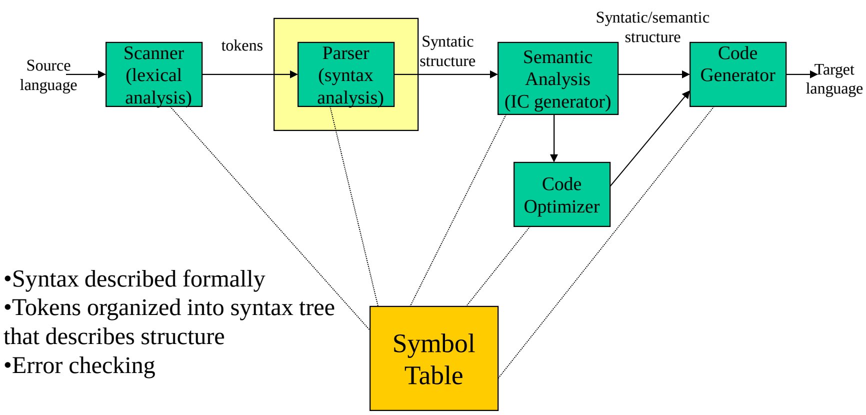
Lecture 4a: Parsing

COSC 4316

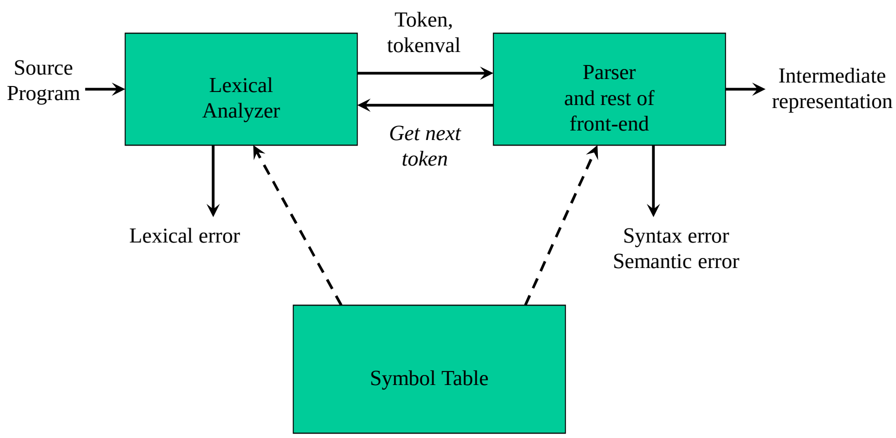
(grateful acknowledgement to Robert van Engelen and Elizabeth White for some of the material from which these slides have been adapted)

Syntax Analysis - Parsing



•Syntax described formally
•Tokens organized into syntax tree that describes structure
•Error checking

Position of a Parser in the Compiler Model

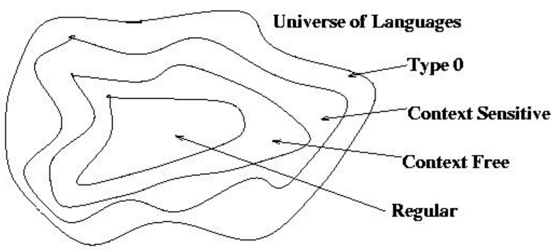


Syntax Described by CF Grammars (BNF)

- Advantages of CFGs
  - Precise, easily understood syntactic specification of a language
  - Automatic construction of parsers
  - Makes syntactic ambiguities more obvious
  - Allows extension of language to be done more readily

Static Analysis - Parsing

We can use context free grammars to specify the syntax of programming languages.



Role of the Parser

- Obtain a string of tokens from the lexical analyzer
- Verify that the string can be generated by the grammar for the source language
- Report syntax errors (intelligibly)
- Recover from some syntax errors

3 Types of Parsers

- Universal parsing algorithms
  - Examples: CYK algorithm or Earley's algorithm
    - May be used for any CF grammar
    - Too inefficient for practical use
- Top down parsers (often constructed by hand)
  - Build parse trees from the root down to the leaves
  - Works with certain classes of grammars (e.g. LL grammars – *later*)
- Bottom up parsers (often build with automated tools)
  - Build parsers from the leaves up to the root
  - Work with a broader class of grammars (LR)

Error Handling

- Errors **will** occur
- A good compiler should assist in identifying and locating errors
  - *Lexical errors*: important, compiler can easily recover and continue
  - *Syntax errors*: most important for compiler, can almost always recover
  - *Static semantic errors*: important, can sometimes recover
  - Goal: detection of an error *as soon as possible* without further consuming unnecessary input
  - How: detect an error as soon as the prefix of the input does not match a prefix of any string in the language (the *viable prefix property*)

Error Recovery Strategies

- *Bail out* (sudden death)
  - Stop when first error found
- *Panic mode*
  - Discard input until a token in a set of designated *synchronizing tokens* is found (e.g., a semicolon or a **}**)
- *Phrase-level recovery*
  - Perform local correction on the input to repair the error
  - Easy to do in predictive parsing because you know what is expected (**match**)
- *Error productions*
  - Augment grammar with productions for erroneous constructs
- *Global correction*
  - Choose a minimal sequence of changes to obtain a global least-cost correction

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Context-Free Grammars (Recap)

- Context-free grammar is a 4-tuple
  $G = (N, T, P, S)$ where
  - T is a finite set of tokens (*terminal* symbols)
  - N is a finite set of *nonterminals*
  - P is a finite set of *productions* of the form
    $A \to \beta$
    where $A \in N$ and $\beta \in (N \cup T)^*$
  - $S \in N$ is a designated *start symbol*

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Notational Conventions Used

- Terminals
  a,b,c,… $\in T$ (lowercase letters early in the alphabet)
  specific terminals: **0**, **1**, **id**, **+**
- Nonterminals (uppercase letters early in the alphabet)
  $A,B,C,\ldots \in N$
  specific nonterminals: *expr*, *term*, *stmt*
- Grammar symbols (uppercase letters late in the alphabet)
  $X,Y,Z \in (N \cup T)$
- Strings of terminals (lowercase letters late in the alphabet)
  $u,v,w,x,y,z \in T^*$
- Strings of grammar symbols (Greek letters)
  $\alpha,\beta,\gamma \in (N \cup T)^*$

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Derivations

- The *one-step derivation* is defined by
  $\alpha\ A\ \beta \Rightarrow \alpha\ \gamma\ \beta$
  where $A \rightarrow \gamma$ is a production in the grammar
- In addition, we define
  - $\Rightarrow$ is *leftmost* $\Rightarrow_{lm}$ if α does not contain a nonterminal
  - $\Rightarrow$ is *rightmost* $\Rightarrow_{rm}$ if β does not contain a nonterminal
  - Transitive closure $\Rightarrow^*$ (zero or more steps)
  - Positive closure $\Rightarrow^+$ (one or more steps)

