Solutions to Selected Computer Lab Problems and Exercises in Chapter 17 of *Statistics and Data Analysis for Financial Engineering, 2nd ed.* by David Ruppert and David S. Matteson

Problem 1. Selected output is listed below. We can see that the p-values for the intercepts are 0.42, 0.14, 0.39, 0.23, 0.91, 0.41, and 0.31. Since all are large, in particular, well above 0.05, we can accept that all of the alphas are zero.

```
> summary(fit_reg)
Response GM_AC :
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.0685     0.0850   -0.81     0.42
market        1.2025     0.1253    9.60   <2e-16 ***

Response F_AC :
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.0977     0.0665   -1.47     0.14
market        1.2378     0.0980   12.63   <2e-16 ***
---

Response UTX_AC :
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.0296     0.0343    0.86     0.39
market        0.9766     0.0506   19.31   <2e-16 ***
---

Response CAT_AC :
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.0568     0.0477    1.19     0.23
market        1.4098     0.0703   20.06   <2e-16 ***
---
%
Response MRK_AC :

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.00745    0.06767   -0.11     0.91
market       0.74991    0.09971    7.52  1.8e-13 ***
---

Response PFE_AC :
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.0402     0.0487   -0.83     0.41
market        0.9496     0.0717   13.24   <2e-16 ***


Response IBM_AC :
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.0323     0.0319   -1.01     0.31
market        0.8212     0.0470   17.48   <2e-16 ***
---
```

Problem 2. The following commands estimate the expected excess returns in both ways.

```
betas=fit_reg$coeff[2,]
betas * mean(market)
apply(stockExRet,2,mean)

> betas * mean(market)
 GM_AC   F_AC UTX_AC CAT_AC MRK_AC PFE_AC IBM_AC
0.0233 0.0240 0.0190 0.0274 0.0146 0.0184 0.0159

> apply(stockExRet,2,mean)
  GM_AC    F_AC  UTX_AC  CAT_AC  MRK_AC  PFE_AC  IBM_AC
-0.0452 -0.0736  0.0485  0.0842  0.0071 -0.0218 -0.0163
```

We see that the estimates from the one-factor model are similar to each other and all are positive. The estimates using sample means are dissimilar to each other and four are negative while three are positive. Thus, the one-factor model produces much less variable estimates of mean returns.

Problem 3.
```
> res = residuals(fit_reg)
> options(digits=3)
> cor(res)
          GM_AC     F_AC   UTX_AC  CAT_AC  MRK_AC  PFE_AC   IBM_AC
GM_AC   1.00000  0.50911  0.03967  0.0202 -0.0472 -0.0188  0.00785
F_AC    0.50911  1.00000 -0.00714  0.0289  0.0128  0.0114  0.03575
UTX_AC  0.03967 -0.00714  1.00000  0.1498 -0.0154 -0.1110 -0.06949
CAT_AC  0.02023  0.02895  0.14977  1.0000 -0.0757 -0.0650 -0.08342
MRK_AC -0.04715  0.01279 -0.01540 -0.0757  1.0000  0.2833 -0.07817
PFE_AC -0.01877  0.01142 -0.11103 -0.0650  0.2833  1.0000 -0.04606
IBM_AC  0.00785  0.03575 -0.06949 -0.0834 -0.0782 -0.0461  1.00000
```

The residual correlation matrix is above. There is a reasonably high residual correlation of 0.51 between GM and F, which makes sense since they are both in automotive industry. There is also a moderate residual correlation of 0.28 between Pfizer and Merck, which again is sensible since both are pharmaceutical companies.

Problem 4. According to the one-factor model, the covariance matrix is $\boldsymbol{\beta}\boldsymbol{\beta}^{\mathsf{T}}\sigma_M^2$ $+\mathrm{diag}(\sigma_{\epsilon,1}^2, \ldots, \sigma_{\epsilon,7}^2)$ where $\boldsymbol{\beta}$ is the vector of the seven betas, σ_M^2 is the variance of the market excess return, and $\sigma_{\epsilon,i}^2$ is the residual variance of the for the jth stock. This matrix is estimated as follows. Notice the double use of `diag`, first to extract the diagonal elements of the residual covariance matrix and second to put these into a diagonal matrix—the net effect is to zero-out the non-diagonal elements.

```
> cov_capm = betas %*% t(betas) * var(market) + diag(diag(cov(res)))
> cov_capm
     GM_AC  F_AC UTX_AC CAT_AC MRK_AC PFE_AC IBM_AC
[1,] 5.512 0.686  0.541  0.781  0.416  0.526  0.455
[2,] 0.686 3.673  0.557  0.804  0.428  0.542  0.468
[3,] 0.541 0.557  1.230  0.634  0.338  0.427  0.370
[4,] 0.781 0.804  0.634  2.441  0.487  0.617  0.534
[5,] 0.416 0.428  0.338  0.487  3.329  0.328  0.284
[6,] 0.526 0.542  0.427  0.617  0.328  2.004  0.359
[7,] 0.455 0.468  0.370  0.534  0.284  0.359  0.992
```

Problem 5. We see from the output below that R^2 is 0.357, so the percentage of the excess return variance for UTX due to the market is estimated as 35.7%

```
Response UTX_AC :

Call:
lm(formula = UTX_AC ~ market)

Residuals:
    Min      1Q  Median      3Q     Max
-3.8194 -0.5149 -0.0154  0.4938  6.3806

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.0296     0.0343    0.86     0.39
market        0.9766     0.0506   19.31   <2e-16 ***
---

Residual standard error: 0.89 on 670 degrees of freedom
Multiple R-squared: 0.357,      Adjusted R-squared: 0.356
F-statistic:  373 on 1 and 670 DF,  p-value: <2e-16
```

Problem 6.

```
> 4*betas
 GM_AC   F_AC UTX_AC CAT_AC MRK_AC PFE_AC IBM_AC
  4.81   4.95   3.91   5.64   3.00   3.80   3.28
```

Exercise 1.

$$\beta = \frac{16 - 5.5}{11 - 5.5} = 1.909.$$

Exercise 2a. The proportion of the investment in the market portfolio is

$$\frac{.11 - .03}{.14 - .03} = 0.7273.$$

The proportion in the riskfree asset is $1 - w = 0.2727$.

Exercise 2b. $0.12w = 0.08727$

Exercise 4. In (7.8), let $\boldsymbol{Y} = (R_{1,t}, \ldots, R_{N,t})^{\mathsf{T}}$, $w_1 = (0, \ldots, 0, 1, 0, \ldots, 0)^{\mathsf{T}}$ with the 1 in the jth position, and $\boldsymbol{w}_2 = (w_{1,M} \ldots, w_{N,M})^{\mathsf{T}}$. Then $\mathrm{COV}(\boldsymbol{Y})\boldsymbol{w}_1 = (\sigma_{1,j}, \ldots, \sigma_{N,j})^{\mathsf{T}}$ and (17.16) follows.

Exercise 5. False. The market only compensates investors for the market component of the volatility, which is $|\beta_j|\sigma_m$. Investors are not compensated for the nonmarket component of the risk.

Exercise 6a. $\beta_P = (0.9 + 1.1 + 0.6)/3 = 0.8667$.

Exercise 6b. $\beta_P^2(0.014) + (0.010 + 0.015 + 0.011)/9 = 0.01452$.

Exercise 6c. $\beta_P^2(0.014)/0.01452 = 0.7244$.