# Chapter 8 : Hypothesis Testing When Sigma is Unknown

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# Student's t Distribution

**Student's *t* Distribution**

If $X_1, \ldots, X_n$ is a random sample from a normal population $N(\mu, \sigma)$ and

$$\overline{X} = \frac{1}{n} \sum X_i \qquad \text{and} \qquad S^2 = \frac{\sum (X_i - \overline{X})^2}{n - 1}$$

then the distribution of

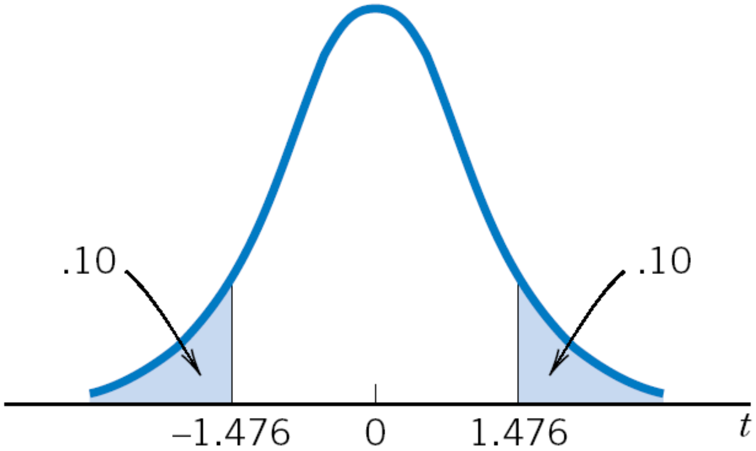
$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

is called **Student's *t* distribution with $n - 1$ degrees of freedom**

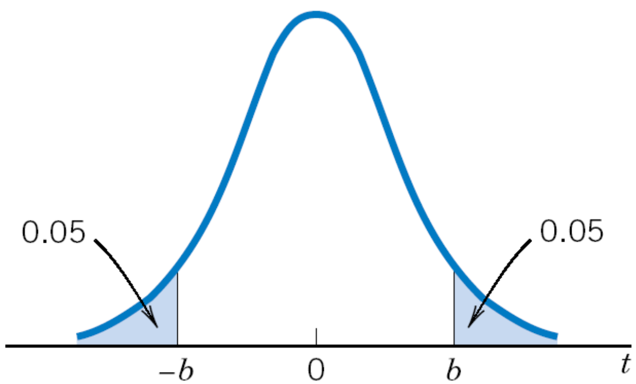
# How to get the value of t distribution from table

