Probability
Chapter 4_Part_B

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Conditional Probability

Let A and B be any two events from the same sample space S. We denote the **conditional probability** of event A, given that event B has occurred by $\boldsymbol{P(A|B)}$,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

where $\boldsymbol{P(B)>0}$.

Equivalent relation is called the **multiplication law of probability** and is given by

$$P(A \cap B) = P(A \mid B) * P(B)$$

Example: Conditional Probabilities

Eye Color	Female	Male	TOTAL
Brown	20	5	25
Blue	15	30	45
Green	30	50	80
TOTAL	65	85	150

Example (continue)

Eye Color	Female	Male	TOTAL
Brown	20	5	25
Blue	15	30	45
Green	30	50	80
TOTAL	65	85	150

$$P(Brown \mid Female) = \frac{P(BrownandFemale)}{P(Female)} = \frac{20/150}{65/150} = \frac{4}{13}$$

Example (continue)

Eye Color	Female	Male	TOTAL
Brown	20	5	25
Blue	15	30	45
Green	30	50	80
TOTAL	65	85	150

$$P(Female \mid Brown) = \frac{20/150}{25/150} = \frac{4}{5} = 0.8$$

Example 12 (textbook)

Complementary alternative medicine (CAM), including acupuncture, yoga, and massage has become more populat. By combining information in tables, we obtain information concerning use of CAM in the past year and weight class based on body mass index. The proportion in the various categories appear in the table below:

TABLE 1 Body Weight and Complementary and Alternative Medicine

	Underweight	Healthy weight	Overweight	Obese	Total
CAM	.01	.13	.12	.12	.38
No CAM	.02	.19	.21	.20	.62
Total	.03	.32	.33	.32	1.00

Example (Ctd.)

A. What is the probability that a person selected at random from this population will have used complementary and alternative medicine (CAM) in the past year?

P(CAM) = ??

B. A person selected at random is found to be overweight. What is the probability that this person use complementary and alternative medicine (CAM) in the past year?

$$P(CAM \mid Overweight) = \frac{P(CAMandOverweight)}{P(Overweight)} = \frac{0.12}{0.33} = 0.364$$

Example:

Table 4.7 gives the classification of all employees of a company given by gender and college degree.

	College Graduate (*G*)	Not a College Graduate (*N*)	Total
Male (*M*)	7	20	27
Female (*F*)	4	9	13
Total	11	29	40

If one of these employees is selected at random for membership on the employee-management committee, what is the probability that this employee is a female and a college graduate?

Solution

We are to calculate the probability of the intersection of the events *F and G*.