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Derivations

- A derivation is an alternative to constructing a parse tree.
- We view a production as a *rewriting rule*.
- The sequence of replacements is called a *derivation.*
- If $S \Rightarrow^* \alpha$, then α is said to be a *sentential form* of the CFG, G
- The *language generated by G* is defined by
  $L(G) = \{w \in T^* \mid S \Rightarrow^+ w\}$
  and w is called a sentence in L(G)
  (a sentential form with no non-terminals)

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Derivations

- When deriving a token sequence, if more than one nonterminal is present, we have a choice of which to replace next.
- One convention:
  - Leftmost derivation –
    - Choose the leftmost possible nonterminal at each step.
    - $\Rightarrow_{lm}$ $\quad \Rightarrow^*_{lm}$ $\quad \Rightarrow^+_{lm}$
- A sentential form produced via a leftmost derivation is called a ***left sentential form***

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Derivation (Example)

S → P (S) | <u>var</u> R
P → <u>func</u> | ε
R → + S | ε

Expressions of variables and functions

- A leftmost derivation of <u>func</u> (<u>var</u> + <u>var</u>) is

  S $\Rightarrow_{lm}$ P (S) $\Rightarrow_{lm}$ **<u>func</u>** (S) $\Rightarrow_{lm}$ **<u>func</u>** (**<u>var</u>** R)
  $\Rightarrow_{lm}$ **<u>func</u>** (**<u>var</u>** + S) $\Rightarrow_{lm}$ **<u>func</u>** (**<u>var</u>** + **<u>var</u>** R)
  $\Rightarrow_{lm}$ **<u>func</u>** (**<u>var</u>** + **<u>var</u>**)

Derivations

- Analogous to leftmost derivations, we have ***rightmost derivations***.
- These seem less intuitive, but they correspond to a large class of parsers (the *bottom-up parsers*.)
- Leftmost derivations are usually associated with top-down parsing.
- Rightmost derivations are sometimes called ***canonical derivations***.

CFG Examples

Indicates a production

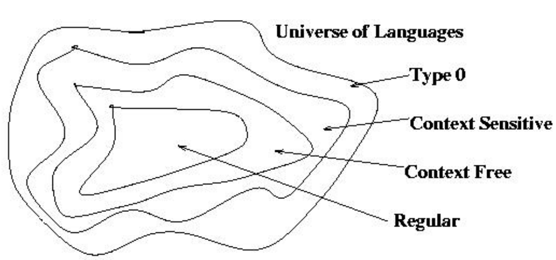
T = {+,-,0..9}, N = {L,D}, S = L
L → L + D | L – D | D
D → 0 | … | 9

Shorthand for multiple productions

T={ (,) }, N = {L}, S = L
L → (L) L
L → ε

Languages

Regular	A → a B, C → ε
Context free	A → α
Context sensitive	αAβ → αγβ
Type 0	α → β



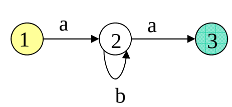
Any regular language can be expressed using a CFG

Starting with a NFA:

- For each state S_i in the NFA
  - Create non-terminal A_i
  - If transition $(S_i,a) = S_k$, create production $A_i \rightarrow a\ A_k$
  - If transition $(S_i,\varepsilon) = S_k$, create production $A_i \rightarrow A_k$
  - If S_i is a final state, create production $A_i \rightarrow \varepsilon$
  - If S_i is the NFA start state, $s = A_i$
- *What does the existence of this algorithm tell us about the relationship between regular and context free languages?*

NFA to CFG Example

ab*a



$A_1 \rightarrow a\ A_2$

$A_2 \rightarrow b\ A_2$

$A_2 \rightarrow a\ A_3$

$A_3 \rightarrow \varepsilon$

Writing Grammars

When writing a grammar (or RE) for some language, the following must be true:

1. All strings generated are in the language.
2. Your grammar produces all strings in the language.

Try these:

- Integers divisible by 2
- Legal postfix expressions
- Floating point numbers with no extra zeros
- Strings of 0,1 where there are more 0 than 1 (hard)

Regular Expressions vs. CFGs

- A grammar in which every production is of the form:

$A \rightarrow w\,B$ ($w, x \in T^*$ and $A, B \in N$)

or $A \rightarrow x$

is called a ***right-linear grammar***. (*Left-linear grammar* defined analogously.)

- It can be proven that any right-linear grammar generates a regular language, and *vice versa*.

Regular Expressions vs. CFGs

- Why use regular expressions to denote the lexical syntax of a language if we could use CFGs instead?
  - Lexical rules are simple – don't use a chainsaw to prune a rose
  - Regular expressions are more concise and easier to understand than CFGs
  - Easier to generate a lexical analyzer from a regular expression than an *arbitrary* grammar.
  - Promotes modularity of the front end.

Parsing

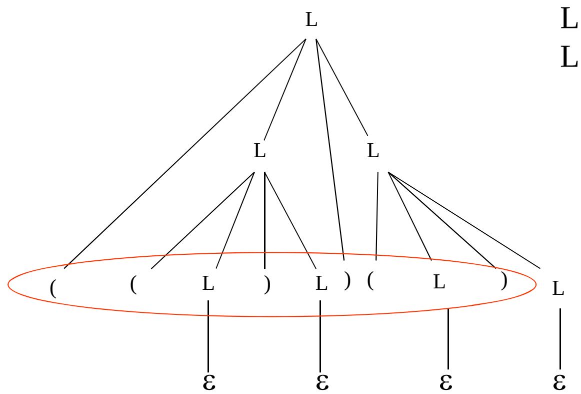
- The task of parsing is figuring out what the parse tree looks like for a given input and language.
- If a string is in the given language, a parse tree must exist.
- However, just because a parse tree exists for some string in a given language doesn't mean a given algorithm can find it.

