Probability Theory Chapter 4 (part A)

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Definitions

An **experiment** is the process of observing a phenomenon that has variation in its outcomes.

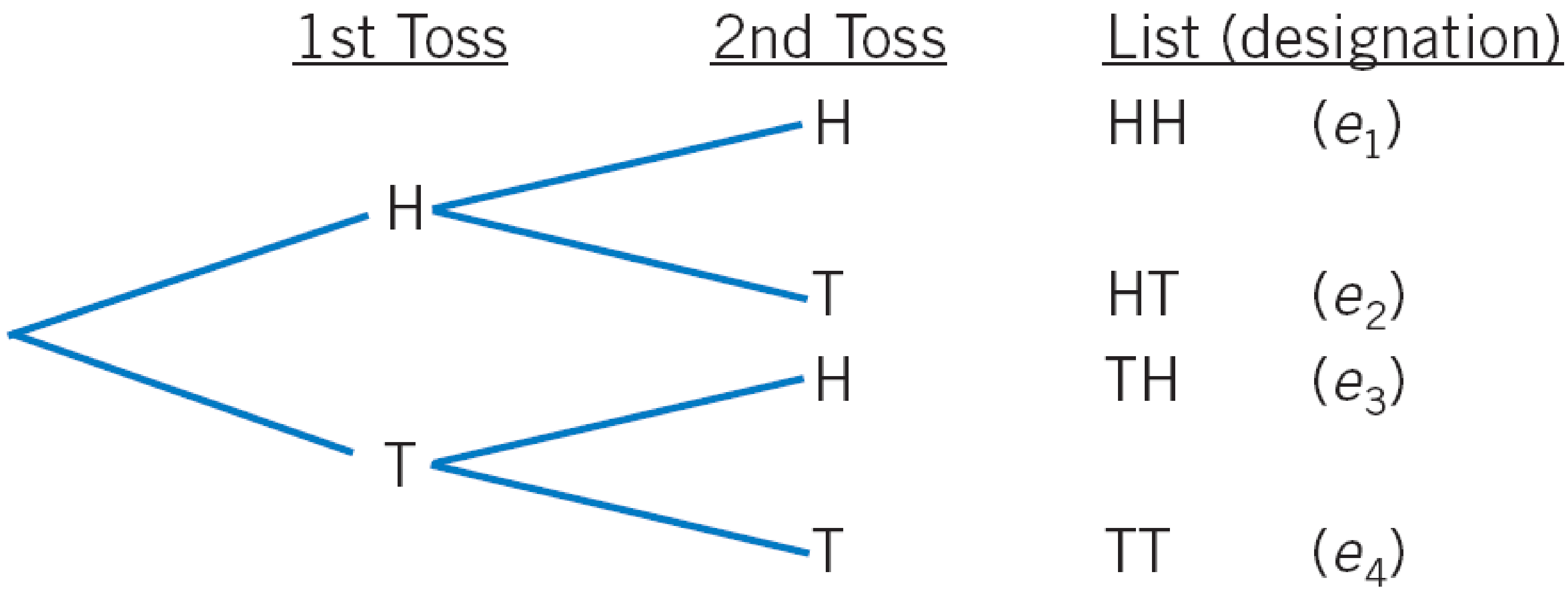
The **sample space** associated with an experiment is the collection of all possible distinct outcomes of the experiment.

Each outcome is called an **elementary outcome,** a **simple event,** or an **element of the sample space.**

An **event** is the set of elementary outcomes possessing a designated feature.

An event A occurs when any one of the elementary outcomes in A occurs.

A Tree diagram and Events for Coin Tossing



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What Is Probability?

When we use probability in a statement, we're using a *number between 0 and 1* to indicate the likelihood of an event.

Probability is a numerical measure between 0 and 1 that describes the likelihood that an event will occur. Probabilities closer to 1 indicate that the event is more likely to occur. Probabilities closer to 0 indicate that the event is less likely to occur.

$P(A)$, read "P of A," denotes the **probability of event** A.

If $P(A) = 1$, event A is certain to occur.

If $P(A) = 0$, event A is certain not to occur.

What Is Probability?

