Solutions to Selected Computer Lab Problems and Exercises in Chapter 19 of *Statistics and Data Analysis for Financial Engineering, 2nd ed.* by David Ruppert and David S. Matteson

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Problem 2. $\text{VaR}^t(0.05)$ is 75.31 and $\text{ES}t(0.05)$ is 122.1. See the output below.

```
> mu = as.numeric(res$estimate['m'])
> lambda = as.numeric(res$estimate['s'])
> nu = as.numeric(res$estimate['df'])
> qt(alpha, df=nu)
[1] -2.292
> dt(qt(alpha, df=nu), df=nu)
[1] 0.048
>
> Finv = mu + lambda * qt(alpha, df=nu)
> VaR = -S * Finv
> options(digits=4)
> VaR
[1] 75.31
> den = dt(qt(alpha, df=nu), df=nu)
> ES = S * (-mu + lambda*(den/alpha)
+           * (nu+qt(alpha, df=nu)^2 )/(nu-1))
> ES
[1] 122.1
```

Problem 5. $\text{VaR}^t(0.05) = 63.4$ and $\text{ES}^t(0.05) = 89.45$.

```
> mu = as.numeric(fitted(pred))
> lambda = as.numeric(sigma(pred)/sqrt( (nu)/(nu-2) ))
> Finv = mu + lambda * qt(alpha, df=nu)
> VaR = -S * Finv
> options(digits=4)
> VaR
[1] 63.4
> den = dt(qt(alpha, df=nu), df=nu)
> ES = S * (-mu + lambda*(den/alpha)
+           * (nu+qt(alpha, df=nu)^2 )/(nu-1))
> ES
[1] 89.45
```

To understand why VaR and ES have decreased, we should look at certain quantities from the ARMA+GARCH model which are below.

```
> mu
[1] 0.0006347
> lambda
[1] 0.008483
> nu
[1] 5.997
> Finv
[1] -0.01585
```

These should be compared with the same quantities from the unconditional model:

```
> mu
[1] 0.0005275
> lambda
[1] 0.008443
> nu
[1] 3.209
> Finv
[1] -0.01883
```

We see that the major change when using the ARMA+GARCH model is that `nu` is larger and consequently `Finv` is smaller than with the marginal model.

It makes sense that `nu` has decreased when we switch to the ARMA-GARCH model, because the condtional heteroscedasticity of the GARCH model "explains" some of the tail weight.

Exercise 5. The key ideas are that the mean is additive and that the standard deviation is subadditive, or, mathematically, $\mu_{X+Y} = \mu_X + \mu_Y$ and $\sigma_{X+Y} \le \sigma_X + \sigma_Y$ for any random variables X and Y.

The proof first notes that $\mathrm{VaR}(\alpha)$ for portfolio i is

$$\mathrm{VaR}(P_i, \alpha) = -S_i\mu_i - S_i\sigma_i z_\alpha$$

for $i = 1, 2$.

Then $\mathrm{VaR}(\alpha)$ for $P_1 + P_2$ is

$$\begin{aligned}
\mathrm{VaR}(P_1 + P_2, \alpha) &= -(S_1\mu_1 + S_2\mu_2) - \sqrt{S_1^2\sigma_1^2 + S_2\sigma_2^2 + 2\rho S_1 S_2 \sigma_1\sigma_2}\, z_\alpha \\
&\le -(S_1\mu_1 + S_2\mu_2) - \sqrt{S_1^2\sigma_1^2 + S_2\sigma_2^2 + 2 S_1 S_2 \sigma_1\sigma_2}\, z_\alpha \\
&= \mathrm{VaR}(P_1, \alpha) + \mathrm{VaR}(P_2, \alpha),
\end{aligned}$$

since $\rho \le 1$ and $-z_\alpha > 0$. (Note: $-z_\alpha > 0$ is true only if $\alpha < 1/2$ but this is a very reasonable assumption since VaR is about tail events.)