Section 8.5

Testing $\mu_1 - \mu_2$ (Independent Samples)

Focus Points

- Identify independent samples and sampling distributions.
- the test Compute the sample test statistic and P-value for $\mu_1 - \mu_2$ and conclude.

Independent Samples

Independent Samples

Many practical applications of statistics involve a comparison of two population means or two population proportions.

With dependent samples, we could pair the data and then consider the difference of data measurements d.

In this section, we will turn our attention to tests of differences of means from *independent samples*.

We will see new techniques for testing the difference of means from independent samples.

Independent Samples

First, let's consider independent samples.

We say that two sampling distributions are *independent* if there is no relationship whatsoever between specific values of the two distributions.

Example 11 – *Independent Samples*

A teacher wishes to compare the effectiveness of two teaching methods.

Students are randomly divided into two groups:

The first group is taught by method 1; the second group, by method 2.

At the end of the course, a comprehensive exam is given to all students, and the mean score $\bar{x}_1$ for group 1 is compared with the mean score $\bar{x}_2$ for group 2. Are the samples independent or dependent?

Example 11 – *Solution*

Because the students were *randomly* divided into two groups, it is reasonable to say that the $\bar{x}_1$ distribution is independent of the $\bar{x}_2$ distribution.

Example 12 – *Dependent sample*

We have considered a situation in which a shoe manufacturer claimed that for the general population of adult U.S. citizens, the average length of the left foot is longer than the average length of the right foot.

To study this claim, the manufacturer gathers data in this fashion: Sixty adult U.S. citizens are drawn at random, and for these 60 people, both their left and right feet are measured.

Let $\bar{x}_1$ be the mean length of the left feet and $\bar{x}_2$ be the mean length of the right feet. Are the $\bar{x}_1$ and $\bar{x}_2$ distributions independent for this method of collecting data?

Example 12 – *Solution*

In this method, there is only *one* random sample of people drawn, and both the left and right feet are measured from this sample.

The length of a person's left foot is usually related to the length of the person's right foot, so in this case the $\bar{x}_1$ and $\bar{x}_2$ distributions are *not* independent.

In fact, we could pair the data and consider the distribution of the differences, left foot length minus right foot length. Then we would use the techniques of paired difference tests.

Part I: Testing $\mu_1 - \mu_2$ When σ_1 and σ_2 Are Known

Part I: Testing $\mu_1 - \mu_2$ When σ_1 and σ_2 Are Known cont'd

Procedure

1. In the context of the application, state the *null and alternate hypotheses* and set the *level of significance* α. It is customary to use $H_0: \mu_1 - \mu_2 = 0$.
2. Use $\mu_1 - \mu_2 = 0$ from the null hypothesis together with $\bar{x}_1, \bar{x}_2, \sigma_1, \sigma_2, n_1$, and n_2 to compute the *sample test statistic*.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

3. Use the standard normal distribution and the type of test, one-tailed or two-tailed, to find the *P-value* corresponding to the sample test statistic.
4. *Conclude the test*. If P-value $\leq \alpha$, then reject H_0. If P-value $> \alpha$, then do not reject H_0.
5. *Interpret your conclusion* in the context of the application.

Example 13 – *Testing the difference of means (σ_1 and σ_2 known)*

A consumer group is testing camp stoves. To test the heating capacity of a stove, it measures the time required to bring 2 quarts of water from 50 F to boiling (at sea level). Two competing models are under consideration.

Ten stoves of the first model and 12 stoves of the second model are tested. The following results are obtained.

Model 1: Mean time $\overline{x}_1$ = 11.4 min; σ_1 = 2.5 min; n_1 = 10

Model 2: Mean time $\overline{x}_2$ = 9.9 min; σ_1 = 3.0 min; n_2 = 12

Example 13 – *Testing the difference of means (σ_1 and σ_2 known)*

cont’d

Assume that the time required to bring water to a boil is normally distributed for each stove.

Is there any difference (either way) between the performances of these two models? Use a 5% level of significance.

Example 13 – *Solution*

(a) State the null and alternate hypotheses and note the value of α.

Let μ_1 and μ_2 be the means of the distributions of times for models 1 and 2, respectively. We set up the null hypothesis to state that there is no difference:

$$H_0: \mu_1 = \mu_2 \qquad \text{or} \qquad H_0: \mu_1 - \mu_2 = 0$$

The alternate hypothesis states that there is a difference:

$$H_1: \mu_1 \neq \mu_2 \qquad \text{or} \qquad H_1: \mu_1 - \mu_2 \neq 0$$

The level of significance is $\alpha = 0.05$.

