Solutions to Selected Computer Lab Problems and Exercises in Chapter 20 of *Statistics and Data Analysis for Financial Engineering, 2nd ed.* by David Ruppert and David S. Matteson

Problem 1a.
```
> effectiveSize(univ_t.coda)
        k        mu     sigma       tau
 955.2276 1413.1482 1500.0000  808.5236
```

sigma mixes best since it has the larger N_{eff}

Problem 1b. tau mixes worst since it has the smallest N_{eff}

Problem 1c.
```
> summary(univ_t.coda)

Iterations = 1002:2000
Thinning interval = 2
Number of chains = 3
Sample size per chain = 500

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

           Mean        SD  Naive SE Time-series SE
k     2.489e+01 11.751537 3.034e-01      3.812e-01
mu    9.384e-03  0.003580 9.244e-05      9.686e-05
sigma 6.848e-02  0.002733 7.057e-05      7.061e-05
tau   2.405e+02 24.279612 6.269e-01      8.619e-01

2. Quantiles for each variable:

           2.5%       25%       50%       75%     97.5%
k     7.653e+00 1.483e+01 2.316e+01  34.21638  47.76387
mu    2.205e-03 7.006e-03 9.362e-03   0.01169   0.01649
sigma 6.345e-02 6.663e-02 6.832e-02   0.07027   0.07402
tau   2.009e+02 2.235e+02 2.373e+02 255.48430 294.81731
```

Using the 2.5% and 97.5% percentiles, the 95% equal-tails interval is (7.6, 47.8).

The function `HPDinterval()` produces a high posterior density interval for each of the three chains. These are (8.2, 47.9), (7.5, 45.3), and (7.5, 48.1).

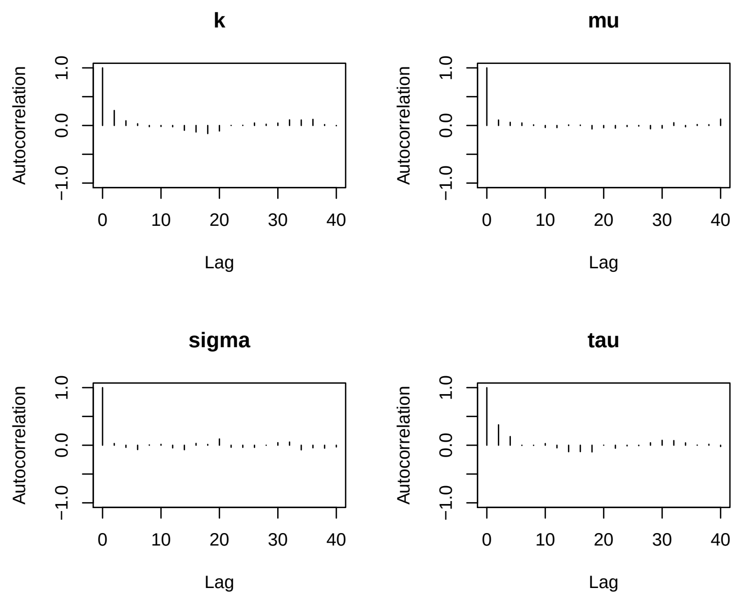
```
> options(digits=2)
> HPDinterval(univ_t.coda)
[[1]]
        lower   upper
k     8.2e+00  47.898
```

```
mu    1.6e-04   0.016
sigma 6.4e-02   0.075
tau   2.0e+02 283.877
attr(,"Probability")
[1] 0.95

[[2]]
        lower   upper
k     7.5e+00  45.277
mu    3.7e-03   0.016
sigma 6.3e-02   0.074
tau   1.9e+02 286.009
attr(,"Probability")
[1] 0.95

[[3]]
        lower   upper
k     7.5e+00  48.142
mu    1.8e-03   0.016
sigma 6.4e-02   0.074
tau   2.1e+02 287.658
attr(,"Probability")
[1] 0.95
```

Below the three chains are combined and the resulting posterior interval is (7.4, 47).

```
> k1 = univ_t.coda[[1]][,1]
> k2 = univ_t.coda[[2]][,1]
> k3 = univ_t.coda[[3]][,1]
> k = c(k1,k2,k3)
> HPDinterval(as.mcmc(k))
     lower upper
var1   7.4    47
attr(,"Probability")
[1] 0.95
```

Problem 2a. From the ACF plots, it appears that tau mixes worst and k mixes best. These results to not agree with the results form the N_{eff} values. It should be noted that all of the parameters mix rather well, so it is difficult to determine which parameters mix best and worst.



