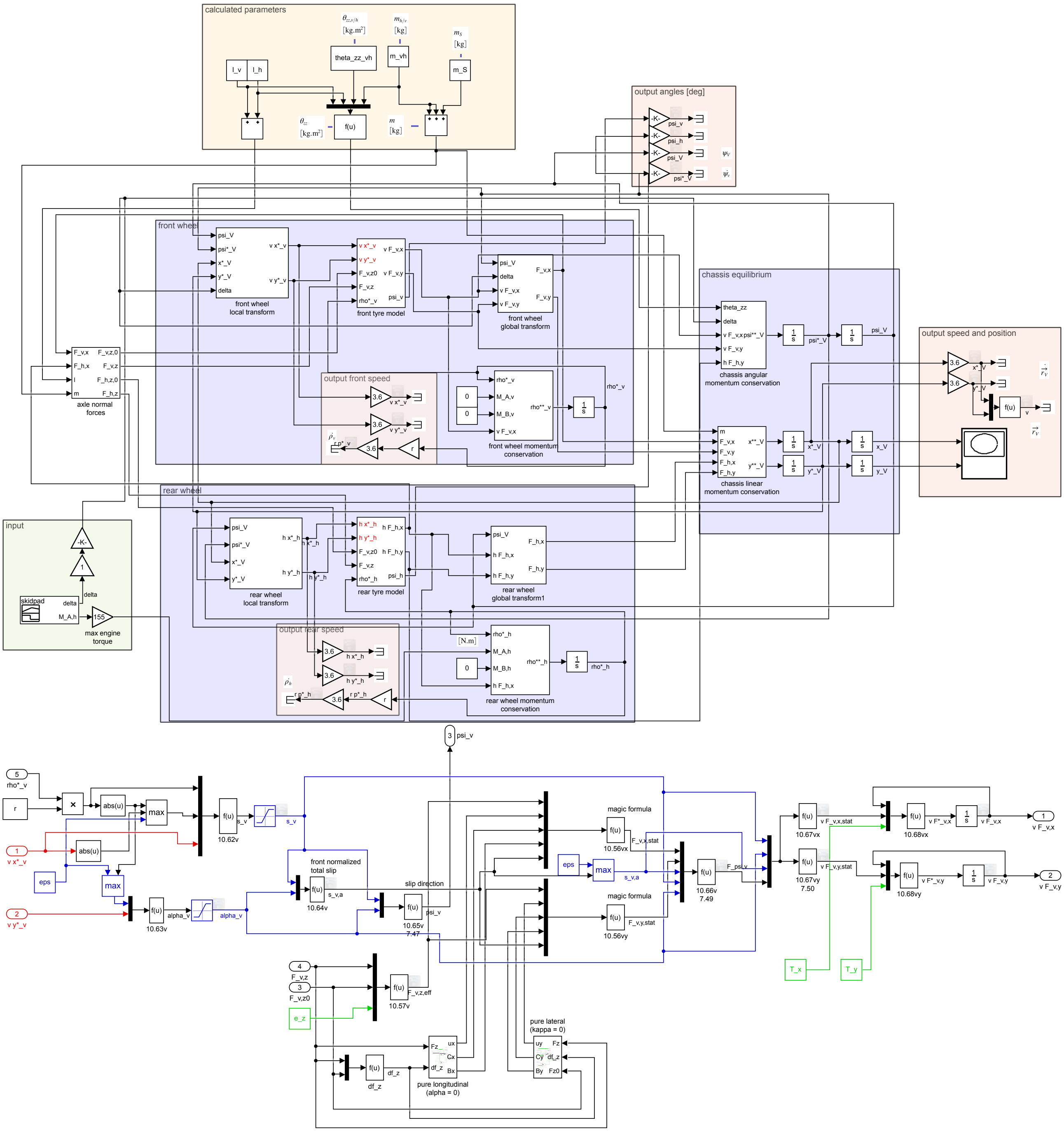


Fig. 10.9 Nonlinear single track model–top view

Figure 2: nonlinear single track model from [2]