Example 13 – *Solution*

cont'd

(b) *Check Requirements* What distribution does the sample test statistic follow? The sample test statistic z follows a standard normal distribution because the original distributions from which the samples are drawn are normal. In addition, the population standard deviations of the original distributions are known and the samples are independent.

(c) Compute $\bar{x}_1 - \bar{x}_2$ and then convert it to the sample test statistic z. We are given the values $\bar{x}_1 = 11.4$ and $\bar{x}_2 = 9.9$. Therefore, $\bar{x}_1 - \bar{x}_2 = 11.4 \quad 9.9 = 1.5$. To convert this to a z value, we use the values $\sigma_1 = 2.5$, and $\sigma_2 = 3.0$, $n_1 = 10$. and $n_2 = 12$. From the null hypothesis, $\mu_1 - \mu_2 = 0$

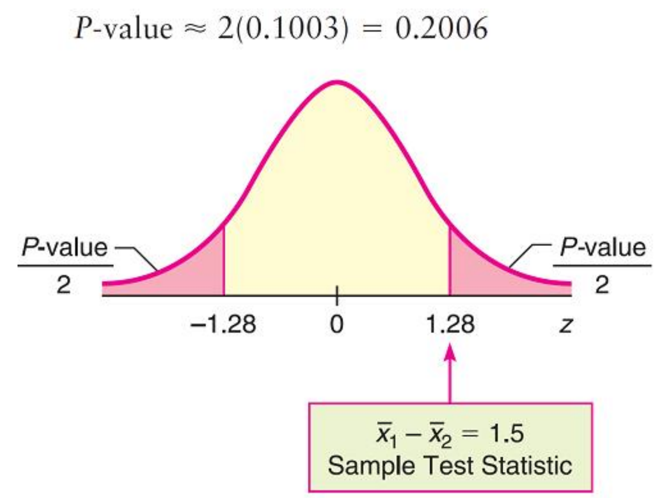
Example 13 – *Solution*

cont'd

$$z = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} = \frac{1.5}{\sqrt{\dfrac{2.5^2}{10} + \dfrac{3.0^2}{12}}} \approx 1.28$$

(d) Find the *P*-value and sketch the area on the standard normal curve. Figure 8-16 shows the *P*-value.

P-value $\approx 2(0.1003) = 0.2006$



P-value
Figure 8-16

Example 13 – *Solution*

(e) Conclude the test.

The P-value is 0.2006 and $\alpha = 0.05$. Since P-value $> \alpha$, do not reject H_0.

(f) *Interpretation* Interpret the results.

At the 5% level of significance, the sample data do not indicate any difference in the population mean times for boiling water for the two stove models.

Part II: Testing $\mu_1 - \mu_2$ When σ_1 and σ_2 Are Unknown

Part II: Testing $\mu_1 - \mu_2$ When σ_1 and σ_2 Are Unknown

To test $\mu_1 - \mu_2$ when σ_1 and σ_2 are unknown, we use distribution methods for estimating $\mu_1 - \mu_2$.

In particular, if the two distributions are normal or approximately mound-shaped, or if both sample sizes are large , we use a Student's *t* distribution.

Let's summarize the method of testing $\mu_1 - \mu_2$.

Part II: Testing $\mu_1 - \mu_2$ When σ_1 and σ_2 Are Unknown

Procedure:

HOW TO TEST $\mu_1 - \mu_2$ WHEN σ_1 AND σ_2 ARE UNKNOWN

Requirements

Procedure

Obtain two independent random samples from populations 1 and 2, where

$\bar{x}_1$ and $\bar{x}_2$ are sample means from populations 1 and 2

s_1 and s_2 are sample standard deviations from populations 1 and 2

n_1 and n_2 are sample sizes from populations 1 and 2

If you can assume that both population distributions 1 and 2 are normal or at least mound-shaped and symmetric, then any sample sizes n_1 and n_2 will work. If you cannot assume this, then use sample sizes $n_1 \geq 30$ and $n_2 \geq 30$.