Probability assignments

1. A probability assignment based on **intuition** incorporates past experience, judgment, or opinion to estimate the likelihood of an event.
2. A probability assignment based on **relative frequency** uses the formula

$$\text{Probability of event} = \text{relative frequency} = \frac{f}{n} \qquad (1)$$

   where f is the frequency of the event occurrence in a sample of n observations.
3. A probability assignment based on **equally likely outcomes** uses the formula

$$\text{Probability of event} = \frac{\text{Number of outcomes favorable to event}}{\text{Total number of outcomes}} \qquad (2)$$

Calculate the probability and write whether each event is *certain*, *likely*, *unlikely*, or *impossible*.

1. The probability of tossing a *7* on a standard six-sided die.

2. A glass jar contains 13 red marbles. Describe the probability of picking a red marble.

3. A glass jar contains 30 marbles. The jar has purple and red marbles. There are 4 red marbles.

a. Describe the probability of picking a purple marble.

b. Describe the probability of picking a red marble.

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Example – *Using a Sample Space*

Human eye color is controlled by a single pair of genes (one from the father and one from the mother) called a *genotype*. Brown eye color, B, is dominant over blue eye color, ℓ. Therefore, in the genotype Bℓ, consisting of one brown gene B and one blue gene ℓ, the brown gene dominates. A person with a Bℓ genotype has brown eyes.

If both parents have brown eyes and have genotype Bℓ, what is the probability that their child will have blue eyes? What is the probability the child will have brown eyes?

Example 2 – *Solution*

To answer these questions, we need to look at the sample space of all possible eye-color genotypes for the child. They are given in Table 4-1.

	Mother	
Father	B	ℓ
B	BB	Bℓ
ℓ	ℓB	$\ell\ell$

Eye Color Genotypes for Child

Table 4-1

Example 2 – *Solution*

cont'd

According to genetics theory, the four possible genotypes for the child are equally likely.

Therefore, we can use Formula (2) to compute probabilities. Blue eyes can occur only with the $\ell\ell$ genotype, so there is only one outcome favorable to blue eyes.

By formula (2),

$$P(\text{blue eyes}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{4}$$

Example 2 – *Solution*

cont'd

Brown eyes occur with the three remaining genotypes: BB, Bℓ, and ℓB.

By formula (2),

$$P(\text{brown eyes}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{4}$$

What Is Probability?

The **sum** of the probabilities of all simple events in a sample space must equal 1.

We can use this fact to determine the probability that an event will not occur. For instance, if you think the probability is 0.65 that you will win a tennis match, you assume the probability is 0.35 that your opponent will win.

The *complement* of an event A is the event that A *does not occur*. We use the notation A^c to designate the complement of event A.

What Is Probability?

Figure 4-1 shows the event *A* and its complement A^c.

Father	Mother	
	B	ℓ
B	BB	Bℓ
ℓ	ℓB	$\ell\ell$

Eye Color Genotypes for Child

Table 4-1

Notice that the two distinct events *A* and A^c make up the entire sample space. Therefore, the sum of their probabilities is 1.

What Is Probability?

The **complement of event** A is the event that A *does not occur*. A^c designates the complement of event A. Furthermore,

1. $P(A) + P(A^c) = 1$
2. $P(\text{event } A \text{ does } \textit{not} \text{ occur}) = P(A^c) = 1 - P(A)$ (3)

Example 3 – *Complement of an Event*

The probability that a college student who has not received a flu shot will get the flu is 0.45. What is the probability that a college student will *not* get the flu if the student has not had the flu shot?

Solution:

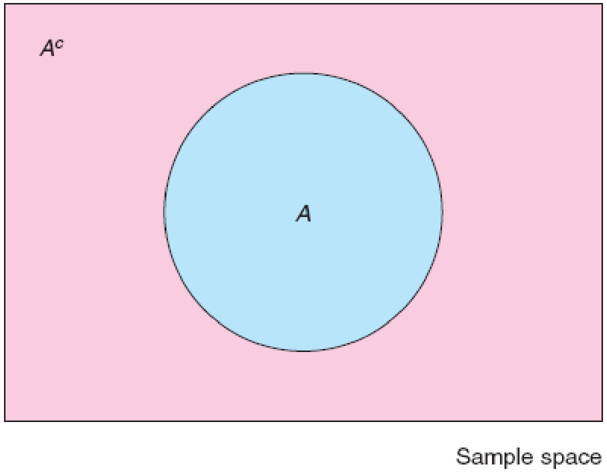
In this case, we have

P(will get flu) = 0.45

P(will *not* get flu) = 1 – P (will get flu)