			constant	unit	description	configuration	a1	a4	c2	c3	c5
wheel (tire+rim)	nonlinear single track model	rim + tyre	m_vh	kg	wheel mass		7.44	6.374	5.443	6.577	5.307
			theta_vh	kg.m^2	tyre rolling inertia		0.40	0.28	0.17	0.26	0.16
			theta_zz_vh	kg.m^2	tyre z inertia		0.22	0.16	0.10	0.15	0.10
			r	m	tyre outer radius		0.260	0.235	0.203	0.229	0.203
		eff. load	e_z		effective load degressive parameter				0		
			T_x	s	settling time of the tires during fast changes of course or velocity				0.02		
			T_y	s					0.02		
	tyre magic formula	pure longitudinal (SA=0)	PCX1		Shape factor Cfx for longitudinal force		1.278793	0.509153		1.441082	
			PDX1		Longitudinal friction Mux at Fznom		2.824687	5.447613		3.097078	
			PDX2		Variation of friction Mux with load		-0.723840	-0.441576		-0.758640	
			PKX1		Longitudinal slip stiffness Kfx/Fz at Fznom		-60.730470	-54.143220		-43.330700	
			PKX2		Variation of slip stiffness Kfx/Fz with load		30.189940	-7.051341		2.939219	
			PKX3		Exponent in slip stiffness Kfx/Fz with load		0.005985	0.171439		-0.253608	
		pure lateral (SR=0)	PCY1		Shape factor Cfy for lateral forces		1.389410	1.459571	0.735002	0.829714	1.598327
			PDY1		Lateral friction Muy		-2.509494	-2.470600	-2.985970	-3.151897	-2.294378
			PDY2		Variation of friction Muy with load		0.283008	0.628891	0.243913	0.129540	0.334261
			PKY1		Maximum value of stiffness Kfy/Fznom		73.211130	37.798840	49.328500	55.598290	54.820040
single track	geom.		l_v	m	distance between S and the front axle (vehicle reference frame)		0.78				
			l_h	m	distance between S and the rear axle (vehicle reference frame)		0.77				
			h_S	m	height of S (ground reference frame)		0.3				
			m_S	kg	suspended mass		90				
			g	m/s^2	acceleration of gravity		9.81				
			eps		avoid singularity		0.001				

