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A Stable Trot Strategy of Quadruped Robot Based on Capture Point

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***Abstract*— Trotting is a kind of dynamic moving style for quadrupedal robot, which can make the robot move in fast and efficient way. However, the posture balance and the walking stability are two difficult problems for the robot control. In this paper, the dynamic model of the robot is obtained by Lagrange method. Based on the QR decomposition of the external force mapping matrix of the support leg, the constraints consistent dynamic model is obtained. The process of the quadruped robot trot is approximated as the locomotion of the linear inverted pendulum(LIPM). Based on the principle of the capture point for the biped robot, the trajectory of the swing leg during the trot is calculated. Through the method of joint torque optimization, inverse dynamic of robot is calculated, and each joint torque is obtained. The simulation results show that the method proposed in this paper can make the quadrupedal robot trot in a stable style.**

Keywords—Quadrupedal robot, LIPM, Capture point, Inverse dynamic

INTRODUCTION

Quadrupedal robots have shown a potential to used for mobile robot to realize locomotion in the complex environment which the wheel robot cannot pass through. For this idear, Many quadrupedal robots have been developed, such as BigDog [1], StarlETH [2-4] and HyQ [5] are advance quadrupedal robotic systems in the world. These robot can move stably in the environment of indoors and outdoors. Recently, the robot MIT Cheetah 1 and 2, was developed with a unique electric actuation system, which are used for the study of high speed quadrupedal locomotion [6], [7], These robot can achieve a fast and high-energy-efficiency trotting gait [8] and a dynamic quadrupedal bounding gait [9]. StarlETH robot use the control approach of hierarchical operational space control based on floating body dynamics to make the robot trot. For the HyQ robot, the desired trajectories of the robot's foot are calculated based on the algorithm of central patten generator (CPG) and implemented on the robot using feedforward torques provided by floating-base inverse dynamics with PD control. For the robot of MIT Cheetahs, the control method is based on virtual model control to calculate the joint torque.

In this paper, a floating body dynamic model of quadruped robot is obtained, and based on QR decomposition of he supporting leg force mapping matrix, the constraints consistent dynamic model of the robot is calculated. Using the model of linear inverted pendulum, the movement process of quadruped robot trotting is approximated, and the trajectory of the underactuated degree of freedom can be predicted. According to the principle of capturability of biped robot, the capture point which can make the robot trotting process can make the robot trot in a stable style can be calculated, and based on the capture point, the trajectory of swing leg is planned. According to the constraint of center of mass, the swing leg, the supporting leg and let joint torque as the optimization object, the problem of inverse dynamic can be solved. joints torques of the quadruped robot are calculated, and apply these torque to each joint of the robot, the control of the robot trotting is achieved. According to the simulation results, the proposed control method can make the robot trot in a stable way. The step length of the quadrupedal robot converges asymptotically.

This paper is organized as follows: dynamic model of the quadruped robot have been established in Section II. According to principle of capturability of robot, the control method is introduced in Section III. In Section IV, a simulation result is shown that the control method in this paper can stabilize the robot locomotion. In. Section V gives the conclusions of the paper.

MODEL

Figure 1 is the structure of the quardrupedal robot, which has four legs, and each leg has 3 degrees of freedom, 2 degrees of freedom are in the hip joint, one for roll degree of freedom and one for pitch degree of freedom, one pitch degree of freedom is in the crus joint. All the joints are actuated by the motor and gear.

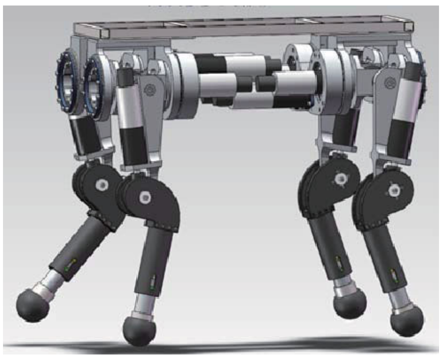

Fig. 1. The structure of the quadruped robot

Let $[x \quad y \quad z]^T$ be the center of mass of the trunk, and let $[\psi \quad \theta \quad \phi]^T$ be the Euler angles of the trunk, Let the degree of freedom of the trunk be $q_b = [x \quad y \quad z \quad \psi \quad \theta \quad \phi]^T$, and $q_u \in R^{12\times1}$ degree of freedom of joints. Based on the Lagrange method, The dynamic model of the robot can be written as:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=Bu+E(q)^T F_{ext}, \tag{1}$$

where $M(q)\in R^{18\times18}$ is the inertia matrix, $C(q,\dot{q})\in R^{18\times18}$ is the Coriolis and centrifugal matrix, $G(q)\in R^{18\times1}$ is the gravitational vector, $q=[q_b^T \quad q_u^T]^T$ is the generalized coordinates, $u\in R^{12\times1}$ is the torque vector, $B\in R^{18\times12}$ is the joint torque mapping matrix.

