Karnaugh Maps (K-maps) - "Car-no"

Karnaugh maps were first applied to circuits in a 1953 paper

"The Map Method for Synthesis of Combinational Logic Circuits"
by Maurice Karnaugh.

K-maps give us a graphical technique for the representation and simplification of a Boolean expression.

A Karnaugh map is made up of 2^n boxes, representing the 2^n entries of a truth table, arranged in a rectangular array (actually a torus would be more accurate). The correct positioning of the boxes is critical to the use of a Karnaugh map. Each box represents a binary sequence. Any two boxes which are adjacent horizontally or vertically must correspond to binary sequences differing in only one bit.

Example : 2 Variable K-map

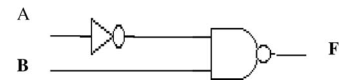
B \ A	0	1
0	$\bar{A}\cdot\bar{B}$	$A\bar{B}$
1	$\bar{A}B$	AB

Note: Each box represents a minterm. Each row a single term. Each column a single term.

Row 1 : represents $\bar{B}$ Row 2 : represents B

Column 1 : represents $\bar{A}$ Column 2 : represents A

Consider the circuit:



The corresponding truth table:

A	B	$\bar{A}$	$F=\overline{\bar{A}B}$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	0	1

Then we use the Karnaugh map by putting a 1 in a square which corresponds to 1 in the truth table.

B \ A	0	1
0	1 $\bar{A}\cdot\bar{B}$	1 $A\bar{B}$
1	$\bar{A}B$	1 AB

We can now show F as a sum of minterms:

$$F = \bar{A}\bar{B} + A\bar{B} + AB$$

Indeed we can even easily represent F as a negation of a sum of minterms:

$$F = \bar{\bar{F}} = \overline{\bar{A}\,\bar{B} + A\bar{B} + AB} = \bar{A}B$$

But the real power comes when we look at blocks of 2^n entries.

B \ A	0	1
0	1	1
1		1

The horizontal oval represents if B=0 it doesn't matter what A is, we get a 1. This two-cell block is $\bar{B}$.

Likewise, the vertical oval represents if A=1 it doesn't matter what B is, we get a 1. This two-cell block is A.

Using this reasoning,

$$F = A + \bar{B}$$

Note : This could have been done using *Algebra*

$$\begin{aligned} F &= \bar{A}\,\bar{B} + A\bar{B} + AB \\ &= (\bar{A} + A)\bar{B} + AB \\ &= \bar{B} + AB \quad \text{recall } (X + \bar{X}Y = X + Y) \\ &= \bar{B} + A \end{aligned}$$

The 3-varialble K-map:

C \ AB	00	01	11	10
0	$\bar{A}\,\bar{B}\,\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\,\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

Individual cells represent minterms.

Blocks of two cells represent represent 2 term Products

Some examples:

C \ AB	00	01	11	10
0	$\bar{A}\,\bar{B}\,\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\,\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

The oval represents $\bar{A}\,\bar{B}$

C \ AB	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

The oval above represents $B\bar{C}$

C \ AB	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

The two corners represent $\bar{B}\,\bar{C}$

Blocks of 4 cells represent individual variables (or their complements)

C \ AB	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

The 4 cells above represent $\bar{A}$

C \ AB	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

The 4 cells above represent $\bar{B}$

Lets use a Karnaugh map to help simplify

$$F = \bar{A}\,\bar{B}\,\bar{C} + \bar{A}\,\bar{B}\,C + A\bar{B}C + \bar{A}B\bar{C}$$

C \ AB	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

We put ones in the corresponding positions:

C \ AB	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$ 1	$\bar{A}B\bar{C}$ 1	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	1 $\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	1 $A\bar{B}C$

Notice the vertical oval is not adding any new information. Adding its corresponding term will be redundant so we will omit it.

$$F = \bar{A}\bar{C} + \bar{B}C$$

For our last 3 variable example, consider reducing the function:

$$F = \bar{A}\,\bar{B}\,\bar{C} + \bar{A}\,B\,C + A\,\bar{B}\,C$$

C \ AB	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

Putting the 1's in

C \ AB	00	01	11	10
0	1 $\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	1 $\bar{A}BC$	ABC	1 $A\bar{B}C$

Notice there are not adjacent cells. This particular function can not be reduced.

4 variable K-Map

CD \ AB	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

Individual cells represent minterms. 2 cell blocks represent 3 variable terms. 4 cell blocks represent 2 variable terms. 8 cell blocks represent 1 variable terms. (We only group 2^k terms at a time)

Example: Use a K-map to simplify $F = AB + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}\bar{D}$

We will start by identifying the appropriate cells –

CD \ AB	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

RULE OF THUMB: To achieve minimization include the smallest number of blocks having each having the largest possible number of cells.

CD \ AB	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

CD \ AB	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}B\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$A\bar{B}\bar{C}\bar{D}$
01	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}B\bar{C}D$	$AB\bar{C}D$	$A\bar{B}\bar{C}D$
11	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$ABCD$	$A\bar{B}CD$
10	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}BC\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$

$F = A\bar{C}\bar{D} + AB + BD$