4 Variable K-maps (cont.)

Example: Simplify

$$F = \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}D + AB\overline{C}D + \overline{A}BCD + ABCD$$
$$+\overline{A}\,\overline{B}C\overline{D} + A\overline{B}C\overline{D}$$

CD \ AB	00	01	11	10
00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$\overline{A}B\overline{C}\cdot\overline{D}$	$AB\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}\,\overline{D}$
01	$\overline{A}\,\overline{B}\,\overline{C}D$	$\overline{A}B\overline{C}D$	$AB\overline{C}D$	$A\overline{B}\,\overline{C}D$
11	$\overline{A}\,\overline{B}CD$	$\overline{A}BCD$	$ABCD$	$A\overline{B}CD$
10	$\overline{A}\,\overline{B}C\overline{D}$	$\overline{A}BC\overline{D}$	$ABC\overline{D}$	$A\overline{B}C\overline{D}$

With appropriate grouping:

CD \ AB	00	01	11	10
00	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$\overline{A}B\overline{C}\,\overline{D}$	$AB\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}\,\overline{D}$
01	$\overline{A}\,\overline{B}\,\overline{C}D$	$\overline{A}B\overline{C}D$	$AB\overline{C}D$	$A\overline{B}\,\overline{C}D$
11	$\overline{A}\,\overline{B}CD$	$\overline{A}BCD$	$ABCD$	$A\overline{B}CD$
10	$\overline{A}\,\overline{B}C\overline{D}$	$\overline{A}BC\overline{D}$	$ABC\overline{D}$	$A\overline{B}C\overline{D}$

Thus $F = \overline{B}\cdot\overline{D} + BD$

Example: Using a K-map to write a sum of products as a product of sums.
Convert $F = ABC + \overline{C}D + \overline{A}BD$ into a product of sums form.

CD \ AB	00	01	11	10
00	$\overline{A}\overline{B}\overline{C}\overline{D}$	$\overline{A}B\overline{C}\cdot\overline{D}$	$AB\overline{C}\overline{D}$	$A\overline{B}\overline{C}\overline{D}$
01	$\overline{A}\overline{B}\overline{C}D$	$\overline{A}B\overline{C}D$	$AB\overline{C}D$	$A\overline{B}\overline{C}D$
11	$\overline{A}\overline{B}CD$	$\overline{A}BCD$	$ABCD$	$A\overline{B}CD$
10	$\overline{A}\overline{B}C\overline{D}$	$\overline{A}BC\overline{D}$	$ABC\overline{D}$	$A\overline{B}C\overline{D}$

$\overline{F}$

CD \ AB	00	01	11	10
00	$\overline{A}\overline{B}\overline{C}\overline{D}$	$\overline{A}B\overline{C}\overline{D}$	$AB\overline{C}\overline{D}$	$A\overline{B}\overline{C}\overline{D}$
01	$\overline{A}\overline{B}\overline{C}D$	$\overline{A}B\overline{C}D$	$AB\overline{C}D$	$A\overline{B}\overline{C}D$
11	$\overline{A}\overline{B}CD$	$\overline{A}BCD$	$ABCD$	$A\overline{B}CD$
10	$\overline{A}\overline{B}C\overline{D}$	$\overline{A}BC\overline{D}$	$ABC\overline{D}$	$A\overline{B}C\overline{D}$

$CD \backslash AB$	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

$\bar{F} = \bar{C}\cdot\bar{D} + \bar{B}\cdot C + \bar{A}\cdot\bar{D}$ A sum of products form

$$
\begin{aligned}
F &= \bar{\bar{F}} = \overline{\bar{C}\cdot\bar{D} + \bar{B}\cdot C + \bar{A}\cdot\bar{D}} \\
&= \overline{\bar{C}\,\bar{D}}\;\;\overline{\bar{B}\,C}\;\;\overline{\bar{A}\,\bar{D}} \\
&= (C + D)(B + \bar{C})(A + D)
\end{aligned}
$$

and F is now in the Product of Sums form.

Use a Karnaugh Map to design a circuit with NAND logic

Problem: A flame detector, smoke detector and two high temperature detectors are situated in a room to produce fire detection system. Because of the number of false alarms, a fire is registered only when two or more sensors simultaneously are triggered. An output of 1 on a sensor indicates fire while 0 indicates no fire.