$$B=\frac{\partial q_u}{\partial q}, \tag{2}$$

$F_{ext}\in R^{3n\times1}$ is ground force vector, $E(q)^T\in R^{18\times3n}$ ground force mapping matrix， n is the number of support leg.

$$E(q)=\frac{\partial P_{st}(q)}{\partial q}, \tag{3}$$

where, $P_{st}\in R^{3\times n}$ is the position matrix of the support foot。

Based on the QR decomposition of the matrix $E(q)^T$

$$E(q)^T=[Q_c \quad Q_u]\begin{bmatrix}R\\ \mathbf{0}\end{bmatrix}, \tag{4}$$

where， R is the upper right triangular matrix，combined to （1）the constraints consistent dynamic model is obtained

$$Q_u^T(M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q))=Q_u^T Bu. \tag{5}$$

With the constraints consistent dynamic model is obtained，ground force make on sense for the dynamic.

CONTROL METHOD

The locomotion of trot is shown in Figure 2:

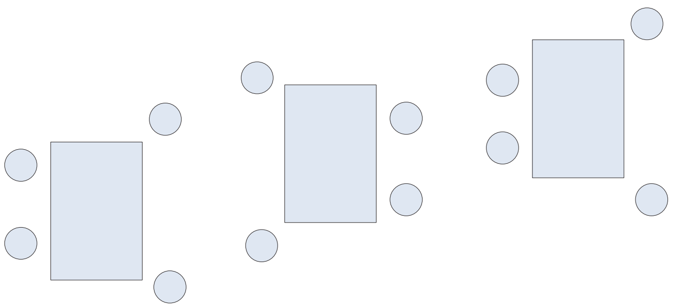

Fig. 2. The trot locomotion style of quadrupedal robots

Trot is a kind of dynamic gait. At each moment, each diagonal two legs composed of support legs, the other two legs are swing legs. At the end of each step, swing legs touch the ground, and exchanged with the support legs. Let left front leg be simplified as L-F, right front leg is simplified as R-F, left behind leg is simplified as L-B, and right behind leg is simplified as R-B, the sequence of trot is shown in Figure 3:

In the trot process, two support legs of the robot is unable to form the support polygon on the ground, there is a degree of freedom of the trunk is underactuated. The principle of the capturability is need to control the underactuated degree of freedom. According to the capture point, the trajectory of each swing leg of the robot is planning, and the movement of the underactuated degrees of freedom can converge to a limit cycle, and the trot achieves a dynamic stability.

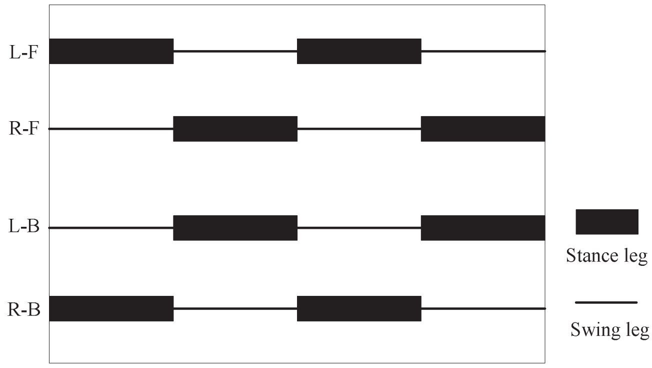

Fig. 3. The sequence diagram of trot

Let the degrees of freedom of the robot along the X direction be the underactuated degrees of freedom. The movement of the center of mass can be approximated as the movement of the linear inverted pendulum. The model of the linear inverted pendulum is shown in Figure 4:

