$\LaTeX$ Test File

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Section 1 Maxwell's Equations

§1.1 Integral Format

Maxwell's Equations (in forms of **Integral**):

$$\oiint_S \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{S} = \sum q = \int_V \rho \mathrm{d}V, \tag{1}$$

$$\oiint_S \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0, \tag{2}$$

$$\oint_L \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I + I_{\mathrm{d}} = \int_S \boldsymbol{j} \cdot \mathrm{d}\boldsymbol{S} + \int_S \frac{\partial \boldsymbol{D}}{\partial t} \cdot \mathrm{d}\boldsymbol{S}, \tag{3}$$

$$\oint_L \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -\int_S \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{S}. \tag{4}$$

Here, (1) states for the **Gauss Theorem** in an **Electric Field**, while (2) states for the **Gauss Theorem** in an **Magnetic Field**. (3) states for the relationship between **A Changing Electric Field** and a magnetic field, or **Ampere's Circulation Theorem**. (4) states for the relationship between **A Changing Magnetic Field** and a electric field, or **Faraday's Theorem of induction**.

Section 2 Partial Derivative

§2.1 Definition

Let $t = f(x, y, \ldots)$, the **Partial Derivative** of f towards x is

$$f'_x = \partial_x f = D_x f = D_1 f = \frac{\partial}{\partial x} f = \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y, \ldots) - f(x, y, \ldots)}{\Delta x}.$$

Define vector $\boldsymbol{a} = (x, y, \ldots), \hat{\boldsymbol{e}}_{\boldsymbol{x}} = (1, 0, \ldots)$, therefore

$$\frac{\partial}{\partial x} f = \lim_{x \to 0} \frac{f(\boldsymbol{a} + h\boldsymbol{e}_{\boldsymbol{x}}) - f(\boldsymbol{a})}{h}.$$

§2.2 Gradient

Define **Gradient** as following:

$$\text{grad } f(\boldsymbol{a}) = \nabla f(\boldsymbol{a}) = \left(\left.\frac{\partial f}{\partial x}\right|_{\boldsymbol{a}}, \left.\frac{\partial f}{\partial y}\right|_{\boldsymbol{a}}, \dots \right).$$

We usually deine Gradient as following in a 3-Dimensional Space:

$$\text{grad } = \nabla = \left[\frac{\partial}{\partial x}\right] \hat{\boldsymbol{e}}_{\boldsymbol{x}} + \left[\frac{\partial}{\partial y}\right] \hat{\boldsymbol{e}}_{\boldsymbol{y}} + \left[\frac{\partial}{\partial z}\right] \hat{\boldsymbol{e}}_{\boldsymbol{z}}.$$

§2.3 Directional Derivative

Define the **Directional Derivative** along vector $\boldsymbol{v} = (v_1, v_2, \dots)$,

$$\nabla_{\boldsymbol{v}} f(\boldsymbol{a}) = \lim_{x \to 0} \frac{f(\boldsymbol{a} + h\boldsymbol{v}) - f(\boldsymbol{a})}{h}.$$

§2.4 Laplace Operator

Define the **Laplace Operator** as following:

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla \cdot \nabla = \nabla^2.$$

§2.5 Divergence

Define the **Divergence** of a vector as following: (it outputs a value)

$$\text{div } \boldsymbol{v} = \nabla \cdot \boldsymbol{v} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (v_x, v_y, v_z) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}.$$