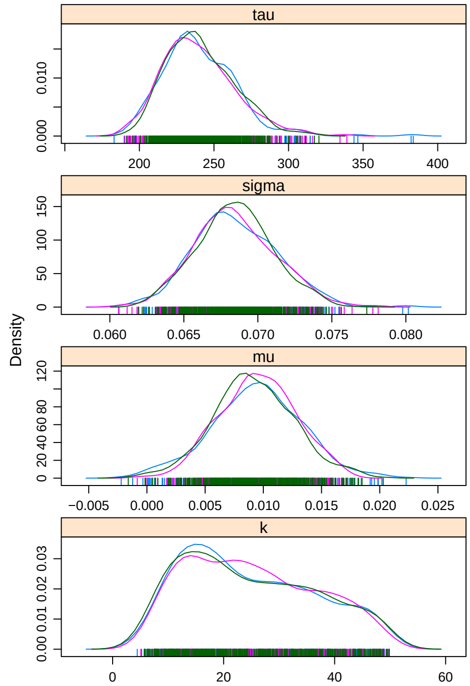
Problem 2b. The following code extracts the degrees of freedom parameter from each chain, combines the three samples, and computes the sample skewness and kurtosis of the combined sample.

```
> k1 = univ_t.coda[[1]][,1]
> k2 = univ_t.coda[[2]][,1]
> k3 = univ_t.coda[[3]][,1]
> k = c(k1,k2,k3)
> std_k = (k-mean(k)) / sqrt(mean((k-mean(k))^2))
> options(digits=4)
> mean(std_k^3)
[1] 0.3544
> mean(std_k^4)
[1] 2.018d_k^4)
[1] 5.012
```

Skewness and kurtosis can also be computed using functions in the `moments` package.

```
> library(moments)
> skewness(k)
[1] 0.3544
> kurtosis(k)
[1] 2.018
```

Problem 3. The degrees of freedom parameter (`k`) has the most skewed posterior density.



Problem 7. The BUGS code is below and is in the file `arma11.bug`.

```
model{
for (i in 2:N)
{
   w[i] <- y[i] - phi * y[i-1] - theta * w[i-1]
}
w[1] ~ dnorm(0, 0.01)
for (i in 2:N)
{
   y[i] ~ dnorm(phi * y[i-1] + theta * w[i-1], tau)
}
phi ~ dnorm(0, 0.001)
theta ~ dnorm(0, 0.001)
tau ~ dgamma(0.01 ,0.0001)
sigma <- 1/sqrt(tau)
}
```

The `R` program is below.