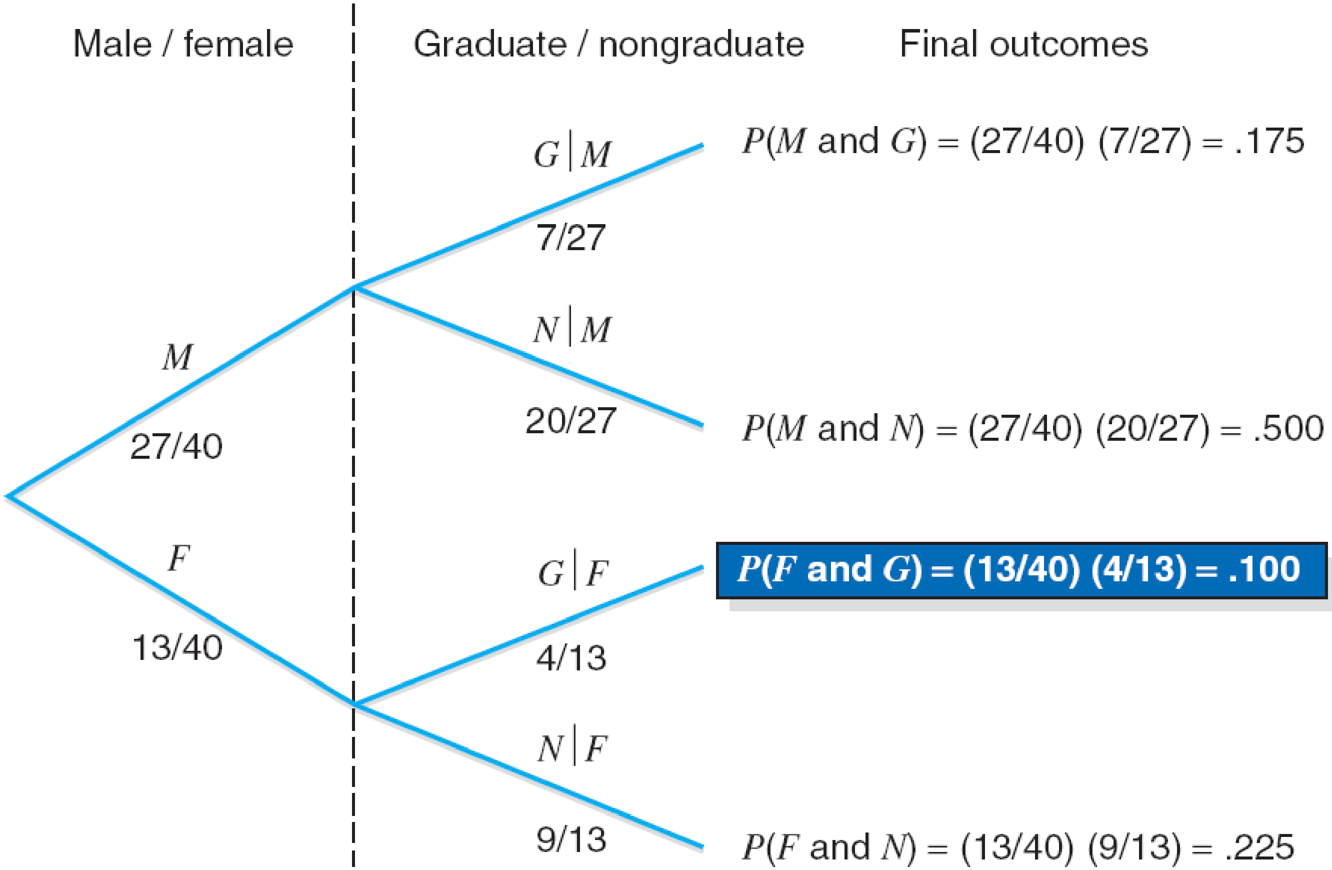
$P(F \text{ and } G) = P(F)\,P(G\,|F)$

$P(F) = 13/40$

$P(G\,|F) = 4/13$

$$P(F \text{ and } G) = P(F)\,P(G\,|F) = (13/40)(4/13) = .100$$

Tree diagram for joint probabilities.



Dependent Events

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be any two events from the sample space $\boldsymbol{S}$. The two events are **dependent** if the probability of the occurrence of the second event is conditional, depending on the first. Given the events A and B are dependent, then the probability of A and B (i.e. the probability of the intersection, $\boldsymbol{A \cap B}$) is given by,

$$\boldsymbol{P(A \cap B) = P(A) \times P(B|A) = P(B) \times P(A|B)}$$

Independent Events

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be any two events from $\boldsymbol{S}$. The event $\boldsymbol{A}$ is said to be **independent** of event $\boldsymbol{B}$ if the occurrence or nonoccurrence of one event has no effect on the probability of the occurrence of the other event.

That is, the probability of $\boldsymbol{A}$ and $\boldsymbol{B}$ is given by,

$$P(A \cap B) = P(A) \times P(B)$$

That is, the probability of the intersection of the events $\boldsymbol{A \cap B}$ is equal to the product of the probabilities of $\boldsymbol{A}$ and $\boldsymbol{B}$.