Procedure

1. In the context of the application, state the *null and alternate hypotheses* and set the *level of significance* α. It is customary to use H_0: $\mu_1 - \mu_2 = 0$.
2. Use $\mu_1 - \mu_2 = 0$ from the null hypothesis together with $\bar{x}_1$, $\bar{x}_2$, s_1, s_2, n_1, and n_2 to compute the *sample test statistic*.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

Part II: Testing $\mu_1 - \mu_2$ When σ_1 and σ_2 Are Unknown

cont'd

Procedure

The sample test statistic distribution is approximately that of a Student's t with *degrees of freedom d.f. = smaller* of $n_1 - 1$ and $n_2 - 1$.

Note that statistical software gives a more accurate and larger *d.f.* based on Satterthwaite's approximation (see Problem 27).

3. Use a Student's t distribution and the type of test, one-tailed or two-tailed, to find the *P-value* corresponding to the sample test statistic.
4. *Conclude the test*. If P-value $\leq \alpha$, then reject H_0. If P-value $> \alpha$, then do not reject H_0.
5. *Interpret your conclusion* in the context of the application.

Example 14 – *Testing the difference of means (σ_1 and σ_2 unknown)*

Two competing headache remedies claim to give fast-acting relief. An experiment was performed to compare the mean lengths of time required for bodily absorption of brand A and brand B headache remedies.

Twelve people were randomly selected and given an oral dosage of brand A.

Another 12 were randomly selected and given an equal dosage of brand B.

Example 14 – *Testing the difference of means (σ_1 and σ_2 unknown)*

cont'd

The lengths of time in minutes for the drugs to reach a specified level in the blood were recorded.

The means, standard deviations, and sizes of the two samples follow.

Brand A: $\bar{x}_1 = 21.8$ min; $s_1 = 8.7$ min; $n_1 = 12$

Brand B: $\bar{x}_2 = 18.9$ min; $s_2 = 7.5$ min; $n_2 = 12$

Example 14 – *Testing the difference of means (σ_1 and σ_2 unknown)*

cont'd

Past experience with the drug composition of the two remedies permits researchers to assume that both distributions are approximately normal.

Let us use a 5% level of significance to test the claim that there is no difference between the two brands in the mean time required for bodily absorption.

Also, find or estimate the *P*-value of the sample test statistic.

Example 14(a) – *Solution*

α = 0.05. The null hypothesis is

$$H_0\text{: } \mu_1 = \mu_1 \qquad \text{or} \qquad H_0\text{: } \mu_1 - \mu_2 = 0$$

Since we have no prior knowledge about which brand is faster, the alternate hypothesis is

$$H_1\text{: } \mu_1 \neq \mu_2 \qquad \text{or} \qquad H_1\text{: } \mu_1 - \mu_2 \neq 0$$

Example 14(b) – *Solution*

cont'd

Check Requirements What distribution does the sample test statistic follow? We use a Student's t distribution with $d.f.$ = smaller sample size – 1 = 12 – 1 = 11.

Note that both samples are of size 12. Degrees of freedom can be computed also by Satterthwaite's approximation.

A Student's t distribution is appropriate because the original populations are approximately normal, the population standard deviations are not known, and the samples are independent.

Example 14(c) – *Solution*

cont'd

Compute the sample test statistic.

We're given $\overline{x}_1 = 21.8$ and $\overline{x}_2 = 18.9$, so the sample difference is $\overline{x}_1 - \overline{x}_2 = 21.8 - 18.9 = 2.9$.