Table 1: simulation parameters summary table



3.2 Magic formula tire models from TTC data with Optimum Tire

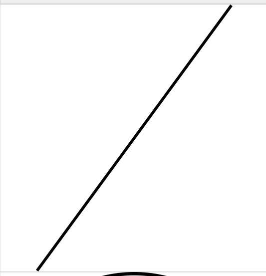
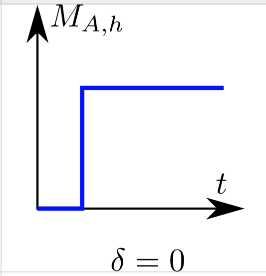
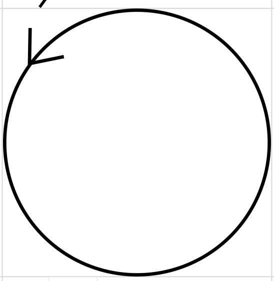
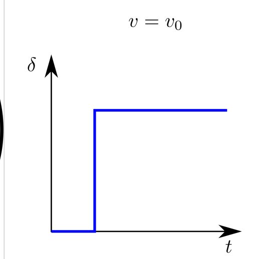
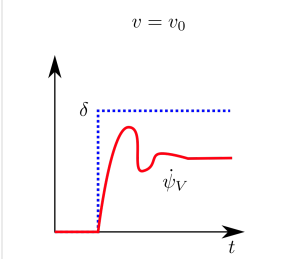
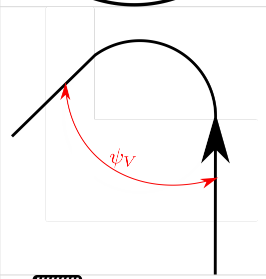
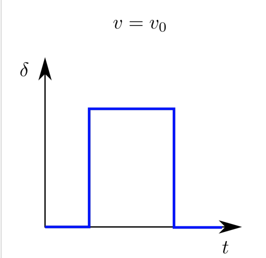
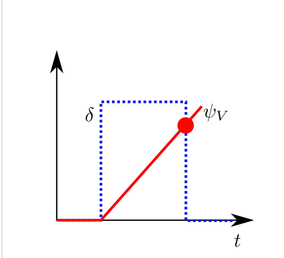
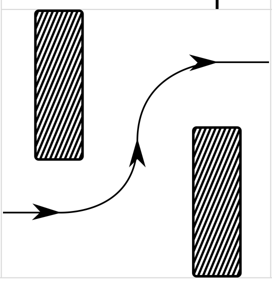
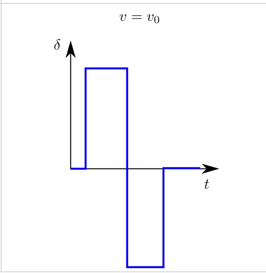
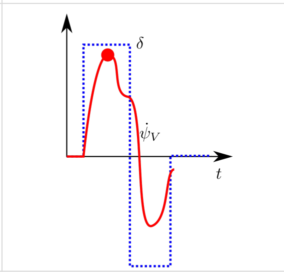
In order to obtain the magic formula parameters required by the single track model, TTC data was analyzed and fitted through the help of Optimum Tire. Only tires chosen in sec.2 where analyzed. Optimum Tire greatly facilitated the fitting process and provided a great tool for validating the calculated magic formula model against raw data points. In tab.2 validation plots are presented for each fitted tire. The calculated models were exported to `.TIR` format to easily save ad access magic formula coefficients.

TTC pure longitudinal data (SA = 0) for both tyre 2 and 5 were not available due to the small tire size. Tire 3 pure longitudinal model was therefore used to simulate tire 2 and 5 with their own remaining parameters (green area in table 1).

A tire effective load model proposed in [2] was implemented as well. However tire steady state response time could not be estimated and the same value was applied in every simulation (yellow area in table 1).

3.3 Test cases list

Following the implementation of the nonlinear single track model and the calculation of the magic formula tire models, a list of test cases was created in order to define different simulation scenarios. The table below presents such tests with respect to the competition events and the input-output variables. A first test T1 was done with no steering input and constant rear thrusting torque $M_{A,h}$. Max speed was measured after a 4-seconds simulation. Test T2's goal was to measure the stabilized angular yaw velocity $\dot{\psi}_V$ at the skidpad event. To do so a step steering input was applied at a given velocity. Tests T3 and T4 tried to quantify the vehicle agility at the autocross/endurance event. The overall vehicle direction ψ_V and angular velocity $\dot{\psi}_V$ were evaluated after a rapid steering input.

test	event	schema	input	output
T1	acceleration			max speed
T2	skidpad			
T3	autocross / endurance			
T4				

3.4 Results

Wheel configurations a1, a4, c2, c3 and c5 simulation results are presented in the first part of table 3. In the highlighted area results were obtained using the same pure longitudinal tire model as specified in sec.3.2.