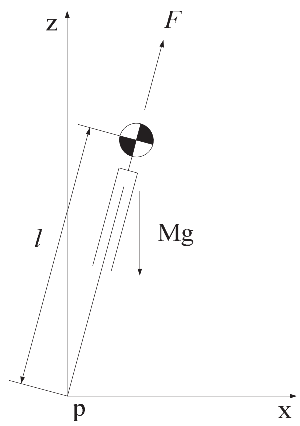

Fig. 4. The linear inverted pendulum model

The dynamic of the linear inverted pendulum model is shown as

$$\ddot{x}=\frac{g}{z_c}(x-p), \tag{6}$$

where, x is the degree of freedom along the X direction, g is the acceleration of gravity, z_c is the height of the center of mass, p is the support point, and in this paper, $p=0$. Solve the formulation(6), the trajectory of x is show as:

$$x(t)=x(0)\cosh(t/T_c)+T_c\dot{x}(0)\sinh(t/T_c) \tag{7}$$

$$\dot{x}(t)=x(0)/T_c\sinh(t/T_c)+\dot{x}(0)\cosh(t/T_c) \tag{8}$$

$$T_c=\sqrt{z_c/g} \tag{9}$$

$$\sinh(t/T_c) = \frac{e^{t/T_c} - e^{-t/T_c}}{2} \tag{10}$$

$$\cosh(t/T_c) = \frac{e^{t/T_c} + e^{-t/T_c}}{2} \tag{11}$$

where, t is the time , $x(0)$ is the initial state of $x(t)$, $\dot{x}(0)$ is the initial state of $\dot{x}(t)$. Let t_c the moment of the exchange between the swing leg and the supporting leg, $x(t_c)$ can be obtained, Let l_d be given distance between the center of mass relative and the support point p at the end of each step, T_d is the cycle of the trot. The capture point can be calculated as:

$$x_p = x(t_c) - \frac{(l_d - T_c\dot{x}(t_c)\sinh(t/T_c))}{\cosh(T_d/T_c)} \tag{12}$$

Let t_i be the moment at the beginning of each step, calculate $s = (t - t_i)/(t_c - t_i)$. Let x_f be the position in X direction of swing legs at the beginning of the current step. Combine x_f, x_p and s, a bezier curve in X direction can be planned, and trajectories of swing leg are planned as

$$P_{sg} = \begin{bmatrix} b(x_f, x_p, s) \\ 0 \\ (h/2)(1 + \cos(\pi(2s-1))) \end{bmatrix}, \tag{13}$$

where, h is the height of the swing leg. Let P_{st} be positions of support legs, the constraint of the legs are shown as

$$\begin{cases} \ddot{P}_{sw} = -k_p(P_{sw} - P_{sg}) - k_d(\dot{P}_{sw} - \dot{P}_{sg}) \\ \ddot{P}_{st} = 0 \\ \dot{P}_{st} = 0 \end{cases} \tag{14}$$

where, k_p and k_d are parameters of PD controller of the swing legs. The constraint of the trunk is :

$$\begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\psi} \\ \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} = -K_p \begin{bmatrix} y \\ z - z_c \\ \psi \\ \theta \\ \phi \end{bmatrix} - K_d \begin{bmatrix} \dot{y} \\ \dot{z} \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}, \tag{15}$$

where, K_p and K_d are parameters of PD controller of the trunk

Let（5），（14）and（15）be equation constraints , and solve the optimization problem $\min(\|u\|^2)$ as follow:

Let $E(q)_b = \partial[y \quad z \quad \psi \quad \theta \quad \phi]/\partial q$, and $\ddot{P}_{sw}$ can be written as $\ddot{P}_{sw} = E(q)_{sw}\ddot{q} + J(q,\dot{q})_{sw}$, Similarly, $\ddot{P}_{st} = E(q)_{st}\ddot{q} + J(q,\dot{q})_{st}$, the equation constraint is:

$$A\begin{bmatrix} \ddot{q} \\ u \end{bmatrix} = b, \tag{16}$$

where,

$$A = \begin{bmatrix} -Q_u^T M(q) & Q_u^T B \\ E(q)_{st} & 0_{6\times12} \\ E(q)_{sw} & 0_{6\times12} \\ E(q)_b & 0_{5\times12} \end{bmatrix}, \quad b = \begin{bmatrix} Q_u^T(C(q,\dot{q})\dot{q} + G(q)) \\ -J(q,\dot{q})_{st} \\ \ddot{P}_{sw} - J(q,\dot{q})_{sw} \\ [\ddot{y} \quad \ddot{z} \quad \ddot{\psi} \quad \ddot{\theta} \quad \ddot{\phi}]^T \end{bmatrix}$$