Parse Trees

The parse tree for some string in a language that is defined by the grammar G as follows:

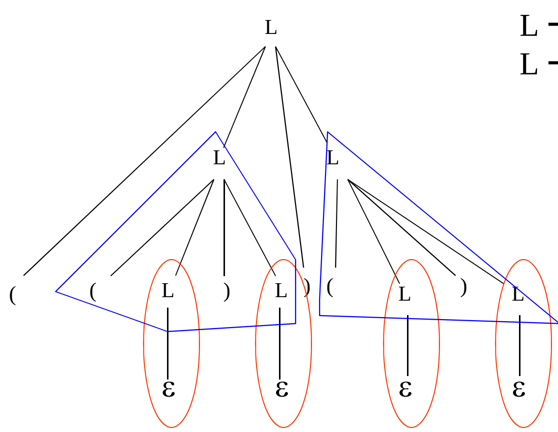
- The root is the start symbol of G
- The leaves are terminals or ε. When visited from left to right, the leaves form the input string
- The interior nodes are non-terminals of G
- For every non-terminal A in the tree with children $B_1 \ldots B_k$, there is some production $A \rightarrow B_1 \ldots B_k$

Parse Tree for (())()



$L \rightarrow (L) L$
$L \rightarrow \varepsilon$

Parse Tree for (())()



$L \rightarrow (L) L$
$L \rightarrow \varepsilon$

Parse Trees & Derivations

- A derivation is a *linear representation* of a parse tree.
- Equivalently, a parse tree is a *graphical representation* of a derivation.

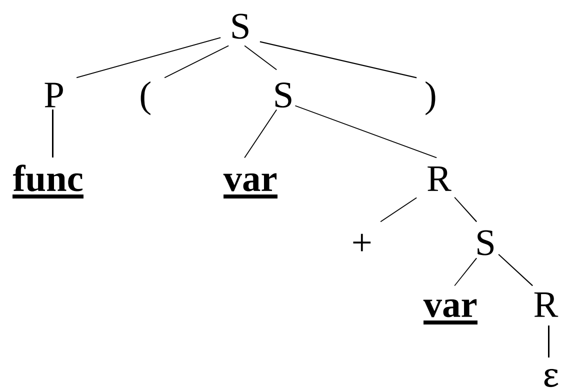
For the grammar and the string
func (**var** + **var**):

$S \rightarrow P\ (\ S\ )\ |\ \mathbf{var}\ R$

$P \rightarrow \mathbf{func}\ |\ \varepsilon$

$R \rightarrow +\ S\ |\ \varepsilon$

$$
\begin{aligned}
S &\Rightarrow_{lm} P\ (\ S\ ) \Rightarrow_{lm} \mathbf{func}\ (\ S\ ) \Rightarrow_{lm} \mathbf{func}\ (\ \mathbf{var}\ R) \\
&\Rightarrow_{lm} \mathbf{func}\ (\ \mathbf{var} + S\ ) \\
&\Rightarrow_{lm} \mathbf{func}\ (\ \mathbf{var} + \mathbf{var}\ R\ ) \\
&\Rightarrow_{lm} \mathbf{func}\ (\ \mathbf{var} + \mathbf{var}\ )
\end{aligned}
$$



Single Step Derivation

Definition: Given $\underline{\alpha\ A\ \beta}$ (with α,β in $(V_n \cup V_t)^*$) and a production $\underline{A \rightarrow \gamma}$,

$\alpha\ A\ \beta \Rightarrow \alpha\ \gamma\ \beta$ is a **single step derivation**.

Examples:

$L + D \Rightarrow L - D + D$ $\quad$ $L \rightarrow L - D$

$(\ L\ )\ (\ L\ ) \Rightarrow (\ (\ L\ )\ L\ )\ (\ L\ )$ $\quad$ $L \rightarrow (L)\ L$

Greek letters ($\alpha,\beta,\chi,\ldots$) denote a (possibly empty) sequence of terminals and non-terminals.

Derivations

Definition: A sequence of the form:

$w_0 \Rightarrow w_1 \Rightarrow \ldots \Rightarrow w_n$

is a **derivation** of w_n from w_0 ($w_0 \Rightarrow^* w_n$)

L	production L → (L) L
⇒ (L) L	production L → ε
⇒ ( ) L	production L → ε
⇒ ( )	

$L \Rightarrow^* ()$

If w_i has non-terminal symbols, it is referred to as *sentential form*.

$L \Rightarrow^* (()) ()$

L	production L → (L) L
⇒ (L) **L**	production L → (L) L
⇒ (L) (L) **L**	production L → ε
⇒ (**L**) (L)	production L → (L) L
⇒ ((**L**) L) (L)	production L → ε
⇒ (( ) L) (**L**)	production L → ε
⇒ (( ) **L**) ( )	production L → ε
⇒ (( )) ( )	

- L(G), the language generated by grammar G is $\{w \text{ in } T^*: S \Rightarrow^* w$, for start symbol $S\}$
- Both **( )** and **(( )) ( )** are in L(G) for the following grammar.
  - L → **(** L **)** L
  - L → ε

Leftmost Derivations

- Recall that a leftmost derivation is one where the leftmost nonterminal is always chosen
- If a string is in a given language (i.e. a derivation exists), then a leftmost derivation *must* exist
- Rightmost derivation defined as you would expect

Leftmost Derivation for (())()

L	production L → (L) L
⇒ (**L**) L	production L → (L) L
⇒ ((**L**) L) L	production L → ε
⇒ (( ) **L**) L	production L → ε
⇒ (( )) **L**	production L →(L) L
⇒ (( )) (**L**) L	production L → ε
⇒ (( )) ( ) **L**	production L → ε
⇒ (( )) ( )	

L → **(** L **)** L
L → ε

Rightmost Derivation for (())()

L	production L → (L) L
⇒ (L) **L**	production L → (L) L
⇒ (L) (L) **L**	production L → ε
⇒ (L) (**L**)	production L → ε
⇒ (**L**) ( )	production L → (L) L
⇒ ((L) **L**) ( )	production L → ε
⇒ ((**L**)) ( )	production L → ε
⇒ (( )) ( )	

L → **(** L **)** L
L → ε

Ambiguity

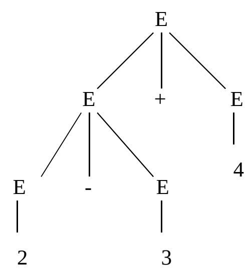
- An ***ambiguous*** grammar is one in which two (or more) parse trees or leftmost derivations exist for *some string in the language*

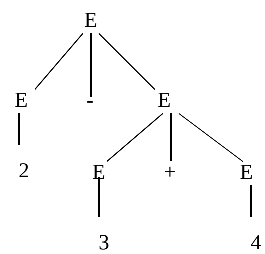
$E \rightarrow E + E$

$E \rightarrow E - E$

$E \rightarrow 0 \mid \ldots \mid 9$

$2 - 3 + 4$





- Two leftmost derivations

E	$\Rightarrow$	$E + E$	E	$\Rightarrow$	$E - E$
	$\Rightarrow$	$E - E + E$		$\Rightarrow$	$2 - E$
	$\Rightarrow$	$2 - E + E$		$\Rightarrow$	$2 - E + E$
	$\Rightarrow$	$2 - 3 + E$		$\Rightarrow$	$2 - 3 + E$
	$\Rightarrow$	$2 - 3 + 4$		$\Rightarrow$	$2 - 3 + 4$

- We must either write unambiguous grammars **or** have *disambiguating* rules.