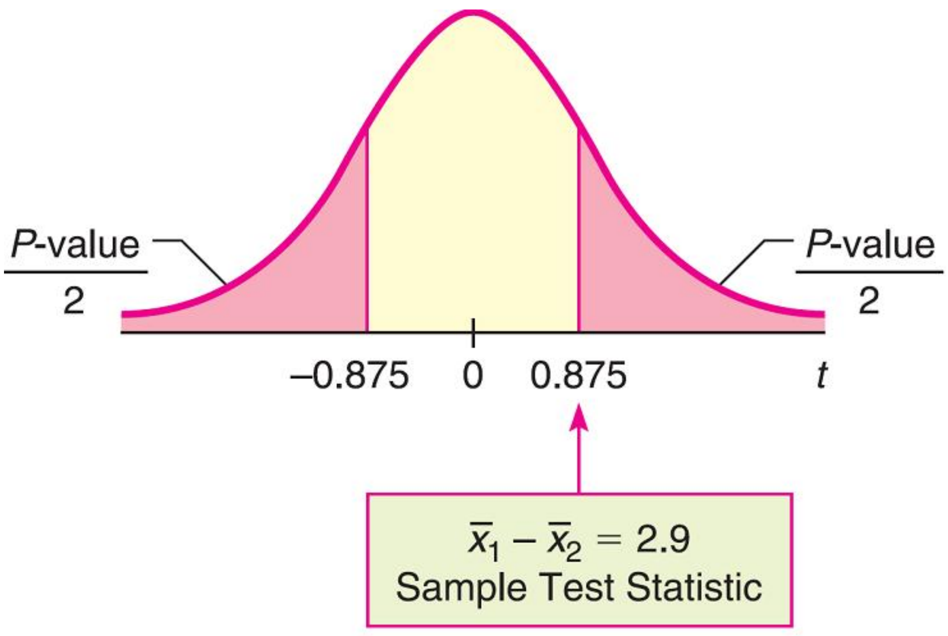
Using $s_1 = 8.7$, $s_2 = 7.5$, $n_1 = 12$,$n_2 = 12$, and $\mu_1 - \mu_2 = 0$ from H_0, we compute the sample test statistic.

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \frac{2.9}{\sqrt{\dfrac{8.7^2}{12} + \dfrac{7.5^2}{12}}} \approx 0.875$$

Example 14(d) – *Solution*

cont'd

Estimate the *P*-value and sketch the area on a *t* graph. Figure 8-18 shows the *P*-value.



P-value

Figure 8.18

Example 14(d) – *Solution*

cont'd

The degrees of freedom are *d.f* = 11(since both samples are of size 12). Because the test is a two-tailed test, the *P*-value is the area to the right of 0.875 together with the area to the left of –0.875. In the Student's *t* distribution table (Table 6 of Appendix II), we find an interval containing the *P*-value.

one-tail area	0.250		0.125
✓ two-tail area	0.500		0.250
d.f. = 11	0.697	↑ Sample $t = 0.875$	1.214

Excerpt from Table 6, Appendix II

Table 8-13

Example 14(d) – *Solution*

cont'd

Find 0.875 in the row headed by *d.f.* = 11. The test statistic 0.875 falls between the entries 0.697 and 1.214.

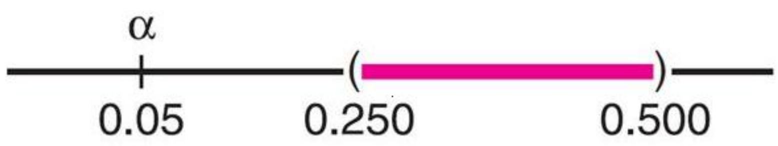
Because this is a two-tailed test, we use the corresponding *P*-values 0.500 and 0.250 from the *two-tail* area row (see Table 8-13, Excerpt from Table 6). The *P*-value for the sample *t* is in the interval

$$0.250 < P\text{-value} < 0.500$$

Example 14(e) – *Solution*

cont'd

Conclude the test.



Since the interval containing the *P*-value lies to the right of α = 0.05, we fail to reject H_0.

Note:
Using the raw data and a calculator Satterthwaite's approximation for the degrees of freedom *d.f* $\approx$ 21.53, the *P*-value $\approx$ 0.3915 This value is in the interval we computed.

