Solutions to Selected Computer Lab Problems and Exercises in Chapter 13 of *Statistics and Data Analysis for Financial Engineering, 2nd ed.* by David Ruppert and David S. Matteson

© 2016 David Ruppert and David S. Matteson.

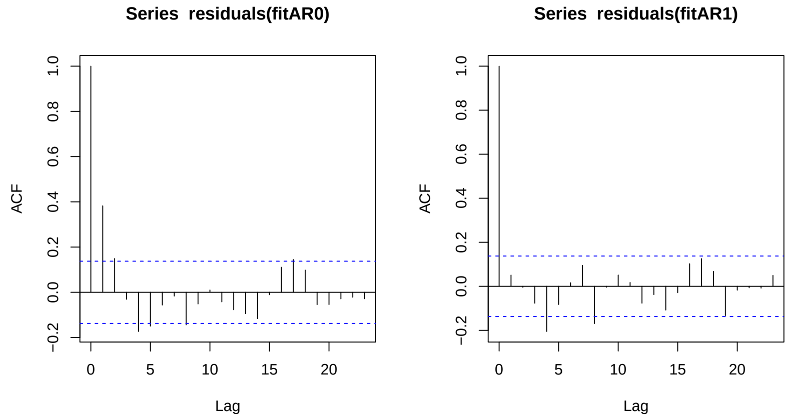
Problem 9. The function `lm()` does not output AIC so I fit the model with no correlation using the function `arima()`. The code is below. I printed the coefficients of the AR0 and LM fits to verify that they were the same.

```
fitLM = lm(unemp~invest+government)
fitAR0 = arima(unemp, order=c(0,0,0), xreg=cbind(invest, government))
fitAR1 = arima(unemp, order=c(1,0,0), xreg=cbind(invest, government))
options(digits = 4)
fitAR0$coef
fitLM$coef
fitAR0$aic
fitAR1$aic
par(mfrow=c(1,2))
acf(residuals(fitAR0))
acf(residuals(fitAR1))
```

The output below:

```
> options(digits = 4)
> fitAR0$coef
 intercept     invest government
  0.078443  -0.006985  -0.005964
> fitLM$coef
(Intercept)      invest  government
   0.078443   -0.006985   -0.005964
> fitAR0$aic
[1] 138.9
> fitAR1$aic
[1] 86.85
```

We see that the AR1 model has a much smaller AIC and so is a better fit by this criterion.



Problem 10. To convert AIC to BIC, one adds $(\log(n) - 2)\,p$ to AIC. Here p is the number of parameters. This is done below. We see that the AR1 model still fits better than the no correlation model, so the conclusion from the previous problem is unchanged.

```
> n = length(unemp)
> n
[1] 203
> fitAR0$aic + (log(n)-2)*2
[1] 145.5
> fitAR1$aic + (log(n)-2)*3
[1] 96.79
```

Problem 12. (a) The value of p is 3 and the estimate of $\mathbf{\Phi}_1$ is below.

```
, , 1
            r      y        pi
r  -0.145511 16.206 -0.098192
y   0.000517  0.292  0.000949
pi  0.182083  5.028 -0.805748
```

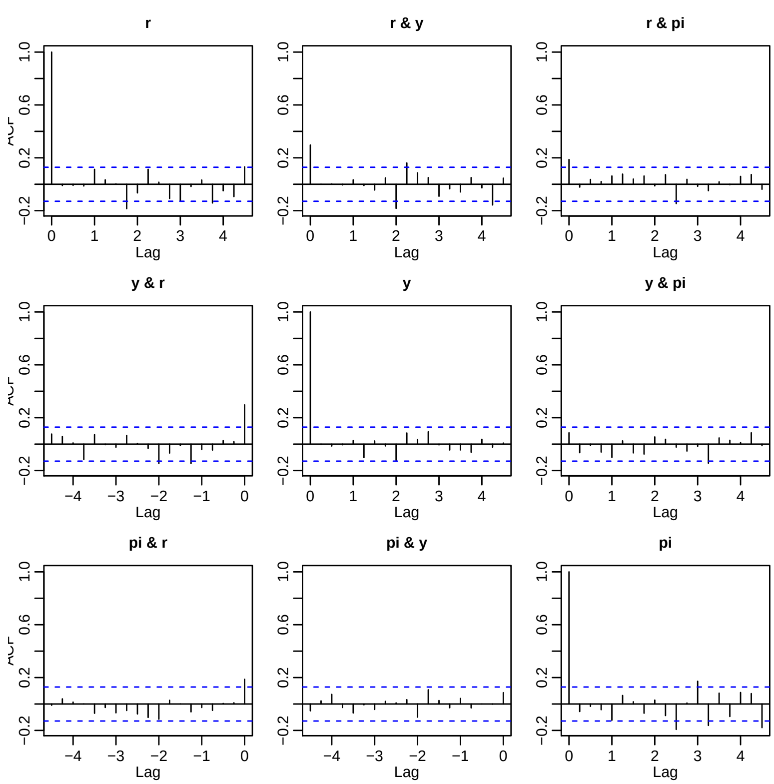
Similarly, the estimate of $\mathbf{\Phi}_j,\ j = 2, \ldots, 3$ are below.

```
, , 2

          r      y        pi
r  -0.22720 12.795  0.032919
y  -0.00198  0.178  0.000382
pi  0.00166 -3.039 -0.685300

, , 3

           r       y        pi
r   0.227678 -2.7642  0.030225
y  -0.000772 -0.0163  0.000416
pi  0.154094 13.2299 -0.513556
```

(b) The estimated covariance matrix of $\boldsymbol{\epsilon}_t$ is below.

```
$var.pred
         r        y       pi
r  0.73758 2.09e-03 0.108699
y  0.00209 6.76e-05 0.000495
pi 0.10870 4.95e-04 0.461793
```

(c) The sample autocorrelations and cross-correlation functions are in the plot below.



There is some indication of correlations in the plots, so a multivariate Ljung-Box test would be helpful. There are 3 Φ- matrices, each 3×3, so `df.adj` is $3 * 3^2 = 27$. I used `lag=10`. Other values of `lag` give similar results. The small p-value indicates statistically significant correlations. As usual, statistical significance should not be confused with practical significance. None correlations in the plot are very large, so the model might be useful for prediction or other applications.

```
> source("SDAFE2.R")
> var1Res = na.omit(as.matrix(var1$resid))
> mLjungBox(var1Res,df.adj=27,lag=10)
   K  Q(K) d.f. p-value
1 10 102.7   63   0.001
```

Note on degrees of freedom: Since `lag` is 10, there are 90 correlations and therefore a total of 90 degrees of freedom, with 27 degrees of freedom used for estimation of the autoregression matrices. Then, there are $63 = 90 - 27$ degrees of freedom available to test for the existence of residual autocorrelation.

Exercise 1. Both seasonal and non-seasonal differencing.

With only non-seasonal differencing, the ACF has periodic behavior that decays only slowly to 0, and seasonal differencing is needed to remove this behavior. With only seasonal differencing, the ACF decays only slowly to 0 which indicates nonstationarity.

Exercise 2. Seasonal differencing only.

With only seasonal differencing, the ACF decays to 0 quickly and has no periodic behavior, so the seasonally differenced time series appears stationary with no seasonality.

Exercise 3. Only non-seasonal differencing.

There is no appearance of seaonality in any of the ACFs, but non-seasonal differencing is needed since the ACF of the undifferenced series decays slowly to 0.