- An ambiguous grammar can sometimes be made unambiguous:

  $E \rightarrow E + T \mid E - T \mid T$

  $T \rightarrow 0 \mid \ldots \mid 9$

  enforces the correct associativity

- Precedence can be specified as well:

  $E \rightarrow E + T \mid E - T \mid T$

  $T \rightarrow T * F \mid T / F \mid F$

  $F \rightarrow (E) \mid 0 \mid \ldots \mid 9$

Another example of ambiguity

- **if-else** in Java

```
if (expr)
    if (expr)
      stmt;
    else
       stmt;
```

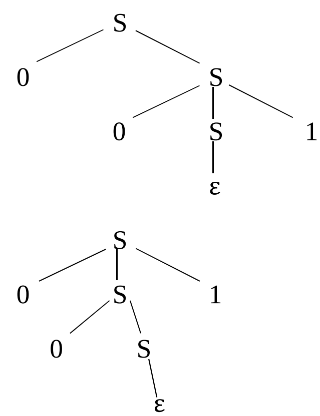
- Which **if** is the **else** associated with?
- The last one, but we can't specify that via the definition.
- Could fix this by requiring the use of an **endif** keyword

Yet another example of ambiguity

- **L = { $0^i 1^j$** | $i \geq j \geq 0$} (0 ≡ if, 1 ≡ else)
- Expressed by the grammar

  $S \rightarrow 0\ S \mid 0\ S\ 1 \mid \varepsilon$

Parse trees for **001**

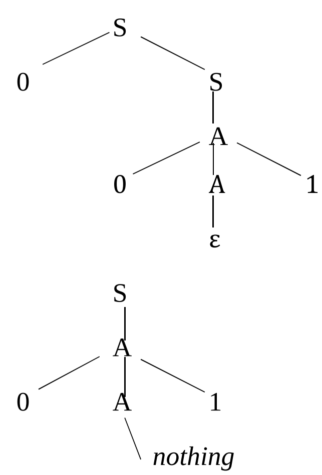


Disambiguating rule: match each 1 with the closest unmatched 0

Disambiguating rule incorporated into the grammar

- $S \rightarrow 0\ S \mid A$
- $A \rightarrow 0\ A\ 1 \mid \varepsilon$

Parse trees for **001**



Disambiguating rule: match each 1 with the closest unmatched 0

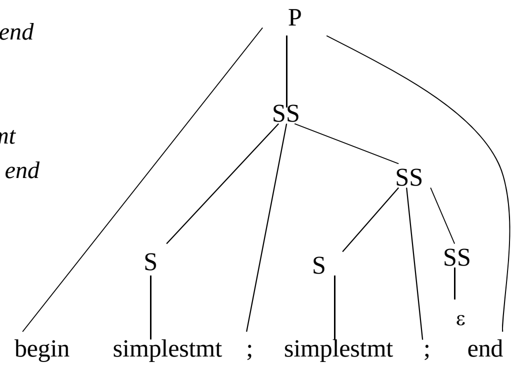
Grammar still has issues, because we can't use a predictive parser – cant predict whether the 0 is matched or unmatched.

The dangling else is really a language design issue.

Conclusion: If you ever design a programming language, you need to know the issues involved in parsing that language!

Input: **`begin simplestmt; simplestmt; end`**

P→ *begin* SS *end*
SS → S ; SS
SS → ε
S → *simplestmt*
S → *begin* SS *end*



Top Down (LL) Parsing

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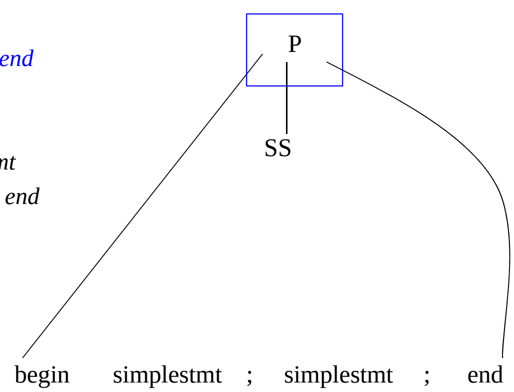

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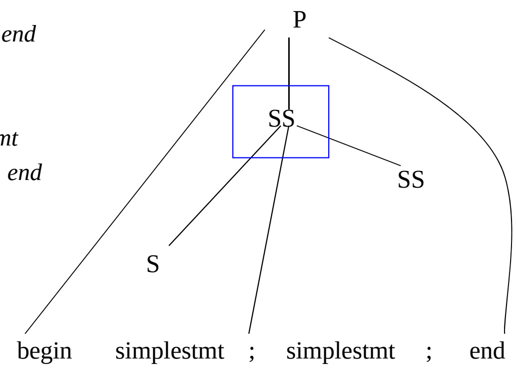

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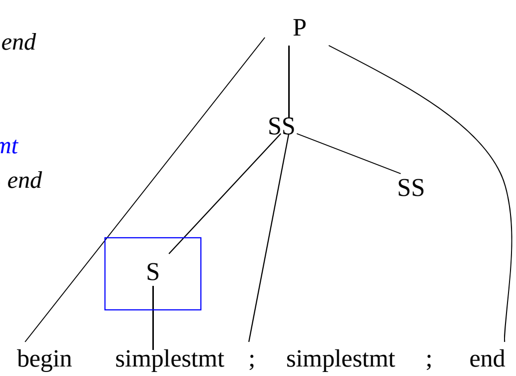

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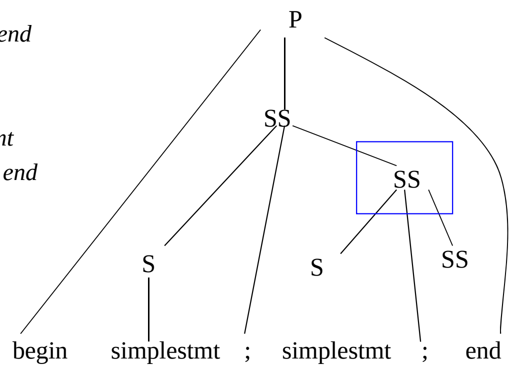