If the outputs of the four sensors are labeled A,B,C and D then the truth table for F (FIRE!) would look like

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$F = \overline{A}\overline{B}CD + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD + A\overline{B}\cdot\overline{C}D + A\overline{B}C\overline{D} +$$
$$A\overline{B}CD + AB\overline{C}\cdot\overline{D} + AB\overline{C}D + ABC\overline{D} + ABCD$$

We indicate F on a K-map

CD \ AB	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

And if we group terms together

CD \ AB	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

So the simplified version of F is

$$F = AB + CD + BD + AC + AD + BC$$

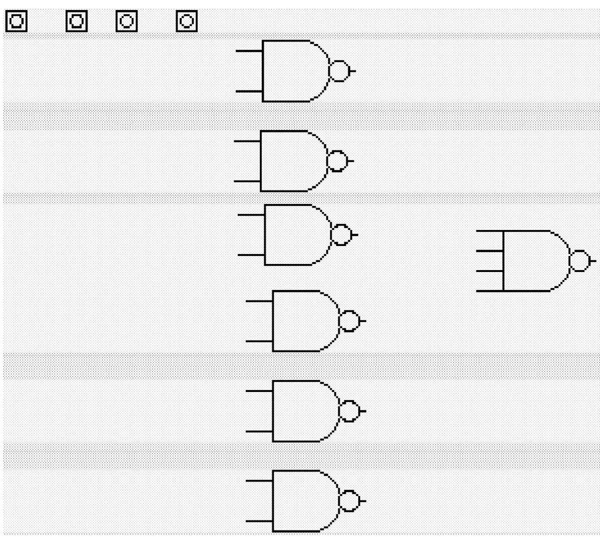
To eliminate the OR's

$$F = \overline{\overline{F}} = \overline{\overline{AB + CD + BD + AC + AD + BC}}$$

$$= \overline{\overline{A\,B}\;\overline{C\,D}\;\overline{B\,D}\;\overline{A\,C}\;\overline{A\,D}\;\overline{B\,C}}$$

And we see we can use six 2-input NAND gates and 1 six input NAND to produce our circuit

We go to Digital Works to set up our circuit and . . .



we see we have no 6 input NAND gates available to us!

Looking at the expression again

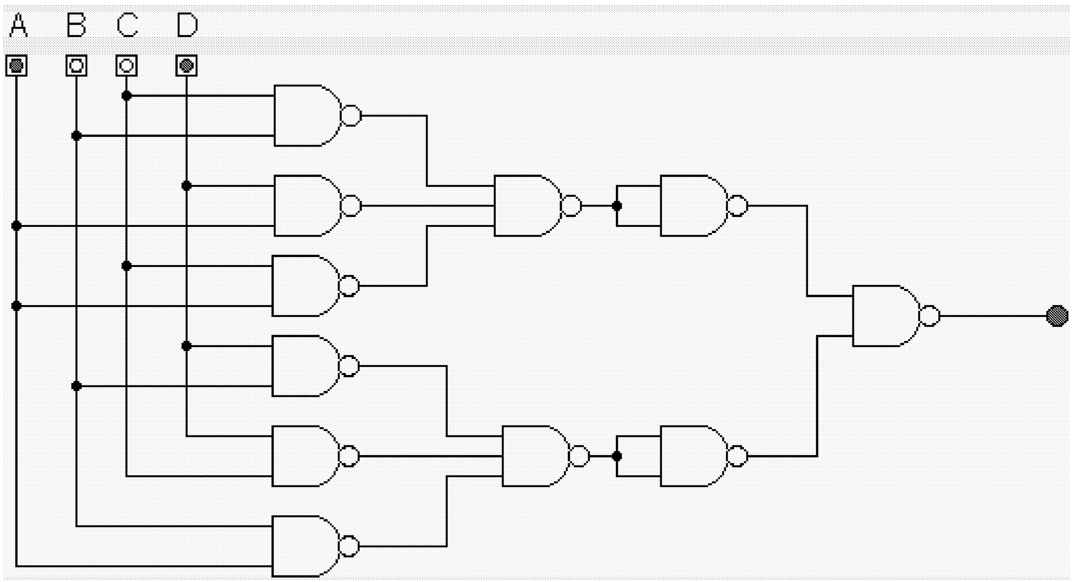
$$F = \overline{\overline{A\,B}\;\overline{C\,D}\;\overline{B\,D}\;\overline{A\,C}\;\overline{A\,D}\;\overline{B\,C}}$$