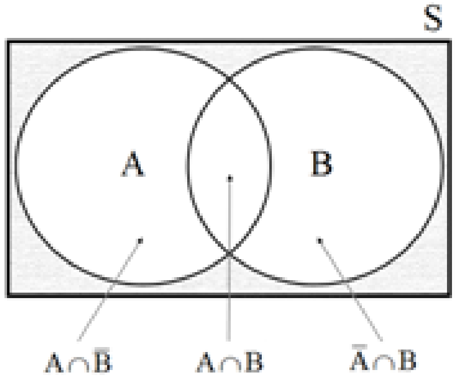
= 1 – 0.45

= 0.55



General Addition Rule

If $\boldsymbol{A}$ and $\boldsymbol{B}$ are any events of the sample space $\boldsymbol{S}$, then

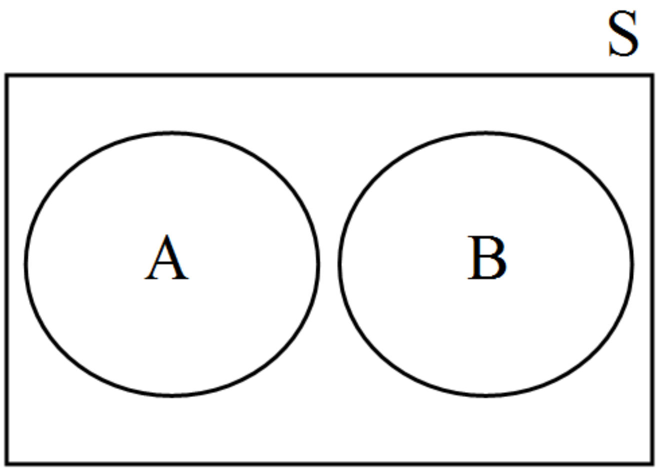


$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Addition Rule for Mutually Exclusive Events

If $\boldsymbol{A}$ and $\boldsymbol{B}$ are any **mutually exclusive** events of the sample space $\boldsymbol{S}$, then

$$P(A \cup B) = P(A) + P(B)$$



Since, $P(A \cap B) = 0$

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Example

1. Compute the probability of the union of events; that is $P(A \cup B)$

a. P(*A*) = 5/13, P(*B*) = 8/13, and $P(A \cap B) = 4/13$

b. P(*A*) = 1/5, P(*B*) = 2/6, and $P(A \cap B) = 2/15$

2. You roll a die that is numbered 1 to 6. What is the probability that you roll a 4 or a 5.

3. You toss a coin two times. What is the probability that you get one head and one tail?

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Example

Consider the following two events for an application filed by a person to obtain a car loan:

A = event that the loan application is approved

R = event that the loan application is rejected

What is the joint probability of A and R?

The two events A and R are mutually exclusive. Either the loan application will be approved or it will be rejected. Hence,

$$P(A \text{ and } R) = \mathbf{0}$$

Example (continue)

4. There are four elementary outcomes in a sample space. If $P(e_1)=0.3$, $P(e_2)= 0.4$, $P(e_3)= 0.2$, what is the probability of $P(e_4)=?$ (Hint: the sum of probabilities of all events =1)

5. In a survey of 50 students, 30 used brand *A*, 20 used brand *B*, and 12 used both brands *A* and *B*.

a. What proportion (probability) used either brand *A* or brand *B*.

b. What proportion (probability) used neither brand *A* nor brand *B*; that is not brand *A* and not brand *B*. *(Hint:* $\mathbf{P}(\overline{\mathbf{A} \cup \mathbf{B}})$*)*

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Example (Class work)

John runs a computer software store. Yesterday he counted 127 people who walked by his store, 58 of whom came into the store. Of the 58, only 25 bought something in the store.

a. Estimate the probability that a person who walks by the store will enter the store.

b. Estimate the probability that a person who walks into the store will buy something.

c. Estimate the probability that a person who walks by the store will come in and buy something.

d. Estimate the probability that a person who comes into the store will buy nothing.