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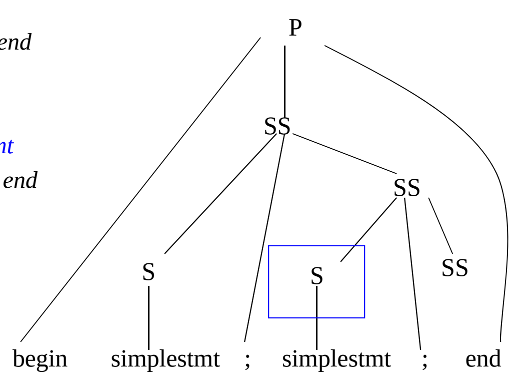

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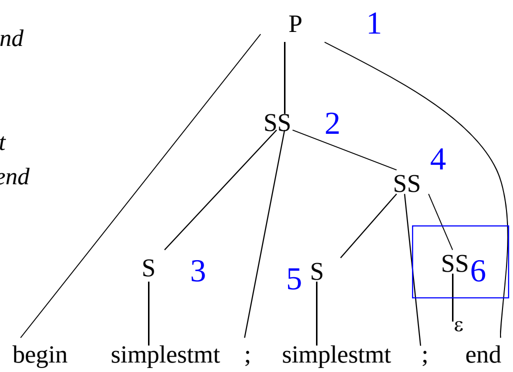
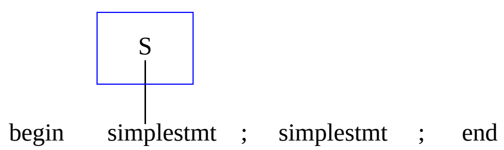


Bottomup (LR) Parsing

P→ *begin* SS *end*
SS → S *;* SS
SS → ε
S → *simplestmt*
S → *begin* SS *end*



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SS → ε
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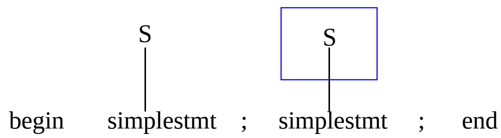

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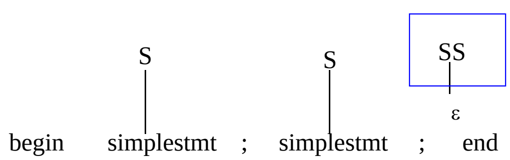

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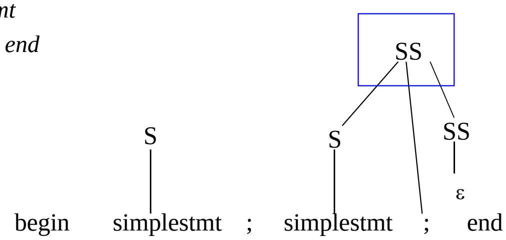

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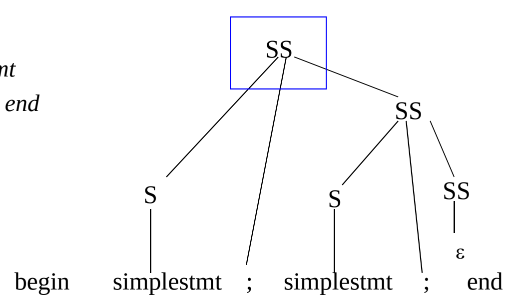

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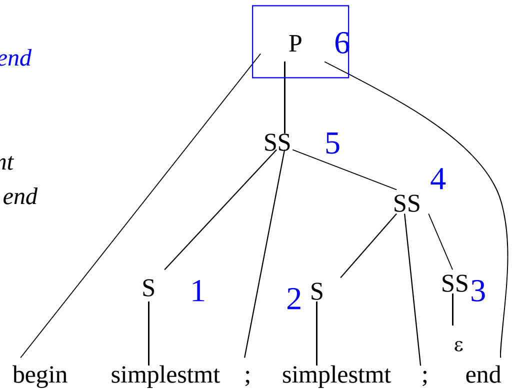

Left Recursion (Recap)

- Productions of the form

  $A \rightarrow A\ \alpha$

  $\quad\quad | \ \beta$

  are left recursive
- When one of the productions in a grammar is left recursive then a predictive parser loops forever on certain inputs

Left Recursion (Recap)

- Replace

  $A \to A\ \alpha$
  $\quad\ \ \mid \beta$

  with

  $A \to \beta\ R$
  $R \to \alpha\ R \mid \varepsilon$

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Indirect Left Recursion

- What about indirect left recursion?

  $B \to D\ \alpha$
  $D \to B\ \beta$

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General Left Recursion Elimination Method

Arrange the nonterminals in some order $A_{1,}$ $A_{2,}$ … A_n

for i = 1, …, n **do**
  for j = 1, …, i-1 **do**
    for each production $A_i \rightarrow A_j\ \alpha$ and $A_j \rightarrow \beta$ **do**
      Replace $A_i \rightarrow A_j\ \alpha$ with $A_i \rightarrow \beta\ \alpha$
    endfor
  endfor
  eliminate any direct *left recursion* among A_i
endfor

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Example Left Recursion Elimination

$A \rightarrow B\ \mathbf{a} \mid \mathbf{b}\ C$
$B \rightarrow B\ \mathbf{c} \mid A\ \mathbf{d}$
$C \rightarrow \mathbf{e} \mid \mathbf{f}$

Choose arrangement: $A = A_1$, $B = A_2$, $C = A_3$

A unchanged
$B \rightarrow B\ \mathbf{c} \mid B\ \mathbf{a}\ \mathbf{d} \mid \mathbf{b}\ C\ \mathbf{d}$
$B \rightarrow \mathbf{b}\ C\ \mathbf{d}\ D$ *(eliminating direct left recursion)*
$D \rightarrow \mathbf{c}\ D \mid \mathbf{a}\ \mathbf{d}\ D \mid \varepsilon$
C unchanged

Result:
$A \rightarrow B\ \mathbf{a} \mid \mathbf{b}\ C$
$B \rightarrow \mathbf{b}\ C\ \mathbf{d}\ D$
$C \rightarrow \mathbf{e} \mid \mathbf{f}$
$D \rightarrow \mathbf{c}\ D \mid \mathbf{a}\ \mathbf{d}\ D \mid \varepsilon$

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Left Factoring

- Most problems with predictive parsing are either (a) left recursion (which we have already dealt with) or (b) common prefixes
- Example:
  - *stmt* $\rightarrow$ **if** *expr* **then** *seq-of-stmts* **endif**
  - *stmt* $\rightarrow$ **if** *expr* **then** *seq-of-stmts* **else** *seq-of-stmts* **endif**
- Solution: Rewrite the production to defer the decision until we have enough information to make the right choice.