**Percentage Points of *t* distributions**

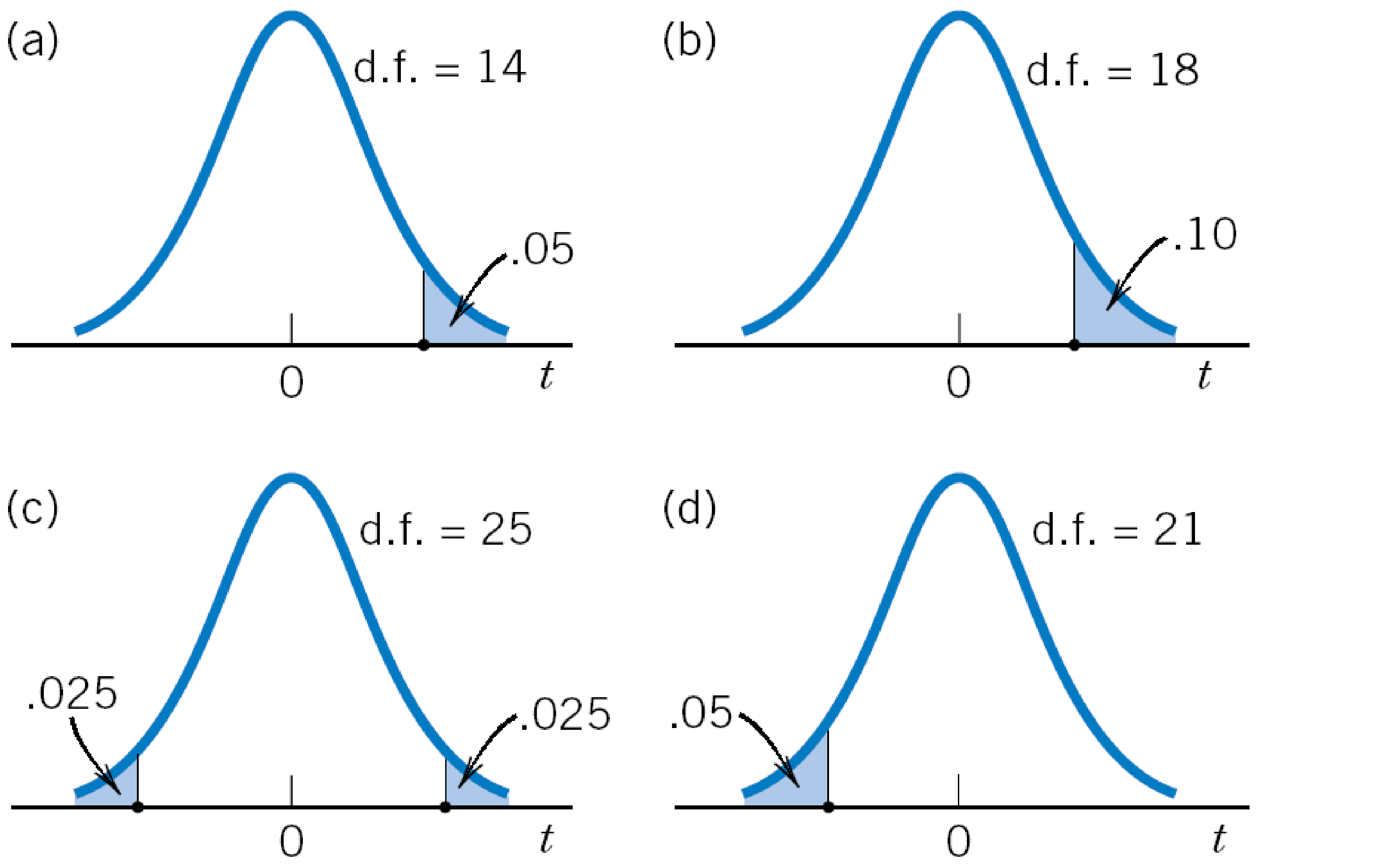
| d.f. \ α | ⋯ | .10 |
|---|---|---|
| . | . | . |
| . | . | . |
| . | . | . |
| 5 | ⋯ | 1.476 |




The upper and lower .10 points of the *t* distribution with d.f. = 5.



Examples: Name the t percentiles shown and find the values



# Part II: Testing $\mu$ when $\sigma$ is unknown

- **Procedure:**

1. In the context of the application, state the *null and alternate hypotheses* and set the *level of significance* $\alpha$.
2. Use $\bar{x}$, $s$, and $n$ from the sample, with $\mu$ from $H_0$, to compute the *sample test statistic.*

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \quad \text{with degrees of freedom } d.f. = n - 1$$

3. Use the Student's $t$ distribution and the type of test, one-tailed or two-tailed, to find (or estimate) the *P-value* corresponding to the test statistic.
4. *Conclude* the test. If $P$-value $\leq \alpha$, then reject $H_0$. If $P$-value $> \alpha$, then do not reject $H_0$.
5. *Interpret your conclusion* in the context of the application.

# Example

A city health department wishes to determine if the mean bacteria count per unit volume of water at a lake beach is within the safety level of 200. A researcher collected 10 water samples of unit volume and found the bacteria counts to be

| | | | | |
|---|---|---|---|---|
| 175 | 190 | 205 | 193 | 184 |
| 207 | 204 | 193 | 196 | 180 |

Do the data strongly indicate that there is no cause for concern? Use $\alpha$=0.05.

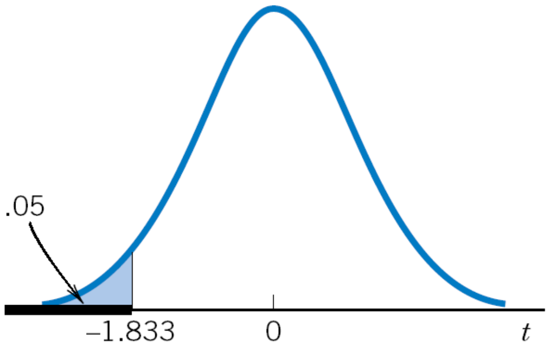
A) State the null hypothesis. $\boldsymbol{H_0: \mu} = 200$

B) State the alternative hypothesis. $\boldsymbol{H_1: \mu} < 200$

C) Since the counts are spread over a wide range, an approximation by a continuous distribution is not unrealistic for inference about the mean. Assuming further that the measurements constitute a sample from a normal population, we employ the T distribution

# Example

D) State the critical value for the hypothesis test. $\boldsymbol{T_{0.05}} = -1.833$ at 9 degree of freedom.




