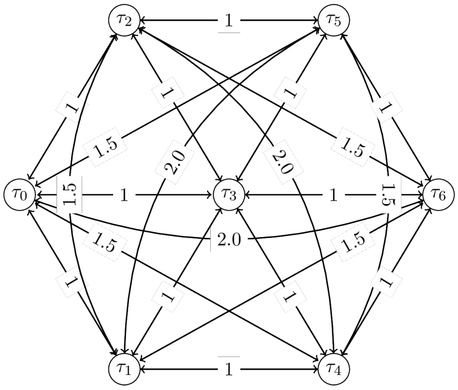
Spatial task allocation and scheduling problem in heterogeneous multi-robot team

Notation

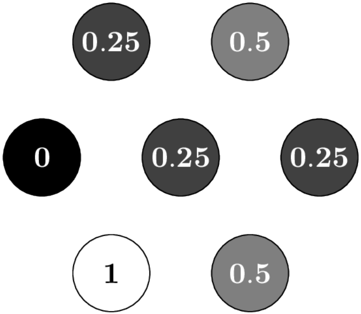
- $\mathcal{A}$: Agents
- $\mathcal{T}$: Tasks
- $G = (\mathcal{T}, E)$: Traversability graph
  $(i, j) \in E$ if task j can be scheduled right after i.
  we can simplify the formulation and assume a complete graph $E = \mathcal{T} \times \mathcal{T}$.
- R_τ: Reward associated to the initial workload of τ;
  In the non-atomic setting, if by the end of the mission a fraction p of the workload of task τ has been perfomed, then we consider that the team obtained a reward equal to $p \cdot R_\tau$ from that task.
- $\varphi_k(\tau)$: Efficiency models; fraction of task τ that k can perform in one time step.
- t_{ij}: travelling time between tasks i and j.
- τ_0 is chosen as starting point, meaning that agents start at the location of this task. In the following, we also assume that τ_0 does not require service, therefore we can specify $\varphi_k(0) = 0$ for all agents $k \in \mathcal{A}$. Also, the reward $R_{\tau_0} = 0$.
- T : time horizon in number of steps

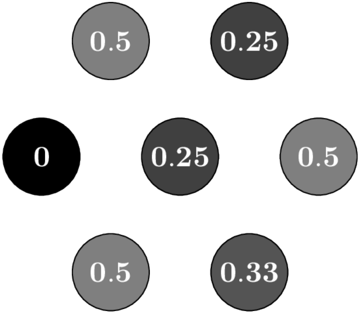
Example



- Traversability graph (complete), with associated travelling times (symmetric). E.g.,
  - $t_{01} = t_{02} = t_{03} = t_{13} = t_{14} = 1,$
  - $t_{05} = t_{04} = t_{12} = t_{16} = 1.5$

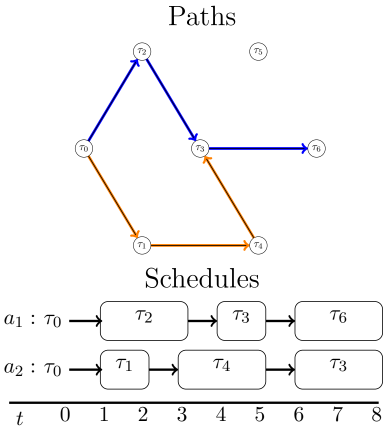
Example





- Efficacy model for agent 1, e.g.,
  - $\varphi_1(\tau_0) = 0$,
  - $\varphi_1(\tau_1) = 1$,
  - $\varphi_1(\tau_2) = 0.25$.
  - i.e., agent 1 can not do task τ_0, can do τ_1 in a single time step, and can do $\frac{1}{4}$ of τ_2 in one time step.

- Efficacy model for agent 2, e.g.,
  - $\varphi_2(\tau_0) = 0$,
  - $\varphi_2(\tau_1) = 0.5$,
  - $\varphi_2(\tau_2) = 0.5$.
  - i.e., agent 2 can not do task τ_0, can do $\frac{1}{2}$ of τ_1 and τ_2 in one time step.



Total Reward: 47.5

- This is the optimal solution for a time horizon $T = 8$ time steps.
- We considered two agents a_1, a_2.
- A solution can be represented by a set of *paths*, and a set of *schedules*.
- A path shows the sequence of tasks that an agent perform (or partially does).
- A schedule indicates the amount of time that an agent spends in doing (or partially doing) each task in its path.
- Here the paths are:
  - For a_1: $< \tau_0, \tau_2, \tau_3, \tau_6 >$.
  - For a_2: $< \tau_0, \tau_1, \tau_4, \tau_3 >$.
- The schedules, number of time steps at each task:
  - For a_1, $\tau_2 = \tau_6 = 2$, $\tau_3 = 1$.
  - For a_2: $\tau_1 = 1$, $\tau_3 = \tau_4 = 2$.