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Left Factoring

- Replace productions

  $A \rightarrow \alpha\ \beta \mid \alpha\ \gamma$

  with

  $A \rightarrow \alpha\ A_R$

  $A_R \rightarrow \beta \mid \gamma$
- e.g.
  - *stmt* $\rightarrow$ **if** *expr* **then** *seq-of-stmts opt_end*
  - *opt_end* $\rightarrow$ **endif | else** *seq-of-stmts* **endif**

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Non-context-free Language Constructs

- Not all syntactic rules are expressible using CFGs.
  - *e.g.*, "variables must be declared before they are used" cannot be exptessed in a CFG.
  - *or*, "the number of formal parameters for a function must equal the number of actual parameters."
- In practice, syntactic details that cannot be represented in a CFG are considered part of the ***static semantics*** and deferred to the semantic analysis phase.

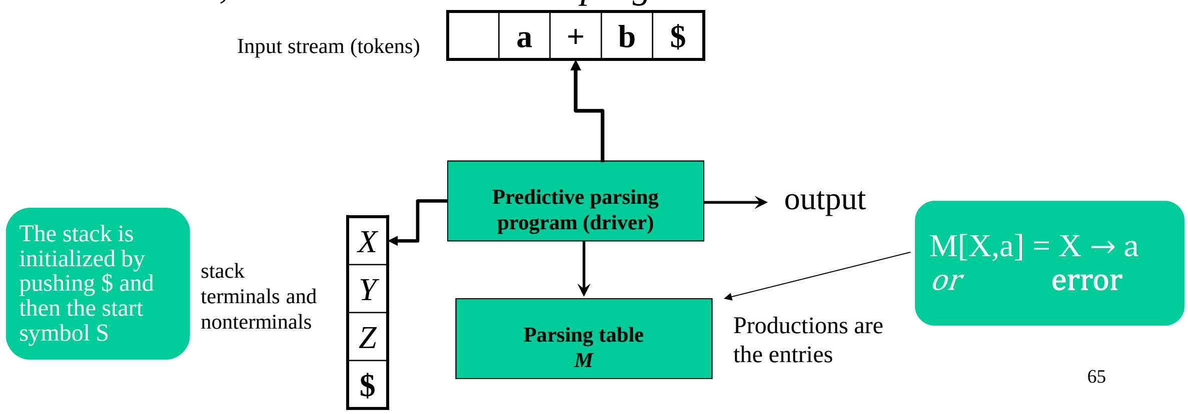
Top-Down Parsing

- General Algorithms:
  - LL (top down)
    - (We looked at elementary recursive descent parsing in module 2)
  - LR (bottom up)
- Both algorithms are driven by the input grammar and the input to be parsed
- LL(1) grammars are those suitable for predictive parsing
- "LL(1)"≡ scans input from **L**eft to right, **L**eftmost derivation, **1** token lookahead.

An LL(1) grammar can always be parsed top-down without backtracking.

Non-Recursive Predictive Parsing: Table-Driven Parsing

- Given an LL(1) grammar $G = (N, T, P, S)$ construct a table $M[A,a]$ for $A \in N$, $a \in T$ and use a *driver program* with a *stack*



Predictive Parsing Algorithm

- If X = **a** = \$ ··· parser halts. Success!
- If X= **a** ≠ \$
  - – Pop X off stack
  - – Advance input pointer
- If X is a non-terminal, look up M[X,**a**]
  - – If, for example M[X,**a**] = $X \rightarrow UVW$, push WVU on the stack (U on top)

Predictive Parsing Program (Driver)

push(**$**)
push(S)
a := *lookahead*
repeat
    X := pop()
    if X is a terminal or X = **$** **then**
        match(X)   *// moves to next token and a := lookahead*
    else if $M[X,a] = X \rightarrow Y_1 Y_2 \ldots Y_k$ **then**
        push($Y_k, Y_{k-1}, \ldots, Y_2, Y_1$)   *// such that Y1 is on top*
        … invoke actions and/or produce IR output …
    else   error()
    endif
until X = **$**

Example Table

$E \rightarrow E + T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow (\ E\ ) \mid \mathbf{id}$

Eliminate left recursion
$E \rightarrow T\ E_R$
$E_R \rightarrow +\ T\ E_R \mid \varepsilon$
$T \rightarrow F\ T_R$
$T_R \rightarrow *\ F\ T_R \mid \varepsilon$
$F \rightarrow (\ E\ ) \mid \mathbf{id}$

Nevermind how the parse table is built just yet

	id	**+**	*****	**(**	**)**	**$**
E	$E \rightarrow T\ E_R$			$E \rightarrow T\ E_R$		
E_R		$E_R \rightarrow +\ T\ E_R$			$E_R \rightarrow \varepsilon$	$E_R \rightarrow \varepsilon$
T	$T \rightarrow F\ T_R$			$T \rightarrow F\ T_R$		
T_R		$T_R \rightarrow \varepsilon$	$T_R \rightarrow *\ F\ T_R$		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \varepsilon$
F	$F \rightarrow \mathbf{id}$			$F \rightarrow (\ E\ )$		

Parsing Example

	id	+	*	(	)	$
E	$E \to T\,E_R$			$E \to T\,E_R$		
E_R		$E_R \to +\,T\,E_R$			$E_R \to \varepsilon$	$E_R \to \varepsilon$
T	$T \to F\,T_R$			$T \to F\,T_R$		
T_R		$T_R \to \varepsilon$	$T_R \to *\,F\,T_R$		$T_R \to \varepsilon$	$T_R \to \varepsilon$
F	$F \to \mathbf{id}$			$F \to (\,E\,)$		

Stack	Input	Production applied
$\$E$	**id+id*id$**	$E \to T\,E_R$
$\$E_R T$	**id+id*id$**	$T \to F\,T_R$
$\$E_R T_R F$	**id+id*id$**	$F \to \mathbf{id}$
$\$E_R T_R$**id**	**id+id*id$**	
$\$E_R T_R$	**+id*id$**	$T_R \to \varepsilon$
$\$E_R$	**+id*id$**	$E_R \to +\,T\,E_R$
$\$E_R T +$	**+id*id$**	
$\$E_R T$	**id*id$**	$T \to F\,T_R$
$\$E_R T_R F$	**id*id$**	$F \to \mathbf{id}$
$\$E_R T_R$**id**	**id*id$**	
$\$E_R T_R$	***id$**	$T_R \to *\,F\,T_R$
$\$E_R T_R F *$	***id$**	
$\$E_R T_R F$	**id$**	$F \to \mathbf{id}$
$\$E_R T_R$**id**	**id$**	
$\$E_R T_R$	**$**	$T_R \to \varepsilon$
$\$E_R$	**$**	$E_R \to \varepsilon$
$	**$**	

- So, the big question:
  How do we compute the parse tables?