Where, $[\ddot{y} \quad \ddot{z} \quad \ddot{\psi} \quad \ddot{\theta} \quad \ddot{\phi}]^T$ is calculated by (15) and, $\ddot{P}_{sw}$ is calculated by (14). The optimization problem can be rewritten as:

$$\begin{cases} \min(\|u\|^2) \\ st : A[\ddot{q}^T \quad u^T]^T = b \end{cases} \tag{17}$$

Solving（17）, joints torques can be calculated as follow:

$$\begin{bmatrix} \ddot{q} \\ u \end{bmatrix} = \begin{bmatrix} S & A^T \\ A & 0_{29\times29} \end{bmatrix}^{-1} \begin{bmatrix} 0_{30\times1} \\ b \end{bmatrix}, \tag{18}$$

where, $S = \begin{bmatrix} 0_{18\times18} & 0_{18\times12} \\ 0_{12\times18} & I_{12\times12} \end{bmatrix}$, I is the unit matrix.

Based on $\ddot{q}$ and u, ground force can be calculated as:

$$F_{ext} = R^{-1}Q_c^T(Bu - M(q)\ddot{q} - C(q,\dot{q})\dot{q} - G(q)), \tag{19}$$

SIMULATION

According to the control method introduced in the last section, the quadrupedal robot is simulated, the result is shown as follow. Let $L = |P_{sw} - P_{st}|$, the trajectoy of L during the robot trot is shown as Figure 5:

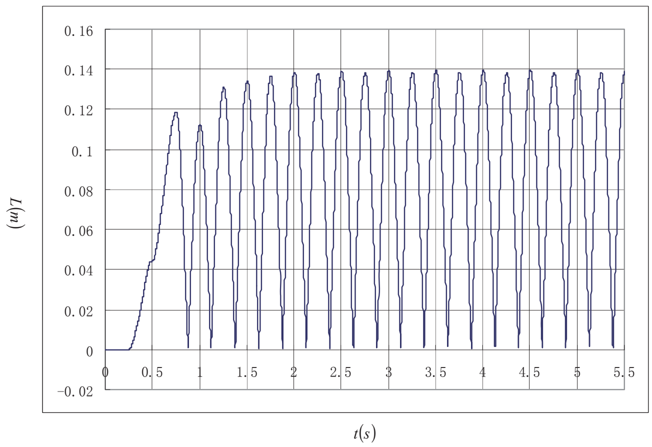

Fig. 5. The trajectory of $L = |P_{sw} - P_{st}|$

The trajectory of θ during the motion of trot is shown as:

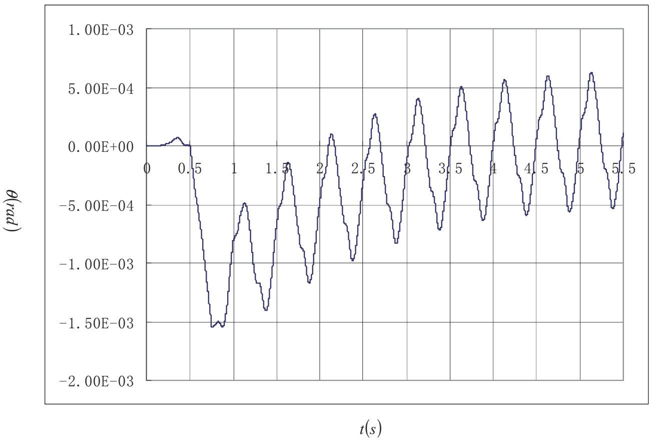

Fig. 6. The trajectory of θ

The trajectory of ϕ during the motion of trot is shown as:

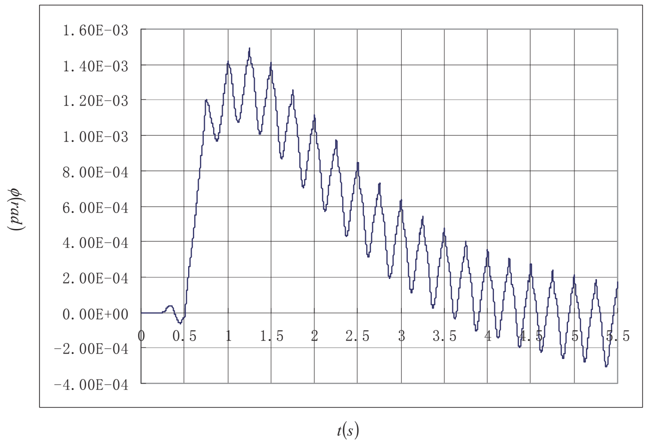

Fig. 7. The trajectory of ϕ

Let q_{h1}, q_{t1} and q_{f1} are the hip joint, thigh joint and crus joint for the left front leg. The limit cycle of q_{h1}, q_{t1} and q_{f1} are shown in Figure 8, Figure 9 and Figure 10.

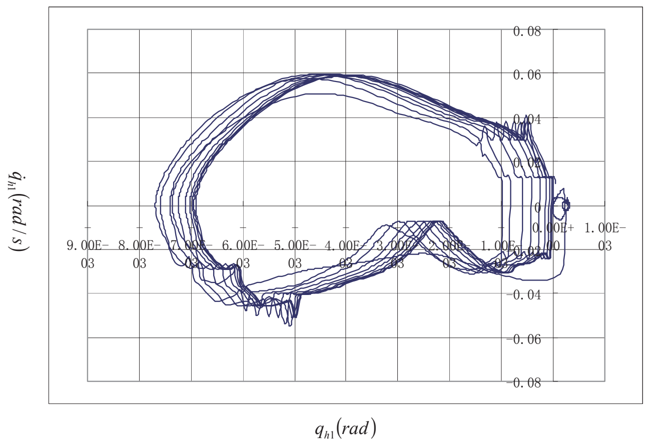

Fig. 8. The linit cycle of q_{h1}

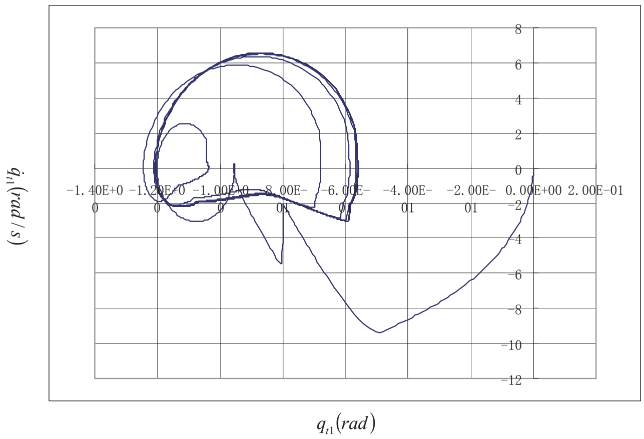

Fig. 9. The limit cycle q_{t1}

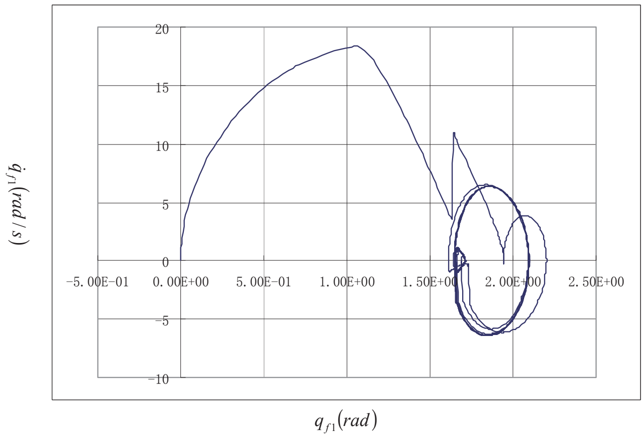

Fig. 10. The limit cycle of q_{f1}

CONCLUSION

In this paper, a control method to stabilize locomotion of trot for the quadrupedal robot is studied. First, the constraints consistent dynamic model of the quadrupedal robot has been introduced. Second, using the calculation method of capture point basing on model of the linear inverted pendulums, the contact point in the ground for the swing leg, which make the quadrupedal robot trot in the given step length can be predicted, with PD control and the inverse dynamic method, the torques of each joints can be obtained. Finally, with method in this paper, a simulation of trot is done. The simulation result shows that the quadrupedal robot can trot in a stable manner.

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