We obtained these summary statistics by using the formula:

$$\overline{X} = \frac{1}{n}\sum X_i \quad \text{and} \quad S^2 = \frac{\sum (X_i - \overline{X})^2}{n-1}$$

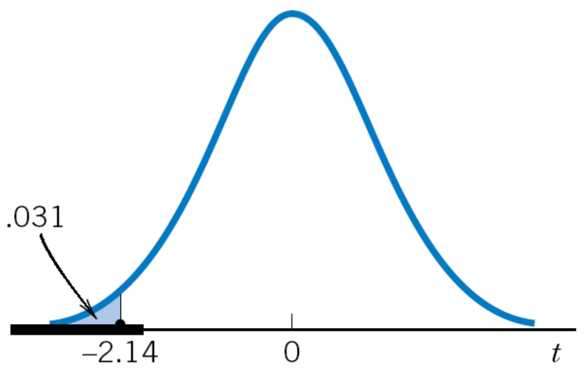
E) Test Statistic

For test statistics we need $\bar{X} = 192.7$ *and* $S = 10.81$

Test Statistic T = $\frac{\bar{X}-\mu}{S/\sqrt{n}} = \frac{192.7-200}{10.81/\sqrt{10}}$ = -2.14 with 9 degree of freedom.

# Example

F) P-value = P[T≤-2.14] = 0.031




*P*-value for left-sided rejection region *T* statistic.

G) What is your conclusion?

# Example

The management at Massachusetts Savings Bank is always concerned about the quality of service provided to its customers. With the old computer system, a teller at this bank could serve, on average, 22 customers per hour. The management noticed that with this service rate, the waiting time for customers was too long. Recently the management of the bank installed a new computer system in the bank, expecting that it would increase the service rate and consequently make the customers happier by reducing the waiting time. To check if the new computer system is more efficient than the old system, the management of the bank took a random sample of 29 hours and found that during these hours the mean number of customers served by tellers was 27 per hour with a standard deviation of 2.5. Testing at the 1% significance level, would you conclude that the new computer system is more efficient than the old computer system? Assume that the number of customers serve per hours follow normal distribution.

# Solve

- $H_0 : \mu = 22 \quad H_1 : \mu > 22$

# Example

- The drug 6-mP (6-mercaptopurine) is used to treat leukemia. The following data represent the remission times (in weeks) for a random sample of 21 patients using 6-mP

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 7 | 32 | 23 | 22 | 6 | 16 | 34 | 32 | 25 | 11 |
| 20 | 19 | 6 | 17 | 35 | 6 | 13 | 9 | 6 | 10 | |

- (Reference: E. A. Gehan, University of Texas Cancer Center).

The sample mean is $\bar{X} \approx 17.1$ weeks, with sample standard deviation $s \approx 10.0$. Let $x$ be a random variable representing the remission time (in weeks) for all patients using 6-mP.

# Example 4

cont'd

- Assume the *x* distribution is mound-shaped and symmetric. A previously used drug treatment had a mean remission time of $\mu$ = 12.5 weeks.

- Do the data indicate that the mean remission time using the drug 6-mP is different (either way) from 12.5 weeks? Use
$\alpha$ = 0.01.

# Example 4 – *Solution*

(a) Establish the null and alternate hypotheses.

Since we want to determine if the drug 6-mP provides a mean remission time that is different from that provided by a previously used drug having $\mu = 12.5$ weeks,

$H_0$: $\mu = 12.5$ weeks and $H_1$: $\mu \neq 12.5$ weeks

(b) *Check Requirements* What distribution do we use for the sample test statistic $t$? Compute the sample test statistic from the sample data.

The $x$ distribution is assumed to be mound-shaped and symmetric.

# Example 4 – *Solution*

cont'd

- Because we don't know $\sigma$, we use a Student's $t$ distribution with *d.f.* = 20. Using $\bar{x} \approx 17.1$ and $s \approx 10.0$ from the sample data, $\mu = 12.5$ from $H_0$, and $n = 21$

$$t \approx \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$\approx \frac{17.1 - 12.5}{10.0/\sqrt{21}}$$