we realize we can "group" as follows

$$F = \overline{\overline{\overline{\overline{AB}\,\overline{CD}\,\overline{BD}}}\;\overline{\overline{\overline{AC}\,\overline{AD}\,\overline{BC}}}}$$

A 2-input NAND, two 3-input NANDs, two 2-input NANDs (as inverters), six 2-input NANDs. Each of which is readily available!

$$F = \overline{\overline{\overline{\overline{AB}\,\overline{CD}\,\overline{BD}}}\;\overline{\overline{\overline{AC}\,\overline{AD}\,\overline{BC}}}}$$



Karnaugh Maps and don't care conditions

Example: An air conditioning system has two input sensors. One input, C, is from a cold-sensing thermostat. If the temperature is below 15° C it registers 1 (true) otherwise it registers 0 (false). The other input, H, is from a heat-sensing thermostat. If the temperature is above 22°C it registers 1 otherwise it registers 0.

Take a look at the input portion of the truth table:

C	H	Interpretation
0	0	15 °≤ Temp ≤ 22 °
0	1	Temp >22 °
1	0	Temp<15 °
1	1	Not possible, sensor failure

Often we encounter a state that cannot exist. We will refer to this as a ***don't care condition***.

In terms of design, we are often free to specify the output of the function for a corresponding don't care condition.

There are of course exceptions to this : An impossible state occurring under fault conditions could indicate circuit failure. If assigning an input to a fault condition leads to a dangerous situation we are ethically bound to not assign an output which could cause harm.

For our discussion we assume we are free to assign a value of our choice as the output for a Boolean function corresponding to don't care conditions.

Example: Consider the function with the following truth table

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	XXX
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	XXX
1	1	1	0	0
1	1	1	1	1

The entry of XXX indicates an impossible input condition.

CD \ AB	00	01	11	10
00	$\overline{A}\overline{B}\overline{C}\overline{D}$	$\overline{A}B\overline{C}\overline{D}$	$AB\overline{C}\overline{D}$	$A\overline{B}\overline{C}\overline{D}$
01	$\overline{A}\overline{B}\overline{C}D$	$\overline{A}B\overline{C}D$	XXX $AB\overline{C}D$	XXX $A\overline{B}\overline{C}D$
11	$\overline{A}\overline{B}CD$	$\overline{A}BCD$	$ABCD$	$A\overline{B}CD$
10	$\overline{A}\overline{B}C\overline{D}$	$\overline{A}BC\overline{D}$	$ABC\overline{D}$	$A\overline{B}C\overline{D}$

Treating the XXX as 0's for a moment $F = \overline{A}B\overline{C}D + \overline{A}BCD + ABCD$

Which does simplify to $F = \overline{A}BD + BCD$

In this case we really don't care what the value is in positions

$$AB\overline{C}D,\ A\overline{B}\overline{C}D$$

So we will pick them so as to produce the most simplified expression for F.

CD \ AB	00	01	11	10
00	$\overline{A}\overline{B}\overline{C}\overline{D}$	$\overline{A}B\overline{C}\overline{D}$	$AB\overline{C}\overline{D}$	$A\overline{B}\overline{C}\overline{D}$
01	$\overline{A}\overline{B}\overline{C}D$	$\overline{A}B\overline{C}D$	XXX $AB\overline{C}D$	XXX $A\overline{B}\overline{C}D$
11	$\overline{A}\overline{B}CD$	$\overline{A}BCD$	$ABCD$	$A\overline{B}CD$
10	$\overline{A}\overline{B}C\overline{D}$	$\overline{A}BC\overline{D}$	$ABC\overline{D}$	$A\overline{B}C\overline{D}$

And we can write $F = BD$