Example 14(f) – *Solution*

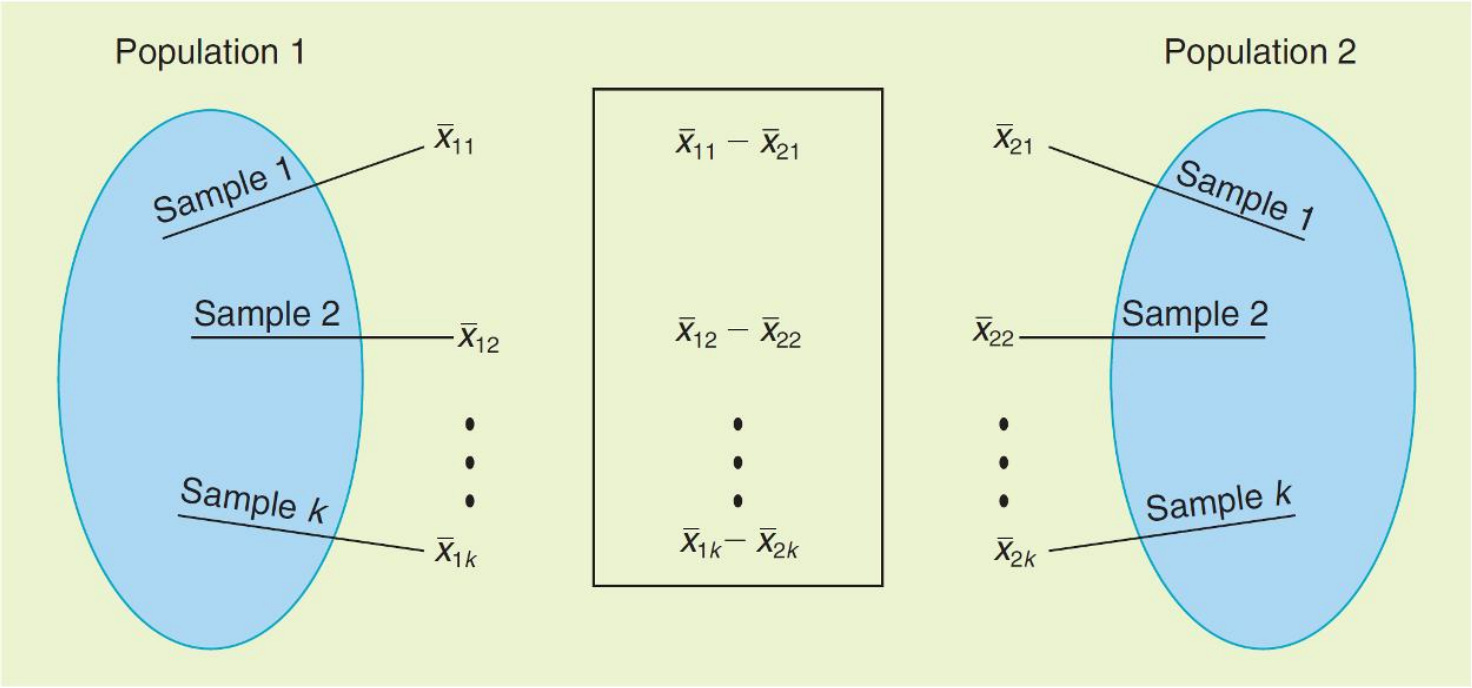
cont'd

Interpretation Interpret the results. At the 5% level of significance, there is insufficient evidence to conclude that there is a difference in mean times for the remedies to reach the specified level in the bloodstream.

Confidence Intervals for $\mu_1 - \mu_2$ (σ_1 and σ_2 known)

Confidence Intervals for $\mu_1 - \mu_2$ (σ_1 and σ_2 known)

Figure 7-7 illustrates the sampling distribution of $\bar{x}_1 - \bar{x}_2$.



Sampling Distribution of $\bar{x}_1 - \bar{x}_2$

Figure 7-7

Example 8 – *Confidence interval for $\mu_1 - \mu_2$, σ_1 and σ_2 known*

In the summer of 1988, Yellowstone National Park had some major fires that destroyed large tracts of old timber near many famous trout streams.

Fishermen were concerned about the long-term effects of the fires on these streams.

However, biologists claimed that the new meadows that would spring up under dead trees would produce a lot more insects, which would in turn mean better fishing in the years ahead.

Guide services registered with the park provided data about the daily catch for fishermen over many years.

Example 8 – *Confidence interval for $\mu_1 - \mu_2$, σ_1 and σ_2 known*

cont'd

Ranger checks on the streams also provided data about the daily number of fish caught by fishermen.

Yellowstone Today (a national park publication) indicates that the biologists' claim is basically correct and that Yellowstone anglers are delighted by their average increased catch.

Suppose you are a biologist studying fishing data from Yellowstone streams before and after the fire. Fishing reports include the number of trout caught per day per fisherman.

Example 8 – *Confidence interval for $\mu_1 - \mu_2$, σ_1 and σ_2 known*

cont'd

A random sample of $n_1 = 167$ reports from the period before the fire showed that the average catch was $\bar{x}_1 = 5.2$ trout per day. Assume that the standard deviation of daily catch per fisherman during this period was $\sigma_1 = 1.9$.

Another random sample of $n_2 = 125$ fishing reports 5 years after the fire showed that the average catch per day was $\bar{x}_2 = 6.8$ trout. Assume that the standard deviation during this period was $\sigma_2 = 2.3$.