Example
Independent
Dependent

INDEPENDENT

Two cards are selected from a standard deck of 52 cards **with replacement**, what is the probability that the two cards selected are Hearts.

$$P(H \text{ and } H) = \frac{13}{52} \times \frac{13}{52} = \frac{1}{16}$$

DEPENDENT

Two cards are selected from a standard deck of 52 cards **without replacement**, what is the probability that the two cards selected are Hearts.

$$P(H \text{ and } H) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$$

Properties: AND means MULTIPLY

Given two events, *A* and *B*, are **independent** then

$$P(A\,\text{and}\,B) = P(A) \times P(B)$$

Given two events, *A* and *B*, are **dependent** then

$$P(A\,\text{and}\,B) = P(A) \times P(B \mid A)$$

Two events A and B are **independent** if

$$P(A \mid B) = P(A)$$

Equivalent conditions are

$$P(B \mid A) = P(B)$$

or

$$P(AB) = P(A)P(B)$$

Example 12 ctd

In the example 12 above. Are the two events, use of CAM and Overweight independent?

Since $P(CAM) = 0.38$ and $P(CAM|Overweight) = 0.364$.
Because these two probabilities are different, CAM and Overweight are not independent.

Recall: if $P(A|B) = P(A)$, then A and B are independent.

Example

Let F_1 represent the event that student passes French 101

and F_2 be student passes French 102.

Given that the probability of passing French 101 is 0.77

i.e. $P(F_1) = 0.77$

and the probability of passing French 102,

given you passed French 101 is 0.90

i.e. $P(F_2 \mid F_1) = 0.90$

What is the probability you passed both French 101 and 102?

$$P(F_1 \cap F_2) = P(F_1) \times P(F_2 \mid F_1)$$

$$= 0.77 \times 0.90 = 0.693$$

Example 14 (textbook)

There are 25 pens in a container on your desk. Among them 20 write well but 5 have defective ink cartridges. You select 2 pens to take to a business appointment. Calculate the probability that:

1. Both pens are defective

$$P(D1 \text{ and D2}) = \tfrac{5}{25} \times \tfrac{4}{24} = \tfrac{1}{30} = 0.033$$

2. One pen is defective but the other writes well.

$$P(D1 \text{ and } G2) + P(G1 \text{ and D2}) = \tfrac{5}{25} \times \tfrac{20}{24} + \tfrac{20}{25} \times \tfrac{5}{24} = \tfrac{1}{6} + \tfrac{1}{6} = 0.333$$

Example 17 (textbook)

In the above example (14), suppose that one pend is drawn at random. It is returned to the box and then another pen is drawn at random. What is the probability that both draws produce pens that will not write?

$$P(D1 \text{ and } D2) = \frac{5}{25} \times \frac{5}{25} = \frac{1}{25} = 0.04$$

Is this probability dependent or independent? Explain.

Example

Based on the data of the Center for Health Statistics, the 2009 birth rates in 50 states are grouped in the following frequency Table

Let A be the event that the birth Rate is Under 14, B be the event that the birth rate is under 18 but not under 12, And C is the 16 or over.

Are events A and C are independent?
Are events A and B are independent?

Birth Rate Per thousand	Number of Cases
10-12	4
12-14	12
14-16	29
16-18	4
18 and over	1

Example

A box contains a total of 100 DVDs that were manufactured on two machines. Of them, 60 were manufactured on Machine I. Of the total DVDs, 15 are defective. Of the 60 DVDs that were manufactured on Machine I, 9 are defective. Let *D* be the event that a randomly selected DVD is defective, and let *A* be the event that a randomly selected DVD was manufactured on Machine I. Are events *D* and *A* independent?

From the given information,

$$P(D) = 15/100 = .15 \quad \text{and}$$

$$P(D \mid A) = 9/60 = .15$$

Hence,

$$P(D) = P(D \mid A)$$

Consequently, the two events, D and A, are independent.

