

**Solutions to Selected Computer Lab Problems and Exercises
in Chapter 16 of *Statistics and Data Analysis for Financial
Engineering, 2nd ed.* by David Ruppert and David S.
Matteson**

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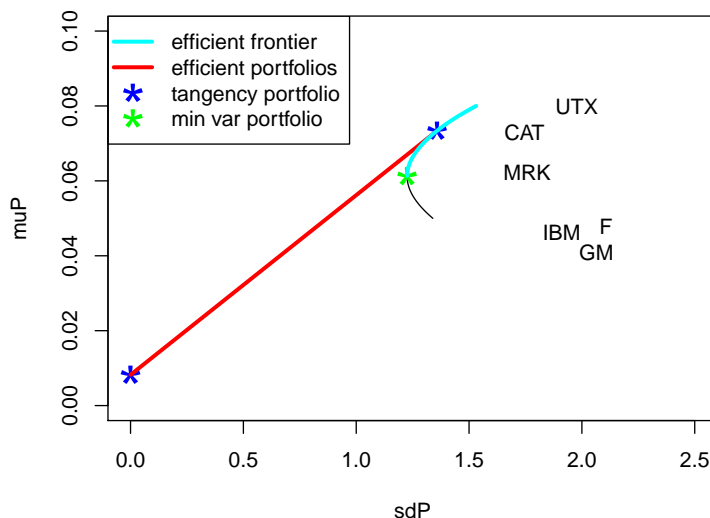
Problem 1. The R program continues the code shown above and shown below.

```
M = length(mean_vect)
library(quadprog)
Amat = cbind(rep(1,M),mean_vect,diag(1,nrow=M),-diag(1,nrow=M))
muP = seq(min(mean_vect)+.02,max(mean_vect)-.02,length=10)
muP = seq(.05,0.08,length=300)
sdP = muP
weights = matrix(0,nrow=300,ncol=M)
for (i in 1:length(muP))
{
  result =
    solve.QP(Dmat=cov_mat,dvec=rep(0,M), Amat=Amat,
      c(1,muP[i],rep(-.1,M),rep(-.5,M)), meq=2)
  sdP[i] = sqrt(2*result$value)
  weights[i,] = result$solution
}
plot(sdP,muP,type="l",xlim=c(0,2.5),ylim=c(0,.1))
mufree = 3/365
points(0,mufree,cex=3,col="blue",pch="*")
sharpe =( muP-mufree)/sdP
ind = (sharpe == max(sharpe)) # locates the tangency portfolio
weights[ind,] # weights of the tangency portfolio
lines(c(0,sdP[ind]),c(mufree,muP[ind]),col="red",lwd=3)
points(sdP[ind],muP[ind],col="blue",cex=3,pch="*")
ind2 = (sdP == min(sdP))
points(sdP[ind2],muP[ind2],col="green",cex=3,pch="*")
ind3 = (muP > muP[ind2])
lines(sdP[ind3],muP[ind3],type="l",xlim=c(0,.25),
  ylim=c(0,.3),col="cyan",lwd=3)
text(sd_vect[1],mean_vect[1],"GM")
text(sd_vect[2],mean_vect[2],"F")
text(sd_vect[3],mean_vect[3],"UTX")
text(sd_vect[4],mean_vect[4],"CAT")
text(sd_vect[5],mean_vect[5],"MRK")
text(sd_vect[6],mean_vect[6],"IBM")
legend("topleft",c("efficient frontier","efficient portfolios",
  "tangency portfolio","min var portfolio"),
  lty=c(1,1,NA,NA),
  lwd=c(3,3,1,1),
  pch=c("","","*","*"),
```

```
col=c("cyan","red","blue","green"),
pt.cex=c(1,1,3,3)
)
```

The last few lines of code produce a legend. You were not expected to include a legend on your own plot, but you can use this example in the future when you are asked to provide a legend. R's help will give you more information about legends.

Here is the plot:



Problem 2. Let ω be the weight for the risk-free asset, μ_f be the risk-free rate, and μ_T be the expected return of the tangency portfolio. Then ω solves $0.07 = \omega\mu_f + (1-\omega)\mu_T$. The following continuation of the R program computes the weights for the six stocks and the risk-free asset. The last line checks that the weights sum to 1.

```
options(digits=3)
omega = (.07 - muP[ind]) / (3/265 - muP[ind])
omega
1-omega
(1-omega)*weights[ind]
omega + sum((1-omega)*weights[ind])

> options(digits=3)
> omega = (.07 - muP[ind]) / (3/265 - muP[ind])
> omega
[1] 0.0544
> 1-omega
[1] 0.946
> (1-omega)*weights[ind]
```

```
[1] -0.08622 -0.00275  0.31707  0.36283  0.30209  0.05255
> omega + sum((1-omega)*weights[ind])
[1] 1
```

We see that the weight for the risk-free asset is 0.054 and the weights for the six stocks are -0.08622 , -0.00275 , 0.31707 , 0.36283 , 0.30209 , 0.05255 . The first two stocks are sold short.

Problem 3. Yes, Black Monday was October 19, 1987 and data go from January 2, 1987 to Sept 1, 2006. Black Monday is the 202th day in the original data set or the 201st day of returns.

If you look in the spread sheet you will see huge price declines that day. The returns that day were:

```
returns[201,]
[1] -21.0 -18.2 -21.7 -15.7 -13.0 -23.5
```

Exercise 1a.

$$0.03 = (0.023)w + (0.045)(1-w) = 0.045 - 0.022w \Rightarrow w = 0.015/0.022 = 0.6818.$$

The portfolio is 68.18% in asset A and 31.82 in asset B.

Exercise 1b. We need to find w that solves

$$5.5 = 6w^2 + 11(1-w)^2 + (2)\sqrt{(6)(11)}(0.17)w(1-w)$$

The solutions are 0.411 and 0.940. These give expected returns of 3.60 and 2.43, respectively, so the largest expected return is achieved by $w = 0.411$. The algebra was done in R as follows:

```
> poly1 = c(-5.5,0,0)
> poly2 = c(0,0,6)
> poly3 = 11*c(1,-2,1)
> poly4 = 2*sqrt(6*11)*0.17*c(0,1,-1)
> poly = poly1+poly2+poly3+poly4
> roots = polyroot(poly)
> roots
[1] 0.411+0i 0.940-0i
> Re(2.3 * roots + 4.5 * (1-roots))
[1] 3.60 2.43
```

Exercise 2. $2/7 = 0.2857$ in risk-free, 0.4643 in C, and 0.2500 in D.

Exercise 5. The equation

$$\mathcal{R}_P = w_1\mathcal{R}_1 + \cdots + w_N\mathcal{R}_N \quad (1)$$

is true if \mathcal{R}_P is a net or gross return, but (1) not in general true if \mathcal{R}_P is a log return. However, if all the net returns are small in absolute value, then the log returns are approximately equal to the net returns and (1) will hold approximately.

Let us go through an example first. Suppose that $N = 3$ and the initial portfolio has \$500 in asset 1, \$300 in asset 2, and \$200 in asset 3, so the initial price of the portfolio is \$1000. Then the weights are $w_1 = 0.5$, $w_2 = 0.3$, and $w_3 = 0.2$. (Note that the number of shares being held of each asset and the price per share are irrelevant. For example, it is immaterial whether asset 1 is \$5/share and 100 shares are held, \$10/share and 50 shares held, or the price per share and number of shares are any other values that multiply to \$500.) Suppose the gross returns are 2, 1, and 0.5. Then the price of the portfolio at the end of the holding period is

$$500(2) + 300(1) + 200(0.5) = 1400$$

and the gross return on the portfolio is $1.4 = 1400/1000$. Note that

$$1.4 = w_1(2) + w_2(1) + w_3(0.5) = (0.5)(2) + (0.3)(1) + (0.2)(0.5).$$

so (1) holds for gross return. Since a net return is simply the gross return minus 1, if (1) holds for gross returns then it holds for net returns, and vice versa. The log returns in this example are $\log(2) = 0.693$, $\log(1) = 0$, and $\log(0.5) = -\log(2) = -0.693$. Thus, the right hand side of (1) when $\mathcal{R}_1, \dots, \mathcal{R}_N$ are log returns is

$$(0.5 - 0.2)(0.693) = 0.138$$

but the log return on the portfolio is $\log(1.4) = 0.336$ so (1) does not hold for log returns. In this example, equation (1) is not even a good approximation because two of the three net returns have large absolute values.

Now let us show that (1) holds in general for gross returns and hence for net returns. Let P_1, \dots, P_N be the prices of assets 1 through N in the portfolio. (As in the example, P_j is the price per share of the j th asset times the number of shares in the portfolio.) Let R_1, \dots, R_N be the net returns on these assets. The j th weight is equal to the ratio of the price of the j th asset in the portfolio to the total price of the portfolio which is

$$w_j = \frac{P_j}{\sum_{i=1}^N P_i}.$$

At the end of the holding period, the price of the j th asset in the portfolio has changed from P_j to $P_j(1 + R_j)$, so that the gross return on the portfolio is

$$\frac{\sum_{j=1}^N P_j(1 + R_j)}{\sum_{i=1}^N P_i} = \sum_{j=1}^N \left(\frac{P_j}{\sum_{i=1}^N P_i} \right) (1 + R_j) = \sum_{i=j}^N w_j(1 + R_j),$$

which proves (1) for gross returns.

Problem 5a. $(0.0047)w + (0.0065)(1 - w) = 0.005$ so that the estimated efficient portfolio is 57.14% stock 1 and 42.86% stock 2.

Problem 5b. $(0.5714)^2(0.012) + (0.4286)^2(0.023) + 2(0.5714)(0.4286)(0.0058) = 0.01098$.

Problem 5c. Actual expected return is $(0.5714)(0.0031) + (0.4286)(0.0074) = 0.004943$.

Actual variance of return is

$$(0.5714)^2(0.017) + (0.4286)^2(0.025) + 2(0.5714)(0.4286)(0.0059) = 0.01303.$$