Solutions to Selected Computer Lab Problems and Exercises in Chapter 6 of *Statistics and Data Analysis for Financial Engineering, 2nd ed.* by David Ruppert and David S. Matteson

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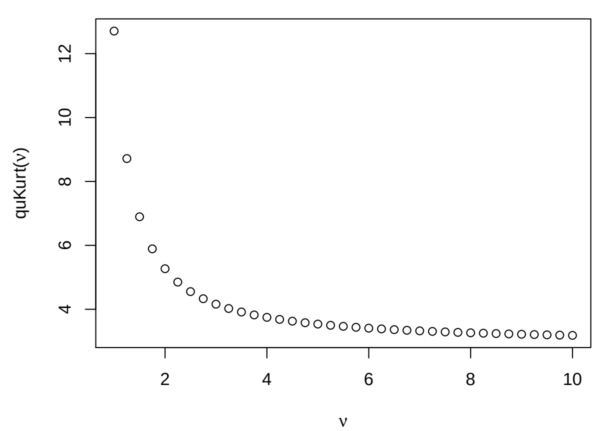
Problem 1. The MLE of ν is 2.99. To have a finite skewness $\nu > 3$ is required, and to have a finite kurtosis $\nu > 4$ is needed, so the t distribution with this value of ν does not have a finite skewness or kurtosis. (Of course, with $\widehat{\nu}$ just below 3, it is possible that the true value of ν exceeds 3. However, a t-distribution with ν equal to the MLE does not have a finite skewness.)

```
> fit_skewt$estimate
    mean       sd       nu       xi
0.000386 0.016116 2.988917 1.033262
```

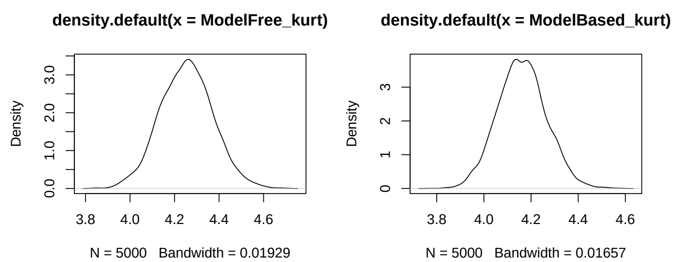
Problem 2.

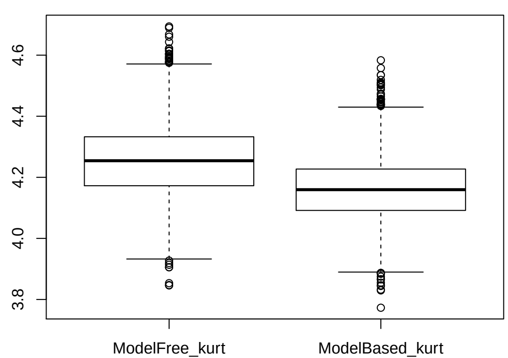
```
nu = seq(1,10,by=0.25)
p1=0.025
p2=0.25
numerator = qt(1-p1,df=nu)-qt(p1,df=nu)
denominator = qt(1-p2,df=nu)-qt(p2,df=nu)
plot(nu,numerator/denominator,ylab=expression(paste("quKurt(",nu,")")),
   xlab=expression(nu))
```



Problem 3. The R code and plots are below. The model-free bootstrap distribution of quKurt has a somewhat higher mean and variance compared to the model-based distribution. Both distributions seem symmetric and light-tailed. See you can see, the boxplots are better suited for comparing the two samples.

```
nboot = 5000
ModelFree_kurt  = rep(0, nboot)
ModelBased_kurt = rep(0, nboot)
set.seed("5640")
for (i in 1:nboot)
{
  samp_ModelFree = sample(bmwRet[,2], n, replace = TRUE)
  samp_ModelBased = rsstd(n, fit_skewt$estimate[1],
                          fit_skewt$estimate[2],
                          fit_skewt$estimate[3], fit_skewt$estimate[4])
  ModelFree_kurt[i] = quKurt(samp_ModelFree)
  ModelBased_kurt[i] = quKurt(samp_ModelBased)
}
par(mfrow = c(1,2))
plot(density(ModelFree_kurt))
plot(density(ModelBased_kurt))
par(mfrow = c(1,1))
boxplot(cbind(ModelFree_kurt,ModelBased_kurt))
```





Problem 4. The output below shows that the 90% confidence intervals using the model-free and model-based bootstraps are (4.065, 4.447) and (3.997, 4.332), respectively.

```
> options(digits=4)
> quantile(ModelFree_kurt,c(0.05,0.95))
   5%   95%
4.065 4.447
> quantile(ModelBased_kurt,c(0.05,0.95))
```

```
   5%   95%
3.997 4.332
```

Problem 5. The 90% BC_a confidence interval is (4.113, 4.497).

The BC_a interval is shifted to the right compared to the percentile intervals. The BC_a interval corrects for bias and for the variance of the estimator depending on the value of the parameter, and, in contrast, the percentile intervals do not make these corrections. Therefore, it is not surprising that the BC_a interval differs from the percentile intervals.

```
> library(bootstrap)
> bca = bcanon(bmwRet[,2],5000,quKurt)
> bca$confpoints
     alpha bca point
[1,] 0.025     4.077
[2,] 0.050     4.113
[3,] 0.100     4.152
[4,] 0.160     4.183
[5,] 0.840     4.416
[6,] 0.900     4.449
[7,] 0.950     4.497
[8,] 0.975     4.540
```

Exercise 1. We see that for 90% of the samples

$$0.71 \leq \frac{s_{b,\text{boot}}}{s} \leq 1.67.$$

By the bootstrap approximation, the probability that

$$0.71 \leq \frac{s}{\sigma} \leq 1.67$$

is approximately 0.9. Therefore

$$\frac{1}{1.67} \leq \frac{\sigma}{s} \leq \frac{1}{0.71}$$

with approximate 0.9 probabilty. The confidence interval is (0.1856, 0.4366) using

$$0.1856 = \frac{s}{1.67} \leq \sigma \leq \frac{s}{0.71} = 0.4366.$$

Exercise 3. (a) $\text{BIAS}_{\text{boot}} = 1.283 - 1.323 = -0.04$.

(b) $\text{MSE}_{\text{boot}} = \text{BIAS}^2_{\text{boot}} + s^2_{\text{boot}} = 0.04^2 + 0.2386^2 = 0.0585$.