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Example

Identify the following :

1. P(A) = 0.4. Is P(A) correct? => Yes. Why? ____________
2. P(A) = -0.35. Is P(A) correct? => No. Why? ____________
3. Determine which of the following probability distributions are plausible; if the probability distribution is implausible, state why?

a) E_1 and E_2 are mutually exclusive events with respective probabilities $P(E_1) = 1/4$ and $P(E_2)=3/4$. Plausible since the sum of probability of events =1.

b) $E_{1,}$ $E_{2,}$ and E_2 are mutually exclusive events with respective probabilities $P(E_1) = 0.145$ and $P(E_2)=0.521$, and $P(E_3)=0.333$

c) $E_{1,}$ $E_{2,}$ and E_2 are mutually exclusive events with respective probabilities $P(E_1) = 1/2$ and $P(E_2)=1/4$, and $P(E_3)=-2/3$

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Example

Based on the data of the Center for Health Statistics, the 2009 birth rates in 50 states are grouped in the following frequency table

If one state is selected at random, what is the probability that the birth rate there is:

A) Under 14?

B) Under 18 but not under 12?

C) 16 or over?

Birth Rate Per thousand	Number of Cases
10-12	4
12-14	12
14-16	29
16-18	4
18 and over	1

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Example 7 – *Mutually Exclusive Events*

Laura is playing Monopoly. On her next move she needs to throw a sum bigger than 8 on the two dice in order to land on her own property and pass Go. What is the probability that Laura will roll a sum bigger than 8?

Solution:

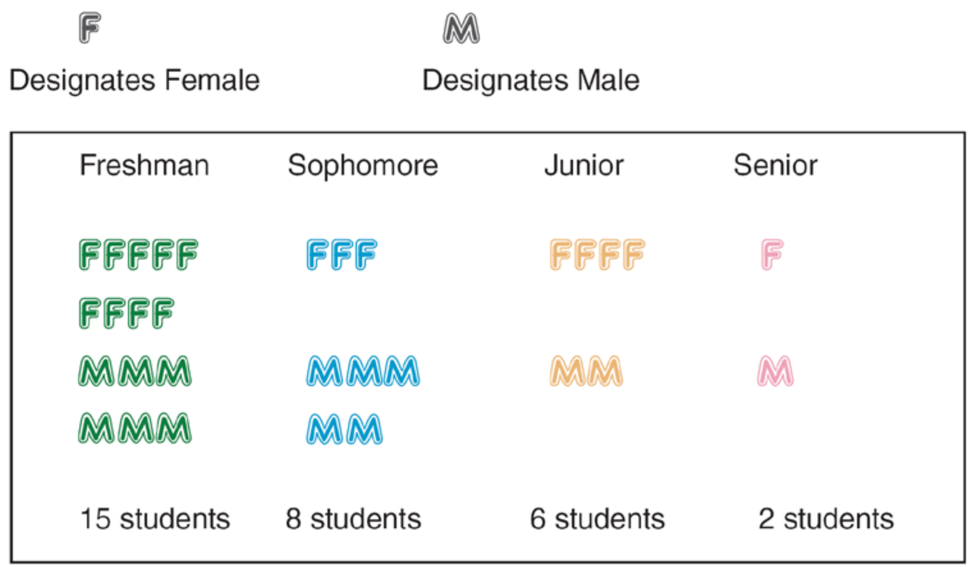
When two dice are thrown, the largest sum that can come up is 12. Consequently, the only sums larger than 8 are 9, 10, 11, and 12.

These outcomes are mutually exclusive, since only one of these sums can possibly occur on one throw of the dice.

Solve

Example 6 – *Probability of Events Combined with Or*

Consider an introductory statistics class with 31 students. The students range from freshmen through seniors. Some students are male and some are female. Figure 4-5 shows the sample space of the class.



Sample Space for Statistics Class

Figure 4-5

Example 6 – *Probability of Events Combined with Or*

cont'd

(a) Suppose we select one student at random from the class. Find the probability that the student is either a freshman or a sophomore.

$$= \frac{23}{31} \approx 0.742$$

Example 6 – *Probability of Events Combined with Or* cont'd

(b) Select one student at random from the class. What is the probability that the student is either a male or a sophomore?