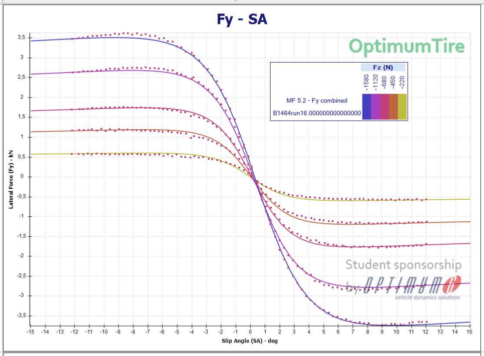
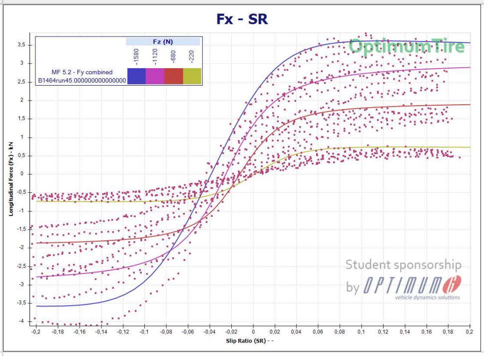
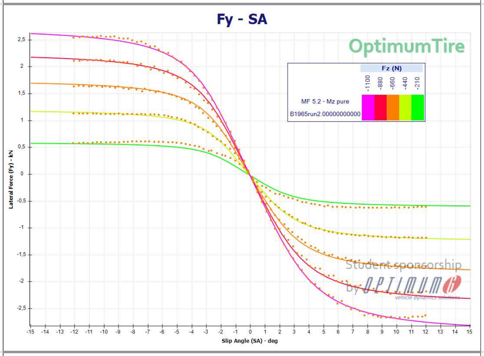
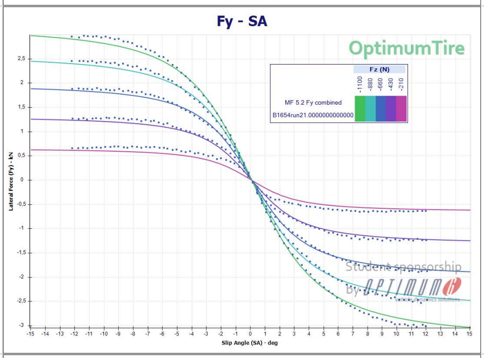
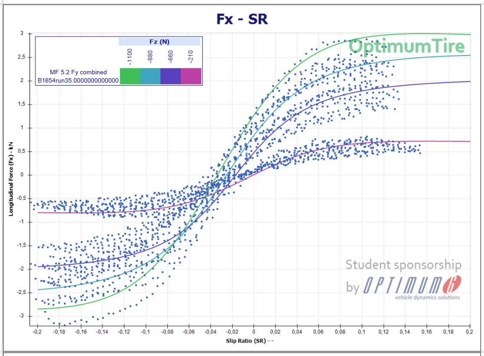
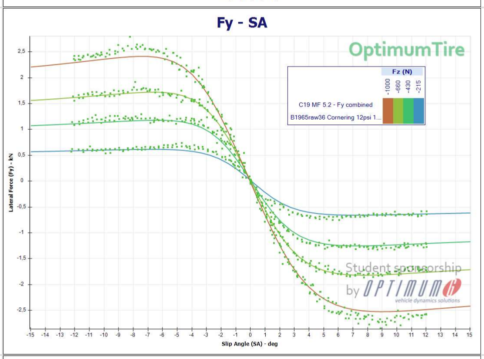
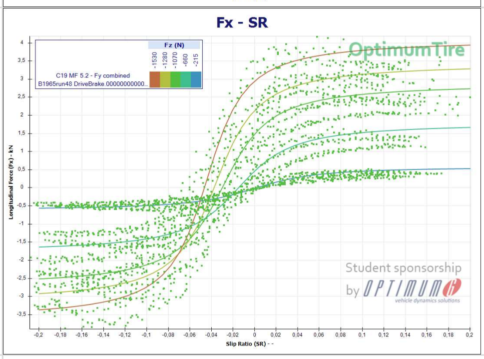
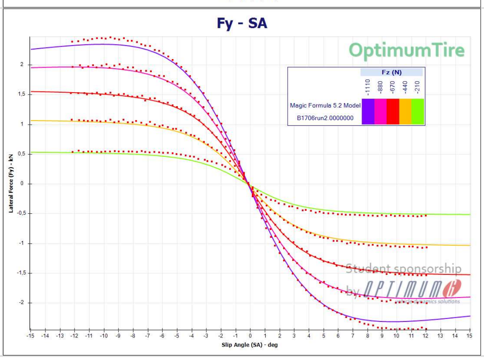
IA = 0deg - P=0.8 bar		
tyre	Fy -SA	Fx - SR
1		
2		
3		
4		
5		

