Chapter 8 (Hypothesis Testing)

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Stating Hypotheses

Null hypothesis H_0: This is the statement that is under investigation or being tested. Usually the null hypothesis represents a statement of "no effect," "no difference," or, put another way, "things haven't changed."

Alternate hypothesis H_1: This is the statement you will adopt in the situation in which the evidence (data) is so strong that you reject H_0. A statistical test is designed to assess the strength of the evidence (data) against the null hypothesis.

Types of Tests

Types of statistical tests

A statistical test is:

left-tailed if H_1 states that the parameter is less than the value claimed in H_0

right-tailed if H_1 states that the parameter is greater than the value claimed in H_0

two-tailed if H_1 states that the parameter is different from (or not equal to) the value claimed in H_0

Testing of Hypothesis about a population mean

The goal of testing statistical hypothesis is to determine if a claim about some feature of the population parameter is strongly supported by the sample data.

Null hypothesis:

$$H_0 : \mu = \mu_0$$

Alternate hypothesis

$$H_a : \mu \neq \mu_0$$

$$H_a : \mu > \mu_0$$

$$H_a : \mu < \mu_0$$

Testing of Hypothesis about a population mean

Decisions

Either

Reject H_0 and conclude that H_1 is substantiated

or

Retain H_0 and conclude that H_1 fails to be substantiated

Since hypothesis testing is based on observed sample statistics computed on n observations, the decision is always subject to error.

Summary

Type I Error ➔Rejecting a true null hypothesis

Type II Error ➔Failing to reject a false null hypothesis

Power ➔The probability of correctly detecting a false null hypothesis
(That is, Power = 1 – β)

Alpha Value ➔Probability of rejecting a true null hypothesis

Beta Value ➔Probability of failing to reject a false null hypothesis

	H_0 True	H_0 False
Reject	Type I Error Alpha and p-value	✔ POWER
Fail to Reject	✔ Confidence	Type II Error Beta

Hypothesis Tests of μ, Given x Is Normal and σ Is Known

Procedure:

Requirements The x distribution is *normal* with known standard deviation σ.

Then $\bar{x}$ has a normal distribution. The standardized test statistic is

$$\textbf{test statistic} = z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

where $\bar{x}$ = mean of a simple random sample

μ = value stated in H_0.

n = sample size

Two approaches to make decision

The random variable $\overline{X}$ whose value serves to determine the action is called the **test statistic.**

A **test of the null hypothesis** is a course of action specifying the set of values of a test statistic $\overline{X}$, for which H_0 is to be rejected.

This set is called the **rejection region** of the test.

The ***P*–value** is the probability, calculated under H_0, that the test statistic takes a value equal to or more extreme than the value actually observed.

Example: Sigma known

Unfortunately, arsenic occurs naturally in some ground water (*Reference: Union Carbide Technical Report*). A mean arsenic level of 8 parts per billion (ppb) is considered safe for agricultural use. A well in Texas is used to water cotton crop. This well is tested on a regular basis for arsenic. A random sample of 37 tests gave a sample mean of 7.2. Based on previous studies, the standard deviation of arsenic level is observed to be 1.9, does this information indicate that the mean level of arsenic in this well is less than 8 ppb? Use 1% level of significance.

Example

a) Identify the underlying distribution.
Since $n \geq 30$, the underlying distribution is approximately normal.

b) State the null hypothesis. H_o: $\mu = 8$

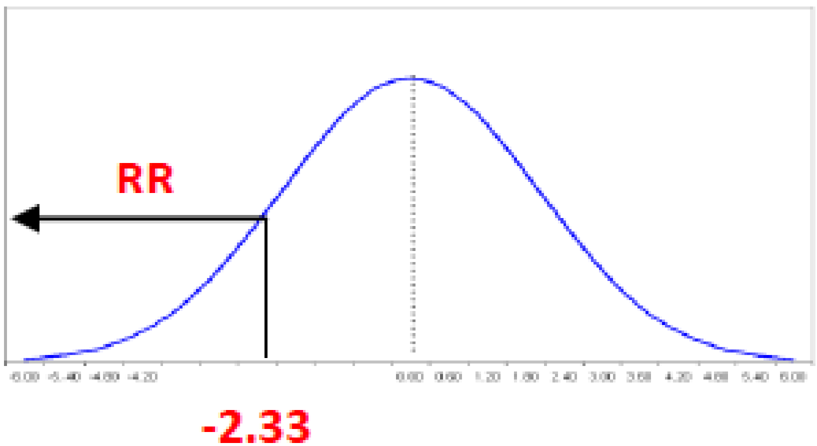
c) State the alternative hypothesis. H_1: $\mu \leq 8$

d) Circle one: <u>One Tail Test</u> / Two Tail Test.

e) State the critical value for the hypothesis test $Z_\alpha = -2.33$

Example

f. Illustrate graphically the rejection region.



Rejection Region R: Z≤ -2.33

g.
Compute Test Statistic

$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{7.2-8}{1.9/\sqrt{37}} = -2.56$$

Example ctd.

h) Find the p-value for the test statistic.

P-value = $P(Z<-2.56)=0.005$

i) Give the significant statement for the hypothesis test: Since P-value is less than alpha level (0.01). At the ____**1%**____ level of significance, there is _**sufficient**_________ evidence to reject the null hypothesis. This means we have enough evidence to say that that the mean level of arsenic in this well is less than 8 ppb

Example

A student at a large midwestern university questioned n=40 students concerning the amount of time they spent doing community service during the past month. The mean amount of time is 4.55 hours. The previous studies indicates that the standard deviation is 5.17. Do these times indicate that the population mean time is different from 2.6 hours at 5% level of significance?

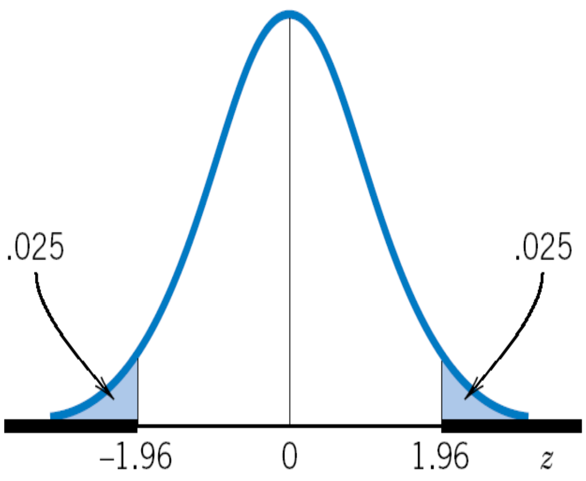
Example:

1. Identify the underlying distribution:

2. Write null and alternative hypothesis
H_o: $\mu = 2.6$ versus H_1: $\mu \neq 2.6$

3. Is this one tail or two tail test?

4. Find the critical value and draw the rejection region
Note: **Rejection Region R: $|Z| \geq Z_{\alpha/2}$**



5. Find the Test Statistic

Example:

6. Find the p-value of the test statistic:

P-value = $2*P(Z>2.39) = 2*0.0084= 0.0168$

7. Conclude your finding:

Since the obtained p-value (0.0168) is less than alpha level(0.05), we reject our null hypothesis and conclude that the population mean time to community service is different from 2.6 hours.

Exercise 20 (Textbook page 451) (Right Tail test example)

Guided Exercise 3 (Page 445 textbook)