- **We're going to need some more theory, folks, so hang on to your hats.**

FIRST Sets

FIRST(α) is the set of all terminal symbols that can begin some sentential form in a derivation that starts with α

$$\alpha \Rightarrow \ldots \Rightarrow a\ \beta$$

(also include ε if $\alpha \Rightarrow^* \varepsilon$

- FIRST(α) = $\{\mathbf{a} \in T \mid \alpha \Rightarrow^* \mathbf{a}\beta\} \cup \{\varepsilon \text{ if } \alpha \Rightarrow^* \varepsilon\}$
- Example:

simple → **integer** | **char** | **num dotdot num**

FIRST(*simple*) = { **integer**, **char**, **num** }

Computing FIRST

- To compute FIRST(X) for any single grammar symbol X:
  1. If X is a terminal, FIRST(X) = {X}
  2. If X → ε is a production, add ε to FIRST(X)
  3. If X is a nonterminal and $X \rightarrow Y_1\ Y_2\ \ldots\ Y_n$ is a production, add **a** to FIRST(X) **if** $a \in$ FIRST(Y_i) **and**

     $\varepsilon \in \text{FIRST}(Y_1) \cap \text{FIRST}(Y_2) \cap \ldots \cap \text{FIRST}(Y_{i-1})$

(i.e, $Y_1\ Y_2\ \ldots\ Y_{i-1} \Rightarrow^* \varepsilon$)

Computing FIRST

- To compute FIRST(α) where $\alpha = X_1 X_2 \ldots X_n$:
  1. Add non-ε symbols of FIRST(X_1)
  2. If ε ϵ FIRST(X_1) then add non-ε symbols of FIRST(X_2).
  3. As long as ε ϵ FIRST(X_{i-1}) then add non-ε symbols of FIRST(X_i).
  4. If ε is a member of ***all*** the FIRST(X_i) sets then add ε to FIRST(α)

Example 1

- S → a S e
- S → B
- B → b B e
- B → C
- C → c C e
- C → d

Start with the 'simplest' non-terminal

- FIRST(C) = {c,d}
- FIRST(B) =
- FIRST(S) =

Example 1

- S → a S e
- S → B
- B → <u>b</u> B e
- B → <u>C</u>
- C → c C e
- C → d

- FIRST(C) = {c,d}
- FIRST(B) = {b,c,d}
- FIRST(S) =

Now that we know FIRST(C) …

Example 1

- S → <u>a</u> S e
- S → <u>B</u>
- B → b B e
- B → C
- C → c C e
- C → d

- FIRST(C) = {c,d}
- FIRST(B) = {b,c,d}
- FIRST(S) = {a,b,c,d}

Example 2

- P → i | c | n T S
- Q → P | a S | d S c S T
- R → b | ε
- S → e | R n | ε
- T → R S q

- FIRST(P) = {i,c,n}
- FIRST(Q) =
- FIRST(R) = {b,ε}
- FIRST(S) =
- FIRST(T) =

Example 2

- P → i | c | n T S
- Q → P | a S | d S c S T
- R → b | ε
- S → e | R n | ε
- T → R S q

- FIRST(P) = {i,c,n}
- FIRST(Q) = {i,c,n,a,d}
- FIRST(R) = {b,ε}
- FIRST(S) =
- FIRST(T) =

Example 2

- P → i | c | n T S
- Q → P | a S | d S c S T
- R → b | ε
- S → $\underline{\text{e}}$ | $\underline{\text{R n}}$ | $\underline{\varepsilon}$
- T → R S q

- FIRST(P) = {i,c,n}
- FIRST(Q) = {i,c,n,a,d}
- FIRST(R) = {b,ε}
- FIRST(S) = {e,b,n,ε}
- FIRST(T) =

Note:
$S \Rightarrow R\ n \Rightarrow n$ because $R \Rightarrow^* \varepsilon$

Example 2

- P → i | c | n T S
- Q → P | a S | d S c S T
- R → b | ε
- S → e | R n | ε
- T → $\underline{\text{R S q}}$

- FIRST(P) = {i,c,n}
- FIRST(Q) = {i,c,n,a,d}
- FIRST(R) = {b,ε}
- FIRST(S) = {e,b,n,ε}
- FIRST(T) = {b,c,n,q}

Note:
$T \Rightarrow R\ S\ q \Rightarrow S\ q \Rightarrow q$
because both R and $S \Rightarrow^* \varepsilon$

Example 3

- S → a S e | S T S
- T → R S e | Q
- R → r S r | ε
- Q → S T | ε

- FIRST(S) =
- FIRST(R) =
- FIRST(T) =
- FIRST(Q) =

Example 3

- S → a S e | S T S
- T → R S e | Q
- R → r S r | ε
- Q → S T | ε

- FIRST(S) = {a}
- FIRST(R) = {r, ε}
- FIRST(T) = {r,a, ε}
- FIRST(Q) = {a, ε}

FOLLOW Sets

- FOLLOW(A) is the set of terminals (including end of file - \$) that may follow non-terminal A in some sentential form.
- FOLLOW(A) = $\{a \in T \mid S \Rightarrow^+ \alpha A a \beta\} \cup \{\$\}$ if $S \Rightarrow^+ \gamma A$
- For example, consider $L \Rightarrow^+ (())(L)L$

  Both ')' and end of file can follow L
- NOTE: ε is ***never*** in FOLLOW sets

Computing FOLLOW(A)

1. If S is the start symbol, put \$ in FOLLOW(S)
2. Productions of the form $B \rightarrow \alpha\ A\ \beta$,

   Add FIRST(β) – $\{\varepsilon\}$ to FOLLOW(A)

   INTUITION: <u>Suppose $B \rightarrow AX$ and FIRST(X) = {c}</u>

   $S \Rightarrow^+ \alpha\ B\ \beta \Rightarrow \alpha\ A\ X\ \beta \Rightarrow^+ \alpha\ A\ c\ \delta\ \beta$

   = FIRST(X)

3. Productions of the form B → α A or

B → α A β where $\beta \Rightarrow^* \varepsilon$ (i.e, $\varepsilon \in$ FIRST(β))

Add everything in FOLLOW(B) to FOLLOW(A)

INTUITION:

– Suppose B → Y A

$S \Rightarrow^+ \alpha\ B\ \beta \Rightarrow \alpha\ Y\ A\ \beta$

FOLLOW(B)

– Suppose B → A X and X $\Rightarrow^* \varepsilon$

$S \Rightarrow^+ \alpha\ B\ \beta \Rightarrow \alpha\ A\ X\ \beta \Rightarrow^* \alpha\ A\ \beta$