Example 8(a) – *Confidence interval for $\mu_1 - \mu_2$, σ_1 and σ_2 known*

cont'd

Check Requirements For each sample, what is the population? Are the samples dependent or independent? Explain. Is it appropriate to use a normal distribution for the $\bar{x}_1 - \bar{x}_2$ distribution? Explain.

Solution:

The population for the first sample is the number of trout caught per day by fishermen before the fire.

The population for the second sample is the number of trout caught per day after the fire. Both samples were random samples taken in their respective time periods.

Example 8(a) – *Solution*

cont'd

There was no effort to pair individual data values. Therefore, the samples can be thought of as independent samples.

A normal distribution is appropriate for the $\bar{x}_1 - \bar{x}_2$ distribution because sample sizes are sufficiently large and we know both σ_1 and σ_2.

Compute a 95% confidence interval for $\mu_1 - \mu_2$, the difference of population means.

Example 8(b) – *Confidence interval for $\mu_1 - \mu_2$, σ_1 and σ_2 known*

cont'd

Compute a 95% confidence interval for, the difference of population means.

Solution:

Since n_1 = 167, $\bar{x}_1$ = 5.2, σ_1 = 1.9, n_2 = 125, $\bar{x}_2$ = 6.8, σ_2 = 2.3, and $z_{0.95}$ = 1.96, then

$$E = z_c\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Example 8(b) – *Solution*

$$= 1.96\sqrt{\frac{(1.9)^2}{167} + \frac{(2.3)^2}{125}}$$

$$\approx 1.96\sqrt{0.0639}$$

$$\approx 0.4955$$

$$\approx 0.50$$

Example 8(b) – *Solution*

cont'd

The 95% confidence interval is

$$(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$$

$$(5.2 - 6.8) - 0.50 < \mu_1 - \mu_2 < (5.2 - 6.8) + 0.50$$

$$-2.10 < \mu_1 - \mu_2 < -1.10$$

Example 8(c) – *Confidence interval for $\mu_1 - \mu_2$, σ_1 and σ_2 known*

cont'd

Interpretation What is the meaning of the confidence interval computed in part (b)?

Solution:

We are 95% confident that the interval –2.10 to –1.10 fish per day is one of the intervals containing the population difference $\mu_1 - \mu_2$, where μ_1 represents the population average daily catch before the fire and μ_2 represents the population average daily catch after the fire.

Example 8(c) – *Solution*

cont'd

Put another way, since the confidence interval contains only *negative values*, we can be 95% sure that $\mu_1 - \mu_2 < 0$. This means we are 95% sure that $\mu_1 < \mu_2$.

In words, we are 95% sure that the average catch before the fire was less than the average catch after the fire.

Confidence Intervals for $\mu_1 - \mu_2$ When σ_1 and σ_2 Are Unknown)

Confidence interval for $\mu_1 - \mu_2$, When σ_1 and σ_2 Are unknown

When σ_1 and σ_2 are unknown, we turn to a Student's *t* distribution.

As before, when we use a Student's *t* distribution, we require that our populations be normal or approximately normal (mound-shaped and symmetric) when the sample sizes and are less than 30.

We also replace σ_1 by s_1 and σ_2 by s_2. Then we consider the approximate *t* value attributed to Welch.

$$t \approx \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

Example 9 – *Confidence interval for $\mu_1 - \mu_2$, When σ_1 and σ_2 unknown*

Alexander Borbely is a professor at the Medical School of the University of Zurich, where he is director of the Sleep Laboratory. Dr. Borbely and his colleagues are experts on sleep, dreams, and sleep disorders.

In his book *Secrets of Sleep*, Dr. Borbely discusses brain waves, which are measured in hertz, the number of oscillations per second.

Rapid brain waves (wakefulness) are in the range of 16 to 25 hertz. Slow brain waves (sleep) are in the range of 4 to 8 hertz.

During normal sleep, a person goes through several cycles (each cycle is about 90 minutes) of brain waves, from rapid to slow and back to rapid.

During deep sleep, brain waves are at their slowest. In his book, Professor Borbely comments that alcohol is a *poor* sleep aid.

In one study, a number of subjects were given 1/2 liter of red wine before they went to sleep. The subjects fell asleep quickly but did not remain asleep the entire night.