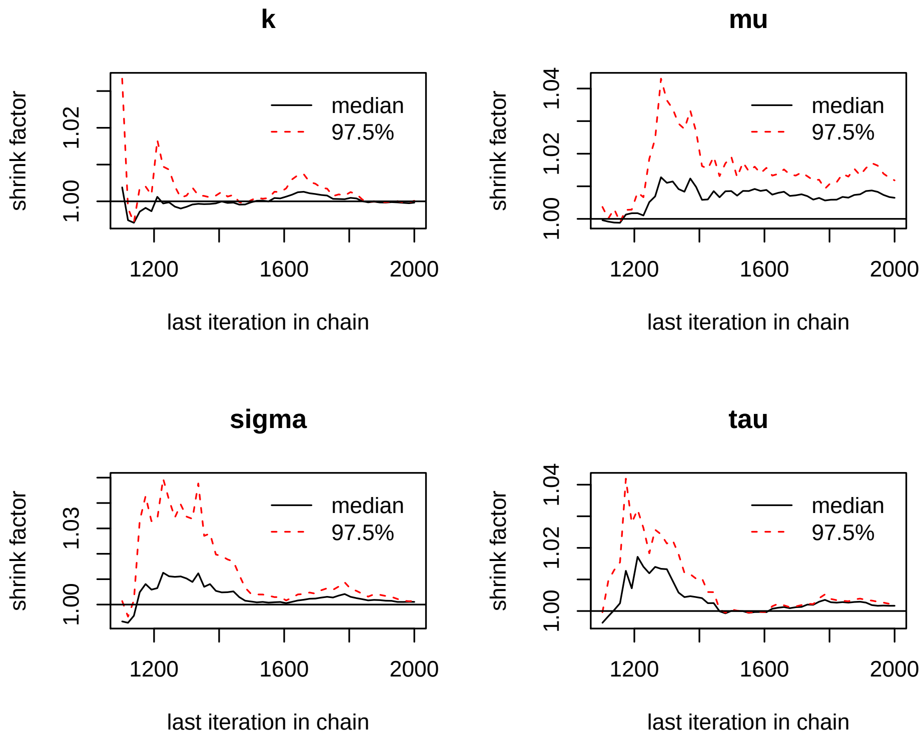
```
library(rjags)
set.seed(5640)
N=600
y = arima.sim(n = N, list(ar = .9, ma = -.5), sd = .4)
y = as.numeric(y)
arma11.sim_data=list(y = y, N = N)
```

```
inits.arma11 =function(){list(phi = rnorm(1, 0 , 0.3),
   theta=rnorm(1,-0.5, 0.1), tau=runif(1,5,8))}

arma11 <- jags.model("arma11.bug", data = arma11.sim_data,
   inits = inits.arma11,
   n.chains = 3, n.adapt = 1000, quiet = FALSE)
nthin = 5
arma11.coda = coda.samples(arma11, c("phi", "theta", "sigma"),
   n.iter = 500 * nthin, thin = nthin)
summary(arma11.coda)
effectiveSize(arma11.coda)
gelman.diag(arma11.coda)

gelman.plot(univ_t.coda)
```

The output is below.

```
> summary(arma11.coda)

Iterations = 1005:3500
Thinning interval = 5
Number of chains = 3
Sample size per chain = 500

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

        Mean     SD Naive SE Time-series SE
phi    0.914 0.0240 0.000620       0.000680
sigma  0.392 0.0115 0.000297       0.000324
theta -0.569 0.0456 0.001178       0.001309

2. Quantiles for each variable:

        2.5%    25%    50%    75%  97.5%
phi    0.864  0.899  0.915  0.931  0.958
sigma  0.371  0.384  0.392  0.400  0.416
theta -0.656 -0.601 -0.571 -0.541 -0.472

> effectiveSize(arma11.coda)
  phi sigma theta
 1271  1284  1217
> gelman.diag(arma11.coda)
Potential scale reduction factors:

      Point est. Upper C.I.
phi            1       1.00
sigma          1       1.01
```

```
theta          1       1.01

Multivariate psrf

1
```



(a) The chains mix rather well and all effective sample sizes are over 1,200. In comparison, the actual sample size is not much larger, only 1,500, since there 3 chains each of size 500 after thinning. The Gelman diagnostics are close to 1 indicating good mixing and an adequate burn-in.

(b) The posterior intervals are below. They differ slightly between chains. The posterior interval for ϕ is (0.87, 0.96) for all three chains and contains the true value, 0.9. The interval for θ is $(-0.66, -0.48)$ for the second and third chain and $(-0.68, -0.50)$ for the first. All three of the intervals contain the true value, -0.5.

```
> HPDinterval(arma11.coda)
[[1]]
        lower   upper
phi    0.8729  0.9622
sigma  0.3707  0.4147
theta -0.6679 -0.4961
attr(,"Probability")
```

```
[1] 0.95

[[2]]
        lower   upper
phi    0.8685  0.9579
sigma  0.3678  0.4128
theta -0.6588 -0.4768
attr(,"Probability")
[1] 0.95

[[3]]
        lower   upper
phi    0.8676  0.9600
sigma  0.3686  0.4130
theta -0.6662 -0.4825
attr(,"Probability")
[1] 0.95
```