CSE 5693 Machine Learning

HW3 Artificial Neural Network Learning

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Written Assignment

(a) **4.1. What are the values of weights w0, w1, and w2 for the perceptron whose decision surface is illustrated in Figure 4.3? Assume the surface crosses the x1 axis at -1, and the x2 axis at 2.**

Answer:

Decision boundary is E: w0 + w1x1 + w2x2 > 0 and pass by points P1 (-1, 0) and P2 (0, 2). We also have P3 (-2, 3) as a positive instance and P4 (0, 0) as a negative instance.

Substituting P1 and P2 in E, we get:

w0 + -w1 =0 => **w1 = w0**

w0 + 2*w2 = 0 => **w2 = -1/2 * w0**

So, we have E is also w0 + w0x1 – 1/2*w0*x2 = 0.

Moreover, since P3 is a positive example and P4 is a negative example, for the perceptron decision boundary, we find **w0 ≤ -1**. The solution is then:

For all w0 ≤ -1, w1 = w0, w2 = -1/2 * w0.

An instance would be w0 = -1, w1 = -1, w2 = 1/2

(b) **4.2. Design a two-input perceptron that implements the boolean function A ^ - B. Design a two-layer network of perceptrons that implements A XOR B.**

Answer:

A	B	H1 = A ^ - B
0	0	0
0	1	0
1	0	1

1	1	0

Let our perceptron be of the form w0 + w1A + w2B > 0 with a step activation function which return 1 when w0 + w1A + w2B > 0 and 0 otherwise. The perceptron takes for input the values A and B. To solve for our perceptron, let's find w0, w1, w2.

For A = 0, B = 0, the result should be 0 so w0 + 0 + 0 ≤ 0, so let's take **w0 = -1**

For A = 0, B = 1, the result should be 0 so -1 + w2 ≤ 0, so let's take **w2 = -1**

For A = 1, B = 0, the result should be 1 so -1 + w1 > 0, so let's take **w1 = 2**

For A = 1, B = 0, the result should be 0 so -1 + 2 -1 ≤ 0, which it already is so our choice of w0, w1, w2 are valid for implementing A ^ -B.

A	B	A XOR B
0	0	0
0	1	1
1	0	1
1	1	0

A XOR B = (A ^ -B) v (-A ^ B) so we have the combination of three 2 inputs perceptrons. 2 on the first layer, and one on the second layer.

1st Layer:

We already found from previous part of the question that (A ^ -B) is implemented by a perceptron with w00 = -1, w01 = 2, w02 = -1.

A	B	H2 = -A ^ B
0	0	0
0	1	1
1	0	0
1	1	0

For part (-A ^ B), we can flip the value of w1 and w2 and we get w10 = -1, w11 = -1, w12 = 2

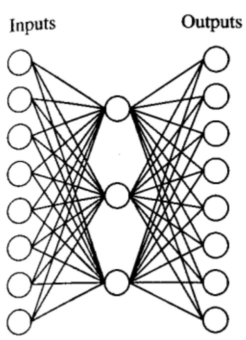
2nd Layer is a disjunction so we have

H1	H2	H1 v H2

0	0	0
0	1	1
1	0	1
1	1	1

For the disjunction, we can use the table and get w20 = -1, w21 = 2, w22 = 2 for the output layer

(c) 4.9. Recall the 8 x 3 x 8 network described in Figure 4.7. Consider trying to train a 8 x 1 x 8 network for the same task; that is, a network with just one hidden unit. Notice the eight training examples in Figure 4.7 could be represented by eight distinct values for the single hidden unit (e.g., 0.1,0.2, . . . ,0.8). Could a network with just one hidden unit therefore learn the identity function defined over these training examples? Hint: Consider questions such as "do there exist values for the hidden unit weights that can create the hidden unit encoding suggested above?" "do there exist values for the output unit weights that could correctly decode this encoding of the input?" and "is gradient descent likely to find such weights?"



Input		Hidden Values				Output
10000000	→	.89	.04	.08	→	10000000
01000000	→	.15	.99	.99	→	01000000
00100000	→	.01	.97	.27	→	00100000
00010000	→	.99	.97	.71	→	00010000
00001000	→	.03	.05	.02	→	00001000
00000100	→	.01	.11	.88	→	00000100
00000010	→	.80	.01	.98	→	00000010
00000001	→	.60	.94	.01	→	00000001

Answer: Ran the program with 1 unit hidden layer to find out.

Yes, 1 one hidden unit layer can theoretically learn to identify function defined over these training examples. Possible values for hidden weights and output weights exists and are part of the hypothesis space. The issue is the likelihood of GD to find them without falling into a local minimum along the way. In conclusion, it's possible with 1 hidden unit but mostly will perform much worse than with 3 hidden units.

(d) With the programming assignment:

i. **discuss the hidden values in testIdentity using 3 and 4 hidden units (Why do 4 hidden units also work? What do the hidden values represent? Any significant difference in the number of iterations to convergence and why?)**

Answer: With 3 hidden inputs, the network is able to learn the binary encoding needed to computer the identities. With 4 hidden unites, the network is still able to compute the identities, but this time with 4 parameters that together are able to index every one of the 8 values. This is due to the fact that increasing the number of parameters doesn't reduce the number of features or ways the network can encode the inputs. The hidden values represent 4 parameters encoding of the input. In term of iterations to converge, it takes less iterations for 3 hidden units than 4 hidden units and this is due to the fact that, 4 hidden units has more parameters that the network has to learn, resulting in a lengthier time to convergence.

ii. **compare performance of using validation set to not using it in testIrisNoisy. Include a plot for the comparisons.**

Answer:

We can see that will validation applied (k-fold cross validation), the network is able to perform better with respect to the corrupted data. K-fold cross validation does improve the performance of the network by reducing overfitting.