Example Independent Selection

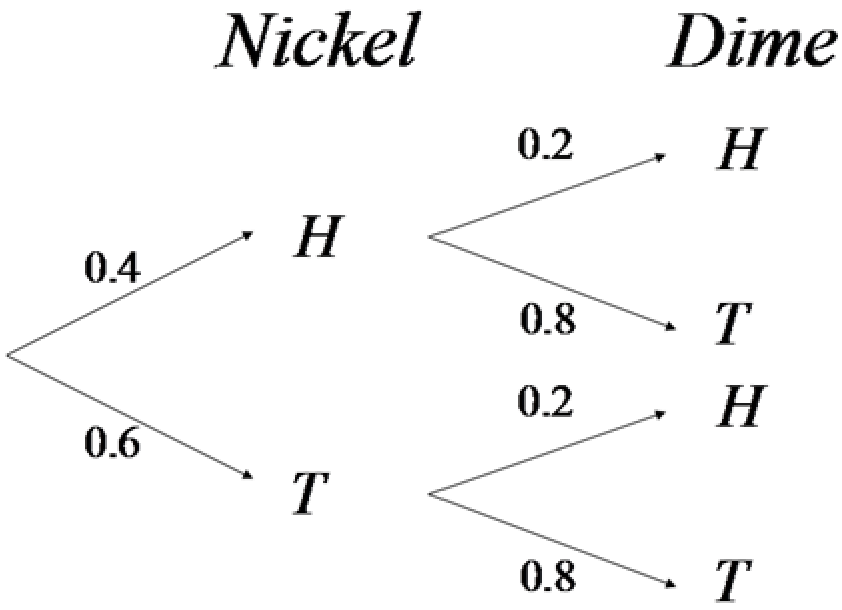
Two unfair coins (nickel and dime) are tossed. Given

P(Head on Nickel) = 0.4

and

P(Head on Dime) = 0.2,

what are the probabilities associated with the unfair coins?



P(HH)=0.4×0.2=0.08

P(HT)=0.4×0.8=0.32

P(TH)=0.6×0.2=0.12

P(TT)=0.6×0.8=0.48

The Rule of Combination

Suppose that r objects are drawn **without replacement**, without **order** being preserved, from a collection of ***n*** distinct objects. Then the sample space consists of

$$C(n,r) = \binom{n}{r} = C_n^r = {}_nC_r = \frac{n!}{r!(n-r)!}$$

sample points; that is, the number of ways to combine without order ***r*** distinct objects; **combinations.**

Combination Formula

$$C(n,r) = {}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

A factorial is a counting method that uses consecutive whole numbers as factors.

The factorial symbol is !

Examples

7! = 7x6x5x4x3x2x1 = **5040**

Note:

1! = 1

0!=1

C(n,n) = 1 **Ex: C(5,5)= ……..**

C(n,1) = n **Ex: C(5,1) =………**

C(n,0)= 1 **Ex: C(5,0)=…………**

Example Combinations

How many ways can you combine 3 out of 4 letters?

L={A,B,C,D}

Combination:
Here n = 4 and r = 3. Hence using the formula in the previous page

$$\binom{4}{3} = \frac{4!}{(4-3)!3!}$$

$$= \frac{(4\times3\times2\times1)}{1x(3\times2\times1)}$$

$$=4$$

ABC ABD ACD BCD

Example

1. How many 3-person committees can be chosen from a group of 9 people?

2. Given 5 men and 3 women, what is the probability that two men and one woman are selected to form a committee.

Practice:

1. Given events A and B are not mutually exclusive events then
   $P(A \cup B) =$
   $P(A \mid B) =$
2. Given events A and B are mutually exclusive events then
   $A \cap B =$
   $P(A \cap B) =$
   $P(A \cup B) =$
   $P(A \mid B) =$
3. Given events A and B are independent events then
   $P(A \cap B) =$
   $P(A \mid B) =$