Table 2: Tire magic formula validation. Notation of sec-2 is used.

test	variable	unit	a1		a4		c2		c3		c5	
MF temperature		°C	55		60		70		60		60	
rear axle torque		Nm	50	60	50	60	50	60	50	60	50	60
T1	SR		0.016	0.018	0.037	0.043	0.032	0.038	0.027	0.033	0.032	0.038
	x*_V	km/h	27	32.3	29.8	35.6	35.6	42.5	30.7	36.7	35.8	42.7
speed		km/h	30	40	30	40	30	40	30	38	30	40
T2	psi*_V (stabilized)	deg/s	173	175	174	173	175	177	174	176	181	176
T3	psi_V	deg	109	136	118	141	107	142	113	140	114	140
T4	psi*_V (max)	deg/s	200	227	250	262	250	320	274	330	215	267

test	a1	a4	c2	c3	c5
T1	59.3	65.4	78.1	67.4	78.5
T2	174.4	173.3	176.4	175.4	177.5
T3	127.9	134.1	131.5	131.9	132.2
T4	218.9	258.4	299	313.2	251.4
	580.5	631.2	685	687.9	639.6

Table 3: nonlinear single track model simulation result

4 Conclusion

4.1 Wheel configuration performance

In order to choose a final wheel configuration, a selection criteria was needed. The idea of summing all of the speed results from the different simulations was taken but a different coefficient was applied to distinguish between 30km/h and 40 km/h runs. Since the TTC data fitted in the magic formula was taken at an average speed of 11m/s, a 0.7 coefficient was applied to the 40Km/h runs of test T2, T3 and T4. The performance of the i-th configuration p_i was thus calculated by:

$$p_i = \dot{x}_V^{T1,50} + \dot{x}_V^{T1,60} + 0.3\left[\dot{\psi}_V^{T2,30} + \psi_V^{T3,30} + \dot{\psi}_V^{T4,30}\right] + 0.7\left[\dot{\psi}_V^{T2,40} + \psi_V^{T3,40} + \dot{\psi}_V^{T4,40}\right]$$

The second part of table 3 presents the results of such criteria and displays 3 different performance levels:

- Hoosier 20.5/7.5-13 with 13 inch rims is the worst solution
- Continental C19 with magnesium rims and Avon 16/7-10 with aluminum rims are interesting solutions. The first one remains a 13 inch configuration whereas the second is the lightest one
- Hoosier 18/7.5-10 and Hoosier 16/7.5-10 with aluminum rims are the best 10" solution among the analyzed configurations

4.2 Future work

A complete magic formula 5.2 model could be implemented in this model since Optimum Tire makes coefficients easily available. This would eventually include tire static properties at SR=0 and SA=0. An engine torque model as well as a secondary transmission model could be implemented as proposed by [2]. This would allow to simulate more accurately any acceleration test. Moreover, a brake model could be implemented as specified in [2] to allow the simulation of a braking event with the possibility of a steering input. Finally, simulation results should be compared to real data from the past vehicle's on-board computer.

References

[1] D.B.; Dupac M. Marghitu: *Advanced Dynamics*. Springer, 2012, ISBN 9781461434740.

[2] Bardini Schramm, Hiller: *Vehicle Dynamics: Modeling and Simulation*. 2018, ISBN 9783662544822.

[3] Hans B. Pacejka and Igo Besselink: *Tire and Vehicle Dynamics*. Elsevier Ltd., third edition, 2012.