Example 9 – *Confidence interval for $\mu_1 - \mu_2$, When σ_1 and σ_2 unknown*

cont'd

Toward morning, between 4 and 6 A.M., they tended to wake up and have trouble going back to sleep.

Suppose that a random sample of 29 college students was randomly divided into two groups.

The first group of $n_1 = 15$ people was given 1/2 liter of red wine before going to sleep. The second group of $n_2 = 14$ people was given no alcohol before going to sleep.

Everyone in both groups went to sleep at 11 P.M. The average brain wave activity (4 to 6 A.M.) was determined for each individual in the groups.

Assume the average brain wave distribution in each group is moundshaped and symmetric.

The results follow:

Group 1 (x_1 values): n_1 = 15 (with alcohol)

Average brain wave activity in the hours 4 to 6 A.M.

16.0	19.6	19.9	20.9	20.3	20.1	16.4	20.6
20.1	22.3	18.8	19.1	17.4	21.1	22.1	

For group 1, we have the sample mean and standard deviation of

$$\bar{x}_1 \approx 19.65 \text{ and } s_1 \approx 1.86$$

Group 2 (x_2 values): n_2 = 14 (no alcohol)

Average brain wave activity in the hours 4 to 6 A.M.

8.2	5.4	6.8	6.5	4.7	5.9	2.9
7.6	10.2	6.4	8.8	5.4	8.3	5.1

For group 2, we have the sample mean and standard deviation of

$$\bar{x}_2 \approx 6.59 \text{ and } s_2 \approx 1.91$$

Example 9(a) – *Confidence interval for $\mu_1 - \mu_2$, When σ_1 and σ_2 unknown*

cont'd

Check Requirements Are the samples independent or dependent? Explain. Is it appropriate to use a Student's *t* distribution to approximate the $\bar{x}_1 - \bar{x}_2$ distribution? Explain.

Solution:

Since the original random sample of 29 students was randomly divided into two groups, it is reasonable to say that the samples are independent. A Student's *t* distribution is appropriate for the $\bar{x}_1 - \bar{x}_2$ distribution because both original distributions are mound-shaped and symmetric.

We don't know population standard deviations, but we can compute s_1 and s_2.

Example 9(b) – *Confidence interval for $\mu_1 - \mu_2$, When σ_1 and σ_2 unknown*

cont'd

Compute a 90% confidence interval for $\mu_1 - \mu_2$, the difference of population means.

Solution:

First we find $t_{0.90}$. We approximate the degrees of freedom *d.f.* by using the smaller of $n_1 - 1$ and $n_2 - 1$. Since n_2 is smaller *d.f.* = $n_2 - 1 = 14 - 1 = 13$. This gives us $t_{0.90} \approx 1.771$. The margin of error is then

$$E \approx t_c \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 1.771\sqrt{\frac{1.86^2}{15} + \frac{1.91^2}{14}} \approx 1.24$$

Example 9(b) – *Solution*

cont'd

The *c* confidence interval is

$$(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$$

$$(19.65 - 6.59) - 1.24 < \mu_1 - \mu_2 < (19.65 - 6.59) + 1.24$$

$$11.82 < \mu_1 - \mu_2 < 14.30$$

After further rounding we have

11.8 hertz $< \mu_1 - \mu_2 <$ 14.3 hertz

Example 9(c) – *Confidence interval for $\mu_1 - \mu_2$, When σ_1 and σ_2 unknown*

cont'd

Interpretation What is the meaning of the confidence interval you computed in part (b)?

Solution:

μ_1 represents the population average brain wave activity for people who drank 1/2 liter of wine before sleeping.

μ_2 represents the population average brain wave activity for people who took no alcohol before sleeping. Both periods of measurement are from 4 to 6 A.M.

Example 9(c) – *Solution*

cont'd

We are 90% confident that the interval between 11.8 and 14.3 hertz is one that contains the difference $\mu_1 - \mu_2$.

It would seem reasonable to conclude that people who drink before sleeping might wake up in the early morning and have trouble going back to sleep.

Since the confidence interval from 11.8 to 14.3 contains only *positive values*, we could express this by saying that we are 90% confident that $\mu_1 - \mu_2$ is *positive*.

Example 9(c) – *Solution*

cont'd

This means that $\mu_1 - \mu_2 > 0$. Thus, we are 90% confident $\mu_1 > \mu_2$ that (that is, average brain wave activity from 4 to 6 A.M. for the group drinking wine was more than average brain wave activity for the group not drinking).