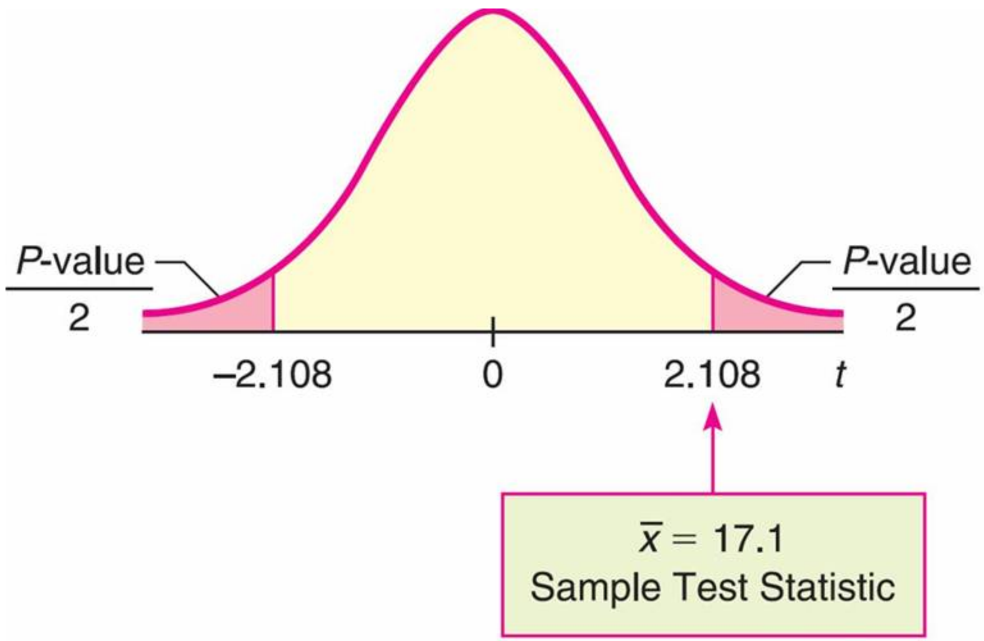
$$\approx 2.108$$

# Example 4 – *Solution*

cont'd

- (c) Find the *P*-value or the interval containing the *P*-value.

  Figure 8-5 shows the *P*-value. Using Table 6 of Appendix II, we find an interval containing the *P*-value.



# Example 4 – *Solution*

cont'd

- Since this is a two-tailed test, we use entries from the row headed by *two-tail area.* Look up the $t$ value in the row headed by $d.f. = n - 1 = 21 - 1 = 20$.

- The sample statistic $t = 2.108$ falls between 2.086 and 2.528. The $P$-value for the sample $t$ falls between the corresponding two-tail areas 0.050 and 0.020. (See Table 8-5.) $0.020 < P\text{-value} < 0.050$

| one-tail area | ... | ... |
|---|---|---|
| ✓ two-tail area | 0.050 | 0.020 |
| $d.f. = 20$ | 2.086 | 2.528 |
| | ↑ Sample $t = 2.108$ | |

Excerpt from Student's t Distribution (Table 6, Appendix II)
**Table 8-5**

# Example 4 – *Solution*

cont'd

- (d) Conclude the test.

  The following diagram shows the interval that contains the single *P*-value corresponding to the test statistic. Note that there is just one *P*-value corresponding to the test statistic. Table 6 of Appendix II does not give that specific value, but it does give a range that contains that specific *P*-value.


As the diagram shows, the entire range is greater than $\alpha$. This means the specific *P*-value is greater than $\alpha$, so we cannot reject $H_0$.

# Example 4 – *Solution*

cont'd

- (e) *Interpretation* Interpret the results in the context of the problem. At the 1% level of significance, the evidence is not sufficient to reject $H_0$. Based on the sample data, we cannot say that the drug 6-mP provides a different average remission time than the previous drug.

# Example

- Pyramid Lake is on the Paiute Indian Reservation in Nevada. The lake is famous for cutthroat trout. Suppose the claim is that the average length of the trout is 19 inches. However, in a random sample of 15 fish caught, the mean length was 18.5 inches with a sample standard deviation of 3.2 inches. Treating these data from sample of 15 normal, test that the average length of trout caught is different than the 19 inches at the 5% level of significance?

# Pyramid Lake (cont)

$H_0 : \mu = \_\_\_\_ in$ $\quad n = 15 \quad \alpha = 0.05$

$H_a : \mu \neq \_\_\_\_ in$ $\quad s = 3.2in$

Find Critical Value and Draw Rejection Region

$$Test\_Statistic(t) = \frac{18.5 - 19}{3.2 / \sqrt{15}} \approx -\_\_\_\_\_\_; df = 14$$

$$p - \text{value} =$$

Conclusion: