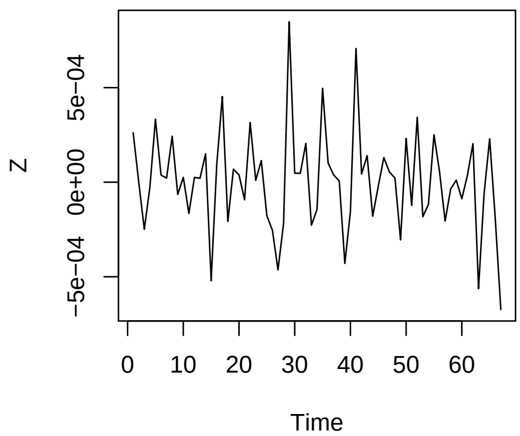
Solutions to Selected Computer Lab Problems and Exercises in Chapter 15 of *Statistics and Data Analysis for Financial Engineering, 2nd ed.* by David Ruppert and David S. Matteson

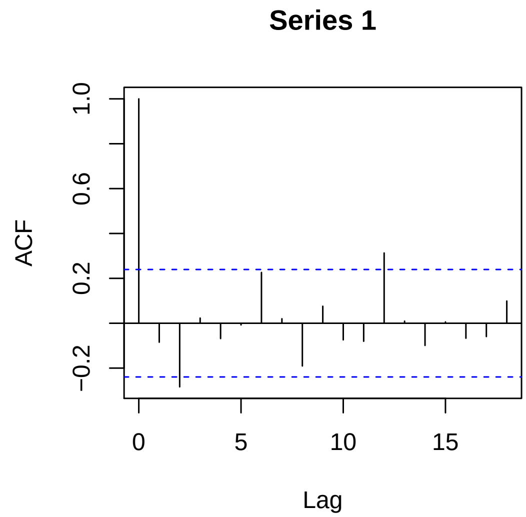
Problem 3. The output is below. If we use the 1% cutoff, then we accept that r $\leq$ 4 but reject that r $\leq$ 3 and so conclude that the number of cointegration vectors is r = 4.

```
> library(urca)
> mk.maturity = read.csv("mk.zero2.csv", header=T)
> options(digits=3,width=60)
> yields.cajo = ca.jo(mk.maturity[,2:11])
> summary(yields.cajo)

######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , with linear trend

Eigenvalues (lambda):
 [1] 0.7518 0.6630 0.5981 0.5498 0.4675 0.4263 0.3486 0.2474
 [9] 0.1162 0.0218

Values of teststatistic and critical values of test:

          test 10pct  5pct 1pct
r <= 9 |  1.43   6.5  8.18 11.6
r <= 8 |  8.03  12.9 14.90 19.2
r <= 7 | 18.48  18.9 21.07 25.8
r <= 6 | 27.87  24.8 27.14 32.1
r <= 5 | 36.12  30.8 33.32 38.8
r <= 4 | 40.96  36.2 39.43 44.6
r <= 3 | 51.87  42.1 44.91 51.3
r <= 2 | 59.25  48.4 51.07 57.1
r <= 1 | 70.71  54.0 57.00 63.4
r = 0  | 90.57  59.0 62.42 68.6

Eigenvectors, normalised to first column:
(These are the cointegration relations)

          M.1.l2   M.2.l2  M.3.l2  M.4.l2 M.5.l2    M.6.l2
M.1.l2    1.0000   1.0000   1.000   1.000    1.0    1.0000
M.2.l2   -4.7344   0.0688  -5.226  -2.685   -1.1   -3.7018
M.3.l2   10.4480 -16.1326  14.436  -1.216  -14.9    4.7046
```

```
M.4.l2  -11.4070  55.1551 -27.040  11.825   59.1   -2.0077
M.5.l2    0.0861 -95.6143  37.192 -14.876  -92.7   22.2520
M.6.l2   16.0781  92.4013 -38.015  -1.399   18.5  -93.9857
M.7.l2  -18.4791 -46.0499  23.988  15.680  130.5  156.3148
M.8.l2    6.4942  11.3213  -4.385  -0.994 -160.4 -122.2777
M.9.l2    1.9199  -3.1913  -2.660 -14.820   70.3   37.7554
M.10.l2  -1.4045   1.0592   0.708   7.487  -10.2   -0.0391
          M.7.l2 M.8.l2 M.9.l2 M.10.l2
M.1.l2    1.0000   1.00    1.0   1.000
M.2.l2   -0.0754  -2.81  -13.8  -0.215
M.3.l2  -16.5754  -2.03   72.0 -16.884
M.4.l2   36.8685  17.14  -99.1  39.882
M.5.l2  -19.4532 -22.91  -29.7 -20.449
M.6.l2  -14.0171   2.50   46.0 -32.564
M.7.l2    8.9956   8.83  254.0  42.682
M.8.l2   13.2719  20.93 -427.4  -1.593
M.9.l2  -12.1269 -39.08  258.6 -23.037
M.10.l2   2.0773  16.42  -61.9  11.081

Weights W:
(This is the loading matrix)

       M.1.l2 M.2.l2 M.3.l2 M.4.l2  M.5.l2 M.6.l2 M.7.l2
M.1.d   11.41 0.0231   2.93 -6.142  1.1392 -2.056 -1.108
M.2.d    6.09 0.3243   4.68 -1.920  0.3127 -1.732 -1.555
M.3.d    3.61 0.6442   5.03 -0.994  0.0122 -1.526 -1.644
M.4.d    3.51 0.9286   4.65 -1.621 -0.0172 -1.325 -1.482
M.5.d    4.02 1.0595   4.08 -2.191 -0.0374 -1.143 -1.286
M.6.d    4.49 0.9969   3.57 -2.271 -0.1221 -0.997 -1.151
M.7.d    4.85 0.8585   3.15 -2.108 -0.2187 -0.913 -1.060
M.8.d    5.01 0.7304   2.90 -1.942 -0.2757 -0.890 -0.978
M.9.d    4.97 0.6658   2.85 -1.912 -0.2788 -0.935 -0.883
M.10.d   4.85 0.6689   2.89 -2.035 -0.2280 -1.015 -0.765
       M.8.l2    M.9.l2 M.10.l2
M.1.d  -0.196 -0.069935   0.104
M.2.d  -0.718  0.000369   0.152
M.3.d  -0.819  0.039527   0.168
M.4.d  -0.707  0.057404   0.170
M.5.d  -0.586  0.067680   0.178
M.6.d  -0.517  0.077499   0.190
M.7.d  -0.466  0.087693   0.202
M.8.d  -0.416  0.097029   0.212
M.9.d  -0.363  0.104393   0.217
M.10.d -0.335  0.109826   0.218

> Z =  as.matrix(mk.maturity[,2:11]) %*% yields.cajo@V[,1]
> par(mfrow=c(1,2))
> ts.plot(Z)
> acf(Z)
```

The time series and ACF plots of Z are below. Neither plot shows any sign of nonstationarity.





Problem 5. The output is below. If we test that r = 0, then the test statistic (8.48) is below the 10% cutoff (12.91), so we accept the null hypothesis that r = 0, that is, that there are no cointegration vectors.

If there was a cointegration vector then there would be a portfolio of Coke and Pepsi stock whose price was mean-reverting and this portfolio would be an arbitrage opportunity. Arbitrage opportunities are rare, so it is not surprising that we did not find a cointegrating vector.

```
> summary(ca.jo(CokePepsi))

######################
# Johansen-Procedure #
######################

Test type: maximal eigenvalue statistic (lambda max) , with linear trend

Eigenvalues (lambda):
[1] 0.0057313636 0.0004665101

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 1 | 0.69  6.50  8.18 11.65
r = 0  | 8.48 12.91 14.90 19.19

Eigenvectors, normalised to first column:
(These are the cointegration relations)
```

```
                KO.Adjusted.l2 PEP.Adjusted.l2
KO.Adjusted.l2        1.000000      1.00000000
PEP.Adjusted.l2      -1.070219     -0.04649554

Weights W:
(This is the loading matrix)

               KO.Adjusted.l2 PEP.Adjusted.l2
KO.Adjusted.d     0.003396076   -0.0010994476
PEP.Adjusted.d    0.013419081   -0.0004738957
```