FOLLOW(B)

Assume the first non-terminal is the start symbol

Example 4

- S → a S e | B
- B → b B C f | C
- C → c C g | d | ε

- FIRST(C) = {c,d,ε}
- FIRST(B) = {b,c,d,ε}
- FIRST(S) = {a,b,c,d,ε}

- FOLLOW(C) =
- FOLLOW(B) =
- FOLLOW(S) = {\$}

Using rule #1

Example 4

- $S \rightarrow a\ \underline{S\ e} \mid B$
- $B \rightarrow b\ \underline{B\ C\ f} \mid C$
- $C \rightarrow c\ \underline{C\ g} \mid d \mid \varepsilon$

- FIRST(C) = {c,d,ε}
- FIRST(B) = {b,c,d,ε}
- FIRST(S) = {a,b,c,d,ε}

- FOLLOW(C) = {f,g}
- FOLLOW(B) = {c,d,f}
- FOLLOW(S) = {$,e}

Using rule #2

Example 4

- $S \rightarrow a\ S\ e \mid \underline{B}$
- $B \rightarrow b\ B\ C\ f \mid \underline{C}$
- $C \rightarrow c\ C\ g \mid d \mid \varepsilon$

- FIRST(C) = {c,d,ε}
- FIRST(B) = {b,c,d,ε}
- FIRST(S) = {a,b,c,d,ε}

- FOLLOW(C) =
  {f,g} ∪ FOLLOW(B)
  = {c,d,e,f,g,$}
- FOLLOW(B) =
  {c,d,f} ∪ FOLLOW(S)
  = {c,d,e,f,$}
- FOLLOW(S) = {$, e }

Using rule #3

Example 5

- $S \rightarrow A\ B\ C \mid A\ D$
- $A \rightarrow \varepsilon \mid a\ A$
- $B \rightarrow b \mid c \mid \varepsilon$
- $C \rightarrow D\ d\ C$
- $D \rightarrow e\ b \mid f\ c$

- FIRST(D) = {e,f}
- FIRST(C) = {e,f}
- FIRST(B) = {b,c,ε}
- FIRST(A) = {a, ε}
- FIRST(S) = {a,b,c,e,f}

- FOLLOW(S) =
- FOLLOW(A) =
- FOLLOW(B) =
- FOLLOW(C) =
- FOLLOW(D) =

Example 5

- $S \rightarrow A\ B\ C \mid A\ D$
- $A \rightarrow \varepsilon \mid a\ A$
- $B \rightarrow b \mid c \mid \varepsilon$
- $C \rightarrow D\ d\ C$
- $D \rightarrow e\ b \mid f\ c$

- FIRST(D) = {e,f}
- FIRST(C) = {e,f}
- FIRST(B) = {b,c,ε}
- FIRST(A) = {a, ε}
- FIRST(S) = {a,b,c,e,f}

- FOLLOW(S) = {$}
- FOLLOW(A) = {b,c,e,f}
- FOLLOW(B) = {e,f}
- FOLLOW(C) = {$}
- FOLLOW(D) = {$}

Example 6

- $S \rightarrow (A) \mid \varepsilon$
- $A \rightarrow T\ E$
- $E \rightarrow \&\ T\ E \mid \varepsilon$
- $T \rightarrow (A) \mid a \mid b \mid c$

- FIRST(T) = {(,a,b,c}
- FIRST(E) = {&, ε }
- FIRST(A) = {(,a,b,c}
- FIRST(S) = {(, ε}

- FOLLOW(S) =
- FOLLOW(A) =
- FOLLOW(E) =
- FOLLOW(T) =

Example 6

- $S \rightarrow (A) \mid \varepsilon$
- $A \rightarrow T\ E$
- $E \rightarrow \&\ T\ E \mid \varepsilon$
- $T \rightarrow (A) \mid a \mid b \mid c$

- FIRST(T) = {(,a,b,c}
- FIRST(E) = {&, ε }
- FIRST(A) = {(,a,b,c}
- FIRST(S) = {(, ε}

- FOLLOW(S) = {$}
- FOLLOW(A) = {) }
- FOLLOW(E) =

FOLLOW(A) = {) }
- FOLLOW(T) =

FIRST(E) $\cup$ FOLLOW(A) $\cup$ FOLLOW(E) = {&,)}

Example 7

- $E \rightarrow T\ E'$
- $E' \rightarrow +\ T\ E' \mid \varepsilon$
- $T \rightarrow F\ T'$
- $T' \rightarrow *\ F\ T' \mid \varepsilon$
- $F \rightarrow (\ E\ ) \mid id$

- FOLLOW(E) =
- FOLLOW(E') =
- FOLLOW(T) =
- FOLLOW(T') =
- FOLLOW(F) =

- FIRST(F) = FIRST(T) = FIRST(E) = {(,id}
- FIRST(T') = {*,ε}
- FIRST(E') = {+,ε}

Example 7

- $E \rightarrow T\ E'$
- $E' \rightarrow +\ T\ E' \mid \varepsilon$
- $T \rightarrow F\ T'$
- $T' \rightarrow *\ F\ T' \mid \varepsilon$
- $F \rightarrow (\ E\ ) \mid id$

- FOLLOW(E) = {$,)}
- FOLLOW(E') = FOLLOW(E) = {$,)}
- FOLLOW(T) = FIRST(E') ∪ FOLLOW(E) ∪ FOLLOW(E') = {+,$,)}
- FOLLOW(T') = FOLLOW(T) = {+,$,)}
- FOLLOW(F) = FIRST(T') ∪ FOLLOW(T) ∪ FOLLOW(T') = {*,+,$,)}

- FIRST(F) = FIRST(T) = FIRST(E) = {(,id}
- FIRST(T') = {*,ε}
- FIRST(E') = {+,ε}

Using FIRST and FOLLOW to Write a Recursive Descent Parser

$$
\begin{aligned}
\textit{expr} &\rightarrow \textit{term rest} \\
\textit{rest} &\rightarrow + \textit{ term rest} \\
&\mid \mathbf{-} \textit{ term rest} \\
&\mid \varepsilon \\
\textit{term} &\rightarrow \mathbf{id}
\end{aligned}
$$

procedure *rest*();
begin
    if *lookahead* in FIRST(+ *term rest*) **then**
        match('+'); *term*(); *rest*()
    else if *lookahead* in FIRST(- *term rest*) **then**
        match('-'); *term*(); *rest*()
    else if *lookahead* in FOLLOW(*rest*) **then**
        return
    else error**()**
end;

FIRST(+ *term rest*) = { **+** }
FIRST(- *term rest*) = { **-** }
FOLLOW(*rest*) = { **$** }

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